

SAMPLE OPTION PRICING

Valuation 6103

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SPY: OCT 17. 2022 EOD

ETF for S&P500

- Option expiry 12.30.22
 - 230 Call options expiring that day
- same maturity options

Computed
from BSM

Solve for sigma that gives market
value of call

Contract Name	Last Trade Date	Strike	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
SPY221230C00150000	2022-06-27 12:02PM EDT	150	241	249.83	251.47	0	-	1	6	257.61%
SPY221230C00160000	2022-06-29 12:26PM EDT	160	220.66	251.73	252.76	0	-	1	3	282.99%
SPY221230C00170000	2022-06-29 9:41AM EDT	170	210.9	237.06	238.58	0	-	1	1	252.01%
SPY221230C00190000	2022-09-23 2:39PM EDT	190	176	181.05	182.66	0	-	1	2	89.64%
SPY221230C00200000	2022-10-17 10:26AM EDT	200	168.31	171.01	172.46	0	-	2	41	82.65%
SPY221230C00210000	2022-05-23 10:44AM EDT	210	185	168.34	169.22	0	-	1	0	112.21%
SPY221230C00230000	2022-06-22 10:47AM EDT	230	150.41	169.1	170.7	0	-	66	43	157.93%
SPY221230C00240000	2022-09-27 1:38PM EDT	240	124.6	131.91	132.49	0	-	1	2	64.12%
SPY221230C00250000	2022-09-22 9:34AM EDT	250	129.02	122.21	122.76	0	-	2	3	60.61%
SPY221230C00260000	2022-10-14 2:26PM EDT	260	100.86	112.34	112.89	0	-	1	31	56.29%
SPY221230C00270000	2022-07-14 2:13PM EDT	270	110.87	158.24	159.01	0	-	3	4	182.34%
SPY221230C00275000	2022-10-05 2:39PM EDT	275	104.94	97.94	98.53	0	-	1	2	51.62%
SPY221230C00280000	2022-09-16 9:38AM EDT	280	107.54	80.42	82.52	0	-	1	16	0.00%
SPY221230C00285000	2022-10-12 11:30AM EDT	285	77.98	88.3	88.9	0	-	1	0	49.16%
SPY221230C00290000	2022-10-10 11:48AM EDT	290	75.24	83.42	84	0	-	1	17	47.03%
SPY221230C00295000	2022-10-14 2:27PM EDT	295	67.7	78.77	79.38	0	-	1	2	45.77%
SPY221230C00300000	2022-10-17 10:12AM EDT	300	71.01	73.85	74.49	0	-	10	125	43.63%
SPY221230C00304000	2022-10-03 12:32PM EDT	304	65.5	70.33	70.95	0	-	2	2	42.96%
SPY221230C00305000	2022-10-10 12:12PM EDT	305	61.31	69.2	69.79	0	-	2	42	42.01%
SPY221230C00310000	2022-10-12 9:33AM EDT	310	53.66	64.7	65.33	0	-	1	0	40.96%
SPY221230C00315000	2022-10-05 2:45PM EDT	315	67.62	60.1	60.69	0	-	10	20	39.33%
SPY221230C00320000	2022-10-17 11:27AM EDT	320	52.16	55.63	56.22	0	-	2	211	38.02%
SPY221230C00321000	2022-10-11 11:40AM EDT	321	47.07	54.74	55.36	0	-	1	1	37.82%

