Pairs Trading and Cointegration

Cointegration definitions

Ey(t) - white noise N(0, Sy)

Ex(t) - white noise N(0, Sx)

y(t) and x(t) are non-stationary time series, but y(t) - g * x(t) is stationary

y(t) and x(t) are cointegrated

Error Correction representation

$y(t) - y(t-1) = Ay * (y(t-1) - g * x(t-1)) + Ey (t)$

$x(t) - x(t-1) = Ax * (y(t-1) - g * x(t-1)) + Ex (t)$

Spread = $y(t-1) - g * x(t-1)$

Ay - convergence rate to “long term equilibrium”

Sign(Ay) = -1 * Sign(Ax)

Stock and Watson (Common Trend Model)

$y(t) = Ny\ (t) + Ey(t)$ (1)

$z(t) = Nz(t) + Ez(t)$ (2)

Ny, Nz - non-stationary trends Ey, Ez - stationary components

$y(t) - g\ z(t)$ - cointegrated combination

Expanding (1) and (2) we find $Ny(t) = g * Nz(t)$

The stochastic trend of the first series must be the same as a trend of the second series up to a multiplier

Cointegrated Stock Prices

$\mathrm{Log}(Pa(t)) - \mathrm{Log}(Pa(t-1)) = Aa * (\mathrm{Log}(Pa(t-1) - g * \mathrm{Log}(Pb(t-1)) + Ea(t)$

$\mathrm{Log}(Pb(t)) - \mathrm{Log}(Pb(t-1)) = Ab * (\mathrm{Log}(Pa(t-1) - g * \mathrm{Log}(Pb(t-1)) + Eb(t)$

Find Aa, Ab, and g

Long 1 share of A and short g shares of B, return of the portfolio:

$(\mathrm{Log}(Pa(t+1)) - \mathrm{Log}(Pa(t)) - g * (\mathrm{Log}(Pb(t+1)) - \mathrm{Log}(Pb(t))) = \mathrm{Spread}(t+1) - \mathrm{Spread}(t)$

Trading Strategy

Log(Pa(t)) - g * Log(Pb(t)) = m - D   (Buy) (1)

Log(Pa(t+1)) - g * Log(Pb(t+1)) = m + D (Sell)

Expected Profit 2 * D

Consider two stocks A and B that are cointegrated with the following data:

Cointegration Ratio = 1.5   → g

Delta used for trade signal = 0.045 ---> D

Bid price of A at time t = $19.50

Ask price of B at time t = $7.46

Trading Strategy (2)

Ask price of A at time t + i = $20.10

Bid price of B at time t + i = $7.17

100 bps = 1%

Average bid-ask spread for A = .0005 percent (5bps (basis points))

Average bid-ask spread for B = .0010 percent (10 basis points)

The Strategy We first examine if trading is feasible given the average bid-ask spreads.

Average trading slippage = (0.0005 + 1.5 × 0.0010) = .002 (20 basis points)

Trading Strategy (3)

This is smaller than the delta value of 0.045.

Trading is therefore feasible.

At time t, buy shares of A and short shares of B in the ratio 1:1.5.

Spread at time t = log (19.50) – 1.5 × log (7.46) = –0.045

At time t + i, sell shares of A and buy back shares the shares of B. Spread at time t + i = log (20.10) – 1.5 × log (7.17) = 0.045

Total return = return on A + g × return on B = log (20.10) – log(19.50) + 1.5 × (log(7.46) – log(7.17)) = 0.3 + 1.5 × 4.0 = .09 (9 percent)

Trading Strategy (4)

Identify stock pairs that could potentially be cointegrated. This process can be based on the stock fundamentals or alternately on a pure statistical approach based on historical data. Our preferred approach is to make the stock pair guesses using fundamental information.

Once the potential pairs are identified, we verify the proposed hypothesis that the stock pairs are indeed cointegrated based on statistical evidence from historical data. This involves determining the cointegration coefficient and examining the spread time series to ensure that it is stationary and mean reverting.

We then examine the cointegrated pairs to determine the delta. A feasible delta that can be traded on will be substantially greater than the slippage encountered due to the bid-ask spreads in the stocks. We also indicate methods to compute holding periods.

Pairs Selection

Common Trend Model and APT

$\mathrm{Log}(P(t)) = n(t) + e(t)$, $n(t)$ - stochastic trend

$\mathrm{Log}(P(t)) - \mathrm{Log}(P(t-1)) = (n(t) - n(t-1)) + (e(t) - e(t-1))$

$R(t) = Rc(t) + Rs(t)$

$Rc(t)$ - return due to non-stationary trend component

$Rs(t)$ - return due to stationary residual component

APT and cointegration

Observation 1: A pair of stocks with the same risk factor exposure profile satisfies the necessary conditions for cointegration.

Condition 1 Now let us consider two stocks A and B with risk factor exposure vectors g * x and x, respectively.

