FIXED INCOME SECURITIES

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Constant Maturity Treasury (CMT)

- Constant maturity is the theoretical value of a U.S. Treasury that is based on recent values of auctioned U.S. Treasuries. The value is obtained by the U.S. Treasury on a daily basis through interpolation of the Treasury yield curve which, in turn, is based on closing bid-yields of actively-traded Treasury securities. It is calculated using the daily yield curve of U.S. Treasury securities.
- Constant maturity yields are often used by lenders to determine mortgage rates. The one-year constant maturity Treasury index is one of the most widely used, and is mainly used as a reference point for adjustable-rate mortgages (ARMs) whose rates are adjusted annually.

Yield Interpolation Example

- Assume we parametrize the continuous yield curve by three important YTM. Instantaneous, 5 years, and 10 years $(y_0, y_5\ and\ y_{10})$
- YTM for any other maturity is a function of these three yields
- Let us Assume Linear Model
  - $y_t = y_0 + \frac{t}{5}(y_5 - y_0) \qquad 0 \le t \le 5$
  - $y_t = y_5 + \frac{t-5}{5}(y_{10} - y_5) \qquad 5 \le t \le 10$
- If we were able to observe $(y_0, y_5\ and\ y_{10})$, then we can construct of a continuous yield curve perfectly given the above linearity assumption.
- However we only observe coupon bearing bond prices
- Our objective is estimate $(y_0, y_5\ and\ y_{10})$, from observed bond prices

Yield Interpolation Example

- Assume we have bond that pays annual coupon c with the first payment at $0 < t_0 < 1$ and last coupon and par of 100 at $t_0 + 6$
- The price of this bond P_0 can be written as:
- $P_0 = \sum_{i=0}^{6} c \left[\frac{1}{1+y_{t_0+i}}\right]^{t_0+i} + 100 \left[\frac{1}{1+y_{t_0+6}}\right]^{t_0+6}$

Yield Interpolation Example

- In this linear model we approximate y_{t_o+i} by $\hat{y}_{t_o+i}$ where $\hat{y}_{t_o+i}$
  - $\hat{y}_{t_o+i} = y_0 + \frac{(t_0+i)}{5}(y_5 - y_0)\ for\ i = 0,1,2,3,4$
  - $\hat{y}_{t_o+i} = y_5 + \frac{(t_0+i-5)}{5}(y_{10} - y_5)\ for\ i = 5,6$
- With this approximation we obtain $\hat{P}_0$ which is only function of three yields $y_0, y_5\ and\ y_{10}$
  - $\hat{P}_0 = \sum_{i=0}^{6} c\left[\frac{1}{1+\hat{y}_{t_0+i}}\right]^{t_0+i} + 100\left[\frac{1}{1+\hat{y}_{t_0+6}}\right]^{t_0+6}$
  - $e_0(y_0, y_5, y_{10}) = P_0 - \hat{P}_0$
- Estimate y_0, y_5, y_{10} by minimizing $e_0(y_0, y_5, y_{10})$ in some metric (MSE)

Yield Interpolation Example

- If we have "n“ bonds we will have n approximation errors
- $e_i(\hat{y}_0) = P_i - \hat{P}_i \quad i = 0, \dots, n-1$
- Estimate y_0, y_5, y_{10} to minimize the joint errors in some metrics for example
- $\hat{y}_0, \hat{y}_5, \hat{y}_{10} = min_{y_0, y_5, y_{10}} \sum_{i=0}^{n-1} [e_i(y_0, y_5, y_{10})]^2$