Coordination Dynamics and Policy Interventions in a Networked Stag-Hunt Model

Model-Based Decision Making

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1 Introduction

Coordination problems appear in technology adoption and infrastructure investment: individual incentives depend on others, so systems can settle into different equilibria and become sensitive to initial conditions. This can produce persistent coordination failure even when a higher-adoption outcome is socially preferable.

The *Stag-Hunt game* captures this trade-off between a risky, payoff-dominant option and a safe but inefficient alternative (Skyrms, 2004). When interactions occur on networks, outcomes also depend on local reinforcement, clustering, and connectivity patterns (Jackson, 2008). This report studies a networked evolutionary Stag-Hunt adoption model with infrastructure feedback, focusing on: (i) how X_0 and payoffs shape tipping, (ii) how topology shifts coordination outcomes, and (iii) when subsidies or seeding interventions are effective and cost-efficient.

2 The Stag-Hunt Game and Model Setup

2.1 The Stag-Hunt Game

The Stag-Hunt payoff matrix is shown in Table 1. Mutual adoption yields the highest payoff, but only if both coordinate; mutual non-adoption yields a lower guaranteed payoff.

Table 1: Payoff matrix of the Stag-Hunt game, with $a > b > 0$.

	Adopt (A)	Not Adopt (N)
Adopt (A)	(a, a)	$(0, b)$
Not Adopt (N)	$(b, 0)$	(b, b)

The game has two pure Nash equilibria: mutual non-adoption (risk-dominant) and mutual adoption (payoff-dominant). Adoption becomes individually attractive only when a sufficient fraction of others adopt, which matches coordination settings such as shared infrastructure and standards adoption (Morris and Shin, 2000).

2.2 Networked Evolutionary Dynamics

Agents occupy a network and repeatedly interact with neighbors. Payoffs come from local Stag-Hunt interactions and are increased by an infrastructure feedback term that raises the adoption payoff as aggregate adoption grows. Strategies update through a logit best-response rule (probabilistic best-response), producing stochastic outcomes near tipping.

Key parameters varied in the experiments are: initial adoption X_0, payoff ratio a/b, infrastructure feedback β_I, and network topology (grid, small-world, Erdős–Rényi, Barabási–Albert). Results are aggregated across multiple random seeds using final adoption and success probabilities (and time-to-threshold where relevant).

2.3 Experimental Design Choices

Table 2 lists baseline settings. The goal is to study regimes where coordination is *not* predetermined, so tipping and policy leverage are visible. The horizon $T = 250$ is long enough for convergence while keeping sweeps over many conditions feasible, and $N = 200$ is large enough to show clustering and diffusion effects without excessive computation.

Payoffs are scaled with $b = 1.0$. The baseline payoff ratio (via `ratio`) is set so adoption is initially risky because infrastructure starts low ($I_0 = 0.05$), but can become attractive as infrastructure grows. Infrastructure updates with rate $g_I = 0.10$, so reinforcement builds gradually and requires sustained coordination. The logit update with $\tau = 1.0$ keeps dynamics payoff-sensitive but non-deterministic, motivating probability-based evaluation across seeds. While I_0 is held fixed in this report, increasing I_0 would mechanically raise early adoption payoffs and shift the tipping boundary in a manner similar to increasing β_I. Fixing I_0 at a low level keeps the baseline near-critical.

Table 2: Baseline parameter values used in all experiments unless stated otherwise.

Parameter	**Value**	**Description**
T	250	Simulation horizon
N	200	Number of agents (network nodes)
b	1.0	Payoff for mutual non-adoption
g_I	0.10	Infrastructure adjustment rate
I_0	0.05	Initial infrastructure level
β_I	2.0	Infrastructure feedback strength
a_0/b	2.1	Baseline payoff ratio
Strategy update	Logit	Probabilistic best-response rule
τ	1.0	Noise parameter in logit update

3 Baseline Dynamics and Tipping Structure

3.1 Adoption as a Function of Initial Conditions and Payoffs

Figure 1 shows final adoption as a function of X_0 and a/b. There is a clear transition between failed coordination and near-complete adoption. Low X_0 combined with weak incentives leads to the risk-dominant non-adoption equilibrium, while sufficiently high X_0 or strong incentives lead to the payoff-dominant equilibrium.

