

Master Thesis

ON SCALABLE DEEP LEARNING AND PARALLELIZING GRADIENT DESCENT

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Preface

This thesis is submitted as a final requirement for the Master of Science degree at the Department of Data Science & Knowledge Engineering of Maastricht University, The Netherlands. The subject of study originally started as a pilot project with Jean-Roch Vlimant, Maurizio Pierini, and Federico Presutti of the EP-UCM group (CMS experiment) at CERN. In order to handle the increased data rates of LHC Run 3 and High Luminosity LHC, the CMS experiment is considering to construct a new architecture for the High Level Trigger based on Deep Neural Networks. However, they would like to significantly decrease the training time of the models as well. This would allow them to tune the neural networks more frequently. As a result, we started to experiment with various state of the art distributed optimization algorithms. Which resulted in the achievements and insights presented in this thesis.

I would like to express my gratitude to several people. First and foremost, I would like to thank my promotores, Gerasimos Spanakis, and Rico Möckel for their expertise and suggestions during my research, which drastically improved the quality of this thesis. Furthermore, I would also like to thank my friends, colleagues and scientists at CERN for their support, feedback, and exchange of ideas during my stay there. It was a very motivating and inspiring time in my life. Especially the support and experience of my CERN supervisors, Zbigniew Baranowski, and Luca Canali, was proven to be invaluable on multiple occasions. I would also like to thank them for giving me the personal freedom to conduct my own research. Finally, I would like to thank my parents and grandparents who always supported me, and who gave me the chance to explore the world in this unique way.

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Abstract

Abstract here.

Summary

Summary here.

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Abbreviations and Notation

η	Static learning rate
η_t	Learning rate with respect to time t .
λ	Communication period, or frequency of commits to the parameter server.
$\mathcal{L}(\theta ; \mathbf{x} ; \mathbf{y})$	Loss function with respect to parametrization θ , input \mathbf{x} , and expected output \mathbf{y} .
τ	Staleness
θ_t^k	Parametrization of worker k at time t .
$\tilde{\theta}_t$	Center variable, or central parametrization maintained by the parameter server.
\triangleq	Is defined as
$J(\theta)$	Loss with respect to parameterization θ .
m	Mini-batch size
n	Number of parallel workers.
ADAG	Asynchronous Distributed Adaptive Gradients
ASGD	Asynchronous Stochastic Gradient Descent
CERN	European Organization for Nuclear Research
CMS	Compact Muon Solenoid
EASGD	Elastic Averaging Stochastic Gradient Descent
GD	Gradient Descent
HE	Hardware Efficiency
HEP	High Energy Physics
HL-LHC	High Luminosity Large Hadron Collider
LHC	Large Hadron Collider
MNIST	Mixed National Institute of Standards and Technology database
PS	Parameter Server
SE	Statistical Efficiency
SGD	Stochastic Gradient Descent
TE	Temporal Efficiency

Chapter 1

Introduction

In this chapter we introduce the main concept, and problems surrounding the parallelization of gradient descent. We familiarize the reader with the topic and some notation by providing some context why someone would like to apply said technique. Furthermore, in Section 1.4, we summarize the problem statement and provide several research questions which will guide the research in this work. Finally, we conclude this chapter in Section 1.5 with a brief outline of the thesis.

1.1 Motivation

In recent years it has been shown that being able to train large and deep neural networks result in state-of-the-art performance [16, 4], especially regarding unsupervised feature learning and image recognition. However, consider the required time, and cost of the infrastructure that would be required in order to train a large model in a reasonable amount of time. Furthermore, it is not only the training time and cost of the infrastructure which need to be taken into consideration, but also the volume of the data. The amount of information that will be gathered will be an increasing important factor in the next few years. Not only with respect to big technology companies and government organizations, but also scientific surveys with limited budgets. These scientific surveys will generate more experimental data than ever [1, 7], and will have to process and analyze that data. To solve the problem of increased computational workloads and budget freezes, the High Energy Physics (HEP) community is exploring and researching machine learning approaches to fit physics problems [2, 14, 12] with the intention to improve detection quality, or reduce computational constraints.

However, the sheer size of these datasets severely impacts the training time of the models. In order to resolve this issue, one could sample some representative subset of the data to reduce the training time. The disadvantage of this approach is that some instances, i.e., data points, might not appear in the final training set. This is especially a problem in Deep Learning, where models usually benefit from having access to a lot of training data due to the high dimensionality of the parametrization [4]. To resolve this issue, Dean et al. [4] introduce two new paradigms to decrease the training time of a large model. The two paradigms, *Model Parallelism*, briefly discussed in Section 1.2, and *Data Parallelism*, discussed in Section 1.3, are inherently different ways of decreasing the training time of a model.

The first paradigm, *Model Parallelism*, is intuitively the most straightforward paradigm since it deals with the parallelization of the computations within a *single* model, i.e., how to parallelize the computations of a single model over multiple machines, or multiple processes. The second paradigm, which will be the main focus of this thesis, is *Data Parallelism*. As stated above, the main concept of data parallelism will be discussed in detail in Section 1.3. However, for completion, think of Data Parallelism as a technique to *parallelize gradient descent*. This is done by allocating n processes over possibly n different machines, and splitting the training set into n *partitions*, or *data shards*. For further convenience, we will call such a process a *worker*. In the next step, we assign a single distinct partition to a worker. Meaning, the worker will not be able to fetch training data from other partitions

since those have been assigned to different workers. However, in certain data parallel settings, it is beneficial to actually consume data from other partitions, once a worker has finished its partition. Finally, the goal of these workers is to work together, and optimize the parameters of a central model.

A lot of different distributed optimization schemes have been suggested in recent years [17, 4, 5]. Most of the recent contributions try to push the limits of asynchronous data parallelism, discussed in Section 2.3, by simply *annealing* the gradients with respect to some hyperparameter to improve the convergence when the number of workers increases. This suggests that there is an intrinsic limit to asynchronous data parallelism, as suggested by [13]. As a result, why don't we simply reduce the number of parallel workers if we reduce the impact of the gradient updates by means of annealing anyway? The approach of reducing the number of parallel workers in such a situation has been suggested by [5], where they perform a *grid-search* of the training hyperparameters (this includes the number of workers) in order to provide the optimal hyperparameters within a training epoch. However, the disadvantage of this technique is that after every epoch, or a specific number of iterations, a grid-search of the hyperparameters has to be performed in order to obtain the optimal configuration of the hyperparameters to ensure convergence.

This brings us to the main motivation behind this work. We intent to obtain a better understanding of *asynchronous* Data Parallelism by building upon previous work, and combine it with novel insights to construct a new distributed optimization scheme without introducing new hyperparameters, or relying on grid-searches to optimize the configuration of existing hyperparameters.

1.2 Model Parallelism

In *model parallelism*, a single model is distributed over multiple machines [4]. The performance benefits of distributing a deep network across multiple machines mainly depends on the structure of the model. Models with a large number of parameters typically benefit from access to more CPU cores and memory, up to the point where communication costs, i.e., propagation of weight updates and synchronization mechanisms, dominate [4].

Let us start with a simple example in order to illustrate this concept more clearly. Imagine having a perceptron, as depicted in Figure 1.1. In order to parallelize this efficiently, we can view a neural network as a dependency graph, where the goal is to minimize the number of synchronization mechanisms, assuming we have unlimited resources. Furthermore, a synchronization mechanism is only required when a node has more than 1 *variable* dependency. A variable dependency is a dependency which can change in time. For example, a bias would be a *static* dependency, because the value of a bias remains constant over time. In the case for the perceptron shown in Figure 1.1, the parallelization is quite straightforward. The only synchronization mechanism which should be implemented resides in output neuron since $y \triangleq \sigma(\sum_i w_i x_i)$ where σ is the activation function of the output neuron.

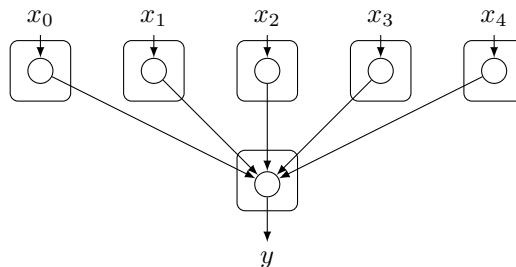


Figure 1.1: A perceptron partitioned using the *model parallelism* paradigm. In this approach every input node is responsible for accepting the input x_i from some source, and multiplying the input with the associated weight w_i . After the multiplication, the result is sent to the node which is responsible for computing y . Of course, this node requires a synchronization mechanism to ensure that the result is consistent. The synchronization mechanism does this by waiting for the results y depends on.

1.3 Data Parallelism

Data parallelism is an inherently different methodology of optimizing parameters. As stated above, it is a technique to reduce the overall training time of a model. In essence, data parallelism achieves this by having n workers optimizing a central model, and at the same time, processing n different shards (partitions) of the dataset in parallel over multiple workers¹. The workers are coordinated in such a way that they optimize the parametrization of a central model, which we denote by $\bar{\theta}_t$. The coordination mechanism of the workers can be implemented in many different ways. Nevertheless, a popular approach to coordinate workers in their task to optimize the central objective, is to employ a centralized *Parameter Server* (PS). The sole responsibility of the parameter server is to aggregate model updates coming from the workers (*worker commits*), and to handle parameter requests (*worker pulls*). In general, there are several approaches towards data parallelism, where some do not require a parameter server. However, all approaches can be categorized into two main groups, i.e., *Synchronous Data Parallelism*, and *Asynchronous Data Parallelism*.

Synchronous Data Parallelism can be usually identified by the presence of one or multiple locking mechanisms. As in Software Engineering, the purpose of these locking mechanisms is to preserve the consistency of the state of a model. As an example, let us consider mini-batch parallelism in Figure 1.2 for a moment. Despite it is trivial to implement locally, one could view mini-batch parallelism as an instance of synchronous data parallelism. First and foremost, mini-batch parallelism is a data parallel technique because we split the mini-batch into several partitions where every partition is consumed by its own worker to produce the sum of the gradients, or *accumulated gradient*, as a result. Finally, mini-batch parallelism is synchronous in nature because in order to compute θ_{t+1} , we need to obtain the averaged gradients $\frac{\sum_i \nabla_\theta \mathcal{L}(\theta_t; \mathbf{x}_i; \mathbf{y}_i)}{m}$, which is actually the sum of the accumulated gradients of all workers, divided by the number of training instances m in the original mini-batch. As a result, the synchronization barrier is present right before the averaging of the accumulated gradients, since these are the intermediary results we have to wait for before applying a gradient update.

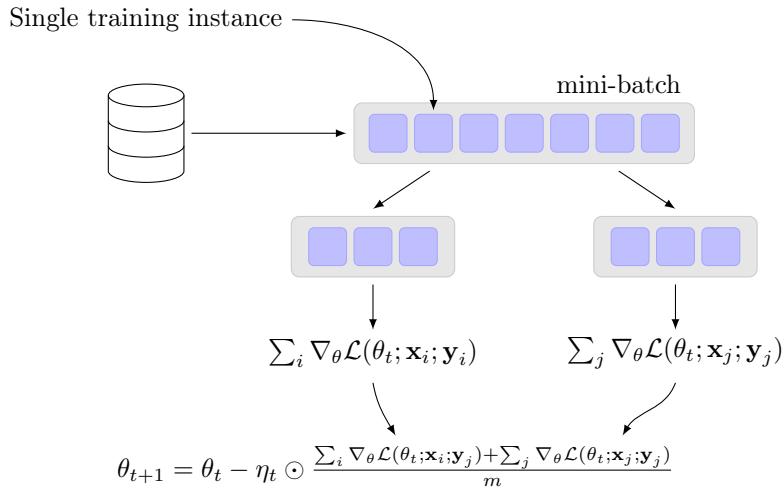


Figure 1.2: Mini-batch parallelism could be viewed as an instance of synchronous data parallelism without a centralized parameter server. Given a mini-batch size of m , we split the mini-batch into several partitions, where a specific worker is responsible for the computation of its own partition. The synchronous nature of this approach lies within the aggregation of the computed gradients, i.e., the results of all workers need to be aggregated, and afterwards averaged in order to integrate the current gradient into the model.

¹As stated in Section 1.1, a worker is a process on a single machine. However, it is possible that multiple workers share the same machine. Nevertheless, one could construct the distribution mechanism (even manually) in such a way every worker will be placed on a different machine.

However, consider what happens when the computational resources on your machine are constrained, or even fully utilized? That is, due to the limited amount of available CPU cores (or even GPU's) your parallelization of the mini-batch computation doesn't scale very well on your machine. The most straightforward solution would be to purchase more performant hardware, but this is not always possible, not only from a financial perspective, but also from a physical one. An alternative approach would be to solve the problem like a distributed system. In order to make this particular approach work, we need a parameter server to coordinate the workers. Since this still is a synchronous approach, the locking mechanism in this particular case is the parameter server itself, since the parameter server will not be able to send the next parameterization θ_{t+1} of the model to the workers because the parameter server can only compute the θ_{t+1} once it received, and processed all accumulated gradients as shown in Figure 1.3. Yet, if one or multiple machines encounter some unmodeled load, for example, because an other user is running a CPU intensive program, then the synchronization mechanism might actually be a serious bottleneck, because during that time the reserved resources on other machines are not being utilized. This effect becomes even more prominent when the infrastructure is *non-homogeneous*, i.e., multiple machines with different hardware in the same computing cluster cause the workers to be out-of-sync (on top of the unmodeled system behaviour), which in turn results in more waits enforced by the parameter server as it acts as a locking mechanism in synchronous data parallelism.

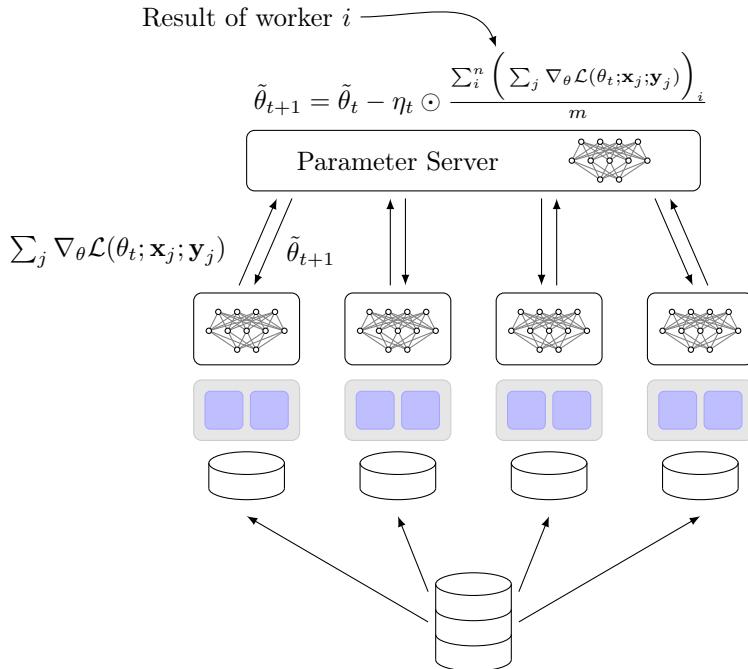


Figure 1.3: Distributed mini-batch data parallelism with n parallel workers, with a total mini-batch size of m . At the start, the data is split into n partitions, and all workers get a copy of the central model with parameterization $\tilde{\theta}_0$. When the training starts, every worker i computes the accumulated gradient $\sum_j \nabla_{\theta} \mathcal{L}(\theta_t; \mathbf{x}_j; \mathbf{y}_j)$ based on its part of the mini-batch, which is then *committed* (send) to the parameter server. After the parameter server receives all accumulated gradients, it averages them, and then applies the batched gradient to the model in order to produce $\tilde{\theta}_{t+1}$. After the new parameterization is available, the workers will be able to *pull* (download) $\tilde{\theta}_{t+1}$, and repeat the procedure described above.

This of course begs the question, *can we eliminate the need for a locking mechanism to prevent unnecessary waits of the workers?* There are some significant initial counter-arguments against removing the synchronization barrier. The most profound issue by removing the synchronization barrier, and thus obtaining *Asynchronous Data Parallelism*, is that the parameter server will incorporate gradients using a simple queuing model (FIFO) based on older parameterizations of the central variable, as shown in Figure 1.4.

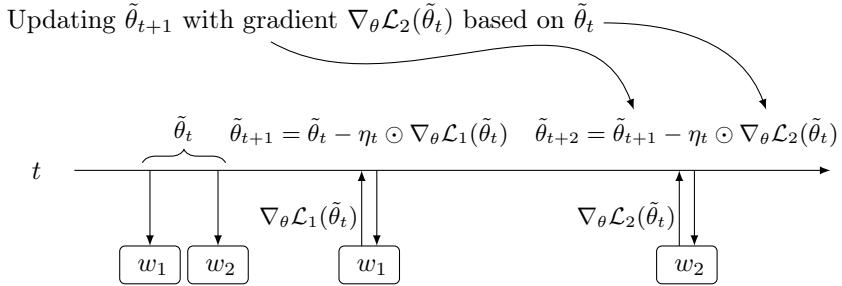


Figure 1.4: In Asynchronous Data Parallelism workers compute and commit gradients to the parameter server asynchronously. This has as a side-effect that some workers are computing, and thus committing, gradients based on old values. These gradients are called *stale gradients* in literature. In this particular example there are 2 workers w_1 , and w_2 . At the start optimization process, the pull the most recent parameterization from the parameter server $\tilde{\theta}_t$. Now all workers start computing the gradients asynchronously based on the pulled parametrization. However, since the parameter server incorporates gradients into the center variable asynchronously as a simple queuing (FIFO) model, other workers will update the center variable with gradients based on an older value, as shown in the figure above. Finally, assuming that the computing cluster is homogeneous, we can derive from this figure that the expected staleness of a gradient update is $\mathbf{E}[\tau] = (n - 1)$.

However, experiments have shown that removing the synchronization barrier actually allows models to converge [4, 17, 5], even when most workers update the central variable using a gradient based on an outdated parameterization of the central variable. An other issue, which only has been formalized recently, is *Implicit Momentum* or *Asynchrony Induced Momentum* [13]. The main reasoning behind implicit momentum, which will be discussed in detail in Section ??, is based on a very simple but powerful idea. The idea states that *memory arises from asynchrony*. Intuitively, this implies that “memory” of previous gradients is preserved due to *stale gradient* updates. This can be observed directly from Figure 1.4, where the update of w_2 is actually updating the central variable with a gradient identical to $\nabla_{\theta}\mathcal{L}_1(\tilde{\theta}_t)$ if we assume that both workers computed the gradient based on the same input data. This is a clear indication that asynchronous data parallelism is *implicitly* (because of the asynchronous nature of the approach) adding *momentum* which is proportional to the number of workers, since adding more workers actually *adds* more “memory” about previous parameterizations. The authors formalize this by probabilistically estimating the expected change between $\tilde{\theta}_{t+1}$ and $\tilde{\theta}_t$. Using some additional assumptions, such as the expected staleness $\mathbf{E}[\tau] = (n - 1)$, and geometrically distributed staleness, the authors are able to formalize the expected update in an asynchronous setting between update t and $t + 1$, which is shown in Equation 2.1.

$$\mathbf{E}[\tilde{\theta}_{t+1} - \tilde{\theta}_t] = \left(1 - \frac{1}{n}\right)\mathbf{E}[\tilde{\theta}_t - \tilde{\theta}_{t-1}] - \frac{\eta}{n}\mathbf{E}\nabla_{\theta}\mathcal{L}(\tilde{\theta}_t; \mathbf{x}; \mathbf{y}) \quad (1.1)$$

Using Equation 2.1, we can immediately derive the term which describes the implicit momentum induced by asynchrony, which is $(1 - \frac{1}{n})$. This result actually suggests that there is a limit to asynchronous optimization: since every problem has some optimal momentum term, which implies that there is an optimal number of asynchronous workers for a specific problem. In order to push the limits of asynchronous optimization, the authors propose various techniques to reduce the abundant amount of implicit momentum. One approach is to apply a *grid-search* to find the optimal hyperparameterization for a given epoch, this also includes the number of workers. Despite that this technique finds the optimal hyperparameterization for a given epoch, the disadvantage is that after every fixed number of iterations, a grid-search of the hyperparameters has to be performed to ensure (optimal) convergence. This is actually in accordance with training in non-data parallel settings, where one starts with a smaller momentum hyperparameter because the gradients at the start will be relatively large compared to the gradients near an optimum, where usually one benefits from a larger momentum hyperparameter. From this intuition we can actually deduce that when the gradient updates are large compared to the gradients close to an optimum, an optimizer does not benefit from high parallelism because the

workers are committing gradients which were based on a parametrization which is “far” from the current central variable, thus obtaining implicit momentum. Furthermore, one could eliminate the need for the gridsearch by constructing a distributed optimization scheme which is based on the idea described in Hypothesis 1.

Hypothesis 1 (H1): *Workers only contribute efficiently to the central objective when they commit gradients which are based on variables close to the central variable.*

This implies that in the presence of high parallelism, only gradient updates which are based on variables close to the *current* central variable matter. This intuition is strengthened in Figure 1.5, where a straggler is causing the central variable to converge to a different solution as opposed to the one it was heading to first. A lot of methodologies have been suggested to handle the straggler problem, however, most approaches suggest a synchronous bounded staleness approach [3, 6]. As a result, the error introduced by staleness is limited [6]. Nevertheless, in gradient-based approaches, the straggler problem can be approached from a different perspective. By incorporating Hypothesis 1, we could eliminate additional engineering efforts since stale updates, and thus stragglers, are built in the optimization procedure.

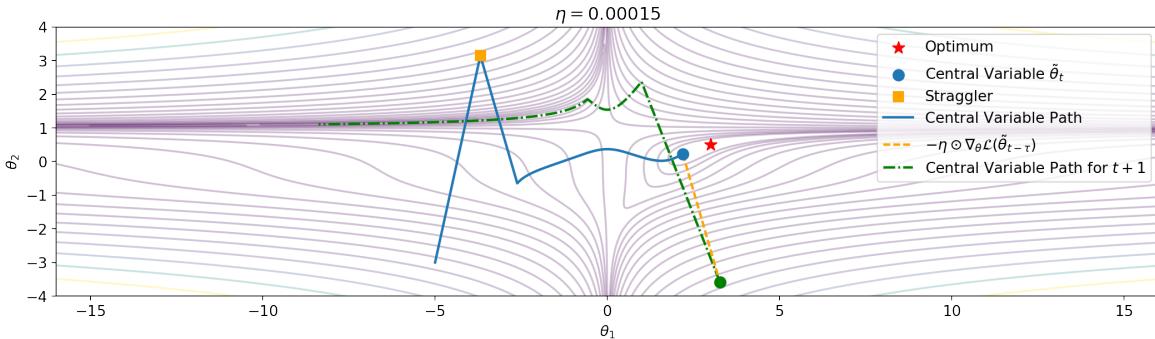


Figure 1.5: Asynchronous optimization procedure applied to Beale’s function. In this experiment we introduce a straggler programmatically (square) at the start of the optimization process. Despite the fact that this is a low probability event (large staleness compared to the number of workers) in real-world situations, the effect we describe here is present in any form of asynchronous optimization. When the straggler in this figure *commits* its gradient update to the parameter server, the central variable $\tilde{\theta}_t$ will be updated using $-\eta_t \odot \nabla_{\theta} \mathcal{L}(\tilde{\theta}_{t-\tau})$ with staleness τ to form $\tilde{\theta}_{t+1}$. This update causes the central variable to converge to a different optimum, and additionally, increases the error of the central variable (green circle). Furthermore, to actually converge to the other optimum, additional computational work has to be performed. This situation drives our intuition presented in Hypothesis 1.

One could of course argue, why not use a smaller number of workers since we are annealing the gradients which are based on non-local variables anyway, thus wasting computational resources on those machines? This is certainly a valid argument. However, let us first consider the hyperparameter grid-search approach suggested by [13]. As mentioned above, despite the fact that the grid-search technique will find the optimal hyperparameters for the current parameterization, it doesn’t mean that these hyperparameters are still optimal during the duration of the training process. Furthermore, to actually obtain the optimal hyperparameters, some certain amount of time needs to be spent in order to find them. This is actually quite problematic, since one actually wishes to reduce training time by applying data parallel techniques. In our approach, which will be discussed extensively in Chapter 4, this is not the case since the gradients will be annealed dynamically based on the curvature of the error space and current parametrization of the central variable, i.e., there is no need for a hyperparameter grid-search during the training process.

Now some general approaches and important issues regarding data parallelism have been addressed, and the reader has gained some intuition on the subject, we can formalize data parallelism and use the notation in the following chapters. In order to formalize data parallelism, let us assume we have a dataset D , which contains our training data, and that we are able to distribute dataset D over n different workers $\mathcal{W} = \{w_1, \dots, w_n\}$. Where every worker $w_i \in \mathcal{W}$ holds a copy of the central model, thus, a copy of the parameterization of the central model $\tilde{\theta}_0$. Furthermore, we denote the parameterization of a particular worker k at time t by θ_t^k . Of course, if a worker wants to contribute to the optimization of the central model, the worker needs to be able to relay update information and retrieve the most recent parameterization of the central model. This is done by instantiating a parameter server, where workers will be able to *commit* their updates, and *pull* the most recent parameterization of the central model. The parameterization of the central model is called the *central variable*, which we denote by $\tilde{\theta}_t$. In the final preparation step, before the actual training starts, D will be split into roughly n equally sized partitions $\mathcal{P} = \{p_1, \dots, p_n\}$, where $|p_i| \approx \frac{1}{|D|}$, and where p_i will be assigned to the corresponding worker w_i .

In general, all data parallel approaches share a similar training procedure, i.e., every worker computes some variable which is communicated with the parameter server to update the central model. In most cases, this variable represents some change $\Delta\theta$ which needs to be applied to the central variable $\tilde{\theta}_t$. However, some approaches such as [17], actually require that the complete worker parametrization θ_t^k is sent to the parameter server. To abstract this specific optimizer detail, we denote the variable that is sent to the parameter server by v . This procedure is described in Algorithm 1 and 2.

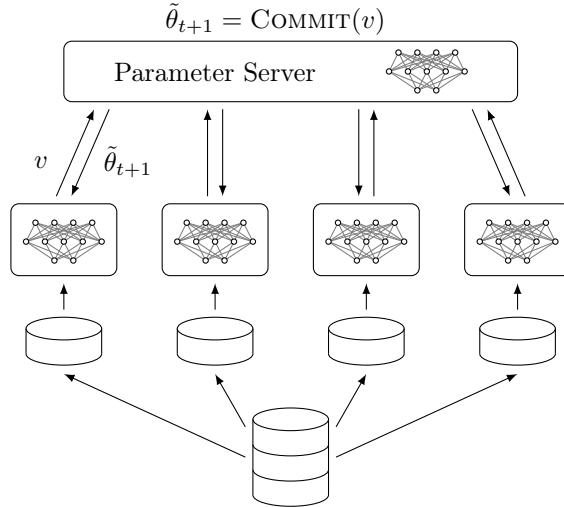


Figure 1.6: Schematic representation of a data parallel approach. In this methodology we spawn n workers (not necessarily on different machines), and assign a data shard (partition) of the dataset to every worker. Using this data shard, a worker i will iterate through all mini-batches to produce a gradient, $\nabla_{\theta}\mathcal{L}_i(x)$, for every mini-batch \mathbf{m} . Next, a variable v is constructed on the worker which is send to the parameter server. The parameter server will incorporate the variable using a COMMIT mechanism to produce the next central parametrization $\tilde{\theta}_{t+1}$.

