Econ 430: Homework1

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Python

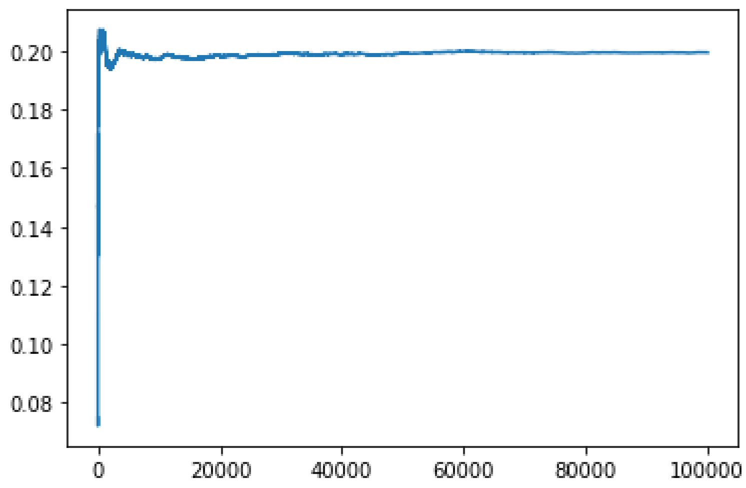
Import library

```
import numpy as np
import matplotlib.pyplot as plt
import math
import random
import sympy
import bootstrapped.bootstrap as bs
import bootstrapped.stats_functions as bs_stats
import scipy.stats as st
```

Exercise 1

```
x_e1 = np.random.exponential(0.2, size=100000)
M_e1 = np.array([np.mean(x_e1[0:i+1]) for i in range(100000)])
plt.plot(M_e1)
```

```
[<matplotlib.lines.Line2D at 0x1a1c481f860>]
```



From the plot, we can know that they are converging to 0.2.

To calculate the speed of converging, I set a value named "err", use a for loop to find when the sequence can reach the range of [5-err,5+err] (when both two values can reach [5-err,5+err])

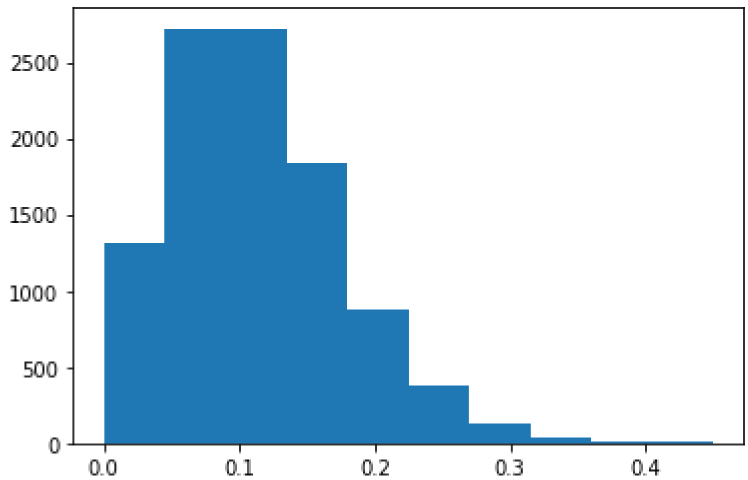
```
err = 0.001
for i in range(100000):
    if abs(M_e1[i]-0.2) < err and abs(M_e1[i+1]-0.2) < err:
        print("How quickly? Steps: ",i)
        break
```

```
How quickly? Steps:  164
```

Exercise 2

```
x_e2 = np.zeros([20,10000])
for i in range(20):
    x_e2[i,:] = np.random.binomial(10,0.01,size = 10000)
M_20 = np.array([np.mean(x_e2[:,i]) for i in range(10000)])
plt.hist(M_20, bins = 10)
```

```
(array([1306., 2717., 2721., 1832.,  873.,  372.,  125.,   41.,    9.,
           4.]),
 array([0.   , 0.045, 0.09 , 0.135, 0.18 , 0.225, 0.27 , 0.315, 0.36 ,
        0.405, 0.45 ]),
 <a list of 10 Patch objects>)
```



This graph reaches the max point in 0.05. This graph firstly rise up and then drop down. There is little number of points when M_20 reaches 0.3.