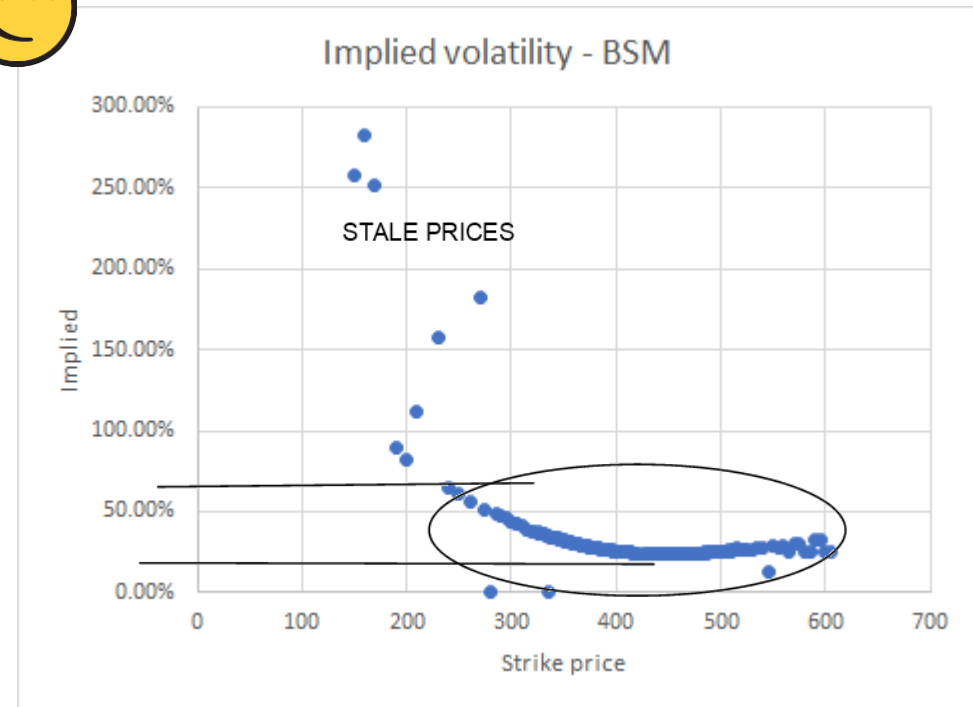
If BSM were true, every call would produce the SAME implied volatility.