Nevertheless, the most common and one of the earliest asynchronous optimization algorithm is DOWNPOUR [4]. In this Master Thesis, we use DOWNPOUR as a baseline performance indicator for all our experiments with other distributed optimization algorithms. In essence, workers are continuously committing gradients to the parameter server using Equation 2.11. After a gradient has been committed to the parameter server, the worker will *pull* the most recent parameterization from the parameter server in order to be consistent with the updates of other workers, as in Algorithm 1 and 2.

$$\Delta\theta^k = -\eta_t \odot \nabla_{\theta}\mathcal{L}(\tilde{\theta}_{t-\tau}; \mathbf{x}; \mathbf{y}) \quad (1.2)$$

Once the parameter server received the update $\Delta\theta^k$ from worker k , the parameter server will simply add (since the worker already negated the gradient) the update to the current central variable in order to produce $\tilde{\theta}_{t+1}$, this is described by Equation 1.3.

$$\tilde{\theta}_{t+1} = \tilde{\theta}_t + \Delta\theta^k \quad (1.3)$$

Furthermore, in order to examine the scaling abilities of the optimization algorithms we discuss in the following chapters, we need a measure to express how well they are performing in a given scenario. To measure this, one could use more traditional metrics such as *statistical efficiency* and *hardware efficiency* [5, 13].

Statistical efficiency (SE) describes the number of iterations that are required to obtain a desired result. In the context of Machine Learning, statistical efficiency describes the number of model updates that have to be performed in order to acquire a desired accuracy. However, in order to obtain some metric about a specific optimization algorithm, we need a baseline to compare against. As stated above, the baseline algorithm we use in this work is DOWNPOUR. Once we evaluated an algorithm ϵ , we compute the statistical efficiency of ϵ compared to DOWNPOUR, as described by Equation 1.4.

$$\frac{SE(\text{DOWNPOUR})}{SE(\epsilon)} \quad (1.4)$$

Hardware efficiency (HE) on the other hand, describes the amount of time it takes to execute a single iteration of a loop. In our work, this denotes the time it takes to complete a *single* epoch. Nonetheless, during this work, we experimented with several optimization algorithms which actually compute several gradients locally, and preprocess them before transmitting the update to the parameter server [17]. In these cases, if someone would employ statistical or hardware efficiency to obtain a performance indicator compared to an other algorithm, they will have a clear advantage since a significantly smaller number of central variable updates occur. Furthermore, usually these algorithms also spend less time consuming all the data since parameter server updates occur less frequent. Moreover, the network is also less saturated due to the reduced number of parameter server updates. In order to have a non-biased metric among different distributed optimization algorithms, we should look at the time it takes to obtain a desired accuracy. We call this *temporal efficiency*.

Temporal efficiency (TE) characterizes the amount of time a process, or a collection of processes, requires in order to obtain a desired accuracy. Temporal efficiency is in effect proportional to statistical efficiency, i.e., $SE(\epsilon) \propto TE(\epsilon)$. However, this is only the case when algorithm ϵ actually transmits an update to the parameter server after a worker computed a gradient (in an asynchronous setting). This is in contrast with algorithms such as [17], where some additional samples are evaluated locally, before an update is transmitted to the parameter server.

Algorithm 1 Describes the general asynchronous optimization procedure of a worker in a data parallel setting. The worker will be identified with a certain index k , the other parameter $p_k \in \mathcal{P}$, is the data partition which has been assigned to worker k .

```

1: procedure WORKER( $k, p_k$ )
2:    $\theta_0^k \leftarrow \text{PULL}()$ 
3:    $t \leftarrow 0$ 
4:   while not converged do
5:      $\mathbf{m} \leftarrow \text{FETCHNEXTMINIBATCH}(p_k)$ 
6:      $\theta_{t+1}^k \leftarrow \theta_t^k - \eta_t \odot \nabla_\theta \mathcal{L}(\theta_t^k; \mathbf{m})$        $\triangleright$  Optimization step, could be [9], or other optimizer.
7:      $v \leftarrow \text{PREPARECOMMIT}()$ 
8:     COMMIT( $v$ )
9:      $\theta_t^k \leftarrow \text{PULL}()$ 
10:     $t \leftarrow t + 1$ 
11:   end while
12: end procedure

```

Algorithm 2 Initialization and variable handling procedures of a parameter server. Before the distributed optimization starts, the INITIALIZEPARAMETERSERVER procedure is called to initialize the local parameters, given the parametrization θ of the specified model. We would like to note that t maintained by the parameter server, is different from the t variable specified in Algorithm 1.

```

1: procedure INITIALIZEPARAMETERSERVER( $\theta$ )
2:    $\tilde{\theta}_0 \leftarrow \theta$ 
3:    $t \leftarrow 0$ 
4: end procedure
5:
6: procedure COMMIT( $v$ )
7:    $\tilde{\theta}_{t+1} \leftarrow \text{APPLYCOMMIT}(v)$ 
8:    $t \leftarrow t + 1$ 
9: end procedure
10:
11: procedure PULL()
12:   return  $\tilde{\theta}_t$ 
13: end procedure

```

1.4 Problem Statement

In recent years it has been shown that being able to train large deep neural networks on vasts amount of data yield state-of-the-art classification performance. However, training these models usually takes days, or in some cases, even weeks. In order to significantly reduce the training time of these models, Jeff Dean (Google) introduced a new paradigm to train neural networks in a distributed fashion, i.e., model – and data parallelism, which is an initial attempt to tackle this problem.

Despite the relatively old research (2012), few efforts have been made to fully understand the implications, or to significantly improve the parallel gradient descent algorithm (DOWNPOUR) proposed by Dean et al. Furthermore, most research focusses on limiting the error introduced by staleness by introducing some synchronization barrier, and thus limiting the amount of asynchrony. Despite this, only recently a sound theoretical argument has been made to understand asynchronous data parallelism [13]. Using this, and understanding this contribution, we try to push the limits of asynchronous data parallelism even further.

As stated above, being able to train a model on a vast amount of data generally improves the statistical performance of a model since the model will have access to many (different) examples to train on. This is in particular the case at CERN, where the experiments collected in the order of 100 PetaBytes of particle collisions in the past years. Machine Learning approaches, and Deep Learning in particular, could potentially help in data reconstruction in the upcoming runs of LHC where particles will generate a huge amount of hits in the detector where it would be infeasible to reconstruct the particle tracks using traditional techniques (combination of a Kalman filter and Runge–Kutta methods). However, due to the petabyte scale of the data, current data parallelism will not be able to train the model in a reasonable amount of time. Therefore, we think it is important to push the current limits of data parallelism even further in order to reduce the overal training time even further.

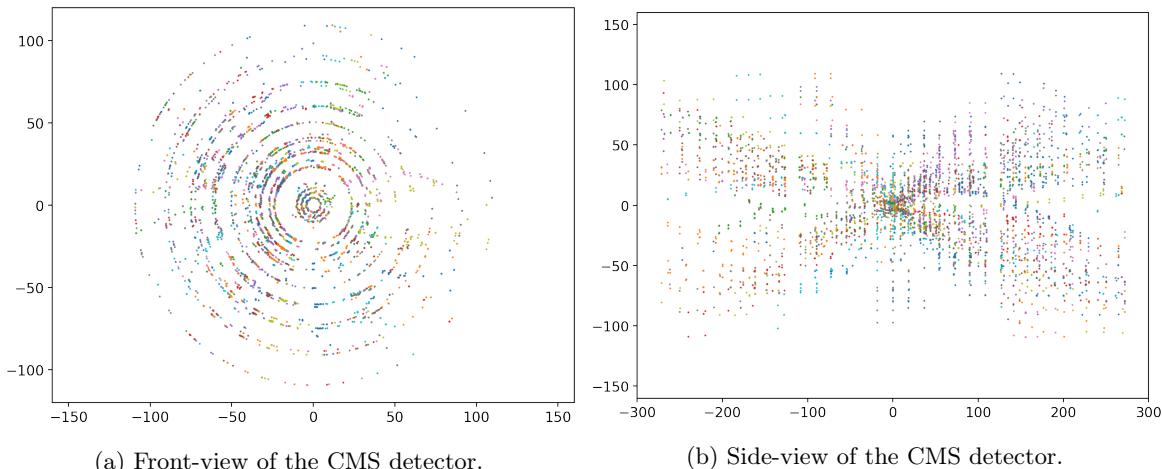


Figure 1.7: Reconstructed particle hits in the CMS detector. The collision point (vertex) is originated at $(0,0,0)$. The inner part of the detector is called the *pixel silicon* detector. This is a high resolution tracking mechanism which is able to handle the highly saturated environment of the inner part of the detector. The more outward part in this particular figure is the *silicon strip* detector. The silicon strip detector basically consists out of blocks with orthogonally positioned silicon strips which activate when a particle passes through them. Toghether, the strips produce a 3-dimensional coordinate. We would like to note that the hits do not actually represent the track of the particle, but it is rather a set of hits which will be used to compute the helix (track) parameters using the Runge–Kutta method.

1.5 Thesis Outline

In this chapter we introduced the main concept, and problems surrounding the parallelization of gradient descent. We familiarized the reader with the topic and some notation by providing some context why someone would like to apply said technique.

Chapter 2 will discuss several distributed optimization methods, i.e., their strengths, and pitfalls. Furthermore, we give some context in order to understand why some of these methods have been proposed, and how they perform in these situations.

Accumulated Gradient Normalization, our first contribution, appears in Chapter 3. We propose said technique to allow for local exploration in the error space to provide better updates to the parameter server. In general, this technique reduces the communication overhead, thus taking into account communication constraints which other techniques try to solve in a different way. Furthermore, in contrast to previous approaches, our approach is not sensitive to hyperparametrization.

In Chapter 4 we introduce our optimizer ADAG, or *Asynchronous Distributed Adaptive Gradients*, which is an asynchronous data parallel approach which uses the contributions from Chapter 3, and implements Hypothesis 1. We examine how *Accumulated Gradient Normalization* is assisting the optimization process to check for potential synergies.

When all theory has been presented, we validate our results and hypotheses by performing some experiments on traditional datasets such as MNIST, and CIFAR to have a baseline comparison to validate against different methods. But also on CERN-specific problems, such as track reconstruction as mentioned in Section 1.4.

Finally, we conclude the thesis in Chapter 6 by summarizing the contributions that have been made, and the empirical results from our experiments.

We use the following research questions to guide our study of parallelizing gradient descent:

1. When do workers contribute positively to the central variable during training?
2. Why does asynchronous EASGD diverge when a small communication frequency is used, and converges with a large communication frequency?
3. Why does *Accumulated Gradient Normalization* converge faster, compared to equivalently sized mini-batches?

Chapter 2

Related Work

In this chapter, we introduce several concepts and techniques related to Distributed Deep Learning on which this works builts upon. We start in Section 2.1 with a recap of all methods and techniques we have discussed in Chapter 1. Afterwards, we continue with a discussion of synchronous followed by an examination of asynchronous optimization methods such as DOWNPOUR and closely related extensions. Furthermore, we address several issues such as *asynchrony induced momentum* which are related to asynchronous optimization. We also consider several approaches which provide a possible solution to these issues.

2.1 Introduction

For all practical applications, Stochastic Gradient Descent (SGD) and derrivatives are the best tools from the numerical optimization toolbox for neural networks. However, applying SGD in its pure form, that is, updating the parameters after evaluating a training sample, is a computationally intensive process. An initial approach for speeding up SGD in terms of convergence with respect to training time was to compute the gradients of several samples, a *mini-batch*, and average them. This approach has several advantages, the first being that a larger mini-batch will result in less noisy updates, as more “evidence” of the surrounding error space will provide a better gradient update. The second advantage being the increased computational parallelism, since all sub-gradients (gradients of the training samples in the mini-batch) are based upon the same parametrization of the model. As a result, the parallelization of the gradient computation is quite straightforward. For instance, for every training sample in a mini-batch, one could allocate a thread, a process, or even a different machine (see Figure 1.3) to compute the gradients in parallel. However, a blocking mechanism is required in order to sum all gradients, average them, and finally update the parametrization of the model. This process is depicted in Figure 1.2. As discussed in Chapter 1, mini-batch parallelism is an instance of *synchronous data parallelism*. Although many synchronous optimization schemes share a similar structure, we discuss other instances of synchronous data parallelism in particular in Section 2.2 since these optimization schemes incorporate gradients and worker parameterizations into the central variable differently compared to mini-batch parallelism.

Nevertheless, a significant, but albeit technical issue in synchronous optimization is when a single or multiple workers are slowed down for some reason, e.g., due to high CPU load, or bandwidth consumption, other workers will have to wait before they can continue with step $t + 1$. As a result, the allocated resources are not fully utilized. This particular issue is known in literature as the *straggler* problem. However, this problem can be mitigated with by using a *homogeneous* hardware configuration. For instance, when one would employ 2 different GPU’s running at different clock speeds, a *heteregenous* hardware configuration, then the CPU will always have to wait for a particlur GPU since it runs at a lower clock speed causing the complete training procedure to be slowed down¹. Furthermore, we could argue that there is a limit to synchronous data parallelism because simply *adding more workers to the*

¹A chain is only as strong as its weakest link.

problem implicitly increases the size of the mini-batch. As a result, when applying synchronous data parallelism, one is not parallelizing gradient descent in a typical sense, but rather parallelizing the computations within a step. Of course, one could even increase the parallelism within synchronous data parallelism even further by applying model parallelism as discussed briefly in Section 1.2. Nevertheless, while such an implementation is definitely possible, it might be more cost-aware from an economical perspective to just let the model train for a longer period of the compared to actually implementing the training procedure described above. Furthermore, even with if one would implement said training method, there is still a limit to the amount of parallelism due to the structure of the computation graph, and communication cost between devices which have to be taken into account. This of course begs the question if it is actually possible to push the limits of asynchrony, and thereby reducing the training time even further. Or from a different perspective, is there a more trivial method besides implementing the above training procedure to reduce the training time.

Several approaches [4, 6, 3, 15, 17, 11, 8] have been suggested over the past years which accomplish exactly this. All these methods are instances of *asynchronous data parallelism*, discussed in Section 1.3. In contrast to synchronous data parallelism, asynchronous methods can be identified by the *absence* of a blocking mechanism which is present in synchronous data parallelism. Despite the fact that this method resolves the waiting time induced by stragglers, it introduces a closely related but persistent issue. More formally, the *staleness* issue is due to the fact that all n workers update the central variable in an asynchronous fashion. Meaning, from the moment a worker k is done computing an update $\Delta\theta^k$ based upon parameterization of the central variable $\tilde{\theta}_t$, it will commit $\Delta\theta^k$ to the parameter server, and afterwards continue with the next mini-batch. Because of this behaviour, it is possible that a number of central variable updates τ occurred during the time worker k was computing $\Delta\theta^k$. As a result, instead of obtaining $\tilde{\theta}_{t+1}$ by applying $\Delta\theta^k$, worker k is actually applying $\Delta\theta^k$ to $\tilde{\theta}_{t+\tau}$. Which is not ideal, since $\Delta\theta^k$ is based on parametrization $\tilde{\theta}_t$. From [13] we know that increasing the number of workers actually increases the amount of staleness τ since $\mathbf{E}[\tau] = (n - 1)$ under a *homogeneous* hardware configuration and a simple queuing model². This result is validated empirically in one of our experiments, shown in Figure 2.1.

A side-effect of updating the central variable with stale updates in an asynchronous fashion, is that stale updates carry information about previous states of the central variable. Which is to be expected since worker updates are based on older parameterizations of the central variable. Using this intuition, the authors in [13] show formally that in a regular asynchronous SGD setting, like DOWNPOUR, stale updates behave like *momentum*. Furthermore, their formalization can even describe the amount of *implicit momentum*, described in Equation 2.1, which is present in an asynchronous optimization procedure. Furthermore, when applying (Nesterov) momentum in a traditional optimization setting, i.e., sequential parameter updates, one needs to specify the amount of momentum. This is usually denoted by a hyperparameter μ . However, we would like to note that in an asynchronous setting, the hyperparameter μ_s from Equation 2.1, is not explicitly defined in the optimizer, but arises from the number of asynchronous workers. As a result, Equation 2.1 is merely descriptive.

$$\mu_s = \left(1 - \frac{1}{n}\right) \quad (2.1)$$

In a previous paragraph we said that there is a limit to mini-batch parallelism, since adding more workers to the problem implicitly increases the size of a mini-batch. However, we observe in Figure 2.1 in accordance with [13], that there might be a limit to asynchronous optimization as well. However, the authors in [13] assume that gradients coming from the workers are not adaptive³, as can be deducted from their proof. The question begs, can we push asynchronous optimization even further? We answer this question in Chapter 3, and Chapter 4, by introducing new techniques using a better, and more intuitive understanding of parameter staleness.

²With a simple queuing model we intent that updates $\Delta\theta^k$ are incorporated into the central variable in a queuing fashion.

³Meaning, they are not modified with respect to some (hyper)parameter.

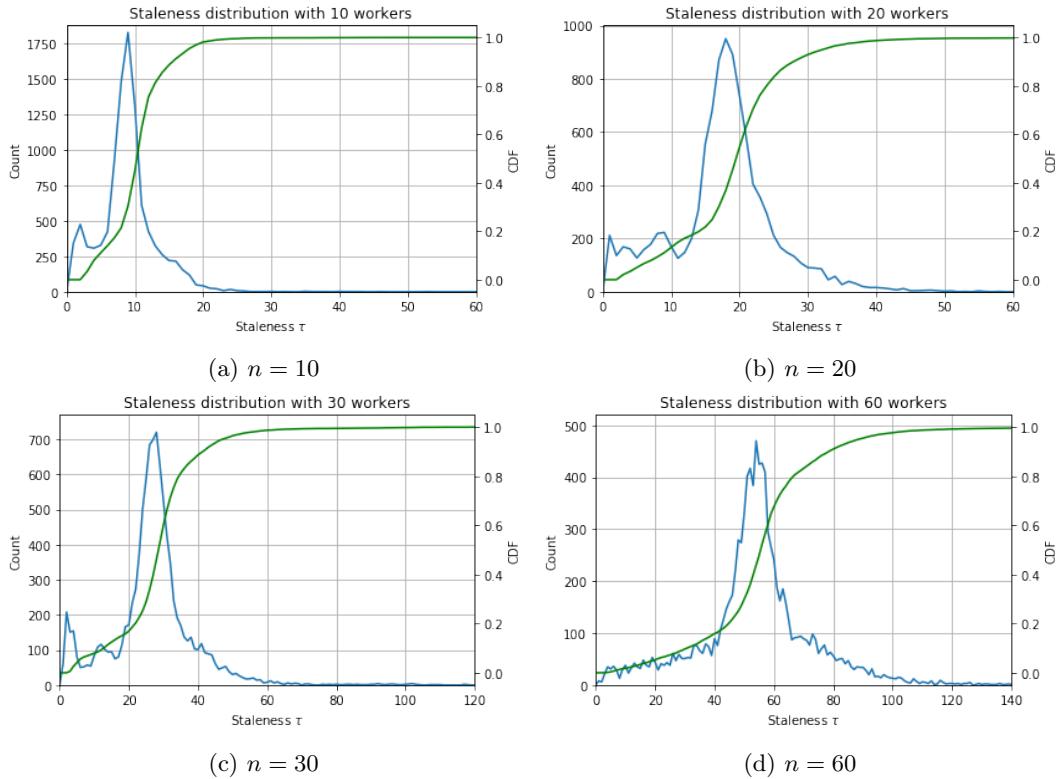


Figure 2.1: These figures show the staleness distribution during a training procedure using a differing number of parallel workers. For every central variable update, we record the staleness τ , and increment the number of occurrences of this particular staleness by 1. Thus effectively building a histogram showing the staleness distribution during the training. With this, we experimentally validate the observations of [13] that $E[\tau] = (n - 1)$. Furthermore, the claim that staleness is geometrically distributed during training also holds (right half of the distribution).

2.2 Synchronous Data Parallelism

2.2.1 Model Averaging

As the name suggests, model averaging optimizes the central variable by simply averaging the parametrizations of the workers after λ (which could be the amount of data in a single epoch) steps until all data has been consumed. As mentioned before, hyperparameter λ denotes the number of *local* steps that have to be performed before the results are communicated with the parameter server. The optimization procedure in the worker and parameter server are quite straightforward, and are described in Equation 2.2 and Equation 2.3 respectively. However, Equation 2.2 has the disadvantage that a lot of communication with the parameter server has to be performed, since after every *local* worker update all parameters have to be shipped to the parameter server to apply Equation 2.3. Furthermore, this approach has several other issues that will be discussed later. But for now we can resolve this issue by delaying a commit to the parameter server by doing several *local* (λ) updates before sending θ_t^k to the parameter server, this is shown in Algorithm 3.

$$\theta_{t+1}^k = \theta_t^k - \eta_t \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}_t^k; \mathbf{y}_t^k) \quad (2.2)$$

$$\tilde{\theta}_{t+1} = \frac{1}{n} \sum_{i=1}^n \theta_t^i \quad (2.3)$$

Contrary to other synchronous methods, model averaging does not reduce the training time in general. In fact, it requires more resources to achieve the same results since the central variable is set to be the average of all workers. This is shown in Figure 2.2 (a) since all workers follow the same first-order path, synchronize, and average the parameters after λ steps to start again from the averaged parameterization, which is in this case the central variable. However, what happens if we initialize the parameterizations of the workers randomly? At first, all workers will do some work locally, but after λ steps, the parameterizations of the workers are averaged. As a result, all workers share the same parameterization in the next step, which brings us again to our initial scenario as shown in Figure 2.2 (b). Furthermore, when applying random initialization, we could say a “warmup” period is required since all workers need to converge a particular solution before convergence can take place. This intuition is strengthened in Figure 2.3.

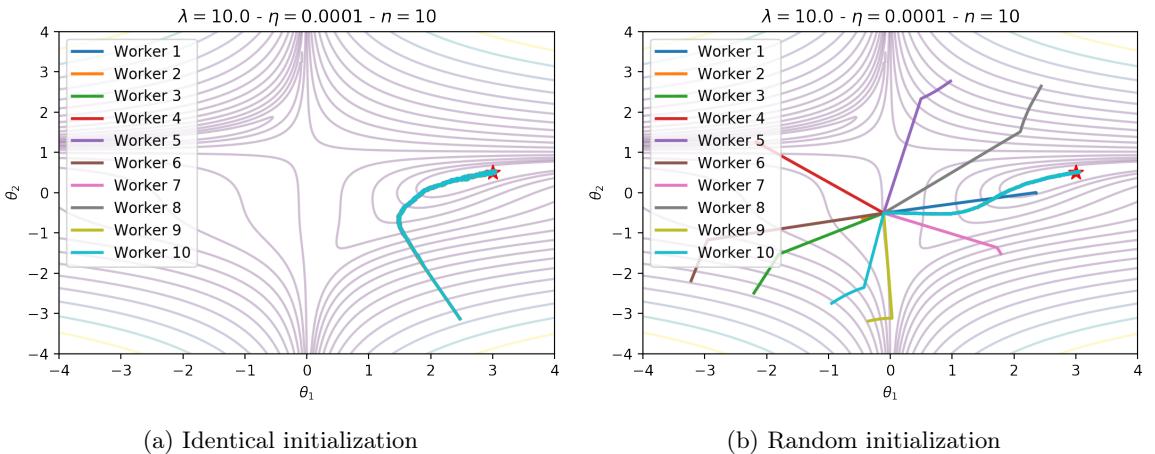


Figure 2.2: In this Figure we show the difference between identical initialization (a), and random initialization (b). In essence, both methods require roughly the same amount of time as a sequential optimization algorithm, i.e., not distributed, while using more resources. However, the difference here is that using random initialization requires a “warmup” period (a single parameter server update), before the actual optimization process can start. In order to simulate the stochastic noise of mini-batch gradient descent, we added a noise term to our gradient computations which was sampled from $X \sim \mathcal{N}(\mu = 0, \sigma^2 = 10.0)$ to ensure that some deviation from the central variable was possible.

