Predicting the direction of stock market prices using random forest

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ABSTRACT *Predicting trends in stock market prices has been an area of interest for researchers for many years due to its complex and dynamic nature. Intrinsic volatility in stock market across the globe makes the task of prediction challenging. Forecasting and diffusion modeling, although effective can't be the panacea to the diverse range of problems encountered in prediction, short-term or otherwise. Market risk, strongly correlated with forecasting errors, needs to be minimized to ensure minimal risk in investment. The authors propose to minimize forecasting error by treating the forecasting problem as a classification problem, a popular suite of algorithms in Machine learning. In this paper, we propose a novel way to minimize the risk of investment in stock market by predicting the returns of a stock using a class of powerful machine learning algorithms known as ensemble learning. Some of the technical indicators such as Relative Strength Index (RSI), stochastic oscillator etc are used as inputs to train our model. The learning model used is an ensemble of multiple decision trees. The algorithm is shown to outperform existing algorithms found in the literature. Out of Bag (OOB) error estimates have been found to be encouraging.*

1.   Introduction

Predicting the trends in stock market prices is a very challenging task due to the many uncertainties involved and many variables that influence the market value in a particular day such as economic conditions, investors' sentiments towards a particular company, political events etc. Because of this, stock markets are susceptible to quick changes, causing random fluctuations in the stock price. Stock market series are generally dynamic, non-parametric, chaotic and noisy in nature and hence, stock market price movement is considered to be a random process with fluctuations which are more pronounced for short time windows. However, some stocks usually tend to develop linear trends over long-term time windows. Due to the chaotic and highly volatile nature of stock behavior, investments in share market comes with high risk. In order to minimize the risk involved, advanced knowledge of stock price movement in the future is required. Traders are more likely to buy a stock whose value is expected to increase in the future. On the other hand, traders are likely to refrain from buying a stock whose value is expected to fall in the future. So, there is a need for accurately predicting the trends in stock market prices in order to maximize capital gain and minimize loss. Among the major methodologies used to predict stock price behavior, the following are particularly noteworthy: (1) Technical Analysis, (2) Time Series Forecasting (3) Machine Learning and Data Mining (Hellstrom and Holmstromm (1998)) and (4) modeling and predicting volatility of stocks using differential equations (Saha, Routh and Goswami (2014)). This paper mainly focuses on the third approach as the data sets associated with stock market prediction problem are too big to be handled with non-data mining methods. (Widom (1995))

   Application of Machine learning models in stock market behavior is quite a recent phenomenon. The approach is a departure from traditional forecasting and diffusion type methods. Early models used in stock forecasting involved statistical methods such as time series model and multivariate analysis (Gencay (1999), Timmermann and Granger (2004), Bao and Yang (2008)). The stock

price movement was treated as a function of time series and solved as a regression problem. However, predicting the exact values of the stock price is really difficult due to its chaotic nature and high volatility. Stock prediction performs better when it is treated as classification problem instead of a regression problem. The goal is to design an intelligent model that learns from the market data using machine learning techniques and forecast the future trends in stock price movement. The predictive output from our model may be used to support decision making for people who invest in stock markets. Researchers have used a variety of algorithms such as SVM, Neural Network, Naive Bayesian Classifier etc. We will discuss the works done by other authors in the next section.

2.   Related Work

The use of prediction algorithms to determine future trends in stock market prices contradict a basic rule in finance known as the Efficient Market Hypothesis (Fama and Malkiel (1970)). It states that current stock prices fully reflect all the relevant information. It implies that if someone were to gain an advantage by analyzing historical stock data, then the entire market will become aware of this advantage and as a result, the price of the share will be corrected. This is a highly controversial and often disputed theory. Although it is generally accepted, there are many researchers who have rejected this theory by using algorithms that can model more complex dynamics of the financial system (Malkiel (2003)).

Several algorithms have been used in stock prediction such as SVM, Neural Network, Linear Discriminant Analysis, Linear Regression, KNN and Naive Bayesian Classifier. Literature survey revealed that SVM has been used most of the time in stock prediction research. Li, Li and Yang (2014) have considered sensitivity of stock prices to external condition. The external conditions taken into consideration include daily quotes of commodity prices such as gold, crude oil, nature gas, corn and cotton in 2 foreign currencies (EUR, JPY). In addition to that, they collected daily trading data of 2666 U.S stocks trading (or once traded) at NYSE or NASDAQ from 2000-01-01 to 2014-11-10. This dataset includes everyday open price, close price, highest price, lowest price and trading volume of every stock. Features were derived using the information from the historical stock data as well as external variables which were mentioned earlier in this section. It was found that logistic regression turned out to be the best model with a success rate of 55.65%. In Dai and Zhang (2013), the training data used in their research was 3M Stock data. The data contains daily stock information ranging from 1/9/2008 to 11/8/2013 (1471 data points). Multiple algorithms were chosen to train the prediction system. These algorithms are Logistic Regression, Quadratic Discriminant Analysis, and SVM. These algorithms were applied to next day model which predicted the outcome of the stock price on the next day and long term model, which predicted the outcome of the stock price for the next n days. The next day prediction model produced accuracy results ranging from 44.52% to 58.2%. Dai and Zhang (2013) have justified their results by stating that US stock market is semi-strong efficient, meaning that neither fundamental nor technical analysis can be used to achieve superior gain. However, the long-term prediction model produced better results which peaked when the time window was 44. SVM reported the highest accuracy of 79.3%. In Xinjie (2014), the authors have used 3 stocks (AAPL, MSFT, AMZN) that have time span available from 2010-01-04 to 2014-12-10. Various technical indicators such as RSI, On balance Volume, Williams %R etc are used as features. Out of 84 features, an extremely randomized tree algorithm was implemented as described in Geurts and Louppe (2011), for the selection of the most relevant features. These features were then fed to an rbf Kernelized SVM for training. Devi, Bhaskaran and Kumar (2015) has proposed a model which uses hybrid cuckoo search with support vector machine (With Gaussian kernel). Cuckoo search method is an optimization technique used to optimize the parameters of support vector machine. The proposed model used technical indicators such as RSI, Money Flow Index, EMA, Stochastic Oscillator and MACD . The data used in the proposed system consists of daily closing prices of BSE-Sensex and CNX - Nifty from Yahoo finance from January 2013 to July 2014. Giacomel, Galante and Pareira (2015) proposes a trading agent based on a neural network ensemble, that predicts if one stock is going to rise or fall. They evaluated their model in two databases: The North American and the Brazilian stock market. Boonpeng and Jeatrakul (2016) implemented a One vs All and One vs One neural network to classify Buy, hold or Sell data and compared their performance with a traditional neural network. Historical data of Stock Exchange of Thailand (SET) of seven years (date 03/01/2007 to 29/08/2014) was selected. It was found that OAA-NN performed better than OAO-NN and traditional NN models, producing an average accuracy of 72.50%.