✓ S, X, r, T, σ, S
BSM
↓
call for any X

Contract Name	Last Trade Date	Strike	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
SPY221230C00330000	2022-10-17 11:27AM EDT	330	47.07	54.74	55.36	0	-	1	1	37.82%
SPY221230C00340000	2022-10-03 12:32PM EDT	340	65.5	70.33	70.95	0	-	2	2	42.96%
SPY221230C00350000	2022-10-10 12:12PM EDT	350	61.31	69.2	69.79	0	-	2	42	42.01%
SPY221230C00360000	2022-10-12 9:33AM EDT	360	53.66	64.7	65.33	0	-	1	0	40.96%
SPY221230C00370000	2022-10-05 2:45PM EDT	370	67.62	60.1	60.69	0	-	10	20	39.33%
SPY221230C00380000	2022-10-17 11:27AM EDT	380	52.16	55.63	56.22	0	-	2	211	38.02%
SPY221230C00390000	2022-10-11 11:40AM EDT	390	47.07	54.74	55.36	0	-	1	1	37.82%
SPY221230C00400000	2022-10-03 12:32PM EDT	400	65.5	70.33	70.95	0	-	2	2	42.96%
SPY221230C00410000	2022-10-10 12:12PM EDT	410	61.31	69.2	69.79	0	-	2	42	42.01%
SPY221230C00420000	2022-10-12 9:33AM EDT	420	53.66	64.7	65.33	0	-	1	0	40.96%
SPY221230C00430000	2022-10-05 2:45PM EDT	430	67.62	60.1	60.69	0	-	10	20	39.33%
SPY221230C00440000	2022-10-17 11:27AM EDT	440	52.16	55.63	56.22	0	-	2	211	38.02%
SPY221230C00450000	2022-10-11 11:40AM EDT	450	47.07	54.74	55.36	0	-	1	1	37.82%
SPY221230C00460000	2022-10-03 12:32PM EDT	460	65.5	70.33	70.95	0	-	2	2	42.96%
SPY221230C00470000	2022-10-10 12:12PM EDT	470	61.31	69.2	69.79	0	-	2	42	42.01%
SPY221230C00480000	2022-10-12 9:33AM EDT	480	53.66	64.7	65.33	0	-	1	0	40.96%
SPY221230C00490000	2022-10-05 2:45PM EDT	490	67.62	60.1	60.69	0	-	10	20	39.33%
SPY221230C00500000	2022-10-17 11:27AM EDT	500	52.16	55.63	56.22	0	-	2	211	38.02%
SPY221230C00510000	2022-10-11 11:40AM EDT	510	47.07	54.74	55.36	0	-	1	1	37.82%
SPY221230C00520000	2022-10-03 12:32PM EDT	520	65.5	70.33	70.95	0	-	2	2	42.96%
SPY221230C00530000	2022-10-10 12:12PM EDT	530	61.31	69.2	69.79	0	-	2	42	42.01%
SPY221230C00540000	2022-10-12 9:33AM EDT	540	53.66	64.7	65.33	0	-	1	0	40.96%
SPY221230C00550000	2022-10-05 2:45PM EDT	550	67.62	60.1	60.69	0	-	10	20	39.33%
SPY221230C00560000	2022-10-17 11:27AM EDT	560	52.16	55.63	56.22	0	-	2	211	38.02%
SPY221230C00570000	2022-10-11 11:40AM EDT	570	47.07	54.74	55.36	0	-	1	1	37.82%
SPY221230C00580000	2022-10-03 12:32PM EDT	580	65.5	70.33	70.95	0	-	2	2	42.96%
SPY221230C00590000	2022-10-10 12:12PM EDT	590	61.31	69.2	69.79	0	-	2	42	42.01%
SPY221230C00600000	2022-10-12 9:33AM EDT	600	53.66	64.7	65.33	0	-	1	0	40.96%
SPY221230C00610000	2022-10-05 2:45PM EDT	610	67.62	60.1	60.69	0	-	10	20	39.33%
SPY221230C00620000	2022-10-17 11:27AM EDT	620	52.16	55.63	56.22	0	-	2	211	38.02%
SPY221230C00630000	2022-10-11 11:40AM EDT	630	47.07	54.74	55.36	0	-	1	1	37.82%
SPY221230C00640000	2022-10-03 12:32PM EDT	640	65.5	70.33	70.95	0	-	2	2	42.96%
SPY221230C00650000	2022-10-10 12:12PM EDT	650	61.31	69.2	69.79	0	-	2	42	42.01%