The narrow boundary between regimes indicates bistability. Near this boundary, outcomes vary strongly across runs, producing intermediate values in the phase diagram. The boundary also shows a trade-off: higher a/b lowers the critical mass needed for adoption. These near-critical settings are used later to test policy leverage where outcomes are uncertain.

3.2 Sensitivity to Infrastructure Feedback

Figure 2 varies infrastructure feedback β_I. For low β_I, adoption stays near zero across runs, indicating reinforcement is too weak to escape non-adoption. At intermediate β_I, outcomes become heterogeneous: some runs fail while others tip to high adoption, consistent with stochastic tipping near a basin boundary. For large β_I, outcomes concentrate near full adoption and variance decreases, indicating stabilization of the high-adoption equilibrium.

Overall, β_I acts as a control parameter that shifts the probability and stability of coordination, rather than increasing adoption smoothly. This matters for policy: moderate reinforcement

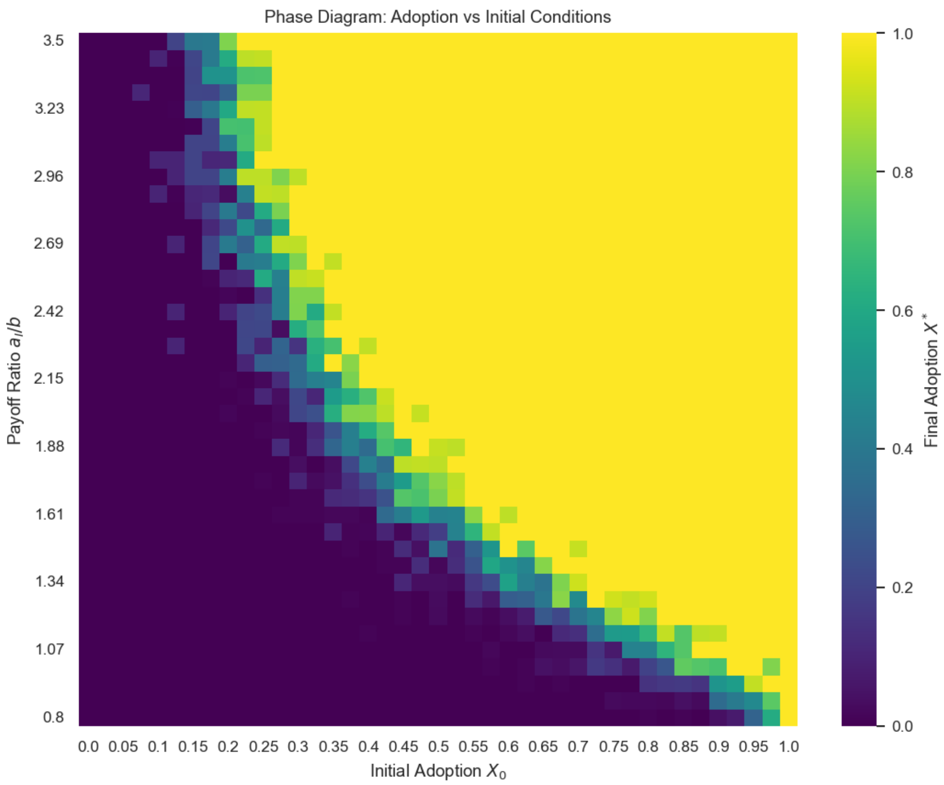


Figure 1: Final adoption as a function of X_0 and a/b.

can still yield uncertain outcomes, while strong reinforcement can qualitatively change system behavior.