The literature survey helps us conclude that Ensemble learning algorithms have remained unexploited in the problem of stock market prediction. We will be using an ensemble learning method known as Random Forest to build our predictive model. Random forest is a multitude of decision trees whose output is the mode of the outputs from the individual trees.

The remainder of the paper is organized as follows. Section 3 discusses about data and the operations implemented on data that include cleaning, pre-processing, feature extraction, testing for linear separability and learning the data via random forest ensemble. Section 4 traces the algorithm by using graph description language and computes the OOB error. Section 5 contains a brief outline on OOB error and convergence estimate. The next section documents the results obtained, followed by a comparative study establishing the superiority of the proposed algorithm. We conclude by summarizing our work in section 7.1.
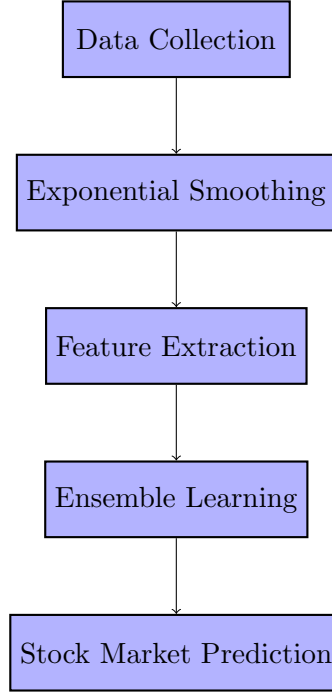
3.   Methodology and Analysis

Fig 1: Proposed Methodology

The learning algorithm used in our paper is random forest. The time series data is acquired, smoothed and technical indicators are extracted. Technical indicators are parameters which provide insights to the expected stock price behavior in future. These technical indicators are then used to train the random forest. The details of each step will be discussed in this section.

3.1   Data Preprocessing

The time series historical stock data is first exponentially smoothed. Exponential smoothing applies more weightage to the recent observation and exponentially decreasing weights to past observations. The exponentially smoothed statistic of a series Y can be recursively calculated as:

$$S_0 = Y_0 \tag{1}$$

$$for\ t > 0,\ S_t = \alpha * Y_t + (1 - \alpha) * S_{t-1} \tag{2}$$

where α is the smoothing factor and $0 < \alpha < 1$. Larger values of α reduce the level of smoothing. When $\alpha = 1$, the smoothed statistic becomes equal to the actual observation. The smoothed statistic

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S_t can be calculated as soon as two observations are available. This smoothing removes random variation or noise from the historical data allowing the model to easily identify long term price trend in the stock price behavior. Technical indicators are then calculated from the exponentially smoothed time series data which are later organized into feature matrix. The target to be predicted in the ith day is calculated as follows:

$$target_i = Sign(close_{i+d} - close_i) \tag{3}$$

where d is the number of days after which the prediction is to be made. When the value of $target_i$ is +1, it indicates that there is a positive shift in the price after d days and -1 indicates that there is a negative shift after d days. The $target_i$ values are assigned as labels to the ith row in the feature matrix.

3.2   Feature Extraction

Technical Indicators are important parameters that are calculated from time series stock data that aim to forecast financial market direction. They are tools which are widely used by investors to check for bearish or bullish signals. The technical indicators which we have used are listed below

Relative Strength Index

The formula for caculating RSI is:

$$RSI = 100 - \frac{100}{1 + RS} \tag{4}$$

$$RS = \frac{Average\ Gain\ Over\ past\ 14\ days}{Average\ Loss\ Over\ past\ 14\ days} \tag{5}$$

RSI is a popular momentum indicator which determines whether the stock is overbought or oversold. A stock is said to be overbought when the demand unjustifiably pushes the price upwards. This condition is generallyt interpreted as a sign that the stock is overvalued and the price is likely to go down. A stock is said to be oversold when the price goes down sharply to a level below its true value. This is a result caused due to panic selling. RSI ranges from 0 to 100 and generally, when RSI is above 70, it may indicate that the stock is overbought and when RSI is below 30, it may indicate the stock is oversold.

Stochastic Oscillator

The formula for calculating Stochastic Oscillator is:

$$\%K = 100 * \frac{(C - L14)}{(H14 - L14)} \tag{6}$$

where,
    C = Current Closing Price
    L14 = Lowest Low over the past 14 days
    H14 = Highest High over the past 14 days
Stochastic Oscillator follows the speed or the momentum of the price. As a rule, momentum changes before the price changes. It measures the level of the closing price relative to low-high range over a period of time.

Williams %R

Williams %R is calculated as follows:

$$\%R = \frac{(H14 - C)}{(H14 - L14)} * -100 \tag{7}$$

where,
  C = Current Closing Price
  $L14$ = Lowest Low over the past 14 days
  $H14$ = Highest High over the past 14 days

Williams %R ranges from -100 to 0. When its value is above -20, it indicates a sell signal and when its value is below -80, it indicates a buy signal.

Moving Average Convergence Divergence

The formula for calculating MACD is:

$$MACD = EMA_{12}(C) - EMA_{26}(C) \tag{8}$$

$$SignalLine = EMA_9(MACD) \tag{9}$$

where,
  $MACD$ = Moving Average Convergence Divergence
  C = Closing Price series
  EMA_n = n day Exponential Moving Average

EMA stands for Exponential Moving Average. When the MACD goes below the SingalLine, it indicates a sell signal. When it goes above the SignalLine, it indicates a buy signal.

Price Rate of Change

It is calculated as follows:

$$PROC(t) = \frac{C(t) - C(t-n)}{C(t-n)} \tag{10}$$

where,
  $PROC(t)$ = Price Rate of Change at time t
  $C(t)$ = Closing price at time t

It measures the most recent change in price with respect to the price in n days ago.