Exercise 3

(a)

Question about (X1,...X9)'s distribution: what is the parameter of binomial? 1 or 9?
I use np.random.binomial(1,0.4,size) to generate the sample.

```
x_e3 = np.zeros([9,100000])
for i in range(9):
    x_e3[i,:] = np.random.binomial(1,0.4,size = 100000)
Y = np.sum(x_e3,axis = 0)
mostclose = 0
dict_close = {}
for i in range(int(min(Y)),int(max(Y)+1)):
    for j in range(int(min(Y)),int(max(Y)+1)):
        value_temp = np.shape(Y[Y<=i])[0]/100000 + np.shape(Y[Y>=j])[0]/100000
        if value_temp <= 0.1 and value_temp > mostclose:
            dict_close[value_temp] = [i,j]
            mostclose = value_temp
print("The best [c1,c2] is ",dict_close[max(dict_close.keys())])
```

```
The best [c1,c2] is  [1, 7]
```

(b)

```
print("The size of test is ",max(dict_close.keys()))
```

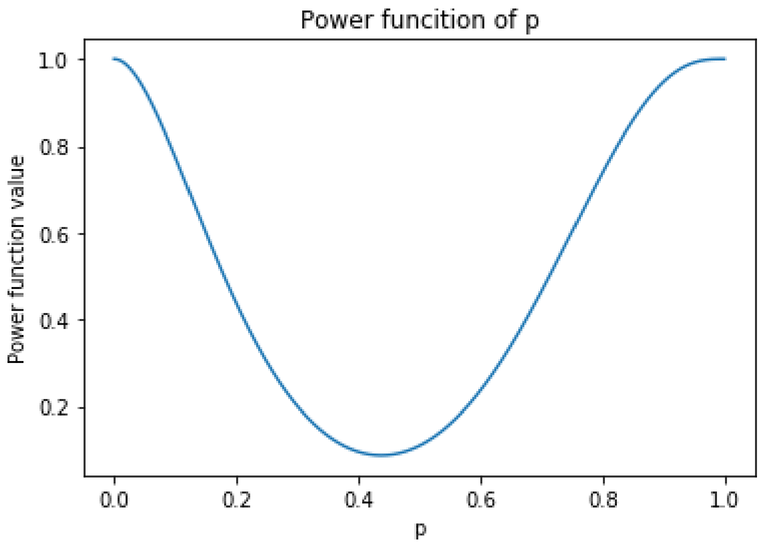
```
The size of test is  0.09340999999999999
```

(c)

The power function: Y<=1 and Y>= 7

```
from scipy.special import comb
e3cx = np.linspace(0,1,1001)
e3result = np.zeros([1,1001])
for i in range(np.shape(e3cx)[0]):
    e3result[0,i] = sum([comb(9,j)*(e3cx[i]**j)*((1-e3cx[i])**(9-j)) for j in
[0,1,7,8,9]])
plt.plot(e3cx, e3result[0,:])
plt.xlabel("p")
plt.ylabel("Power function value")
plt.title("Power funcition of p")
```

```
Text(0.5, 1.0, 'Power funcition of p')
```

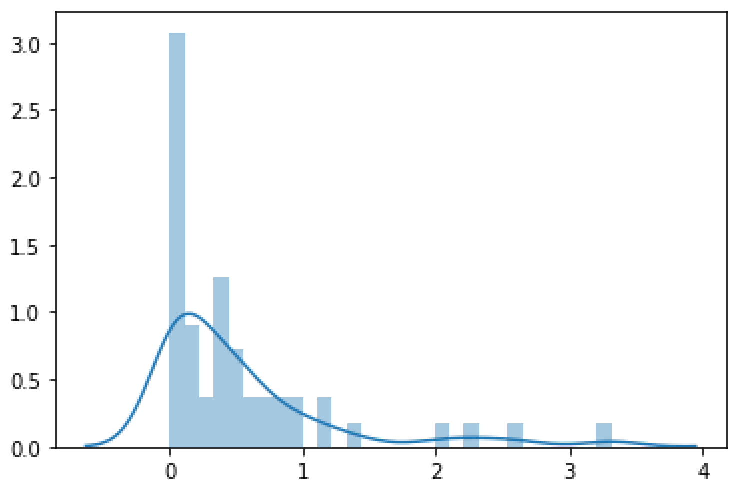


Exercise 4

(a)

```
import seaborn as sns
e4b = np.loadtxt(r"C:\Users\chenyixian\Desktop\Homework\430\Prob4_data.txt")
sns.distplot(e4b,bins = 30)
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x6d3a59ea58>
```