Algorithm 3 Describes the worker procedure with local worker exploration. The worker will be identified with a certain index k and will be initialized by assigning the central variable ($\tilde{\theta}_t$) to the worker, or the worker variable can be randomized at the start of the optimization procedure. Furthermore, we introduce a hyperparameter λ , which is the number of local updates that have to be performed in order the worker parameterization θ_t^k is communicated with the parameter server.

```

1: procedure MODELAVERAGINGWORKER( $k$ )
2:    $\theta_0^k \leftarrow \text{PULL}()$  or  $\text{RANDOM}()$                                  $\triangleright$  Worker variable  $\theta_0^k$  can be randomized.
3:    $t \leftarrow 0$ 
4:   while not converged do
5:      $i \leftarrow 0$ 
6:     for  $i < \lambda$  do
7:        $\mathbf{x}, \mathbf{y} \leftarrow \text{FETCHNEXTMINIBATCH}()$ 
8:        $\theta_{t+1}^k \leftarrow \theta_t^k - \eta_t \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}; \mathbf{y})$      $\triangleright$  Optimization step, could be [9], or other optimizer.
9:        $i \leftarrow i + 1$ 
10:       $t \leftarrow t + 1$ 
11:    end for
12:    COMMIT( $\theta_t^k$ )
13:    WAITFOROTHERWORKERS()
14:     $\theta_t^k \leftarrow \text{PULL}()$ 
15:  end while
16: end procedure

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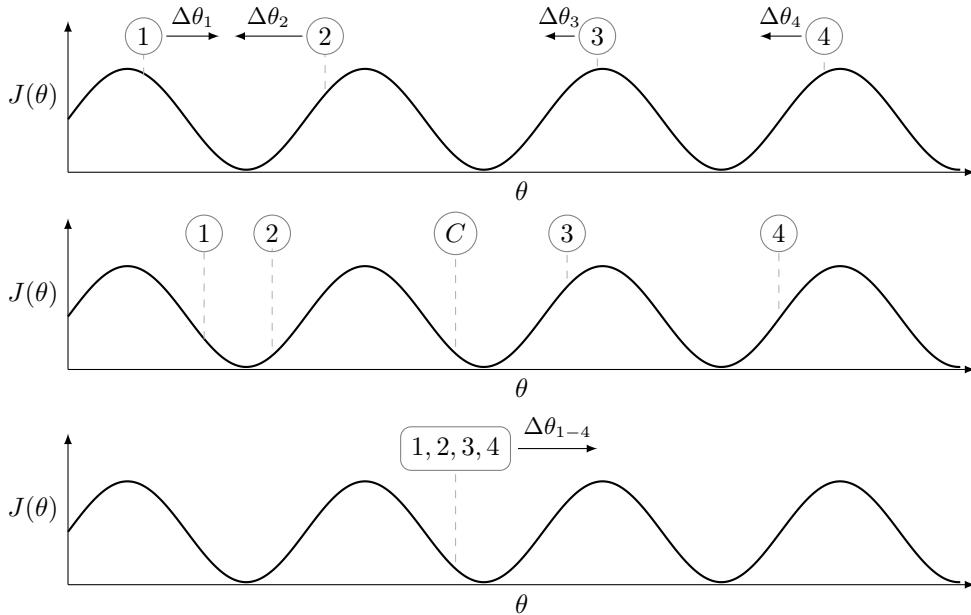


Figure 2.3: This figure explains the intuition behind model averaging. In the first state, all workers, w_1 , w_2 , w_3 , and w_4 , are uniformly initialized over the hypothesis space. Using the local parametrizations, every worker obtains an update Δw_i , and applies it locally. Afterwards, all workers send their parametrizations to the parameter server which will average them to obtain a central variable, which is depicted by C in this particular figure. Finally, all workers fetch the most recent central variable, and start computing new gradients based for the following iteration. Furthermore, what can be observed directly from this figure, is that when the workers do not agree on a *local neighborhood*, the central variable will not be able to converge. This is additional support for Hypothesis 1.

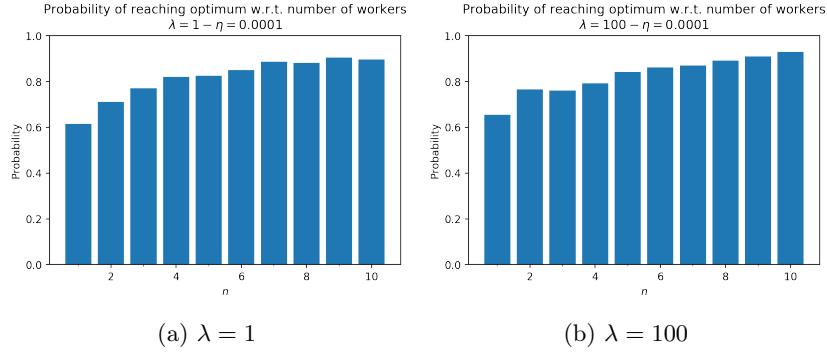


Figure 2.4: Probability distribution extracted from several Monte-Carlo simulations under different conditions. We find that the probability of reaching the optimum (Beale’s function) increases when the number of random initialized workers increases. Despite the fact that we observe that more exploration (λ) yields a better statistic, we believe that this result is not significant due to the relatively low number of simulations (1000 per worker per hyperparameter).

Nevertheless, what is interesting about the random initialization of workers, is that when we increase the number of workers, the probability that we will find a better solution (minima) compared to a sequential optimization process increases. However, this also depends on the curvature of the error space. For example, in Figure 2.4 we use Beale’s function to obtain our statistic. However, from the plots we can deduce that the curvature of the error space is slightly biased towards the global minimum if first-order gradients are used. If the error space was not biased towards a specific minima, the statistic for a single worker under different hyperparameterizations should be 50%.

Furthermore, what is the role of the exploration parameter λ ? Does it contribute to the optimization process besides reducing the amount of communication with the parameter server by increasing the amount of local work? In principle this would help to optimization process since more exploration of the parameter space occurs, and as a result, better updates are applied. Furthermore, remember what we said in Section 2.1 on synchronous data parallelism, that effectively increasing the number of workers implicitly increases the size of the mini-batch. Yet, in this particular case there is a subtle difference, i.e., local exploration of the parameter space. As a result, it is not implicitly increasing the size of the mini-batch since some form of local exploration occurs. As a result, *the averaged central variable will produce a less-noisy consensus based on the (local) exploration of the error space*. However, the problem lies in the fact when different sets of workers enter different minima, possibly because of *too much exploration of the error space*, as depicted in Figure 2.3. Because if this happens, then the averaging step could potentially reset the situation instead of continuing exploring a single minima.

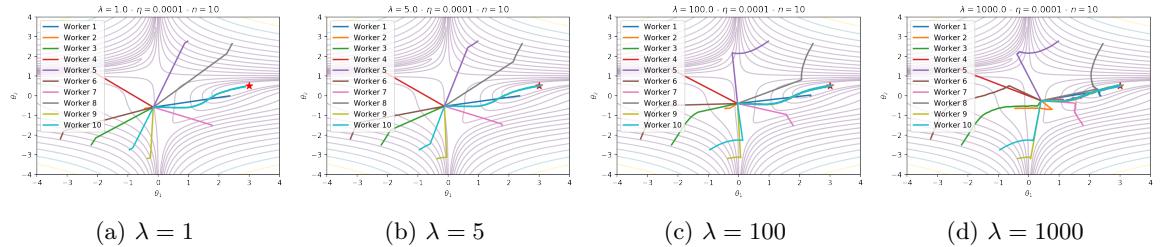


Figure 2.5: Workers which have been randomly initialized (experiments use same initialization) with different values for λ (communication frequency, or number of local worker iterations). Since the workers are initially randomized of the hypothesis space, the central variable in the first averaging step will be shifted towards the global minima because of the amount of local exploration, and due to the bias of the error space as shown in Figure 2.4.

2.2.2 Elastic Averaging SGD

Elastic Averaging SGD, or EASGD [17], is a distributed optimization algorithm designed with communication constraints in mind. In essence, EASGD could be viewed as an extension of model averaging described in Section 2.2.1 with the difference that the workers, and the central variable are coordinated using an *elastic force* [17] instead of averaging the workers after a fixed amount of steps. This means that instead of simply transmitting the parametrization or a gradient to the parameter server, the workers commit an *elastic difference* which is defined as $\eta_t \rho(\theta_t^k - \tilde{\theta}_t)$ where ρ is the *elasticity* hyperparameter. Intuitively, ρ describes the amount of exploration that can be performed by the workers with respect to the central variable $\tilde{\theta}_t$.

Assuming $\lambda = 1$, the worker update and central variable update are described in Equation 2.4, and Equation 2.5 respectively. What is different compared to most other optimization schemes, is that the worker update described in Equation 2.4 has a second component, i.e., the *elastic difference*. Furthermore, note that compared to other distributed optimization schemes, the workers in EASGD do not synchronize their parameterization with the central variable as shown in Algorithm 4, but rather update the central variable using the elastic difference, and than use the new central variable as a new reference point to compute the following elastic differences.

$$\theta_{t+1}^k = \theta_t^k - \eta_t \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}_t^k; \mathbf{y}_t^k) - \eta_t \rho(\theta_t^k - \tilde{\theta}_t) \quad (2.4)$$

$$\tilde{\theta}_{t+1} = \tilde{\theta}_t + \eta_t \sum_{i=0}^n \rho(\theta_t^i - \tilde{\theta}_t) \quad (2.5)$$

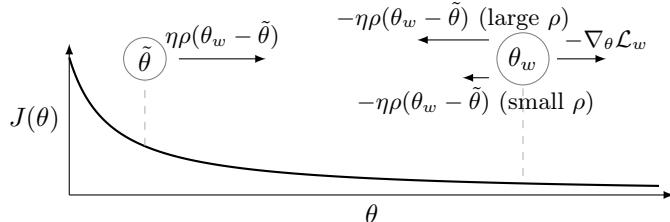


Figure 2.6: In this example we would like to make the intuition behind Elastic Averaging SGD clear by describing it as a classical physics problem. In this particular figure we have 1 worker, θ_w ($n = 1$, $\lambda = 1$), and a central variable $\tilde{\theta}$. The y -axis represents the error with respect to a certain parameterization θ . Using Algorithm 4, the worker performs λ steps to compute the next value of θ_w using Equation 2.4. As stated before in Section 2.2.2, Equation 2.4 holds 2 components, i.e., the regular negated first-order gradient ($-\eta_t \nabla_{\theta} \mathcal{L}_w$), and the *negated* elastic difference ($-\eta_t \rho(\theta_w - \tilde{\theta})$). As always, the negated first-order gradient represents the steepest slope with respect to the current error (loss), and parameterization. However, the *negated* elastic difference actually points towards the central variable. The magnitude of the elastic difference is controlled by hyperparameter ρ . This implies that a large value of ρ *strongly* limits the amount of exploration a worker can perform with respect to the central variable since the magnitude of the elastic difference vector will be proportionally larger compared to a small ρ . Finally, the central variable is updated using Equation 2.5 and the elastic difference coming from all workers. Furthermore, we would like to note that the elastic difference during the central variable update is *not* negated. Meaning that the central variable is optimized with respect to the “pull” of all workers, with the effect that workers which are ahead in the optimization process, will have a larger elastic force, causing the central variable to be influenced more strongly by distant workers. Nevertheless, at the same time, distant workers are pulled towards the central variable using an equal but opposite force *and* the negated gradient, i.e., the net force of the elastic difference and the negated gradient combined results either in a step towards a minimum, a null-operation, or a step in the direction of the central variable.

Algorithm 4 Worker procedure of synchronous EASGD. This algorithms accepts several hyperparameters, the first being the number of local computations λ , the exploration hyperparameter ρ , and the dynamic learning rate η_t .

```

1: procedure EASGDPARTNER( $k$ )
2:    $\theta_0^k \leftarrow \tilde{\theta} \leftarrow \text{PULL}()$ 
3:    $t \leftarrow 0$ 
4:   while not converged do
5:      $i \leftarrow 0$ 
6:     for  $i < \lambda$  do
7:        $\mathbf{x}, \mathbf{y} \leftarrow \text{FETCHNEXTMINIBATCH}()$ 
8:        $\theta_{t+1}^k \leftarrow \theta_t^k - \eta_t \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}; \mathbf{y})$ 
9:        $i \leftarrow i + 1$ 
10:       $t \leftarrow t + 1$ 
11:    end for
12:     $\mathcal{E} = \eta_t \rho (\theta_t^k - \tilde{\theta})$ 
13:     $\theta_{t+1}^k = \theta_t^k - \mathcal{E}$ 
14:    COMMIT( $\mathcal{E}$ )
15:    WAITFOROTHERWORKERS()
16:     $\tilde{\theta} \leftarrow \text{PULL}()$ 
17:     $t \leftarrow t + 1$ 
18:  end while
19: end procedure

```

However, there is an interesting property about the elastic difference which is rather problematic for the communication frequency λ . Using the intuition from Figure 2.6, we can deduce that Equation 2.4 has an additional solution for 0 besides when $\nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}_t^k; \mathbf{y}_t^k) = 0$. This solution has profound implications on λ , and as a result, on the allocated computing resources. Because if this situation occurs, then all additional computations are wasted. Since during that time, the central variable is not updated, and the worker are not updated. In order to describe this formally, let us consider the cases for which the EASGD worker update rule is 0, as shown in Equation 2.6.

$$-\eta_t \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}_t^k; \mathbf{y}_t^k) - \eta_t \rho (\theta_t^k - \tilde{\theta}_t) = 0 \quad (2.6)$$

Since we know that when a worker evaluates a 0-gradient, i.e., $\nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}_t^k; \mathbf{y}_t^k) = 0$, then the EASGD worker update rule is also 0. Since no change in the parameterization of the worker took place, and as a result, there is no change in the value of the elastic difference. Unless $\lambda > 1$, and in step $t - 1$, $\nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}_t^k; \mathbf{y}_t^k) \neq 0$. In this case, a net force will be applied in the direction of the central variable, that is, the worker will move back towards the central variable.

Nevertheless, let us focus our efforts on the case when Equation 2.6 is satisfied. First, let us assume that $\nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}_t^k; \mathbf{y}_t^k) \neq 0$. As a result, the only way that the condition described in Equation 2.6 can be satisfied is when the *non-negated* elastic difference equals the gradient update as shown in Equation 2.7.

$$-\eta_t \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}_t^k; \mathbf{y}_t^k) = \eta_t \rho (\theta_t^k - \tilde{\theta}_t) \quad (2.7)$$

Using Equation 2.7, we can derive a condition, described in Equation 2.8, where the amount of computational work is not wasted, and introduce an early-stopping mechanism to prevent such waste.

$$\|-\eta_t \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}_t^k; \mathbf{y}_t^k)\| > \|\eta_t \rho (\theta_t^k - \tilde{\theta}_t)\| \quad (2.8)$$

One might ask why the elastic difference in Equation 2.8 needs to be smaller than the gradient term in order for the worker to move towards a minima? Remember from Figure 2.6 that the worker only moves (backwards or forwards) whenever Equation 2.6 is not satisfied. As a result, we can deduce that a worker only moves towards a minima when Equation 2.8 is met until the *equilibrium* condition

in Equation 2.7 is satisfied. At this point, a worker could stop any further local computations since additional computations would be wasted anyway. Of course, assuming the mini-batch size m is sufficiently large to eradicate any noise originating from the gradients. In order to proof this result empirically, we conducted several experiments with ordinary hyperparameters ($\lambda = 30$, $\eta = 0.00001$, $n = 5$) for different values of ρ summarized in Figure 2.7 and Figure 2.8. What we observe in both cases is that the workers initially perform a large amount of exploration due to the relatively large value of λ . Since λ has a large value, the workers do not often synchronize with the central variable causing them to reach the equilibrium condition described in Equation 2.7 because the gradients are nog large enough to satisfy Equation 2.8.

To summarize, if the gradients produced by the workers are not large enough to produce a net force in the direction of a minima, then the workers are in an equilibrium with the elastic difference causing additional computational work to be wasted since the elastic difference will only lower if the distance between the worker and the central variable gets smaller. An obvious solution to this problem would be to lower the value of λ or ρ . However, lowering the value of λ causes the worker to communicate more frequently with the parameter server, even if the equilibrium condition is not met. Since EASGD is designed with communication constraints in mind, this is not an ideal solution. Furthermore, lowering ρ is also not ideal since this would reduce the equilibrium condition even further, causing workers to deviate further from the central variable. As mentioned before in Section 2.2.1 this is not desired because the possibility exists that different sets of workers will fall into different minima. This is especially problematic in EASGD since the parameterizations of the workers are not synchronized with the central variable, but are coordinated using the elastic difference. For example, imagine the case when two sets of workers (equal in number) are in two different minima with the central variable being in the middle of these two sets. Furthermore, assume that the elastic difference in these two sets of workers is identical and Equation 2.7 is satisfied. This implies that the central variable and the workers are not able to move, even after a synchronization step, which in turn results in the fact that the central variable is not able to converge.

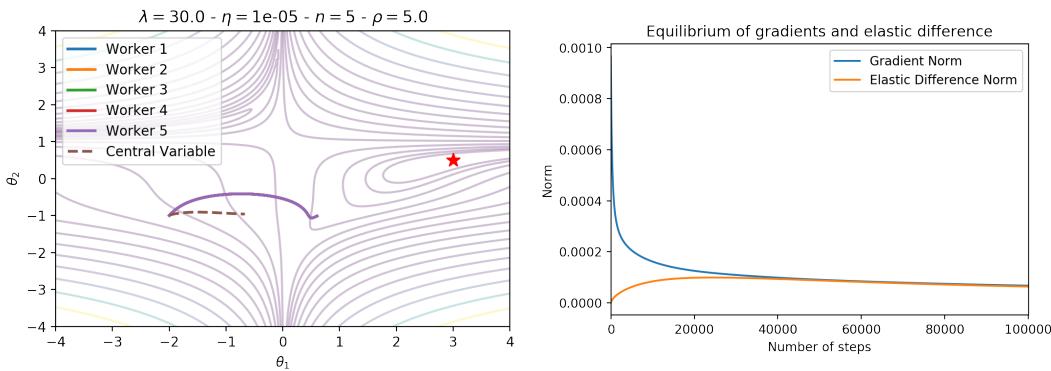


Figure 2.7: In this experiment we use $\rho = 5$, as suggested by the authors [17]. We observe that at some point the elastic difference is starting to influence the first-order gradients causing the workers to bend downwards, which is not present in Figure 2.8. This “bend” is due to a significant elastic force pointing in the direction of the central variable. If we would decompose the vector in its main components, i.e., θ_1 and θ_2 , we would find that the θ_2 component is significant enough to cause the worker to bend downwards since the elastic force is getting stronger proportional to the distance between the worker and the central variable and hyperparameter ρ . Furthermore, the equilibrium plot shows that the worker does a lot of initial exploration, while the central variable slowly follows the workers based on the averaged elastic difference. After 40000 steps, we see that the workers reach the equilibrium condition. As a result, any computational work done by the workers within a communication window is wasted since the distance between the workers and the central variable needs to grow smaller for the elastic force to shrink. However, because the elastic force is shrinking proportionally to the distance between the workers and the central variable, it allows the workers to explore *slightly* more of the hypothesis space since the workers already met the equilibrium condition.

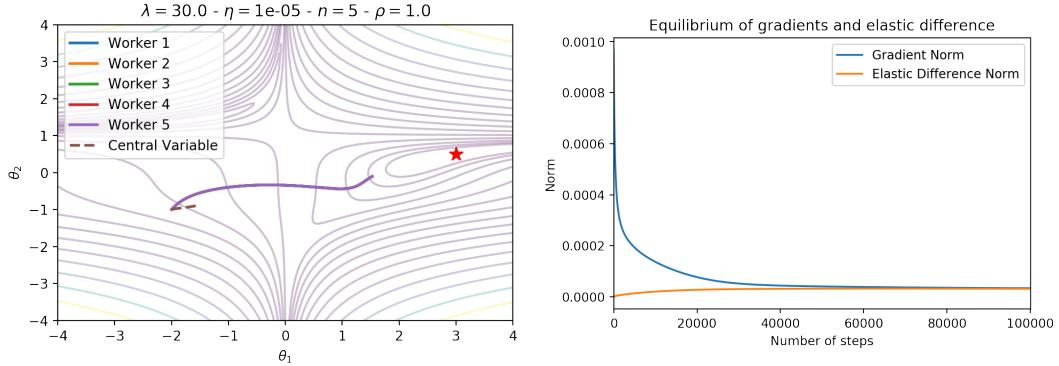


Figure 2.8: In this experiment we reduce the value of the exploration hyperparameter ρ while all other hyperparameters from our experiment in Figure 2.7 remain the same. After running the experiment we do not observe the ‘‘bend’’ which we initially observed in Figure 2.7. However, this was expected since the elastic force is not that prominent due to the reduced value of hyperparameter ρ . Nevertheless, we still observe that the workers reach the equilibrium condition, with the difference that the central variable shows slower convergence behavior compared to Figure 2.7.

To improve convergence of the central variable in EASGD, and reduce the amount of wasted computational resources, we implemented Equation 2.9 based on Equation 2.8 as a measure to stop wasting further computations.

$$\|-\eta_t \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}_t^k; \mathbf{y}_t^k)\| \leq \|\eta_t \rho(\theta_t^k - \tilde{\theta}_t)\| \quad (2.9)$$

However, we observed from our experiments that this exact condition is rarely satisfied. Of course, we made the assumption in Equation 2.7 that the parameterization of a worker does *not* change if a worker settles in an equilibrium. Of course, before reaching the equilibrium, the gradient is non-zero, this implies that there is a change in the parameterization of a worker causing the elastic difference to be modified. With the result that the net force applied to the worker is *approaching* the equilibrium. As a result, the net force applied to a worker is gradually becoming smaller.

In order to anticipate for a declining net force, and as a result, the gradually approaching equilibrium, we communicate the elastic difference with the parameter server if Equation 2.10 is met.

$$\left(\|\eta_t \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}_t^k; \mathbf{y}_t^k)\| - \|\eta_t \rho(\theta_t^k - \tilde{\theta}_t)\| \right) < \eta_t \quad (2.10)$$

Using the same experimental configuration as in Figure 2.7, with the difference that Equation 2.10 has been implemented as an early-stopping mechanism to communicate the elastic difference with the parameter server. Using Equation 2.10, we can see that the proposed early stopping mechanism benefits the central variable in terms of convergence with respect to the default experimental configuration in Figure 2.7.

What is remarkable about the equilibrium plot in Figure 2.9 is the presence of a sudden drop in the norm of the elastic difference vector which is absent in Figure 2.7 despite the fact that the workers follow similar paths. This is due to the early stopping mechanism which pulling the central variable closer towards the workers causing the decline in the norm of the elastic difference. Furthermore, since at this point the gradients are relatively small, a lot of local work can be done while at the same time the central variable will be able to keep the elastic distance small.

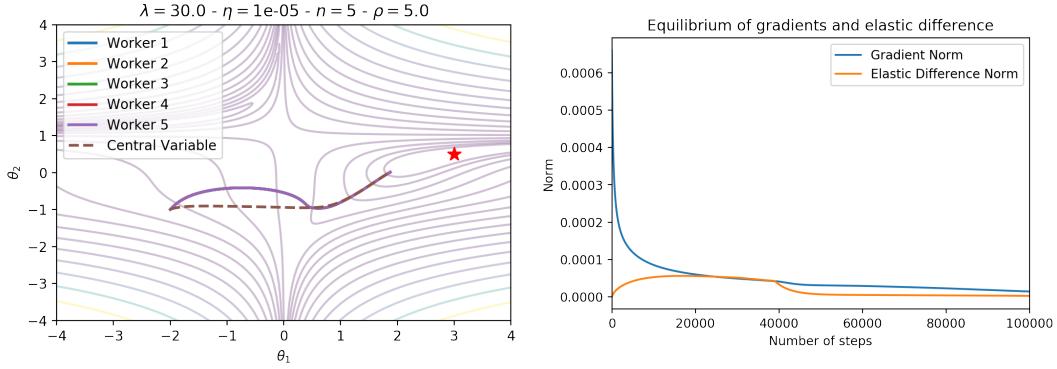


Figure 2.9: EASGD using the early stopping mechanism described in Equation 2.10. Compared to Figure 2.7, which uses the same experimental configuration, we observe that the central variable benefits in terms of convergence from increased communication when the early stopping condition is met.

Since the early stopping condition effectively communicates the elastic difference with the parameter server, is it not in essence increasing the communication frequency, i.e., lowering λ ? In effect it isn't, since communication only takes place when the workers and the central variable approach an equilibrium condition. To show this, we conducted several experiments with and without the early stopping mechanism shown in Figure 2.10 and Figure 2.11.

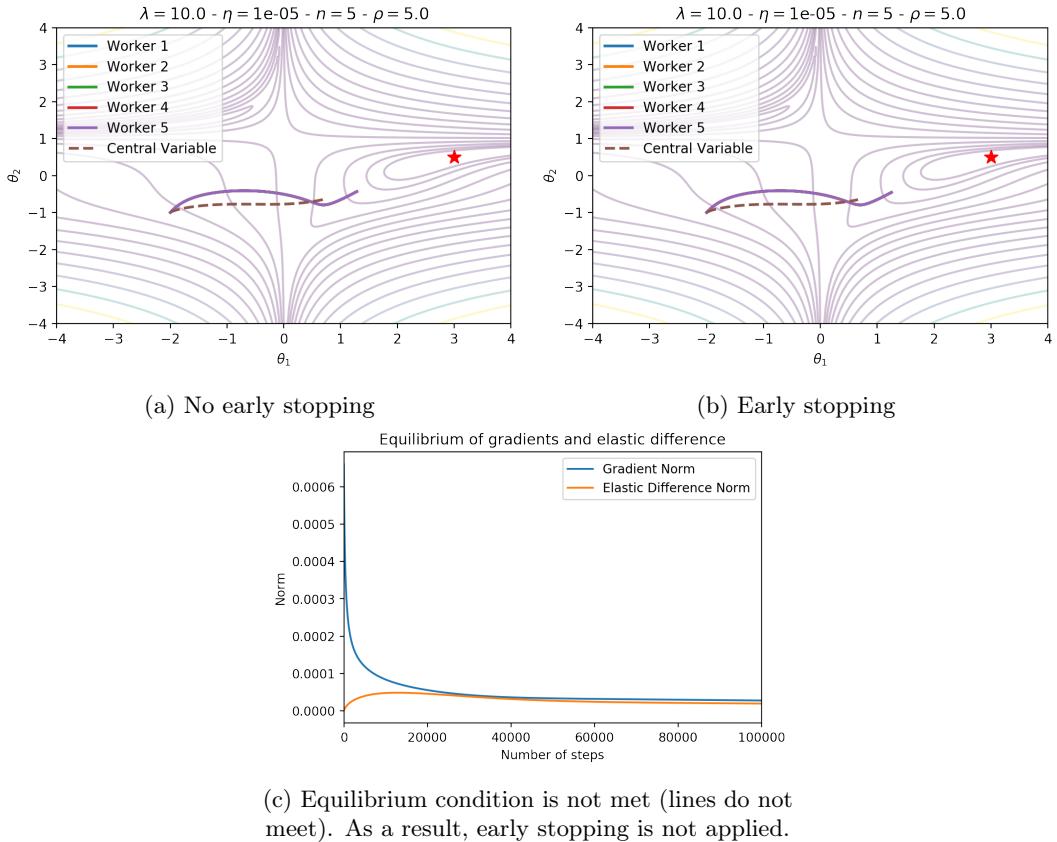


Figure 2.10: $\lambda = 10$

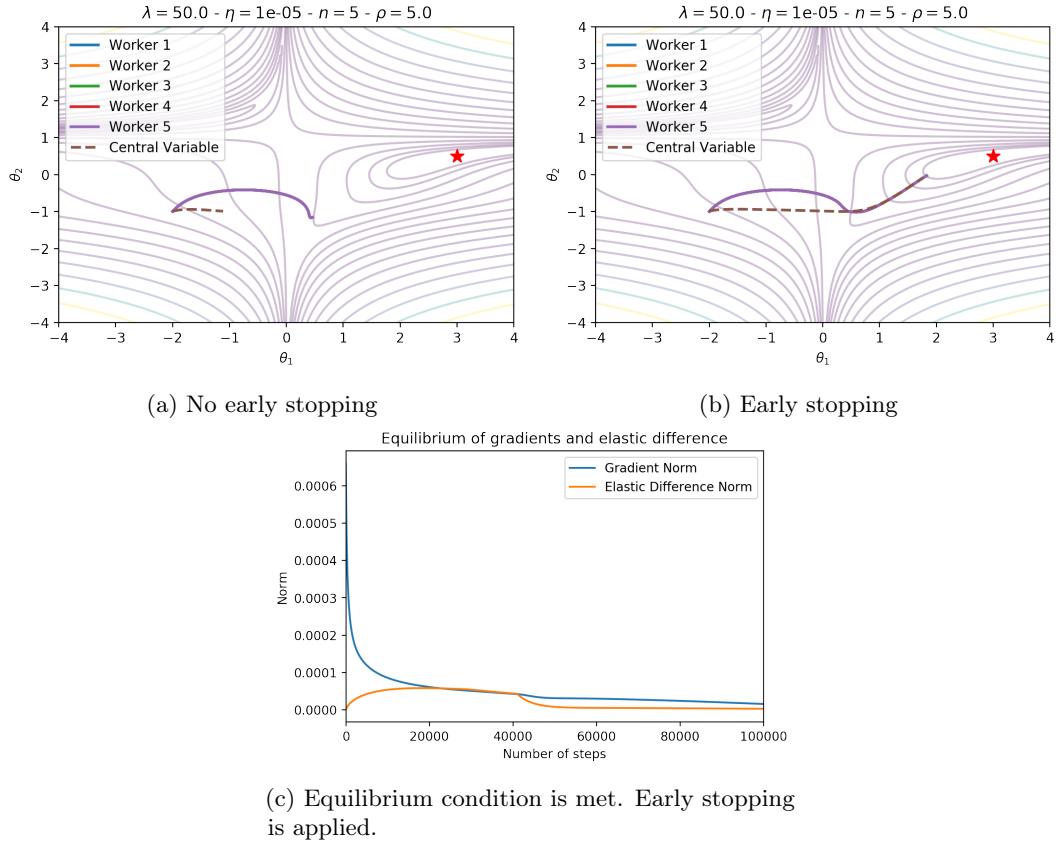


Figure 2.11: $\lambda = 50$

An other interesting phenomena is when EASGD is approaching a minima. Compared to other synchronous optimizers such as mini-batch parallelism, or model averaging, EASGD requires a significant amount of additional steps to actually converge to the minima because of the equilibrium condition as shown in Figure 2.12. Remember, if the workers are approaching the equilibrium condition, a small net force is applied to the worker because the *negated* elastic difference counter-acts the advances of the workers to minimize the variance of the workers with respect to the central variable.

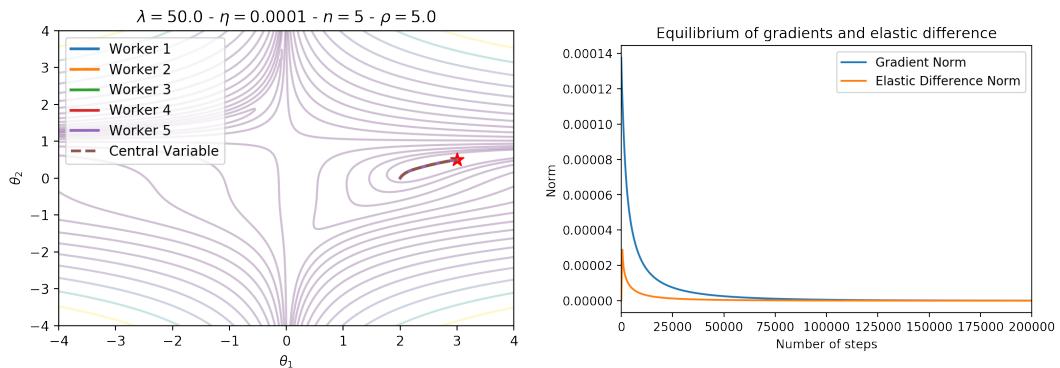


Figure 2.12: Slow converge of EASGD at minima due to the equilibrium condition. In this figure we apply early stopping but due to the small gradients close to the minima it does not have any significant effects.

2.3 Asynchronous Data Parallelism

2.3.1 DOWNPOUR

Waits induced by blocking mechanisms in synchronous data parallelism significantly reduce the hardware utilisation during training, especially in non-homogeneous hardware configurations. As mentioned in Section 2.1, this can be resolved by simply removing the synchronizations barriers, as suggested by [4]. The algorithm the authors proposed is called DOWNPOUR, and is shown in Algorithm 5.

DOWNPOUR is an intuitively a very simple algorithm. As before, we have n different workers optimizing a central variable $\tilde{\theta}_t$ with the difference that every worker will *commit* a gradient after every mini-batch in an asynchronous fashion, and after a commit has been performed, the worker will synchronize with the parameter server by *pulling* the most recent parameterization of the central variable.

$$\theta_{t+1}^k = \theta_t^k - \eta_t \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}_t^k; \mathbf{y}^k) \quad (2.11)$$

$$\tilde{\theta}_{t+1} = \tilde{\theta}_t - \eta_t \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}_t^k; \mathbf{y}^k) \quad (2.12)$$

As mentioned in Chapter 1, due to the asynchronous optimization one does not have the issue with idle workers due to synchronizations mechanisms. Instead, there is a more severe problem related to the parameterizations of the workers. Since workers will commit gradients to the central variable in an asynchronous manner, other workers will commit gradients based on older parameterizations of the central variable. As a result, the gradients of these workers are *stale* as shown in Figure 1.4 and Figure 2.13. An additional, but related issue to parameter staleness, is that increasing the amount of asynchronous workers, increases the staleness in the system as shown in Figure 2.1.

Algorithm 5 Worker procedure of DOWNPOUR, which is a parallelized extension of SGD.

