Bachelor Thesis - Determining Profitable Pairs Trading Strategies under Cointegration

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Abstract

The purpose of this paper is to determine if pairs trading is still a profitable trading strategy, in line with current literature, it still is. How it answers the question is multifaceted, we do so by investigating different models under various parameters and implementing different strategies under a cointegrated framework. The data used in this study are all tradable securities on the NYSE, NASDAQ, ARCA and AMEX. The results in the study show that using moving windows to regularly update trading signals and using a GARCH(1,1) model to adjust trading bands with a conditional volatility produces the highest winning ratio of winning pairs and best overall performance. The study is limited as trading costs are not accounted for and also more investigative studies could have been carried out on other estimated parameters and particular trading strategies could have been extended.

Keywords: Pairs trading, Cointegration, Kalman filter, GARCH, mean-reversion, trading strategies

Contents

1 Introduction

Wall Street in the 21st century is said to be dominated by mathematicians, physicists and computer scientists employed by hedge funds or through propriety trading. These Wall Street wizards use data driven algorithms and have made the intuitions of the so-called 'Kings of Wall Street' irrelevant. As The Wall Street Journal puts it, "Prognosticators imagined a time when data-driven traders who live by algorithms rather than instincts would become the kings of Wall Street. That day has arrived." (Gregory & Bradley, 2019).

The success of many trading algorithms depend on the quality of the predictions of stock price movements. Predictions of stock prices are generally less accurate than a portfolio of stocks. A classical strategy that tries to make the most of the predictability of a portfolio of stocks or securities is pairs trading, where a linear combination of two assets is traded. At the heart of this strategy is how these two assets co-move (Alvaro et al., 2015).

Although more complicated models can be applied to pairs trading strategies, it is fundamentally quite simple. The series obtained through a linear combination of two assets, is called the spread. If there is a large enough deviation from its supposed mean, a miss-pricing is identified, then a profit can be made when the spread reverts back to its mean, under the efficient market hypothesis.

Pairs trading can be put into a class of strategies called **statistical arbitrage**. A true statistical arbitrage strategy produces returns in excess of the risk-free rate with zero risk. However, pairs trading strategies are not entirely true statistical arbitrage in the sense that they rely on the joint predictability of the assets and hence are not risk free.

In the literature, it is said that pairs trading strategies were first implemented by Nunzio Tartaglia at Morgan Stanly in the mid 1980's, who assembled a team of mathematicians, computer scientists and physicists that achieved great success by 1987. However, the team started to produce large losses, and so, the team dissolved in 1989 and the members of the team went their own ways or joined different funds, and the knowledge of pairs trading spread into the market (Vidyamurthy, 2004).

Given the inherent proprietary nature of this trading strategy, it became a popular research topic the 90's and early 00's, where it gradually faded. Huck & Afawubo (2015), states that there has been a sudden burst of interest in recent years, including several books. The academic literature can be divided into two main approaches, although Huck & Afawubo (2015) and Krauss (2017) might argue differently. The first approach, was the minimum sum squared distance (SSD), being one of first approaches to be implemented. Followed by, the cointegration. For this study, we will use the cointegration approach and will not dwell on the other possible approaches. In tandem with recent literature, in this study we also pose the question if pairs trading is a still profitable strategy. Moreover, this study differs from other literature, in several aspects. First, we study the performance of pairs trading under different models, the normal cointegrated approach, but repeat it using a Kalman filter. Second these models are studied under various parameters as a sensitivity analysis. Third, we study if certain estimated parameters can provide any insight on performance. Finally, we also study the performance when adjusting trading signals to conditional volatility using a $GARCH$ model, similar to Miah et al. (2016).

The rest of the this study is organised as follows. Section 2 provides a theoretical background with key concepts that will be used for trading, section 3 describes the methodology and the various parameters that will be used. Section 4 provides the results for the various parameters under the two models that are implemented. Section 5 hopes to find any insight between estimated parameters, from the linear relationship between pairs of course, and performance. Section 6 is where we implements conditional volatility to trading signals. Finally, section 7 concludes our study.

2 Theory

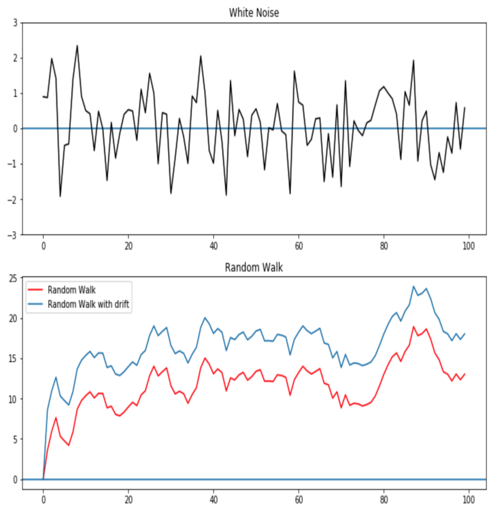
The theoretical key concept of this thesis is cointegration, the works of Shumway & Stoffer (2017) and Zivot & Wang (2007) are used for theoretical discussion on time series and key ideas from Vidyamurthy (2004) will be used for pairs trading and on spread dynamics. We start this section with an introduction to times series and gradually build up to the theoretical models that will be implemented for our trading strategies.

2.1 Preliminaries

2.1.1 Time Series

Time series can be defined a set of random variables that are indexed in the order of time from where they are first obtained, as $x_t, t = 0, 1, 2, ...$. In general, a collection of random variables indexed by some $t, t \in \mathbb{Z}$, is usually know as a *stochastic process*. The realizations of the stochastic process are usually the observed or measured values. i.e. the observations of a stock price. The simplest example of a time series is a **white noise**, it is an indexed collection where the realizations are obtained from an independent and identical distribution, $x_t = \epsilon_t \sim \mathcal{WN}(0, \sigma_w^2)$.

Figure 1: Example of white noise and a random walk (and with drift)



2.1.2 ARMA models

However, in most cases it would not be enough to describe a stochastic process as a simple white noise process, and so other models are needed. For instance, **autoregression** and **moving averages**. Also known as AR(p) and MA(q) models, where p and q are the number of lags or previous observational values used:

AR(p):

$$x_t = \theta_1 x_{t-1} + \theta_2 x_{t-2} + ... + \theta_p x_{t-p} + \epsilon_t = \sum_{i=1}^{p} \theta_i x_{t-i} + \epsilon_t \tag{1}$$

AR(p) with drift:

$$x_t = \mu + \theta_1 x_{t-1} + \theta_2 x_{t-2} + ... + \theta_p x_{t-p} + \epsilon_t = \mu + \sum_{i=1}^{p} \theta_i x_{t-i} + \epsilon_t \tag{2}$$

AR(1) - Random walk: It would be safe to say that stock prices act like a random walk, see figure 1. That is, the cumulative sum of a white noise process:

$$x_t = \mu + \theta_1 x_{t-1} + \epsilon_t \tag{3}$$

A necessary condition is that $\theta = 1$ and setting $x_0 = \mu$, let μ be some constant, such that a random walk (see figure 1) with drift is:

$$x_t = \sum_{i=1}^{t-1} \mu + \sum_{i=1}^{t-1} \epsilon_{t-i} + \epsilon_0, \epsilon_t \sim \mathcal{WN}(0, \sigma_\epsilon) \tag{4}$$

The other popular time series model is the **MA(q)**, where a time series can be expressed as q number of previous white observations:

MA(q)

$$y_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + ... + \theta_1 \epsilon_{t-q} \tag{5}$$

Combining these models, an ARMA representation of a time series model can be expressed as:

$$x_t = \mu + \theta_1 x_{t-1} + ... + \theta_p x_{t-p} + \psi_1 \epsilon_{t-1} + ... + \psi_q \epsilon_{t-q} + \epsilon_t \tag{6}$$

Then $\mathbb{E}(x_t) = \mu$. We define the **autocovariance function** as:

$$\gamma_s = cov(x_t, x_{t-s}) = \mathbb{E}[(x_t - \mu)(x_{t-s} - \mu_t)] \tag{7}$$

Only if x_t is stationary. $\gamma_0 = var(x_t), \forall t$ and we define the **autocorrelation function** as

$$\rho_s = \frac{cov(x_t, x_{t-s})}{\sqrt{var(x_t) var(x_{t-s})}} = \frac{\gamma_s}{\gamma_0} \tag{8}$$

Where $\gamma_0 = var(x_t)$. It can be observed that through substitution an AR(1) process can be expressed

as an MA(∞) process. Moreover, suppose we can go back infinitely back in time, then we can express an AR(p) as MA(∞), and, MA(q) can be expressed as an AR(∞), (Zivot & Wang, 2007).
Moreover, Zivot & Wang (2007) state that an ARMA(p, q) can take the form:

$$x_t - \mu = \phi_1(x_{t-1} - \mu) + ... + \phi_p(x_{t-p} - \mu) + \epsilon_t + \theta_1\epsilon_{t-1} + ... + \theta_q\epsilon_{t-q} \tag{9}$$

We introduce the lag operator as L, where $Lx_t = x_{t-1}$ and $L^2x_t = L.x_{t-1} = x_{t-2}$. Then, we define $\phi(L) = 1 - \phi_1L - ... - \phi_pL^p$ and $\theta(L) = 1 + \theta_1L + ... + \theta_qL^q$. Then (9) may be expressed as:

$$\phi(L)(x_t - \mu) = \theta(L)\epsilon_t \tag{10}$$

Now suppose we have an $AR(p)$ process in its mean-adjusted form:

$$x_t - \mu = \phi_1(x_{t-1} - \mu) + ... + \phi_1(x_{t-p} - \mu) + \epsilon_t \tag{11}$$

or, in lag notation:

$$\phi(L)(x_t - \mu) = \epsilon_t \tag{12}$$

where $\phi(L) = 1 - \phi_1L - ... - \phi_pL^p$. Then, we define - as in Zivot & Wang (2007) (3.9) - the *characteristic polynomial*:

$$\phi(z) = 1 - \phi_1z - \phi_2z^2 - ... - \phi_pz^p \tag{13}$$

2.1.3 Unit root and Stationarity

We introduce the concept of an unit root and stationarity. Understanding these concepts allows us to make behavioural assumptions on the series and how the series can develop over time. We start by using definition 1.6 of Shumway & Stoffer (2017). A time series is **strictly stationary** if the probabilistic behaviour of every collection of values is identical to that of some time shifted set. In other words:

$$\mathbb{P}(x_{t_1} \leq c_1, ..., x_{t_k} \leq c_k) = \mathbb{P}(x_{t_1+h} \leq c_1, ..., x_{t_k+h} \leq c_k) \tag{14}$$

In particular:

$$\mathbb{P}(x_s \leq c) = \mathbb{P}(x_t \leq c) \tag{15}$$

Thus any points in s and t the distribution is the same. That is, its moments do not change over time. However, these are very strong assumption and do not apply in most real life cases. Thus we consider the weaker definition of (1.7) in Shumway & Stoffer (2017), **weakly stationary**:

- The time series of x_t is a finite variance process.
- The mean remains constant over time - $\mathbb{E}(x_t) = \mu_t = \mu$.
- The autocovariance function in (7) depends only on the difference of s and t, |s - t|.
- Henceforth, weakly stationary will be referred as stationary.

Coming back to the characteristic equation (13) of the $AR(p)$ process. It is said that (13) is stationary when all the roots of $\phi(z)$ lie outside the unit circle. In practice for stationarity it is necessary that $|\phi_1 + ... + \phi_p| < 1$.

We repeat the same concepts for an $AR(1)$ process. Suppose we have the following mean-adjusted form:

$$x_t - \mu = \phi(x_{t-1} - \mu) + \epsilon_t \tag{16}$$

Then the *characteristic equation* for the $AR(1)$ is:

$$\phi(z) = 1 - \phi z \tag{17}$$

it has a root for $z = \frac{1}{\phi}$. Stationarity is satisfied if the absolute value of the root of the characteristic equation is greater than one. $|\frac{1}{\phi}| > 1$ or $|\phi| < 1$.

Thus so far a stationary time series, $\mathcal{I}(0)$, fluctuates around its mean with a finite variance that does not depend upon time. These fluctuations around the mean is also known as **mean-reverting**; tendency to return to its mean. Where its auto correlations die out fairly rapidly. A nonstationary, $\mathcal{I}(1)$, series wanders widely with no finite mean or variance. Where $\mathcal{I}(d)$, represents the order of integration such that you would have to difference the time series d times in order to make it stationary, see figure 2 - in most real applications we deal with either $d = 0$ or $d = 1$, rarely $d = 2$. We have seen that stationarity is satisfied provided that the root of a time series lies outside the unit circle. Thus, we have the following hypothesis to determine if there is an unit root or not i.e. to determine whether a time series is stationary or not.

$$\begin{aligned} H_0 : \phi = 1(\text{unit root in } \phi(z) = 0) \Rightarrow y_t \sim \mathcal{I}(1) \\ H_1 : |\phi| < 1 \Rightarrow y_t \sim \mathcal{I}(0) \end{aligned}$$

There are several tests developed to determine an unit root such as the (Augmented) Dickey Fuller test, Phillips-Perron test, KPSS test or the Johansen test.

2.2 Cointegration

Cointegration was first introduced by Engle and Granger in 1987, for which they later received a Nobel Prize in economics in 2003. It describes a statistical relationship between non-stationary time series that are of order $I(d)$, where a linear combination of these times series would create a single stationary time series.

Let, $Y_t = (y_{1,t}, ..., y_{n,t})'$ denote a (n x 1) vector of $I(1)$ time series. Then Y_t is said to be cointegrated if there exists an (n x 1) vector $\beta = (\beta_1, ..., \beta_n)$ such that:

$$\beta' Y_t = \beta_1 y_{1,t} + ... + \beta_n y_{n,t} = \mu + \epsilon_t, \text{ is of order } I(0) \tag{18}$$

It can be argued that for a cointegrated time series, it has a long-run equilibrium and that the relations between the time series can not drift too far apart, as the underlying economic forces will

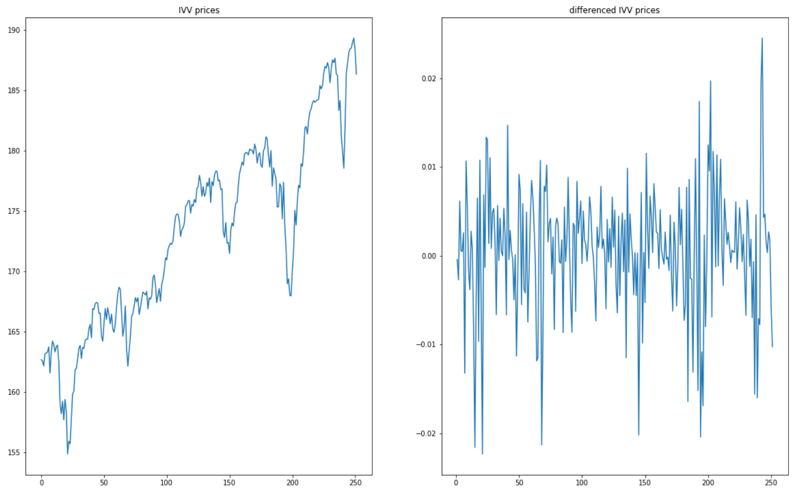


Figure 2: left: IVV ETF following a random walk, right: The same ETF differenced.

unwind deviations back to the long-run equilibrium. Where, μ is the long-run equilibrium and ϵ_t represents the cointegration error, which denotes any short term deviations at time t, it is important to note that ϵ_t is a *white noise process*, we see that because, suppose we solve for ϵ_t, then:

$$\epsilon_t = \beta' Y_t - \mu \tag{19}$$

Then for the right hand side to be stationary, i.e. the cointegrated time series minus a constant value, ϵ_t needs to be $I(0)$. Taking the expected value $\mathbb{E}(\epsilon_t) = \mathbb{E}(\beta' Y_t) - \mu = 0$.

