FIXED INCOME SECURITIES

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Change of Probability Measure

- Assume two periods (0,1)
- In period 1 there are "n" possible states $i = 1, \dots, n$
- $S \in \{s_1, \dots, s_n\}$
- Assume we have two probability set (*probability measure)* P & $\tilde{P}$ that is :
- $P(s_i) = \pi_{\boldsymbol{i}} > 0 \quad \& \quad \tilde{P}(s_i) = \tilde{\pi}_i > 0$
- Let X be a random variable, under probability measures P & $\tilde{P}$, the expected value of X is $E(X)$ & $\tilde{E}(X)$ respectively:

$$E(X) = \sum_{i=1}^{n} x_i \pi_i$$

$$\tilde{E}(X) = \sum_{i=1}^{n} x_i \, \tilde{\pi}_i$$

Change of Probability Measure

- Let Z be another random variable, under the probability set $(probablity\ measure)\ P$, the expected value of ZX is $E(ZX)$ is:

$$E(ZX) = \sum_{i=1}^{n} z_i x_i \pi_i$$

- Define $Z \in \{z_1, \ldots, z_n\}$ as $z_i = \frac{\tilde{\pi}_i}{\pi_i} > 0$ (ratios of two state probability) , under probability measures P, the expected value of Z is $E(Z) = 1$, since $E(Z) = \sum_{i=1}^{n} z_i \pi_i = \sum_{i=1}^{n} \frac{\tilde{\pi}_i}{\pi_i} \pi_i = 1$
- And
- $E(ZX) = \sum_{i=1}^{n} z_i x_i \pi_i = \sum_{i=1}^{n} \frac{\tilde{\pi}_i}{\pi_i} x_i \pi_i = \sum_{i=1}^{n} x_i\, \tilde{\pi}_i = \tilde{E}(X)$

Change of Probability Measure

- Now let us work backward.
- Assume we are under probability measure P and we have random variables $(Z, X_1, \dots, X_k)$ and we are interested to find expected value of random variable ZX_j under P:
- $E\left(ZX_j\right) = \sum_{i=1}^{n} z_i x_{ji} \pi_i$
- If random variable Z has property such that $z_i > 0 \;\&\; E(Z) = 1$
- Then we can define a new probability measure $\tilde{P}$ with
$$\tilde{P}(s_i) = \tilde{\pi}_i = z_i \pi_i$$
- Then :
- $E\left(ZX_j\right) = \sum_{i=1}^{n} z_i x_{ji} \pi_i = \sum_{i=1}^{n} z_i x_{ji} \frac{\tilde{\pi}_i}{z_i} = \sum_{i=1}^{n} x_i \tilde{\pi}_i = \tilde{E}\left(X_j\right)$

Two Periods & Two States Economy 1 Asset

- Assume Two Period Economy (period 0 and period 1)
- Assume in period 1, there are 2 possible states of outcome (u,d)
- Assume in period 0, we have 1 security with price p_{10} and payout p_{1s} in period 1. $(s = u \; or \; d)$

$$P_0 = [p_{10}] \;\&\; P_1 = [p_{1u} \quad p_{1d}]$$

- Assume an investor would like to have w_{1u} and w_{1d} in period1, states $u \;\&\; d$ respectively. $W_1 = [w_{1u} \quad w_{1d}]'$
- Is there a portfolio, $X_0 = [x_{10}]$ at time 0 that the investor can held to achieve his objective of W_1 in period one.
- $w_{1u} = x_{10}p_{1u}$ and $w_{1d} = x_{10}p_{1d}$ or $W_1 = X_0 \, P_1$
- Since we have two equation w_{1u} and w_{1d} and one control x_{10}, in general the investor won't be able to achieve the W_1 objective

Two Periods & Two States Economy 2 Assets

- How about two securities with period 0 prices p_{10} & p_{20} and period 1 payout :

$$P_1 = \begin{bmatrix} p_{1u} & p_{1d} \\ p_{2u} & p_{2d} \end{bmatrix} \quad \& \quad P_0 = \begin{bmatrix} p_{10} \\ p_{20} \end{bmatrix}$$

- Holding portfolio X_0:

$$X_0 = [x_{10} \quad x_{20}]$$

- The period 1 payoff will be $W_1 = X_0\, P_1$:

$$w_{1u} = x_{10}p_{1u} + x_{20}p_{2u} \quad \& \quad w_{1d} = x_{10}p_{1d} + x_{20}p_{2d}$$

