Options in Valuation

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VL10 Video Lecture 10

Valuation for Financial Engineers

Option terminology

Futures = obligation

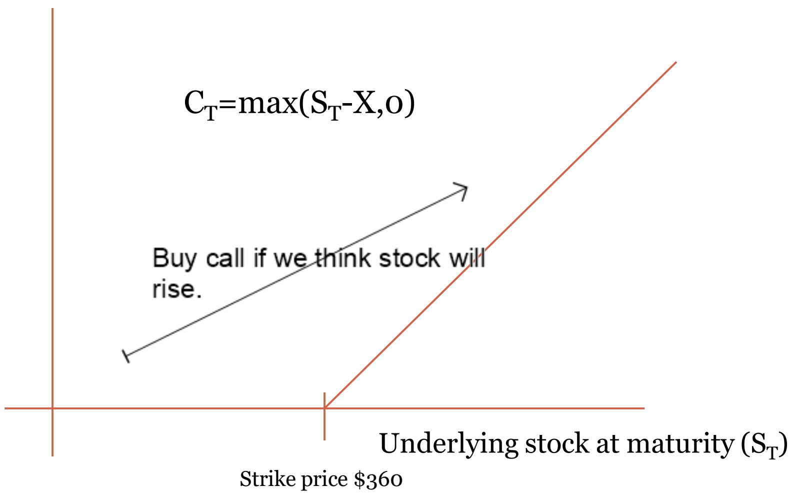
- ***Financial options*** on securities and futures contracts trade on exchanges
  - Options trade on equities, bond futures, interest rate futures, commodity & currency futures
- The right to buy an underlying asset is a ***call option***
- The price of the option is the ***premium***
- The agreed price for future purchase of the underlying is the ***strike price*** or ***exercise price***
- The seller or ***writer*** of a call option has an obligation and unlimited liability, since he must deliver the underlying if the option buyer wants it
- The right to sell an underlying asset is a ***put option***
- The last possible exercise date is called the ***option maturity***, and the length of time until that date is the ***term***.
- The **option payout** at maturity is the value assuming correct exercise.
  - The **net option payout** incorporates the value of the option premium
- An option than can be exercised at any time prior to or at maturity is called an ***American option***, and one that can be exercised only at maturity is called a ***European option***.
- The ***intrinsic value*** of an option is the value if it can be exercised today.
  - The extrinsic value, or time value, is the difference between the full premium and the intrinsic value

Example: AAPL

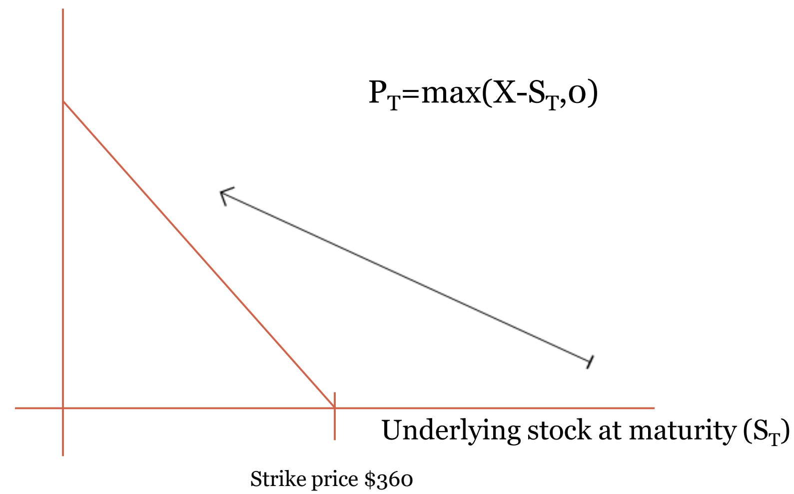
- Suppose Apple Inc.’s shares are trading at $350.
- You own a call option that expires in 17 days with a strike price of $360.
- What is the value of the call option at maturity as a function of the stock price?
  - Show a graph and a formula
  - Repeat for the put option

EUROPEAN

CALL OPTION



PUT OPTION



Name these payoff diagrams

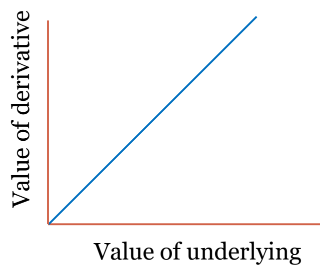
Use these words to name the strategies defined by the blue line(s), such as "Long call net"

Long (+)
Short (–)

Call
Put
Futures
Stock

Gross
Net

1



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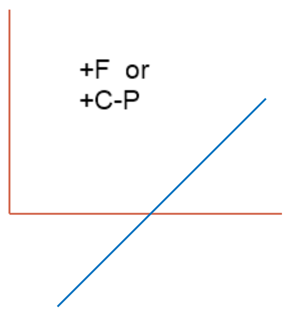

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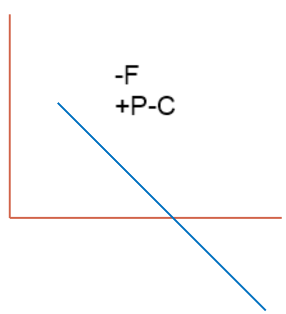

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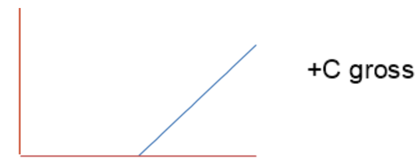