2.2.1 Cointegration in context of pairs trading

Moreover, there does not only exist an unique β for which the combination of time series is cointegrated. Since for any scalar c the linear combination $c\beta' Y_t = \beta^* Y_t \sim I(0)$. Hence, some *normalization* assumption is required to uniquely identify β'. A typical normalization is:

$$\beta' = (1, -\beta_2, ..., -\beta_n) \tag{20}$$

so that

$$\beta' Y_t \rightarrow y_{1,t} - \beta_2 y_{2,t}, ..., -\beta_n y_{n,t} = \mu + \epsilon_t \sim I(0) \tag{21}$$

or

$$\beta' Y_t \rightarrow y_{1,t} = \mu + \beta_2 y_{2,t}, ..., +\beta_n y_{n,t} + \epsilon_t, \text{ where } \epsilon_t \sim I(0) \tag{22}$$

There error term ϵ_t is oftern referred to as the *disequilibrium error* or the *cointegrating residual*. In the long-run equilibrium, the disequilibrium error ϵ_t is zero and the long-run equilibrium relationship is:

$$\beta' Y_t \rightarrow y_{1,t} = \mu + \beta_2 y_{2,t}, ..., + \beta_n y_{n,t} \tag{23}$$

Suppose we have two stocks that are of order $I(1)$, that have the following relation:

$$stock_{1,t} = \alpha + \beta stock_{2,t} + u_t \text{ where } u_t \sim \mathcal{N}(0, \sigma_u^2) \tag{24}$$

Let the cointegrating vector be $\beta' = (1, -\beta)$, then we create a single stationary time series, also known as the spread:

$$spread_t = stock_{1,t} - \beta stock_{2,t} = \alpha + u_t \sim I(0) \tag{25}$$

We can estimate the parameters $\{\alpha, \beta, u_t\}$ by means of OLS regression:

$$\hat{u}_t = stock_{1,t} - \hat{\alpha} - \hat{\beta} stock_{2,t} \tag{26}$$

$$\hat{s}pread_t = \hat{s}tock_{1,t} - \hat{\beta} stock_{2,t} = \hat{\alpha} + \hat{u}_t \sim I(0) \tag{27}$$

The long-run equilibrium is $\hat{\alpha}$ and short term deviations are $\hat{u}_t$ and $\mathbb{E}(spread_t) = \mathbb{E}(stock_{1,t} - \hat{\beta} stock_{2,t}) = \hat{\alpha}$.

In order to determine whether the spread is stationary, we use the **Engle - Granger 2 step approach** as proposed in their work in 1987, (Engle & Granger, 1987). The first step is to determine their order of integration; specifically to test for unit-root. The second step consists of estimating the long-run equilibrium between the two time series by the use of OLS regression as defined in (23). Once the parameters are estimated, we test whether the residuals, $\hat{u}_t$ is stationary or not through the use of the use of the ADF test. If, $\hat{u}_t$ is stationary, then the two time series is said to be *stationary*. However, we do not carry out the first step as it is safe to assume that the time series that make up the spread are $I(1)$.

2.2.2 Correlation and Cointegration

The first step in our method of selecting pairs is through correlation, see section 3. Although correlation and in fact very high correlation (like our pairs) might be tempting to consider a causal relationship, it should not as problems might arise, for example a *spurrious regression*. This occurs when two independent non-stationary series are regressed, where from the surface might suggest a strong relation such as a significant t value and high R^2. However, the estimated coefficient will not converge to zero, but instead converges to a distribution to a non-normal variable that is not necessarily centered at zero, which would not be the case if the series to be stationary, (Zivot & Wang, 2007).

If two times series where cointegrated its estimated parameter, $\hat{\beta}$ will converge to its true value, β, and so regression with $I(1)$ data would only make sense when the data are cointegrated. Because under cointegration a long-run relationship can be determined with the entire data set, without having to difference the data, and so not losing valuable information. Although correlation does indicate some co-movement, it lacks stability.

Therefore, as discussed in section 3, correlation is used as primary step to determine co-movement. The second step would be to test for cointegration. Since, cointegration relations are defined in the long-run, it has beneficial applications to trading - so to say in theory at least - such as pairs trading.

2.3 GARCH models

Financial markets often exhibit volatility clustering, that is a time series would experience periods of high or low volatility. Zivot & Wang (2007) comments that with economic and financial data, time-varying volatility is more common than constant volatility, and in order to accurately model of time-varying are of great importance. This where we introduce the GARCH model, *Generalized Auto Regressive Conditional Heteroskedasticity*, which was first attributed by Tim Bollerslev in 1986. Then we define the GARCH(p, q) model as:

$$y_t = \mu + \epsilon_t \tag{28}$$
$$\epsilon_t = z_t \sigma_t \tag{29}$$
$$\sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} b_j \sigma_{t-j}^2 \tag{30}$$

Where μ is the mean of y_t and z_t is normal identical and independently distributed, and ϵ_t is $i.i.d$ with mean zero. The conditional variance is of the series is denoted by σ_t^2 . For simplicity we assume that ϵ_t is normally distributed with mean zero and variance of 1. Zivot & Wang (2007) argues that using a GARCH(1,1) with 3 parameters is adequate enough, Miah et al. (2016) modelled volatility in daily stock returns and found that using GARCH(1,1) is the best specification than other GARCH(p,q) models, where $a_0 > 0$ and $a_i, b_j \geq 0, \forall i, j$. So, we rewrite (30) as:

$$\sigma_t^2 = a_0 + a_1 \epsilon_{t-1}^2 + b_1 \sigma_{t-1}^2 \tag{31}$$

For the model in (41) to be stationary it is required that $a_1 + b_1 < 1$. For estimation of the parameters, further discussion can be found in Zivot & Wang (2007, Chapter 7.4).

2.4 Kalman Filter

A special case of a linear regression model that subsumes its own category, are state space models or the dynamic linear model. In this section we will briefly introduce the state space model and then the Kalman filter. In general, state space models can be defined by two main concepts. First, there is hidden or latent process, say x_t, called the *state* process. Second, are the observations themselves, say y_t, and are independent given x_t such that the observations y_t are dependent on the given state x_t. See figure 3 below. Zivot & Wang (2007) and Shumway & Stoffer (2017) provide further treatment on state space models.

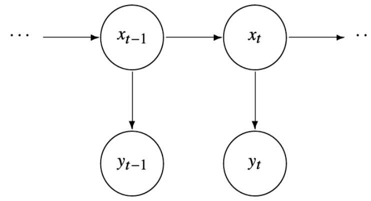


Figure 3: Diagram of a state space model - Diagram taken from Shumway & Stoffer (2017)

State space models have many modern applications ranging from its first application on the Apollo 11 mission to GPS tracking to applications in finance. The application of the Kalman filter in this research is applied to the spread as defined in (26). Note that if t are the estimated values of state process and n equals the number of observations then filtering is when $t < n$ and filtering when $t = n$.
We will use the notation used in (Shumway & Stoffer, 2017, Chapter 6) to describe state space models and the Kalman filter.

We introduce the *state equation* as a linear Gaussian state space model in its basic form, which matches our assumption that our spread follows an AR(1) process, and so we only consider a univariate case.

$$x_t = \phi x_{t-1} + \omega_t \tag{32}$$

Where ω_t is normal independent and identically distributed with zero mean and variance Q such that, $\omega_t \sim$ iid $\mathcal{N}(0, Q)$. We assume that the process starts with $x_0 \sim \mathcal{N}(\mu_0, \sigma_0)$. We do not observe the *state* process directly but only a linear transformed version of it with an added noise:

$$y_t = A_t x_t + v_t,\ v_t \sim iid\mathcal{N}(0, R) \tag{33}$$

Where A_t and R are scalars, as in an univariate case. (33) Is also known as the *observation equation*.

For the Kalman filter, with some slight different notation, we define the conditional estimation of the state process and covariance, given information up to $t-1$, as:

$$x_{t|t-1} = \mathbb{E}(x_t | y_{1:t-1}) \tag{34}$$
$$P_{t|t-1} = \mathbb{E}\{(x_t - x_{t|t-1})(x_t - x_{t|t-1})' | y_{1:t-1}\} \tag{35}$$

Then the Kalman filter, for $t = 1, ..., n$ is recursively updated, we assume the following initial conditions; $x_0 = \mu_0$ and $P_0 = \sigma_0$. Then:

$$x_{t|t-1} = \phi x_{t-1|t-1} \tag{36}$$
$$P_{t|t-1} = \phi P_{t-1|t-1} \phi' + Q \tag{37}$$

with

$$x_{t|t} = x_{t|t-1} + K_t(y_t - A_t x_{t|t-1}) \tag{38}$$

$$P_{t|t} = [I - K_t A_t] P_{t|t-1} \tag{39}$$

where

$$K_t = P_{t|t-1} A_t' [A_t P_{t|t-1} A_t' + R]^{-1} \tag{40}$$

When the spread of a pair of securities is stationary, the pairs are cointegrated, that is to say a long-run relationship is identified. We implement the Kalman filter on the spread to estimate the 'true' state of the spread and compare its performance.

3 Methodology

The data used for this research is a premium dataset from Quandl, and consists of end of the day prices both un- and adjusted prices from all listed securities the on NASDAQ, AMEX, NYSE and ARCA with stock history dating from 1996 to 2020. We used the Quantopian Zipline backtesting software to trade pairs. The initial capital used for each traded pair is set at 100.000, although no commission or sliggape is set during the research, the default commission is set at $ 0.001 per share in the backtest package. Due to the nature of the data, the backtest algorithm only trades once per day, and closes all positions at the end of the trading period.

3.1 Pairs Selection

To determine which pairs to trade we used the trading year of 2014 as a formation period and the subsequent years are used for trading. In the formation period we created a pipeline of all listed securities on the previously mentioned exchanges, from the data there were more than 6700 possible securities to trade, checking for cointegration on each possible pairs would be computationally exhaustive as there are more than 20 million possible pairs to be made. Vidyamurthy (2004), infers that a cointegrated system needs it constituents to be perfectly correlated. Thus, from our pipeline of of 6700+ tradable securities, we created a 6718×6718 correlation matrix and obtained 2472 pairs that are highly correlated (between 0.99 and 1). From this list of pairs we were interested in the top 250 most correlated pairs. However, due to some technical reasons, we could only investigate 239 pairs which consist of 324 individual securities, see appendix 8.1.

Although correlation does suggest co-movement, it does not imply cointegration. From the possible candidates we implemented the 2 step Engle and Granger approach to determine which ones are cointegrated, where we estimate a linear relationship and perform the ADF test on the residuals. From 239 pairs we can then either determine if the spread is stationary or not i.e. whether the spread cointegrated or not. If the reader permits, we shall refer to pairs that have cointegrated spread as either *stationary pairs* or *coinntegrated pairs*. We perform backtests on both these stationary and non-stationary pairs and see how their cointegration relationship changes over time. We believe it is worthwhile to see how well these two subsets perform against each other. The philosophy behind it is that breaks in cointegration relationships does occur. In table 42, we experience such structural breaks within the trading periods. Similarly, we experience pairs that are non-stationary become stationary at some point in time. Furthermore, these breaks are measured annually, but this is not to say that several breaks can occur during the trading period and thus potentially influencing the performance of such pairs annually and during trading periods. In order to prevent spread from wandering off, due to a structural break, we set a *stop* position at 3 standard deviations.

Having identified the pairs to trade, we implement three different trading strategies. First, we trade the aforementioned pairs, hereafter to be named the Basic model. Second, Vidyamurthy (2004), also proposes different spread dynamics to model the spread, and so, we will implement the Kalman filter on the spread. Third and final, is that we implement a similar trading strategy as done by Miah et al. (2016), section 6, by creating trading bands that are based on the conditional volatility that we estimated through a $GARCH(1,1)$ model. In addition, we repeat these models under various parameters. Unlike other existing literature, Huck (2013), points out that among the main academic literature there are three main parameters that are always considered; the formation period, trading period and spread deviations. This is where this thesis differs, the parameters that will be used to compare the performance for all models are: Time horizon, Adjusting bands i.e. deviations and moving windows.

3.2 Trading Rules

For trading to happen we have to establish some metric to determine exit and entry signals. We have defined the spread in (24), that is cointegration determined by the ADF test on the residual values of the OLS. We normalize the spread, called the *zscore*. Note that unless specified, the parameters used in this research are all based on the formation period. It is of interest to see if cointegration relationships in the formation period also holds in subsequent years, as we believe that there is a long term relationship. However, we must also be aware that it is possible for breaks in the cointegrated relationship due to price shocks. We define the *zscore* as follows:

- $spread_t = stock_{1,t} - \hat{\beta} stock_{2,t}$
- mean of formation period: $\hat{\mu}_f = \frac{1}{252} \sum_{k=1}^{s} spread_k$
- standard deviation of formation period: $\hat{\sigma}_f = \sqrt{\frac{\sum_{k=1}^{s} (spread_k - \mu_f)}{252}}$
- let t, be a trading day, then we define the $zscore_t$ (normalization of the spread) as : $zscore_t = \frac{spread_t - \mu_f}{\sigma_f}$ [1].

The *zscore* creates signals to trade i.e. deviations from the zscore. We denote the such deviations by Δ. In addition, we placed a stopping position at 3 standard deviations from the mean to act as a stopping mechanism in order to protect the portfolio in case the spread wanders off and does not come back to its mean. Furthermore, we have an exit position at 0.05 deviations from the mean - or in other words, we observe mean-reversion. Otherwise the algorithm either longs or shorts the spread.

- Short the spread if: $zscore_t > \Delta$. This indicates that the first stock is overvalued in terms of the second stock, so we *short* one unit $stock_{1,t}$ and *long* $\hat{\beta} stock_{2,t}$ units.
- Short the spread: $zscore_t < -\Delta$. Indicating that the first stock is undervalued in terms of the second stock, so we *short* $stock_{1,t}$ and *long* $\hat{\beta}$ units of $stock_{2,t}$. Shorting $-\hat{\beta}$ units.

[1] a zscore of 1, implies 1 standard deviation from the mean, -1 is a negative deviation from the mean: $zscore \sim \mathcal{N}(0,1)$

- Considering the weights, although we have 1 unit of $stock_{1,t}$ and $\hat{\beta}$ units of $stock_{2,t}$. We have to adjust it to the weights at which we hold these stocks in our portfolio, we denote the weights of $stock_{1,t}$ and $stock_{2,t}$ as w_1 and w_2:
  - we have the following constraints: $w_1 + w_2 = 1$ and $w_1 = \hat{\beta} w_2$
  - So we set, $w_2 = \frac{1}{1+\hat{\beta}}$ and $w_1 = 1 - w_2$, as the weights to either long or short the stocks in our pair.

Figure 4: Example of the trading rule under the Basic Model

On the 50th trading day, the algorithm would have *longed* the spread, and for every other subsequent day it would re-adjust weights of holding those stocks or ETF's until day 159, where it closes all positions and would create a new entry positions on day 164.

Figure 5: Example of the trading rule under Kalman filter

3.3 Parameters

For each model we perform a sensitivity analysis on various parameters. We investigate over three main parameters. First, how pairs perform over an increasingly longer time interval, we trade for the years {2015, 2016, 2017, 2018}. Second, changing the bands, i.e. the deviations from the *zscore*: $\Delta \in \{0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2\}$. Third and final, using moving windows, we use windows of historical data - $mw \in \{15, 30, 45, 60, 90, 120, 252\}$ days. This would allow the *zscore* to be dynamically update it self over time, and possibly capture more relevant behaviour rather than a *fixed zscore*. Then we define the updated as, $zscore'$:

- We use these moving window to dynamically update the *zscore* and so the signals. Then we define the mean and standard deviation as:
- $\mu'_f = \frac{\sum_{k=N-mw+(t-1)}^{t-1} spread_k}{mw}$
- and similarly for the standard deviation, $\sigma'_f = \sqrt{\frac{(\sum_{k=N-mw+(t-1)}^{t-1} spread_k - \mu'_f)}{mw}}, \forall t \geq 1$. Where N is the number of trading days, and $t \in \mathbb{N}$.
- Then for our updated $zscore' = \frac{spread_t - \mu'_f}{\sigma'_f}$

4 Backtest Results

In this section we start by providing some tools to evaluate the performance of the 239 pairs to be investigated, the backtests on the parameters and models repeated for every single pair with a supposed capital of \$100.000. The results are presented as follows; we provide the overall results from all pairs traded; then split the results into stationary and non-stationary pairs; finally from the two subset, we further divide it into 'winning' pairs. Furthermore, from the 239 pairs obtained during the formation period 176 pairs are stationary and 63 non-stationary.

4.1 Performance Metrics

We use the Quantopian Zipline package for python to implement the backtest strategies. It returns several indicators of the performance of the trading strategy. For this thesis, we use the annual returns, market alpha and beta, the Sharpe ratio and the max drawdown as performance evaluations [2].