```

1: procedure DOWNPOURWORKER( $k$ )
2:    $\theta_0^k \leftarrow \tilde{\theta} \leftarrow \text{PULL}()$ 
3:    $t \leftarrow 0$ 
4:   while not converged do
5:      $\mathbf{x}, \mathbf{y} \leftarrow \text{FETCHNEXTMINIBATCH}()$ 
6:      $g \leftarrow -\eta_t \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}; \mathbf{y})$ 
7:     COMMIT( $g$ )
8:      $\theta_{t+1}^k \leftarrow \text{PULL}()$ 
9:      $t \leftarrow t + 1$ 
10:   end while
11: end procedure
```

Nevertheless, contrary to EASGD, DOWNPOUR is not designed with communication constraints in mind. As a result, increasing the number of workers in an already saturated environment will not reduce the training time since whenever a worker computes a gradient, it will commit the result to the parameter server. To illustrate this, imagine having several workers committing highly dimensional gradients to the parameter server. Since it takes some time to incorporate these gradients into the central variable, other workers will have to wait for their turn in the queue. To reduce the side-effects (waits) in this particular situation, [11] proposes to before committing the gradient, a worker should send a small control message to check if the queue is currently occupied, i.e., gradients of other workers are being incorporated into the central variable. If this is the case, do not commit the gradient, but fetch the next mini-batch, compute the gradient, and accumulate the computed gradient with the previously computed gradient, and send a new control message

An obvious problem with this method is that the possibility exists that after x number of retries, the central variable queue is still occupied by other workers. Following the naive procedure, the zealous method should keep accumulating new gradients. However, this is problematic at step

$x + 1$, when the queue is available. Imagine the possibility that other workers, and the central variable converged to a different minima. If our worker would commit the accumulated gradient in this particular situation, then the central variable would not be close to its minima as it was before. In order to reduce the amount of staleness in such a situation, one could introduce a limit to the amount of local steps that can be made in an attempt to reduce staleness as shown in Algorithm 6.

However, this technique introduces an additional hyperparameter. Several other approaches which handle this problem in a different way without introducing new hyperparameters, and additionally giving some intuition and insight into parameter staleness in Deep Learning are discussed in detail in Chapter 3 and Chapter 4.

Algorithm 6 Zealous worker procedure of DOWNPOUR, inspired by [11].

```

1: procedure ZEALOUSDOWNPOURWORKER( $k$ )
2:    $\theta_0^k \leftarrow \tilde{\theta} \leftarrow \text{PULL}()$ 
3:    $t \leftarrow 0$ 
4:   while not converged do
5:      $i \leftarrow 0$ 
6:      $g \leftarrow 0$ 
7:     while  $i < \lambda$  do
8:        $\mathbf{x}, \mathbf{y} \leftarrow \text{FETCHNEXTMINIBATCH}()$ 
9:        $g \leftarrow g - \eta_t \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}; \mathbf{y})$ 
10:      if not QUEUEOCCUPIED( ) or  $i = (\lambda - 1)$  then
11:        COMMIT( $g$ )
12:         $\theta_{t+1}^k \leftarrow \text{PULL}()$ 
13:         $i \leftarrow \lambda$                                  $\triangleright$  Satisfy stopping condition.
14:      end if
15:       $t \leftarrow t + 1$ 
16:       $i \leftarrow i + 1$ 
17:    end while
18:  end while
19: end procedure

```

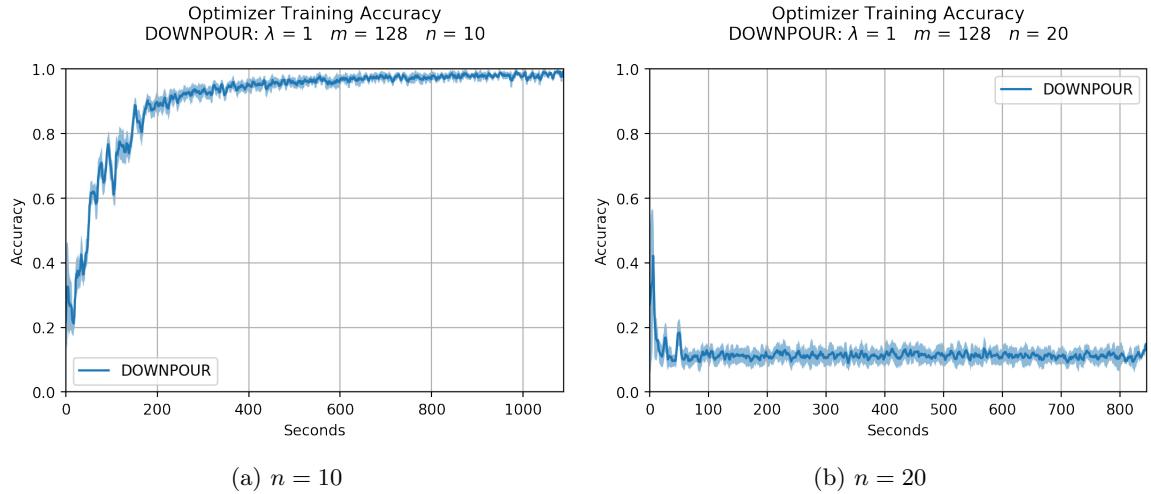


Figure 2.13: Divergence due to number ($n = 20$) of asynchronous workers in the optimization process [13] and not dealing with parameter staleness in a more intelligent way. Lowering the number workers ($n = 10$) causes the central variable to converge.

2.3.2 Dynamic SGD

The intrinsic reason that asynchronous methods like DOWNPOUR cannot guarantee convergence is that these method directly incorporate local updates (good) and stale updates (bad) in the central variable [8]. To combat this, one could incorporate staleness information of a worker to have a per-worker learning which decays proportionally stale gradients proportional to the staleness [8]. As a result, Equation 2.13 will better cope with parameter staleness in contrast to DOWNPOUR.

$$\tilde{\theta}_{t+1} = \tilde{\theta}_t - \frac{\eta_t}{\tau_i} \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}_t^k; \mathbf{y}_t^k) \quad (2.13)$$

However, we would like to note that in a homogeneous hardware configuration the gradients will be scaled down proportional to the number of asynchronous workers as $\mathbf{E}[\tau] = (n - 1)$ [13]. As a result, the expected scalar that will be applied to a worker gradient in a homogeneous setting is summarized in Equation 2.14.

$$\mathbf{E}\left[\frac{1}{\tau}\right] = \left(\frac{1}{n - 1}\right) \quad (2.14)$$

Furthermore, as we will see in Chapter 4, this technique scales down gradients with respect to the number of stale steps, while the issue of staleness mainly arises from the *distance between parameterizations*. For example, imagine a plateau where the workers have to pass through in order to converge to a minima. Since progress is slow in a plateau, thus the distance between the workers and the central variable will remain relatively small, it is non-sensical to scale down the gradients with respect to the number of stale steps, since these gradients provide good information because they are *close to the “old” central variable*.

Nevertheless, in order to capture the staleness information, some additional engineering is required to keep track of worker staleness since the worker themselves do not possess this information. Contrary to [8], we propose a different architecture to capture the staleness information. In our approach we exchange messages between the parameters server and workers. These messages can obtain additional information beside the parameterization of a worker or the central variable. During the initialization phase of the training procedure, we keep track of a *parameter server clock*. Basically, this clock holds the number of updates that have been applied to the central variable. Furthermore, in order to obtain the staleness of every worker, and apply the scaling factor described in Equation 2.13, the parameter server needs to keep track of the last *clock tick* when a worker i sent an update. It does so by maintaining a hashmap, or a different datastructure, to associate the last update clock tick with a particular worker. Furthermore, we scale the parameterizations on the parameter server since this will ensure the consistency of the staleness information because it is possible that an other worker might commit an update in the meantime (CAP theorem), and reduce the amount of blocking mechanisms. Nevertheless, for (very) large networks there might be an issue when insufficient processing power is allocated to preprocess the queue in an efficient manner.

An additional requirement of this architecture, is that the parameter server needs to update the worker datastructure whenever a worker *pulls* the central variable (because the worker will compute a gradient based on the parameterization of the central variable it just pulled, and this is required to compute the number of updates that happened in between). Then, whenever a worker commits a gradient to the central variable, the parameter server just needs to compute the difference between the last recorded clock time of this particular worker and the current value of the parameter server clock. Where the last recorded clock tick of a particular worker can be retrieved from the data structure mentioned above. The complete pseudo-code for these procedures can be found in Algorithm 7 and Algorithm 8. However, an alternative approach which does not require an additional data structure to be maintained by the parameter server, is to simply send the current clock tick together with the parameterization of the central variable to the worker during a *pull*. Similar to our previous technique, the scaling will still happen at the side of the parameter server in order to preserve staleness consistency. However, when the worker commits a gradient to the parameter server, the parameter server does not have to do additional searches in the worker dictionary since the value of the parameter server clock

at the time of the central variable pull is added to the commit message. As a result, the difference between the current clock value and the given clock value, thus the number of stale updates, can be derived easily. However, this requires the worker procedure to be slightly modified whereas the worker procedure described in Algorithm 7 does not contain any modifications to extract staleness information.

Algorithm 7 Worker procedure of DYNSGD.

```

1: procedure DYNSGDWORKER( $k$ )
2:    $\theta_0^k \leftarrow \tilde{\theta} \leftarrow \text{PULL}()$ 
3:    $t \leftarrow 0$ 
4:   while not converged do
5:      $i \leftarrow 0$ 
6:      $a \leftarrow 0$ 
7:     while  $i < \lambda$  do
8:        $\mathbf{x}, \mathbf{y} \leftarrow \text{FETCHNEXTMINIBATCH}()$ 
9:        $g \leftarrow \eta_t \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}; \mathbf{y})$ 
10:       $a \leftarrow a - g$ 
11:       $\theta_{t+1}^k \leftarrow \theta_t^k - g$ 
12:       $t \leftarrow t + 1$ 
13:       $i \leftarrow i + 1$ 
14:    end while
15:    COMMIT( $a$ )
16:     $\theta_{t+1}^k \leftarrow \text{PULL}()$ 
17:  end while
18: end procedure

```

Algorithm 8 Parameter server procedures of DYNSGD.

```

1: procedure DYNSGDPARAMETERSERVER
2:    $\tilde{c} \leftarrow 0$                                  $\triangleright$  Parameter server clock
3:    $\tilde{m} \leftarrow \tilde{c}$                           $\triangleright$  Initialize staleness datastructure
4:
5:   procedure HANDLEPULL( $k$ )                       $\triangleright k$  denotes the worker identifier
6:      $\tilde{m}[k] = \tilde{c}$ 
7:     return  $\tilde{\theta}_{\tilde{c}}$ 
8:   end procedure
9:
10:  procedure HANDLECOMMIT( $k, \Delta\theta^k$ )
11:     $\tau \leftarrow \tilde{c} - \tilde{m}[k]$ 
12:     $\tilde{\theta}_{\tilde{c}+1} = \tilde{\theta}_{\tilde{c}} + \frac{1}{\tau+1} \odot \Delta\theta^k$            $\triangleright +1$  to prevent division by 0
13:     $\tilde{c} \leftarrow \tilde{c} + 1$ 
14:  end procedure
15:
16: end procedure

```

Experimental Validation of Dynamic SGD

To validate the claims made by [8], we conducted several experiments on the MNIST [10] dataset with different hyperparameterizations. Using an equivalent experimental configuration as with DOWNPOUR in Section 2.3.1, and identical hyperparameterizations, we confirm that DYNSGD is able to cope with an increased amount of staleness due to asynchrony compared to DOWNPOUR as shown in Figure 2.14. Furthermore, due to the high communication frequency in Figure 2.14, a significant amount of communication took place which reduced the throughput of the workers. To combat this, we decreased the communication frequency (thus increasing λ) which allows for more local work to be

done before incorporating the *accumulated gradient* into the central variable, as shown in Figure 2.15. As expected, the training time was reduced significantly. However, due to the increased amount of local work, the gradients that were submitted to the parameter server became proportionally larger as well. Contrary to the expectations of the authors in [8], DYNSGD is not able to handle this amount of staleness. As a result, this is an indication that staleness is a more “deeper” problem, especially in the context of Deep Learning, which will be addressed in Chapter 4.

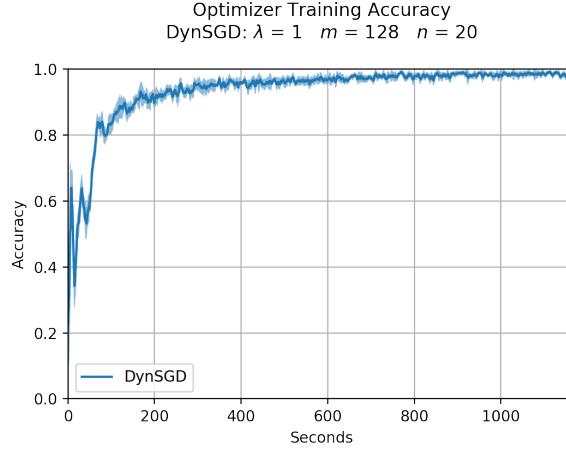


Figure 2.14: In this experiment we applied 20 asynchronous workers on equally shared 5 epochs worth of MNIST training data. Since this is an identical experimental configuration for which DOWNPOUR was used, this experiment verifies the claims of the authors that DYNSGD is able to handle staleness better.

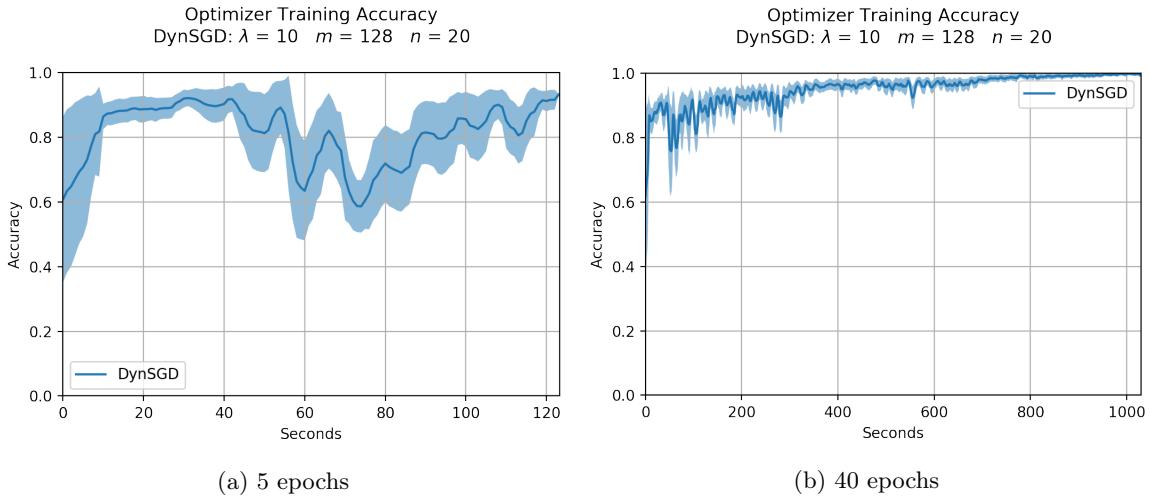


Figure 2.15: Since network-communication is a blocking factor in terms of CPU usage, we decided to test several iterations of local exploration before committing the accumulated gradient to the parameter server. In our first experiment (a), we copied the experimental configuration from Figure 2.14, with the difference that we decreased the communication frequency. Due to the decreased communication frequency, more local work was done, which had the result that the norm of the committed gradients to the parameter were larger. This had profound implications on convergence stability of the central variable, which only reached 90% training accuracy while using the same amount of data, but, spending less time. In order to have a “fair” comparison with Figure 2.14 in terms of training time, we increased the total amount of training data to 40 epochs, which resulted in a training accuracy of 99.7%, compared to 98.1% in Figure 2.14.

2.3.3 Asynchronous Elastic Averaging SGD

In Section 2.2.2 we discussed the synchronous version of EASGD. To reduce the effect of blocking mechanisms, which is profound in synchronous data parallelism, the authors proposed [17] an asynchronous extension of EASGD which is called AEASGD, or Asynchronous Elastic Averaging SGD. In essence, nothing changed for the worker procedure shown in Equation 2.15 with the exception of the blocking mechanism. However, contrary to other optimization algorithms discussed in this chapter (which commit first-order gradients), AEASGD is optimizing the central variable by incorporating the elastic differences of the workers in an asynchronous fashion, as shown in Equation 2.16 (for intuition, see Figure 2.6).

$$\theta_{t+1}^k = \theta_t^k - \eta_t \odot \nabla_\theta \mathcal{L}(\theta_t^k; \mathbf{x}_t^k; \mathbf{y}_t^k) - \eta_t \rho(\theta_t^k - \tilde{\theta}_t) \quad (2.15)$$

$$\tilde{\theta}_{t+1} = \tilde{\theta}_t + \eta_t \rho(\theta_t^k - \tilde{\theta}_t) \quad (2.16)$$

Since EASGD and its variants are designed with communication constraints in mind, and allow for more exploration of the local hypothesis space [17], we can significantly reduce the communication frequency, and thus further reducing the training time. However, as in any asynchronous optimizer, AEASGD mitigates staleness by pulling the most recent central variable in order to compute a more up-to-date elastic difference as shown in Algorithm 4.

Algorithm 9 Worker procedure of AEASGD. Note how the pull of the central variable occurs *before* the computation of the elastic difference.