On Balance Volume

This technical indicator is used to find buying and selling trends of a stock. The formula for calculating On balance volume is:

$$OBV(t) = \begin{cases} OBV(t-1) + Vol(t) & \text{if } C(t) > C(t-1) \\ OBV(t-1) - Vol(t) & \text{if } C(t) < C(t-1) \\ OBV(t-1) & \text{if } C(t) = C(t-1) \end{cases} \tag{11}$$

where,
  $OBV(t)$ = On Balance Volume at time t
  $Vol(t)$ = Trading Volume at time t
  $C(t)$ = Closing price at time t
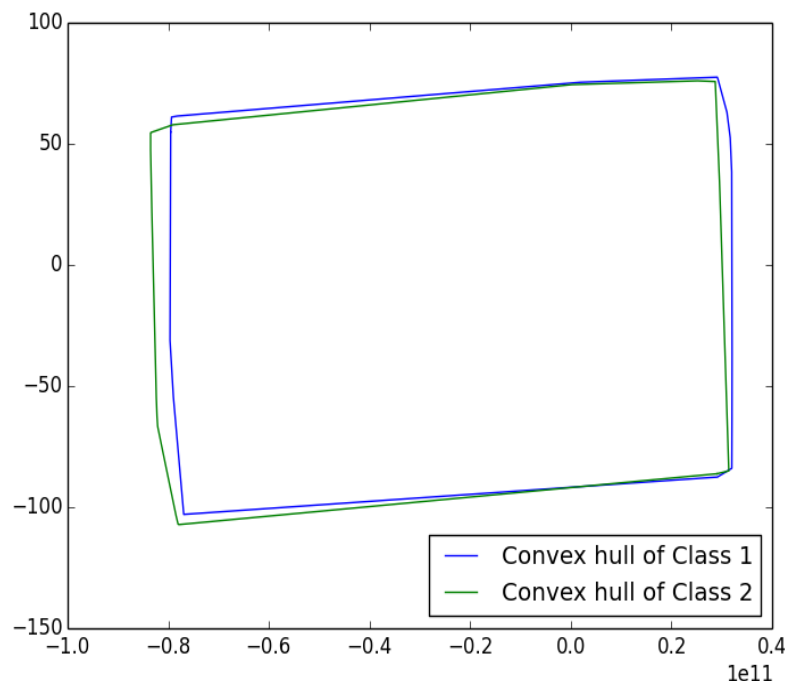
3.3   Test for linear separability

Fig 2: Test for linear separability

Before feeding the training data to the Random Forest Classifier, the two classes of data are tested for linear separability by finding their convex hulls. Linear Separability is a property of two sets of data points where the two sets are said to be linearly separable if there exists a hyperplane such that all the points in one set lies on one side of the hyperplane and all the points in other set lies on the other side of the hyperplane.

Mathematically, two sets of points X_0 and X_1 in n dimensional Euclidean space are said to be linearly separable if there exists an n dimensional normal vector W of a hyperplane and a scalar k, such that every point $x \in X_0$ gives $W^T x > k$ and every point $x \in X_1$ gives $W^T x < k$. Two sets can be checked for linearly separability by constructing their convex hulls.

The convex hull of a set of points X is its subset which forms the smallest convex polygon that contains all the points in X. A polygon is said to be convex if a line joining any two points on the polygon also lies on the polygon. In order to check for Linear Separability, the convex hulls for the two classes are constructed. If the convex hulls intersect each other, then the classes are said to be linearly inseparable. Principle component analysis is performed to reduce the dimensionality of the extracted features into two dimensions. This is done so that the convex hull can be easily visualized in 2 dimensions. The convex hull test reveals that the classes are not linearly separable as the convex hulls almost overlap. This observation concludes that Linear Discriminant Analysis cannot be applied to classify our data and hence, providing a stronger justification to why Random Forest Classifier is used. Another important reason is that since each decision trees in the forest operate on the random subspace of the feature space, it leads to the automatic selection of the most relevant subset of features. Before discussing the RF algorithm, we will be looking at some key definitions in the following section.

3.4   Key Definitions

Assume there are n data points $D = \{(x_i, y_i)\}_{i=1}^{n}$ and feature vectors $\{x_i\}_{i=1}^{n}$ with stated outcomes. Each feature vector is d- dimensional.

Definition 1: We define a classification tree where each node is endowed with a binary decision if $x_i <= k$ or not ; where k is some threshold. The topmost node in the classification tree contains all the data points and the set of data is subdivided among the children of each node as defined by the classification . The process of subdivision continues until every node below has data belonging to one class only. Each node is characterized by the feature, x_i and threshold k chosen in such a way that minimizes diversity among the children nodes. This is often referred as gini impurity.

Definition 2: $X = (X_1, ..., X_d)$ is an array of random variables defined on probability space called as random vectors. The joint distribution of $X_1, ..., X_d$ is a measure on μ on R^d, $\mu(A) = P(X \in A)$, $A \in R^d$ where $d = 1, ..., m$. For example, Let $x = (x_i, ..., x_d)$ be an array of data points. Each feature x_i is defined as a random variable with some distribution. Then the random vector X has joint distribution identical to the data points, x.

Definition 3: Let us represent $h_k(x) = h(x|\theta_k)$ implying decision tree k leading to a classifier $h_k(x)$. Thus, a random forest is a classifier based on a family of classifiers $h(x|\theta_1), ..., h(x|\theta_k)$, built on a classification tree with model parameters θ_k randomly chosen from model random vector θ. Each classifier, $h_k(x) = h(x|\theta_k)$ is a predictor of the number of training samples. $y =^+_- 1$ is the outcome associated with input data, x for the final classification function, $f(x)$.

Next, we describe the working of the Random Forest learner by exploiting the key concepts defined above.

3.5   Random Forest

Decision trees can be used for various machine learning applications. But trees that are grown really deep to learn highly irregular patterns tend to overfit the training sets. A slight noise in the data may cause the tree to grow in a completely different manner. This is because of the fact that decision trees have very low bias and high variance. Random Forest overcomes this problem by training multiple decision trees on different subspace of the feature space at the cost of slightly increased bias. This means none of the trees in the forest sees the entire training data. The data is recursively split into partitions. At a particular node, the split is done by asking a question on an attribute. The choice for the splitting criterion is based on some impurity measures such as Shannon Entropy or Gini impurity.

Gini impurity is used as the function to measure the quality of split in each node. Gini impurity at node N is given by

$$g(N) = \sum_{i \neq j} P(\omega_i)P(\omega_j) \tag{12}$$

where $P(\omega_i)$ is the proportion of the population with class label i. Another function which can be used to judge the quality of split is Shannon Entropy. It measures the disorder in the information content. In Decision trees, Shannon entropy is used to measure the unpredictability in the information contained in a particular node of a tree (In this context, it measures how mixed the population in a node is). The entropy in a node N can be calculated as follows

$$H(N) = -\sum_{i=1}^{i=d} P(\omega_i)log_2(P(\omega_i)) \tag{13}$$

where d is number of classes considered and $P(\omega_i)$ is the proportion of the population labeled as i. Entropy is the highest when all the classes are contained in equal proportion in the node. It is the lowest when there is only one class present in a node (when the node is pure).