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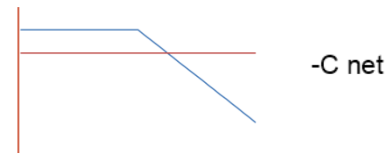

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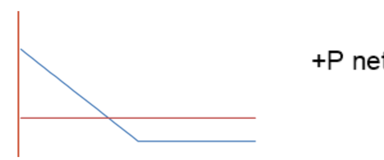

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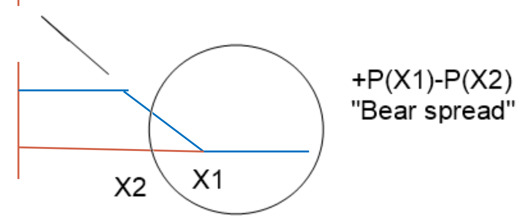

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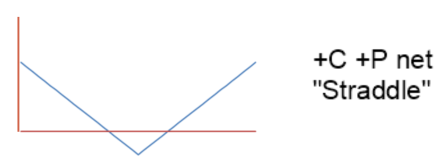
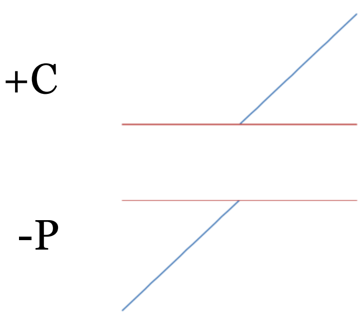
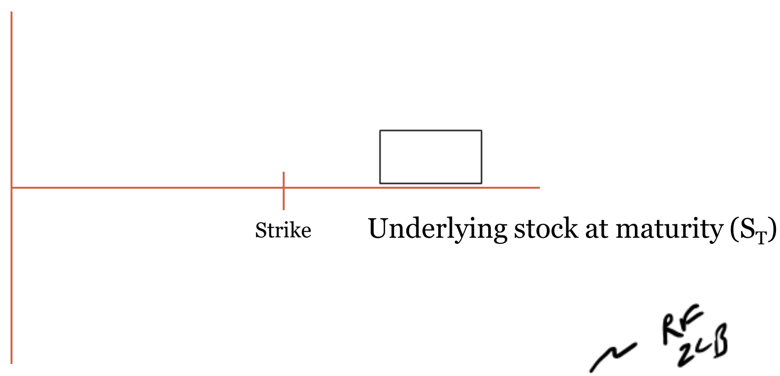


Put-Call Parity in Theory

- Show the payoff diagram for the position +C-P at maturity
- What can you conclude?





RF ZCB

$+S + P = +C + XB$

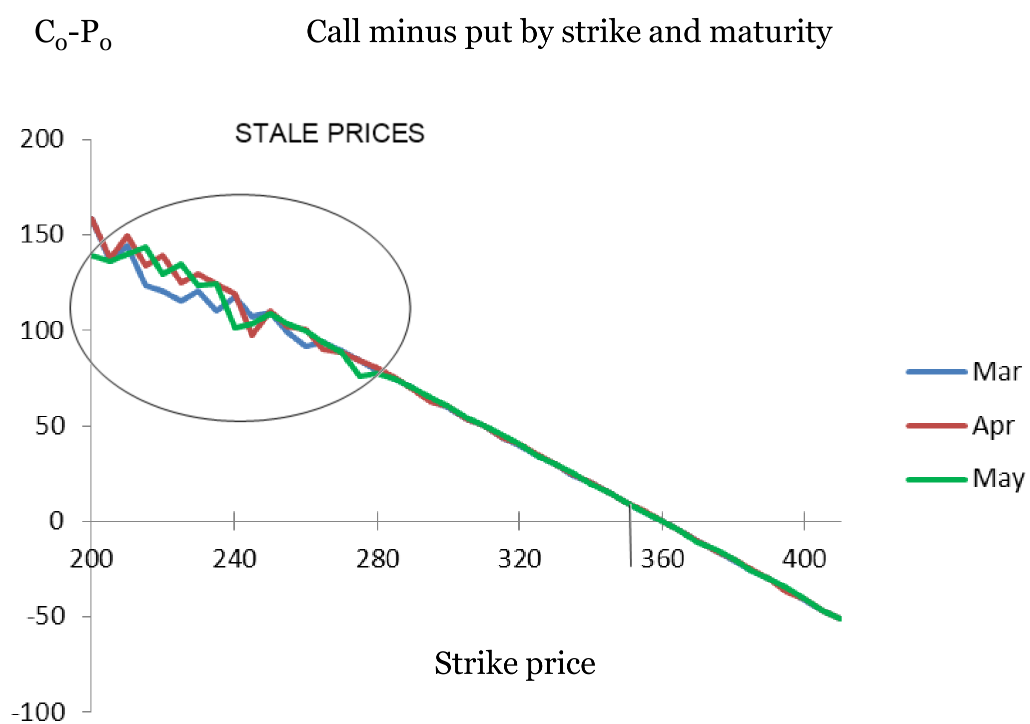
$+P = +C + XB - S$

synethetic put

- This relationship is called put-call $\boxed{\text{parity}}$ for European options
- The equation shows that calls can be priced from puts and vice-versa

Put-Call Parity using AAPL Data

- Long call plus short put forms a straight line
- For European options
  - $C - P = S - D - XB$
    - synthetic long forward
  - $P - C = XB - S + D$
    - synthetic short forward
  - $P = C - S + D + XB$
    - synthetic put
- Interpretations
  - Value equation
  - Risk equation
  - Strategy equivalence +/-
- For American options
  - $S - D - X \leq C - P \leq S - XB$