- Since we have two equation w_{1u} and w_{1d} and two control x_{10}, x_{20}in general the investor can solve for a UNIQUE portfolio X_0 to achieve the W_1 objective (Complete Market)
- $X_0 = W_1 P_1^{-1}$ with period 0 price $W_0 = X_0\, P_0 = W_1 P_1^{-1} P_0$

Two Periods & Two States Economy 3 Assets

- Consider 3 securities with period 0 prices p_{10}, p_{20} & p_{30} and period 1 payout :

$$P_0 = \begin{bmatrix} p_{10} \\ p_{20} \\ p_{30} \end{bmatrix} \quad \& \quad P_1 = \begin{bmatrix} p_{1u} & p_{1d} \\ p_{2u} & p_{2d} \\ p_{3u} & p_{3d} \end{bmatrix}$$

- Holding portfolio X_0:

$$X_0 = [x_{10} \quad x_{20} \quad x_{30}]$$

- The period 1 payoff will be $W_1 = X_0\, P_1$:

$$w_{1u} = x_{10}p_{1u} + x_{20}p_{2u} + x_{30}p_{3u} \;\&\; w_{1d} = x_{10}p_{1d} + x_{20}p_{2d} + x_{30}p_{3d}$$

- Since we have two equation w_{1u} and w_{1d} and 3 controls x_{10}, x_{20} and x_{30}in general the investor can solve for multiple portfolios X_0 to achieve the W_1 objective

Two States Economy : 1 & 2 Assets

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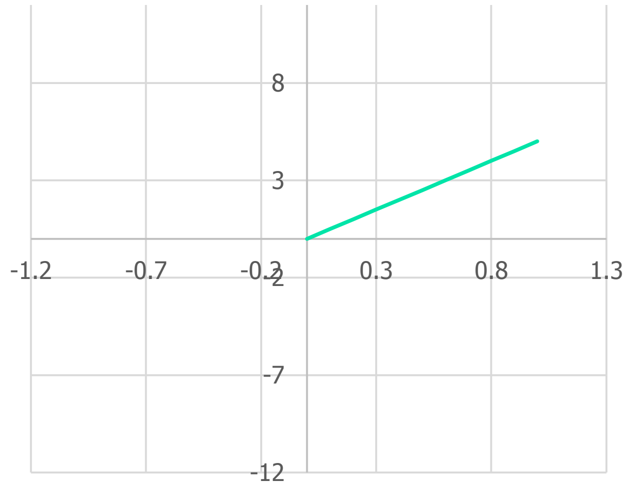

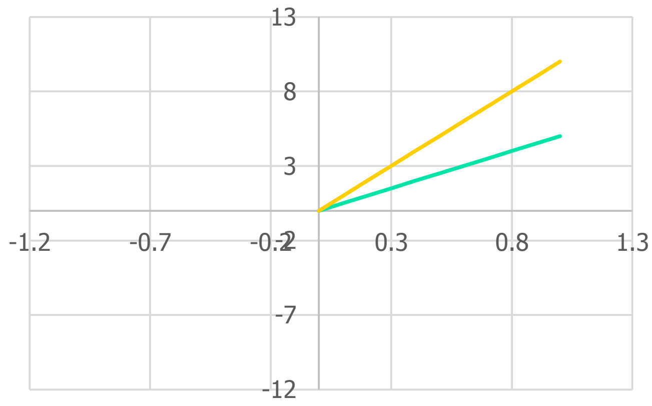

Two Periods & "S" States Economy "N" Assets

- Assume Two Period Economy (period 0 and period 1)
- Assume in period 1, there are S possible states of outcome
- Assume in period 0, we have N securities with prices P_0 and payout P_1 in period 1. $(p_{is} \quad i = 1, .., N \; and \; s = 1, \dots, S)$
- $P_0 = \begin{bmatrix} p_{10} \\ \vdots \\ p_{N0} \end{bmatrix} \; P_1 = \begin{bmatrix} p_{11} & \cdots & p_{1s} \\ \vdots & \ddots & \vdots \\ p_{N1} & \cdots & p_{Ns} \end{bmatrix}$