The obvious heuristic approach to choose the best splitting decision at a node is the one that reduces the impurity as much as possible. In order words, the best split is characterized by the highest gain in information or the highest reduction in impurity. The information gain due to a split can be calculated as follows

$$\Delta I(N) = I(N) - P_L * I(N_L) - P_R * I(N_L) \tag{14}$$

where $I(N)$ is the impurity measure (Gini or Shannon Entropy) of node N, P_L is the proportion of the population in node N that goes to the left child of N after the split and similarly, P_R is the

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proportion of the population in node N that goes to the right child after the split. N_L and N_R are the left and right child of N respectively.

At the heart of all ensemble machine learning algorithms is Bootstrap aggregating, also known as bagging. This method improves the stability and accuracy of learning algorithms. At the same time, it also reduces variance and overfitting which is a common problem while constructing Decision trees. Given a sample dataset D of size n, bagging generates B new sets of size n' by sampling uniformly from D with replacement. With this knowledge, we can now summarize the algorithm of random forest classifier as follows

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Algorithm 1 Random Forest Classifier

1: **procedure** RANDOMFORESTCLASSIFIER(D)   ▷ D is the labeled training data
2:      $forest$ = new Array()
3:      **for do** $i = 0$ to B
4:          D_i = Bagging(D)   ▷ Bootstrap Aggregation
5:          T_i = new DecisionTree()
6:          $features_i$ = RandomFeatureSelection(D_i)
7:          T_i.train($D_i, features_i$)
8:          $forest$.add(T_i)
9:      **end for**
10:      **return** $forest$
11: **end procedure**

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4. Tracing the RF algorithm

In this section we will trace the Random Forest algorithm for a particular test sample. To begin with, we trained a random forest using the Apple Dataset for a time window of 30 days. We generated graph description language files describing the forest. The output of this process is 30 .dot files that corresponds to 30 decision trees in the random forest. These files are found in found in `https://drive.google.com/open?id=0B98OlHZhHCf1Y0s1Q3AwbjVCWGM`. Next, we wrote a python script that reads all the .dot files and traces the RF algorithm for a test sample.

4.1 Graph Description Language

Graph Description Language is a structured language that is used to describe graphs that can be understood both by humans and computers. It can be used to describe both directed and undirected graphs. A graph description language begins with the graph keyword to define a new graph and the nodes are defined within curly braces. The relationship between nodes are specified using double hyphen (–) for an undirected graph and arrows (-¿) for a directed graph. The following is an example examples of a Graph Description Language.

```
graph  graphname {
      a -- b -- c;
      b -- d;
   }
```

4.2 Trace output

For the sake of convenience, we will be showing the trace of only 3 trees out of 30 trees in the forest. For trace of the entire forest, check: `https://drive.google.com/open?id=0B98OlHZhHCf1T3dvNDJsVzFfaFE`. We take a test sample with the following features and run our trace script.

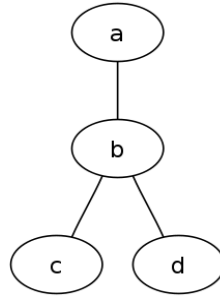- RSI: 91.638968318801957
- Stochastic Oscillator: 88.201032228068314

Fig 3: An undirected graph

- *Williams: -11.798967771931691*
- *Moving Average Convergence Divergence: 5.9734426013145026*
- *Price Rate of Change: 0.11162583681857041*
- *On Balance Volume: 6697423901.3580704*

For Tree 0:
At node 0:(MACD=5.97344260131) <= -8.6232?
False: Go to Node 10
At node 10:(Stochastic Oscillator=88.2010322281) <= 80.9531?
False: Go to Node 134
At node 134:(MACD=5.97344260131) <= 10.2566?
True: Go to Node 135
At node 135:(RSI=91.6389683188) <= 97.9258?
True: Go to Node 136
At node 136:(MACD=5.97344260131) <= -1.7542?
False: Go to Node 140
At node 140:(Price Rate Of Change=0.0412189145804) <= 0.0708?
True: Go to Node 141
At node 141:(MACD=5.97344260131) <= 9.2993?
True: Go to Node 142
At node 142:(MACD=5.97344260131) <= 7.7531?
True: Go to Node 143
At node 143:(On Balance Volume=23722858211.2) <= 24938491904.0?
True: Go to Node 144
At node 144:(Williams=-11.7989677719) <= -15.7228?
False: Go to Node 154
Leaf Node 154 is labeled as Rise

For Tree 1:
At node 0:(MACD=5.97344260131) <= -8.3677?
False: Go to Node 8
At node 8:(RSI=91.6389683188) <= 42.5156?
False: Go to Node 56
At node 56:(MACD=5.97344260131) <= 9.888?
True: Go to Node 57
At node 57:(Price Rate Of Change=0.0412189145804) <= 0.0612?
True: Go to Node 58
At node 58:(Stochastic Oscillator=88.2010322281) <= 80.3201?
False: Go to Node 108
At node 108:(On Balance Volume=23722858211.2) <= 19212566528.0?
False: Go to Node 110
At node 110:(Price Rate Of Change=0.0412189145804) <= -0.0245?
False: Go to Node 112
At node 112:(Price Rate Of Change=0.0412189145804) <= 0.0554?

*   True: Go to Node 113*
At node 113:(On Balance Volume=23722858211.2) <= 19823968256.0?

False: Go to Node 115
Leaf Node 115 is labeled as Rise

For Tree 2:
At node 0:(Stochastic Oscillator=88.2010322281) <= 13.7965?
False: Go to Node 10
At node 10:(On Balance Volume=23722858211.2) <= 26287554560.0?
True: Go to Node 11
At node 11:(Price Rate Of Change=0.0412189145804) <= -0.0649?
False: Go to Node 19
At node 19:(Stochastic Oscillator=88.2010322281) <= 80.6269?
False: Go to Node 121
At node 121:(MACD=5.97344260131) <= 12.8322?
True: Go to Node 122
At node 122:(MACD=5.97344260131) <= 3.3398?
False: Go to Node 132
At node 132:(Price Rate Of Change=0.0412189145804) <= 0.0816?
True: Go to Node 133
At node 133:(Price Rate Of Change=0.0412189145804) <= 0.08?
True: Go to Node 134
At node 134:(RSI=91.6389683188) <= 97.229?
True: Go to Node 135
Leaf Node 135 is labeled as Rise

29 of the trees in forest predict a rise in price while a single tree predicts a fall in price. As a result, the output of the ensemble is Rise. This prediction matches with the actual label assigned to the test sample. Each tree recursively divides the feature space into multiple partitions and each partition is given a label that indicates whether the closing price will rise or fall after 30 days. Looking at the decision trees in the forest, it is really hard to fathom why the data is split on a particular attribute, especially when the same attribute may be used to split the data further down along the tree. To understand why a particular split is chosen at a node, we need to to be familiar with the concept of impurity measures such as Shannon Entropy and Gini impurity. The decision rules learned by the trees may not be easily understood due to complexities in the underlying pattern of the training data. This is where random forests lose some favor with a technically minded person who likes to know what is under the hood.