(b)

```
e4b = np.loadtxt(r"C:\Users\chenyixian\Desktop\Homework\430\Prob4_data.txt")
alpha4 = sympy.symbols('alpha4')
beta4 = sympy.symbols('beta4')
sympy.solve([alpha4/beta4 - np.mean(e4b),alpha4/beta4**2 - np.std(e4b)**2],
[alpha4,beta4])
```

```
[(0.580541866754351, 1.08184910505451)]
```

(c)

```
e4b = np.loadtxt(r"C:\Users\chenyixian\Desktop\Homework\430\Prob4_data.txt")
sample4 = np.random.choice(e4b,1000,replace = True)
s4mean = bs.bootstrap(sample4, stat_func = bs_stats.mean)
s4std = bs.bootstrap(sample4, stat_func = bs_stats.std)
print("In bootstrap: \n mean is ",s4mean,"\nstd is ",s4std,"\n")
#[2:6] aims to pick the mean/std value in the bootstrap-typ edata
s4mean = float(str([s4mean])[2:6])
s4std = float(str([s4std])[2:6])
alpha4c = s4mean/(s4std**2)
lambda4c = alpha4c/s4mean
print("In bootstrap: \n alpha is ",alpha4c,"\nlambda is ",lambda4c,"\n")
#95% confidence interval

cinternal = st.t.interval(1-0.05, len(sample4)-1, loc=np.mean(sample4),
scale=st.sem(sample4))
print("95% confidence interval is ",cinternal)
```

```
In bootstrap:
 mean is  0.519344    (0.478460075, 0.5590814500000001)
std is  0.6578065138503875    (0.6046334577807261, 0.7137667840610815)

In bootstrap:
 alpha is  1.202365811110416
lambda is  2.3166971312339424
```

The value of alpha and beta is larger when I use bootstrap than the value when I use method moments.

Exercise 5

(a)

```
label_e5 = list(range(1,10001))
sample_a = random.sample(label_e5,500)
```

(b)

```
label_e5 = list(range(1,10001))
sample_b = []
for i in range(500):
    sample_b.append(random.sample(label_e5,1)[0])
```

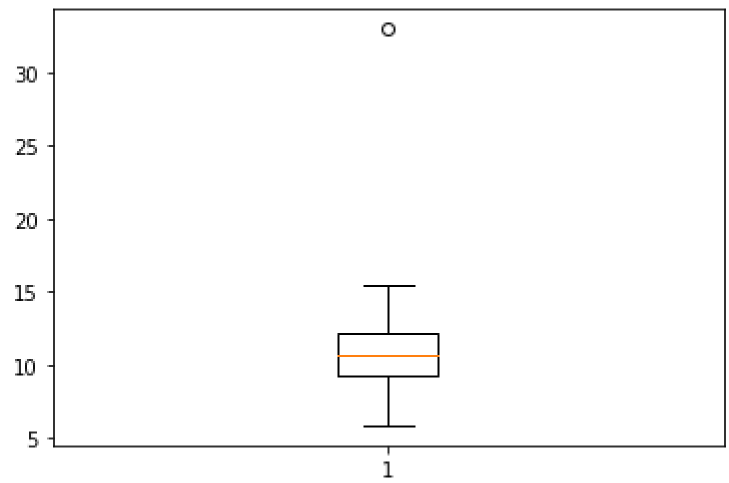
I have a question.
Is random.sample iid.? some articles tell that random.sample can satisify the requirement of iid.

