MSc Thesis Progress Report

Meeting with Kai Zehmisch — February 2, 2026

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Executive Summary

Topic: Computational investigation of Viterbo's Conjecture boundary cases after HK-O 2024 counterexample.

Timeline: Target March 2026 submission (~8 weeks remaining). Schedule at risk due to unresolved correctness blockers (#155/#144).

Current Phase: Algorithm implementation complete, correctness validation in progress.

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1. Project Overview

1.1 Research Question

Viterbo's Conjecture (systolic ratio ≤ 1 for convex bodies) was disproved by Haim-Kislev & Ostrover (2024) with a 10-facet Lagrangian product polytope achieving sys ≈ 1.047.

This thesis asks: What is the true boundary? Can we discover refined conjectures through systematic computational exploration?

1.2 Approach

1. **Implement capacity algorithms** — compute $c_{EHZ}(K)$ for polytopes in $\mathbb{R}^4$
2. **Build polytope dataset** — systematic families + random sampling
3. **Run experiments** — characterize where counterexamples occur
4. **Formulate conjectures** — based on computational evidence

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2. Completed Work

2.1 Algorithm Implementation (Rust)

Algorithm	Status	Complexity	Applicability
HK2017	Implemented, known bugs (#155)	$O(F!)$	Any polytope, $F \leq 10$ facets
Tube	Implemented, known bugs (#155)	Branch-and-bound	Polytopes without Lagrangian 2-faces
Billiard	Design complete, impl pending	$O(E^6)$	Lagrangian products only

Test status: Fixture tests pass (known polytopes); random polytope tests fail (#155).

Validation status: No cross-algorithm validation. Fixture tests pass individually: - HK2017 on tesseract: c = 4.0 (verified) - Tube on cross-polytope: c = 1.0 (verified)

But no overlapping polytope tested by both algorithms.

2.2 Experiments Run (findings not yet validated)

Experiment	Preliminary Finding
benchmark_hk2017	Runtime $\approx 1\mu s$/permutation, linear scaling ($R^2 > 0.99$)
algorithm_inventory	Mahler bound exactly saturated by tesseract/cross-polytope dual pair
runtime_performance_analysis	Three algorithm hotspots identified (40-45% in core loop)

Note: These findings depend on algorithm correctness, which is not yet established.

2.3 Thesis Writing

Chapter	Lines	Status
math.tex	483	Core definitions complete

Chapter	Lines	Status
algorithms.tex	652	Tube needs polish; HK2017 + Billiard simpler but missing; Clarke proof not yet migrated; figures need digital redraw
counterexample.tex	67	Stub
experiments.tex	29	Stub
conjectures.tex	9	Stub
intro.tex	14	Stub (can draft early)

Also completed: Clarke duality talk (given, proof to be migrated).

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3. Current Blockers

3.1 Critical Path

Correctness validation (highest priority):

```
#155 Random polytope test failures (both HK2017 and Tube)
#144 No cross-validation between HK2017 and Tube
  ↓ blocks confidence in
All experiments
```

Billiard algorithm path:

```
#92 Billiard thesis section (has design notes, needs formal writeup)
  ↓ blocks
#93 Billiard test suite (includes triangle×triangle validation)
  ↓ blocks
#112 Algorithm performance comparison
```

Status: Active troubleshooting on #155+#144. Root cause under investigation.

3.2 Planned Experiments (blocked by #155/#144)

Issue	Experiment	Description
#96	algorithm-comparison	HK2017 vs Tube on non-Lagrangian polytopes
#100	billiard-hko-orbit	Validate HK-O pentagon counterexample

Issue	Experiment	Description
#101	random-polytope-sys	How rare are counterexamples?
#102	lagrangian-product-polygons	Regular polygon products study
#105	dataset-dimension-reduction	PCA/UMAP on polytope features
#106	sys-ratio-optimization	Gradient flow toward maximum sys

All experiments blocked until algorithm correctness is established.

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4. Roadmap

4.1 Milestones

Milestone	Target	Status
M4: Algorithm Toolbox	—	6/12 issues closed
M6: Dataset Characterized	—	1/5 issues closed
M8: Thesis Submission	End of March	3/5 issues closed

4.2 Remaining Work (Rough Effort Estimates)

Legend: - **Agent hrs** — parallelizable, results in wait time (can run multiple agents) - **Jörn hrs** — not parallelizable, the actual bottleneck

Task	Agent	Jörn	Blocked By
#155+#144 Random polytope + cross-validation fix	8	2	— (in progress)
#92 Billiard section writeup	2	6	—
#93 Billiard test suite	12	3	#92
#96 Algorithm comparison	2	0.5	—

Task	Agent	Jörn	Blocked By
#100 HKO orbit validation	2	0.5	—
#101 Random polytope sys distribution	2	0.5	—
#102 Lagrangian product polygons	8	0.5	—
Thesis writing + editing	16	96	experiments done
Total	**~52**	**~109**	

Critical path: ~109 Jörn-hours cannot be parallelized. With ~8 weeks remaining and part-time availability, this is the binding constraint.

4.3 Velocity

Project timeline: Thesis started ~Oct 14, 2025. Repo created Nov 15, 2025.

Phase	Dates	Commits
Setup	Nov 10 - Dec 1	83
Algorithm dev	Dec 1 - Dec 28	58
Steady progress	Jan 5 - Jan 25	120
Agent sprint	**Jan 26 - Jan 31**	**253**
Total		**509**

Key observation: Agent sprint produced many commits in few days (253 in 6 days vs 261 in prior ~2.5 months).