SPY221230C00660000	2022-10-12 9:33AM EDT	660	53.66	64.7	65.33	0	-	1	0	40.96%
SPY221230C00670000	2022-10-05 2:45PM EDT	670	67.62	60.1	60.69	0	-	10	20	39.33%
SPY221230C00680000	2022-10-17 11:27AM EDT	680	52.16	55.63	56.22	0	-	2	211	38.02%
SPY221230C00690000	2022-10-11 11:40AM EDT	690	47.07	54.74	55.36	0	-	1	1	37.82%
SPY221230C00700000	2022-10-03 12:32PM EDT	700	65.5	70.33	70.95	0	-	2	2	42.96%
SPY221230C00710000	2022-10-10 12:12PM EDT	710	61.31	69.2	69.79	0	-	2	42	42.01%
SPY221230C00720000	2022-10-12 9:33AM EDT	720	53.66	64.7	65.33	0	-	1	0	40.96%
SPY221230C00730000	2022-10-05 2:45PM EDT	730	67.62	60.1	60.69	0	-	10	20	39.33%
SPY221230C00740000	2022-10-17 11:27AM EDT	740	52.16	55.63	56.22	0	-	2	211	38.02%
SPY221230C00750000	2022-10-11 11:40AM EDT	750	47.07	54.74	55.36	0	-	1	1	37.82%
SPY221230C00760000	2022-10-03 12:32PM EDT	760	65.5	70.33	70.95	0	-	2	2	42.96%
SPY221230C00770000	2022-10-10 12:12PM EDT	770	61.31	69.2	69.79	0	-	2	42	42.01%
SPY221230C00780000	2022-10-12 9:33AM EDT	780	53.66	64.7	65.33	0	-	1	0	40.96%
SPY221230C00790000	2022-10-05 2:45PM EDT	790	67.62	60.1	60.69	0	-	10	20	39.33%
SPY221230C00800000	2022-10-17 11:27AM EDT	800	52.16	55.63	56.22	0	-	2	211	38.02%
SPY221230C00810000	2022-10-11 11:40AM EDT	810	47.07	54.74	55.36	0	-	1	1	37.82%
SPY221230C00820000	2022-10-03 12:32PM EDT	820	65.5	70.33	70.95	0	-	2	2	42.96%
SPY221230C00830000	2022-10-10 12:12PM EDT	830	61.31	69.2	69.79	0	-	2	42	42.01%
SPY221230C00840000	2022-10-12 9:33AM EDT	840	53.66	64.7	65.33	0	-	1	0	40.96%
SPY221230C00850000	2022-10-05 2:45PM EDT	850	67.62	60.1	60.69	0	-	10	20	39.33%
SPY221230C00860000	2022-10-17 11:27AM EDT	860	52.16	55.63	56.22	0	-	2	211	38.02%
SPY221230C00870000	2022-10-11 11:40AM EDT	870	47.07	54.74	55.36	0	-	1	1	37.82%
SPY221230C00880000	2022-10-03 12:32PM EDT	880	65.5	70.33	70.95	0	-	2	2	42.96%
SPY221230C00890000	2022-10-10 12:12PM EDT	890	61.31	69.2	69.79	0	-	2	42	42.01%
SPY221230C00900000	2022-10-12 9:33AM EDT	900	53.66	64.7	65.33	0	-	1	0	40.96%
SPY221230C00910000	2022-10-05 2:45PM EDT	910	67.62	60.1	60.69	0	-	10	20	39.33%
SPY221230C00920000	2022-10-17 11:27AM EDT	920	52.16	55.63	56.22	0	-	2	211	38.02%
SPY221230C00930000	2022-10-11 11:40AM EDT	930	47.07	54.74	55.36	0	-	1	1	37.82%
SPY221230C00940000	2022-10-03 12:32PM EDT	940	65.5	70.33	70.95	0	-	2	2	42.96%
SPY221230C00950000	2022-10-10 12:12PM EDT	950	61.31	69.2	69.79	0	-	2	42	42.01%
SPY221230C00960000	2022-10-12 9:33AM EDT	960	53.66	64.7	65.33	0	-	1	0	40.96%
SPY221230C00970000	2022-10-05 2:45PM EDT	970	67.62	60.1	60.69	0	-	10	20	39.33%
SPY221230C00980000	2022-10-17 11:27AM EDT	980	52.16	55.63	56.22	0	-	2	211	38.02%
SPY221230C00990000	2022-10-11 11:40AM EDT	990	47.07	54.74	55.36	0	-	1	1	37.82%