It should be noted here that our algorithm converges as the number of trees in the forest increases. We calculated out of bag (OOB) error of the classifier with respect to the apple dataset for proof of convergence. In the table given below, the first column indicates the the time window after which the prediction is to be made, the second column indicates the number of trees in the forest, the third column indicates the size of the training sample used and the last column is the OOB error rate.

Trading Period (Days)	No. of Trees	Sample Size	OOB error
30	5	6590	0.241729893778
30	25	6590	0.149165402124
30	45	6590	0.127617602428
30	65	6590	0.123672230653
60	5	6545	0.198472116119
60	25	6545	0.0890756302521
60	45	6545	0.0786860198625
60	65	6545	0.0707410236822
90	5	6500	0.191384615385
90	25	6500	0.0741538461538
90	45	6500	0.0647692307692
90	65	6500	0.0555384615385

Fig 4: OOB error calcuation

As observed, the error rate decreases as the number of trees in the forest is increased. More details

about error rates and convergence will be discussed in the next section.

5.   OOB error and Convergence of the Random Forest

Given an ensemble of decision trees $h_1(X), h_2(x), h_3(x), ..., h_k(x)$, as in Breiman (2001) we define margin function as

$$mg(X,Y) = av_k I(h_k(X) = Y) - max_{j \neq Y} av_k I(h_k(X) = j) \tag{15}$$

where X, Y are randomly distributed vectors from which the training set is drawn. Here, $I(.)$ is the indicator function. The generalization error is given by

$$PE^* = P_{X,Y}(mg(X,Y) < 0) \tag{16}$$

The X and Y subscripts indicate that probability is calculated over X, Y space. In random forests, the kth decision tree $h_k(x)$ can be represented as $h(x, \theta_k)$ where x is the input vector and θ_k is the bootstrapped dataset which is used to train the kth tree. For a sequence of bootstrapped sample sets $\theta_1, \theta_2, ..., \theta_k$ which are generated from the original dataset θ, it is found that PE^* converges to

$$P_{X,Y}(P_\theta(h(X,\theta) = Y) - max_{j \neq Y} P_\theta(h(X,) = j) < 0) \tag{17}$$

The proof can be found in appendix I in Breiman (2001). To practically prove this theorem with respect to our dataset, the generalization error is estimated using out of bags estimates Bylander and Hanzlik (1999). The out of Bag (OOB) error measures the prediction error of Random forests algorithm and other machine learning algorithms which are based on Bootstrap aggregation.
Note: *The average margin of the ensemble of classifiers is the extent to which the average vote count for the correct class flag exceeds the count for the next best class flag.*

5.1   Random forest as ensembles: An analytical exploration

As defined earlier, a Random Forest model specifies θ as classification tree marker for $h(X|\theta)$ and a fixed probability distribution for θ for diversity determination in trees is known.
The margin function of an RF is:

$$margin_{RF}(x,y) = P_\theta(h(x|\theta) = y) - max_{j \neq y} P_\theta(h(x|\theta) = j) \tag{18}$$

The strength of the forest is defined as the expected value of the margin:

$$s = E_{x,y}(margin_{RF}(x,y)) \tag{19}$$

The generalization error is bounded above by Chebyshev's inequality and is given as

$$Error = P_{x,y}(margin_{RF}(x,y) < 0) \leq P_{x,y}(|margin_{RF}(x,y) - s| \geq s) \leq \frac{var(margin_{RF}(x,y))}{s^2} \tag{20}$$

Remark: *We know that the average margin of the ensemble of classifiers is the extent to which the average vote count for the correct class flag exceeds the count for the next best class flag. The Strength of the forest is the expected value of this margin. When the margin function gives a negative value, it means that an error has been made in classification. The generalization error is the probability that the margin is a negative value. Since margin itself is a random variable, equation (20) shows that it is bounded above by its variance divided by the square of the threshold. As the strength of the forest grows, error in classification decreases.*
we present below, the Chebyshev's inequality as the inspiration for the error bound.

Chebyshev's inequality: *Let X be any random variable (not necessarily non-negative) and $C > 0$. Then,*

$$P(|X - E(X)| \geq c) \leq \frac{var(x)}{c^2} \tag{21}$$

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Remark: *It's easy to relate the inequality to the error bound of the Random Forest learner.*

Proof of Chebyshev's inequality: *We require a couple of definitions before the formal proof.*

   A) Indicator Random Variable

$$I(X \geq c) = \begin{cases} 1 & if\ X \geq c \\ 0 & Otherwise \end{cases} \tag{22}$$

   B) Measurable space

$$A = \{x \in \Omega | X(x) \geq c\} \tag{23}$$

$$E(X) = \sum_{x \in A} P(x)X(x) = \mu \tag{24}$$

Proof:

$$Define, A = \{x \in \Omega | X(x) - E(x) \geq c\}$$

$$Thus,\ var(X) = \sum_{x \in \Omega} P(X = x)(X(x) - E(x))^2$$

$$= \sum_{x \in A} P(X = x)(X(x) - E(x))^2 + \sum_{x \notin A} P(X = x)(X(x) - E(x))^2 \geq 0$$

$$\geq \sum_{x \in A} P(x = x)(X(x) - E(x))^2$$

$$\geq P(X = x)c^2 \quad since, X(x) - E(x) \geq C; \forall x \in A$$

$$= c^2 P(A) = c^2 P(|X - E(X)| \geq c)$$

$$=> \frac{var(X)}{c^2} \geq P(|X - E(X)| \geq c)$$

Remark: *This means that the probability of the deviation of a data point from its expected value being greater than c, a threshold, is bounded above by the variance of the data points divided by the square of the threshold, c. As c increases, the upper bound decreases which implies the probability of a large deviation of a data point from its expected value is less likely.*