Notes:
D = PV of dividends
B = e^{-rT}, the ZCB price

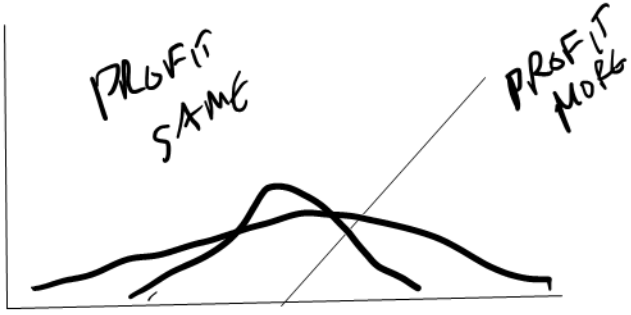
Comparative statics

- How do these factors affect the value of calls and puts?
  - Rising stock prices

    $S\uparrow \Rightarrow C\uparrow, P\downarrow$

  - Increasing risk in the stock price

    $\sigma\uparrow \Rightarrow C\uparrow, P\uparrow$ I ♡ RISK



  - Higher strike prices

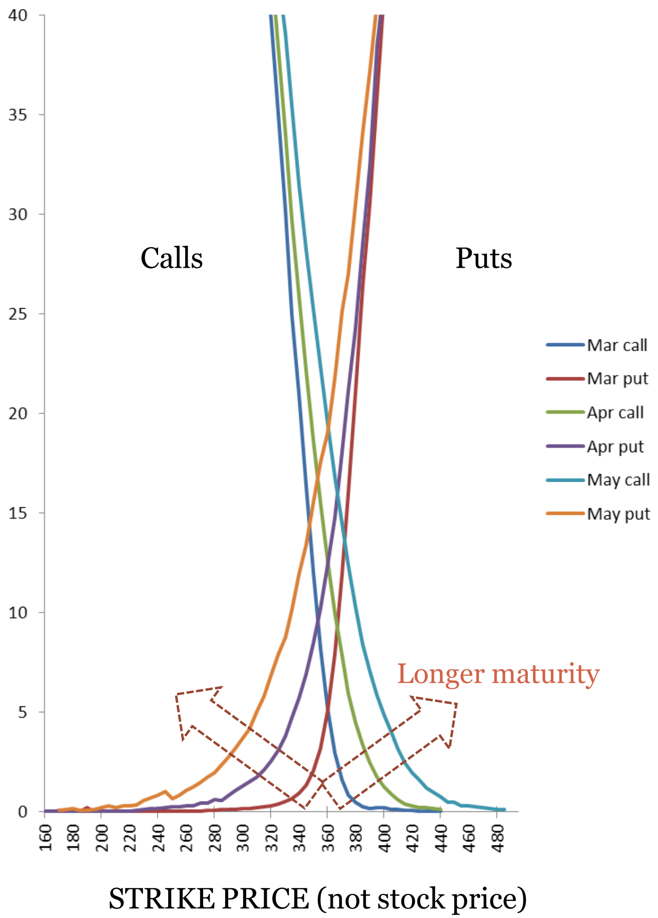
    $X_1 < X_2$ $C(X_1) > C(X_2)$ $P(X_1) < P(X_2)$

  - Increasing time to maturity of the option

    $T_1 < T_2$ $C(T_2) > C(T_1)$

Sample option prices

Apple (AAPL) option prices as of 3/5/11



NOTES:
American style options on CBOE

Puts ↑ at higher strikes

Calls ↓ at higher strikes

Puts and calls ↑↑ with maturity

How do people use options?

- Speculative

  Retail

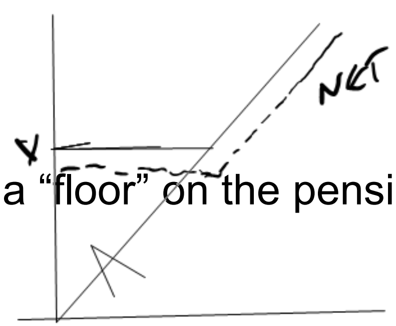
  - Benefit from rising prices using leverage and having downside protection
    - buy a call
  - Benefit from falling prices using leverage and having downside protection
    - buy a put
  - Belief that options are overvalued or undervalued

    Institutionals
    - Buy low, sell high!

    "sales trader"
  - Designing a custom option strategy to take advantage of a view
    - e.g. “I think AAPL will rise $20 in the next month, but not more”
- Hedging

  +S+P (X)

  - Protecting against losses
    - A pension fund buys put options on the market to put a “floor” on the pension fund value



Example of a directional trade

AAPL OPTION CHAIN

Apple Inc. (AAPL)

Last	Change	% Change	Volume	52-Week High	52-Week Low	03/02/11 NASDAQ 10:10 a.m. EST USD
349.82	0.51	0.15%	2,571,811	364.90(02/16/11)	199.25(05/06/10)	