Market Alpha and Beta:
The annual returns are self evident. The Quantopian Zipline package estimates the market alpha and beta through linear regression:

$$R_p = \alpha + \beta_m r_m \tag{41}$$

Note that this is different from the classical CAPM model:

$$\mathbb{E}(R_p) = R_f + \beta(\mathbb{E}(R_m) - R_f) \tag{42}$$

The market beta shows how the portfolio moves with the market. It is a measure of risk, if beta is greater than 1, the portfolio is riskier than the market and vice versa. The alpha can be quantified as how the portfolio performed against the market index. Another interpretation of alpha is that it is a measure of the active return on an investment. An alpha of 1 % means that the return on investment was 1% better than the market, and vice versa. If the alpha was zero, then the investment has earned too little for its risk. As we shall see, although a pair might perform very well, if it has a negative alpha it means that too much risk was taken. Pairs trading is also know as a market neutral strategy, this is because long and short positions are taken, cancelling out the effects of market movements and so we expect a beta of 0, or at least very close to 0. However, this is not always the case, in some cases we do experience a very low market beta. Nevertheless, in general, a higher alpha is preferred and is used to analyse the performance. The value of what is considered to be a 'good' alpha is dependent of the risk preference of an investor.

The Sharpe Ratio:

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p} \tag{43}$$

Was introduced by William F. Sharpe. It is a measure of risk adjusted return, in the sense that you subtract the risk free rate from the portfolio return [3]. Generally, the higher the Sharpe ratio, the more attractive the risk-adjusted return.

[2]Other measures such as the Sortino ratio and daily returns are also worth investigating. However, due to the scope of the research and time limitations we will not look into them. If the reader is interested the full databases of all the results, it can be requested.

[3]Quantopian uses the standard US treasury Bills, the year is dependent on how many year the backtest is run. So if the backtest is run for a year, then it uses the 1 - year US treasury bill.

Max Drawdown:
This indicator measures largest observed loss from a peak to a trough of a portfolio, before a new peak is attained. It is a measure of downside risk over a specified time period. There are 2 main preferences on the what is considered an acceptable max drawdown (mdd); a mdd should be between 5% and 30%, or should be at most half of your returns, for example if your return is 20% then having a mdd of at most 10 % is considered acceptable. However, the disadvantage is that it only shows the highest loss given a time period, it does not reflect the frequency of losses, nor does provide any information if the portfolio provides any returns or losses.

Table 1: SPY Cumulative Returns

Cumulative Return of SPY	
Year	Cumulative returns (%)
2015	0.145601
2016	0.159945
2017	0.299156
2018	0.581077

4.2 Basic Model results

4.2.1 Backtest results on Time Horizon

Table 2: Average Yearly Results

year	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
2015	57963.27	-0.42	3.44	0.81	0.32	-1.37	-0.29
2016	91027.78	-0.09	11.65	0.02	1.10	-1.30	-0.62
2017	210866.04	1.11	22.47	-19.39	2.33	-1.32	-0.85
2018	-96563.21	-1.97	121.07	-14.20	1.34	-1.23	-1.29

Table 3: Average Stationary Pairs

year	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
2015	41413.25	-0.59	4.12	0.38	0.09	-1.12	-0.31
2016	50110.98	-0.50	13.09	-0.75	1.20	-0.99	-0.68
2017	208667.07	1.09	26.26	-23.74	3.10	-1.01	-0.93
2018	138911.04	0.39	141.70	-17.30	1.86	-0.93	-1.30

Table 4: Average Non-stationary pairs

year	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
2015	95067.33	-0.05	0.43	1.78	0.82	-1.92	-0.25
2016	182760.59	0.83	7.47	1.74	0.90	-2.00	-0.50
2017	215795.99	1.16	9.78	-9.64	0.60	-2.01	-0.66
2018	-624481.28	-7.24	50.91	-7.27	0.19	-1.91	-1.26

There are 63 non-stationary pairs and 176 stationary pairs, combing to 239 pairs. However, during the backtesting, only 201 of those traded, comprising of 139 stationary and 62 non-stationary pairs. Following this, we observe that on average holding a pairs trading portfolio for 3 years would provide the highest returns with significant earnings that out outperforms the market. However, this comes with the risk of a high drawdowns along with other exorbitant metrics. These results are obtained due to several extreme backtest results.[4]. It seems that on average pairs trading does not seem profitable with significant under performance in all the other metrics.

Dividing the pairs into stationary and non-stationary camps, we can observe some improvement. That is for stationary pairs, we see positive returns, that also beat the market, the longer the pairs are traded - 2017 and 2018 respectively. However, this comes at much risk as can be seen in table 3. In addition, we observe a similar pattern for the non-stationary pairs for the trading years of 2016 and 2017. Albeit with much risk, table 4. Unexpectedly, non-stationary pairs outperformed stationary pairs for the first 3 trading years in terms of returns and better overall performance metrics, except for the Sharpe ratio.

Table 5: Stationary positive counts for yearly backtests

year	winning pairs	pos %	avg. perf	avg. alpha	avg. beta	avg. Sharpe	avg. max draw
2015	28	0.201439	129161.27	0.35	-0.04	0.78	-0.15
2016	37	0.266187	518031.16	-0.86	3.42	0.49	-0.74
2017	46	0.330935	1142302.77	-69.92	9.25	0.39	-0.57
2018	43	0.309353	4110539.07	-56.21	5.20	0.29	-0.80

[4]Although there are no outliners for the trading year of 2015, they come up in subsequent years. In 2016, the pair DBS vs SLV made a return of approximately $3.985 million, with a market alpha and beta of -126.89 and 34.86 and a mdd of 100%

Table 6: Non-stationary counts for yearly backtests

year	winning pairs	pos %	avg. perf	avg. alpha	avg. beta	avg. Sharpe	avg. max draw
2015	13	0.209677	138695.78	0.37	0.31	0.97	-0.12
2016	17	0.274194	500804.13	3.51	1.52	0.55	-0.63
2017	15	0.241935	848247.92	-39.65	2.10	0.38	-1.23
2018	17	0.274194	878475.74	2.33	1.91	0.32	-1.04

Insofar, non-stationary pairs seem to outperform stationary pairs. It is when we look at the extremes where we observe any real difference i.e. the winning pairs. The results in table 5 and 6 tells us that when the pairs do win, they significantly outperform the market with enormous returns - in both cases we experience many outliners. The returns does come with its risks. It seems that the ratio of winning pairs seem approximately the same, ranging from 20 % to 30 %. With more winning pairs as the trading interval increases. In the stationary camp, we see that they experience a much lower market alpha and a higher market beta, suggesting that these pairs are inherently more riskier and move more with the market index [5]. Moreover, stationary pairs do have a much smaller mdd and Sharpe ratio compared to the non-stationary pairs, suggesting that the risk-adjusted returns are lower, but do not face as much large drops in the portfolio. It could worthwhile to look at the results again if large outliners are removed.

4.2.2 Backtest results on changing the Bands

Table 7: Average results for changing the bands

band	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
0.25	84242.77	-0.16	5.05	0.59	0.14	-1.51	-0.35
0.50	72049.46	-0.28	4.69	1.55	-0.81	-1.45	-0.38
0.75	56237.77	-0.44	3.44	4.03	-1.63	-1.36	-0.35
1.00	58377.42	-0.42	3.42	0.81	0.31	-1.37	-0.29
1.25	58161.78	-0.42	3.42	0.40	0.12	-1.40	-0.28
1.50	57383.27	-0.43	3.43	-8.51	2.55	-1.33	-0.28
1.75	48183.76	-0.52	3.56	0.64	-0.58	-1.32	-0.36
2.00	53988.58	-0.46	3.46	1.24	0.23	-1.31	-0.29

[5] We use the returns of the SP500 SPY EFT index as the benchmark

Table 8: Average Stationary pairs for bands

band	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
0.25	89221.05	-0.11	5.96	1.24	0.07	-1.24	-0.34
0.50	64865.44	-0.35	5.62	0.60	-0.06	-1.14	-0.43
0.75	41129.46	-0.59	4.10	2.71	-0.47	-1.07	-0.38
1.00	42244.27	-0.58	4.09	0.38	0.09	-1.12	-0.31
1.25	41447.60	-0.59	4.09	0.29	-0.00	-1.13	-0.30
1.50	40490.61	-0.60	4.09	-12.54	3.50	-1.03	-0.30
1.75	26306.67	-0.74	4.25	1.11	-0.66	-1.02	-0.43
2.00	34590.64	-0.65	4.13	1.98	0.49	-1.00	-0.33

Table 9: Average Non-stat pairs for bands

band	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
0.25	72921.19	-0.27	1.72	-0.87	0.29	-2.12	-0.38
0.50	88387.32	-0.12	0.61	3.72	-2.51	-2.14	-0.29
0.75	90597.00	-0.09	0.61	7.03	-4.27	-2.02	-0.28
1.00	95067.33	-0.05	0.43	1.78	0.82	-1.92	-0.25
1.25	96173.06	-0.04	0.42	0.66	0.39	-1.99	-0.24
1.50	95800.45	-0.04	0.42	0.66	0.38	-1.96	-0.24
1.75	97936.50	-0.02	0.43	-0.43	-0.37	-1.95	-0.20
2.00	98103.25	-0.02	0.43	-0.43	-0.36	-1.99	-0.19

The backtest results are based on trading year of 2015. Similar to the *yearly* performance we have 203 pairs that traded, composed of 141 stationary pairs and 62 non-stationary pairs. Subsequently, from table 7, the overall returns are negative, as we increase the bands, or in other words delaying the trading signal, the performance get worse in terms of return on investment. However, the other indicators suggests that they are more stable. With much lower mdd's, market beta's and higher market alpha's, setting the trading signal at 0.75 deviations from the *zscore*, produces an average alpha of 4, but causing a loss of an average of 44%. Indicative that solely looking at an alpha does not necessarily translate into a good investment. Similar to the *yearly* results, the non-stationary pairs seem to out perform the stationary pairs in almost all metrics and obtaining relative small losses, in most cases only a few percent in comparison to much larger losses up to 74% for stationary pairs.

Table 10: Average positive counts for Stationary pairs for changing the Bands

band	winning pairs	pos %	avg. perf	avg. alpha	avg. beta	avg. Sharpe	avg. max draw
0.25	36	0.255319	313046.56	3.75	0.16	0.85	-0.30
0.50	32	0.226950	275651.38	1.39	-0.69	0.75	-0.28
0.75	34	0.241135	128047.49	-4.08	-5.35	0.69	-0.23
1.00	28	0.198582	129161.27	0.35	-0.04	0.78	-0.15
1.25	27	0.191489	122783.35	0.24	-0.10	0.75	-0.11
1.50	27	0.191489	124228.38	-69.82	18.88	0.69	-0.17
1.75	25	0.177305	120851.87	-0.35	-0.85	0.69	-0.18
2.00	24	0.170213	125158.02	0.35	0.17	0.81	-0.15

Table 11: Average positive counts for Non-stationary pairs for changing the Bands

band	winning pairs	pos %	avg. perf	avg. alpha	avg. beta	avg. Sharpe	avg. max draw
0.25	11	0.177419	180244.39	0.61	0.17	1.11	-0.19
0.50	11	0.177419	139471.03	0.39	0.22	0.98	-0.13
0.75	14	0.225806	136650.41	0.35	0.30	0.95	-0.12
1.00	13	0.209677	138695.78	0.37	0.31	0.97	-0.12
1.25	12	0.193548	142344.52	4.09	1.98	1.11	-0.48
1.50	17	0.274194	128714.96	2.88	1.39	0.82	-0.35
1.75	17	0.274194	132449.36	-1.10	-1.38	0.61	-0.24
2.00	17	0.274194	132078.83	-1.11	-1.37	0.53	-0.24

Looking at the 'winning' pairs, and in tandem with the previous tables. We see a dichotomy in the results. The return on investment is higher for cointegrated pairs if the trading signals are shortened, with 0.25 and 0.5 deviations from the *zscore*. While, for the non-cointegrated pairs, the return on investment along with the other metrics improve as we increase the signal to trade i.e. larger band values. Compared, to the yearly results, the overall performance metrics have improved in terms of a more 'realistic' view, with significant lower max drawdowns, improved market alpha and beta's and higher Sharpe ratio's, especially for the stationary winning pairs.

4.2.3 Backtest results on changing Moving Windows

Table 12: Average performance for moving windows

window	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
15	120231.67	0.20	3.46	1.20	0.24	-0.46	-0.32
30	103720.43	0.04	3.35	-6.32	-3.93	-0.46	-0.26
45	81759.97	-0.18	1.69	0.50	-0.10	-0.37	-0.23
60	93123.74	-0.07	0.64	0.28	-0.43	-0.31	-0.17
90	90597.30	-0.09	0.64	1.05	-0.05	-0.29	-0.18
120	90593.65	-0.09	0.67	0.21	-0.38	-0.24	-0.19
252	97990.96	-0.02	0.75	0.33	-0.46	-0.14	-0.16

Table 13: Average performance of Stationary Pairs for moving windows

window	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
15	137144.60	0.37	4.07	1.87	0.09	-0.49	-0.33
30	113884.32	0.14	3.95	1.11	-0.24	-0.50	-0.27
45	82415.60	-0.18	1.90	1.32	0.11	-0.41	-0.22
60	98590.48	-0.01	0.35	0.61	-0.64	-0.36	-0.15
90	94714.54	-0.05	0.32	0.72	-0.49	-0.37	-0.16
120	93995.08	-0.06	0.46	0.53	-0.52	-0.32	-0.17
252	100784.93	0.01	0.86	0.64	-0.61	-0.26	-0.15

Table 14: Average performance of Non-Stationary Pairs for moving windows

window	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
15	80860.57	-0.19	1.05	-0.38	0.59	-0.36	-0.29
30	80060.22	-0.20	1.00	-23.62	-12.52	-0.36	-0.24
45	80233.75	-0.20	1.04	-1.40	-0.59	-0.30	-0.24
60	80397.88	-0.20	1.03	-0.47	0.05	-0.18	-0.23
90	81012.91	-0.19	1.06	1.82	0.97	-0.12	-0.24
120	82675.57	-0.17	1.00	-0.54	-0.07	-0.06	-0.22
252	91486.96	-0.09	0.43	-0.41	-0.09	0.14	-0.19

We have the same amount of pairs that were able to trade. With 141 stationary pairs and 62 non-stationary pairs. The average performance under a moving window framework brings a more promising result, a much improvement on the performance metrics and returns. With profits achieved on either small moving windows or a very large one, 15, 30 and 252 days respectively, for stationary pairs. Regarding the non-stationary pairs, although for all bands they experience a loss, the losses decrease as the size of the moving windows increase. Moreover, under this framework, in general, we see the results that we would expect. That cointegrated pairs perform better than the ones that are not, where they have been out performed on every metric, tables 13 and 14.

Table 15: Average positive counts of Stationary Pairs for moving windows

window	winning pairs	pos %	avg. perf	avg. alpha	avg. beta	avg. Sharpe	avg. max draw
15	60	0.422535	245281.16	3.23	-0.67	0.95	-0.35
30	61	0.429577	186727.48	0.37	-0.75	0.64	-0.24
45	66	0.464789	113975.11	0.16	0.02	0.73	-0.10
60	66	0.464789	113394.88	0.15	0.06	0.73	-0.08
90	59	0.415493	108049.62	0.10	0.09	0.65	-0.08
120	67	0.471831	110870.18	0.12	0.02	0.58	-0.08
252	67	0.471831	123067.30	0.23	-0.05	0.73	-0.05

Table 16: Average positive counts of Non-Stationary Pairs for moving windows

window	winning pairs	pos %	avg. perf	avg. alpha	avg. beta	avg. Sharpe	avg. max draw
15	25	0.409836	119039.82	0.17	0.17	0.95	-0.07
30	27	0.442623	107861.64	0.08	0.10	0.64	-0.06
45	30	0.491803	109230.82	0.10	0.09	0.66	-0.07
60	33	0.540984	106610.58	0.07	0.02	0.63	-0.07
90	34	0.557377	107246.41	0.09	-0.08	0.64	-0.08
120	35	0.573770	107612.82	0.09	-0.07	0.71	-0.07
252	32	0.524590	109001.46	0.09	-0.12	0.99	-0.06

These results become clearer when we look at the 'winning' pairs. Firstly, the ratio of profit making pairs have nearly doubled, compared to other parameters. Second, we observe the same high returns for the moving windows of 15, 30 and 252 days with either acceptable or even lower max drawdowns. Finally, we observe that for the other metrics that stationary pairs outperform the non-stationary pairs, and the market in some cases[6]. It can be argued that first; the stationary pairs that win, have a stronger mean-reverting relationship; second, by regularly updating the *zscore* would capture more relevant behaviour of the inner workings of the underlying stocks that would allow for better performance. Moreover, it is worth investigating how these pairs perform if we increase the time intervals, as we start observing outliners in 2016 and in the subsequent years.

