Risk Model in details

- Statistical Models
- Fundamental Models (BARRA)
- Macro Models

Estimation example: CAPM model

$E(R_i) = R_f + B_i * (E(R_m) - R_f) + E(Alpha)$, $R_f = 0$

$B_i = Cov(R_i, R_m) / Var(R_m)$

Intuition:

R(i,t) - stock return for stock “i”

$R(i,t) = (Price(i,t+1) - Price(i,t))/Price(i,t)$

Question: How to define market?

- Market cap weighted S&P500
- Equal weighted index
- Statistical methodology

Estimation example (1): CAPM

Construct index equal weighted index

MarketReturn(t) = 1/N Sum(R(i,t), i=1...N)

Run regression to estimate betas:

R(i,t) = Bi*MarketReturn + e(i,t)

e(i,t) ~ N(0, Sigma(i))

Use OLS to estimate Bi

Other choices for market proxy are possible

Determining market structure from the data

PCA (Principal Component Analysis) approach

Consider time window: $t=0, \ldots. T$ for a universe of N stocks

$R(i,t)$ ----> $N \times T$ matrix

Average return $A(i) = \langle R(i,t) \rangle = 1/T \; \mathrm{Sum}(R(i,t), t=1...T)$

Volatility $S(i) = 1/(T-1) \; \mathrm{Sum}((R(i,t)-A(i))^2 , t = 1...T)$

Define standardized return $Y(i,t) = R(i,t) / S(i)$

Correlation Matrix

$G(i,j) = 1/(T-1)\ \mathrm{Sum}(Y(i,t)*Y(j,t)\ ,\ t=1...T)$

$\mathrm{Rank}(G) <= \min(N, T)$

Eigenvalues: $L1 > L2 >= L3 >= \dots.\ LN > 0$

Eigenvectors: $F1, F2, \dots..$

$G\ F1 = L1\ F1$ etc ….

Variance explained by “M” eigenvectors: $1/N\ \mathrm{Sum}(Li,\ i=1...M)$

Spectral Representation of Correlation Matrix

G ---> N x N matrix

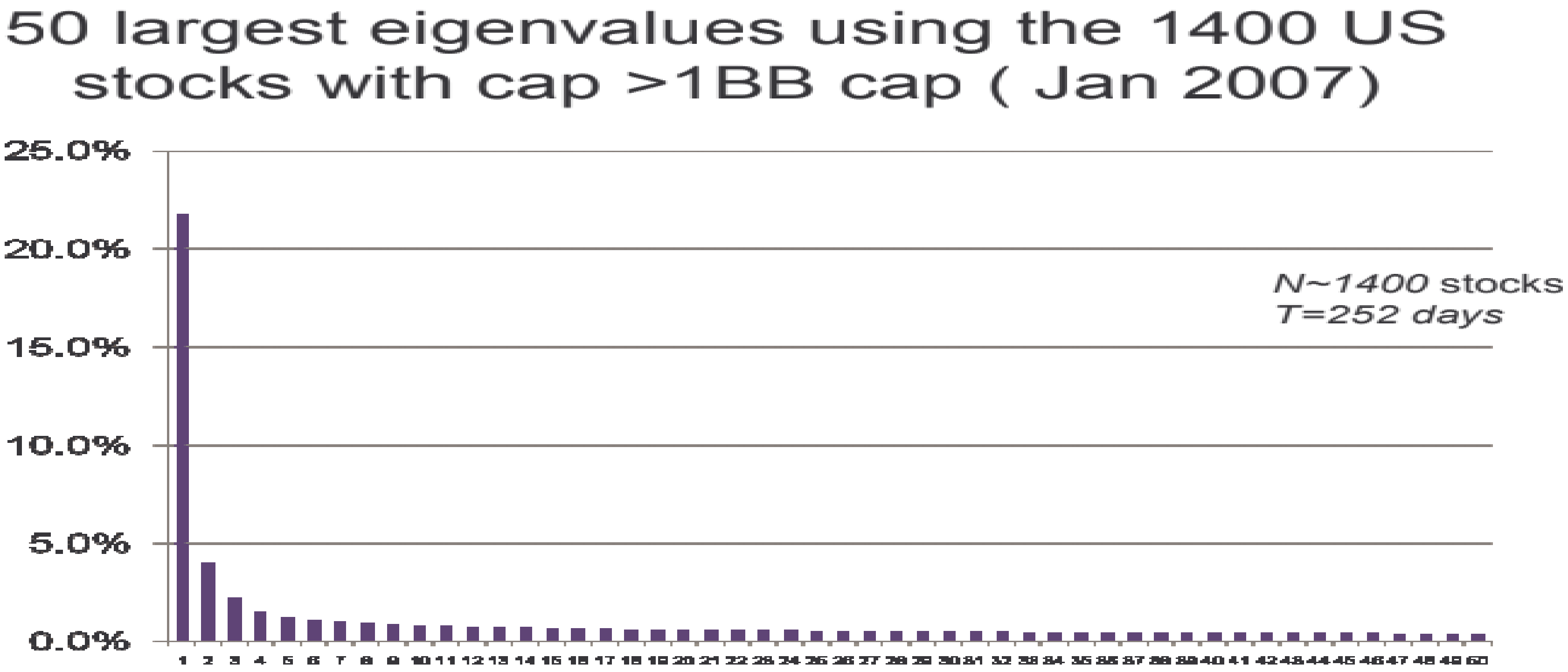
F(i,k) ---> eigenvector corresponding to eigenvalue L(k)