Options Show all months

March 2011 Options: Expand | Hide — In The Money

Calls						Strike Price	Puts					
Last	Chg	Bid	Ask	Volume	Open Int.		Last	Chg	Bid	Ask	Volume	Open Int.
50.60	+1.55	49.65	50.10	1	164	300.00	0.04	0.00	0.03	0.10	1	1,402
44.50		44.75	46.00	0	29	305.00	0.05	-0.01	0.03	0.13	5	708
39.60		39.70	40.10	0	181	310.00	0.08	-0.01	0.01	0.15	4	1,306
34.50		34.75	35.10	0	60	315.00	0.21	+0.06	0.10	0.20	3	1,272
30.45	+1.25	29.85	30.10	22	257	320.00	0.12	-0.07	0.14	0.18	88	4,003
25.10		24.95	25.40	0	212	325.00	0.18	-0.14	0.16	0.19	97	3,124
20.70	+0.85	19.90	20.20	37	1,107	330.00	0.24	-0.24	0.23	0.24	344	4,378
15.95	+0.75	15.00	15.40	1	1,223	335.00	0.36	-0.43	0.38	0.41	258	5,368
10.54	-0.16	10.60	10.75	204	2,313	340.00	0.85	-0.49	0.79	0.84	2,430	7,549
6.70	-0.10	6.55	6.70	1,246	3,242	345.00	1.83	-0.71	1.72	1.76	1,757	6,027
LAST TRADE						349.8064	as of 3/2/2011 10:10 AM					
3.45	-0.36	3.45	3.55	1,806	7,211	350.00	3.60	-0.90	3.55	3.60	1,531	5,298
1.50	-0.30	1.45	1.51	1,781	9,767	355.00	6.45	-0.95	6.45	6.65	528	1,947
0.53	-0.23	0.52	0.54	1,622	9,936	360.00	10.60	-0.70	10.65	10.85	123	589
0.21	-0.08	0.19	0.20	865	7,347	365.00	14.85	-1.00	15.15	15.40	120	271
0.12	-0.01	0.09	0.11	84	3,882	370.00	20.55		19.95	20.35	0	317
0.07	0.00	0.06	0.08	6	2,473	375.00	26.05		25.05	25.50	0	237
0.07		0.02	0.10	0	718	380.00	30.55		29.85	30.40	0	159
0.04		0.02	0.04	0	164	385.00			33.75	36.45	0	0
0.06			0.04	0	144	390.00	41.00		39.55	40.40	0	26
			0.03	0	0	395.00			43.75	46.40	0	0
0.05			0.12	0	20	400.00			48.95	51.35	0	0

Source: wsj.com

- These quotes are as of 3/2/11
- CBOE Options expire on Saturday following the third Friday of the contract month (17 days)
- Apple is trading at 350. If you think Apple is going to trade at 370 in 17 days, which option offers the highest return?

  A. Buy the stock
  20/350 ____%

  B. Buy a call struck at the money
  20/3.55-1 ____% (20-3.55)/3.55

  C. Buy a call struck at 355
  15/1.51-1 ____%

  D. Buy a call struck at 360
  10/0.54-1 ____%

Answers
6%
463%
893%
1752%

- **Incredible leverage with downside protection, but high likelihood of losing entire premium**
- **NOTE: These returns depend on being right!**

Two dimensions of option trading

- Some people trade options to take a view on the underlying stock value

	Call option	Put option
Stock rises	C↑	P↓
Stock falls	C↓	P↑

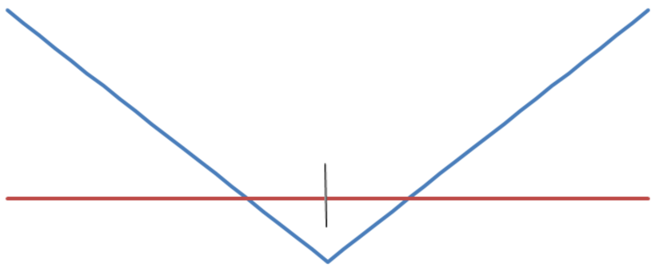
- Others think that the *risk* of the stock is going to change, or that the options are mispriced
  - An increase in risk causes calls and puts to increase in value
  - Why? The option owner's upside is higher, and their downside is limited
  - This improves the average outcome

	Call option	Put option
Volatility rises	C↑	P↑
Volatility falls	C↓	P↓

Trade stock & trade volatility at the same time

Example of a volatility trade

- M&A arbitrage
  company being put up for sale
  - A company is being put into play, and the stock price has risen
  - Competition for company plus risk of failure
  - You think volatility is increasing, no sense of direction
  - What will you do?
- Answer
  - Buy calls and puts
  - Same strike = STRADDLE
  - Sell after they revalue
    - Do not hold until expiration



Combining direction and volatility

view

$S\uparrow, \sigma\downarrow$

- Pharma company
  - You believe Pfizer is going to be successful in getting FDA approval for a new blockbuster drug
  - This will cause the stock to rise and uncertainty about the future stock values to fall

- What strategy?
  - Buy stock? Ok, not great!
    - Benefit from appreciation but not from volatility
  - Buy call? not good
    - Leveraged stock play, but will lose if volatility drops
  - Sell straddle? ok not great
    - Benefit from falling volatility, but no directional benefit
  - Sell put

$S\uparrow \Rightarrow P\downarrow$ \$ $\quad \sigma\downarrow \Rightarrow P\downarrow$ \$

Fill in the option strategy table!

- Use simple combinations of long and short, call, put and forward positions

YOUR VIEW	σ↑	No view	σ↓
S↓	+P	-F +P-C	-C
No view	+C+P		-C-P
S↑	+C	+F +C-P	-P

Fill in +C, -C, +P, -P, +F, -F, or combine any two of these

Key takeaways:

1. Each box implies a different strategy
2. Trade options if you have a view on volatility
3. Without a view, there is no trade!