$$where \; p_{ij} is \; payout \; of \; security \; i \; in \; state \; j$$

- Investor holding portfolio X_0 in period 0 :

$$X_0 = \begin{bmatrix} x_{10} & \cdots & x_{N0} \end{bmatrix}$$

- Will result in period 1 payoff : $W_1 = X_0 \, P_1$

Two Periods & "S" States Economy "N" Assets

- $W_1' = \begin{bmatrix} x_{10}p_{11} + \ \dots + \ x_{N0}p_{N1} \\ \vdots \\ x_{10}p_{1S} + \ \dots + \ x_{N0}p_{NS} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} x_{i0}p_{i1} \\ \vdots \\ \sum_{i=1}^{N} x_{i0}p_{iS} \end{bmatrix}$
- The period 0 value of this portfolio is :

$$W_0 = X_0 \, P_0 = \sum_{i=1}^{N} x_{i0}p_{i0}$$

- The payouts in period 1 are $W_1 = [w_{11} \quad \dots \quad w_{1S}]$

$$W_1 = X_0 \, P_1$$

$$w_{1j} = \sum_{i=1}^{N} x_{i0}p_{is} \quad j = 1, \dots, S$$

Two Periods & "S" States Economy "N" Assets

- Can Investor achieve any desired period 1 pay off :

$$W_1^* = [w_{11}^* \quad \dots \quad w_{1s}^*]$$

- In general if $N = S$ and P is full rank then Investor can achieve ANY GENERIC period 1 payoff matrix W_1^* by selecting portfolio $X_1^* = W_1^* P_1^{-1}$(Complete Market). Period 0 cost of portfolio :

$$W_0^* = X_0^* P_0 = W_1^* P_1^{-1} P_0$$

- If $N < S$, less securities than states then rank of $P_1 < S$ and P_1^{-1} dose not exist and we *can not* achieve ANY GENERIC consumption matrix W^* by selecting any portfolio X^* since it may not exist (Incomplete Market)

Arrow-Debreu Security

- Let X_{0i}^{AD} be period "0" asset such that its period 1, payoff is W_{1i}^{AD} where $W_{1i}^{AD} = \left[w_{1i1}^{AD} = 0 \ldots w_{1ii}^{AD} = 1 \ldots w_{1iS}^{AD} = 0\right]$
- Here investor will receive $1 iff next period realized state is "i" otherwise if any other state was realized the payoff is 0.
- Then X_{0i}^{AD} is called basic security or Arrow-Debreu security. (Note that we have "S" of these securities).
- If X_{0i}^{AD} can be constructed from portfolio of existing assets 1,…,N then period "0" price of these securities are :
- $W_{1i}^{AD} = X_{0i}^{AD}\ P_1 \qquad or \qquad X_{0i}^{AD} = W_{1i}^{AD}\ P_1^{-1}$
- $p_{i0}^{AD} = X_{0i}^{AD}\ P_0 = \sum_{j=1}^{N} x_{0ij}^{AD}\ p_{j0}$ where $X_{0i}^{AD} = \left[x_{0i1}^{AD}\ \ldots x_{0iN}^{AD}\right]$
- Note we can construct X_{0i}^{AD} if P_1^{-1} exist (Complete Market)

Arrow-Debreu Security

- Assume an expected utility maximizing agent want to maximize his life long utility function by choosing trading strategy $X = (X_0, \dots, X_T)$, where $X_i = (x_{i1}, \dots, x_{iN})$ are period 'i' investment in asset 1,…N.

$$Max \sum_{i=0}^{T} \beta^i E_o[U(c_i(s_i))] \text{ subject to}$$

$$c_i(s_i) + P_i(s_i)'X_i(s_i) = e_i(s_i) + P_i(s_i)'X_{i-1}(s_{i-1})$$

- Assume T = 1, (i.e. only two period, 0 and 1)

$$Max\ U(c_0) + \beta^1 E_o[U(c_1(s_1))]$$

subject to :

$$c_0 + P_0{}'X_0 = e_0$$

$$c_1(s_1) = e_1(s_1) + P_1(s_1)'X_0$$

Arrow-Debreu Security

- Let us assume we have only two assets and S states in period 2

$$P_0 = (p_{01}, p_{02}) \; \& \; P_1(s) = (P_{11}(s), P_{12}(s))$$

- Let period 0 risk free rate be $(r_f - 1)$ [for example if rate is 5% then $(r_f = 1.05)$] , and assume asset "1" is risk free asset where $p_{01} = 1$. Therefore in period 1 the price or payoff of this asset is r_f, irrelevant of which state "s" will be realized or:

$$P_{11}(s) = r_f \; for \; s = 1, \dots, S \; ; \; P_{11} = (r_f, \dots, r_f).$$