4.3 Kalman Filter Results

4.3.1 Kalman Filter backtest results on Time Horizon

Table 17: Average yearly backtest results under Kalman Filter

year	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
2015	98909.18	-0.01	0.25	-0.37	0.09	-0.48	-0.10
2016	83762.05	-0.16	3.04	-0.29	0.10	-0.35	-0.23
2017	62128.38	-0.38	5.52	-1.51	-0.15	-0.24	-0.36
2018	-6328.07	-1.06	9.32	-0.73	0.47	-0.18	-0.85

[6] Outperform the SPY ETF

Table 18: Average yearly backtest results for Stationary Pairs under Kalman Filter

year	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
2015	97143.41	-0.03	0.20	-0.53	0.15	-0.55	-0.11
2016	73217.67	-0.27	3.60	-0.43	0.18	-0.40	-0.26
2017	42715.54	-0.57	6.58	-2.17	-0.21	-0.31	-0.42
2018	-42299.81	-1.42	11.00	-0.94	0.66	-0.21	-1.10

Table 19: Average yearly backtest results for Non-Stationary Pairs under Kalman Filter

year	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
2015	102990.69	0.03	0.32	0.02	-0.07	-0.31	-0.08
2016	108135.14	0.08	0.86	0.03	-0.10	-0.24	-0.16
2017	107000.70	0.07	0.97	0.03	-0.01	-0.08	-0.22
2018	76819.74	-0.23	2.69	-0.24	0.01	-0.13	-0.28

The Kalman Filter acts as a smoother, with the idea to filter out 'possible' noise that the stock market creates and to estimate the 'true' underlying state of the spread. The data shows that 202 from the 239 pairs are traded, with 141 stationary and 61 non-stationary pairs. On average none of the pairs delivered positive returns with 2015 being close. Moreover, as the time interval increases, the worse the returns get. However, the other metrics are relatively more stable with higher alphas, lower betas, higher Sharpe ratios and lower mdd's.

We further look at the data by dividing the pairs traded in two camps. On average, the stationary pairs perform in a similar fashion to the overall average. Looking at tables 18 and 19, we observe that non-stationary pairs preform better than stationary pairs in terms of revenue and other performance metrics.

Table 20: Average yearly backtest results of positive counts under a Kalman Filter for Stationary Pairs

year	winning pairs	pos %	avg. perf	avg. alpha	avg. beta	avg. Sharpe	avg. max draw
2015	44	0.312057	106567.15	0.04	-0.19	0.47	-0.10
2016	47	0.333333	165459.62	0.23	-0.02	0.58	-0.14
2017	55	0.390071	184427.88	-0.13	0.14	0.53	-0.19
2018	54	0.382979	216703.57	0.11	0.23	0.39	-0.32

Table 21: Average yearly backtest results of positive counts under a Kalman Filter for Non-Stationary Pairs

year	winning pairs	pos %	avg. perf	avg. alpha	avg. beta	avg. Sharpe	avg. max draw
2015	19	0.311475	121166.08	0.17	-0.25	0.52	-0.08
2016	20	0.327869	149778.47	0.22	-0.17	0.39	-0.13
2017	22	0.360656	157854.88	0.30	-0.06	0.47	-0.18
2018	24	0.393443	162883.97	0.14	-0.06	0.36	-0.16

Although the Kalman Filter and the Basic model have roughly the same amount of traded pairs. The amount of 'winning' pairs are lower for the Kalman filter. The results seem more stable due to less outliners in the backtests with significant returns. Moreover, we observe, lower levels of beta (suiting to the inherent market neutral strategy), with relative low levels of alpha. Furthermore, we have significantly lower levels of max draw downs and somewhat higher Sharpe ratio's. We observe this pattern in both the Basic model and in comparison to the non-stationary pairs. In addition, we observe a similar pattern, where on average non-stationary pairs outperform stationary pairs, as in the Basic model.

4.3.2 Kalman Filter backtest results on changing Bands

Table 22: Average backtest results for changing Bands under Kalman Filter

band	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
0.25	102817.58	0.03	0.76	-0.22	0.28	-0.48	-0.17
0.50	107247.63	0.07	0.76	0.11	-0.01	-0.46	-0.13
0.75	101040.91	0.01	0.41	-0.02	0.15	-0.44	-0.14
1.00	98925.14	-0.01	0.24	-0.36	0.08	-0.48	-0.10
1.25	102141.65	0.02	0.31	-0.39	-0.08	-0.39	-0.09
1.50	101410.68	0.01	0.22	-0.02	-0.06	-0.32	-0.07
1.75	101060.81	0.01	0.17	0.00	-0.05	-0.31	-0.05
2.00	101477.49	0.01	0.16	0.03	-0.03	-0.31	-0.05

Table 23: Average backtest results for changing Bands under Kalman Filter for Stationary Pairs

band	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
0.25	103899.90	0.04	0.88	-0.32	0.38	-0.58	-0.17
0.50	108945.47	0.09	0.84	0.10	-0.06	-0.54	-0.13
0.75	100247.86	0.00	0.34	0.02	0.23	-0.50	-0.14
1.00	97183.37	-0.03	0.20	-0.53	0.15	-0.55	-0.11
1.25	101098.40	0.01	0.31	-0.58	-0.09	-0.48	-0.10
1.50	100098.24	0.00	0.20	-0.05	-0.06	-0.38	-0.08
1.75	100046.58	0.00	0.14	-0.01	-0.04	-0.42	-0.06
2.00	100719.84	0.01	0.13	0.02	-0.02	-0.41	-0.05

Table 24: Average backtest results for changing Bands under Kalman Filter for Non-Stationary Pairs

band	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
0.25	100321.26	0.00	0.37	0.01	0.04	-0.24	-0.15
0.50	103331.66	0.03	0.54	0.14	0.10	-0.28	-0.15
0.75	102870.03	0.03	0.53	-0.10	-0.05	-0.30	-0.15
1.00	102942.45	0.03	0.32	0.02	-0.06	-0.31	-0.08
1.25	104547.84	0.05	0.30	0.03	-0.05	-0.15	-0.05
1.50	104437.76	0.04	0.28	0.05	-0.06	-0.16	-0.05
1.75	103400.09	0.03	0.22	0.04	-0.07	-0.04	-0.05
2.00	103224.95	0.03	0.22	0.04	-0.06	-0.07	-0.04

Similar to the Basic model, 205 pairs have traded in the course of the backtest. From these pairs there were 143 stationary pairs and 62 non-stationary pairs. Comparing to the Basic, we observe that on average both stationary and non-stationary pairs have positive results, very relative low max drawdowns and market beta's. Furthermore, we observe that non-stationary pairs out perform the stationary pairs by a small margin on all metrics.

Table 25: Average positive counts under Kalman Filter for Stationary Pairs for changing Bands

band	winning pairs	pos %	avg. perf	avg. alpha	avg. beta	avg. Sharpe	avg. max draw
0.25	43	0.300699	144398.38	0.44	-0.09	0.64	-0.11
0.50	40	0.279720	153788.07	0.53	-0.19	0.72	-0.10
0.75	44	0.307692	123370.74	0.18	0.08	0.60	-0.12
1.00	44	0.307692	106567.15	0.04	-0.19	0.47	-0.10
1.25	54	0.377622	112402.06	-0.87	-0.49	0.46	-0.12
1.50	45	0.314685	108725.86	-0.06	-0.23	0.47	-0.12
1.75	37	0.258741	108324.43	0.02	-0.24	0.50	-0.09
2.00	31	0.216783	110310.45	0.16	-0.24	0.60	-0.10

Table 26: Average positive counts under Kalman Filter for Non-Stationary Pairs for changing Bands

band	winning pairs	pos %	avg. perf	avg. alpha	avg. beta	avg. Sharpe	avg. max draw
0.25	24	0.387097	121267.03	0.23	-0.10	0.66	-0.11
0.50	21	0.338710	133488.57	0.28	-0.11	0.65	-0.10
0.75	20	0.322581	134313.00	0.29	-0.10	0.62	-0.12
1.00	19	0.306452	121166.08	0.17	-0.25	0.52	-0.08
1.25	17	0.274194	123406.20	0.19	-0.21	0.62	-0.08
1.50	15	0.241935	125732.34	0.27	-0.35	0.66	-0.11
1.75	15	0.241935	119145.67	0.22	-0.35	0.66	-0.11
2.00	11	0.177419	125133.71	0.29	-0.39	0.75	-0.14

Looking at the winning subset of all traded pairs, we observe a higher ratio of winning pairs. Although the returns aren't as significant, they are promising. The pattern of results in the Basic model follow for the Kalman filter regarding the performance of the stationary and non-stationary pairs. However, there is a small difference, that is, as the band size, i.e. trade signal, increases the ratio of winning pairs decreases and the overall performance has waned relative to the Basic model.

4.3.3 Kalman Filter backtest results on changing Moving Windows

Table 27: Average backtest results under Kalman Filter for Moving Windows

window	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
15	75294.48	-0.25	2.22	0.30	0.13	-0.33	-0.24
30	110556.77	0.11	0.88	0.12	-0.07	-0.35	-0.13
45	98620.55	-0.01	0.29	0.09	-0.24	-0.38	-0.13
60	98249.59	-0.02	0.27	0.13	0.04	-0.35	-0.14
90	98181.16	-0.02	0.27	-0.32	0.13	-0.51	-0.13
120	104485.19	0.04	0.51	-0.01	-0.05	-0.46	-0.12
252	97143.84	-0.03	0.34	0.56	-0.05	-0.55	-0.18

Table 28: Average backtest results under Kalman Filter for Moving Windows for Stationary Pairs

window	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
15	62040.24	-0.38	2.61	0.40	0.18	-0.40	-0.29
30	114859.85	0.15	1.03	0.16	-0.12	-0.33	-0.14
45	96379.54	-0.04	0.22	0.11	-0.34	-0.39	-0.15
60	95917.48	-0.04	0.26	0.17	0.06	-0.36	-0.15
90	95284.39	-0.05	0.23	-0.48	0.19	-0.57	-0.14
120	103868.77	0.04	0.56	-0.04	-0.07	-0.48	-0.13
252	96597.49	-0.03	0.26	0.80	-0.04	-0.64	-0.20

Table 29: Average backtest results under Kalman Filter for Moving Windows for Non-Stationary Pairs

window	average portfolio value	average percentage change	average standard deviation	average alpha	average beta	average Sharpe ratio	average max drawdown
15	107920.31	0.08	0.53	0.05	0.02	-0.15	-0.10
30	99964.57	-0.00	0.19	0.00	0.03	-0.43	-0.09
45	104136.90	0.04	0.42	0.04	0.00	-0.35	-0.10
60	103990.17	0.04	0.29	0.05	0.00	-0.33	-0.10
90	105311.65	0.05	0.35	0.05	-0.02	-0.35	-0.11
120	106002.53	0.06	0.38	0.06	-0.01	-0.39	-0.11
252	98488.71	-0.02	0.49	-0.02	-0.06	-0.33	-0.14

During the backtest under a Kalman Filter we observed that from 239 tradable pairs, only 135 traded of which there 96 stationary pairs and 39 non-stationary. On average, the Kalman filter on the spread only returns positive results on a 30 and 120 day moving window, and an overall stable performance metrics. Comparing the stationary pairs and non-stationary pairs, we see that non-stationary pairs outperform stationary pairs. Moreover, although less pairs are traded, under the belief that this smoothing of the spread reduces trading signals, non-stationary pairs returned more positive returns on average than in the previous model. With, 15, 45, 60, 90 and 120 moving days respectively. Compared to the 30 and 120 day moving windows for stationary. Disconcertingly, the non-stationary pairs, with higher returns, also seem more stable and favourable in terms of the performance metrics. The final tables below, show a similar pattern under the Kalman Filter, we look at the 'winning' subsets of stationary and non-stationary pairs.

Table 30: Average positive counts under Kalman Filter for Stationary Pairs for Moving Windows

window	winning pairs	pos %	avg. perf	avg. alpha	avg. beta	avg. Sharpe	avg. max draw
15	32	0.333333	113143.40	0.20	0.23	0.59	-0.11
30	41	0.427083	144530.32	-0.26	0.13	0.66	-0.16
45	37	0.385417	108276.49	0.10	-0.05	0.57	-0.09
60	39	0.406250	108188.25	0.23	0.23	0.51	-0.12
90	31	0.322917	107945.55	0.11	-0.10	0.49	-0.11
120	35	0.364583	127286.14	0.22	-0.11	0.64	-0.12
252	29	0.302083	113633.36	0.72	1.13	0.45	-0.19

Table 31: Average positive counts under Kalman Filter for Non-Stationary Pairs for Moving Windows

window	winning pairs	pos %	avg. perf	avg. alpha	avg. beta	avg. Sharpe	avg. max draw
15	18	0.461538	124557.52	0.19	-0.04	0.50	-0.10
30	15	0.384615	111339.69	0.12	0.00	0.49	-0.09
45	11	0.282051	129363.39	0.28	-0.08	0.63	-0.15
60	12	0.307692	126300.70	0.30	-0.08	0.72	-0.15
90	14	0.358974	127307.15	0.27	-0.18	0.64	-0.13
120	12	0.307692	133321.45	0.34	-0.24	0.68	-0.16
252	9	0.230769	131151.60	0.23	-0.10	0.62	-0.12

4.4 Overview

Comparing the performance of the traded pairs under different parameters for both models, we experience higher returns for the Basic model than with the Kalman. However, the Kalman filter mirrors the performance for stationary and non-stationary pairs both on average and for the winning ones, but compresses the results. It is believed that the filter reduces the noise that surrounds the spread and therefore trading signals are reduced. Moreover, from the three parameters; adjusting for the time horizon and the bands, we have, surprisingly, seen that non-stationary outperform stationary pairs. This is contrary to what we expected, that is cointegrated pairs have an identified long-run relationship where profits can be obtained when the spread reverts back to its long-run mean. Which can not be necessarily said for pairs that are not cointegrated, as they do not have a supposed long-run equilibrium.

It is when we look at the winning pairs where we see the biggest difference and meet expectations; that stationary pairs outperform non-stationary pairs. In addition, regarding stationary pairs, it is shown that a longer trading horizon does provide with higher returns, albeit with sometimes relative higher risk - 2 to 3 years seem more appropriate. Regarding the bands, for stationary pairs, the data favours shorter signals ranging from 0.25, and in some cases, to 1 standard deviation from the mean[7] of the *zscore*.

Having said that, moving windows provide the most favourable results, on average, using shorter moving windows. Contrary to the previous parameters, stationary pairs outperform non-stationary pairs on average, as well as when looking at the winning pairs, with highest returns using either very short or a long moving window, 15, 30 or 252 days respectively, for the Basic model. When applied to the Kalman filter, bigger windows are necessary; 15, 30, 120 and 252 moving windows. It can be argued that when using moving windows and in tandem with the supposed long term linear relationship, allows the mean-reverting relationship to update and so create better trading signals. The data suggest that the best scales to use for this updating mechanism should be done with either short or large moving windows.

5 Further Analysis

In this section, we are interested in examining the underlying relations between the logarithm of returns against $\hat{\beta}$ and the *mean squared error*. We examine this in a similar fashion as in the previous section. We look at the overall performance, the difference between stationary and non-stationary, for both Models. We refer *mean squared error* as *mse* in rest of the section.

5.1 Log returns vs *mse*

We introduce the mean squared error term below. Coming back to how we defined the spread, and the estimated parameters, we take the residual values:

$$\hat{u}_t = \hat{\alpha} - y_t - \hat{\beta} x_t \tag{44}$$

In theory, a good estimated model should have an *mse* close to 0 and is always non-negative. This is where the observed values equal the estimated values.

[7] Note that the *zscore* has a mean of 0.

$$mse(\delta) = \frac{\sum_{n=1}^{t}(y_i - \hat{y}_n)^2}{n} = \frac{\sum_{n=1}^{t}(y_n - (\hat{\alpha} - \hat{\beta}x_n))^2}{n} = \frac{\sum_{n=1}^{t}\hat{u}_n^2}{n}, \forall t > 1 \qquad (45)$$

Where δ represents the set of estimators for all trading days t and 1_{losing} is an indicator function for whenever the log return is below ln(100.000)=11.512925, for the linear regression below, and returns that have a negative portfolio value at the end of the year are removed. The regression is repeated for all the trading years. It is of the opinion that a cointegrated pair would have a smaller mse, than a non-cointegrated pair, as a long-run equilibrium is identified and expect that this is also holds true to log returns, that is, we expect an inverse relationship between mse and log returns.