Takeaways for this part

- The mathematics of options can be complex, but the idea is simple: Buy the call to get the upside, or buy a put to get the downside, the long option position protects you against losing more than you paid
- Selling options can be very risky if done in isolation
- Over shorter time periods, options give investors exposure to both the underlying value and its perceived risk
- This means you can trade considering both dimensions
- For the rest of this presentation, we will use market option prices as benchmarks for valuing more complex cash flows

Valuing Options & Other Risky Cash Flows

What were you supposed to remember from last week? ☺

Newspaper: "Derivatives are contracts whose values are derived from asset prices"

1. Forward contracts are called “derivatives”
2. The long forward assumes the risk of the underlying and therefore must earn the same risk premium.
3. The long forward contract is similar to buying the underlying with 100% leverage
   - It is different in that you don’t have the use of the asset, including the right to receive any cash flows
4. The value of a forward contract is zero at inception, and the forward price is the amount paid for the underlying on the maturity date
5. Forward prices can be derived from the GVE
6. Without dividends, $F_0 = S_0e^{rT}$; with dividends, $F_0 = S_0e^{(r-\delta)T}$

financial forwards/futures

Two questions for this week

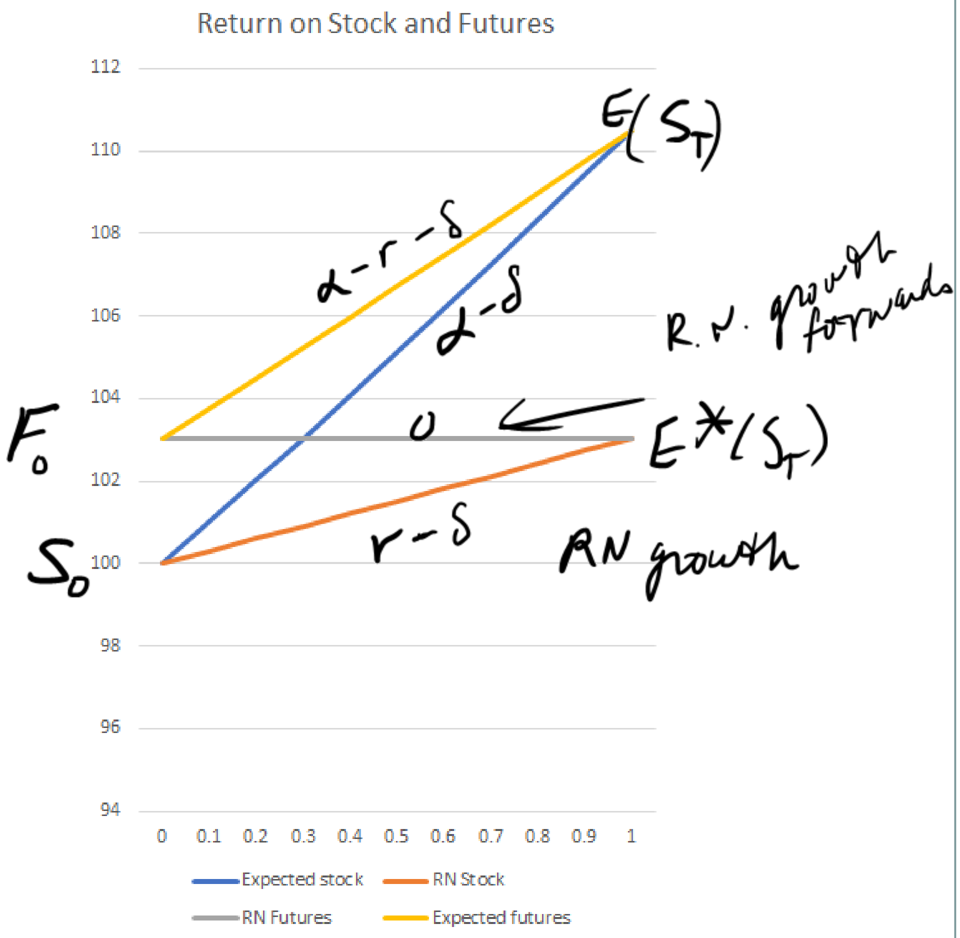
- What is the risk-neutral growth rate for forward/futures contracts?

- What is the expected return on a forward/futures contract?

TV RISK

STOCK $r_i = r_f + \beta_i (r_m - r_f)$

FUTURES $r_i = \beta_i (r_m - r_f)$



Yet another forward price derivation

- Consider a financial forward with no dividends
- Regress the long payoff at maturity on the underlying
  - Payoff = $S_T - F_0$ at time T.
  - The payoff can be written as a "regression" of the payoff on S_T, with $a = -F_0$, $b=1$
- What is the present value?
  - Discount constant at the risk-free rate
  - The present value of S_T is already known to be S_0
  - The present value of the forward payoff is 0 at inception
- Equation
  - $0 = -F_0 e^{-rT} + S_0$
  - $F_0 = S_0 e^{rT}$
- Risk-neutrality
  - Looks like risk doesn't matter
  - Risk is embedded in S_0
  - Use this trick for options