- First Order Conditions are :
- $U'(c_0)P_0 = \beta \, E_0[U'(c_1)P_1]$ where

$$P_0 = (p_{01} \; p_{02}) \; \& \; P_1(s) = [P_{11}(s), P_{12}(s)] \; for \; s = 1, \dots, S$$

Arrow-Debreu Security

- In particular we can write for all assets 1 or 2 :

$$p_{o1} = \beta\, E_0\left[\frac{U'(c_1)}{U'(c_0)} P_{11}\right] = \frac{1}{r_f} E_0\left[\beta r_f \frac{U'(c_1)}{U'(c_0)} P_{11}\right]$$

$$p_{o2} = \beta\, E_0\left[\frac{U'(c_1)}{U'(c_0)} P_{12}\right] = \frac{1}{r_f} E_0\left[\beta r_f \frac{U'(c_1)}{U'(c_0)} P_{12}\right]$$

- Examine the First Equation:

$$p_{o1} = \beta\, E_0\left[\frac{U'(c_1)}{U'(c_0)} P_{11}\right] = \sum_{s=1}^{S} \pi_s \beta \frac{U'(c_1(s))}{U'(c_0)} p_{11}(s) =$$

$$1 = p_{o1} = \sum_{s=1}^{S} \beta r_f \frac{U'\big(c_1(s)\big)}{U'(c_0)} \pi_s$$

Arrow-Debreu Security

- Since Asset 1 is risk free asset with $P_{11}(s) = r_f$ and $p_{o1} \;= 1$
- $1 = \; p_{o1} \; = \sum_{s=1}^{S} \beta r_f \frac{U'(c_1(s))}{U'(c_0)} \pi_s$
- Now Let us examine the random variable $Z = \beta r_f \frac{U'(C_1)}{U'(C_0)}$:
- $z(s) = \; \beta r_f \frac{U'(c_1(s))}{U'(c_0)} > 0 \;\; \& \, E(Z) = \sum_{s=1}^{S} \beta r_f \frac{U'(c_1(s))}{U'(c_0)} \pi_s = 1$
- $p_{oi} \; = \; \beta \, E_0 \left[\frac{U'(c_1)}{U'(c_0)} P_{1i}\right] = \frac{1}{r_f} E_0 \left[\beta r_f \frac{U'(c_1)}{U'(c_0)} P_{1i}\right] = \frac{1}{r_f} E_0 [Z P_{1i}]$
- Define new probability measure $\tilde{P}$
- $\tilde{\pi}_s = z_s \pi_s = \beta r_f \frac{U'(c_1(s))}{U'(c_0)} \pi_s \geq 0 \; and \; \sum_{s=1}^{S} \tilde{\pi}_s = 1$

Arrow-Debreu Security

- $p_{0i} = \frac{1}{r_f} E_0[ZP_{1i}] = \frac{1}{r_f} \tilde{E}_0[P_{1i}]$
- Note since $\tilde{\pi}_s \geq 0 \; and \; \sum_{s=1}^{S} \tilde{\pi}_s = 1$, we can think about $\tilde{\pi}_s$ as some probability set which is related to actual probability by random variable Z.
- Let us assume we have two state $1 \; and \; 2 \;\; \pi_1 = \pi_2 = {}^1/_2$
- Assume $c_1(1) > c_1(2)$ since $U'\big(c_1(s)\big) \geq 0$ and $U^{"}\big(c_1(s)\big) < 0$
- Then $U'\big(c_1(2)\big) \geq U'\big(c_1(1)\big)$ or $\tilde{\pi}_2 \geq {}^1/_2 \geq \tilde{\pi}_1$