. Therefore, it can argued that for stationary pairs we would expect an inverse relationship between log returns and the mse, which can be expressed as θ_1. The other regressors, θ_2 and θ_3, describe the relationship between log returns and losing pairs and their mse. We expect that losing pairs, have a negative coefficient θ_2, and a positive θ_3. The philosophy behind a positive θ_3 is that the spread of losing pairs are believed to have a weaker fit, and so a 'weaker' mean-reverting mechanism. Finally, η, can be interpreted as the 'inherent' (mean) revenue of the traded pairs.

$$Log(return) = \eta + \theta_1.mse + \theta_2.1_{losing} + \theta_3(mse.1_{losing}) + \epsilon, \text{ where } \epsilon \sim N(0,1) \qquad (46)$$

Table 32: Yearly mean log returns[8]

Year	**Mean Log Returns Under Basic Model**
2015	11.462570
2016	11.490808
2017	11.513080
2018	11.486685
Year	**Mean Log Returns Under Kalman**
2015	11.479840
2016	11.505116
2017	11.535019
2018	11.554465

[8] significant at 5%*, f-test for significance against zero+

Table 33: For all Pairs

Regression values Under Basic Model				
Year	η	θ_1	θ_2	θ_3
2015	11.6205*	-3.88e-10	-0.2454*+	-0.0052+
2016	11.7457*	-8.401e-10	-0.4220*+	0.0122+
2017	11.8406*	-1.182e-09	-0.5674*+	-0.0184+
2018	11.8699*	-1.288e-09	-0.6627*+	0.0022+
Regression values under Kalman				
Year	η	θ_1	θ_2	θ_3
2015	11.5421*	-1.054e-10	-0.1047*+	-1.2260*+
2016	11.6591*	-5.276e-10	-0.2686*+	-0.3954+
2017	11.7463*	-8.42e-10	-0.4008*+	-0.4496+
2018	11.8147*	1.089e-09	-0.4938*+	-0.6966+

Table 34: Regression Values for Stationary and Non-Stationary Pairs

Regression values Under Basic Model				
Year	η	θ_1	θ_2	θ_3
2015	11.6020*	-3.215e-10	-0.2301*+	0.0185+
2016	11.6974*	-6.658e-10	-0.3599*+	0.0222+
2017	11.7964*	-1.023e-09	-0.5236 *+	-0.0169+
2018	11.8370*	-1.169e-09	-0.6432*+	0.0093+
Non-Stationary Pairs Under Basic Model				
Year	η	θ_1	θ_2	θ_3
2015	11.7453*	-1.4027	-0.3429*+	0.9818+
2016	11.8501*	9.1164	-0.5225*+	-9.6908+
2017	11.9962*	4.5595	-0.7171*+	-4.6775+
2018	11.8390*	9.4982	-0.5838*+	-9.8639+
Stationary Pairs Under Kalman				
Year	η	θ_1	θ_2	θ_3
2015	11.5341*	-7.645e-11	-0.1099*+	-1.7206+
2016	11.6634*	-5.429e-10	-0.2222*+	-2.7140*+
2017	11.7462*	-8.419e-10	-0.3583*+	-2.9618*+
2018	11.8088*	-1.068e-09	-0.4668*+	-2.7333+
Non-Stationary Pairs Under Kalman				
Year	η	θ_1	θ_2	θ_3
2015	11.5217*	0.2891*	-0.0378*+	-1.4807*+
2016	11.5435*	0.6568*	-0.1940*+	-0.8494*+
2017	11.6571*	0.6198*	-0.3307*+	-0.8745*+
2018	11.7562*	0.5877*	-0.4205*+	-1.1758+

It seems that under the Basic model the inherent revenues are higher than under the Kalman filter, that also increases over time. The hypothesis for θ_1 does match with the data only slightly. However, the negative coefficients are not significant enough for anything decisive. Furthermore, they also fail the F-test to check if they are significant from zero. However, given the small nature of these coefficients, one should not be surprised. On the other hand, θ_2 and θ_3 are significant from 0, θ_2 does meet expectations; that is losing pairs have a negative relation with log returns. Although, the values alternate between positive and negative for θ_3, they are not significant enough, for anything decisive - similarly under the Kalman filter.

A similar pattern follow when looking into stationary and non-stationary pairs. Surprisingly, the inherent value of non-stationary pairs are higher than for stationary pairs. We do observe, negative values for θ_1 in both models for stationary pairs. However, they are neither significant in the regression nor significant enough from 0. For θ_3, the same applies, but are significant from 0. From tables 33 and 34, it seems that only θ_2 meet expectations with statistical significance.

5.2 Log returns vs estimated Beta

$$Log(return) = \phi_1 \hat{\beta} + \phi_2 1_{losing} \hat{\beta} + \epsilon, \text{ where } \epsilon \sim N(0,1) \tag{47}$$

We ignore the constant in the regression of (57), the idea is to capture a more direct relationship between estimated parameter, β. Although, the nature of mse could allow for deductive expectations, it is not necessarily true for $\hat{\beta}$, and thus through (57) we would like to capture the differences between stationary and non-stationary pairs, and between the winning and losing pairs.

Table 35: For all Pairs

Regression values Under Basic Model		
Year	ϕ_1	ϕ_2
2015	2.5194^{*+}	0.6704^{+}
2016	2.7026^{*+}	0.3761^{+}
2017	2.7280^{*+}	0.3355^{+}
2018	2.6761^{*+}	0.4870^{+}
Regression values under Kalman		
Year	ϕ_1	ϕ_2
2015	0.6950^{*+}	1.8743^{+}
2016	2.5536^{*+}	-1.8900^{+}
2017	3.1063^{*+}	-2.4290^{+}
2018	3.1628^{*+}	-2.4979^{+}

The constituents that make up the pairs, are reflected through the spread, which is a linear combination of two securities; that are related to another up to a scalar, $\hat{\beta}$. Under the Basic model we see that the ϕ_1 is roughly stable and significant over time, while under the Kalman filter they increase over time. When looking at the losing pairs,ϕ_2 ,they tend to be positive and significant under the Basic model, but negative and not significant under Kalman.

Table 36: Regression Values for Stationary and Non-Stationary Pairs

Stationary Pairs Under Basic Model		
Year	ϕ_1	ϕ_2
2015	2.3278^{*+}	0.3649^{+}
2016	2.4960^{*+}	0.0874^{+}
2017	2.5430^{*+}	-0.0055^{+}
2018	2.4494^{*+}	0.2103^{+}
Non-Stationary Pairs Under Basic Model		
Year	ϕ_1	ϕ_2
2015	5.2290^{*+}	2.8904^{+}
2016	5.6495^{*+}	2.2813^{+}
2017	5.2711^{*+}	3.0239^{*+}
2018	5.9350^{*+}	1.9583^{+}
Stationary Pairs Under Kalman		
Year	ϕ_1	ϕ_2
2015	0.6042^{*+}	1.6082^{*+}
2016	2.3150^{*+}	-1.7594^{*+}
2017	2.8163^{*+}	-2.2527^{*+}
2018	2.8422^{*+}	-2.2863^{*+}
Non-Stationary Pairs Under Kalman		
Year	ϕ_1	ϕ_2
2015	6.4729^{*+}	1.6230^{+}
2016	6.2527^{*+}	1.6781^{+}
2017	9.7499^{*+}	-3.4580^{*+}
2018	10.4491^{*+}	-4.3720^{*+}

Under the Basic model, stationary pairs have lower coefficients than non-stationary pairs. With almost all coefficients statistically significant except for the losing pairs, ϕ_2, and all coefficients are significant from 0. Regarding, ϕ_1, it is indicative of where securities that make up stationary pairs are a lower scalar to one another compared to non-stationary pairs. We observe a similar pattern for the losing pairs but with statistically insignificant coefficients.

Under the Kalman filter, we see a similar phenomenon, where stationary pairs have lower scalars than non-stationary - non-stationary pairs seem to be changing over time - and are statistically significant over.

6 Pairs trading under GARCH

In this section we, implement a similar trading strategy carried out by Huang & Martin (2019). As mentioned in the theoretical section, we assumed the spread for each pair is an $AR(1)$ process. We use the residuals from the linear regression below and apply a GARCH(1,1) model to obtain the conditional volatility, σ_g. From there we set the bands in which the strategy will trade; we go *short* on the spread when the *zscore* is between $1-\sigma_g$ and $3+\sigma_g$. Vice versa when *longing* the spread [9].

[9]Note that we perform the regression on a daily basis

$$spread_t = \alpha + \beta spread_{t-1} + \epsilon_t, \text{ where } \epsilon_t \sim N(0,1)\ \forall\ t, \text{ trading days} \tag{48}$$

In the figure below the black dashed line is the day the strategy begins its trading, as it marks the end of the formation period of the pairs. Trades are carried out, going long or short on the spread, when the *zscore* is in either the red or green bands. We placed a *stop* positions whenever the *zscore* crosses the second upper band.

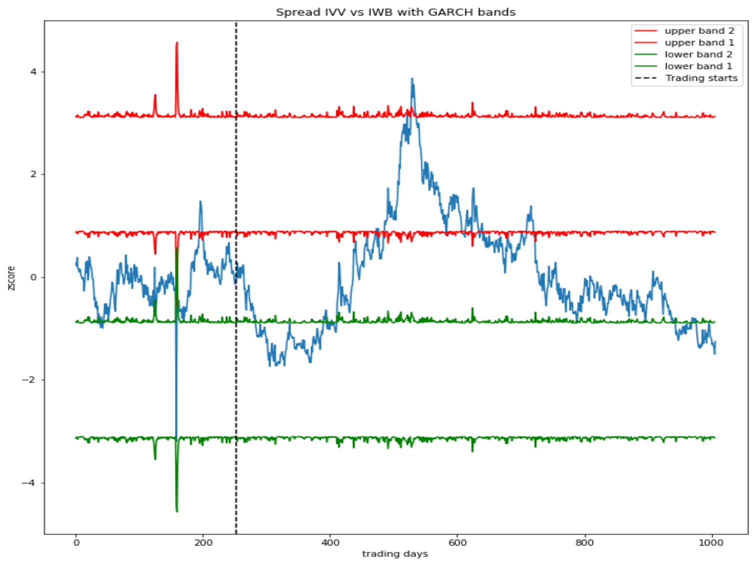


Figure 6: Example of the GARCH trading rule

6.1 Backtest results under GARCH bands

Table 37: Average yearly results Under GARCH

Year	Avg. Portfolio value	Avg. Percentage change	Avg. Standard deviation	Avg. Alpha	Avg. Beta	Avg. Sharpe ratio	Avg. Max drawdown
2015	96947.99	-0.03	0.34	-0.02	0.11	-0.69	-0.16
2016	95012.18	-0.05	3.12	-0.17	-0.16	-0.55	-0.47
2017	185159.09	0.85	10.34	-3.61	-0.59	-0.42	-0.58
2018	528987.15	4.29	71.94	-2.28	-0.19	-0.33	-0.83

Table 38: Average yearly results for Stationary Pairs Under GARCH

Year	Avg. Portfolio value	Avg. Percentage change	Avg. Standard deviation	Avg. Alpha	Avg. Beta	Avg. Sharpe ratio	Avg. Max drawdown
2015	97592.90	-0.02	0.32	-0.00	0.13	-0.71	-0.16
2016	93231.70	-0.07	3.64	-0.31	-0.22	-0.57	-0.55
2017	221630.05	1.22	12.16	-5.10	-0.83	-0.46	-0.68
2018	801017.13	7.01	83.73	-3.20	-0.27	-0.35	-0.97

Table 39: Average yearly results for Non-Stationary Pairs Under GARCH

Year	Avg. Portfolio value	Avg. Percentage change	Avg. Standard deviation	Avg. Alpha	Avg. Beta	Avg. Sharpe ratio	Avg. Max drawdown
2015	95289.65	-0.05	0.38	-0.06	0.07	-0.62	-0.17
2016	99590.57	-0.00	0.83	0.20	-0.00	-0.51	-0.27
2017	91376.63	-0.09	1.16	0.20	0.03	-0.32	-0.32
2018	-170518.52	-2.71	20.54	0.07	0.02	-0.26	-0.47

Under the GARCH model the performance of the pairs favours it over the previous models. On average, we experience lower levels of losses and significant returns. Where relative low losses occurred for the first 2 trading year and high returns in the subsequent years, with a much lower risk factor. In addition to the returns and risk, market beta is lower (pertaining to market neutral strategy) and the Sharpe Ratio, although negative, higher compared to the Basic model, indicating that risk adjusted return is higher under the GARCH model. Moreover, for the winning years, the market alpha have improved significantly, although still negative, suggesting that inherent risk improved as well [10].

In section 4, we saw that there was a tendency for non-stationary pairs to outperform stationary pairs, both during the time horizon and bands. The opposite is true for the GARCH model, where stationary pairs outperform non-stationary pairs. With stationary achieving profitability levels in the third and fourth trading years. While non-stationary pairs perform worse over time.

Table 40: Average yearly results for winning Stationary Pairs Under GARCH

Year	winning pair	pos %	Avg. Perf	Avg. Alpha	Avg. Beta	Avg. Sharpe	Avg. draw	Max-
2015	42	0.259259	121104.98	0.25	-0.02	0.68	-0.12	
2016	48	0.296296	223962.45	-0.01	0.38	0.45	-0.44	
2017	52	0.320988	651809.30	-3.51	0.72	0.37	-0.35	
2018	51	0.314815	2659549.88	-1.11	0.94	0.37	-0.63	

[10] Note that, for example, in the Basic model under yearly performances, we observed an α well below zero, reaching value up to -2377.

Table 41: Average yearly results for winning Non-Stationary Pairs Under GARCH

Year	winning pair	pos %	Avg. Perf	Avg. Alpha	Avg. Beta	Avg. Sharpe	Avg. draw	Max-
2015	20	0.317460	119267.11	0.17	0.01	0.58	-0.07	
2016	17	0.269841	157065.91	1.03	-0.04	0.38	-0.21	
2017	17	0.269841	169663.79	-0.26	-0.09	0.46	-0.18	
2018	19	0.301587	137336.99	0.12	0.05	0.33	-0.19	

Under the GARCH model, the ratio of winning pairs increases, which also increases as the time horizon increases, as well as the returns, for both types of pairs. Moreover, the profitability increases over time with significant returns for stationary pairs, while non-stationary pairs do not experience such high returns [11]

[11] We did not repeat this trading strategy for the other parameters due to time limitations and size of this project.

7 Conclusion

7.1 Literature Review

One of the most, if not the most, cited literature on pairs trading are Gatev et al. (1999) and Gatev et al. (2006). Their works is also the largest, Gatev et al. (1999), performed pairs trading on all US equities between 1962 up to 1997, and then extended their work from 1962 to 2002 in Gatev et al. (2006). Their pairs selection method involved using the *sum squared difference* (SSD), the idea behind using such method is that the stocks that comprise the pair that is traded are suitable substitutes, and so if the market is efficient, any mispricing between such pairs should be corrected. Their works have shown that over time pairs trading have become less profitable, that is, pre-1988 profits levels are (abnormal) higher than post-1988. However, top performing pairs have shown to show excessive return of 12% (and 11% in Gatev et al. (2006)) on average throughout the studies. Gatev et al. (2006) argue that possible larger institutions have a relative advantage as they have better access to leverage and cheaply execute trades. The works of Do & Faff (2010) is another major cited paper. They employ the same research of Gatev et al. (2006) but extend the trading period to 2009. Their results are aligned well with the findings of Gatev et al. (2006), that is, up until the 90's pairs trading performed well and declined in the 90's with frequent negative returns leading into the 00's. However, pairs trading performance picked up in bear markets between 2003-2007 and again in 2007-(mid) 2009. In the aforementioned literature, it is argued that an increase in hedge fund activities is one potential reason for the decline in performance as more arbitrageurs exploiting inefficiency's of the market. However, the increase in market participants only tell part of the story and more research needs to be done in terms arbitrate risks and market inefficiencies. Moreover, Do & Faff (2010), demonstrated that the SSD method is an insufficient method to determine close economic substitutes.

Cointegration constitutes to a more rigorous framework for pairs trading compared to the SSD method. This is due to the more mathematical sound identification of long-run equilibrium relationships, (Krauss, 2017). Vidyamurthy (2004) is among one of the most cited literature on pairs trading under a cointegration framework. He proposes a set of heuristic concepts rather than empirical research, that also applied to this study.

In the domain of cointegration, Huck & Afawubo (2015) is also regarded as one of the most cited amongst the literature. Their work, in a sense, is a slight extension to Do & Faff (2010), as in they compared pairs trading performance under different approaches, that is cointegration, stationary and SDD - with a similar approach in trading rules and selecting stocks to trade as in Do & Faff (2010). Their results were in line with existing literature, where they observed a decline in performance in pairs trading, even when accounting for trading costs. On the other hand, under a cointegration framework, they have shown that the performances are more robust, stable and produce more excessive returns, as well, a high and robust alpha, while accounting for trading costs. Huck & Afawubo (2015), also has shown that pairs trading are sensitive under key parameters, which is also inline with Huck (2013).

