

Dispersion

A Guide for the Clueless

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INTRODUCTION

Once upon a time dispersion trading desks used to be the kings (and queens) of volatility trading in the equity arena (if we ignore the 35 mio USD short vega position by LTCM).

Dispersion desks can handle significant volatility risks in the same way that a basket desk can handle extremely large deltas per instrument or a cap/floor vs. swaption trader can handle extremely large volatility risks per underlying. As a matter of fact, a dispersion trader is essentially a cap/floor vs. swaptions trader, albeit somewhat less structured. Dispersion traders can come into a single stock and sometimes sell significant vegas (read: millions) before anyone knows what is happening.

Dispersion trading was a quite profitable trading activity into the early 2000s.

The reason dispersion trading was quite profitable and relatively risk less up until then is that the market showed a long dispersion bias i.e. realized dispersion was above implied dispersion on average.

Dispersion trading is also known under the moronic pseudonym of volatility arbitrage. The only time there ever was a true arbitrage was in the early 1990s in Europe. Here we saw implied correlations above 1.0, which, needless to say, are sort of a sell. Nobody capitalized on this arbitrage opportunity. Why? Because if you can make money by doing Reversals/Conversions and Jelly Rolls, why would you bother with something more complicated like dispersion?

Nowadays trading dispersion has become less a P/L generator in itself but rather as a position risk management and/or liquidity enhancement tool.

As dispersion exposes a book to correlation, it is a way to hedge exposure certain correlation products such as rainbow options (Mountain Range options in particular). Also, the strategy can be packaged and sold to various investors who have need for that sort of strategy either as a hedge to some other activity or simply as a view on dispersion levels.

Dispersion allows a trader to run very large gammas on an individual underlying level... and hence is every delta trader's wet dream.

BACKGROUND

So what's all this about dispersion anyway? Why is this particular type of trade called dispersion and not, say, basket/constituent volatility spread? Well, dispersion sounds more cryptic, and hence it's easier to impress people with it. Period. And it's Greek in origin, making you sound sophisticated.

Apart from that, what drives this trade is how the constituents of a basket *disperse* themselves around a mean (of zero).

Every major equity derivatives player on the sell side now has a dispersion desk (or had one) and there are plenty of buy-side shops who try to run the strategy. There are even several funds dedicated to this activity.

BASKET OPTIONS: A REVIEW

For those less gifted at memorizing formulas, let's quickly review the make-up of a basket:

$$S_{Basket} = \sum_{i=1}^n w_i S_i \text{ and it's corresponding variance:}$$

$$\sigma_{Basket}^2 = \sum_{i=1}^n w_i^2 \sigma_i^2 + 2 \sum_{i=1}^n \sum_{j=i+1}^n w_i w_j \sigma_i \sigma_j \rho_{ij}$$

Having dispensed with that, we can now look at a basket option. Or an index option, which happens to be a basket option. (Yes, this is pretty obvious, however a lot of people don't realize this until you actually point it out, so, not knowing my audience....).

Now that we know how a index option's volatility (and hence it's price) is formed, one sort of has to wonder if there is maybe some insight to be gained by looking at the individual volatilities and simply calculate the volatility of the index? Sounds good so, we need a bunch of σ_i and... ah shucks... a correlation factor. You could of course call the good folks at Capital Structure Demolition LLC all the time and ask me what it is, but I'm sort of busy, so please don't.

Instead, let's see what happens if we put $\rho=1$.

$$\text{Average Basket Variance } \bar{\sigma}_{Index}^2 = \sum_{i=1}^n w_i \sigma_i^2$$

We get the variance of the basket, which is simply an addition of the variances for the index.

So that would give us an upper bound on our index variance. Which is nifty, because around 1992/93 the European index volatility was above this, so we actually had a nice arbitrage there. As already noted in the preface, not many people did this, most were busy computing Call-Put Parity violations.

But, we can utilize this number and compare it to the actual index vol. Depending on how wide the spread between the two is, we might be able to do some relative value trading around it. This gives us:

$$\text{Dispersion Spread } D = \sqrt{\sum_{i=1}^n w_i \sigma_i^2 - \sigma_{Index}^2}$$

So, now that we have this dispersion spread wriggling around between it's upper ($D=\text{Average Basket Vol}$) and lower bound ($D=0$). Once we have established where D likes to hang around, it becomes natural to put on a spread position.

