Simulation of trading Profit/Loss

$Q_{i,n}$ = investment in stock i at the start of period n

$R_{i,n}$ = dividend-adjusted return of stock over period n

r = Fed Funds rate or reference rate for cash

$r + \delta r$ = interest paid for cash on long stock

$r - \delta r$ = interest received for cash on short stock

ε = market impact + clearing & commissions

E_n = equity in the account at start of period n

Typically, we will assume $\varepsilon = 5$ bps $= 0.0005$,

and $\delta r = 0$, for simplicity

Basic P/L equation

$$E_{n+1} = E_n + r\Delta t E_n + \sum_{i=1}^{N} Q_{i,n} R_{i,n} - r\Delta t \left(\sum_{i=1}^{N} Q_{i,n} \right)$$

$$- \delta r \Delta t \sum_{i=1}^{N} \left| Q_{i,n} \right| - \varepsilon \sum_{i=1}^{N} \left| Q_{i,n+1} - Q_{i,n} \right|$$

$$\Lambda = \frac{\sum_{i=1}^{N} \left| Q_{i,n} \right|}{E_n} = \text{leverage ratio}$$

$$\Lambda = \frac{\text{Long Market Value} + \left| \text{Short Market Value} \right|}{\text{Equity}}$$

Examples of Leverage

Long-only: $$\Lambda = \frac{L}{E}$$

Long-only, Reg T: (margin acct) $$L \le 2E \qquad \therefore \quad \Lambda \le 2$$

130-30 Investment funds $$L = 1.3E, \quad |S| = 0.3E \quad \therefore \quad \Lambda = 1.6$$

Long-short \$-Neutral, Reg T: $$L + |S| \le 2E \quad \therefore \quad \Lambda \le 2$$

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Long-short, Equal target position in each stock $$Q_i \le \pm \frac{\Lambda_{\max} E}{N} \qquad \therefore \sum_i |Q_i| \le \Lambda_{\max} E$$

Sharpe Ratio

$$\mu = \frac{1}{\Delta t N_{\text{periods}}} \sum_{n=1}^{N_{\text{periods}}} \frac{E_n - E_{n-1}}{E_{n-1}}$$

Expected return
over simulation period

$$\sigma^2 = \frac{1}{\Delta t N_{\text{periods}}} \sum_{n=1}^{N_{\text{periods}}} \left(\frac{E_n - E_{n-1}}{E_{n-1}} - \mu \Delta t \right)^2$$

Variance over
simulation period

$$S = \frac{\mu - r}{\sigma}$$

Sharpe Ratio

The Sharpe ratio measures returns above the risk-free rate.

It is independent of the leverage of the strategy (dimensionless).