Arrow-Debreu Security

- For asset 2:
- $p_{o2} = \beta\, E_0\left[\frac{U'(c_1)}{U'(c_0)} P_{12}\right] = \frac{1}{r_f} E_0\left[\beta r_f \frac{U'(c_1)}{U'(c_0)} P_{12}\right] = \frac{1}{r_f} E_0[Z' P_{12}] =$
- $r_f = E_0\left[Z' \frac{P_{12}}{p_{02}}\right] = \tilde{E}_0\left[\frac{P_{12}}{p_{02}}\right] = \sum_{S=1}^{S} \pi_S z_S \frac{p_{12(s)}}{p_{o2}} = \sum_{S=1}^{S} \tilde{\pi}_S \frac{p_{12(s)}}{p_{o2}}$
- Let $r_{12}(s) = \frac{p_{12(s)}}{p_{o2}}$ be the (1+ rate of return) of asset 2 in state s
- $r_f = \sum_{S=1}^{S} \tilde{\pi}_S r_{12}(s) = \sum_{S=1}^{S} \tilde{\pi}_S \frac{p_{12}(s)}{p_{02}}$
- $r_f = \frac{p_{11}}{p01} = \tilde{E}\left[\frac{P_{12}}{p_{02}}\right] = \tilde{E}[R_{12}]$

Arrow-Debreu Security

- Back to N asset model. First Order Conditions are :
- $U'(c_0)P_0 = \beta\, E_0[U'(c_1)P_1]$ where $P_0 = (p_{01}, \dots, p_{0N})$ and $P_1 = (P_{11}, \dots, P_{1N})$ are the price of assets in period 0 and period 1.
- In particular we can write for all assets "i":

$$p_{oi} = \beta\, E_0\left[\frac{U'(c_1)}{U'(c_0)} P_{1i}\right] = \frac{1}{r_f} E_0[Z' P_{1i}]$$

- Let asset "1" is risk free asset with $p_{01} = 1$, then $P_{11} = (r_f, \dots r_f)$ for all s. Time 0 price of asset "1" price :

$$p_{oi} = 1 = \beta\, E_0\left[\frac{U'(c_1)}{U'(c_0)} P_{11}\right] = \sum_{j=1}^{S} \beta r_f \frac{U'\left(c_{1j}\right)}{U'(c_0)} \pi_j = \sum_{j=1}^{S} z_j \pi_j = \sum_{j=1}^{S} \tilde{\pi}_j$$

Arrow-Debreu Security

- And for any other asset

$$1 = \beta\, E_0\left[\frac{U'(c_1)}{U'(c_0)}\frac{P_{1i}}{p_{0i}}\right] = \beta\, E_0\left[\frac{U'(c_1)}{U'(c_0)}R_i\right]$$

- $1 = \beta\left[\sum_{j=1}^{S}\pi_j\frac{U'(c_{1j})}{U'(c_0)}\, r_{ij}\right]$ where $R_i = (r_{i1},\dots,r_{iS}) = (\frac{p_{j1}}{p_{0j}},\dots,\frac{p_{jS}}{p_{0j}})$
- $\frac{(\beta r_f)\,U'(c_{1j})}{U'(c_0)}\pi_j = \tilde{\pi}_j \geq 0$ and $\sum_{j=1}^{S}\frac{(\beta r_f)\,U'(c_{1j})}{U'(c_0)}\pi_j = \sum_{j=1}^{S}\tilde{\pi}_j = 1$
- $\pi_j = \frac{U'(c_0)}{(\beta r_f)\,U'(c_{1j})}\tilde{\pi}_j$ *and* $\left[\beta\sum_{j=1}^{S}\frac{U'(c_0)}{(\beta r_f)\,U'(c_{1j})}\frac{U'(c_{1j})}{U'(c_0)}\tilde{\pi}_j\; r_{ij}\right] = 1$
- $r_f = \sum_{j=1}^{S}\tilde{\pi}_j\, r_{ij} = \tilde{E}(R_i) = \tilde{E}\left(\frac{P_{1i}}{p_{0i}}\right)$
- $p_{0i} = \frac{1}{r_f}\sum_{j=1}^{S}\tilde{\pi}_j\, p_{ij} = \frac{1}{r_f}\tilde{E}(P_i)$

