FIXED INCOME SECURITIES

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Macro-Economic Related Futures Contract (FC)

- Few Good Characteristic of Macro-Economic Related Futures Contract (FC)
- FC that its price is solely function of PRICE of tradable assets.
- The Price of the FC to be primarily influence by Macro Factor that the FC is representing and minimizing idiosyncratic risk. (for example changes in US long term rate, rather than one particular bond)
- As liquid as possible
- Standardize Convention (standard maturity and coupon)
- Long History of standardize price for time series analysis
- Deliverable Contract instead of Cash settle contract.

Macro-Economic Related Futures Contract (FC)

- Macro-Economic Factor Ultra Long Term Default Free Interest Rate.
- On October 20, 2020 There are 19 out standing US government bonds with maturities between 30 & 26 years, coupons between 3.00%-1.25%
- Which one of these bonds should represent the Ultra Long Term Default Free Interest Rate.
- For example let 2/15/2047, 3% coupon be the underlying asset for Future Contract.
- This Bond may not be very liquid.
- There is huge idiosyncratic price risk.

Macro-Economic Related Futures Contract (FC)

- The FC is not standardize. Every day the Maturity is decrease by 1 day and we may not be able to find another 3% coupon bond
- We can’t construct long historical time series of price.
- How to address the above concerns:
  - Standardize the contract, i.e. underlying assets is a always theoretical exactly 30 years 6% US government Bond (note this exact bond usually doesn't exist)
  - Expand the delivery against this contract to include many bonds that their prices are primarily function of macro-economic factor that we concentrating on. (i.e. Factor Ultra Long Term Default Free) . US Bond with maturity greater than 25 years at maturity of the Futures Contract.

Note

- Assume Two Assets with time 0 price $P_A(0) = P_B(0) = 100$.
  - $P_A(0) = \tilde{E}_0\left(\frac{P_A(s_t)}{B(t,s_{t-1})}\right) = P_B(0) = \tilde{E}_0\left(\frac{P_B(s_t)}{B(t,s_{t-1})}\right) = 100$
- We know $probability\ of\ \left(P_A(s_t) = P_B(s_t)\right) = 0$
  - $min\left(\frac{P_A(s_t)}{B(t,s_{t-1})}, \frac{P_B(s_t)}{B(t,s_{t-1})}\right) \leq \frac{P_A(s_t)}{B(t,s_{t-1})}$
  - $min\left(\frac{P_A(s_t)}{B(t,s_{t-1})}, \frac{P_B(s_t)}{B(t,s_{t-1})}\right) \leq \frac{P_B(s_t)}{B(t,s_{t-1})}$
  - $\tilde{E}_0\left(min\left(\frac{P_A(s_t)}{B(t,s_{t-1})}, \frac{P_B(s_t)}{B(t,s_{t-1})}\right)\right) \leq \tilde{E}_0\left(\frac{P_A(s_t)}{B(t,s_{t-1})}\right) = \tilde{E}_0\left(\frac{P_B(s_t)}{B(t,s_{t-1})}\right) = P_A(0) = P_B(0) = 100$
  - $\tilde{E}_0\left(min\left(\frac{P_A(s_t)}{B(t,s_{t-1})}, \frac{P_B(s_t)}{B(t,s_{t-1})}\right)\right) \leq min\left(\tilde{E}_0\left(\frac{P_A(s_t)}{B(t,s_{t-1})}\right), \tilde{E}_0\left(\frac{P_A(s_t)}{B(t,s_{t-1})}\right)\right) = 100$
- $\tilde{E}_0\left(min(x_1, \dots, x_n)\right) \leq min\left(\tilde{E}_0(x_1), \dots, \tilde{E}_0(x_n)\right)$

Futures Contract With Delivery Option

- Time 0, Future Price, maturing at T^* :
- $F(0, T^*) = \tilde{E}\big(P(T^*, S_{T^*})\big)$
- Where $P(T^*, S_{T^*})$ is the price of deliverable asset at maturity
- Note at maturity the Buyer will take the delivery of asset and pays the invoice price of $P(T^*, S_{T^*})$ to the seller.
- If the deliverable asset can be more than one. We need to equalize the final risk adjusted expected invoice price such that at TIME 0, both side would be indifferent to which asset to delivery.
- $\tilde{E}_0[P_0(T^*, S_{T^*})] = \tilde{E}_0[\eta_1 P_1(T^*, S_{T^*})] = \cdots = \tilde{E}_0[\eta_i P_i(T^*, S_{T^*})]$
- $\tilde{E}_0[P_0(T^*, S_{T^*})] = \eta_1 \tilde{E}_0[P_1(T^*, S_{T^*})] = \cdots = \eta_i \tilde{E}_0[P_i(T^*, S_{T^*})]$
- η_j is the Inverse of Conversion Factor for bond j (ICF)