The factor exposure vectors in this case are identical up to a scalar. We denote the factor exposures as

Stock A: gx = (g * x1,g * x2,g * x3,...,g * xn)

Stock B: x = (x1,x2,x3,...,xn)

Returns:

Ra(t) = g * (x1 * b1 + …. xn * bn) + Ea(t), bi - APT factor returns

Rb(t) = (x1 * b1 + ….+ xn * bn) + Eb(t)

APT and cointegration (2)

Signal To Noise Ratio = Volatility of Residual / Volatility Of Common Trends

Testing residuals

Construct the residual time series by applying the equilibrium relationship to the time series of stocks constructed using the close-close method. If the two series are indeed cointegrated, constructing such a time series provides us with a fairly good picture of the oscillations about the equilibrium value. We begin by reviewing the ideal situation. In an ideal situation for tradability, the two stocks would be cointegrated, and the residual series would be stationary. It is therefore desirable that the properties of the residual series exhibit the characteristics of stationary series.

Tradability

Highly mean-reverting series are also characterized by a high frequency of zero-crossings. A zero-crossing is defined as the transition of the time series across its long-run mean. The frequency of zero-crossing is then the number of times we can expect the time series to cross its equilibrium value in unit time. Thus, the zero-crossing frequency provides us with a quantitative characterization for the mean reversion property. Notice that if the zero-crossing rate is very high, then the time to revert to mean is short, implying that the time we need to hold the paired position is small. The signal-to-noise ratio is bound to be good, and we could be more comfortable with the idea that the pair is tradable. Thus, a high zerocrossing rate for the residual series is a preferred trait and directly appeals to our requirements.

Tradability (1)

A high zero-crossing rate is indicative of a stationary series.

Consider the example of Brownian motion a nonstationary series. The distribution of Brownian motion is symmetric about the mean, the zerocrossing event is very infrequent. The theoretical explanation for the phenomenon is well captured by the famous arcsine law for Brownian motion discovered by P. Levy. The law provides us with information on the last passage time, or the last time that the Brownian motion visited zero. More precisely, let us consider a Brownian motion starting at zero, or time $t = 0$ and stopped at time T. If g is the last time when zero is visited, then the probability distribution satisfies

Tradability (2)

P(g < u) = (2 / Pi) * arcsin (Sqrt(u/T))

Density function = (1/Pi) * 1/Sqrt (u * (T-u))

Risk Arbitrage

When a business wants to raise initial capital to finance its operation, it can generally do it two ways. One way is to borrow money from lenders and pay interest on the borrowed capital. This approach is also called issuing bonds or issuing debt. The other approach is to promise a percentage ownership in the business commensurate with the fraction of capital invested. This is known as issuing equity.

A firm may choose to go entirely one route or alternately issue equity for some portion of the capital and issue debt for the remainder. The percentage of debt and equity that comprise the firm's capital is termed capital structure. It is important to note that the study of capital structure and its impact on firm value is a vast subject area in itself. However, for our purpose, the simplistic definition will suffice. We now move on to Corporate event. A corporate event may be described as an action undertaken by a company that affects its shareholders and/or bondholders. Typical corporate events could be paying of dividends, stock splits, tender offers, mergers, exchange offers, spinoffs, and recapitalizations.

Of these events, the paying of dividends and stock splits do not affect capital structure. The others, however, could potentially alter the capital structure of the firm. So, in the context of risk arbitrage we are interested only in those.

Risk Arbitrage 1

Recapitalizations are situations in which companies deliberately decide to alter their capital structure. The current shareholders or bondholders would receive securities or a combination of cash and new securities in exchange for their shares. Spinoffs occur when a firm splits its business into separate units. Current shareholders receive new shares in the spun-off entity in addition to the current shares owned by them.

A tender offer is a situation that occurs when a company decides to acquire another company. Shareholders of the acquired company receive cash in exchange for their shares. In exchange offers and mergers, shareholders of the acquired company receive shares in the acquiring company in exchange for their shares. Sometimes, in the case of mergers and tender offers, the acquired company shares are exchanged for a combination of cash and shares. In all of these situations the commonality lies in the fact that they involve an exchange of one security for another on a scheduled date in the future. Trading on the price disparity between the two exchanged securities is termed risk arbitrage. How does pairs trading figure in all of this? The answer is quite straightforward. Of the two securities involved in the exchange, we buy the lowerpriced security, sell the higher-priced security, and lock in the price difference for our profit. Note that this is possible only when both the securities in question are traded currently in the open market. In the case of recapitalizations and spinoffs, one side of the exchanged securities is issued afresh and cannot be traded before issue. Keeping with the theme of pairs trading, we therefore focus on mergers and exchange offers.

Risk Arbitrage (2)

Fixed Ratio Stock Exchange

The simplest form of a merger transaction is the stock for stock transaction. Shares of the target are exchanged for shares of the bidder in a fixed exchange ratio. This exchange ratio is determined by the accountants and analysts of the merging companies based on the valuation of the target firm. It is, however, self-evident that in such fixed exchanges the price paid for the target varies with the stock price of the bidder.