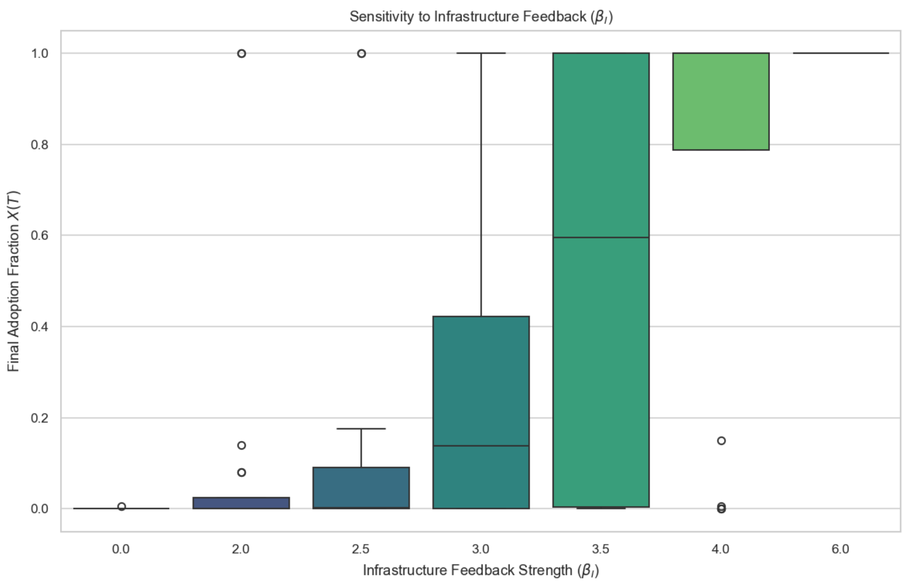


Figure 2: Final adoption vs. infrastructure feedback strength β_I.

4 Network Topology Effects

The four topologies isolate different structural features. Grids capture local interactions and strong clustering; small-world networks add shortcut links; Erdős–Rényi networks provide a random baseline with weak clustering; Barabási–Albert networks add degree heterogeneity through hubs.

Figure 3 shows $P(X(T) \geq 0.8)$ versus X_0 by topology. Grids tip at relatively low X_0 due to local reinforcement but remain less reliable at system scale. Small-world networks show sharper transitions because shortcuts allow local clusters to spread. Erdős–Rényi networks require higher X_0 to initiate coordination but transition sharply once tipping occurs. Barabási–Albert networks show the most right-shifted and gradual transition, suggesting that hubs do not reliably accelerate coordination in this setting. These differences motivate network-specific policy evaluation. These topologies also differ in adoption speed: once tipping occurs, small-world and Erdős–Rényi networks reach high adoption faster than grids, where diffusion is limited by local growth.

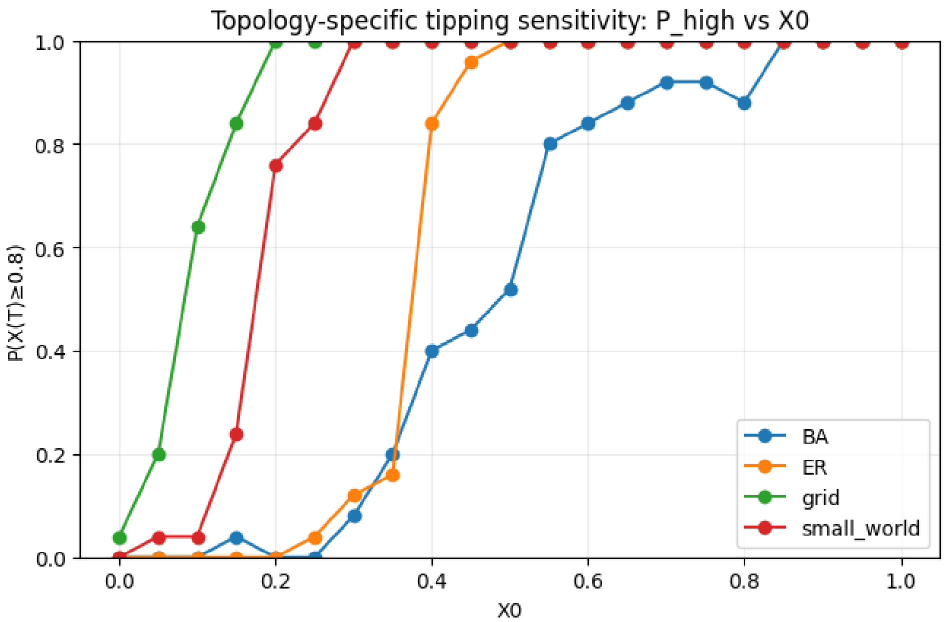


Figure 3: $P(X(T) \geq 0.8)$ vs. X_0 for different network topologies.