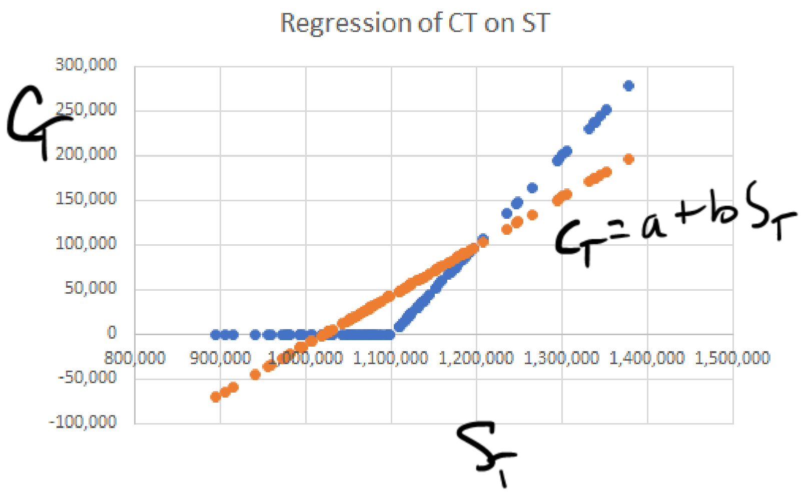
This is called a regression model
or *benchmark model*
…
This is a special case of the GVE.

Using simulation & regression to value options

- Simulate values of the stock using the *risk-neutral* stock simulation formula in Excel

$$S_T = S_0 e^{\left(r-\delta-\frac{1}{2}\sigma^2\right)T+z\sigma\sqrt{T}}$$

- Calculate the call option value at maturity for each S_T using $C_T = \max(S_T - X, 0)$
- Regress C_T on S_T so that $C_T = a + bS_T + \varepsilon$
- Then the present value is
  - $C_0 = ae^{-rT} + bS_0$
- And the standard error is the square root of the variance of the mean response
- Very Important Note:
  - This model is meant for options on securities.
  - There is also a closed form function which can be used without the need for simulation
  - The Black-Scholes function is available on many websites



EXCEL DEMO WITH BLACK-SCHOLES FUNCTION

DELTA

BETA.

control variate technique

$C_0 = -XN(d_2)e^{-rT} + N(d_1)S_0$

Regression or Benchmarking Method

1. Simulate underlying variables

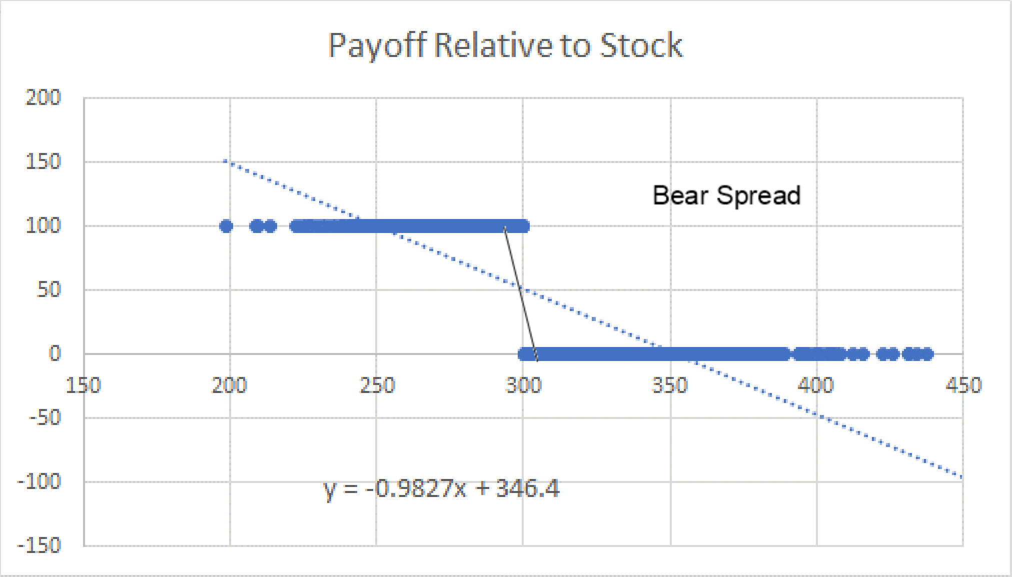
   GBM: Use δ=r (0 growth rate) for futures, starting from futures price

2. Calculate cash flow (simulate N times)
3. Identify hedging instruments and calculate payoffs
4. Regress payoff on hedging instruments
5. Use regression together with time zero values to find the current value of the cash flow
   - Time zero values for constants and futures are taken at the risk-free rate
   - Prices for traded assets and options are taken at time zero values
6. If there are multiple cash flows, then repeat.

Detailed Example: Valuing a binary option

- On 7/3/20, the SPY ETF is trading at $312.23. A hedge fund wants to buy insurance against the market (SPY) falling below 300.
- An investment bank offers a binary put, that pays $100 MM if SPY≤300 on 9/18/20.

- The bank prepares an indicative valuation based on the Black-Scholes model, using σ = 28%, and the dividend rate δ = 1.6%
- Our job is to value the binary put using simulation, and the most accurate model possible.