Fixed Value Stock Exchange

The fixed value stock exchange may be considered an attempt to do a better job at mitigating the variability. The transaction structure also helps to prevent excessive shorting of the bidder stock on the eve of deal announcements. In such transactions, the dollar value of the target stock is fixed. The exchange ratio is then determined based on the stock price of the bidder during what is known as the pricing period. A commonly used approach is to use the average closing price of the bidder stock in the pricing period to determine the exchange ratio. For example, if the price of the target stock is fixed at $10 and the average of the closing prices of the bidder stock during the pricing period is $20, then the exchange ratio is 0.5; that is, two shares of the target stock is good for one share of the bidder stock. Sometimes, to prevent manipulation of the closing price by arbitrageurs, the volume weighted average price during the pricing period is used to determine the ratio. Thus, in the fixed value approach, the exchange ratio is gradually revealed as the prices unfold over the pricing period. Although, on a rare occasion, the average of the closing prices on a predetermined number of randomly chosen days in the pricing period has also been used to determine the exchange

Risk Arbitrage (3)

Stock and Cash Exchange

The effect of bidder stock volatility on the exchange ratio may be further mitigated by paying for the target stock with a combination of cash and securities. Generally speaking, though, the exchange ratio in this case could be determined using either the fixed ratio or the fixed value method.

Example:

Bidder B pays for target T, 70 percent in stock, 30 percent in cash. The share exchange ratio is 0.5; that is, 1/2 a share of B for one share of T. Cash amount paid for the remaining 30 percent is $20. Based on the preceding specification, let us now compute the exchange on a per target stock basis. Share amount: share percentage × ratio = 0.7 × 0.5 = .35 Cash amount: cash percentage × cash value = 0.3 × $20 = $6.0 Thus, for each share of the target the holder of record would receive 0.35 share of the bidder and $6 in cash. Note that while these sets of transaction terms reduce the dependence of the exchange ratio on the price of the bidder stock, paying partly in cash reduces the amount of equity issued and has an effect on the capital structure of the new entity.

Pair trading Spread for Risk Arbitrage

The transaction terms in a merger or exchange offer create a strict parity relationship between the bidder and target stocks. Violations of this parity relationship can be measured based on the stock prices of both the stocks and is called the spread. To see how the spread is calculated, consider the fact that each target share is exchanged for a fixed number of bidder shares or bidder shares plus cash, the value of which we shall call the exchange value of a target share. This can be calculated exactly with the knowledge of the transaction terms and the current price of the bidder stock. Let us say that we also know the current price of the target stock. The spread is now given as

Spread = exchange value of a target share – current price of target share

The magnitude of the spread is indicative of the disparity and is therefore representative of the profit potential in dollar terms on a per-target share basis. Prior to the date of deal completion, the target shares almost always trade at a discount to their exchange value. This implies that the value of the spread as calculated above is usually positive. Let us see why that is. If the spread were negative—that is, the exchange value of a target share is less than the current price of the target share—then there is not much reason to hold the target stock. One could sell the target stock right away and make more money than waiting until deal close. If the spread is zero, then also it makes no sense to wait until deal completion date, as it is better to cash in right now than wait till deal completion to get the same amount of money. In either of the cases, market participants would begin to sell the target stock until the value of the spread is no longer negative or zero. Thus, one can expect the spread to be positive.

Detailed Example

Merger between Intel Corporation (INTC) and Level Communications (LEVL). The deal was announced on March 4, 1999. INTC was the bidder, and LEVL was the target. The exchange ratio was 0.86; that is, every share of LEVL was exchanged for 0.86 share of INTC. The deal was completed on August 10, 1999.

In the example of INTC and LEVL, trade would be to short 0.86 share of INTC against every share of LEVL that we buy. This is done on the day of deal announcement. We would thus pocket about $4.26. When the deal is complete, the positions are reversed. This could be done in one of two ways. One way to do it is to exchange the target shares for the bidder shares and use those to close out the short position on the bidder. Alternately, we could unwind both the positions individually; that is, we sell the shares of LEVL and cover the short position on INTC. Given that the spread value around deal completion is typically zero, we can expect the proceeds from the sale of one share of LEVL would be equal to the price of 0.86 shares of INTC. These proceeds are now used to buy the shares of INTC to cover the short position. Thus, for every share of LEVL we were long, we would have pocketed about $4.26. The following list is an example of a typical rate of return calculation for a deal.

Detailed Example (1)

Deal Details

The spread on the deal is calculated as follows: spread = price of B × 0.35 + 6.0 – price of T

Announcement Terms: 1/2 a share of B to be exchanged for every share of T, that is, ratio = 0.5.

Dividends: No dividends will be paid by B or T

Current stock prices: price(B) = \$20.0, price(T) = \$8.0

Spread: 0.5 × price(B) – price(T) = (0.5 × 20.0) – 8.0 = \$2.0

Estimated Time to Close: 3 – 6 months. (average = 135 days)

The Strategy

1. Purchase two shares of T (\$16.0)

2. Sell Short one share of B \$20.0