Futures Contract With Delivery Option

- At Maturity if the SELLER has the DELIVERY option :
- The seller will receive $F(T^*, T^*)$ and can delivery any of the above deliverable bond and adjusted the ICF $\eta_i P_i(T^*, S_{T^*})$
- The seller profit is : $F(T^*, T^*) \; - \; \eta_i P_i(T^*, S_{T^*})$
- To maximize profit choose j, $MIN(\eta_1 P_1(T^*, S_{T^*}), \dots, \eta_i P_i(T^*, S_{T^*}))$
- Cheapest to Delivery
- At Maturity if the BUYER has the DELIVERY option:
- At the buyer will pay $F(T^*, T^*)$ and can demand delivery any of the above deliverable bond and adjusted the conversion factor
- The Buyer profit: $\eta_i P_i(T^*, S_{T^*}) - F(T^*, T^*)$
- To maximize profit choose j, $MAX(\eta_1 P_1(T^*, S_{T^*}), \dots, \eta_i P_i(T^*, S_{T^*}))$

Bond futures

- The Chicago Board of Trade lists futures on the Treasury bonds on the 30, 20, 10-, 5- and 2-year Treasury notes. They are very liquid ,specially 20 and 10 year futures.

- You should notice that unlike many future contracts they are deliverable. We have two types of settlement: physical and cash settlement. And these futures are physically deliverable. Though, you can close your position with taking opposite position, and very few contracts are actually delivered.

- It is important that for the delivery, you have many options for which bond to deliver. for example, the 30 year T-bond is based on the 6% coupon and maturity of 20 years. But as you can see in the next table ,they are other deliverable bonds with different yields and coupons which you can deliver instead.

Delivery options

Exhibit 1 -- Deliverable Grade for CBOT US Treasury Futures

Futures Contract	Contract Size (Face Value)	Deliverable Grade
Treasury Bonds	$100,000	US Treasury bonds. Both maturity date and, if callable, the first call date must be at least 15 years from the first day of the contract expiration month.
10-Year Treasury Notes	$100,000	US Treasury notes. Remaining term to maturity must be at least 6 years 6 months and no more than 10 years from the first day of the contract expiration month.
5-Year Treasury Notes	$100,000	US Treasury notes. Remaining term to maturity must be at least 4 years 2 months and no more than 5 years 3 months from the first day of the contract expiration month.
2-Year Treasury Notes	$200,000	US Treasury notes. Original term to maturity must be no more than 5 years 3 months from the first day of the contract expiration month. Remaining term to maturity must be at least one year 9 months from the first day of the contract expiration month and no more than 2 years from the last day of the contract expiration month.

Conversion Factor

- In order to make all those bonds standardized, we use conversion factor which is a rate used to adjust differences in bond values for delivery on a U.S. Treasury bond futures contract. Because the contract calls for the equivalent of an 6% coupon, 20-year maturity bond, it is necessary to convert other securities that are acceptable for delivery.

- After calculating the price of those bonds with given conversion factors we can select the cheapest bond to deliver

Future prices

- If you remember our discussion for future contracts we derived the below formula:

$$H(t,T_1,T_2) = \overset{\cong}{E}_t\left[p(T_1,T_2)\right]$$

And we saw that the future price itself, not the discounted one, is martingale . The future price is the time t expectation of the minimum of deliverable zero-coupon bonds' price trading at time T1

$$H(t,T_1) = \overset{\cong}{E}_t\left[\min(\theta_1 p^{(T_1)}, \theta_2 p^{(T_1)}, ..\theta_n p^{(T_1)}\right]$$

Future prices

- based on Jensen 's inequality:

$$H(t,T_1) = \overset{\cong}{E}_t\left[\min(\theta_1 p_1{}^{(T_1)}, \theta_2 p_2{}^{(T_1)}, ..\theta_n p_n{}^{(T_n)}\right] <= \min_t\left[(\theta_1 \overset{\cong}{E}[p_1{}^{(T_1)}], \theta_2 \overset{\cong}{E}[p_2{}^{(T_1)}], .\theta_n \overset{\cong}{E}[p_n{}^{(T_1)}]\right]$$

$$\min_t\left[(\theta_1 \overset{\cong}{E}[p_1{}^{(T_1)}], \theta_2 \overset{\cong}{E}[p_2{}^{(T_1)}], .\theta_n \overset{\cong}{E}[p_n{}^{(T_1)}]\right] = \min\left[(\theta_1 F_1(t,T_1), \theta_2 F_2(t,T_1), .., \theta_n F_n(t,T_1))\right]$$

θ_i : Conversion Factor for the *ith* deliverable bond

For calculating duration, we should consider this option on selecting the best deliverable of bond, and therefore, it is called option adjusted duration. And for the same reason we need to have option adjusted convexity.

