CSCI 1051 Problem Set 3

January 20, 2025

Submission Instructions

Please upload your solutions by **5pm Friday January 24, 2025.**

- You are encouraged to discuss ideas and work with your classmates. However, you **must acknowledge** your collaborators at the top of each solution on which you collaborated with others and you **must write** your solutions independently.
- Your solutions to theory questions must be written legibly, or typeset in LaTeX or markdown. If you would like to use LaTeX, you can import the source of this document here to Overleaf.
- I recommend that you write your solutions to coding questions in a Jupyter notebook using Google Colab.
- You should submit your solutions as a **single PDF** via the assignment on Gradescope.

Problem 1: Image Embeddings

In this problem, we will embed images using an autoencoder that you train from scratch. I recommend that use one of the datasets from the MNIST family.

Part A: Autoencoder Training

Train a small (three or so layers with activations) autoencoder to produce embeddings in two dimensions via reconstruction loss. I suggest writing one class for the encoder and one for the decoder.

Part B: Plots

Take (a subset of) images in your training data, encode them with the encoder portion of the autoencoder and plot them on a scatter plot.

Take a grid of points (about 10 by 10) in the range of points from the prior plot and pass them through your decoder. Now plot the resulting images on a grid based on their latent dimension.

Part C: Variational Autoencoder

Train a variational autoencoder with reconstruction loss and variational loss simultaneously. Remember that the encoder will reproduce two vectors that you will interpret as the mean $\boldsymbol{\mu} \in \mathbb{R}^2$ and standard deviation $\boldsymbol{\sigma} \in \mathbb{R}^2_+$ of the latent vector. You can find the latent vector by computing $\mathbf{z} = \boldsymbol{\mu} + \boldsymbol{\sigma}\boldsymbol{\epsilon}$ where $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{I})$.

The KL divergence (aka cross entropy) loss simplifies to something like

$$\sum_{i=1}^{2} -\log(\sigma_i) + \sigma_i^2 + \mu_i^2. \tag{1}$$

Part D: Variational Plots

Reproduce the same plots that you created above for the variational autoencoder. What do you notice?