Step 1: Simulate S using growth rate r – δ

- $S_T = S_0 e^{(r-\delta-0.5\sigma^2)T+z\sigma\sqrt{T}}; z = normsinv(rand(\ \ ))$

Simple simulation

SPY Option Valuation

Valuation Date 7/3/2020

SPY	312.23
Binary strike	300
Volatility	28.00%
Risk-free rate	0.30%
Dividend rate	1.60%
Maturity date	9/18/2020
Maturity in years	0.21
Exp(-rt)	0.9994
Exp(-dt)	0.9966

Method 1: Uncontrolled simulation

	1K Sims	1MM Sims
Mean	38.08	41.04
StErr	1.54	0.05

1000 Simulations

	z	Stock	Payoff
1	-0.34	296	100
2	0.15	315	0
3	0.42	326	0
4	1.39	369	0
5	-0.89	275	100
6	0.62	334	0
7	-0.27	298	100
8	-0.09	305	0
9	0.43	326	0
10	-0.83	277	100
11	1.95	397	0
12	-0.23	300	100
13	-0.80	279	100

=If(S<=300,100,0)

$StErr = Stdev/\sqrt{1000}$

Step 2: Regress Payout (y) on Stock (x)

- At time T, Payout = a + b Stock
- At time 0, Value = $ae^{-rT} + bS_0$
- Running the regression
  - a = 346.4, b = -0.9827
- Results

Regression method with stock

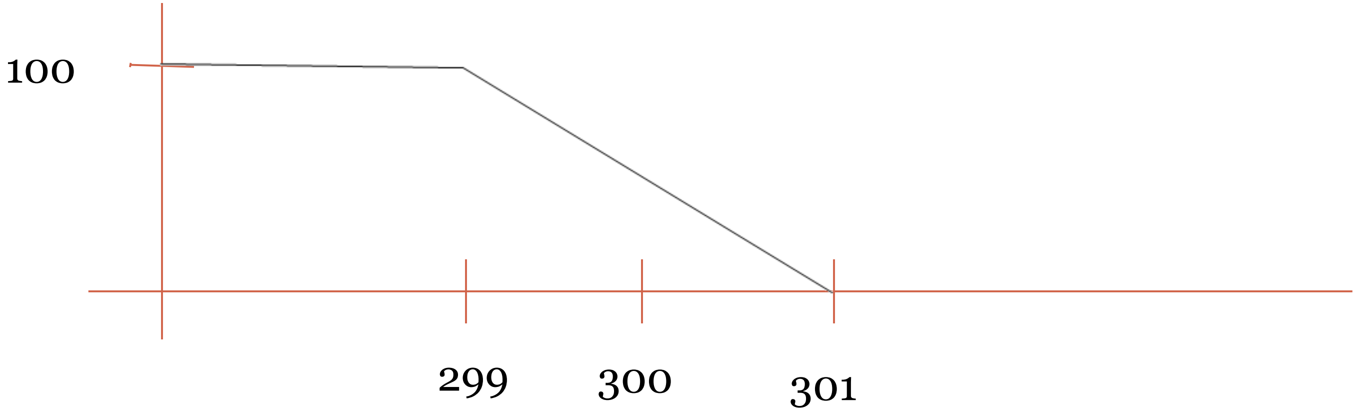
Method 2: Control variate simulation

(Regress payout on stock)

	1K Sims	1MM Sims
Intercept	345.0790	
Slope	-0.9689	
Regression mean	43.35	41.09
Steyx	30.38	
StErr	0.96	0.03

Step 3: Include vanilla options to increase accuracy

- Bear spread:
  - Buy 50 puts with strike price 301
  - Sell 50 puts with strike price 299
- What does the bear spread payout look like?
- How does it compare to the binary payout?



Step 4: Run option regression to optimize fit

- At time T, Payout = $a + b\ Put_T(299) + c\ Put_T(301)$
- At time 0, Value = $ae^{-rT} + b\ Put_0(299) + c\ Put_0(301)$
  - The puts are worth 10.22 and 11.04
- Results

Regression method with options

1000 Simulations (same as first tab)

	Binary	Stock	Put (299)	Put (301)
1	100	257.48	41.52	43.52
2	100	277.71	21.29	23.29
3	0	334.57	0.00	0.00
4	100	262.67	36.33	38.33
5	100	251.92	47.08	49.08
6	100	280.64	18.36	20.36
7	0	317.95	0.00	0.00
8	0	310.22	0.00	0.00
9	0	363.75	0.00	0.00
10	0	308.52	0.00	0.00
11	0	394.25	0.00	0.00
12	100	274.70	24.30	26.30
13	100	274.35	24.65	26.65
14	0	313.52	0.00	0.00

Regression results with two put options

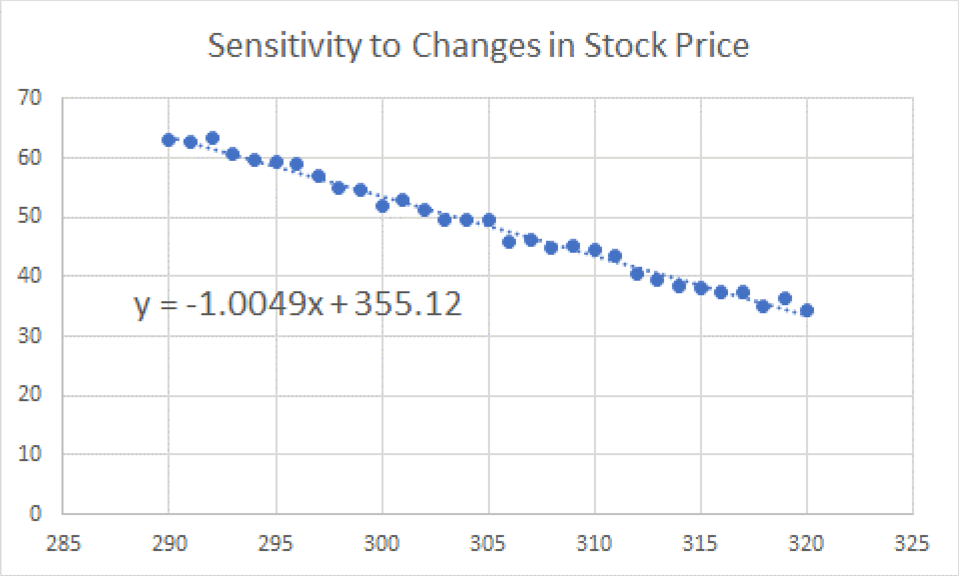
	Const	Put(299)	Put(301)
Coefficients	-0.17	-50.23	50.23
Component values	1.00	10.22	11.04
Overall value	41.02		
StErr (1000)	0.131171		
StErr (1MM)	0.004148		