G(i,j) = Sum(F(i,k)*F(j,k)*L(k), k=1...Rank(G))

A(i, k) ----> factor loading of risk model “i” - stock index, “k” - factor index

A(i, k) = F(i, k) * sqrt(L(k)) / S(i)

Example


50 largest eigenvalues using the 1400 US stocks with cap >1BB cap (Jan 2007)
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
N~1400 stocks
T=252 days

Exercise

Derive 1-factor model using PCA

Compare it with simple minded approach described above

Problems with PCA approach

- Curse of dimensionality
- Low dimensional risk factors (eg. industries) can’t be determined
- Latent factors

Basic structure of Statarb models

R(i,t) = Return Attributed to Risk (aka systematic returns) + Alpha + Unexplained Residual

Return Attributed to Risk Model = Sum(F(i,k) * m(k,t), k = 1…. NRISKFACTORS)

m(k,t) - unpredictable, for example IID variables drawn from N(0, Sm(k))

Sm(k) - volatility of risk factor “k”

Linear Alpha Models

Features (aka alpha factors) X(i,t,p)

"i" - stock label i = 1….N

"t" - time label

"p" - feature index

Alpha(i,t) = Sum(X(i,t,p)*w(p), p =1 …. NumberOfFeatures)

Example of Alpha Factors/Features

Philosophy (Predictability in the markets exists for 2 reasons)::

- Information is processed at different speed by different players (fundamental factors)
- Providing (liquidity) service to other players in the market

Examples of Alpha Factors (2)

Fundamental factors:

- Earning surprise: (ActualEarning - EpsConsensus) / EpsConcensus
- Changes in analyst recommendations: RecValue - PrevRecValue

  RecValue = 2 (Strong Buy), 1 (Buy), 0 (Neutral) -1 (Sell) -2 (Strong Sell)

- Changes in Eps estimates: (EpsConsensus - EpsConsensusPrev) / (EpsConsensus + EpsConsensusPrev)

Examples of Alpha Factors (3)

Technical factors:

- Mean reversion factor: - R(i,t-1)
- Volume spikes: V(i,t) / MovingAverage(V(i,t))

Improving Alpha Factors

- Control outliers: ranking, winsorization
- Orthogonal projection relative to risk factors

Combining different Alpha Factors

- Use Ordinary Linear Regression to find optimal weights for each factor
- Advanced machine learning methods

Risk Factors versus Alpha Factors

The distinction is euphemeral, it depends on your ability to forecast future value of the feature/risk factor return

The most known example is “momentum”

Constructing momentum factor:

Mmnt(i,t) = Rank((P(i,t) - P(i,t-252))/P(i,t-252))

Mmnt(i,t) ---> Mmnt(i,t) - SectorAverage(Mmnt(i,t))

Optimal Trading (Optimization)

Use inputs: Risk Model/Alpha Model/Trading Cost Model

- Maximize return for a given level of risk
- Impose common sense constraints: a) Position limits b) Trading limits c) Sector/Industry exposures d) Other constraints

Optimization Example

- TradingCost = |n(i,t+1) - n(i,t)|*P(i,t)*RelativeSpread(i)
- Alpha1(i, t) = 1d mean reversion
- Alpha2(i,t) = changes in analyst recommendations
- Risk Model: 1 factor CAPM

CombinedAlpha(i,t) = Alpha1(i, t) + Alpha2(i,t)

R(i,t) =   Beta(i)*market(i,t) + CombinedAlpha(i, t) + Residual(i, t)

Optimization (2)

Given previous weights w0(i), find new weights w(i) such that

Max(w(i)*CombinedAlpha(i,t) - TradingCost(|w(i) - w0(i)|)

Sum(w(i)*Beta(i)) = 0

Extra constraints: Sum(w(i)) = 0 market neutrality

w(i) < Position Limit

|w(i) - w0(i) | < Trading Constraint