Excercise 6

(a)

```
data30 = np.random.normal(10,2,size = 30)
data1 = np.random.normal(30,2,size = 1)
data31 = np.concatenate([data30,data1])
plt.boxplot(data31)
```

```
{'whiskers': [<matplotlib.lines.Line2D at 0x1a1be899d68>,
  <matplotlib.lines.Line2D at 0x1a1be899e48>],
 'caps': [<matplotlib.lines.Line2D at 0x1a1be8a8438>,
  <matplotlib.lines.Line2D at 0x1a1be8a8780>],
 'boxes': [<matplotlib.lines.Line2D at 0x1a1be899940>],
 'medians': [<matplotlib.lines.Line2D at 0x1a1be8a8ac8>],
 'fliers': [<matplotlib.lines.Line2D at 0x1a1be8a8e10>],
 'means': []}
```



(b)

I notice that there are only one points outside the box, and the value of box is between 9 and 13. The value of line of boxplot is between 6 to 16.

(c)

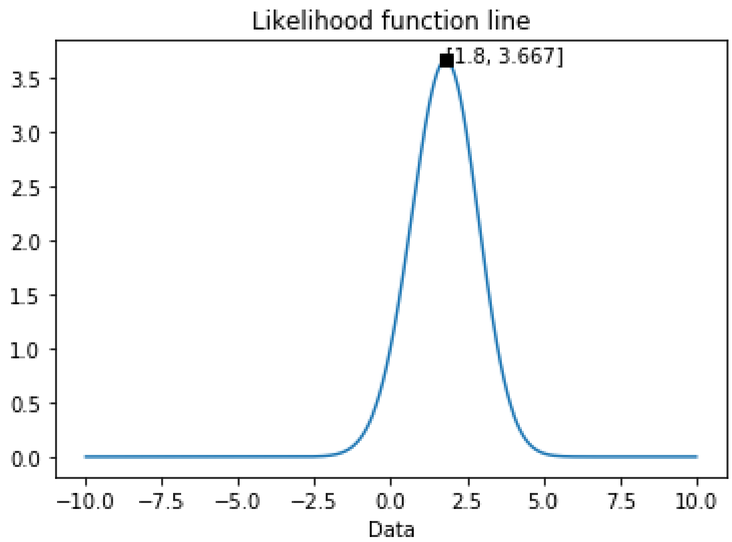
I think the mean, the edge of box can measure the location of distribution,
Because most values is in the box. The edge of box and the mean of it can show what these values are around.

And the end of lines can measure the spread of the distribution. If the line is very long, it shows that the values' distribution and the spread is large. Else, it shows that the values' distribution and the spread is small.

Exercise 7

```python
def expliki(theta):
    return np.exp(-(theta-1)**2/2)+3*np.exp(-(theta-2)**2/2)
datae7 = np.linspace(-10,10,1001)
datae7 = np.delete(datae7,[0])
mle = expliki(datae7)
max_mle = np.argmax(mle)
plt.plot(datae7,mle)
plt.plot(datae7[max_mle],mle[max_mle],'ks')
plt.annotate('['+str(round(datae7[max_mle],3))+',
'+str(round(mle[max_mle],3))+']', xy=(datae7[max_mle],mle[max_mle]))
plt.xlabel("Data")
plt.title("Likelihood function line")
```

```
Text(0.5, 1.0, 'Likelihood function line')
```

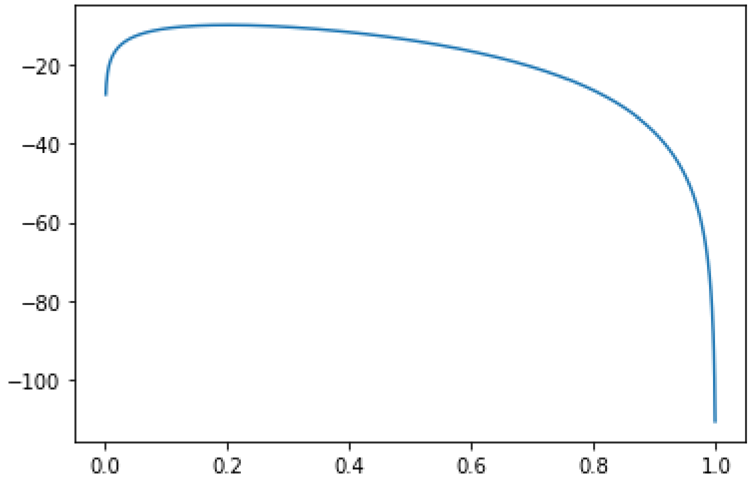


Exercise 8

(a)

```python
def e8(theta):
    return(4*np.log(theta)+16*np.log(1-theta))
datae8 = np.linspace(0,1,1001)
datae8 = np.delete(datae8,[0,1000])
e8data = e8(datae8)
plt.plot(datae8,e8data)
max_index = np.argmax(e8data)
print("MLE, theta is ",round(datae8[max_index],3))
```