```

1: procedure AEASGDWORKER( $k$ )
2:    $\theta_0^k \leftarrow \tilde{\theta} \leftarrow \text{PULL}()$ 
3:    $t \leftarrow 0$ 
4:   while not converged do
5:      $i \leftarrow 0$ 
6:     for  $i < \lambda$  do
7:        $\mathbf{x}, \mathbf{y} \leftarrow \text{FETCHNEXTMINIBATCH}()$ 
8:        $\theta_{t+1}^k \leftarrow \theta_t^k - \eta_t \odot \nabla_\theta \mathcal{L}(\theta_t^k; \mathbf{x}; \mathbf{y})$ 
9:        $i \leftarrow i + 1$ 
10:       $t \leftarrow t + 1$ 
11:    end for
12:     $\tilde{\theta} \leftarrow \text{PULL}()$ 
13:     $\mathcal{E} = \eta_t \rho(\theta_t^k - \tilde{\theta})$ 
14:     $\theta_{t+1}^k = \theta_t^k - \mathcal{E}$ 
15:    COMMIT( $\mathcal{E}$ )
16:     $t \leftarrow t + 1$ 
17:  end while
18: end procedure
```

What is interesting about AEASGD in our experiments, is that given identical hyperparameterizations as other optimizers in this chapter AEASGD is not able to get closer to a minima as can be seen in Figure 2.16. This could potentially be due to the equilibrium condition described in Section 2.2.2. Nevertheless, the claim the authors make that EASGD benefits from more exploration is validated in Figure 2.16 as well. Furthermore, during our experiments we have encountered several interesting observations on AEASGD. The first being when a high communication frequency is used, i.e., $\lambda = 1$, AEASGD shows sign of *divergence*. However, this effect is not always present, and our suspicions are that numerical error might be the root cause of this issue. In order to better understand the properties of the divergence in AEASGD, we conducted several simulations which show similar behaviour. To this date, we do not really have an idea why this is exactly happening, and is subject to further investigation. Furthermore, we would like to declare the following observations, as a result of these simulations:

- Asynchronous optimizers which commit gradients: central variable is “pushed” by workers towards a minima.
 - EASGD: central variable is “pulled” by the workers towards a minima as described in Section 2.2.2.
 - AEASGD: central variable acts as a “mean” of a distribution of workers, where the variance of the workers is controlled by ρ .

The final remarkable observation, shown in Figure 2.17, was that in almost all tests, the validation accuracy of the optimizer was consistently better than the training accuracy. According to us, this is because of the observation mentioned above, i.e., because AEASGD behaves like a moving distribution (more like a particle filter), it will tend to find a flat minima which generalizes better than the training accuracy of the central variable. However, this is pure speculation and is subject to further research.

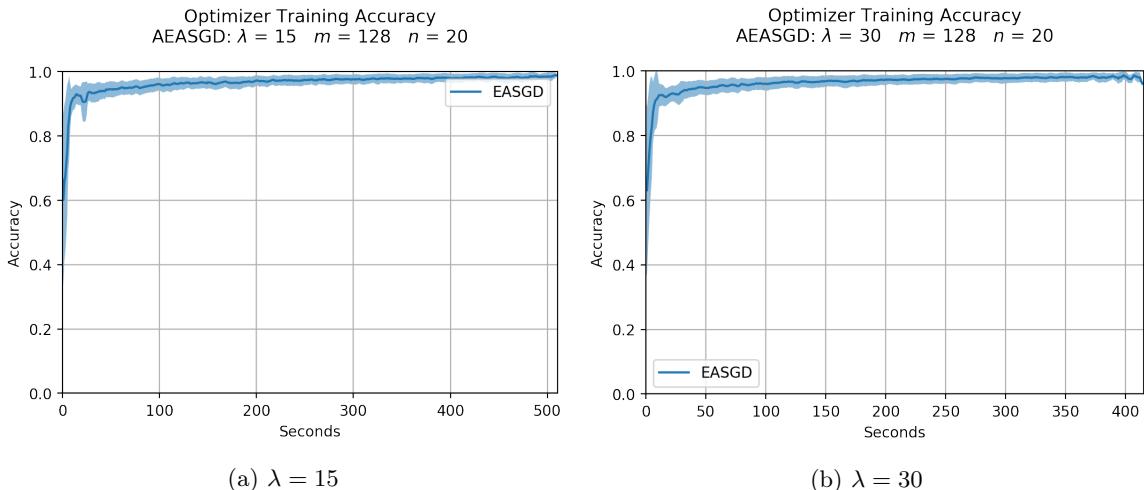


Figure 2.16: Accuracy plots which show the training accuracy of the central variable for different communication frequencies. These results validate the claims of the authors that in general, the central variable benefits from more exploration. However, we would argue that this is true only when sufficient amount of data is available. Furthermore, we would like to note that compared to other optimizers discussed in this Chapter, EASGD seems to converge slowly close to a minima, this might be due to the equilibrium condition described in Section 2.2.2. However, further evidence is required to confirm this.

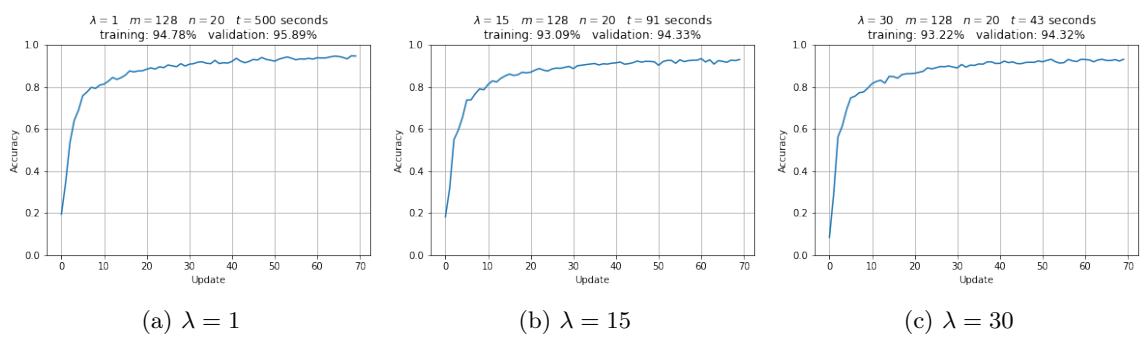


Figure 2.17: Results of a small experiment to show the consistently higher validation accuracy compared to the training accuracy in AEASGD for different values of λ .

2.4 Hybrids

2.4.1 Stale Synchronous Optimizers

Other approaches to deal with staleness in asynchronous optimization have been suggested in the past years [3, 6]. These approaches, typically called Stale Synchronous Optimizers, are based on the concept that staleness needs to be limited within a system. In order to limit the amount of staleness within a system, asynchronous workers can only be a certain number of steps ahead compared to the slowest worker. Hence the name stale synchronous optimizers, since they allow for a limited amount of staleness, and if this particular condition is not met by some workers, then these workers will have to wait until the slower workers caught up. However, as mentioned before, we know from [13] that the expected staleness in a homogeneous system is $E(\tau) = (n - 1)$. As a result, adding more workers to the problem in stale synchronous optimizers is non-sensical, since adding more workers to the problem implicitly increases the amount of staleness. As a result, the limit which is specified will have to be changed whenever a couple of workers are added to the optimization process. For this reason, we will not consider stale synchronous optimizers in this thesis.

Chapter 3

Accumulated Gradient Normalization

This chapter addresses the first contribution of this thesis, which is *accumulated gradient normalization*, or AGN in short. We start by laying out the concept and intuition behind accumulated gradient normalization, and show why AGN provides the central variable with more efficient updates. Finally, to show our claims, we provide several experiments where we compare AGN with different distributed optimizers such as DOWNPOUR [4], AEASGD [17], and DYNNSGD [8].

3.1 Concept and intuition

The main issue with DOWNPOUR [4] is the requirement of constant communication with the parameter server after every gradient computation. Furthermore, as the number of parallel workers increases, DOWNPOUR fails to converge due to the amount of *implicit momentum* [13], as shown in Figure 2.13. To reduce the amount of communication with the parameter server, one could take ideas from EASGD [17], and perform several iterations of local exploration before committing the gradients to the parameter server. However, in the case of algorithms like DOWNPOUR, that is, where gradients are committed to the parameter server in an asynchronous fashion, more local exploration results in proportionally larger gradients, and as a result, complicate the staleness and the implicit momentum problem even further as will be addressed in Chapter 4. To intuitively show why this is an issue, let us consider Figure 3.1. In a regular DOWNPOUR setting, first-order gradients such as in Subfigure (a) are committed to the parameter server. However, when an algorithm allows for a certain amount of local exploration, such as Algorithm 6, the gradient that is committed to the parameter server is an *accumulated gradient* as shown in Subfigure (b).

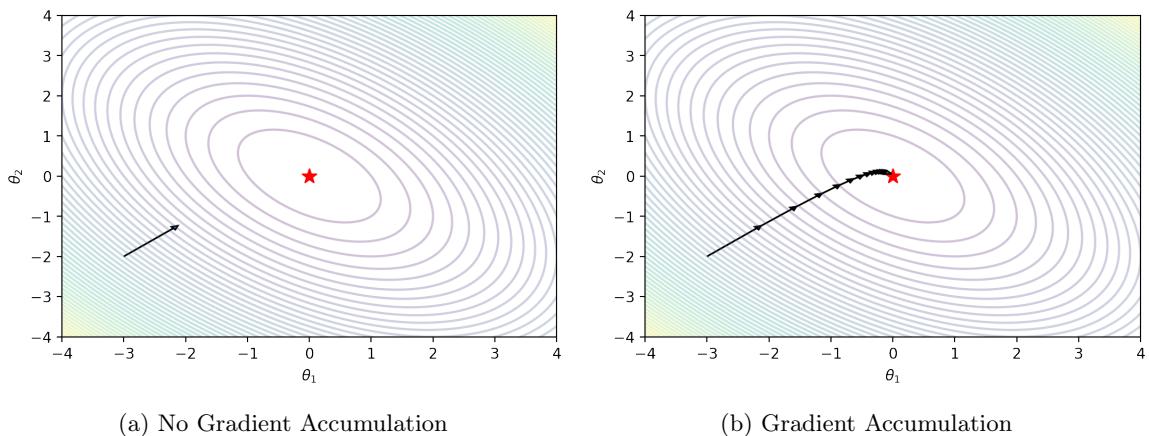


Figure 3.1: This figure shows the difference between regular first-order gradients (a), and accumulated gradients (b). We observe that *accumulated gradients are proportionally larger to the number of exploration steps*. However, they do provide a better direction compared to first-order gradients.

Now, imagine two asynchronous environments, in the first no gradient accumulation is performed, and in the last gradient accumulation takes place. In the environment where no gradient accumulation is performed, as in regular DOWNPOUR, first-order gradients are committed to the parameter server. However, as we have seen in Chapter 2, and in particular Figure 2.13, we saw that DOWNPOUR diverges when the number of asynchronous workers is too high due to the amount of implicit momentum [13]. As a result, careful tuning is required when no adaptive methods are applied. Nevertheless, given the fact that DOWNPOUR converges with $n = 10$ workers in Figure 2.13 and our knowledge about gradient accumulation, i.e., *accumulated gradients that are committed are proportional to the number of exploration steps for every worker, and provide better directions to a minimum*, we would expect that for some amount of local exploration while using the same hyperparameterization (with the exception of local exploration steps λ) DOWNPOUR would diverge again due to the magnitude of the accumulated gradients. This behaviour is illustrated in Figure 3.2.

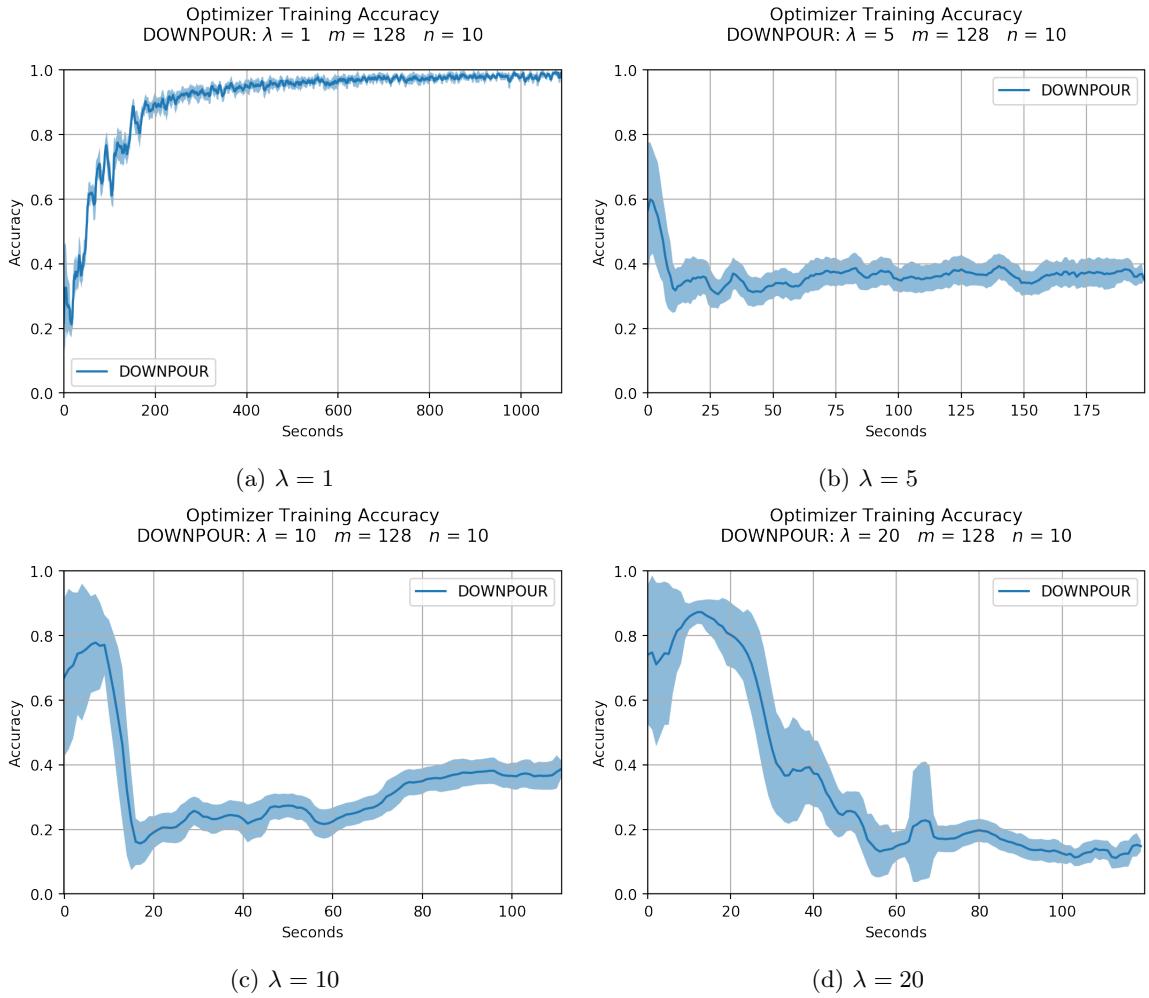


Figure 3.2: Illustration of divergence due to gradient accumulation in DOWNPOUR. In Figure 2.13, we say that for $n = 10$ DOWNPOUR converged to a good solution. In order to reduce the training time, we decrease the communication frequency (increasing λ). However, due to the larger gradients that are committed to the parameter server, which increases the amount of implicit momentum, the central variable is not able to converge as before.

To reduce the magnitude of the accumulated gradients, and thereby reducing the amount of implicit momentum, while at the same time preserving the better direction that has been provided due to the amount of local exploration, we propose to normalize (average) the accumulated gradient with

the amount of local steps that have been performed by the workers (λ), shown in Equation 3.1¹. We call this technique of normalizing the accumulated gradient *Accumulated Gradient Normalization* or AGN. An initial critique of this technique would be that by normalizing the accumulated gradient, AGN would in effect be undoing the work that has been done by a single worker. This seems at first a valid criticism, however, one needs to take into account that AGN is actually using the worker exploration steps to compute a better gradient based on first-order gradients.

$$\Delta\theta = -\frac{1}{\lambda} \sum_{i=0}^{\lambda} \eta_t \frac{1}{m} \sum_{j=0}^{m-1} \nabla_{\theta} \mathcal{L}(\theta_i; x_{ij}; y_{ij}) \quad (3.1)$$

Since AGN is using local steps to compute a better gradient compared to first order gradients, it can also be used under communication constraints like EASGD since less communication with the parameter server is required. In Figure 3.3, we show how a *Normalized Accumulated Gradient* is obtained and applied to the central variable using Equation 3.1 as described in Algorithm 10.

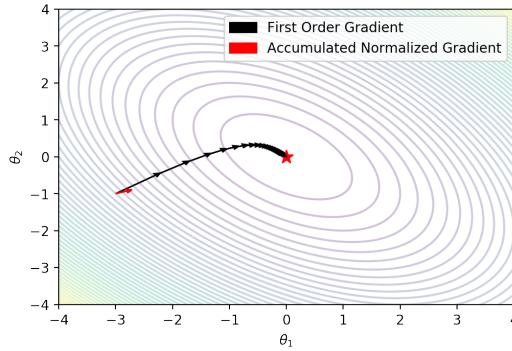


Figure 3.3: After pulling the most recent parameterization of the central variable from the parameter server, the worker starts accumulating λ first order gradients, and applies those gradients locally to explore the surrounding error space. Finally, after λ exploration steps have been performed, the accumulated is normalized w.r.t. λ and send to the parameter server.

Algorithm 10 Worker procedure of AGN.