5.2   OOB error visualization

After creating all the decision trees in the forest, for each training sample $Z_i = (X_i, Y_i)$ in the original training set T, we select all bagged sets T_k which does not contain Z_i. This set contains bootstrap datasets which do not contain a particular training sample from the original training dataset. These sets are called out of bags examples. There are n such sets for each n data samples in the original training dataset. OOB error is the average error for each Z_i calculated using predictions

from the trees that do not contain z_i in their respective bootstrap sample. OOB error is an estimate of generalization error which measures how accurately the random forest predicts previously unseen data. We plotted the OOB error rate for our random forest classifier using the AAPL dataset.
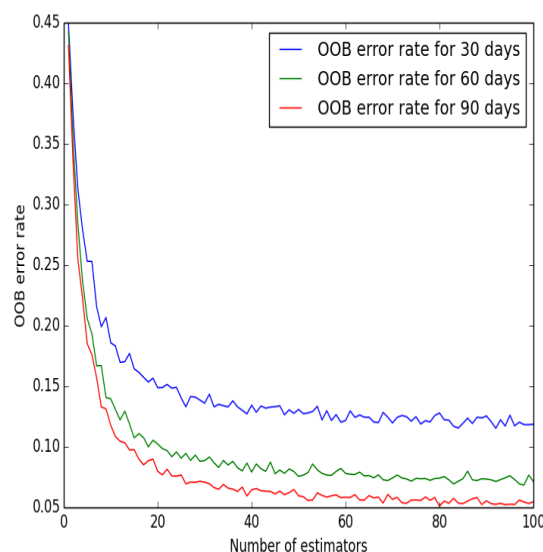

Fig 5: OOB error rate vs Number of estimators

From the above plot, we can see that the OOB error rate decreases dramatically as more number of trees are added in the forest. However, a limiting value of the OOB error rate is reached eventually. The plot shows that the Random Forest converges as more number of trees are added in the forest. This result also explains why random forests do not overfit as more number of trees are added into the ensemble.

6.   Results

Using the prediction result produced by our model we can decide whether to buy or sell our stock. If the prediction is +1, which means the price is expected to rise after n days then the suggested trading decision is to buy the stock. Whereas, if the prediction is -1, it means the price is expected to fall after n days and the suggested trading decision is to sell the stock. Any wrong prediction can cause the trader a great deal of money. Hence, the model should be evaluated for its robustness. The parameters that are used to evaluate the robustness of a binary classifier are accuracy, precision, recall (also known as sensitivity) and specificity. The formula to calculate these parameters are given below:

$$Accuracy = \frac{tp + tn}{tp + tn + fp + fn} \tag{25}$$

$$Precision = \frac{tp}{tp + fp} \tag{26}$$

$$Recall = \frac{tp}{tp + fn} \tag{27}$$

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$$Specificity = \frac{tn}{tn + fp} \qquad (28)$$

where,

tp *= Number of true positive values*
tn *= Number of true negative values*
fp *= Number of false positive values*
fn *= Number of false negative values*

Accuracy measures the portion of all testing samples classified correctly. Recall (also known as sensitivity) measures the ability of a classifier to correctly identify positive labels while specificity measures the classifier's ability to correctly identify negative labels. And precision measures the proportion of all correctly identified samples in a population of samples which are classified as positive labels. We calculate these parameters for the next 1 Month, 2 Months and 3 Months prediction model using AAPL, GE dataset (Which are listed on NASDAQ) and Samsung Electronics Co. Ltd. (Which is traded in Korean Stock Exchange). The results are provided in the tables below:

Trading Period	Accuracy%	Precision	Recall	Specificity
1 month	86.8396	0.881818	0.870736	0.865702
2 months	90.6433	0.910321	0.92599	0.880899
3 months	93.9664	0.942004	0.950355	0.926174

Fig 6: Results for Samsung dataset

Trading Period	Accuracy%	Precision	Recall	Specificity
1 month	88.264	0.89263	0.90724	0.84848
2 months	93.065	0.94154	0.93858	0.91973
3 months	94.533	0.94548	0.96120	0.92341

Fig 7: Results for Apple Inc. dataset

Trading Period	Accuracy%	Precision	Recall	Specificity
1 month	84.717	0.85531	0.87637	0.80968
2 months	90.831	0.91338	0.93099	0.87659
3 months	92.543	0.93128	0.94557	0.89516

Fig 8: Results for GE Dataset

6.1   Receiver Operating Characteristic

The Receiver Operating Characteristic is a graphical method to evaluate the performance of a binary classifier. A curve is drawn by plotting True Positive Rate (sensitivity) against False Positive Rate (1 - specificity) at various threshold values. ROC curve shows the trade- off between sensitivity and specificity. When the curve comes closer to the left-hand border and the top border of the ROC space, it indicates that the test is accurate. The closer the curve is to the top and left-hand border, the more accurate the test is. If the curve is close to the 45 degrees diagonal of the ROC space, it means that the test is not accurate. ROC curves can be used to select the optimal model and discard the suboptimal ones.
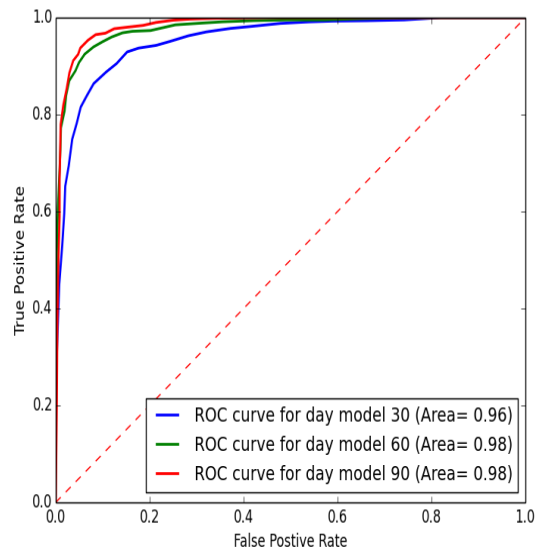
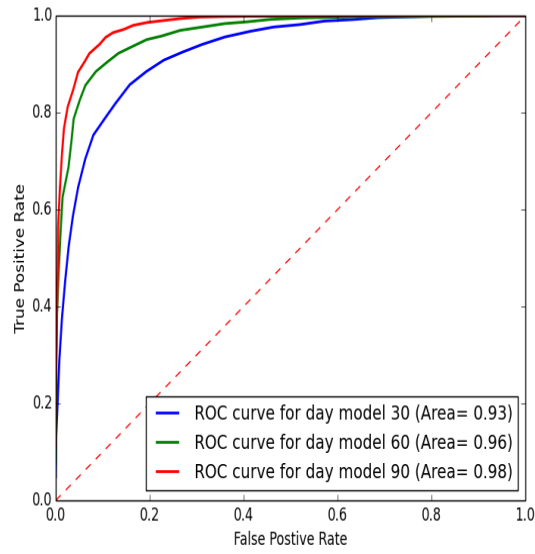Fig 9: ROC curves corresponding to AAPL dataset