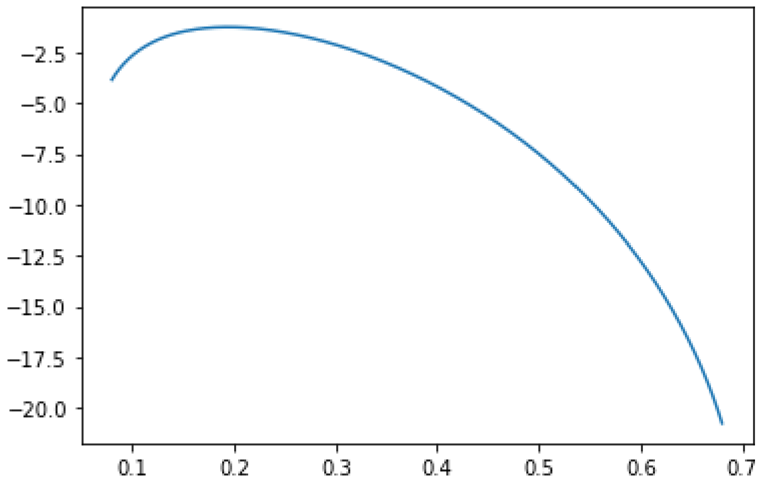
```
MLE, theta is  0.2
```



(b)

```
from scipy.special import comb
def e8b(theta):
    return np.log(comb(50*theta,4)*comb(50-50*theta,16)/comb(50,20))
datae8b = np.linspace(4/50,34/50,1000)
e8bdata = e8b(datae8b)
plt.plot(datae8b,e8bdata)
max_index = np.argmax(e8bdata)
print("MLE, THETA is ",round(datae8b[max_index],3))
```

```
MLE, THETA is  0.194
```



Exercise 9

```
def e9pro(size):
    e9sample = np.zeros([10000,size])
    for i in range(10000):
        e9sample[i,:] = np.random.normal(0,1,size = size)
    e9 = np.zeros([10000,2])
    for i in range(10000):
        e9[i,0] = np.mean(e9sample[i,:])-np.std(e9sample[i,:])/pow(size,0.5)
        e9[i,1] = np.mean(e9sample[i,:])+np.std(e9sample[i,:])/pow(size,0.5)
    pro = np.shape(e9[(e9[:,0]<0) & (e9[:,1]>0)])[0]/10000
    print("With size ",size,", Proportion is ",pro)
e9pro(5)
e9pro(10)
e9pro(100)
```

```
With size  5 , Proportion is  0.5802
With size  10 , Proportion is  0.6332
With size  100 , Proportion is  0.6866
```

Exercise 10

```
sample =
[3.27,-1.24,3.97,2.25,3.47,-0.09,7.45,6.20,3.74,4.12,1.42,2.75,-1.48,4.97,8.00,3
.26,0.15,-3.64,4.88,4.55]
#useing plut-in MLE
mu = np.mean(sample)
sigma = np.std(sample)
print("In plug-in MLE: \n mu is ",mu,"\nsigma is ",sigma,"\n")
#using bootstrapping

samples = np.random.choice(sample,1000,replace = True)
bootmean = bs.bootstrap(samples, stat_func = bs_stats.mean)
bootstd = bs.bootstrap(samples, stat_func = bs_stats.std)
print("In bootstrap: \n mean is ",bootmean,"\nstd is ",bootstd,"\n")

#95% confidence interval

mr = st.t.interval(1-0.05, len(samples)-1, loc=np.mean(samples),
scale=st.sem(samples))
print("95% confidence interval is ",mr)
```

```
In plug-in MLE:
 mu is  2.9
sigma is  2.9214225986666156

In bootstrap:
 mean is  2.9605    (2.777695500000001, 3.1412715)
std is  2.950489103521652    (2.8355049338147267, 3.070758646426798)

95% confidence interval is  (2.7773167116383175, 3.1436832883616828)
```