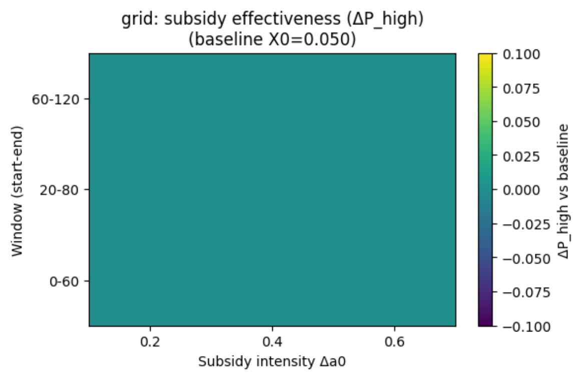
5 Policy Interventions and Coordination Outcomes

Two intervention types are tested: (i) temporary subsidies (time-limited increases in the adoption payoff) and (ii) targeted seeding (forcing a fraction of agents to adopt at time t). Policies are evaluated relative to network-specific near-critical baselines, so improvements reflect genuine leverage rather than trivial success. Performance is measured mainly through changes in coordination probability ΔP_{high}, supported by final adoption and time-to-threshold when successful. Costs are approximated by subsidy intensity$\times$duration or the seeded fraction.

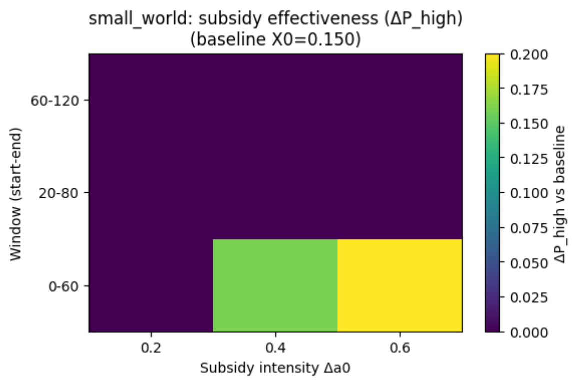
5.1 Subsidy Timing and Intensity

Figure 4 shows subsidy effects across timing windows and intensities. Subsidies are consistently most effective when applied early ($t = 0$–60). Late subsidies ($t = 60$–120) have little effect, consistent with path dependence: once the system moves toward non-adoption, temporary incentives often cannot reverse it.

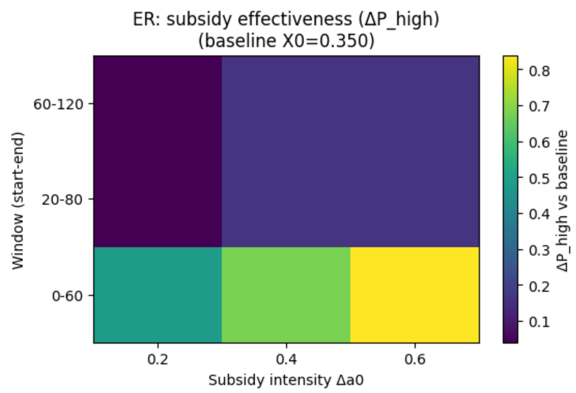
Effects differ by topology. Erdős–Rényi networks show the largest absolute gains under strong early subsidies. Small-world and Barabási–Albert networks improve only for higher intensities, while grids show little response across settings. This aligns with diffusion: in lattices, clusters grow slowly and subsidies expire before a global cascade forms; in more connected networks, early subsidies can propagate quickly and help the system cross the coordination threshold.



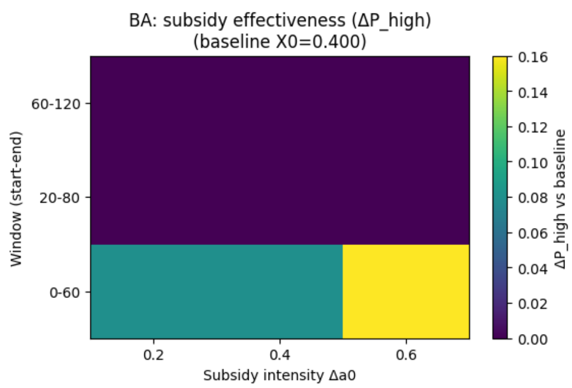
(a) Grid



(b) Small-world



(c) Erdős–Rényi



(d) Barabási–Albert

Figure 4: Subsidy effects on ΔP_{high} by timing window and intensity Δa_0 (relative to near-critical baselines shown in the caption of Figure 4).