Fiz (2014) posed the question if pairs trading under a cointegrated framework are profitable. Huck & Afawubo (2015), proposed further suggestions on whether pairs trading would work in different markets. Fiz (2014), rather than using components of the SP500, the study used components of the Euro Stoxx 50 Index and of the Dow Jones Industry Average in a more recent time frame, between 2001 and 2014. Concluding that pairs trading is still gross profitable, but trading costs significantly reduce profits.

Other empirical works done by Perlin (2009) and Bolgun et al. (2009), implemented pairs trading in more recent times as well, and under a cointegration framework and found that significant excess returns on Brazilian and Turkish financial markets. Leong (2018), implemented pairs trading under a cointegration framework for SP500 large cap and small-cap between 2010 and 2013. When accounting for trade costs, significant profits are achieved for the large-cap components while losses occurred for small-cap pairs. Moreover, he replicated the studies of Perlin (2009) and Bolgun et al. (2009) on the Ibovespa 100 and XU100 and incurred losses. However, a possible explanation would be because it was implemented on a different time frame and different trading rules. Contrary to Do & Faff (2010), Fiz (2014) and Leong (2018) did not find any increases in trading or profits during any periods of bear markets.

In this thesis we extend some of the literature to more recent times. However, this thesis differs from other literature is that we compare various parameters under different spread dynamics, with the exception of the GARCH model. Moreover, we were interested how pairs that have a cointegrated spread would perform against pairs whose spread is not - although all of the pairs are very highly correlated. It could be argued that pairs that are not cointegrated, and so only highly correlated, would be subject to randomness. More specifically, we are also interested to see if pairs that are cointegrated perform better than pairs that are random, or simply highly correlated. We do so by compare average performance of all traded pairs, as some pairs do not trade, we then place the pairs into two camps; either cointegrated or not. Finally, we look at the pairs that have positive returns and compare them. We repeat this under the Basic model and under the Kalman filter for various parameters. However, we separated the results under the GARCH model, due to time limitations and scope of this project, as wee only applied it to the time horizon rather than all parameters.

7.2 Conclusion

The objective of this study is to empirically answer several questions regarding pairs trading. First, if pairs trading is still profitable under a cointegrated framework. Second, if spread dynamics affect performance. Third, how do various parameters affect the performance, including the GARCH. Final, can estimated parameters, as in section 5, provide any insight to performance.

First, under both models when considering the time horizon and bands we observed, unexpectedly, that on average pairs trading does not seem profitable, in fact we observe a pattern in both models where on average pairs that are not cointegrated outperform cointegrated pairs. Only when looking at the winning pairs we see significant returns and that cointegrated pairs outperform pairs that are not. From here we see that pairs should be held approximately 2 to 3 years and shorter trading signals should be used, i.e. 0.25 and 0.5 deviations respectively. Also, in some cases 0.75 and 1.0 deviations are also fine.

Fiz (2014) adjusted bands under a cointegrated framework, he found that overall gross profit decreases and number of trades decreases as the bands are increased, as well as average returns per trade increases. So do our pairs as well, on average either very short or either large bands i.e. 0.25, 0.5 and 2 deviations. However, the results, are not entirely comparable, as we only traded for 1 year.

Every year we check whether the same pairs were cointegrated or not, we observed that cointegrated relationships alternate over the years[12]. Although, we did not test it, it would be naive to assume that structural breaks in cointegrated relationship only happen on an annual basis. In fact, Leong (2018), occasionally experienced structural breaks every two weeks. Which could explain why using a moving window of either 15 or 30 days performed so well. It could be argued that regularly updating the trading signal would capture more relevant information - and as well as the results suggest a 120 or 252 moving window - than a static approach[13]. In addition, it could also be the opposite, pairs that are not cointegrated experience cointegration during certain time intervals during trading periods, which could explain why in some cases it outperforms cointegrated pairs which we predominately saw for when adjusting for bands and trading intervals.

Second, Kalman filter not only reduces the noise of the spread, but also mirrors the pattern of the results under the Basic model, compresses it. Which, in hindsight, makes sense, as the spread is filtered, so become the trading signals and thus the results.

Third, as discussed before, on average it seemed that when adjusting for years traded and the bands, non-cointegrated pairs outperform cointegrated pairs. It is only when looking at winning pairs that we see a difference. The parameter that seemed matched expectations is the *moving window*. Although there are some instances where other parameters outperform *moving windows*, it showed the largest difference between cointegrated pairs and non-cointegrated pairs, and produced highest rate of winning pairs, with much lower market beta and mdd's. It would be interesting to see how these pairs would perform under a larger trading period and if estimated $\{\hat{\alpha}, \hat{\beta}\}$ were to be regularly updated.

In section 6 we implemented a similar trading rule as Miah et al. (2016), where unlike the Basic model, we posed conditional volatility on the bands. The study was limited to only the trading period, due to time limitation. Contrary to their work where Miah et al. (2016) did not obtain excessive returns, we obtained significant returns. Moreover, it also met expectations. That is, pairs where the spread is cointegrated outperformed the pairs whose spread on average and when considering the winning pairs. Possible explanation why we obtained different results is because of a different selecting procedure, pairs traded and trading period. Moreover, the study was not replicated for other the other parameters due to time limitations.

Fourth, when looking at the relationship between *mse* and $\hat{\beta}$ to the log returns, there is some evidence to a relation between log return and *mse* for both models. However, in most cases they are not significant. Having said that, more interesting results are obtained between log returns and $\hat{\beta}$. When regressing for stationary winning and losing pairs i.e. θ_1 and θ_2. We seen that under the Basic model winning pairs seem to have a more stable θ_1 as time progresses, while losing pairs seem to be alternating between a lot, similarly to non-stationary pairs. This could indicate that under the Basic model, winning pairs - that presumably win due to mean-reversion with the possibility of occasional structural breaks - tend to a more stable $\hat{\beta}$ as time progresses. That is, the linear relationship between two pairs that make up the spread are held together better over time. This could explain why when looking at winning pairs, pairs that are cointegrated outperform pairs that are not. However, under the Kalman filter we observe a slightly weaker results (table 36). Furthermore, it would be interesting to see if the same holds for the other parameters and if it could be repeated for the *p-value* obtain from the ADF test.

[12]See appendix 8.1

[13]To clarify, we do not update the *zscore*

There are some limitations to the study, first is that due to technical issues, only 239 pairs of the 250 were investigated. Due to time limitations not all studies were replicated to all parameters. Furthermore, although information is available at the reader's request, other aspects could have been used in the study such as daily trades and returns, Sortino ratio and as well accounting for trading costs.

An unexpected result that we obtained from our pairs selection method was that a large proportion of pairs were made out of ETF's, the tables in the appendix shows that a large proportion of winning pairs in 2015 and 2018 were made out of ETF's whose constituents are made of the SP500, DJIA or Euronext, which are provided by several companies with different variations in investment purposes, such as either large-cap or mid-cap - some of these ETF's are different in nature, but this is left at the discretion of the reader. When trading comparable ETF's, we see in the tables that they have significant returns, it can be argued that they are close substitutes of another. In addition, we also see pairs of different industries, whether ETF or a normal stock, such as pharmaceuticals, oil, healthcare, real estate or commodities such as gold or silver and are not necessarily subject to the same industry as can be seen for the pair *DBS vs SLV* which is an energy company versus a silver trust.

Insofar, we have shown that pairs trading can still be a (gross) profitable trading strategy. However, we do see that from our pairs selected, that approximately half of the pairs traded produced profitable returns for certain parameters. We found that generally the Basic model performs better than the Kalman filter, possibly because of lower volatility as the Kalman filter smooths the spread. Having said that, when applying moving windows we have found that it produces the highest ratio of winning pairs and general good performance. In addition, applying conditional volatility on the bands also produces favourable performance, given that we only performed the strategy on the trading years, it outperformed the Basic model. More interestingly, these methods matched expectations, that is, on average they seem to return profits quicker; on average pairs that are cointegrated perform better than the ones that are not and bigger differences are observed when we look at the strictly winning pairs.

Counts of words

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Words in headers: 152
Words outside text (captions, etc.): 633
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Number of floats/tables/figures: 48
Number of math inlines: 250
Number of math displayed: 44
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  160+10+0 (2/1/0/0) _top_
  633+1+0 (1/0/1/0) Section: Introduction
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182+2+0 (1/0/15/2) Subsection: GARCH models
351+2+9 (1/1/25/6) Subsection: Kalman Filter
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373+2+36 (1/2/35/0) Subsection: Trading Rules
125+1+0 (1/0/13/0) Subsection: Parameters
91+2+0 (1/0/0/0) Section: Backtest Results
460+2+87 (1/1/0/3) Subsection: Performance Metrics
906+20+158 (4/15/5/0) Subsection: Basic Model results
568+25+169 (4/15/0/0) Subsection: Kalman Filter Results
353+1+8 (1/0/2/0) Subsection: Overview
54+2+0 (1/0/8/0) Section: Further Analysis
427+3+22 (1/3/24/3) Subsection: Log returns vs $mse$
233+5+10 (1/2/8/1) Subsection: Log returns vs estimated Beta
136+4+16 (1/1/10/1) Section: Pairs trading under GARCH
216+5+81 (1/5/1/0) Subsection: Backtest results under GARCH bands
0+1+0 (1/0/0/0) Section: Conclusion
878+2+0 (1/0/0/0) Subsection: Literature Review
1222+4+10 (2/0/11/0) Subsection: Conclusion
0+1+0 (1/0/0/0) Section: Appendix
3731+10+4 (1/0/0/0) Subsection: List of pairs traded including which ones pertain to Stationarity
0+12+0 (1/0/0/0) Subsection: Winning Stationary Pairs for the Year of 2015 under the Basic Model
1+12+0 (1/0/0/0) Subsection: Winning Stationary Pairs for the Year of 2018 under the Basic Model

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8 Appendix

8.1 List of pairs traded including which ones pertain to Stationarity

Table 42: List of pairs traded

Security1	Security2	Form. period	Stat '15	Stat '16	Stat '17	Stat '18
ProShares UltraShort MidCap400 (MZZ)	UltraPro Short MidCap400 (SMDD)	NS	NS	NS	NS	NS
Vanguard Extended Duration Treasury (EDV)	ProShares Ultra 20+ Year Treasury (UBT)	NS	NS	NS	NS	NS
iShares Core S&P 500 (IVV)	iShares Russell 1000 (IWB)	S	NS	NS	NS	NS
Invesco DB Gold Fund (DGL)	Aberdeen Standard Physical Gold Shares (SGOL)	S	NS	NS	NS	NS
Invesco S&P 500 Downside Hedged (PHDG)	Barclays ETN S&P VEQTOR ETN (VQT)	S	NS	NS	NS	NS
iShares Russell 3000 (IWV)	Schwab U.S. Broad Market (SCHB)	NS	NS	NS	NS	NS
iShares Russell 1000 (IWB)	SPDR S&P 500 (SPY)	S	NS	NS	NS	NS
iShares Russell 1000 (IWB)	ProShares Ultra S&P500 (SSO)	S	NS	NS	NS	NS
Direxion Daily Healthcare Bull 3X Shares (CURE)	ProShares Ultra Health Care (RXL)	S	NS	NS	NS	NS
iShares MSCI USA Equal Weighted (EUSA)	Vanguard Large-Cap (VV)	S	NS	NS	NS	NS
Sprott Physical Gold Trust ETV (PHYS)	Aberdeen Standard Physical Gold Shares (SGOL)	NS	NS	NS	NS	NS
ProShares Ultrashort FTSE China 50 (FXP)	Direxion Daily FTSE China Bear 3x Shares (YANG)	NS	NS	NS	NS	NS
Schwab U.S. Broad Market (SCHB)	Vanguard Total Stock Market (VTI)	NS	S	NS	NS	NS
Invesco DB Gold Fund (DGL)	SPDR Gold Trust (GLD)	S	NS	NS	NS	NS
WisdomTree U.S. LargeCap Dividend Fund (DLN)	Vanguard High Dividend Yield (VYM)	NS	NS	NS	NS	NS
Fidelity MSCI Utilities Index (FUTY)	SPDR Select Sector Fund - Utilities (XLU)	NS	NS	NS	NS	NS
Invesco CurrencyShares Japanese Yen Trust (FXY)	ProShares Ultra Yen (YCL)	S	NS	NS	NS	NS
Vanguard Utilities (VPU)	SPDR Select Sector Fund - Utilities (XLU)	NS	NS	NS	NS	NS
SPDR Gold Trust (GLD)	Sprott Physical Gold Trust ETV (PHYS)	S	NS	NS	NS	NS
Schwab U.S. Large-Cap Growth (SCHG)	Vanguard S&P 500 Growth (VOOG)	S	NS	NS	NS	NS
Schwab U.S. Broad Market (SCHB)	Vanguard Russell 1000 Index Fund (VONE)	NS	NS	NS	NS	NS
Invesco Financial Preferred (PGF)	Invesco Preferred (PGX)	S	NS	NS	NS	NS
ProShares Short Dow30 (DOG)	ProShares UltraShort Dow30 (DXD)	S	NS	NS	NS	NS
DB 3X Short 25 Year Treasury Bond ETN (SBND)	ProShares UltraPro Short 20 Year Treasury (TTT)	S	NS	NS	NS	NS
United Ses Short Oil Fund (DNO)	DB Crude Oil Double Short ETN due June 1 2038 ...	NS	NS	NS	NS	NS
iShares Core S&P Total U.S. Stock Market (ITOT)	Vanguard S&P 500 (VOO)	S	NS	NS	NS	NS
iShares Russell 1000 (IWB)	Schwab U.S. Broad Market (SCHB)	NS	NS	NS	NS	NS
ProShares Ultra S&P500 (SSO)	Vanguard S&P 500 (VOO)	S	NS	NS	NS	NS
iShares Russell Top 200 (IWL)	Vanguard Mega Cap (MGC)	S	NS	NS	NS	NS
ProShares UltraShort Lehman 20 Year Treasury (...	Direxion Daily 20-Year Treasury Bear 3X (TMV)	NS	NS	NS	NS	NS

Continued on next page

Table 42: List of pairs traded

Security1	Security2	Form. period	Stat '15	Stat '16	Stat '17	Stat '18
Fidelity MSCI Health Care Index (FHLC)	iShares U.S. Healthcare (IYH)	NS	NS	NS	NS	NS
iShares Core S&P 500 (IVV)	ProShares Ultra S&P500 (SSO)	S	NS	NS	NS	NS
Vanguard Russell 1000 Index Fund (VONE)	Vanguard S&P 500 (VOO)	S	NS	NS	NS	NS
Market Vectors Double Short Euro ETN (DRR)	ProShares UltraShort Euro (EUO)	S	NS	NS	S	S
iShares S&P Mid-Cap 400 Growth (IJK)	Vanguard S&P Mid-Cap 400 Growth (IVOG)	S	S	NS	NS	NS
iShares Core S&P U.S. Value ETF (IUSV)	iShares Russell 1000 Value (IWD)	S	NS	NS	NS	NS
ProShares UltraPro Short Russell2000 (SRTY)	Direxion Small Cap Bear 3X Shares (TZA)	NS	NS	NS	NS	NS
iShares Core S&P Total U.S. Stock Market (ITOT)	iShares Core S&P 500 (IVV)	S	NS	NS	NS	NS
Barclays ETN FI Enhanced Europe 50 ETN (FEEU)	SPDR STOXX Europe 50 ETF (FEU)	S	NS	NS	NS	NS
Barclays ETN FI Enhanced Europe 50 ETN (FEEU)	SPDR Portfolio Europe (SPEU)	S	NS	NS	NS	NS
ProShares Ultra S&P500 (SSO)	Vanguard Russell 1000 Index Fund (VONE)	S	NS	NS	NS	NS
ProShares UltraShort Lehman 20 Year Treasury (...	ProShares UltraPro Short 20 Year Treasury (TTT)	NS	NS	NS	NS	NS
Direxion Financial Bull 3X Shares (FAS)	ProShares Ultra Financials (UYG)	NS	NS	NS	NS	NS
iShares Core S&P Total U.S. Stock Market (ITOT)	SPDR S&P 500 (SPY)	S	NS	NS	NS	NS
Vanguard Extended Duration Treasury (EDV)	PIMCO 25 Year Zero Coupon U.S. Treasury Index ...	S	NS	NS	NS	NS
ProShares Short 20+ Year Treasury (TBF)	ProShares UltraShort Lehman 20 Year Treasury (...	NS	NS	NS	NS	NS
SPDR S&P 500 (SPY)	ProShares Ultra S&P500 (SSO)	S	NS	NS	NS	NS
iShares Dow Jones U.S. (IYY)	Vanguard Large-Cap (VV)	NS	NS	NS	NS	NS
ProShares Ultra Gold (UGL)	VS 3X GOLD (UGLD)	S	NS	NS	NS	NS
iShares Core S&P 500 (IVV)	Vanguard Russell 1000 Index Fund (VONE)	S	NS	NS	NS	NS
BB&T Corporation (BBT)	Truist Financial Corporation (TFC)	S	NS	NS	NS	NS
SPDR S&P 500 (SPY)	Vanguard Russell 1000 Index Fund (VONE)	S	NS	NS	NS	NS
iShares 20+ Year Treasury Bond ETF (TLT)	Vanguard Long-Term Government Bond ETF (VGLT)	NS	NS	NS	S	S
ProShares UltraShort QQQ (QID)	ProShares UltraPro Short QQQ (SQQQ)	NS	NS	NS	NS	NS
ProShares UltraShort Lehman 7-10 Year Treasury...	Direxion Daily 10-Yr Treasury Bear 3x Shrs (TYO)	S	NS	NS	NS	NS
iShares 20+ Year Treasury Bond ETF (TLT)	ProShares Ultra 20+ Year Treasury (UBT)	NS	NS	NS	NS	NS
ProShares Ultra QQQ (QLD)	PowerShares QQQ Trust Ser 1 (QQQ)	NS	NS	NS	NS	NS
ProShares Short MidCap400 (MYY)	ProShares UltraShort MidCap400 (MZZ)	S	NS	NS	NS	NS
ProShares UltraShort S&P500 (SDS)	Direxion Daily S&P 500 Bear 3X (SPXS)	NS	NS	NS	NS	NS
WisdomTree U.S. LargeCap Dividend Fund (DLN)	WisdomTree U.S. Total Dividend Fund (DTD)	S	NS	NS	NS	NS