WHAT DRIVES DISPERSION?

Without wanting to go into the technical details (I'm a much more intuitive and visual person), we shall look at the two legs of the spread strategy from a volatility perspective. Rather than decompose the trade into some highfalutin math, let's look at the simple side of things. Say, we are long dispersion, so we are:

- long the volatility of each constituent
- short index volatility

So, that's easy enough, long and short convexity is something we can work with. So what do we want? Unless you are in a rather self-destructive mood, the following would be desirable:

- lots of volatility on the constituents
- no volatility on the index

Say we define volatility as the daily squared rate of return: r^2 . This gives us our realized average basket volatility for the constituent side of the trade:

Realized Average Basket Volatility

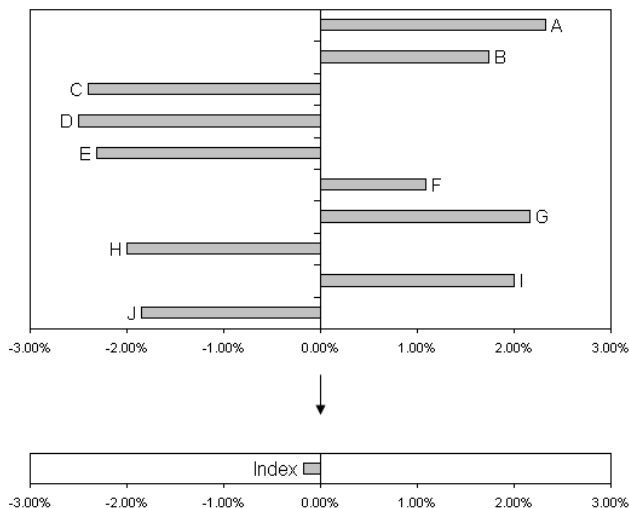
$$\bar{\sigma}_{Realized} = \sqrt{\sum_{i=1}^n w_i r_i^2}$$

And, if we recall that the index is calculated as

$$S_{Index} = \sum_{i=1}^n w_i S_i \text{ it follows that } r_{Index} = \sum_{i=1}^n w_i r_i$$

As we are short vol on the index, we'd like r_{Index} to be as small as possible while keeping the individual r_i at high absolute values. How does that work? Well take a look at the following graphs, which show the rate of change for a day for an index with 10 constituents (A-J) with equal weightings.

day with low correlation

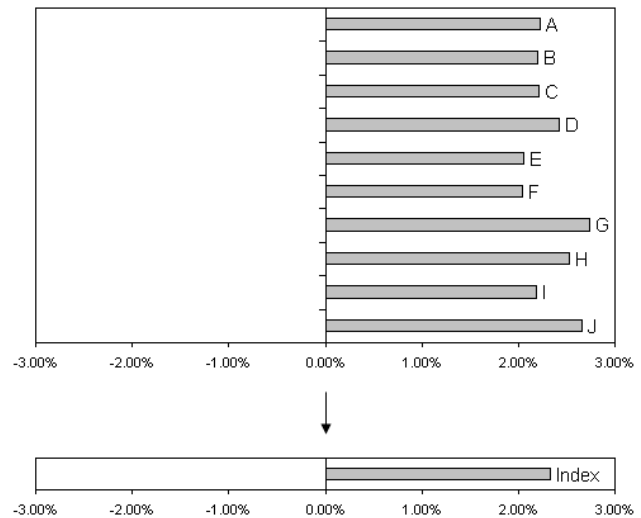


This graph shows a day where all the moves of the constituents seem to be uncorrelated with each other. In other words, they tend to cancel each other out. With our position, we are hopefully re-balancing our deltas on the individual stocks, capturing the volatility there, but we don't have re-balance our deltas on the index (and hence incurring gamma related losses), as it has barely moved.

So, we earn gamma profits in A-J, and earn theta in the index, which we can use to subsidize the theta we are paying on the stocks. The measured dispersion for this day is 33%.

Free lunch? Not quite... what about a day like this?

day with high correlation



The dispersion here is 6%. Lets see... we hit pay dirt on the stock side of things. Uh, but the index side... uh... we are losing a lot of money, almost twice as much than we earn in theta.