Arrow-Debreu Security

- Example : Assume S = 2, in addition assume $\pi_1 = \pi_2 = 0.5$
- Assume $c_{11} \geq c_{12}$ Also assume $U'(c_i) \geq 0$ and $U''(c_i) \leq 0$
- $U'(c_{12}) \geq U'(c_{11}) \geq 0$ and $\widetilde{\pi_2} \geq 0.5 \geq \widetilde{\pi_1}$
- Assume we have N=S Arrow-Debreu Securities with period 0 price :
- $P_0 = (p_{01}, \ldots, p_{0S})$ and period 1 price
- $P_i = (0, .., P_{ii} = 1, .., 0)\ for\ i = 1, \ldots, S\ and\ 0\ ow$
- $p_{0i} = \frac{1}{r_f} \sum_{j=1}^{S} \widetilde{\pi}_j\, p_{ij} = \frac{1}{r_f}\, \tilde{\pi}_i$

Arrow-Debreu Security

- An Example :
- Assume two period (0,1). Assume in period 1 two possible state $S_1(1)$ and $S_1(2)$, with probabilities π_1 and π_2.
- Assume two assets , a risk free asset and risky asset.
- Assume price of risk free asset in period 0 is 1, and in period one $R_f = (1 + r_0)$
- Assume price of risky asset in period 0 is P_0^1 and in period 1 is $P_1^1(1)$ and $P_1^1(2)$ in states 1 and 2 respectively and WLOG assume $P_1^1(1) > P_1^1(2)$
- Assume the Agent Utility function is $U(C_t) = \ln(C_t)$ and the second period endowment $e_1(1) = e_1(2) = 0$

Arrow-Debreu Security

- Assume Agent make an investment decision $X_0 = (x_0^0, x_1^0)$ and consumption decision c_0 to maximize :

  $$Max\ U(c_0) + \beta^1 E_o[U(c_1(s_1))]$$

  subject to :

  $$c_0 + x_0^0 + x_1^0\, P_0^1 = e_0$$

  $$c_1(1) = e_1(1) + x_0^0 R_f + x_1^0\, P_1^1(1)$$

  $$c_1(2) = e_1(2) + x_0^0 R_f + x_1^0\, P_1^1(2)$$

- Note $e_1(1) = e_1(1) = 0$ if $x_1^0 > 0$, then $c_1(1) > c_1(2)$
- FOC:

  $$P_0^1 = \beta\, E_0\left[\frac{U'\big(c_1(s_1)\big)}{U'(c_0)} P_1^1(s_1)\right]$$

Arrow-Debreu Security

- Note since $U(c_t) = \ln(c_t)$, $U'(c_t) = \frac{1}{c_t}$
- $P_0^1 = \beta \left[\pi_1 \frac{c_0}{c_1} P_1^1(1) + \pi_2 \frac{c_0}{c_2} P_1^1(2) \right]$
- And for Risk Free asset
- $1 = \left[\pi_1 \beta R_f \frac{c_0}{c_1} + \pi_2 \beta R_f \frac{c_0}{c_2} \right]$
- Let : $\pi_1 \beta R_f \frac{c_0}{c_1} = z_1 \pi_1 = \widetilde{\pi_1}$ and $\pi_2 \beta R_f \frac{c_0}{c_2} = z_2 \pi_2 = \widetilde{\pi_2}$
- $1 = \left[\pi_1 \beta R_f \frac{c_0}{c_1} + \pi_2 \beta R_f \frac{c_0}{c_2} \right] = \widetilde{\pi_1} + \widetilde{\pi_2}$
- Since $c_1 > c_2$, then $z_1 < z_2$ and $\pi_1 > \widetilde{\pi_1}$ and $\pi_2 < \widetilde{\pi_2}$
- $P_0^1 = \frac{1}{R_f} [\widetilde{\pi_1} P_1^1(1) + \widetilde{\pi_2} P_1^1(2)] < \frac{1}{R_f} [\pi_1 P_1^1(1) + \pi_2 P_1^1(2)]$

Arrow-Debreu Security

- $P_0^1 = \frac{1}{R_f} \tilde{E}\left(P_1^1(s_1)\right) < \frac{1}{R_f} E\left(P_1^1(s_1)\right)$
- $R_f = \tilde{E}\left(\frac{P_1^1(s_1)}{P_0^1}\right) = \tilde{E}\left(R_1^1(s_1)\right) < E\left(R_1^1(s_1)\right)$
- HW:
- Assume two period economy with two states in period 1 and $\pi_1 = \pi_1 = \frac{1}{2}$. Let $\beta = 0.95238$ & $r_f = 1.05$.
- Assume an individual utility function is $U(c_t) = \ln(c_t)$.
- Let the optimal allocation will result in $c_0 = 2$, $c_1 = 3$ & $c_2 = \frac{3}{2}$
- Find $z_1, z_2, \tilde{\pi}_1, \tilde{\pi}_2$
- Assume there are two asset (A,B) with payoffs :