US 10YR NOTE FUT Dec11 TYZ1 130-29+ Trade 10/04/2011 Delivery 12/30/11

Sort Order Decreasing Settle 10/05/2011 Cheapest IRP= -0.020

re-sort Maturity Date Prices in decimal 86 Days Act/ 360

	Cash Security	Price	Source	Conven. Yield	Conver. Factor	Gro/Bas (32nds)	Implied Repo%	Actual Repo%	Net/Bas (32nds)
	Master:								
1)	T 2 $\frac{1}{8}$ 08/15/21	103-02+	BGN	1.783	0.7225	271.586	-32.357	0.070	256.248
2)	T 3 $\frac{1}{8}$ 05/15/21	112-09$\frac{7}{8}$	BGN	1.729	0.7981	250.235	-26.316	0.070	227.334
3)	T 3 $\frac{5}{8}$ 02/15/21	116-26	BGN	1.677	0.8367	232.645	-22.919	0.070	206.164
4)	T 2 $\frac{5}{8}$ 11/15/20	108-04$\frac{3}{8}$	BGN	1.659	0.7728	222.729	-24.480	0.070	203.567
5)	T 2 $\frac{5}{8}$ 08/15/20	108-08$\frac{3}{4}$	BGN	1.619	0.7778	206.157	-22.460	0.070	187.108
6)	T 3 $\frac{1}{2}$ 05/15/20	115-21$\frac{3}{8}$	BGN	1.549	0.8391	185.966	-17.985	0.070	160.262
7)	T 3 $\frac{5}{8}$ 02/15/20	116-18+	BGN	1.507	0.8508	166.073	-15.527	0.070	139.591
8)	T 3 $\frac{3}{8}$ 11/15/19	114-15$\frac{5}{8}$	BGN	1.473	0.8391	148.216	-13.982	0.070	123.446
9)	T 3 $\frac{5}{8}$ 08/15/19	116-03$\frac{7}{8}$	BGN	1.448	0.8582	120.446	-10.469	0.070	93.961
10)	T 3 $\frac{1}{8}$ 05/15/19	112-10$\frac{5}{8}$	BGN	1.410	0.8329	105.190	-9.479	0.070	82.290
11)	T 2 $\frac{3}{4}$ 02/15/19	109-18	BGN	1.379	0.8164	85.692	-7.749	0.070	65.715
12)	T 3 $\frac{3}{4}$ 11/15/18	116-15$\frac{1}{8}$	BGN	1.316	0.8765	55.028	-3.000	0.070	27.449
13)	T 1 $\frac{3}{8}$ 09/30/18	100-07	BGN	1.342	0.7463	80.376	-9.140	0.070	70.574
14)	T 1 $\frac{1}{2}$ 08/31/18	101-06$\frac{1}{4}$	BGN	1.318	0.7607	51.297	-5.158	0.070	40.499
15)	T 4 08/15/18	117-30$\frac{1}{4}$	BGN	1.261	0.8937	30.094	-0.020	0.070	0.815
16)	T 2 $\frac{1}{4}$ 07/31/18	106-07	BGN	1.294	0.8006	44.886	-3.443	0.070	28.631

Hedging

- For hedging purposes it is most desirable to enter future contracts. It is easy to sell futures compared with shorting bond, and the margin is much less for the future contract. But for futures we need to have the option adjusted duration and convexity for calculating duration and convexity of our portfolio.

Here you can see the quotes and specifications of these future contracts:

- http://www.cbot.com/cbot/pub/page/0,3181,830,00.html

Few Notes

- Few Notes : Assume Bond with Cash Flow of CF_i in period i .
- $P_o = \tilde{E}_o \left[\sum_{i=1}^{N} \frac{CF_i(s_i)}{B_i(s_{i-1})}\right]$
- Let us assume we are in period 2 and we received CF_1 and CF_2 in periods 1 & 2 and we have the Bond .(Note that assume we invested CF_1 in MM account.
- Our Wealth in period 2 is :
- $\frac{CF_1}{B_1} B_2(s_1) + CF_2 + \tilde{E}_2 \left[\sum_{i=3}^{N} \frac{CF_i(s_i)}{B_i(s_{i-1})}\right] B_2(s_1) = \frac{CF_1}{B_1} B_2(s_1) + CF_2 + P_2(s_2)$
- $P_o = \tilde{E}_o \left[\sum_{i=1}^{N} \frac{CF_i(s_i)}{B_i(s_{i-1})}\right] = \tilde{E}_o \left[\frac{CF_1}{B_1} \frac{B_2(s_1)}{B_2(s_1)} + \frac{CF_2(s_2)}{B_2(s_1)} + \frac{P_2(s_2)}{B_2(s_1)}\right]$

Few Notes

- Assume we have Future Contract Expiring in Period 2, we Know, the Future price follows a martingale and it is the risk-adjusted or risk-neutral expectation of the price at time of expiration .
- $F_o = \tilde{E}_0[F_1(s_1)] = \tilde{E}_0[F_2(s_2) = P_2(s_2)] = \tilde{E}_0\left[\sum_{i=3}^{N} \frac{CF_i(s_i)}{B_i(s_{i-1})} B_2(s_1)\right]$