Continued on next page

Table 42: List of pairs traded

Security1	Security2	Form. period	Stat '15	Stat '16	Stat '17	Stat '18
iShares U.S. Utilities (IDU)	SPDR Select Sector Fund - Utilities (XLU)	NS	NS	NS	NS	NS
Vanguard Mega Cap (MGC)	Vanguard Large-Cap (VV)	NS	NS	NS	NS	NS
SPDR Series Trust Portfolio S&P 500 Value (SPYV)	Vanguard S&P 500 Value (VOOV)	S	NS	NS	NS	NS
iShares Dow Jones U.S. (IYY)	Vanguard Total Stock Market (VTI)	NS	NS	NS	NS	NS
Vanguard Extended Duration Treasury (EDV)	iShares 20+ Year Treasury Bond ETF (TLT)	S	NS	NS	NS	S
ProShares Ultra S&P500 (SSO)	Vanguard Large-Cap (VV)	S	NS	NS	NS	NS
Invesco DB Gold Fund (DGL)	ProShares Ultra Gold (UGL)	NS	NS	NS	NS	NS
iShares Core S&P Total U.S. Stock Market (ITOT)	ProShares Ultra S&P500 (SSO)	S	NS	NS	NS	NS
iShares Russell 3000 (IWV)	iShares Dow Jones U.S. (IYY)	NS	NS	NS	NS	NS
iShares Core S&P Total U.S. Stock Market (ITOT)	Schwab U.S. Broad Market (SCHB)	NS	NS	NS	NS	NS
iShares 20+ Year Treasury Bond ETF (TLT)	PIMCO 25 Year Zero Coupon U.S. Treasury Index ...	S	NS	NS	NS	NS
First Trust S&P REIT Index Fund (FRI)	Vanguard Real E Se (VNQ)	S	NS	NS	NS	NS
Goldman Sachs Connect S&P Enhanced Commodity T...	iShares GSCI Commodity-Indexed Trust Fund (GSG)	NS	NS	NS	NS	NS
ProShares Short QQQ (PSQ)	ProShares UltraShort QQQ (QID)	NS	NS	NS	NS	NS
Direxion Daily 20-Yr Treasury Bull 3x Shrs (TMF)	ProShares Ultra 20+ Year Treasury (UBT)	NS	NS	NS	NS	NS
Fox Corporation (FOXA)	Fox Corporation (FOXA)	NS	NS	S	S	NS
iShares S&P Mid-Cap 400 Value (IJJ)	Vanguard S&P Mid-Cap 400 Value (IVOV)	S	S	NS	NS	NS
FI Enhanced Gloabl High Yield ETN (FIEG)	Barclays ETN FI Enhanced Global High Yield ETN...	S	NS	NS	S	NS
SPDR Portfolio Long Term Treasury (SPTL)	Vanguard Long-Term Government Bond ETF (VGLT)	S	NS	S	S	S
Invesco DB Oil Fund (DBO)	United Ses 12 Month Oil (USL)	S	NS	NS	NS	NS
Direxion Small Cap Bull 3X Shares (TNA)	ProShares UltraPro Russell2000 (URTY)	NS	S	NS	NS	NS
Babcock & Wilcox Co. (BWC)	BWX Technologies Inc. (BWXT)	NS	NS	NS	NS	NS
VelocityShares Daily 2x VIX Short-Term ETN (TVIX)	ProShares Trust Ultra VIX Short Term Futures ...	S	NS	NS	S	S
iShares U.S. Real E Se (IYR)	ProShares Ultra Real E Se (URE)	NS	NS	NS	NS	NS
iShares Dow Jones U.S. (IYY)	Schwab U.S. Broad Market (SCHB)	NS	NS	NS	S	S
ProShares Ultra Dow30 (DDM)	ProShares UltraPro Dow30 (UDOW)	NS	NS	NS	NS	NS
iShares Dow Jones U.S. (IYY)	Vanguard Russell 1000 Index Fund (VONE)	NS	NS	NS	NS	NS
ProShares UltraShort Dow30 (DXD)	UltraPro Short Dow30 (SDOW)	NS	NS	NS	NS	NS
ProShares UltraShort S&P500 (SDS)	ProShares UltraPro Short S&P500 (SPXU)	NS	NS	NS	NS	NS
iShares Russell 1000 (IWB)	iShares Dow Jones U.S. (IYY)	S	NS	NS	NS	NS
ProShares Ultra QQQ (QLD)	ProShares UltraPro QQQ (TQQQ)	NS	NS	NS	NS	NS
iShares Core S&P Total U.S. Stock Market (ITOT)	iShares Russell 1000 (IWB)	S	NS	NS	NS	NS
Invesco DB Gold Fund (DGL)	DB Gold Double Long ETN due February 15 2038 (...	S	NS	NS	NS	NS
SPDR Portfolio Long Term Treasury (SPTL)	iShares 20+ Year Treasury Bond ETF (TLT)	NS	NS	NS	NS	S
iShares Russell 1000 (IWB)	Vanguard Large-Cap (VV)	S	NS	NS	NS	NS

Continued on next page

Table 42: List of pairs traded

Security1	Security2	Form. period	Stat '15	Stat '16	Stat '17	Stat '18
iShares Core S&P Total U.S. Stock Market (ITOT)	Vanguard Large-Cap (VV)	S	NS	NS	NS	NS
Invesco DB Silver Fund (DBS)	Aberdeen Standard Physical Silver Shares (SIVR)	S	S	NS	NS	NS
iShares S&P 500 Growth (IVW)	SPDR Series Trust Portfolio S&P 500 Growth (SPYG)	S	NS	NS	NS	NS
Direxion Daily S&P 500 Bull 3X Shares (SPXL)	ProShares Ultra S&P500 (SSO)	S	NS	NS	NS	NS
iShares Core S&P Total U.S. Stock Market (ITOT)	Vanguard Russell 1000 Index Fund (VONE)	NS	NS	NS	NS	NS
ProShares UltraShort S&P500 (SDS)	ProShares Short S&P500 (SH)	S	NS	NS	NS	NS
ProShares Trust VIX Short-Term Futures (VIXY)	iPath S&P 500 VIX Short Term Futures TM ETN (VXX)	S	NS	NS	NS	NS
ProShares Trust VIX Short-Term Futures (VIXY)	iPath S&P 500 VIX Short Term Futures TM ETN (V...	S	NS	NS	NS	NS
iShares Europe (IEV)	Vanguard FTSEEuropean (VGK)	S	NS	NS	NS	NS
VelocityShares Daily Long VIX Short-Term ETN ...	iPath S&P 500 VIX Short Term Futures TM ETN (VXX)	S	S	S	S	S
VelocityShares Daily Long VIX Short-Term ETN ...	iPath S&P 500 VIX Short Term Futures TM ETN (V...	S	S	S	S	S
Banco BBVA Argentina S.A. ADS (BBAR)	BBVA Banco Frances S.A. (BFR)	S	NS	S	S	NS
iShares Core S&P Total U.S. Stock Market (ITOT)	iShares Dow Jones U.S. (IYY)	S	NS	NS	NS	NS
Zimmer Biomet Holdings Inc. (ZBH)	Zimmer Holdings Inc. (ZMH)	NS	NS	NS	NS	NS
iShares Russell 1000 (IWB)	Vanguard Russell 1000 Index Fund (VONE)	S	S	NS	NS	NS
ProShares Ultra S&P500 (SSO)	ProShares UltraPro S&P 500 (UPRO)	S	NS	NS	NS	NS
First Trust NASDAQ-100 Equal Weighted Index Fu...	Direxion NASDAQ-100 Equal Weighted Index Share...	S	S	NS	NS	NS
iShares S&P 500 Growth (IVW)	Vanguard S&P 500 Growth (VOOG)	S	S	S	S	NS
Invesco DB Silver Fund (DBS)	iShares Silver Trust (SLV)	S	NS	NS	NS	NS
iShares Russell 1000 Growth (IWF)	Vanguard Russell 1000 Growth Index Fund (VONG)	S	S	S	S	NS
iShares Russell 1000 Value (IWD)	Vanguard Russell 1000 Value Index Fund (VONV)	S	S	S	S	NS
Vanguard Russell 1000 Index Fund (VONE)	Vanguard Large-Cap (VV)	S	NS	NS	NS	NS
Viacom Inc. (VIA)	Viacom Inc. (VIAB)	S	NS	NS	NS	NS
Vanguard Mega Cap (MGC)	SPDR S&P 500 (SPY)	NS	NS	NS	NS	NS
iShares Core S&P 500 (IVV)	Vanguard Mega Cap (MGC)	NS	NS	S	NS	NS
iShares U.S. Energy (IYE)	Vanguard Energy (VDE)	NS	NS	NS	NS	NS
Vanguard Mega Cap (MGC)	Vanguard S&P 500 (VOO)	NS	NS	S	NS	NS
iShares Russell 2000 (IWM)	Vanguard Russell 2000 Index Fund (VTWO)	S	S	S	S	S
Vanguard S&P 500 (VOO)	Vanguard Large-Cap (VV)	S	NS	NS	NS	NS
Sesa Sterlite Ltd ADR (Sponsored) (SSLT)	Vedanta Limited American Depositary Shares (Ea...	NS	NS	NS	NS	NS
iShares Core S&P 500 (IVV)	Vanguard Large-Cap (VV)	NS	NS	NS	NS	NS
SPDR S&P 500 (SPY)	Vanguard Large-Cap (VV)	S	NS	NS	NS	NS
Fidelity MSCI Energy Index (FENY)	iShares U.S. Energy (IYE)	S	NS	NS	NS	NS
Dr Pepper Snapple Group Inc (DPS)	Keurig Dr Pepper Inc. (KDP)	S	NS	NS	NS	NS
ProShares Trust VIX Mid-Term Futures (VIXM)	iPath S&P 500 VIX Mid-Term Futures ETN (VXZ)	S	S	S	NS	NS

Continued on next page

Table 42: List of pairs traded

Security1	Security2	Form. period	Stat '15	Stat '16	Stat '17	Stat '18
ProShares Trust VIX Mid-Term Futures (VIXM)	iPath S&P 500 VIX Mid-Term Futures ETN (VXZ_)	S	S	S	NS	NS
SPDR Series Trust Portfolio S&P 500 Growth (SPYG)	Vanguard S&P 500 Growth (VOOG)	S	NS	NS	NS	S
DB Gold Double Long ETN due February 15 2038 (...	ProShares Ultra Gold (UGL)	NS	NS	NS	NS	NS
AdvisorShares DoubleLine Value Equity (DBLV)	AdvisorShares Wilshire Buyback (TTFS)	NS	NS	NS	NS	NS
Fidelity MSCI Utilities Index (FUTY)	iShares U.S. Utilities (IDU)	NS	NS	NS	NS	NS
ishares Gold Trust (IAU)	Aberdeen Standard Physical Gold Shares (SGOL)	S	S	NS	NS	NS
SPDR Gold Trust (GLD)	ishares Gold Trust (IAU)	S	S	NS	NS	NS
Direxion Mid Cap Bull 3X Shares (MIDU)	UltraPro MidCap400 (UMDD)	S	NS	NS	NS	NS
Fidelity MSCI Financials Index (FNCL)	Vanguard Financials (VFH)	S	NS	S	S	S
Fidelity MSCI COnsumer Staples Index (FSTA)	Vanguard Consumer Staples (VDC)	S	NS	NS	NS	NS
iShares U.S. Utilities (IDU)	Vanguard Utilities (VPU)	NS	NS	NS	NS	NS
Linde plc (LIN)	Praxair Inc. (PX)	S	NS	NS	NS	NS
Direxion Mid Cap Bear 3X Shares (MIDZ)	UltraPro Short MidCap400 (SMDD)	S	NS	NS	NS	NS
iShares Russell 3000 (IWV)	Vanguard Total Stock Market (VTI)	S	S	S	S	NS
Fidelity MSCI Energy Index (FENY)	Vanguard Energy (VDE)	S	S	NS	NS	NS
Fidelity MSCI Utilities Index (FUTY)	Vanguard Utilities (VPU)	S	S	NS	NS	NS
Aberdeen Standard Physical Silver Shares (SIVR)	iShares Silver Trust (SLV)	S	S	NS	NS	NS
Direxion Daily S&P 500 Bear 3X (SPXS)	ProShares UltraPro Short S&P500 (SPXU)	S	S	NS	NS	NS
DDR Corp. (DDR)	SITE Centers Corp. (SITC)	S	NS	NS	NS	NS
VelocityShares Daily Long VIX Short-Term ETN ...	ProShares Trust VIX Short-Term Futures (VIXY)	S	NS	NS	NS	NS
Barclays Bank Plc iPath Exchange Traded Notes ...	United Ses Oil Fund (USO)	NS	NS	NS	NS	NS
CBRE Group Inc Class A (CBG)	CBRE Group Inc Class A (CBRE)	S	NS	NS	NS	NS
SPDR Gold Trust (GLD)	Aberdeen Standard Physical Gold Shares (SGOL)	S	S	S	S	S
Bellatrix Exploration Ltd (Canada) (BXE)	Bellatrix Exploration Ltd (Canada) (BXEFF)	S	NS	NS	NS	NS
Bellatrix Exploration Ltd (Canada) (BXEFD)	Bellatrix Exploration Ltd (Canada) (BXEFF)	S	NS	NS	NS	NS
Direxion Daily 20-Year Treasury Bear 3X (TMV)	ProShares UltraPro Short 20 Year Treasury (TTT)	S	NS	NS	NS	NS
Encana Corporation (ECA)	Ovintiv Inc. (DE) (OVV)	S	NS	NS	NS	NS
ProShares Short VIX Short Term Futures (SVXY)	VelocityShares Daily Inverse VIX Short Term ET...	S	NS	NS	NS	NS
Capri Holdings Limited (CPRI)	Michael Kors Holdings Limited (KORS)	S	NS	NS	NS	NS
AIM ImmunoTech Inc. (AIM)	Hemispherx BioPharma Inc. (HEB)	S	NS	NS	NS	NS
Oxford Resource Partners LP (OXF)	Westmoreland Resource Partners LP representing...	S	NS	NS	NS	NS
Oxford Resource Partners LP (OXF)	Westmoreland Resource Partners LP representing...	S	NS	NS	NS	NS
iShares Core S&P 500 (IVV)	Vanguard S&P 500 (VOO)	S	S	S	NS	NS
iShares Core S&P 500 (IVV)	SPDR S&P 500 (SPY)	S	S	S	NS	NS