```

1: procedure AGNWORKER( $k$ )
2:    $\theta_0^k \leftarrow \tilde{\theta} \leftarrow \text{PULL}()$ 
3:    $t \leftarrow 0$ 
4:   while not converged do
5:      $i \leftarrow 0$ 
6:      $a \leftarrow 0$ 
7:     while  $i < \lambda$  do
8:        $\mathbf{x}, \mathbf{y} \leftarrow \text{FETCHNEXTMINIBATCH}()$ 
9:        $g \leftarrow -\eta_t \odot \nabla_{\theta} \mathcal{L}(\theta_t^k; \mathbf{x}; \mathbf{y})$ 
10:       $a \leftarrow a + g$ 
11:       $\theta_{t+1}^k = \theta_t^k + g$ 
12:       $i \leftarrow i + 1$ 
13:       $t \leftarrow t + 1$ 
14:    end while
15:     $a \leftarrow \frac{a}{\lambda}$                                  $\triangleright$  Accumulated Gradient Normalization step.
16:     $\text{COMMIT}(a)$ 
17:     $\theta_t^k \leftarrow \text{PULL}()$ 
18:  end while
19: end procedure

```

¹Note if $\lambda = 1$, AGN is in essence equivalent to DOWPOUR.

An interesting thought experiment would be what would happen in the case that the workers would not communicate with the parameter server at all, that is, $\lambda = \infty$. How would the normalized accumulated gradients look like in such a situation, described by Equation 3.2?

$$\lim_{\lambda \rightarrow \infty} -\frac{\sum_{i=0}^{\lambda} \eta_t \frac{1}{m} \sum_{j=0}^{m-1} \nabla_{\theta} \mathcal{L}(\theta_i; x_{ij}; y_{ij})}{\lambda} \quad (3.2)$$

In order to completely understand how the worker deltas would look like after $\lambda = \infty$ steps, one first needs to understand the individual components of Equation 3.2. The most inner component, $\eta_t \frac{1}{m} \sum_{j=0}^{m-1} \nabla_{\theta} \mathcal{L}(\theta_i; x_{ij}; y_{ij})$, is just the computation of a mini-batch using $m - 1$ samples, where index i denotes the current step in the gradient accumulation. Please note that a mini-batch can differ for different values of i as training samples are randomly retrieved from the dataset. After computing the gradient based on the mini-batch, the local model will be updated as $\theta_{i+1} = \theta_i - \eta_t \frac{1}{m} \sum_{j=0}^{m-1} \nabla_{\theta} \mathcal{L}(\theta_i; x_{ij}; y_{ij})$. This process goes on for λ steps, while at the end, the accumulated is normalized with respect to λ .

Let us assume we have a smooth convex error space, or a smooth non-convex error space with at least a single minima. Due to the existence of a minima in both cases, first order gradients will eventually converge to, or in the neighbourhood of said minima. Furthermore, we make the assumption that the hyperparameterization during the training procedure will not change. For instance, no learning rate decay after x number of steps. Under these assumptions, it is trivial to realize that applying gradient descent for ∞ steps will cause the parameterization to converge in a minima. Of course, given that the hyperparameterization, and the data allow for convergence to occur. As a result, the term $\sum_{i=0}^{\lambda} \eta_t \frac{1}{m} \sum_{j=0}^{m-1} \nabla_{\theta} \mathcal{L}(\theta_i; x_{ij}; y_{ij})$ is finite, even after applying ∞ steps of mini-batch updates. To simplify our problem, let us denote \vec{c} as the *finite* result of the top term in Equation 3.2 for $\lambda = \infty$. Therefore, we can write Equation 3.2 as Equation 3.3. Furthermore, since \vec{c} is finite, Equation 3.3 can be treated as an instance of $\frac{1}{\infty}$, which approaches 0. Subsequently, Equation 3.3 is 0 in the limit to infinity.

$$\lim_{\lambda \rightarrow \infty} -\frac{\vec{c}}{\lambda} = \vec{0} \quad (3.3)$$

This implies that due to the infinitely low communication frequency, the normalized accumulated gradients will basically be $\vec{0}$. However, what is interesting is that the normalized accumulated gradients directly point towards a minima due to the infinite amount of exploration steps that have been performed. Subsequently, one can view a normalized accumulated gradient when $\lambda = \infty$ as a point, but with a direction. Therefore, if we would allow for infinite steps until convergence, the path the central variable would traverse is a straight line towards the minima, as shown in Figure 3.4.

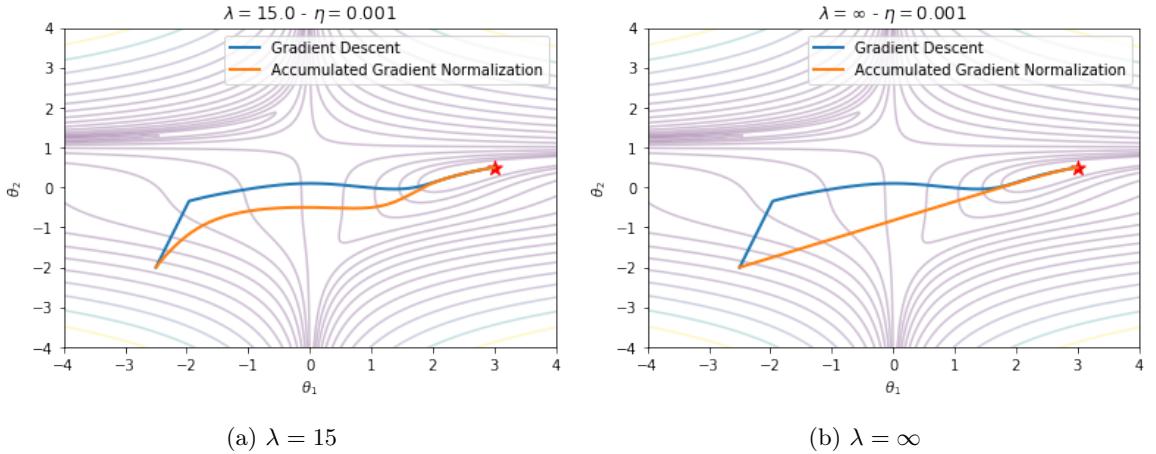


Figure 3.4: AGN for different values of λ . This small experiment shows that when $\lambda = \infty$, the path the central variable traverses is equal to a straight line towards the minima.

The thought experiments described above helps us in several ways if we make several additional assumptions. The first assumption assumes that normalized accumulated gradients with $\lambda = \infty$ can be computed immediately, that is, without a delay. This is of course an unrealistic assumption. However, one needs to consider realistic communication constraints. Given a certain network throughput, what is the amount of local communication that needs to be performed in order for a parameter commit to be “worth it”. As mentioned above, $\lambda = \infty$ is not a very good solution since the normalized accumulated gradient will converge to $\vec{0}$ in the limit. Nevertheless, if the normalized accumulated gradient could be computed immediately, as we assumed, the central variable would traverse the shortest path to a minima, in contrast to first order gradients. Of course, this is not a realistic assumption. Furthermore, this issue is quite similar to *stochastic gradient descent* vs. *mini-batch gradient descent*, since in AGN we also have to make the decision between more frequent parameter updates, and more “local” iterations to compute a better gradient, where better in the case of mini-batch gradient descent means less-noisy.

In most settings, the size of a mini-batch is determined empirically, and is dependent on the noise of the gradients. Furthermore, when using mini-batch gradient descent, a trade-off is made between more frequent parameter updates, i.e., a smaller mini-batch, or more robust and consistent gradients by increasing the size of a mini-batch which results in a more accurate approximation of the first order curvature. This is similar to our situation. Yet, in mini-batch gradient descent you are basically trying to estimate a hyperparameter based on several unknowns, i.e., convergence based on error space and noise of individual gradients. However, AGN is balancing the amount of local computation to produce a better gradient, with the throughput of the network, which is a known variable. For instance, imagine a hypothetical communication infrastructure which is able to apply the commits of the workers directly into the workers with no delay. In this situation, one could apply DOWNPOUR. However, remember from Figure 3.2 that DOWNPOUR does not handle an increased amount of asynchronous parallelism ($n = 20$). As a result, even in an ideal situation DOWNPOUR will not be able to converge due to the amount of implicit momentum.

Nevertheless, the situation in AGN is different as will become apparent in Section 3.2. Contrary to DOWNPOUR, AGN does not commit first order gradients to the parameter server, but rather a normalized sequence of first order gradients which result in better directions towards a minima, as discussed above. Because of this, workers will produce a better gradient in terms of direction with respect to first order gradients using the amount of local computation available to them to handle the communication constraints. Therefore, AGN worker deltas will therefore point more or less in the same direction, and are normalized with respect to the number of exploration steps, which reduces the amount of implicit momentum since first order gradients are not applied to the central variable.

3.2 Experimental Validation

In this Section we evaluate AGN against different distributed optimization algorithms. As before, MNIST [10] is used as a benchmark dataset, and all optimizers use the model described in Appendix .1 with identical parameterization of the weights. Furthermore, we will set the mini-batch size to $m = 128$ in all optimizers, and use *40 epochs* worth of training data that will be equally distributed over all n workers with the exception for DOWNPOUR, which will only use 5 epochs, since DOWNPOUR can not handle accumulated gradients. Our computing infrastructure consists a relatively small cluster of *15 nodes* with a *10Gbps interconnect*, most of them in the same rack, each having 2 Intel® Xeon® CPU E5-2650 v2 @ 2.60GHz CPU’s, where every CPU has 8 cores and 2 threads. No GPU’s are used during training, and no learning rate decay is applied.

Our initial experiment, shown in Figure 3.5, shows the training accuracy of AGN, AEASGD, and DYNNSGD over time. In this experiment, we use a near-optimal hyperparameterization for all optimizers to ensure convergence. Furthermore, we also report the validation accuracy of every trained model based on the validation set that is provided by MNIST. Looking at Figure 3.5, we observe a significant increase in training performance for AGN, both in training accuracy, and in training time when compared to current state-of-the-art algorithms such as AEASGD and DYNNSGD. Furthermore, the claim we made in Chapter 2

that DYNSGD scales the gradients down with respect to staleness τ , which in effect is $\mathbf{E}[\tau] = n - 1$, and thereby not addressing the staleness problem since the expected scaling factor is $(n - 1)^{-1}$ and not the distance between the parameterization of the central variable, and the parameterization of the worker, can be derived from Figure 3.5 and Figure 3.5 (b). Since gradient accumulation is performed in both figures, i.e., $\lambda > 1$, we can tell that DYNSGD has difficulties converging when λ is larger because the gradients that are committed to the parameter server are proportionally larger, as is the case in Figure 3.5 (a). Furthermore, due to the relatively high communication frequency ($\lambda = 10$), DYNSGD will take longer to process all data since more communication with the parameter server is required. However, in the case of Figure 3.5 (a), we see that DYNSGD is equally *temporal efficient* to AEASGD, since DYNSGD takes the *same* amount of time to reach the final training accuracy of AEASGD.

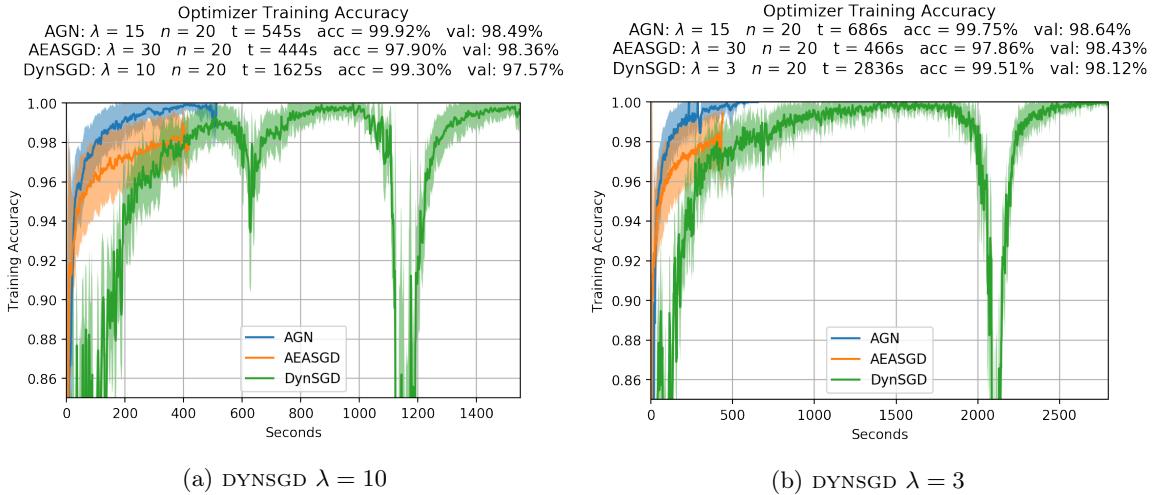


Figure 3.5: In this experiment we train all optimizers on 40 epochs worth of data with a mini-batch of $m = 128$. Due to staleness-handling method of DYNSGD, the optimizer is not able to handle accumulated gradients which results in non-stale accumulated gradients being incorporated directly into the central variable with the disadvantage that other workers are even more stale in terms of parameter distance. Which is the root cause of this divergent behaviour. In Subfigure (b) we reduce the amount of local exploration steps, which in turn reduces the length of the accumulated gradient. Therefore causing other workers to be less stale, and consequently reducing the divergent effects we observed in Subfigure (a). Furthermore, in Chapter 2, we made that claim that AEASGD will not be able to get close to a minima due to the existence of an equilibrium. This behaviour is confirmed as AEASGD is not able to surpass AGN in terms of training accuracy. However, as mentioned in Chapter 2, the validation accuracy of AEASGD is usually higher than the training accuracy. This results further strengthens our suggestion that due to the presence of the equilibrium condition, AEASGD will not overfit the data since the optimizer will not be able to come “close” to a minima.

Furthermore, let us consider the case for DYNSGD when $\lambda = 15$. In this situation, DYNSGD closely resembles AGN since, as mentioned above, the worker deltas in DYNSGD are scaled down with respect to the staleness τ , which results in an expected scaling factor of $(n - 1)^{-1}$. Whereas in AGN, the deltas are always scaled (on worker level) with respect to the communication frequency λ . As a result, the scaling of worker deltas will be *on average* similar to AGN. This begs the question, why do we see such divergent, and noisy behaviour in Figure 3.5, and especially in Figure 3.6? The reason for the lies in the combined effect how DYNSGD deals with staleness, and due to the presence of gradient accumulation. To illustrate this behaviour, consider the case when a DYNSGD worker w is committing an *accumulated gradient*, with staleness 0, i.e., $\tau_w = 0$. In this situation, the parameter server will scale down the gradient with respect to $(0 + 1)^{-1}$, which is 1. As a result, the accumulated gradient that worker w sent to the parameter server will not be scaled down. Of course, this is perfectly reasonable, since there the accumulated gradient that worker w sent was not stale. However, what happens when the other $n - 1$ workers have to commit a gradient? Remember from Equation 2.13 that DYNSGD scales

the gradients down with respect to the number of *stale steps* τ_w . As we will show in Chapter 4, this method is rather naive, because what really matters is the *distance* between updates as suggested in Hypothesis 1. Since the delta worker w committed was not stale, the full accumulated gradient was incorporated into the central variable, causing it to shift with the full magnitude of the delta. Since the length of an accumulated gradient is proportional to the number of local exploration steps, the deltas of other workers will be proportionally more stale in terms of distance, and therefore causing the divergent behaviour shown in Figure 3.5 and Figure 3.6.

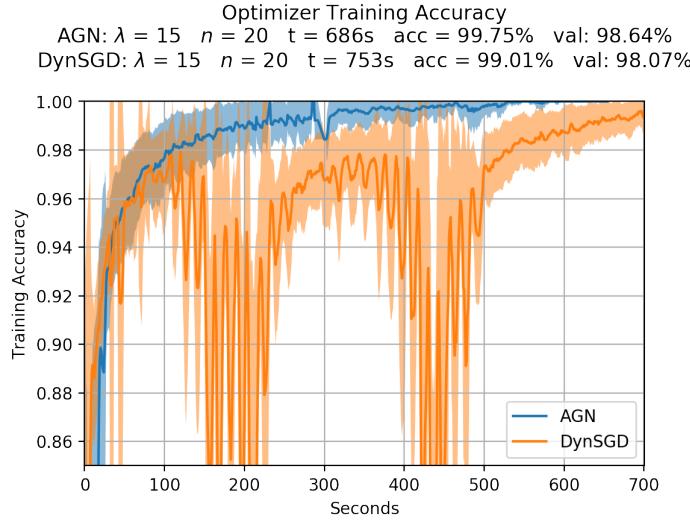


Figure 3.6: Shows the similarity between AGN and DYNSGD by using identical communication frequencies in both optimizers. As a result, DYNSGD worker deltas are *expected* to be similar to AGN deltas since $E[\tau] = n - 1$. However, due to DYNSGD's staleness handling mechanism, which is basically scaling the deltas down by the number of *stale steps*, DYNSGD is not able to prevent divergent behaviour. Contrary to DYNSGD, AGN is not staleness aware. However, AGN does provide more stable and direct gradient updates since the optimizer normalizes the accumulated gradients proportionally to the amount of local exploration.

An additional observation from Figure 3.5 and Figure 3.6, one we made before in Chapter 2, is that the validation accuracy of AEASGD is in accordance with the training accuracy, meaning, the optimizer does not seem to overfit. The reason for this lies, according to us, with the EASGD equilibrium condition, described in Section 2.2.2. The quilibrium condition will prevent the central variable moving too close to a minima, preventing the central variable from overfitting. Since AEASGD and AGN are clearly outperforming DYNSGD in terms of training time and validation accuracy for this particular experiment, the following experiments will only consider AEASGD and AGN. To evaluate the performance of these optimizers under different conditions, we will conduct several experiments using the same hyperparameterizations we mentioned at the start of this Section, which will evaluate the performance of AGN and AEASGD with respect to different (distributed) hyperparameters, i.e., number of workers, and communication frequency. As before, no learning rate decay is applied.

Initial observations from our results, summarized in Table 3.2, indicate that AGN performs better in terms of validation accuracy when a low communication frequency is used (which is a design requirement of (A)EASGD), and a high number of asynchronous workers are deployed ($n \geq 20$). However, looking at the overall training accuracy of both optimizers, we observe that AGN is significantly outperforming AEASGD in all configurations of the distributed hyperparameters. The reason for this might be due to the equilibrium condition of EASGD described in Chapter 2. Furthermore, increasing the number of asynchronous workers results in a significant drop in both training, and validation accuracy in AEASGD.

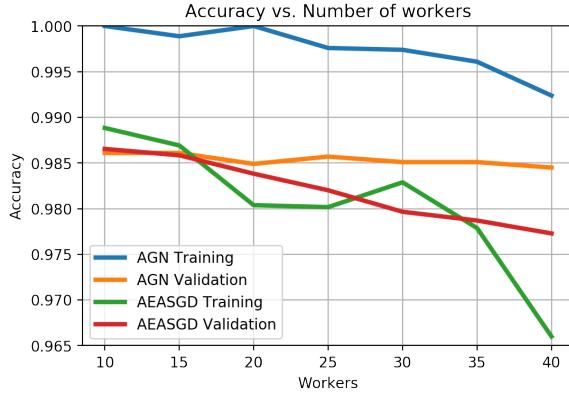


Figure 3.7: Decline of the training accuracy of both AGN and AEASGD as the number of asynchronous workers increases. From these experiments, we observe that AGN is more robust to an increased amount of asynchrony as the training accuracy only starts to decrease from $n = 25$ workers, while the validation accuracy remains stable even with 40 workers.

Contrary to AEASGD, AGN is able to cope more effectively with an increased amount of parallelism, as its training accuracy only starts to decline from $n = 25$ asynchronous workers, while the validation accuracy is barely fluctuating (because AGN is still slightly overfitting), as shown in Figure 3.7. An obvious follow-up question to this result would be to question the fact whether increasing the amount of asynchronous workers really improves the temporal efficiency optimizer, i.e., the amount of time it takes to reach a certain training accuracy. In fact, it does reduce the training time to reach a certain training accuracy, as shown in Figure 3.8. However, several factors have to be taken into account.

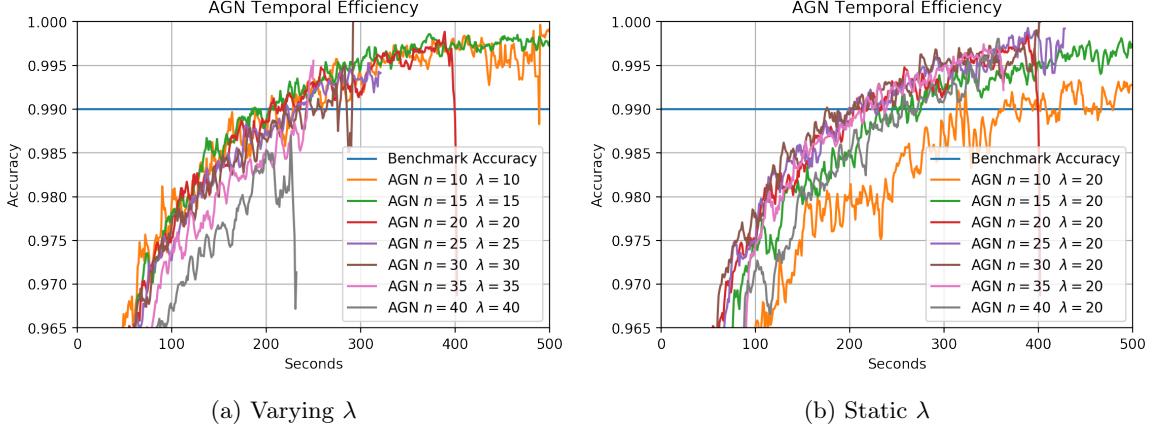


Figure 3.8: Training accuracy plots for different configurations of the distributed hyperparameters. In the case of a varying λ with respect to the number of workers (to minimize the noise of the commits), we observe that optimizers with a higher communication frequency (small λ), is actually benefitting from the more frequent updates with the parameter server. However, as the number of asynchronous workers grows, a low communication frequency increases the noise in the commits due to parameter staleness. Furthermore, if the λ is too large, less frequent parameter server updates occur, which results in a slower convergence rate since more time is spent locally. As a result, a balance is required similar to determining the size of a mini-batch.

The first being an increased amount of staleness that is inserted into the system as the number of asynchronous workers increase. This effect is difficult to mitigate. Previous approaches [8] propose to scale gradient commits down proportionally to the number of stale steps. However, as previously shown, this is not an effective solution since accumulating gradients locally, is in effect making the gradients larger, and as a result, committing accumulated gradients increases the *distance* between

the central variable and other workers². The second and final issue is the balance between updating the central variable with a certain frequency, and the amount of local work to effectively reduce the training time due to high communication costs. In effect, this resembles the situation usually one has when selecting a mini-batch size m , i.e., do we allow for more frequent parameter updates (small m), or do we compute a less noisy first order gradient by increasing m , thereby reducing the frequency of parameter updates and the convergence of a model? To show that this is the case, let us consider Figure 3.10 and Figure 3.11. In Figure 3.10, we evaluate varying values of λ for a specific number of asynchronous workers n . In all cases, we observe configurations using $\lambda = 40$ usually have the slowest convergence rate with respect to other configurations with higher communication frequencies. Furthermore, an additional interesting observation, one which is in accordance with the theory we discuss in Chapter 4, is the fact that high communication frequencies and a larger number of asynchronous workers causes divergent behaviour due to parameter staleness.

Nevertheless, what is really interesting, is why configurations with low communication frequencies actually do converge, in contrast to configurations with high communication frequencies (with respect to the number of workers). Since our definition of staleness relates to the distance between the *current* parameterization of a worker, and the *current* parameterization of the central variable. One can imagine that increasing the number of asynchronous workers, effectively increases the *distance* between the parameterizations of the workers and the *current* central variable due to the queueing model discussed before, i.e., worker deltas are incorporated in the central variable in a queueing fashion. Yet, parameter staleness still does not explain why configurations with low communication frequencies converge, as opposed to configurations with higher communication frequencies. The question begs, is convergence guaranteed due to the amount of local exploration, thus providing the parameter server with a better “direction”, as shown in Figure 3.3. Or, is it due to limit condition described in Equation 3.3, which eventually scales the worker deltas down to 0 as the communication frequency decreases (λ increases)? This is a rather difficult question to answer. However, we are definitely not dealing with the limit condition since one expects this to be the case when AGN approaches a minima, or when λ approaches infinity. As a result, the convergence property of AGN is largely dedicated to the amount of local exploration, which in turn provides the central variable with a more stable, and direct gradient towards a minima. Alternatively, consider the case when no normalization of the accumulated gradient proportional to the amount of local exploration takes place, and thereby pushing large gradients to the parameter server. We showed that this approached has the tendency to diverge despite the fact it provides a better direction, as is the case in AGN. However, due to the magnitude of the worker delta, a lot of staleness (in terms of distance) is induced causing other workers to commit (also large) gradients in an already very stale central variable, which in turn causes the divergence we observe in a non-normalized setting.

Now the argument on convergence rates of AGN with respect to the communication frequency has been made, let us focus on the influence on the amount of asynchronous workers. For this, we turn to Figure 3.11, which shows the accuracy plots for several training configurations with a static communication frequency, and a varying number of asynchronous workers. Initial observations indicate that increasing the number of workers with respect to a static communication frequency, speeds up the training procedure. Of course, given the fact that the right communication frequency has been selected in order to ensure convergence. Again, as stated before, lower communication frequencies yield more stable gradients in the presence of more asynchronous workers. However, from Figure 3.10 and Figure 3.11, can be deduced that configurations with a lower number of asynchronous workers, and with a high communication frequency actually reach a certain benchmark accuracy faster. Therefore, why spent double the amount of computational resources to achieve the same results?

Of course, in such a case one is able to process a lot more training data in a shorter amount of time. However, this is detrimental to the accuracy of the central variable, as more staleness is induced when a larger number of workers is used. Nevertheless, we make this observation for the MNIST [10] dataset. However, if we would use a more challenging dataset such as CIFAR-10(0) or ImageNet,

²Which is our definition of staleness, see Chapter 4

one might actually benefit from an increased amount of parallelism (workers) due to the relatively small parameter updates, which reduces the amount of staleness that workers induce into the central variable.

To compare AGN against AEASGD, we could take our *temporal efficiency* metric which is described in Chapter 1. However, since it basically assumes some *benchmark accuracy* (see Figure 3.8), the metric might be biased because it requires a person to define a benchmark accuracy. To prevent this issue, we redefine *temporal efficiency* in terms of the surface described by a training metric (be it training accuracy, or training loss). This means that for some optimizer a , we have a function $f_a(t)$ which describes the performance of a model at time t , e.g., $f_a(t)$ describes the training accuracy of the model at time t . If we integrate over t , we obtain a surface representing the performance of a model over time. If we would do this for an other optimizer b , and divide the surface of optimizer a by the performance surface of optimizer b , we get a ratio which describes how optimizer a is performing compared to optimizer b . If this ratio is larger than 1, it means that optimizer a is outperforming optimizer b , else, it is the other way around (unless the surfaces are equal of course). However, in order to compute a *fair* surface area, we have to limit the computation to the minimum m of both training times. This is done to prevent that optimizers with a longer training time have a significant advantage, since they have more time to produce a better model. To summarize, we define the temporal efficiency \mathcal{E} of two optimizers a and b as the ratio of their performance surface, as stated in Equation 3.4. Using this new definition of *temporal efficiency*, we can make a more qualitative judgement which optimizer is performing better in different scenarios.

$$\mathcal{E}(a, b) = \frac{\int_0^m f_a(t) dt}{\int_0^m f_b(t) dt} \quad (3.4)$$

Finally, we apply the new definition of *temporal efficiency* to compare AGN against AEASGD and summarize the results in Table 3.1. However, from Figure 3.12 and Figure 3.13, we make the rather strange observation that increasing the amount of asynchrony results in a deterioration of the training accuracy (which is expected since more staleness is induced). However, the rather unexpected property is that increasing the amount of asynchronous workers results in an early *flattening* of the training accuracy. Again, this is due to the equilibrium condition described earlier. Since we increase the amount of asynchrony in the optimization procedure, workers will reach the equilibrium condition faster because the elastic difference is computed based on the most recent parameterization of the central variable. Meaning, as soon as AEASGD is done computing λ iterations, the central variable is pulled to the worker where the elastic difference is computed based on the recently pulled central variable, which is very stale (again, using our definition of staleness) due to the low communication frequency and high number of asynchronous workers. As a result, AEASGD has troubles reaching a better training accuracy.

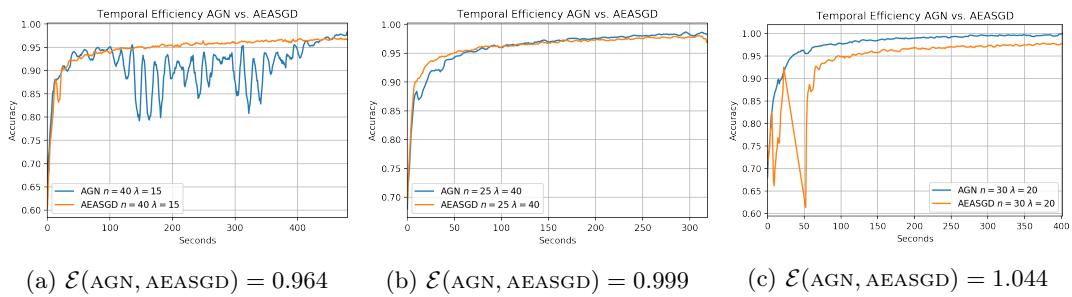


Figure 3.9: Several accuracy plots of AGN and AEASGD. All subfigures show the computed *temporal efficiency* of AGN, which were obtained by applying Equation 3.4.

n	λ	Temporal Efficiency AGN
10	10	1.009
10	15	1.012
10	20	1.003
10	25	0.999
10	30	0.988
10	35	0.990
10	40	0.990
15	10	1.008
15	15	1.015
15	20	1.009
15	25	1.005
15	30	1.000
15	35	0.997
15	40	0.994
20	10	0.983
20	15	1.018
20	20	1.022
20	25	1.009
20	30	1.007
20	35	1.002
20	40	0.997
25	10	0.954
25	15	1.021
25	20	1.017
25	25	1.014
25	30	1.008
25	35	1.003
25	40	0.999
30	10	0.926
30	15	1.017
30	20	1.045
30	25	1.015
30	30	1.012
30	35	1.005
30	40	1.005
35	10	0.881
35	15	0.997
35	20	1.017
35	25	1.020
35	30	1.016
35	35	1.027
35	40	1.012
40	10	1.044
40	15	0.964
40	20	1.027
40	25	1.025
40	30	0.997
40	35	1.009
40	40	1.009

Table 3.1: Temporal efficiency of AGN and AEASGD compared to different distributed hyperparameters. Using this information, we can deduce that AGN is outperforming AEASGD in 69.73% of the cases, which is significantly better. Furthermore, this statistic includes cases which are known where AGN is performing worse, i.e., small amount of asynchrony, low communication frequency, and high amount of asynchrony, and high communication frequency.

n	λ	AGN t	AGN Acc.	AGN Val.	AEASGD t	AEASGD Acc.	AEASGD Val.
10	10	1066.08s	100.00%	98.43%	953.61s	99.22%	98.64%
10	15	864.01s	99.53%	98.54%	846.86s	99.38%	98.69%
10	20	886.34s	99.38%	98.61%	804.07s	98.91%	98.67%
10	25	855.51s	98.75%	98.52%	784.46s	98.91%	98.74%
10	30	886.01s	99.22%	98.50%	930.73s	98.91%	98.64%
10	35	850.87s	98.44%	98.42%	798.74s	99.22%	98.62%
10	40	845.21s	98.75%	98.43%	791.04s	97.66%	98.58%
15	10	979.72s	99.67%	98.61%	813.32s	99.11%	98.67%
15	15	638.13s	99.89%	98.58%	721.47s	98.66%	98.65%
15	20	562.72s	99.89%	98.49%	564.92s	99.00%	98.60%
15	25	580.71s	98.77%	98.50%	542.28s	98.33%	98.62%
15	30	544.97s	98.44%	98.44%	660.58s	98.55%	98.58%
15	35	562.93s	98.66%	98.45%	573.54s	98.33%	98.45%
15	40	561.17s	99.22%	98.42%	566.00s	98.88%	98.52%
20	10	839.94s	99.26%	98.36%	821.12s	97.90%	98.52%
20	15	571.17s	100.00%	98.45%	610.88s	98.52%	98.49%
20	20	432.93s	99.38%	98.47%	510.72s	97.78%	98.39%
20	25	479.72s	99.63%	98.45%	421.50s	97.86%	98.34%
20	30	433.36s	99.42%	98.49%	429.16s	98.36%	98.33%
20	35	418.83s	98.52%	98.44%	409.86s	98.19%	98.33%
20	40	434.86s	98.44%	98.34%	420.46s	97.66%	98.28%
25	10	768.45s	98.62%	97.87%	753.27s	98.26%	98.40%
25	15	524.85s	99.76%	98.57%	540.20s	98.32%	98.32%
25	20	455.92s	99.70%	98.52%	401.17s	97.66%	98.23%
25	25	351.30s	99.10%	98.44%	372.74s	98.44%	98.26%
25	30	365.58s	98.56%	98.48%	364.29s	98.20%	98.14%
25	35	364.04s	98.32%	98.46%	371.01s	97.54%	98.02%
25	40	373.22s	98.05%	98.35%	346.99s	97.72%	98.04%
30	10	778.42s	97.92%	97.25%	764.59s	99.17%	98.28%
30	15	507.43s	99.72%	98.51%	527.42s	98.12%	98.13%
30	20	428.09s	99.74%	98.48%	461.58s	97.92%	98.03%
30	25	341.44s	99.01%	98.48%	334.52s	98.59%	97.96%
30	30	318.41s	99.17%	98.39%	310.19s	97.66%	97.77%
30	35	312.96s	99.17%	98.35%	305.48s	98.28%	97.86%
30	40	316.68s	98.65%	98.30%	343.07s	98.28%	97.73%
35	10	691.38s	96.65%	96.74%	785.00s	98.09%	97.96%
35	15	511.17s	99.01%	98.18%	515.88s	97.91%	98.06%
35	20	390.05s	99.61%	98.51%	405.90s	97.77%	98.01%
35	25	314.71s	99.52%	98.37%	324.66s	97.82%	97.81%
35	30	273.62s	98.67%	98.43%	379.93s	97.63%	97.82%
35	35	276.11s	99.29%	98.13%	357.71s	97.50%	97.84%
35	40	284.50s	98.69%	98.17%	543.86s	97.79%	97.59%
40	10	748.94s	94.17%	95.55%	1256.09s	96.57%	97.99%
40	15	506.25s	95.99%	97.25%	534.42s	96.88%	97.89%
40	20	383.51s	99.24%	98.40%	412.37s	96.65%	97.83%
40	25	308.15s	98.86%	98.39%	347.50s	96.65%	97.67%
40	30	351.54s	98.66%	98.45%	305.50s	96.47%	97.67%
40	35	279.30s	98.73%	98.32%	252.70s	96.32%	97.60%
40	40	257.62s	97.88%	98.18%	250.74s	96.65%	97.45%

Table 3.2: Summary of AGN and AEASGD experiments using different distributed hyperparameters (n and λ). From these experiments we find that AGN performs better in terms of training, and validation accuracy in the presence of a higher number of asynchronous workers, and a reduced communication frequency.

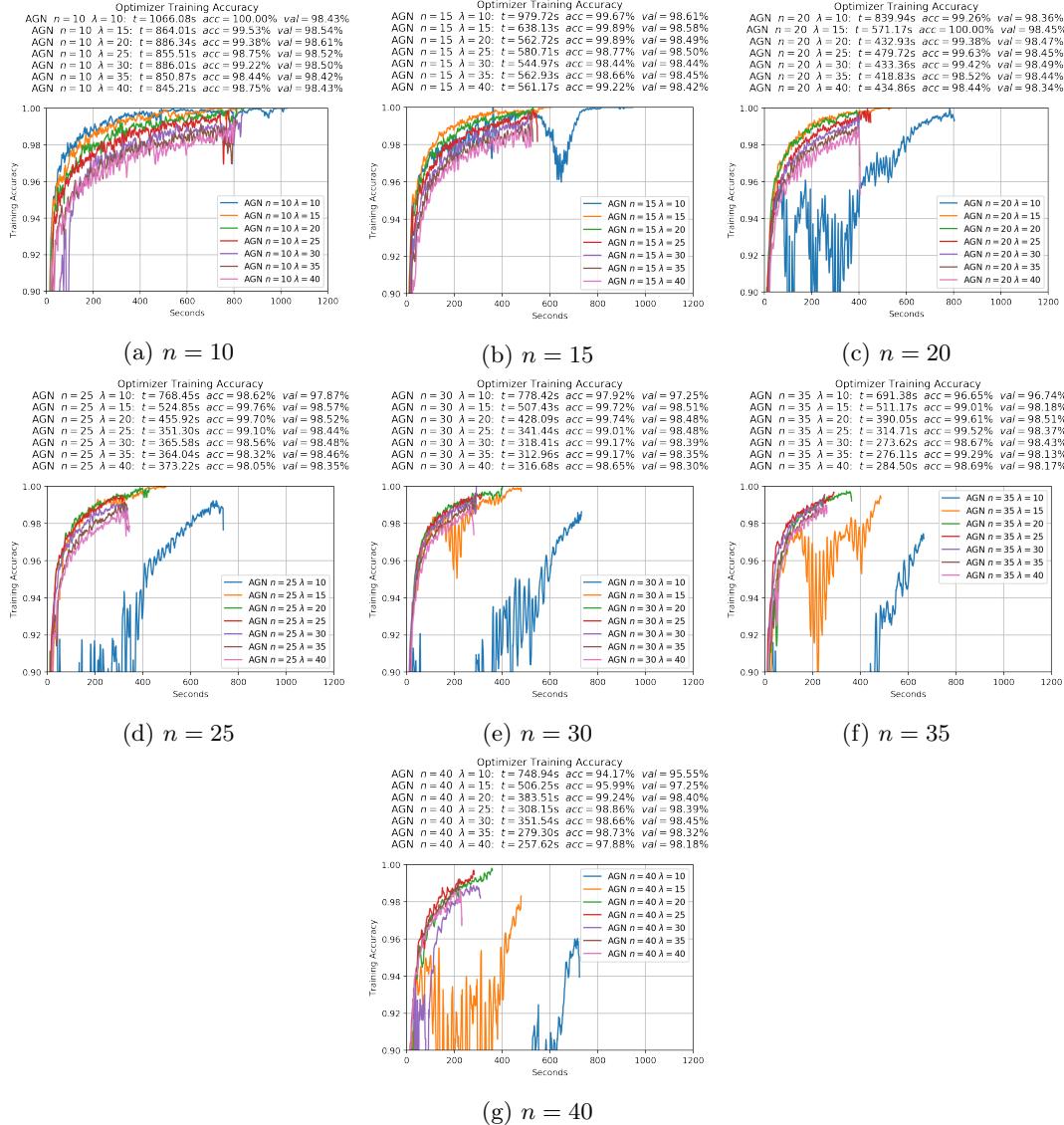


Figure 3.10: This Figure shows several experiments were we fixed the number of workers, but vary the communication frequency. From this we observe that AGN performs well when a relatively equal high communication frequency is used with respect to the number of workers, and derive the following heuristic $\lambda \approx \frac{n+\text{layers in network}}{2}$. Furthermore, increasing the amount of workers, and maintaining a high communication frequency deteriorates the performance of the central variable as well. As a result, a balance between the communication frequency, and the number of asynchronous workers is required.

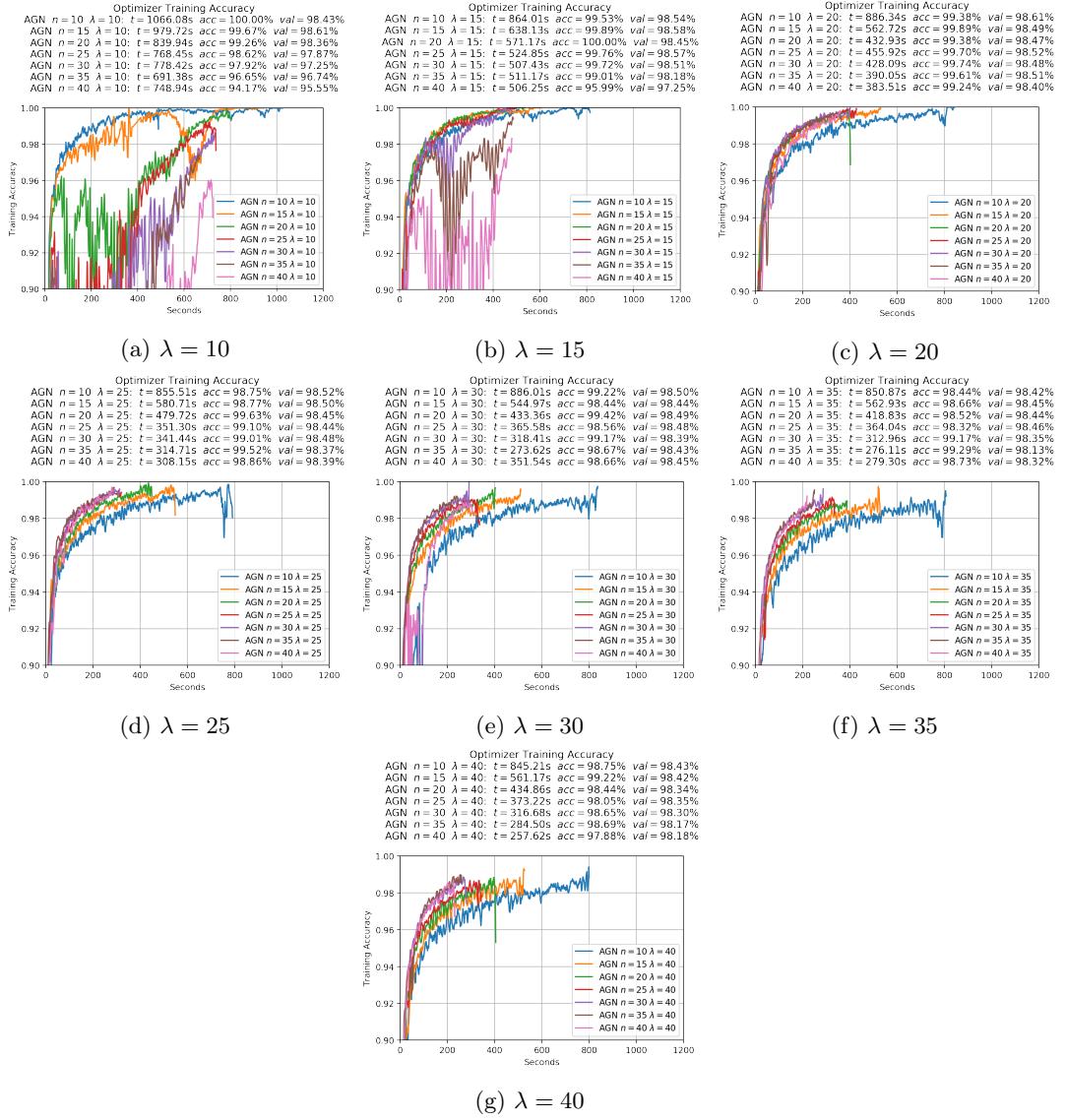


Figure 3.11: In this experiment we clamp the communication frequency, but vary the number of asynchronous workers. Due to the equal communication frequency, we can observe good scaling properties of AGN. In most cases doubling the number of workers, reducing the training time by half and is more temporally efficient. However, for larger number of workers $n > 30$ we do not observe a reduction of training time. This is due to the implementation of our parameter server, which is based on Python threads instead of Python processes, as will be discussed in Chapter 5. Furthermore, note that reducing the amount of computational resources might actually benefit the training accuracy of the central variable, as a smaller number of asynchronous workers reduces the amount of staleness that can be incorporated in the central variable.

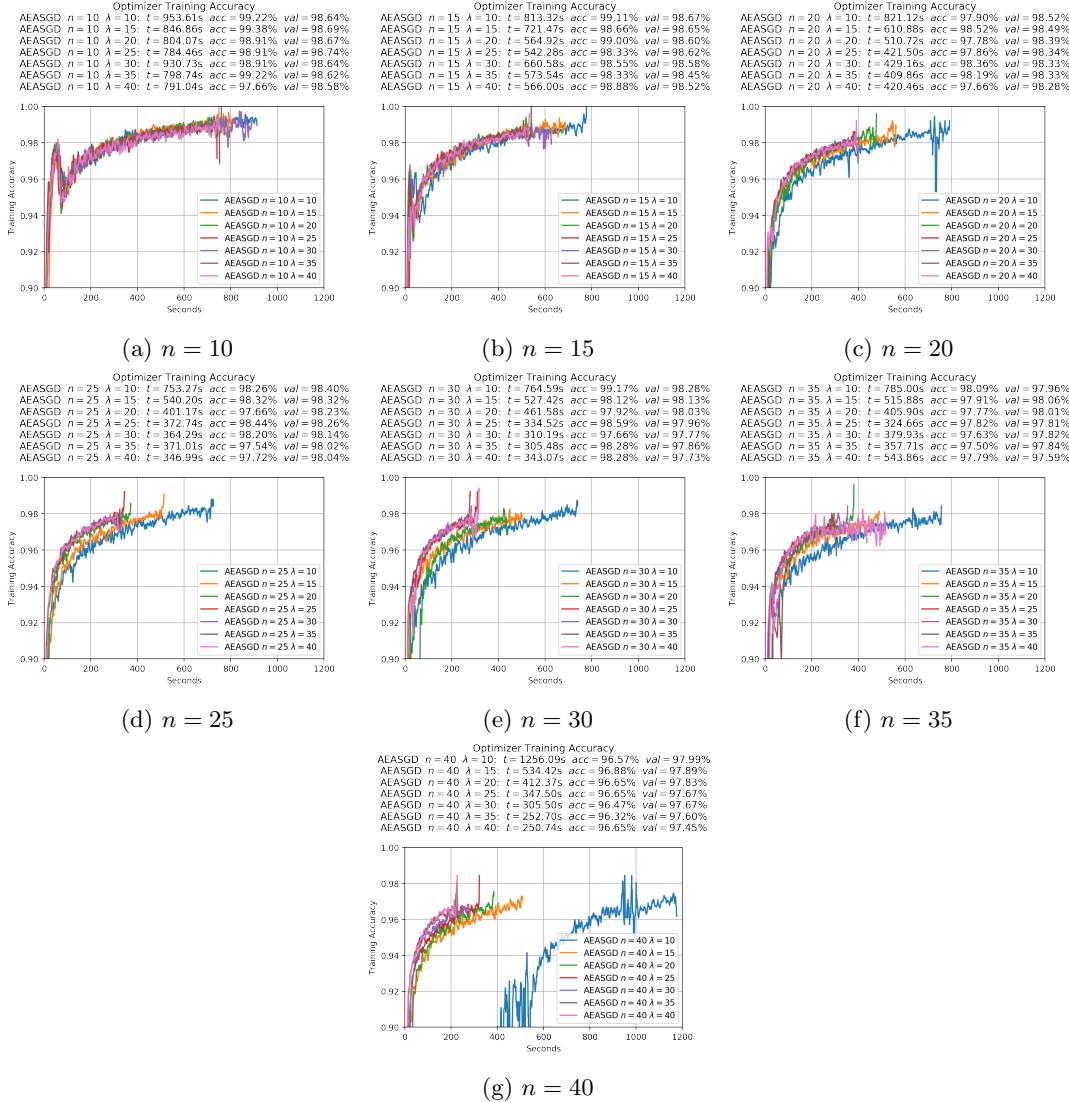


Figure 3.12: AEASGD fixed worker, with varying communication frequency experiments. As suggested by the author, AEASGD usually benefits from an increased amount of local worker (large λ). Furthermore, we observe that initially, AEASGD is quite robust to hyperparameterization ($n = 10$). However, as the number of asynchronous workers increases, and the communication frequency is further reduced, the accuracy plots start to deviate to the point that they *flatten* at a suboptimal training accuracy. Meaning, a lower training accuracy compared to other configurations, where AEASGD reached a significantly higher training accuracy.

3.2. EXPERIMENTAL VALIDATION

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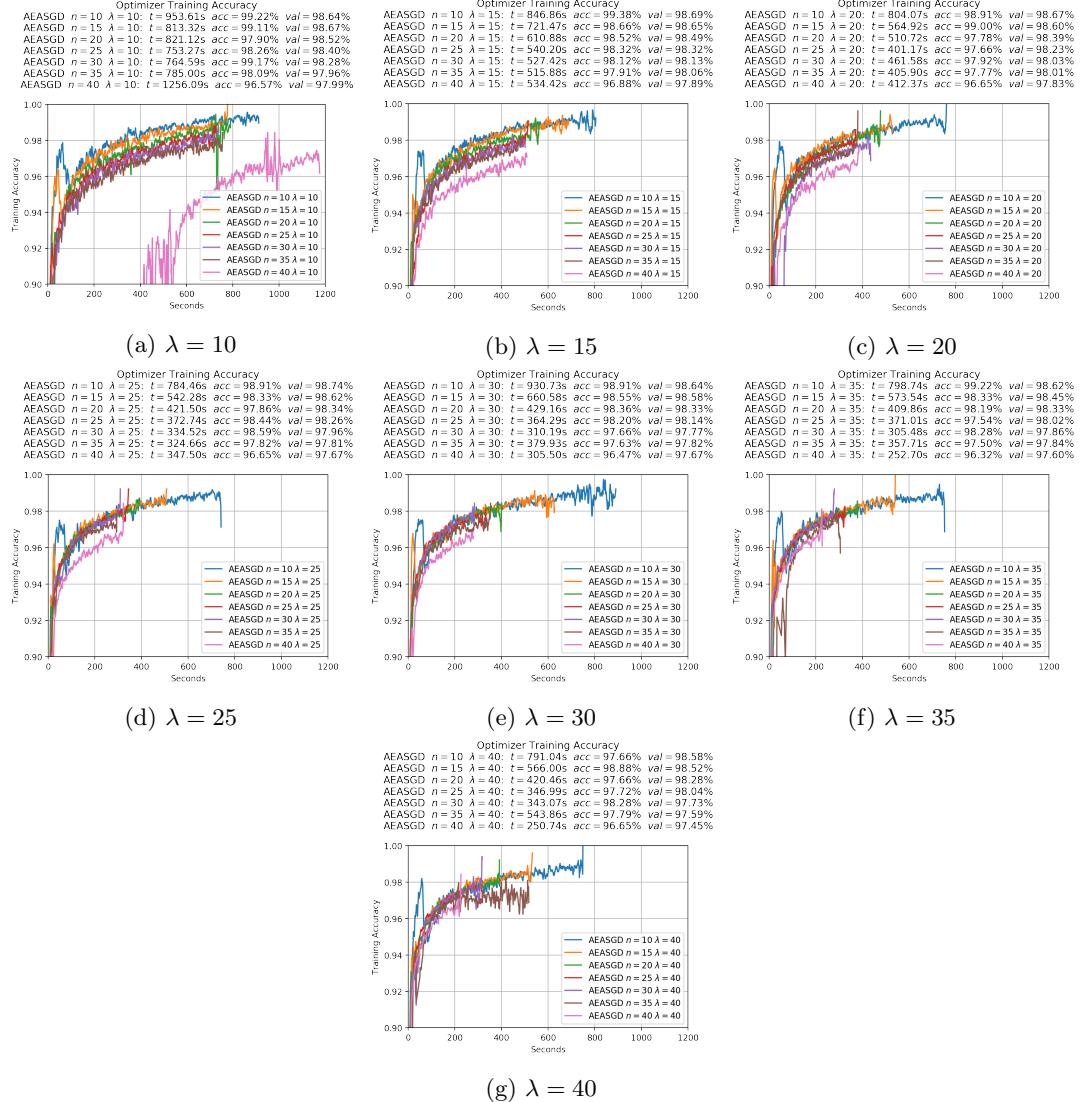


Figure 3.13: In this experiment we clamp the communication frequency, but vary the number of workers. As in Figure 3.12, we do similar observations regarding the flattening of the training accuracy in the presence of a low communication frequency, and a large amount of asynchrony. However, an additional interesting observation which escaped our eye in Figure 3.12, is the large ‘‘bump’’ in training accuracy for $n = 10$. Nevertheless, this bump is always followed up by a significant reduction in training accuracy. Usually one would expect such behaviour in a configuration with a large amount of asynchrony, but this behaviour is not present in said configurations. Furthermore, the reason why we do not observe significant fluctuations in training accuracy, contrary to AGN, is due to hyperparameter ρ , which controls the amount of exploration that can be done in terms of *distance*, as discussed in Chapter 2.

Chapter 4

Asynchronous Distributed Adaptive Gradients

In this chapter we introduce a novel optimizer called ADAG. ADAG, or *Asynchronous Distributed Adaptive Gradients*, is an optimization process designed with data parallel methods in mind. We build upon previous work [4, 5, 9, 17, 8] and incorporate new insights backed up by theory and experimental evidence. We start in Section 4.1 by formalizing the problem setting. Section 4.2 describes our algorithm in detail, supported by intuition and theory. Finally, we experimentally show the effectiveness of our approach in Section 4.3 and give some points for future work in Section 4.4.

4.1 Problem setting

Currently, staleness τ is defined in literature as the number of steps between the current timestep t of the central variable, and the timestep of the central variable which a worker based its gradients upon, which is $t - \tau$. However, as shown in Chapter 2 and Chapter 3, this particular definition of staleness is problematic when one tries to mitigate parameter staleness. To illustrate this, we showed in Chapter 3 that DYN SGD [8], which uses the above definition of staleness, is not able to solve the staleness problem if the learning rate is too high or gradient accumulation takes place, i.e., if the magnitude of the worker deltas is too large. Since DYN SGD fails to deal with staleness efficiently in those situations, one can deduce that using the number of stale steps is a rather naive approach. As a result, the problem of parameter staleness in Deep Learning has an additional dimension.

A step forward in understanding the staleness problem is by gaining intuition on what mechanism is exactly causing the central variable to diverge, or converge more slowly. As mentioned in Chapter 1, divergent behaviour of the central variable is caused by stale workers committing gradients which are based on old parameterizations of the central variable. Again, in this definition we use the term “old”. However, we argue in the following sections that an old central variable is not problematic as long as the *distance* between the old, and the current central variable is small. To strengthen this intuition, let us consider Figure 1.5 from Chapter 1.5. Clearly, the reason why the central variable diverges is because the straggler commits a gradient which was based on a completely different loss (position). Hypothetically, let us consider that only a single other worker committed a gradient causing the central variable to be close to a minima. If one would employ an optimizer like DYN SGD, which uses a definition of staleness based on the number of stale *steps*, the stale gradient would be scaled down by half, causing significant divergent behaviour in the central variable. However, if one would use the *distance* between the current, and the old central variable, and scale the workers deltas proportionally to this distance, one is able to simply ignore the result of the straggler thus preventing divergent behaviour. This intuition is shown in Figure 4.1, which incorporates the delta from the straggler into the central variable using Equation 4.1.

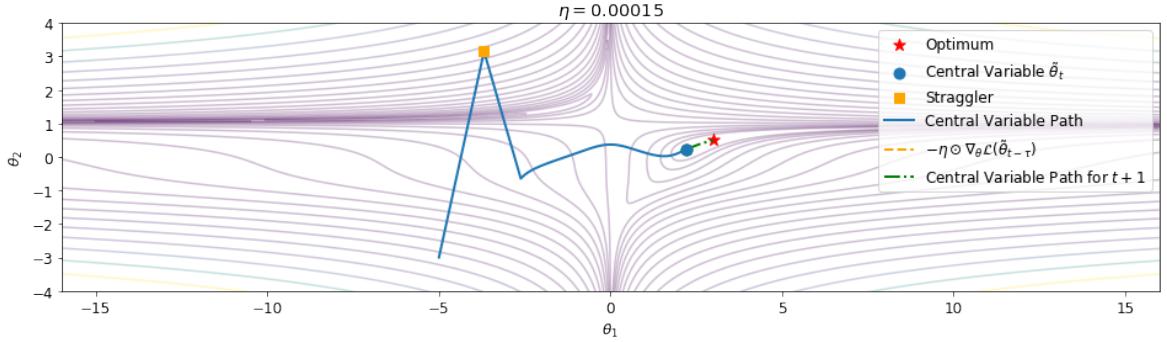


Figure 4.1: Correction of the stale worker delta using Equation 4.1. Contrary to Figure 1.5, the gradient is scaled down significantly in a way it does not deteriorate the performance of the central variable.

An interesting observation from Chapter 3, is that contrary to AGN, AEASGD does not show any divergent behaviour for most configurations, i.e., in terms of distributed hyperparameterizations. This is rather interesting, because it tells us that AEASGD has an intrinsic mechanism to deal with parameter staleness. If we review Equation 2.15, which describes the worker update rule for $\lambda = 1$, we see that the elastic difference, defined as $\eta_t \rho(\theta_t^k - \tilde{\theta}_t)$, incorporates the *distance* between the worker parameterization θ_t^k and the *current* parameterization of the central variable $\tilde{\theta}_t$. AEASGD uses the elastic difference as an *opposing force* of the worker exploration, meaning, it limits the amount of exploration that can be done. As a result, the elastic difference of AEASGD in effect limits the *distance* that can be covered by the worker (unless the gradients suddenly become larger). As a result, AEASGD serves as additional evidence for our notion that staleness should be defined in terms of *distance*, and not in terms of stale steps, as formalized in Definition 4.1.1.

Definition 4.1.1 *Staleness* Given a parameterization for worker k at time t , θ_t^k , based on the central variable $\tilde{\theta}_{t-\tau}$ where τ is the number of stale steps, and a central variable at time t , $\tilde{\theta}_t$. Then, staleness is defined as the difference (distance) between θ_t^k and $\tilde{\theta}_{t-\tau}$.

Using Definition 4.1.1, we can make the deduction that staleness is not really a problem as long as workers commit gradients based on older parameterization of the central variable, which are close to the *current* central variable. This is again additional support for Hypothesis 1, which states that workers only contribute efficiently to the central variable as they remain close to each other, i.e., the variance of the workers amount the parameter space is small.

4.2 Algorithm & Update Rule

This Section fully describes the ADAG update rule, and several architectural decisions that can be made when implementing this technique. Furthermore, using Definition 4.1.1, we show how ADAG can push the limits of asynchronous optimization further, while at the same time eliminating the need for (distributed) hyperparameter gridsearches to ensure convergence.

To describe the ADAG update rule, we use the same terminology and notation used in Definition 4.1.1 to further strengthen the intuition in parameter staleness, and how parameter staleness is used in Equation 4.1. Let us consider the case when $\tau = 0$. In this situation, a worker k computes the gradient based on $\tilde{\theta}_{t-\tau}$ which is equal to $\tilde{\theta}_t$. As a result, the distance between these two parameterizations will be 0, which results in the worker delta $\Delta\theta^k$ to be incorporated into the central variable *as is*, and thereby causing the ADAG update rule to generalize to SGD. In the next step, we add more asynchronous workers to the optimization problem. By doing so, we implicitly increase the staleness as $E[\tau] = (n - 1)$. As a result, parameter staleness in terms of stale steps, τ , is expected to be larger than 0. Consequently, the distance between $\tilde{\theta}_t$ and $\tilde{\theta}_{t-\tau}$ is *expected* to be non-zero causing the worker delta to be scaled down proportionally to the difference of these parameterizations. However, since

parameter changes in Deep Learning usually consist of very small updates, we use the inverse learning rate (η_t^{-1}) to get a sense of the scale at which these updates operate at. Using the inverse learning rate in Equation 4.1, and due to the fact that staleness (in terms of distance) is usually relatively small in Deep Learning since the gradients are scaled with respect to η_t , we find that ADAG scales the gradients more realistically. Since without the inverse learning rate, the scaling would be very small.

$$\tilde{\theta}_{t+1} = \tilde{\theta}_t + \frac{1}{\eta_t^{-1} \|\tilde{\theta}_t - \tilde{\theta}_{t-\tau}\|^2 + 1} \odot \Delta\theta^k \quad (4.1)$$

Nevertheless, one could view the inverse learning rate from a different perspective. First let us denote the set of workers as \mathcal{W} . Then, the staleness term in Equation 4.1 can be written as a sequence of gradients from different workers w , as shown in Equation 4.2.

$$\tilde{\theta}_t - \tilde{\theta}_{t-\tau} = \sum_{i=0}^{\tau} \exists! w \in \mathcal{W} : \eta_t \nabla_{\theta} \mathcal{L}_w(\tilde{\theta}_{t-\tau_w}) \quad (4.2)$$

Furthermore, assuming a static learning rate η , Equation 4.2 can be simplified by moving the learning rate η_t before the summation sign, obtaining Equation 4.3.

$$\tilde{\theta}_t - \tilde{\theta}_{t-\tau} = \eta \sum_{i=0}^{\tau} \exists! w \in \mathcal{W} : \nabla_{\theta} \mathcal{L}_w(\tilde{\theta}_{t-\tau_w}) \quad (4.3)$$

Remember that the scaling of the worker deltas are proportional to Equation 4.4.

$$\eta^{-1} \|\tilde{\theta}_t - \tilde{\theta}_{t-\tau}\|^2 \quad (4.4)$$

Substituting the staleness term in Equation 4.4 for Equation 4.3 gives us:

$$\eta^{-1} \|\eta \sum_{i=0}^{\tau} \exists! w \in \mathcal{W} : \nabla_{\theta} \mathcal{L}_w(\tilde{\theta}_{t-\tau_w})\|^2 \quad (4.5)$$

Earlier, the assumption was made the learning rate is static. As a result, we can cancel the learning rate terms, and thus obtaining Equation 4.6.

$$\eta^{\cancel{\mathcal{W}}} \|\sum_{i=0}^{\tau} \exists! w \in \mathcal{W} : \nabla_{\theta} \mathcal{L}_w(\tilde{\theta}_{t-\tau_w})\|^2 \quad (4.6)$$

This result indicates that the scaling term in Equation 4.1 is proportional to a *unitless* sequence of worker gradients, i.e., not scaled down by a learning rate. As a result, Equation 4.1 is not sensitive to the hyperparameterization of η . Therefore, the scaling term will work at any scale, and still be proportional to the magnitude of the worker gradients.

An additional, but important aspect that needs to be considered is how ADAG keeps track of $\tilde{\theta}_{t-\tau}$. For this we foresee two possible implementations. The first, described in Algorithm 11, keeps track of $\tilde{\theta}_{t-\tau_w}$ for a particular worker w at a worker level. Meaning, worker w keeps track of its local copy θ_t^w , and the original central variable $\tilde{\theta}_{t-\tau}$. Next, when worker w is done computing $\Delta\theta_t^w$, w will commit both $\Delta\theta_t^w$ and $\tilde{\theta}_{t-\tau}$ to the parameter server. Since the parameter server already holds $\tilde{\theta}_t$, the parameter server can now compute the next central variable using Equation 4.1.

Algorithm 11 Implementation of ADAG where the workers are responsible for keeping track of $\tilde{\theta}_{t-\tau}$.