5.2 Targeted Seeding Policies

Seeding directly increases adoption and can move the system across the coordination threshold without relying on a temporary incentive change. Compared to subsidies, seeding is less timing-sensitive. Its impact depends on topology: in Barabási–Albert networks, seeding yields large gains because seeded nodes can influence many others; in grids, seeding tends to create local clusters that may not spread system-wide. This makes seeding especially attractive in networks where influence can propagate beyond local neighborhoods.

5.3 Policy Efficiency and Cost-Effectiveness

Tables 3 and 4 summarize the most cost-efficient and most effective interventions per topology. Table 3 shows that seeding is the most cost-efficient option across all networks: small seeded fractions generate substantial ΔP_{high} at low cost. Table 4 shows that absolute effectiveness can favor subsidies in some networks (notably Erdős–Rényi), but these gains come at high cost and depend on early timing. Overall, subsidies behave like expensive "pushes" that can work in well-connected networks, while seeding provides a more robust and cost-efficient lever.

Table 3: Most cost-efficient policy intervention per network topology, ranked by efficiency $\Delta P_{\text{high}}/\text{cost}$.

Network	Policy type	Policy	X_0	ΔP_{high}	Efficiency
Grid	Seeding	seed(t=10, frac=0.02)	0.05	1.00	50.0
Small-world	Seeding	seed(t=10, frac=0.10)	0.15	0.44	4.4
Erdős–Rényi	Seeding	seed(t=10, frac=0.10)	0.35	0.32	3.2
Barabási–Albert	Seeding	seed(t=10, frac=0.10)	0.40	0.60	6.0

Table 4: Most effective policy intervention per network topology, ranked by absolute improvement in coordination probability ΔP_{high}.

Network	Policy type	Policy	X_0	ΔP_{high}	Cost
Grid	Seeding	seed(t=10, frac=0.02)	0.05	1.00	0.02
Small-world	Subsidy	subsidy(0−60, Δa_0=0.6)	0.15	0.44	36
Erdős–Rényi	Subsidy	subsidy(0−60, Δa_0=0.6)	0.35	0.84	36
Barabási–Albert	Seeding	seed(t=10, frac=0.10)	0.40	0.60	0.10

5.4 Interpretation and Policy Implications

The main implication is that effective policy depends on *when* and *where* leverage is applied. Near tipping points, small targeted actions can shift the system between equilibria. Strong early subsidies can work in well-connected networks but are costly and timing-sensitive. Targeted seeding is cheaper and more robust, especially in networks where seeded agents can influence beyond local clusters. In practice, this supports prioritizing targeted early actions that help the system cross coordination thresholds rather than relying only on broad temporary incentives.

6 Discussion and Limitations

Results depend strongly on initial conditions, feedback strength, and topology, with the highest uncertainty near the tipping boundary. The model is intentionally stylized: logit updates with fixed τ omit learning and heterogeneous preferences; infrastructure is a single global stock and ignores spatial variation and congestion; and networks are static rather than adaptive. Cost measures (subsidy intensity/duration and seeded fraction) allow relative comparisons but do not capture institutional constraints. The findings should therefore be interpreted as identifying qualitative leverage points and failure modes. Future work could add heterogeneous agents, spatial infrastructure, and adaptive networks to test robustness.

7 Conclusion

Coordination outcomes in the networked Stag-Hunt model are strongly non-linear: small changes in X_0, incentives, or timing can shift the system between persistent non-adoption and widespread adoption. Baseline sweeps show bistability and stochastic tipping, and network structure shifts both the tipping location and reliability of success. Policy tests show that early subsidies can strongly increase coordination in well-connected networks but are expensive and timing-sensitive, while targeted seeding is consistently more cost-efficient and less timing-dependent, especially in heterogeneous networks. Overall, policies that push the system across a coordination threshold—aligned with network structure and timing—are most likely to trigger self-sustaining adoption.

References

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