Home Work

- $P^A = \begin{bmatrix} p_1^A = 200 \\ p_2^A = 100 \end{bmatrix}$ & $P^B = \begin{bmatrix} p_1^B = 100 \\ p_2^B = 200 \end{bmatrix}$
- Find the expected pay off assets A and B under probability measures P & $\tilde{P}$
- Using $p_{0i} = \frac{1}{r_f}\tilde{E}(P_i)$ find the period 0, price of assets A and B

Home Work

- Let us assume we have 2 period economy . In period 2 there are three possible state : (u, m, d) (think of (u, m, d), as high, medium and low growth state)
- Find the price if Arrow-Debru securities
  - $W_{1u}^{AD} = \left[w_{1u,u}^{AD} = 1, w_{1u,m}^{AD} = 0, w_{1u,d}^{AD} = 0\right]$
  - $W_{1m}^{AD} = \left[w_{1m,u}^{AD} = 0, w_{1m,m}^{AD} = 1, w_{1m,d}^{AD} = 0\right]$
  - $W_{1d}^{AD} = \left[w_{1d,u}^{AD} = 0, w_{1d,m}^{AD} = 0, w_{1d,d}^{AD} = 1\right]$
- Case 1) Assume we have one risk free(asset 1) and one risky asset(asset 2), where price of these two assets are 100 in period 0.
  - $P_0 = (p_{01} = 100, p_{02} = 100)$
  - $P_{1,1} = (p_{1u} = 105, p_{1m} = 105, p_{1d} = 105)$
  - & $P_{2,1} = (p_{2u} = 110, p_{2m} = 105, p_{2d} = 100)$

Home Work

- Asset 1 and 2 prices are 100 in period 0. Asset 1 prices are 105 in period 1, in states, (u, m, d). Asset 2 prices are $(110,105,100)$in period 1, in states, (u, m, d).
- Case 2) Assume we have one risk free(asset 1) and two risky assets (asset 2, asset 3), where price of these three assets are 100 in period 0.
  - P_0= $(p_{01} = 100, p_{02} = 100, p_{03} = 100)$
  - $P_{1,1}$= $(p_{1u} = 105, p_{1m} = 105, p_{1d} = 105)$
  - $P_{2,1}$= $(p_{2u} = 110, p_{2m} = 105, p_{2d} = 100)$
  - $P_{3,1}$= $(p_{3u} = 120, p_{3m} = 105, p_{3d} = 90)$
- Asset 1, 2 & 3 prices are 100 in period 0. Asset 1 prices are 105 in period 1, in states, (u, m, d). Asset 2 prices are $(110,105,100)$in period 1, in states, (u, m, d). Asset 3 prices are $(120,105,90)$in period 1, in states, (u, m, d).

Home Work

- In Case 1, to find
- $W_{1u}^{AD} = \left[w_{1u,u}^{AD} = 1, w_{1u,m}^{AD} = 0, w_{1u,d}^{AD} = 0\right]$
- We need to set up a portfolio of available assets in period 0, $X_{0,u}^{AD} = \left[x_{0,1u}^{AD}, x_{0,2u}^{AD}\right]$ such that the pay of this portfolio is W_{1u}^{AD} in period 1.
  - $w_{1u,u}^{AD} = x_{0,1u}^{AD} * p_{1u} + x_{0,2u}^{AD} * p_{2u} = 1$
  - $w_{1u,m}^{AD} = x_{0,1u}^{AD} * p_{1m} + x_{0,2u}^{AD} * p_{2m} = 0$
  - $w_{1u,d}^{AD} = x_{0,1u}^{AD} * p_{1d} + x_{0,2u}^{AD} * p_{2d} = 0$
  - $p_{0,u}^{AD} = x_{0,1u}^{AD} * p_{01} + x_{0,2u}^{AD} * p_{2d}$