Context: - Agents available since late 2025, but **parallelization** enabled ~Jan 26 → commit explosion - Before ~Jan 14: blocked on difficult proof (Clarke duality) → led to the talk - Commit-hours undercount proof work, literature review, thinking

Bottleneck shift: Before parallelization, coding time dominated. Now, Jörn's review/direction capacity is the bottleneck.

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5. Risk Assessment

Risk	Likelihood	Impact	Mitigation
Billiard algorithm has deeper bugs	Medium	High	Design notes are thorough; test against HK2017
Experiments yield no new conjectures	Low	Medium	Negative results are still publishable
Time overrun on thesis writing	Medium	Medium	Chapters are scaffolded; experiments generate content

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6. Confidence in Correctness

6.1 What We Have Verified

1. **Known values:** Individual fixture tests pass and match literature:
   - HK2017 on tesseract: $c = 4.0$ (test passes, matches literature)
   - Tube on cross-polytope: $c = 1.0$ (test passes, matches literature)
   - No cross-algorithm comparison exists (no polytope tested by both)
2. **Mathematical properties checked:**
   - Scaling: $c(\lambda K) = \lambda^2 c(K)$
   - Mahler bound: $c(K) \cdot c(K^\circ) \leq 4$
   - Constraint satisfaction: $\sum \beta_i h_i = 1$, $\sum \beta_i n_i = 0$
3. **Test status:** Fixture tests pass; random polytope tests fail (#155).

6.2 Known Gaps (Critical)

[!] Tube lacks effective cross-validation (#144): - The HK2017 vs Tube comparison test can pass with **0 successful comparisons** - Capacity axioms alone don't prove we compute c_{EHZ} vs some other capacity-like function - **Status:** Under investigation (#155+#144).

Other gaps: - Billiard algorithm: design only, not yet implemented/tested - HK thesis simplices claim: not yet verified (blocked on document access) - Large polytopes ($F > 10$): only Tube applicable, no cross-check

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7. Wasteful Efforts Summary

Infrastructure overhead (not quantified): - Claude Code web environment instability (crashes, state loss) - CI/environment setup iterations - Agent workflow refinement (commands, conventions)

Lesson learned: Local development environment (CC CLI) is more stable than CC web. Agent parallelization requires upfront investment in conventions but pays off quickly.

Net assessment: Infrastructure overhead was real but acceptable for a research project exploring new tooling. The agent-assisted workflow now runs smoothly.

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8. Questions for Discussion

1. Is the experiment roadmap appropriately scoped for March deadline?
2. Should billiard algorithm be prioritized, or focus on HK2017/Tube experiments only?
3. Are there specific polytope families Kai recommends investigating?

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9. Experiment Prioritization

Discussion point for Kai: Jörn can estimate implementation effort, but Kai may have better intuition for mathematical interestingness. The table below shows Jörn's effort/utility estimates — Kai's input on "expected mathematical value" could adjust priorities.

Priority Assessment

Issue	Experiment	Utility	Effort	Notes
#96	algorithm-comparison	High	Mid	Cross-validates HK2017 vs Tube
#100	billiard-hko-orbit	High	Mid	Validates HK-O counterexample + non-Lagrangian perturbation for Tube

Issue	Experiment	Utility	Effort	Notes
#101	random-polytope-sys-distribution	**High**	Mid	**Critical:** "How rare are counterexamples?" — informs all downstream experiments
#102	lagrangian-product-polygons	Mid	Low	Systematic study of counterexample family
#105	dataset-dimension-reduction	Low	Low	PCA/UMAP exploration
#106	sys-ratio-optimization	Low	Mid	Gradient flow to max sys
#110	lagrangian-product-random-polygons	Mid	Low	Same as #102 but random polygons (code reuse)
#111	fixed-facet-vertex-count	High	Mid	Verify CH2021 simplex observation (sys <= 3/4 for 5-facet)
#112	algorithm-performance-comparison	High	Mid	Blocked on billiard impl
#113	algorithm-optimization-ablation	Low	High	Performance tuning

Issue	Experiment	Utility	Effort	Notes
#114/#115	ML capacity prediction	Low	High	Similar experiments (merge?); blocked on dataset

Recommended Order

1. **#101** (critical path: informs ML experiments, gives "fraction of counterexamples" result)
2. **#100** (validates the key counterexample)
3. **#96** (cross-validation builds confidence)
4. **#102** → **#110** (code reuse, systematic exploration)
5. **#111** (verify HK simplex claim)
6. Rest as time permits

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10. Full Experiment Inventory

Completed (with FINDINGS.md): | Experiment | Description | |————|————-| | benchmark_hk2017 | Runtime scaling analysis | | algorithm_inventory | Survey existing approaches (not yet reviewed by Jörn) | | runtime_performance_analysis | Profile Tube hotspots (not yet reviewed by Jörn) |

Open experiments: See prioritization table above (§9).

Note: This list is not exhaustive. More experiments will be designed after initial results reveal the Viterbo landscape.

Appendix: Print Thesis Chapters

For detailed algorithm specifications, print: - `packages/latex_viterbo/chapters/algorithms.tex` (Tube algorithm) - `packages/latex_viterbo/chapters/math.tex` (definitions)

For experiment results, print: - `packages/python_viterbo/src/viterbo/experiments/*/FINDINGS.md`