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Table 42: List of pairs traded

Security1	Security2	Form. period	Stat '15	Stat '16	Stat '17	Stat '18
Direxion Daily S&P 500 Bull 3X Shares (SPXL)	ProShares UltraPro S&P 500 (UPRO)	S	NS	NS	NS	NS
SPDR S&P 500 (SPY)	Vanguard S&P 500 (VOO)	S	S	S	S	NS
Dean Foods Company (DF)	Dean Foods Company (DFODQ)	S	S	NS	NS	NS
Advantage Oil & Gas Ltd (AAV)	Advantage Oil & Gas Ltd (AAVVF)	S	NS	NS	NS	NS
Ensco Rowan plc Class A (ESV)	Valaris plc Class A (VAL)	S	NS	NS	NS	NS
Hi-Crush Partners LP representing limited part	Hi-Crush Inc. (HCR)	S	NS	NS	NS	NS
Carbo Ceramics Inc. (CRR)	Carbo Ceramics Inc. (CRRT)	S	NS	NS	NS	NS
Sutor Technology Group Ltd (SUTR)	TOR (TOR)	S	NS	NS	NS	NS
Jacobs Engineering Group Inc. (J)	Jacobs Engineering Group Inc. (JEC)	S	NS	NS	NS	NS
Welltower Inc. (HCN)	Welltower Inc. (WELL)	S	NS	NS	NS	NS
SPDR Bloomberg Barclays Corporate Bond ETF	SPDR Portfolio Corporate Bond (SPBO)	S	NS	NS	NS	NS
Appliance Recycling Centers of America Inc. (JAN)	JanOne Inc. (JAN)	S	NS	NS	NS	NS
Caladrius Biosciences Inc. (CLBS)	NeoStem Incorporated (NBS)	S	NS	NS	NS	S
SPDR Barclays Mortgage Backed Bond ETF (MBG)	SPDR Portfolio Mortgage Backed Bond (SPMB)	S	NS	NS	NS	NS
Regado BioSciences Inc (RGDO)	Tobira Therapeutics Inc. (TBRA)	S	NS	S	S	S
SPDR Barclays Intermediate Term Treasury ETF ...	SPDR Portfolio Intermediate Term Treasury (SPTI)	S	NS	NS	NS	NS
Giga-tronics Incorporated (GIGA)	Giga-tronics Incorporated (GIGAD)	S	NS	NS	NS	NS
iShares U.S. Credit Bond ETF (CRED)	iShares U.S. Credit Bond ETF (USIG)	S	NS	NS	NS	NS
EnerJex Resources Inc. (NEW) (ENRJ)	AgEagle Aerial Systems Inc. (UAVS)	NS	NS	NS	NS	NS
TAL International Group Inc. (TAL_)	Triton International Limited (TRTN)	S	S	NS	NS	NS
SPDR ICE BofAML Broad High Yield Bond ETF (CJNK)	SPDR Portfolio High Yield Bond (SPHY)	NS	NS	NS	NS	NS
Brickell Biotech Inc. (BBI)	Vical Incorporated (VICL)	S	S	S	S	S
Vicon Industries Inc (VCON)	Vicon Industries Inc (VII)	S	NS	NS	NS	NS
Vicon Industries Inc (VCON)	Vicon Industries Inc (VCOND)	S	NS	NS	NS	NS
Westmoreland Resource Partners LP representing...	Westmoreland Resource Partners LP representing...	S	S	S	S	S
Hugoton Royalty Trust (HGT)	Hugoton Royalty Trust (HGTXU)	S	S	S	S	S
PXSV (PXSV)	Invesco S&P SmallCap Value with Momentum (XSVM)	S	S	S	S	S
PXLV (PXLV)	Invesco S&P 500 Value with Momentum (SPVM)	S	S	S	S	S
Bristow Group Inc. (BRS)	Bristow Group Inc. (BRSWQ)	S	S	S	S	S
BioTime Inc. (BTX)	Lineage Cell Therapeutics Inc. (LCTX)	S	S	S	S	S
Mitsubishi UFJ Financial Group Inc. (MTU)	Mitsubishi UFJ Financial Group Inc. (MUFG)	S	NS	NS	NS	NS
NortonLifeLock Inc. (NLOK)	Symantec Corporation (SYMC)	S	S	S	S	S
SandRidge Mississippian Trust I of Beneficial ...	SandRidge Mississippian Trust I of Beneficial ...	S	S	S	S	S
Vislink Technologies Inc. (VISL)	XG Technology Inc (XGTI)	S	NS	NS	S	S
Designer Brands Inc. Class A (DBI)	DSW Inc. (DSW)	NS	S	S	S	S
Gafisa SA S.A. American Depositary Shares (GFA)	Gafisa SA S.A. American Depositary Shares (GF...	S	S	S	S	S
Precigen Inc. (PGEN)	Intrexon Corporation (XON)	S	S	S	S	S
PXLG (PXLG)	Invesco S&P 500 GARP (SPGP)	S	S	S	S	NS

Continued on next page

Table 42: List of pairs traded

Security1	Security2	Form. period	Stat '15	Stat '16	Stat '17	Stat '18
Rexahn Pharmaceuticals Inc. (REXN)	Rexahn Pharmaceuticals Inc. (RNN)	NS	S	NS	S	S
Vina Concha Y Toro (VCO)	Vina Concha Y Toro (VCOYY)	S	S	S	S	S
Cornerstone Building Brands Inc. (CNR)	NCI Building Systems Inc. (NCS)	S	S	S	NS	NS
Novelion Therapeutics Inc. (NVLN)	Novelion Therapeutics Inc. (NVLNF)	S	S	S	S	S
Oxford Square Capital Corp. (OXSQ)	TICC Capital Corp. (TICC)	S	S	S	S	NS
AU Optronics Corp American Depositary Shares ...	AU Optronics Corp American Depositary Shares (...	S	S	S	S	S
Barings BDC Inc. (BBDC)	Triangle Capital Corporation (TCAP)	S	S	S	S	NS
Emerge Energy Services LP representing Limited...	Emerge Energy Services LP representing Limited...	S	S	S	S	S
Emerge Energy Services LP representing Limited...	Emerge Energy Services LP representing Limited...	S	S	S	S	S
Phio Pharmaceuticals Corp. (PHIO)	RXi Pharmaceuticals (RXII)	S	S	S	S	NS
Pain Therapeutics Inc. (PTIE)	Cassava Sciences Inc. (SAVA)	S	S	S	S	S
RAIT Financial Trust of Beneficial Interest (...	RAIT Financial Trust of Beneficial Interest (...	S	NS	NS	NS	NS
CBS Corporation Class B (CBS)	ViacomCBS Inc. (VIAC)	NS	NS	NS	NS	NS
Global X NASDAQ China Technology ETF (CHI_2)	Global X NASDAQ China Technology ETF (QQQC)	S	NS	NS	NS	NS
Neuralstem Inc. (CUR)	Seneca Biopharma Inc. (SNCA)	S	S	S	S	S
Clearway Energy Inc. Class C (CWEN)	NRG Yield Inc. Class C (NYLD)	S	NS	S	S	NS
EVINE Live Inc. (EVLV)	iMedia Brands Inc. (IMBI)	S	NS	NS	NS	NS
Intellipharmaceutics International Inc. (IPCI)	Intellipharmaceutics International Inc - Ordin...	S	S	S	S	S
Key Energy Services Inc. (KEG)	Key Energy Services Inc. (KEGX)	S	S	S	S	S
Key Energy Services Inc. (KEG)	Key Energy Services Inc. (KEGXD)	S	S	S	S	S
Mountain Province Diamonds Inc. (MPVD)	Mountain Province Diamonds Inc. (MPVDF)	S	S	S	S	NS
Piper Sandler Companies (PIPR)	Piper Jaffray Companies (PJC)	S	S	NS	NS	NS
Seadrill Limited (SDRL)	Seadrill Limited (SDRL)	S	NS	NS	NS	S
Superior Energy Services Inc. (SPN)	Superior Energy Services Inc. (SPNV)	S	S	S	S	S
Superior Energy Services Inc. (SPN)	Superior Energy Services Inc. (SPNVD)	S	S	S	S	S
Tile Shop Hldgs Inc. (TTS)	Tile Shop Hldgs Inc. (TTSH)	S	S	S	S	NS
Aegean Marine Petroleum Network Inc. (ANW)	Aegean Marine Petroleum Network Inc. (ANWWQ)	S	S	S	S	S
Atlas Corp. (ATCO)	Seaspan Corporation (SSW)	S	S	S	S	S
Basic Energy Services Inc. (BAS)	Basic Energy Services Inc. (BASX)	S	S	NS	NS	NS
SPDR STOXX Europe 50 ETF (FEU)	SPDR Portfolio Europe (SPEU)	S	S	S	S	S
Liberty Tax Inc. (FRGA)	Liberty Tax Inc. (TAX)	S	S	S	S	S
Government Properties Income Trust (GOV)	Office Properties Income Trust (OPI)	S	S	S	S	NS
Hospitality Properties Trust (HPT)	Service Properties Trust (SVC)	S	S	S	S	NS
Och-Ziff Capital Management Group LLC Class A ...	Sculptor Capital Management Inc. Class A (SCU)	NS	NS	NS	NS	NS
PXMV (PXMV)	Invesco S&P MidCap Value with Momentum (XMVM)	S	S	S	S	S
Ready Capital Corproation (RC)	Sutherland Asset Management Corporation (SLD)	S	S	S	S	NS
Liberty Tax Inc. (TAX)	Liberty Tax Inc. (TAXA)	S	S	S	S	S

8.2 Winning Stationary Pairs for the Year of 2015 under the Basic Model

Pairs	perf_2015	alpha_2015	beta_2015	Sharpe_2015	maxdraw_2015
IWB vs SSO:STAT	155385	0.532510	0.530744	1.036612	-0.066051
EUSA vs VV:STAT	166674	1.200470	1.952547	0.999346	-0.633428
DGL vs GLD:STAT	104922	0.049364	0.028402	0.789826	-0.040936
SSO vs VOO:STAT	129902	0.325945	0.101290	0.798450	-0.085319
IVV vs SSO:STAT	137325	0.370843	0.086302	1.002564	-0.013017
ITOT vs IVV:STAT	100500	0.016054	-0.321265	0.101928	-0.060080
SPY vs SSO:STAT	137329	0.370704	0.091560	1.002972	-0.013284
SSO vs VV:STAT	137734	0.434491	0.342036	0.766705	-0.088545
ITOT vs SSO:STAT	179678	0.755597	1.141976	1.044768	-0.139471
IJJ vs IVOV:STAT	178174	0.776343	-0.890567	1.258618	-0.355039
DBO vs USL:STAT	105307	0.054537	-0.056445	0.983324	-0.072366
IWB vs VV:STAT	100606	0.006547	-0.018439	0.570546	-0.005106
DBS vs SIVR:STAT	100385	0.150889	-0.403114	0.269187	-0.517928
SPXL vs SSO:STAT	151617	0.589235	-0.266881	0.842400	-0.071201
VIXY vs VXX:STAT	101378	0.015241	-0.053136	0.607517	-0.012293
VIXY vs VXX_:STAT	101378	0.015241	-0.053136	0.607517	-0.012293
QQEW vs QQQE:STAT	101888	0.021843	-0.104974	0.562923	-0.029374
FENY vs IYE:STAT	105725	0.065363	-0.299810	0.837599	-0.051440
FUTY vs VPU:STAT	102889	0.034861	-0.145358	0.414047	-0.052926
SUTR vs TOR:STAT	179921	0.899848	0.048738	1.054453	-0.536893
J vs JEC:STAT	101911	0.021529	0.068700	0.250843	-0.074328
CLBS vs NBS:STAT	158875	0.507140	-0.191757	1.788774	-0.129886
RGDO vs TBRA:STAT	180054	1.448688	-0.483901	0.891768	-0.623314
CWEN vs NYLD:STAT	100546	0.005241	0.022661	0.218718	-0.032693
EVLV vs IMBI:STAT	187198	0.668736	-0.058573	2.278413	-0.089733
SPN vs SPNV:STAT	104522	0.164583	-1.145297	0.318811	-0.251225
SPN vs SPNVD:STAT	104522	0.164583	-1.145297	0.318811	-0.251225
RC vs SLD:STAT	100172	0.001821	-0.000747	0.138481	-0.006126

8.3 Winning Stationary Pairs for the Year of 2018 under the Basic Model

Pairs	perf_2018	alpha_2018	beta_2018	Sharpe_2018	maxdraw_2018
PHDG vs VQT:STAT	907530	0.745792	-0.318408	0.095385	-1.315916
IWB vs SSO:STAT	145445	0.096258	0.323909	0.451686	-0.201223
EUSA vs VV:STAT	5.47548e+06	19.138933	-1.725223	0.614946	-2.187434
ITOT vs VOO:STAT	112833	0.040359	-0.042390	0.306376	-0.186707
SSO vs VOO:STAT	108147	0.035178	0.016853	0.176741	-0.242504
IWL vs MGC:STAT	101838	-0.009584	0.188829	0.144746	-0.066769
IVV vs SSO:STAT	120305	0.056171	0.053252	0.323389	-0.140863
DRR vs EUO:STAT	7.02251e+06	-2377.614141	176.206287	-0.494599	-3.456568
IUSV vs IWD:STAT	126920	0.091914	0.018684	0.307046	-0.187812
ITOT vs IVV:STAT	118566	0.051407	-0.029587	0.418916	-0.129813
ITOT vs SPY:STAT	112816	0.031202	0.071103	0.310711	-0.188393
SPY vs SSO:STAT	120049	0.055644	0.053007	0.320621	-0.145341
PST vs TYO:STAT	2.3291e+06	-0.592524	-1.559146	-0.012370	-4.128217
MYY vs MZZ:STAT	1.18718e+07	-24.107724	0.916335	-0.645433	-4.106598
SSO vs VV:STAT	124422	0.083905	0.049682	0.295690	-0.226918
ITOT vs SSO:STAT	164770	0.130921	0.795272	0.463151	-0.304266
FIEG vs FIGY:STAT	1.07257e+06	13.732999	0.772025	0.622786	-1.976664
DBO vs USL:STAT	416135	62.926099	-8.176442	0.429714	-1.780322
DGL vs DGP:STAT	103864	0.013361	0.004819	0.149152	-0.204035
ITOT vs VV:STAT	112848	0.051837	-0.122934	0.249075	-0.187506
IVW vs SPYG:STAT	115525	0.119228	-0.656117	0.236144	-0.222326
SPXL vs SSO:STAT	238715	0.327919	0.009319	0.566030	-0.237074
ITOT vs VONE:STAT	123056	0.061433	0.052047	0.361036	-0.137940
ITOT vs IYY:STAT	128631	0.079021	-0.055598	0.449168	-0.127753
DBS vs SLV:STAT	5.01905e+06	-63.696381	44.402033	-0.460793	-1.000934
VIXM vs VXZ:STAT	112802	0.035903	0.048981	0.266970	-0.185715
VIXM vs VXZ_:STAT	112802	0.035903	0.048981	0.266970	-0.185715
MIDZ vs SMDD:STAT	257042	-3.090283	6.943155	-0.287231	-1.920364
SPXL vs UPRO:STAT	153552	0.247373	0.017791	0.338703	-0.224443
SUTR vs TOR:STAT	233357	0.325331	-0.145199	0.655489	-0.536893
J vs JEC:STAT	111418	0.023625	0.104350	0.326631	-0.108983
CLBS vs NBS:STAT	182259	0.169109	-0.098466	1.083753	-0.129886
RGDO vs TBRA:STAT	141125	0.320119	-0.171850	0.376499	-0.623314
CRED vs USIG:STAT	119816	0.056293	-0.004150	0.348570	-0.104917
PXLV vs SPVM:STAT	113621	0.174279	-1.090341	0.260701	-0.528212
PXLG vs SPGP:STAT	245629	0.176134	1.763549	0.744950	-0.555116
CWEN vs NYLD:STAT	108661	0.023502	-0.023272	0.517435	-0.049130
KEG vs KEGX:STAT	113842	0.035104	-0.004508	0.492754	-0.002390
KEG vs KEGXD:STAT	113842	0.035104	-0.004508	0.492754	-0.002390
MPVD vs MPVDF:STAT	173660	-49.905062	4.163690	-0.552624	-1.221900
SDRL vs SDRL_:STAT	1.3781e+08	2.093266	1.528456	0.041352	-4.274480
SPN vs SPNV:STAT	178331	0.210397	-0.445309	0.710333	-0.251225
SPN vs SPNVD:STAT	178331	0.210397	-0.445309	0.710333	-0.251225

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