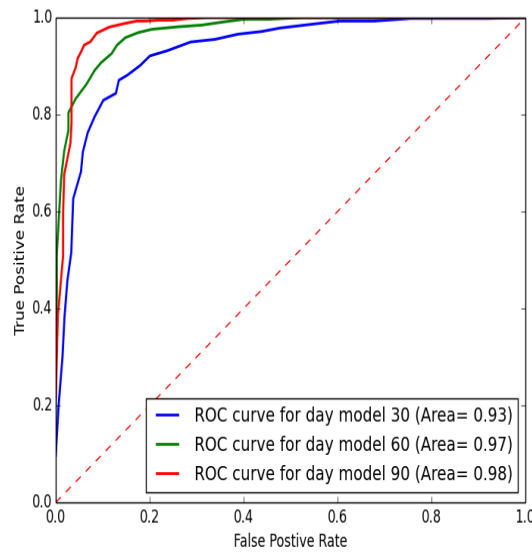Fig 10: ROC curves corresponding to GE dataset

Fig 11: ROC curves corresponding to Samsung dataset

As we can see from the ROC curves, the 90 Day model proves to be the most optimal model. The area under the ROC curve is a really important parameter for evaluating the performance of a binary classifier. Accuracy is measured by the area under the ROC curve. An area of 1 represents an excellent classifier; an area of .5 represents a worthless classifier which produces random outputs. In other words , the area measures discrimination, that is, the ability of the classifier to correctly classify a positive shift and a negative shift in stock prices in case of our problem. T he area under the ROC curve is above 0.9 for all three models using all three datasets. This means our classifier is excellent.

7. Discussion and Conclusion

The robustness and accuracy of the proposed algorithm in contrast with he ones present in literature need to be discussed. We'll perform a comparative analysis between the results found in Dai and Zhang (2013) and Xinjie (2014) with the results produced by our model for the same dataset. In Dai and Zhang (2013), the authors selected 3M stock which contained daily data ranging from 1/9/2008 to 11/8/2013. They have used four supervised learning algorithms, i.e Logistic Regression, Gaussian Discriminant Analysis, Quadratic Discriminant Analysis, and SVM. Their results are summarized in the Fig 12.
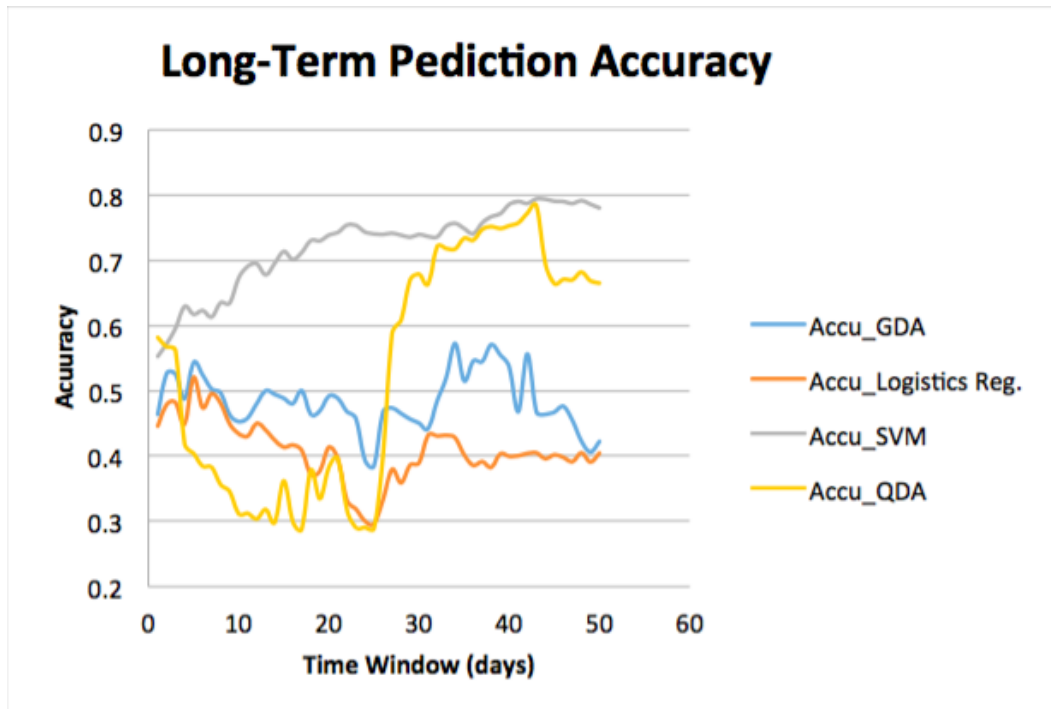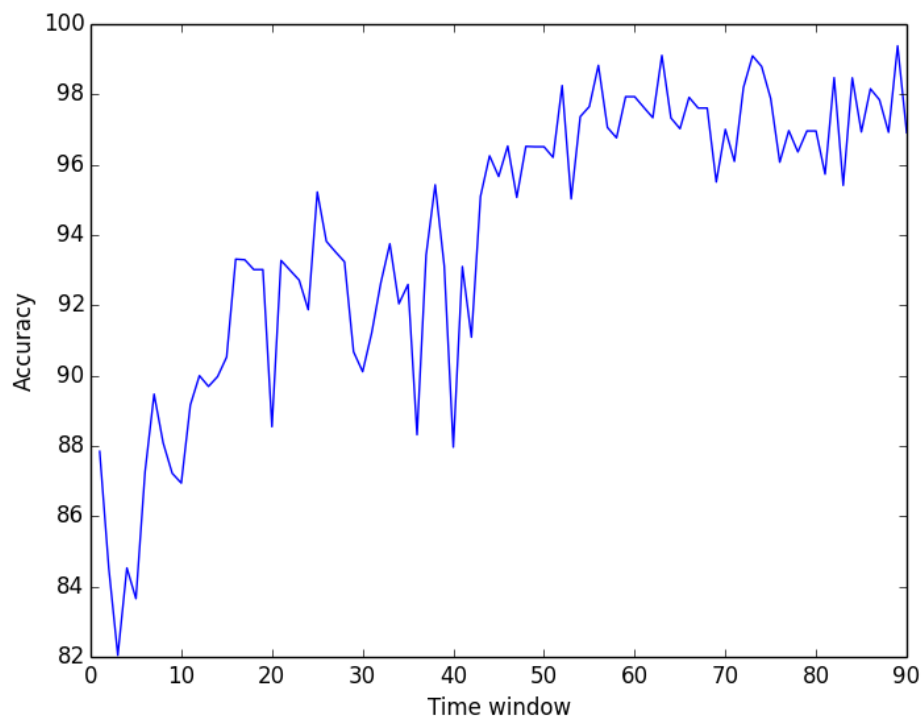
Fig 12: Results from Dai and Zhang (2013)

Fig 13: Results for 3M stock obtained with our model

From the plot in Fig 12:, we can see that for SVM and QDA model, the accuracy increases when the time window increases. Furthermore, SVM gives the highest accuracy when the time window is 44 days (79.3%). Its also the most stable model.