So we can see, the way the stocks move together has real impact on the P/L of this strategy. This is what realized dispersion is about: *the co-movement of the constituents, the way they disperse around the mean (of 0).*

EXPOSURE TO CORRELATION

So, what's this about correlation and hedging correlation and all that?

We assumed in the calculation of the average basket volatility that correlation is 1. Of course it usually isn't, unless all hell breaks loose.

We recall our formula for basket variance and assume that the correlation for all pairs ρ_{ij} is the same, giving us an average correlation for the entire basket. This allows us to solve for this average correlation if we have the volatilities for all constituents and the index.

$$\text{Average Correlation } \bar{\rho} = \frac{\sigma_{Index}^2 - \sum_{i=1}^n w_i^2 \sigma_i^2}{2 \sum_{i=1}^n \sum_{j=i+1}^n w_i w_j \sigma_i \sigma_j}$$

We can compute this for both types of volatilities, both realized and implied. It's easy to see that if we are long dispersion (remember, that means we sell the index and buy the stock vols) we are short correlation and vice versa. So, by establishing a dispersion position we take a position in a bunch of funky greeks, one of which we at Capital Structure Demolition LLC call *zeta*.

LIQUIDITY ENHANCEMENT WITH DISPERSION

So, if you have the likes of Allianz and AIG asking you for markets in single stock options and they happen to be for about five times the average volume in those things, you can either tell them to get lost or you can take down the paper for a bit of edge and do the (more liquid) index side against it, effectively establishing the beginning of a dispersion position. (either long or short). Now you sold 1 million MSFT vega, but you have bought some amount of NDX vega to temporarily stabilize your position. If you can get some other constituents on, you got a nice little dispersion trade going. You can now ease out of this over time, minimizing market impact or show the stock part to your friends and pretend you're the biggest swinging dick on the Street.

Usually this type of trading is less considerate of getting a representative basket going, but works with more traditional hedge parameter computation, such as volatility surface betas. Nevertheless, it is a form of dispersion. We will come back to this point in a minute.

SEX WITH SHEEP

It's good. Give it a try one day.

MONSTER GAMMA

As mentioned in the preface, because you're effectively creating opposite positions in highly correlated greeks, some of the numbers can get pretty large pretty quick... in terms of individual underlyings. It is not uncommon for a dispersion desk to fork out a couple of million as theta in the days leading up to expiration. So, if you have a bit of a delta fetish, that huge gamma can come in very handy to smack the chickens around and make Colonel Sanders look like a chicken rights advocate.

LONG DISPERSION BIAS

As mentioned earlier, there has been (not so much at the moment) a bias towards being long dispersion, meaning if

you were long dispersion, on average, you made more (lost less) money on your long constituent vol than you lost (made) on your short index vol.

Amongst some of the reasons are:

- Demand for index protection: Institutionals will usually buy protection through index vol, hence keeping the index surface higher relative to the average basket vol surface.
- Supply for single stocks volatility: On individual equities, there are proportionally more premium sellers than there are on the index side.
- Index liquidity: As most people have noticed, index futures are pretty liquid... sort of like water, whereas single stocks are more like snot. So, given the fact that index arbitrage bands are wide and that the single stocks can get out of line with the index somewhat, it's easy to see that there is extra volatility there (in terms of price swings) that simply doesn't exist in the index.
- Event risk: As always, people like underestimating the extremes and fun days such as bankruptcies, corporate scandals and mergers/take-overs are pay dirt for long dispersion.

PROBLEMS WITH BEING SHORT DISPERSION

Short dispersion (also called a "Chinese" position) is sort of a hairy thing. As you would be selling the constituent vol and buying the index vol, you expose yourself to a lot of short equity gamma. You'd be tempted to think that you have your long gamma on the index side against it and all is well. Thing is, there are lots of little details (the devil is in the details, as always) that make that trade sort of hard to hold. As in index arbitrage, company bankruptcies and take-over/mergers are some of the things that can cause your book to implode.

Also, you are going against the bias (see above), but this isn't a problem if you can get the trade on at high enough implied dispersion levels.

Of course, this doesn't mean that you should never be short dispersion, merely that you should well be aware of the risks and weigh them against a potential reward.

PROXY HEDGING

Most people don't even try to hold a full basket of constituent vol. Nowadays, that's a disaster waiting to happen. The pricing has become too tight to let you walk around with a truckload of equity vol simply because you want to be running a "complete" dispersion trade.