```

1: procedure ADAGPARAMETERSERVER
2:    $t \leftarrow 0$                                       $\triangleright$  Parameter server clock
3:    $\tilde{\theta}_t \leftarrow \text{RANDOM}()$ 
4:
5:   procedure HANDLEPULL( $k$ )
6:     return  $\tilde{\theta}_t$ 
7:   end procedure
8:
9:   procedure HANDLECOMMIT( $k, \Delta\theta^w, \tilde{\theta}_{t-\tau_k}$ )
10:     $\tilde{\theta}_{t+1} = \tilde{\theta}_t + \frac{1}{\eta_t^{-1} \|\tilde{\theta}_t - \tilde{\theta}_{t-\tau_k}\|^2 + 1} \odot \Delta\theta_t^k$ 
11:     $t \leftarrow t + 1$ 
12:   end procedure
13:
14: end procedure

```

However, an obvious limitation of Algorithm 11 is the increased network usage since two parameterizations have to be shipped to the parameter server, i.e., the worker delta $\Delta\theta_t^k$ and the original central variable parameterization $\tilde{\theta}_{t-\tau}$. To reduce the network usage of Algorithm 11, and thereby reducing the waiting time of the workers, we propose to let the parameter server keep track of worker pulls, i.e., whenever a worker pulls the central variable, the parameter server copies the central variable into a datastructure (for example, a hashmap), and associates the the parameterization of the central variable with the worker which requested the pull at that time. This procedure is roughly described in Algorithm 12. However, we would like to note that despite the fact that Algorithm 12 reduces the communication costs, it increases the memory requirements proportional to the number of concurrent workers, which might be problematic as the number of asynchronous workers is high.

Algorithm 12 Network efficient implementation of ADAG.

```

1: procedure ADAGPARAMETERSERVER
2:    $t \leftarrow 0$                                       $\triangleright$  Parameter server clock
3:    $\tilde{\theta}_t \leftarrow \text{RANDOM}()$ 
4:    $m \leftarrow \text{INITIALIZE}()$             $\triangleright$  Initializes a data structure which keeps track of worker pulls.
5:
6:   procedure HANDLEPULL( $k$ )                          $\triangleright$   $k$  denotes the worker identifier
7:      $m[k] = \tilde{\theta}_t$ 
8:     return  $\tilde{\theta}_t$ 
9:   end procedure
10:
11:  procedure HANDLECOMMIT( $k, \Delta\theta^w$ )
12:     $\tilde{\theta}_{t+1} = \tilde{\theta}_t + \frac{1}{\eta_t^{-1} \|\tilde{\theta}_t - m[k]\|^2 + 1} \odot \Delta\theta_t^k$ 
13:     $t \leftarrow t + 1$ 
14:  end procedure
15:
16: end procedure

```

However, there is a practical problem in Algorithm 11 and Algorithm 12 which tends to be overlooking when viewing ADAG from a theoretical perspective. that is, both Algorithm 11 and Algorithm 12 put severe computational load on the parameter server to ensure consistency of the staleness computation when computing the scaling factor. Remember that in order to ensure staleness consistency, we lock any writes to the central variable during the time the parameter server computes the scaling factor. This induces a significant wait in other workers due to relatively heavy computations which need to be executed by the parameter server. However, taking inspiration from AEASGD [17], we can make

significant improvements regarding computational efficiency if we loosen the consistency constraints. By doing so, we could let the workers compute the scaling factor locally, and then transmit the scaled delta to the parameter server which only has to *add* the delta into the central variable. Furthermore, to reduce the network usage even more, we only pull the central variable only once, as shown in Algorithm 13. Despite the fact that Algorithm 13 is more computationally efficient because the load is parallelized equally over all workers, it has several consistency issues. The first being that the computed scaling factor *might* not reflect reality since other workers *could* have committed their deltas to the parameter server between the time our worker pulled the central variable, and the committed its scaled delta. Furthermore, if this occurs, the worker will start computing gradients based on an already (slightly) stale central variable in the next iteration.

Algorithm 13 Network efficient, and more computational efficient implementation of ADAG. With the side-effect that we loosen the staleness consistency constraints.