INFORMATION IN THE “SMILE”

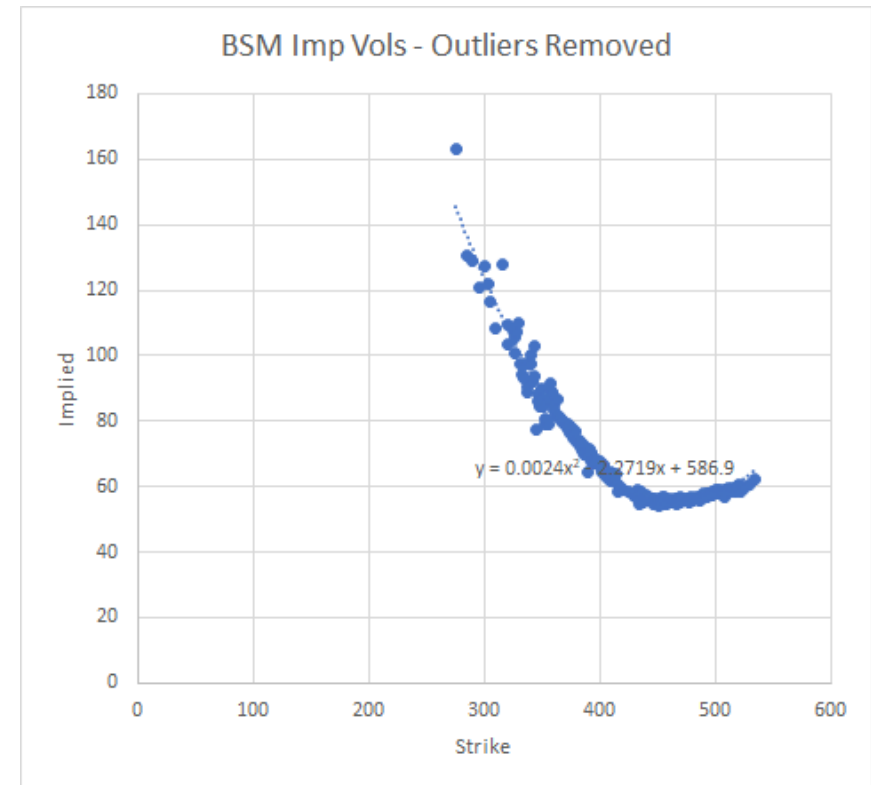
- Implied volatility
 - Using the market price of the option, what volatility level causes the BSM price of the option to equal the market price?
 - This is the implied vol
- Implied vol is to options what YTM is to bonds
 - It is a pricing tool, but some of the meaning is lost since the implied vol differs for every strike
- Often called a “vol smile”
 - Sometimes a smirk!
- The equity option smirk suggests a negatively-skewed risk-neutral distribution of the stock price at expiration



Proof that BSM is mis-specified.oo

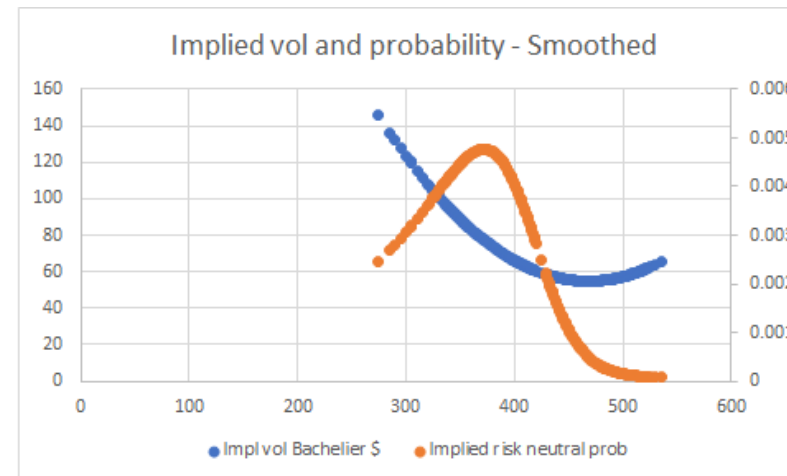
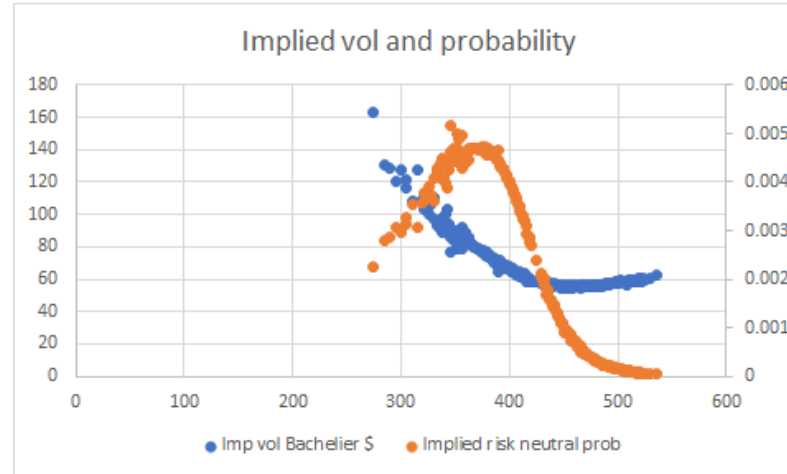
PROBLEMS WITH BSM & EQUITY OPTIONS