So, whether you are using dispersion as a P/L driver or whether you simply want to use it to enhance liquidity, you will (or rather should) look at proxy hedging (unless you are

still trading in 1997... in which case abandon everything and simply buy Nasdaq-100 Futures and hold them until the 24th of March 2000).

What does “proxy hedging” mean in this context? As there is only one (usually) index that you are focusing on (although a good dispersion book merges multiple indices and/or baskets), we can’t really proxy it. However, we can proxy the various constituents. We can either use other constituents from the same basket or external instruments that we happen to like.

This is very similar to the way program trading desks optimize their sub-baskets, in case that is more meaningful to you.

What are some ways to proxy the volatility of a constituent?

How about:

- By fundamental similarity
- By cheapness/richness
- By regression
- Other

Fundamentally similar constituents are stocks from the same sector or with similar exposure. So, if you have two airlines in your basket, why not only choose one (the one that is more attractive (from a future volatility point of view... not because of the comfy first class seats)?

If you are into relative value oriented trading, you can simply choose to substitute those constituents you consider most expensive with those you think are a bargain. This is the beauty of incomplete replication: Plenty of room for improvement on the pricing. How do you determine richness/cheapness? Please... this is a guide, not a manual.

Now, once you made a decision on what you want to replace with what, you will need a way to bring the numbers together in a meaningful way. If you have half a million USD vega on some airline, but you don’t want that particular airlines volatility, how to decide how much vega of something else to buy? Here’s where the proxy hedge ratios come in.

Regression (the usual tool to relate two supposedly related variables) produces proxy hedge ratios to allow you to substitute one constituent for another.

If you are looking to replace the vega of one constituent with the vega of another, several possibilities come to mind:

- Raw Vega:

$$vega_i = \frac{dV}{d\sigma_i}$$

Simply aggregate via standard vega, this obviously ignores the fact that volatilities are imperfectly correlated and/or have different volatilities themselves. Not a good idea, unless you have absolutely nothing else to work with.

- Scaled Vega:

$$Scaled\ Vega_i = vega_i \cdot \sigma_i$$

At least by doing this, we do justice to the fact that things do have different volatilities.

- Factor Vega:

$$factor\ vega_i = \frac{dV}{d\sigma_i} \cdot \frac{d\sigma_i}{dFactor} = vega_i \cdot \beta_{\sigma_i, Factor}$$

This is the preferred way of working out proxy hedge ratios. It does recognize that the volatilities themselves vary with different volatilities (though imperfectly correlated). The factor can be anything... the implied volatility of another stock, for example or the implied volatility of an index. The regression against index volatility makes sense, in that it gives us a broad based reference to measure against. You should now see the similarities to index arbitrage (with non-replicating baskets).

The observant reader might question: Can I proxy hedge the deltas for each constituent, Δ_i ? After all, the index side is so much more liquid. The answer is, yes, but it depends on the circumstances. As a general rule of thumb, the higher the current realized basket correlation, the more you can resort to rebalancing your deltas with the more liquid index.

SUMMARY

Dispersion allows for the mixing of index (basket) volatility and constituent member volatility.

It’s a fairly complex trade where a single book can consist of hundreds (thousands) of positions. For this reason, one needs to have a certain amount of back-office and risk management capability to make it worth one’s while. Also it helps to have decent access to (internal) market makers, as trying to profit from dispersion as a market taker where you are giving up one or two percent in annualized dispersion terms is hard.

There used to be a fairly strong bias towards the long side of dispersion (realized dispersion > implied dispersion, on average). This has eroded over the recent years and, as with

index arbitrage and cap/floor/swaption trading, the money is not being made by running complete replication books, but rather through relative value trading, sub-basket selection and effective forecasting.

Dispersion desks usually have large risk limits as they need room to maneuver. This allows the desks to effectively trade volatility as an outright position as well (unlike basket desks that will not really run around naked). An interesting observation is that implied equity dispersion levels are strongly correlated with general implied

volatility levels. So, for cost and operational reasons, if you want to speculate on the level of implied dispersion, you might want to consider buying or selling index volatility instead, which results in similar P/L developments... and less of a headache.

All in all, there are easier and less complex ways of making more money with less effort. Nevertheless, having a dispersion book can be useful given the right setup.

Thanks for reading all this. I honestly don't know how you managed, I fell asleep while proof-reading half-way through.