Using the same dataset as in Dai and Zhang (2013), we calculated the accuracy for various time widows using the model we have built. The results which we have found is visualized as a graph in Fig 6. As we can see from this graph, the accuracy peaked at 96.92% when the time window is 88 days. This is clearly a better result than the one found in Dai and Zhang (2013).

For our next comparative, we will be looking at the result obtained in Xinjie (2014). The author of Xinjie (2014) have chosen three datasets for the study, i.e AAPL, MSFT and AMZN. The time span of the stock data ranges from 2010-01-04 to 2014-12-10. Xinjie (2014) has used an extremely randomized tree algorithm to select a subset of features from a total of 84 technical indicators. These features are then fed to an SVM with rbf kernel for training to predict the next 3-day, next 5-day, next 7-day and next 10-day trend. The results are given in the table below.

Company/Accuracy	Next 3-day	Next 5-day	Next 7-day	Next 10-day
Apple	73.4%	71.41%	70.25%	71.13%
Amazon	63%	65%	61.5%	71.25%
Microsoft	64.5%	73%	77.125%	77.25%

Fig 14: Results from Xinjie (2014)

We calculated the accuracy result for the same datasets using our prediction model and obtained the results which given in fig 15.

Company/Accuracy	Next 3-day	Next 5-day	Next 7-day	Next 10-day
Apple	85.197%	83.88%	88.11%	92.08%
Amazon	86.51%	88.49%	85.14%	87.46%
Microsoft	84.59%	83.88%	89.47%	86.46%

Fig 15: Results obtained using our model

Devi, Bhaskaran and Kumar (2015) used BSE-SENSEX and CNX-NIFTY datasets to predict next day outcome using SVM with Cuckoo Search optimization. The results are summarized in the bar charts below.
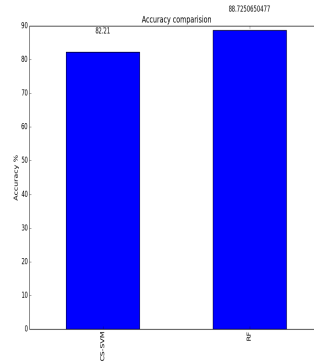

Fig 16: Comparing Accuracies of CS-SVM as obtained in Devi(2015) and RF for BSE-SENSEX
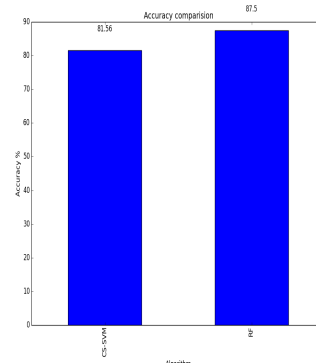
Fig 17: Comparing Accuracies of CS-SVM as obtained in Devi(2015) and RF for BSE-SENSEX

SVM with Cuckoo search optimization performs really well giving accuracy results of above 80%. However, the Random Forest classifier still performs better than the model proposed in Devi, Bhaskaran and Kumar (2015).

From the comparative analysis we have done in this section, we can confidently say that our model outperforms the models as seen in various papers in our literature survey. We believe that this is due to the lack of proper data processing in Li, Li and Yang (2014),Dai and Zhang (2013),Xinjie (2014),Devi, Bhaskaran and Kumar (2015). In this paper, we have performed exponential smoothing which is a rule of thumb technique for smoothing time series data. Exponential smoothing removes random variation in the data and makes the learning process more easier. But in none of the papers we have reviewed, have used exponential smoothing to smooth their data. Another important reason could be the inherent non linearity in data. This fact discourages the use of linear classifiers. However in Li, Li and Yang (2014), the authors have used linear classifier algorithm: Logistic Regression as their supervised learning algorithm which yielded a success rate of 55.65%. We believe that the use of SVM in Dai and Zhang (2013) and Xinjie (2014) is not very wise. Due to that fact that the two classes in consideration (rise or fall) are linearly inseparable, researchers are compelled to use SVM with non linear kernels such as Gaussian kernel or Radial Basis Function. Despite many advantages of SVMs, from a practical point of view, they have some drawbacks. An important practical question that is not entirely solved, is the selection of the kernel function parameters - for Gaussian kernels the width parameter σ - and the value of ϵ in the ϵ loss insensitive function (Horvth (2003) in Suykens et al.).

7.1   Conclusion

Predicting stock market due to its non linear, dynamic and complex nature is really difficult. However in the recent years, machine learning techniques have proved effective in stock forecasting. Many algorithms such as SVM, ANN etc. have been studied for robustness in predicting stock market. However, ensemble learning methods have remained unexploited in this field. In this paper, we have used random forest classifier to build our predictive model and our model has produced really impressive results. The model is proved to be really robust in predicting future direction of stock movement. The robustness of our model has been evaluated by calculating various parameters such as accuracy, precision, recall and specificity. For all the datasets we have used i.e, AAPL, MSFT and Samsung, we were able to achieve accuracy in the range 85-95% for long term prediction. ROC curves were also plotted to evaluate our model. The curves graphically proved the robustness of our model. It was also proved that our classification algorithm converges as more number of trees are added to the random forest.

Our model can be used for devising new strategies for trading or to perform stock portfolio management, changing stocks according to trends prediction. For future work, we could build random forest models to predict trends for short time window in terms of hours or minutes. Ensembles of different machine learning algorithms can also be checked for its robustness in stock prediction. We also recommend exploration of the application of Deep Learning practices in Stock Forecasting. These practices involve learning weight coefficients on large directed and layered graph. Deep Learning models, known earlier as problematic in training, are now being embraced in stock price estimation due to the recent advances.

The model proposed indicates, for the first time, to the best of our knowledge the nonlinear nature of the problem and the futility of using linear discriminant type machine learning algorithms. The accuracy reported is not pure chance but is based solidly on the understanding that the problem is

not linearly separable and hence the entire suite of SVM type classifiers or related machine learning algorithms should not work very well. The solution approach adopted is a paradigm shift in this class of problems and minor modifications may work very well for slight variations in the problem statement.

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