- BSM assumes future stock price distributions are lognormal, therefore positively skewed
 - Empirically, these distributions are negatively skewed
- The lognormal distribution has relatively thin tails
 - In practice, the kurtosis of the market returns is greater, implying thicker tails
- BSM implicitly assumes stock prices are positive
 - However, prices can reach zero in practice
- If BSM were true, then implied vol would be the same for all strikes
 - The smile indicates that the model is incorrectly specified



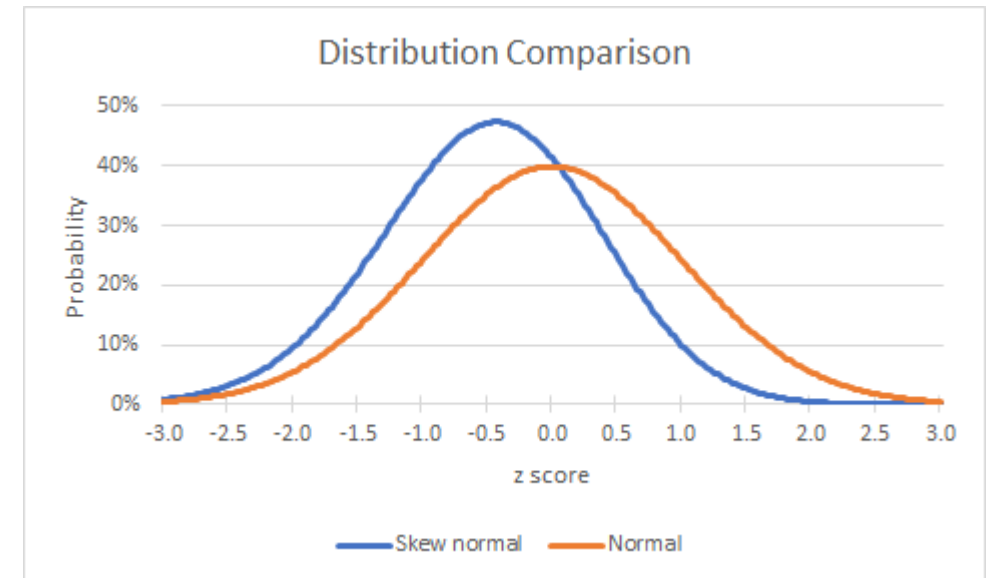
WHAT IF RETURNS ARE NORMAL?

- Use the Bachelier model to compute implied dollar volatilities
- Reduces the skew problem
 - But not really enough
- Does not affect the kurtosis problem
- Best to use a negatively skewed distribution



CORRECTING FOR SKEWNESS & KURTOSIS

- Use the skew-normal distribution to add skewness and kurtosis
- Skewness parameter
 - $\text{Lambda} = -0.93$
- Kurtosis parameter
 - $\text{Gamma} = 0.19$
- Implications
 - Two extra parameters
 - Constant across strikes
 - Better specification
 - Better hedge ratios?



GENERAL EUROPEAN OPTION PRICING FORMULA

- Determine the appropriate risk-neutral probability distribution of future underlying values
 - Substitute the risk-neutral mean for the natural mean
 - This is SD/B , where D is a dividend factor and B is a discount factor
- Calculate or simulate expected option payout at maturity using this probability distribution
- Discount at the risk-free interest rate to find the present value

$\mu \leftarrow \mu^* = S(1+r)$ → RISK NEUTRAL PROBABILITY

$$C_0 = e^{-rT} E(C_T | S_0)$$
$$C_0 = e^{-rT} \int_{-\infty}^{\infty} \max(S_T - X, 0) f^*(S_T) dS_T$$

You can value European call for any probability distribution associated with the underlying stock