```

1: procedure ADAGWORKER( $k$ )
2:    $\tilde{\theta}_{\text{original}} \leftarrow \text{PULL}()$                                  $\triangleright$  Keep a local copy of the central variable
3:    $\theta_t^k \leftarrow \tilde{\theta}_{\text{original}}$ 
4:
5:   procedure COMMIT( $\Delta\theta^w$ )                                          $\triangleright$  Redefine COMMIT operation.
6:      $\tilde{\theta}_t \leftarrow \text{PULL}()$ 
7:      $\Delta\theta^k \leftarrow \frac{1}{\eta_t^{-1}\|\tilde{\theta}_t - \tilde{\theta}_{\text{original}}\|^2 + 1} \odot \Delta\theta^k$ 
8:     SENDTOPARAMETERSERVER( $\Delta\theta^k$ )
9:      $\tilde{\theta}_{\text{original}} \leftarrow \tilde{\theta}_t$ 
10:     $\theta_t^k \leftarrow \tilde{\theta}_{\text{original}}$ 
11:     $t \leftarrow t + 1$ 
12:  end procedure
13:
14: end procedure

```

4.3 Experiments

To demonstrate the effectiveness of Definition 4.1.1, several experiments shall be conducted in the following section regarding the convergence properties of ADAG. Looking back at Chapter 3, we saw that staleness mitigation is critical in highly concurrent environments, as the staleness increases proportionally to the number of workers, as shown in Figure 4.2. Furthermore, without ADAG, we have to decrease the communication frequency even further to ensure that divergence does not occur. However, as a result of the increased amount of local exploration, parameter updates occur less frequently. Which again results in a slower convergence of the central variable, and thereby reducing the *temporal efficiency* of those highly concurrent configurations compared to less concurrent configurations.

Let us start by considering several predictive scenarios to verify this novel understanding of parameter staleness in Deep Learning, and how to deal with it, as shown in Equation 4.1. First and foremost, at the start of any parameterized machine learning training procedure, a model is initialized according to some initialization strategy. Since the probability is *very* low that a randomly initialized parameterization will perform well in terms of classification accuracy, we can make the assumption that the loss during the initial phase of the training will be relatively high. Furthermore, let us assume that there are n workers trying to optimize the central variable in an asynchronous manner, and that ADAG is applied before incorporating the worker deltas into the central variable, i.e., scaling proportional to $n^{-1}\|\tilde{\theta}_t - \tilde{\theta}_{t-\tau}\|^2$ is applied to the incoming worker delta. In this case, we can expect that during the initial phase of the training procedure only a few workers will contribute efficiently to the central variable. With this statement, we are implying that only the deltas of a small fraction of workers will not be scaled down significantly because the loss, and thereby the gradients are quite large at the start of the training procedure, in contrast to the situation when the central variable is close to an optimum.

As a result, other workers will be scaled down significantly, because the parameterization on which they based their gradients on (which all workers started with), is too distant from the current central variable.

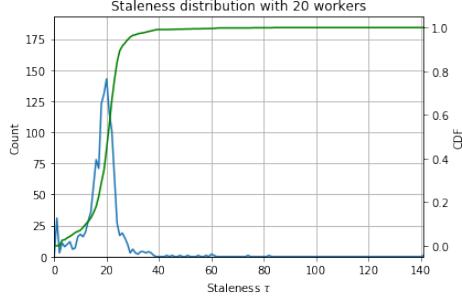


Figure 4.2: Staleness distribution of $n = 20$. This is in accordance with the theory [13], which says that $\mathbf{E}[\tau] = (n - 1)$. Note that the x -axis denotes the staleness in terms of stale steps.

The following prediction we make is related to the convergence of optimizers which employ ADAG. First, we know that stale gradients will be scaled down significantly due to the update rule described in Equation 4.1. Furthermore, since the loss is getting smaller as the training procedure goes on, staleness, in terms of Definition 4.1.1, is implicitly getting smaller as well. As a result, if one would plot the scaling term for every worker, we should observe (on average) a decline in the scaling factor. This would imply that the distance between the central variables, and thereby the worker gradients are small. As a result, worker deltas do not have to be scaled down as much as their gradients are relatively local to the current central variable. Furthermore, in such a situation, the optimization process would benefit from more asynchronous workers since the staleness is relatively small. An additional situation where more asynchronous workers would be beneficial is on a *plateau*, where the losses are very small which implies that the parameter updates are very small. In such a situation, ADAG would barely scale down the gradients. Furthermore, as the workers would “leave” the plateau, worker updates which are too stale would automatically be nullified.

To empirically validate these predictions, we conducted several experiments for which we know AGN has difficulties converging, i.e., high amount of asynchrony and a high communication frequency. Furthermore, to show that Definition 4.1.1 is a better description of staleness compared to the number of steps, we will show that staleness, in number of steps, is not necessarily proportional to the scaling factor. This is a valid assumption to make since the scaling factor *is* proportional to the distance between parameterizations.

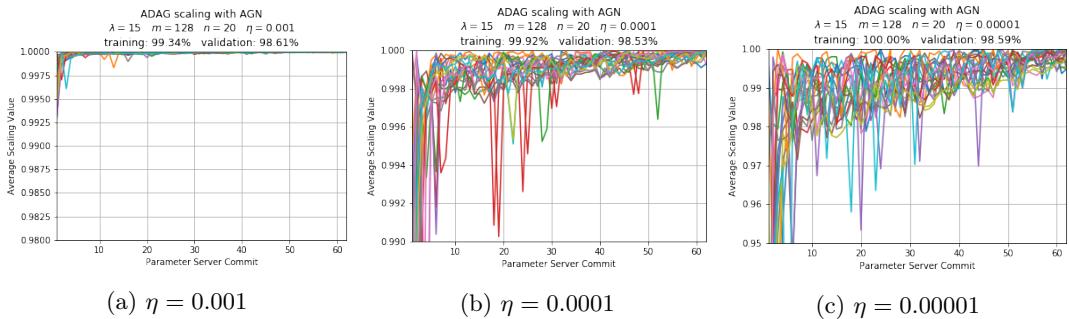


Figure 4.3: Average scaling (of all weights) value of every worker in ADAG with respect to different learning rates. In all Subfigures we observe a significant scaling at the start of the learning procedure (loss is relatively high, as mentioned in our first prediction), while at the end of the training procedure, scaling is not very prominent because the workers converged to a minima.

Figure 4.5 shows the stale steps (top row), together with the average scaling factor (bottom row) of every worker. Since ADAG provides a per-weight scaling factor, we compute the average scaling factor by averaging all scaling factors for every trainable weight. This allows us to get a general overview of how the scaling term behaves under different values of staleness. Looking at Figure 4.5, we observe a significant scaling at the start of the optimization procedure *for most workers*, thus providing evidence for our initial prediction. Furthermore, to show that the number of stale steps is not necessarily proportional to staleness in terms of distance, let us consider *Worker 2* in Figure 4.5 for a moment. Around parameter update step 12, a very stale update was committed to the parameter server, both in distance as in number of steps. However, other updates of *Worker 2* are also quite stale compared to other workers. Yet, the scaling factor for these updates are almost equal to 0, thus indicating that the number of stale steps is not necessarily proportional to the real staleness. Of course, this begs the question why other workers for these particular update numbers have significantly lower scaling factors, while their staleness is lower as well. This can be explained by the fact that *Worker 2* was significantly slower than other workers during the initial phase of the training procedure, which caused the central variable to have moved significantly towards a minima during that time. Of course, since gradients are relatively small in the neighbourhood of a minima, staleness is not actively playing a role in the divergence of the central variable. As a result, worker gradients are not scaled down. This result can be validated by looking at *Worker 8* as well, since *Worker 8* behaves in a similar manner.

Furthermore, Figure 4.5 can serve as additional validation for the statement made in [13], which says that $\mathbf{E}[\tau] = (n - 1)$. If we look at the average staleness of all workers, we can immediately see that the *expected* staleness value of the optimization process is about 20, which is the number of workers used in that particular experiment.

Finally, we apply ADAG to solve the instability issues of AGN in Chapter 3 under highly concurrent conditions with high communication frequencies, i.e., high number of asynchronous workers with frequent parameter updates. As shown in Figure 4.4, we see that AGN initially has some difficulties converging to a minima due to the increased amount of staleness and *implicit momentum* [13]. However, when ADAG is applied, this instability is gone because ADAG scales worker deltas proportional to the distance of the old, and current central variable.

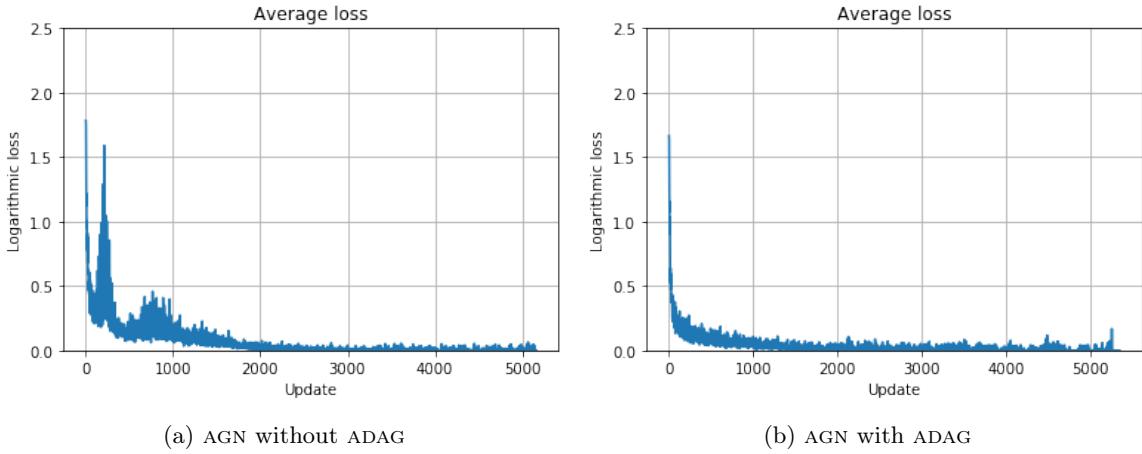


Figure 4.4: In this experiment we apply AGN to the MNIST problem using 30 asynchronous workers. From Subfigure (a) we observe some divergent behaviour of the central variable during the initial phase of the training procedure. However, if we apply ADAG to AGN in these exact same conditions, the divergent behaviour previously observed is absent due to the fact that stale worker deltas are scaled down, or even nullified. Furthermore, since the divergent behaviour is not present in Subfigure (b), AGN with ADAG reaches convergence faster (in terms of updates).

However, scaling worker deltas according to Equation 4.1 might be rather simplistic. Nevertheless, ADAG is effective despite the fact that there might be a better technique. Furthermore, during our

experimentation with ADAG, we found that especially in high staleness environments, Equation 4.8 is not really effective due to the relatively large learning rate, as shown in Figure 4.4. Therefore, we introduce a hyperparameter γ , which controls the scale at which the gradients operate. As a result, we can rewrite Equation 4.1 as Equation 4.7. In our experiments, γ is usually defined as 0.0001. However, if the loss still shows *significant* divergent behaviour, we recommend to reduce the value of γ even further.

$$\tilde{\theta}_{t+1} = \tilde{\theta}_t + \frac{1}{\gamma^{-1}\|\tilde{\theta}_t - \tilde{\theta}_{t-\tau}\|^2 + 1} \odot \Delta\theta^k \quad (4.7)$$

This indicates that there must to be a more intelligent way to define γ . However, due to the time-constraints, this is not considered in this thesis.

4.4 Discussion

Using our knowledge from previous Chapters, we are finally able to construct a distributed gradient-based optimization procedure which is able to handle practical constraints, such as communication costs). Furthermore, by reducing the communication frequency, we increase the amount of computational time that can be spent to produce a better gradient in contrast first order gradients. However, a reduced communication frequency had a significant impact on the convergence rate of the central variable. Therefore, two options are possible:

- Increase the communication frequency.
- Increase the number of asynchronous workers.

However, as discussed in Chapter 3, increasing the communication frequency causes divergent behaviour when a relatively large number of asynchronous workers is present in the optimization process due to the staleness that is inserted. Then, in Chapter 4, we introduced a novel understanding of parameter staleness in terms of distance between the old, and the current central variable. Using this understanding, a per-weight scaling term was constructed, ADAG, which automatically scales stale worker deltas down, or even nullifies them proportional to the staleness of the old central variable, thus preventing divergent behaviour of the central variable. Furthermore, since the scaling term is applied dynamically, a training procedure would automatically benefit from increased parallelism in small loss environments, as workers which were significantly scaled down initially due to the relatively higher loss, will now commit gradients with a larger magnitude, which accelerates the training procedure.

$$\Delta\theta^k = -\frac{1}{\gamma^{-1}\|\tilde{\theta}_t - \tilde{\theta}_{t-\tau}\|^2 + 1} \odot \frac{\sum_{i=0}^{\lambda} \eta_t \frac{1}{m} \sum_{j=1}^{m-1} \nabla_{\theta} \mathcal{L}(\theta_i^k; \mathbf{x}_i; \mathbf{y}_i)}{\lambda} \quad (4.8)$$

$$\tilde{\theta}_{t+1} = \tilde{\theta}_t + \Delta\theta^k \quad (4.9)$$

Using the intuition layed out above, we will now formalize the procedure described in Eqation 4.8 and Equation 4.9. To optimize the throughput of the training procedure, in terms of samples per second. We recommend (in the most complex scenario), distributed mini-batch parallelism, followed by an asynchronous approach using ADAG-AGN to optimize a central variable. Nevertheless, this might be a very complex system. However, this can be simplified significantly for other, less requiring use-cases. For instance, one could simply perform mini-batch parallelism locally by, e.g., distributing the workload over several CPU cores, or even GPUs. Then, once the AGN gradient, and ADAG scaling term from Algorithm 13 are computed, the worker deltas can be sent asynchronously to the parameter server using Equation 4.9.

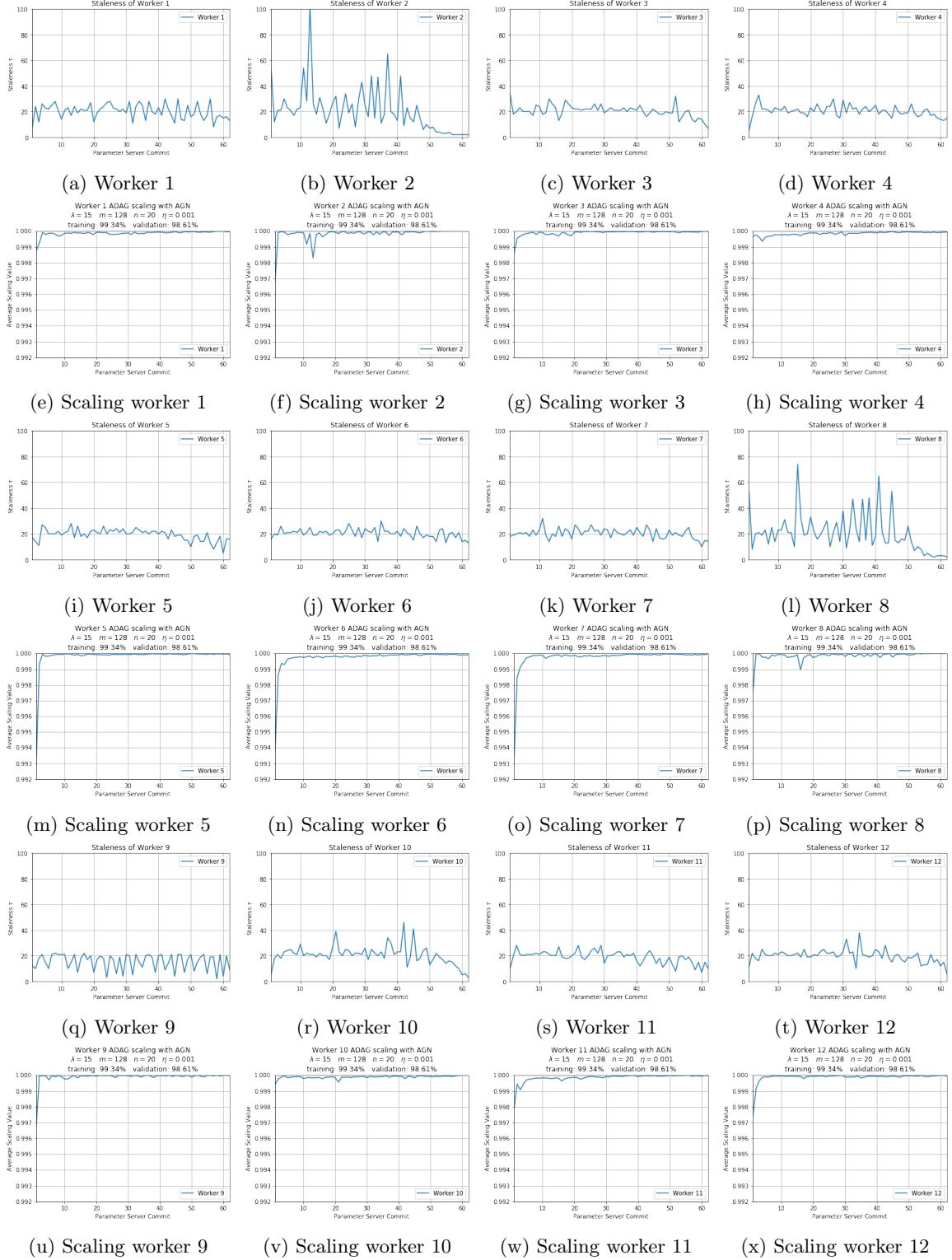


Figure 4.5: Worker staleness and scaling factor of Worker 1 till Worker 12. In the top row, we show the number of stale steps between every parameter update. Whereas in the bottom row, we show the scaling factor corresponding to the same parameter update step.

Chapter 5

Experimental Setup

This chapter describes the experimental setup of our experiments in Chapter 3 and Chapter 4. Furthermore, the architecture of *dist-keras*, which is our Distributed Deep Learning framework based on Apache Spark and Keras, is also introduced.

5.1 Distributed Keras

Distributed Keras, or *dist-keras* in short, is a distributed Deep Learning framework built on top of Apache Spark and Keras with the goal to significantly reduce the training using distributed machine learning algorithms, and allow bigger than memory datasets. This project initially started as a prototype with the CMS collaboration. However, the project has seen several iterations since its start in August 2016, and is still undergoing active development [[dist_keras](#)]. Furthermore, *dist-keras* is designed with a focus on "state-of-the-art" distributed optimization algorithms. We designed the framework in such a way that a new distributed optimizer could be implemented with ease, thus enabling a person to focus on research. Several distributed methods are supported, such as, but not restricted to, the training of ensembles and models using data parallel methods.

5.1.1 Architecture

Before we dive into the architecture of *dist-keras*, let us first discuss several concepts within Apache Spark, since these are heavily used in the framework. First, Apache Spark is a general purpose cluster computing framework using directed acyclic computation graphs to define a sequence of operations. This graph is constructed by a *driver* program, which responsibilities are to send new instructions to worker nodes, or receive results. However, the driver program does *not execute* any intructions from the DAG (directed acyclic graph). The processes which are responsible for the execution of the DAG, are called *executors*. Usually, executors are spawned dynamically using a cluster resource manager. However, it is possible to spawn them manually on every cluster node with the disadvantage that job-dependent configurations (e.g., memory) are not possible.

Nevertheless, an additional important aspect is the way data is handled. Spark usually benefits from a large amount of memory, since it tries to fit as much as possible data in memory to increase the processing throughput. However, if the data does not fit in memory, a flag has to be specified to modify the persistance of the data, meaning, data is allowed to be serialized to disk, and read back into memory when needed. Furthermore, since a computation is defined as a DAG, data can be recomputed in the case of a potential loss due to, for example, data not fitting into memory, or serialized data not fitting on a local disk, or even an unexpected shutdown. Furthermore, to prevent a complete recomputation of data, one can *cache* the data at a specific point. Basically, calling *cache* (checkpointing) on a dataset, tells the executors to keep track of the data in its current form, i.e., not recomputing it. Furthermore, in order to apply modifications, or mapping functions to datasets in a distributed manner, Spark provides an abstraction of data in terms of *Resilient Distributed Dataset*. As discussed before, the term *resilient* implies that the data is able to recover from failures described

above. Furthermore, Spark also provides some syntactic sugar in terms of DataFrames and DataSets to easily manipulate data in a tabular format. However, in contrast to regular tabular entries, data rows are not limited to a specific schema, but can be customized if desired. As a result, we can construct any particular dataset of interest in a Spark DataFrame, and preprocess it for Deep Learning, or other analytical purposes. Furthermore, since Apache Spark provides several utilities to distribute the data among the executors, and stream the data were needed, an architecture can be constructed to feed the data to Deep Learning models in production, or during training.

In general, a *dist-keras* workflow proceeds as follows. Initially, a Keras model is constructed on the machine where the Spark Driver will be allocated, this could be a personal laptop or desktop computer, however, it is recommended that this machine is in nearby proximity of other cluster nodes and has performant hardware including efficient networking capabilities because the parameter server will be allocated on this machine. Nevertheless, despite the fact allocating a single parameter server creates a significant bottleneck on this particular machine, especially when using large models, it proved to be sufficient for our use-cases. However, in essence it should be possible to allocate parameter servers dynamically on different machines since our current parameter server implementations does not have any Spark dependencies. Next, the data is read from a raw datasource, e.g., *csv*, *Apache Parquet*, or *Apache Avro*, or some other data format that is supported. However, at this point the data is spread over several machines, but not all machines involved in the training procedure. To ensure fast delivery of the data to the executor, we *repartition* the data equal to the number of workers which will train the model in parallel. However, due to Spark's *lazy-evaluation* mechanism, the repartitioning (shuffling) of the data is not triggered. To trigger this, we call a Spark *action* such as a *count*, to execute the DAG computation.

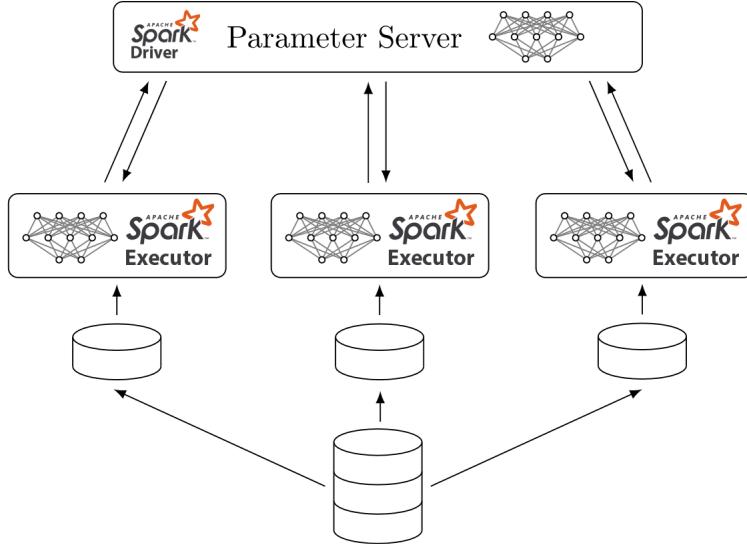


Figure 5.1: Architecture of Distributed Keras and assigned roles during a distributed optimization procedure. As stated before, the Spark Driver will spawn the parameter server on the same machine. However, since our parameter server implementation does not have any Spark dependencies, it can in principle be spawned on any cluster machine.

Before starting the training procedure, all relevant objects such as model weights, optimizer, input column, output column, and so on, are serialized. After the serialization of the training configuration is done, the configuration is transported to the Spark executors, where they can be deserialized and reinstated. At this point, two concurrent threads will be spawned on every executor. The first thread is responsible for the prefetching of the data, since it is possible that the data needs to be

streamed from a different machine. Furthermore, this thread is also responsible for converting the prefetched training samples into the expected format, i.e., *numpy matrices*. The second thread is mainly responsible for the training procedure, shown in Figure 5.1. This thread will implement a specific distributed optimization procedure such as ADAG-AGN, AEASGD, or DOWNPOUR. Nevertheless, during the training procedure, every worker will collect a set of timestamped training metrics (batch accuracy, batch loss). Meaning, after every computation of a mini-batch, the training metrics are timestamped and yielded to Spark for later processing. After all data has been processed, training metrics of all Spark executors are collected and processed to generate central variable accuracy and loss over time.

Finally, when the DAG computation is completed, *dist-keras* fetches the most recent parameterization from the parameter server, and initializes a model with that parameterization. Afterwards, a user can potentially compute validation accuracy in a distributed manner since several utility data *transformers* and *predictors* are provided by *dist-keras*.

5.2 Use-case: CMS Event Identification

To reduce the computational load of collision reconstructions in future LHC runs, the CMS experiment is exploring Machine Learning techniques as a possible approach for accomplishing this. In this particular case, the experiment is evaluating Deep Learning techniques by fitting them to physics problems. High-level problems such as deciding which data to keep in the High Level Trigger given some physical attributes, and other inference problems, are possible applications. Not because of a statistical perspective perce, but mostly to reduce computational complexity. In the following experiments, we mainly occupy ourselves with the reconstruction of *particle tracks*, and identification of *track types* from raw detector *hits*. A particle track, *track* denoted from this point on, is the path that has been traversed by a particle through the detector. The track is reconstructed from a set of hits, which have been triggered (detected) by parts of the detector. The reconstruction of these tracks is a computationally intensive process, since given a set of hits, one needs to minimize the χ^2 error of the track with respect to the set of hits that have been associated with a particular track. However, this is only one aspect of the problem, one first needs to obtain the set of hits which describe a track given all hits within a collision, which also includes the background (false-positives) generated by the detector itself. Currently, the set of hits related to a single track are extracted using a two-pass Kalman filter and a iterative method to find the best fit, with the first pass of the Kalman filter starting on the outer edges of the detector.

An additional problem of applying Deep Learning, or any other Machine Learning approach to the problem of track reconstruction, or event identification, is the petabyte scale data that needs to be dealt with. A simple, and more common solution would be to sample a more manageable fraction of the dataset to train our models on. However, we want the model to be able to extract as many diverse tracks as possible that have been reconstructed over previous years by reconstruction software. As a result, a distributed (data parallel) approach is necessary. However, the data representation is also an important aspect of the problem. Currently, all collisions are stored in the ROOT format, where every reconstructed track for a particular collision (or set of collisions), including the hits of the track, and track parameters can be extracted from. This specific data format is quite problematic, especially taking the petabyte-scale data into account. Furthermore, depending on the modelling, the data needs to be preprocessed in a particular way. One could of course preprocess the complete physics data in a format, shape the data accordingly, and simply copy the data to the machines which will be used during training. However, this is not a very space efficient approach.

A more reasonable, and space efficient approach would be to deliver the preprocessed data to the training procedure in a streaming manner. For example, every worker has a buffer in which it will prefetch and preprocess data coming from the ROOT format in a way the model will understand, e.g., *numpy arrays*. This approach does not have the space inefficiencies the previous approach had. However, there is an increased computational load on the worker nodes due to the prefetching,

preprocessing of the data, and duplicate processing of the same training samples if multiple epochs of training are required. Nevertheless, compared to computation of gradients in large models, this load is neglectable. An additional benefit of this approach is that the same architecture can be used during model development, since data representations can be generated on-the-fly as in Apache Spark, shown in Figure 5.2.

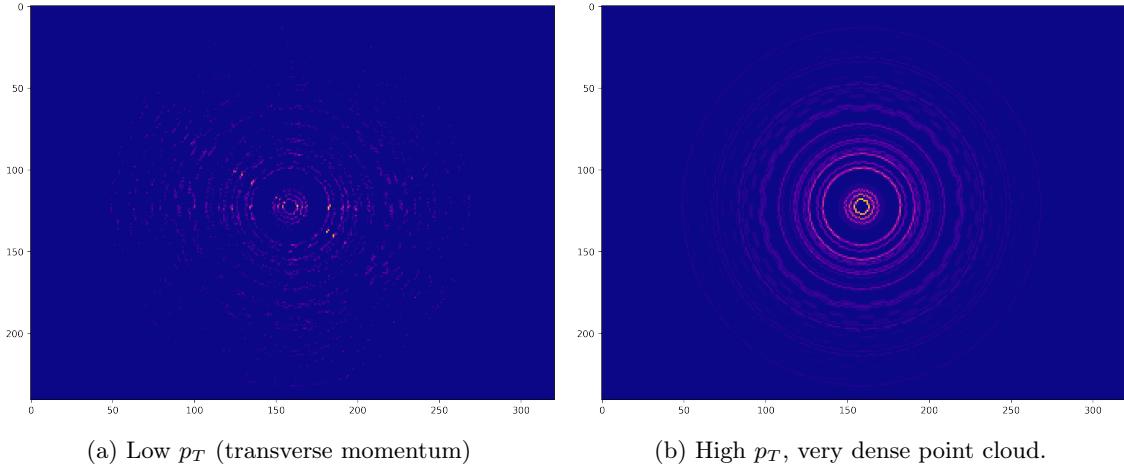


Figure 5.2: Pixel-binned data representation of the hits which occurred in two different collisions.

Therefore, in a final system architecture, we could combine several ideas presented in this thesis to build a system which is able to handle such a data-scale.

Chapter 6

Conclusion

This work

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Appendices

.1 MNIST Model

This Section describes the model that has been used in smaller experiments throughout this thesis. In essence it is a very simple model, the model consists of several layers with relu activations, where finally we apply *softmax* to the output layer.

```
mlp = Sequential()
mlp.add(Dense(1000, input_shape=(784,)))
mlp.add(Activation('relu'))
mlp.add(Dense(2000))
mlp.add(Activation('relu'))
mlp.add(Dense(1000))
mlp.add(Activation('relu'))
mlp.add(Dense(10))
mlp.add(Activation('softmax'))
```