Linest output

50.22706	-50.2339	-0.16925
0.142158	0.147663	0.129417
0.996181	3.068388	#N/A
130024.5	997	#N/A
2448363	9386.762	#N/A

Step 5: Run sensitivity analyses (optional)

- Verify that your solution and intuition are correct
- For each important variable in the model, show how changes in that variable affect the value of the derivative contract
- For example, the binary option value changes constantly as the stock price changes: Each $1 increase in the stock prices reduces the option value by ________



Summary so far

Options

- You need to know these terms:
  - Underlying, call, put, premium, strike, writer, maturity, term, American, European, intrinsic, extrinsic
- Options are used by speculators in directional trades, volatility trades and combination trades.
- Most options are not held to maturity.
- Long option speculators have little to worry about except premium erosion.
- Short option speculators should worry about leverage, margin requirements, large downsides and possible illiquidity.
- Hedgers using options should consider basis risk, volumetric risk and margin risk. The latter applies only for short option positions.

Valuation using Benchmarks

- The easiest way to value an option or any complex cash flow is to simulate the payoffs and benchmark the valuation to the underlying stock via regression
- Additional priced and traded benchmarks may be added to increase the precision of the regression results
- Care must be taken to choose the correct growth rate for any market variables
  - $r - \delta$ for traded assets
  - $r=\delta$ for futures
- Time zero values are treated differently
  - Constant and futures: Discount at risk-free rate
  - Assets and options: Use time zero values
- This is a powerful and versatile valuation technique.

Exotic option #1

- Asian option *(Path-dependent)*
  - Same payout as a regular option, except that the underlying is the average of a sequence of daily prices (usually over one month) instead of the price at the end of the month
- How would you simulate this?
- Why do people use this?

$$F_1 \quad F_2 = F_1 \exp\left(\frac{(0 - \frac{1}{2}\sigma^2)}{252} + \frac{z\sigma}{\sqrt{252}}\right) \quad F_3 \quad F_4 \cdots F_{22}$$

$$A = \text{Avg}(F_1 \ldots F_{22}) \qquad \text{Asian Call} = \max(A - X, 0)$$

$$\text{option payout at } T = a + bF + c \max(F_{22} - X, 0)$$

$$\text{at } 0 = ae^{-rT} + be^{-rT}F + c \cdot \text{call}(X)$$

Exotic option #2

~ difference

- Option on a spread *(Multi-asset)*
  - Pays max(S_1-S_2-X,0), that is, the underlying is the difference between two values

GAS OIL

- How would you simulate this?
- Why do people use this?

OIL REFINERY PROFIT MAX (GAS PRICE − OIL PRICE − COSTS, 0)

S_1 S_2 X

OPTION TO STOP PRODUCTION

SIM S_1, S_2, correlated

$$\text{PAYOFF} = a + b_1 S_{1T} + c S_{2T} + d(\text{OPT } 1) + e(\text{OPT } 2)$$

$$PV = ae^{-rT} + b_1 S_1 + c S_2 + d\, O_1 + c\, O_2$$

Bonus problem

- How would you value an American call option?
- Background
  - When is an American option exercised early
  - What is the shape of the exercise boundary?

Recitation Questions

- How would you compare futures with options?
- What is the maximum loss in a short put or short call position?
- What is the maximum loss on a long put or long call position?
- In what sense are options *leveraged*?

Recitation Questions

- What is the difference between an American & European option?
- What is intrinsic value?
- In what way are puts and calls most similar?
- How would you construct a synthetic long forward position from traded options?

Recitation Questions

- How would you construct a put option from other securities and contracts if the put was not able to be traded?
- What is the effect of rising stock prices on calls and puts?
- What is the effect of rising volatility on calls and puts?
- How would speculators use options in their trading strategies?

Recitation Questions

- How would equity traders use options in their hedging strategies?

- The Black-Scholes-Merton model assumes stock prices follow a geometric random walk. What are the pros and cons of this assumption?

- The BSM model closed form solution for a European call on a non-dividend paying stock is given by $C = SN(d_1) – Xe^{-rT}N(d_2)$. In what sense is this similar to the regression model?

Recitation Questions

- Why does the Black-Scholes-Merton model use r-δ as the growth rate for the simulation of the stock price?

- What are the steps in the regression model for option valuation?

- How are dividends handled in option valuation models?

- How would you price the option that paid off max(S_1-S_2-X,0) at maturity if S_1 and S_2 are two different stock prices? (This is a *spread option*.)

Recitation Questions

- How would you value a basket option?
  - i.e. the underlying asset is basket of securities
- How would you value an Asian option?
  - E.g. a call option on Tesla where the effective stock price is the average stock price over one trading month