



# FIXED INCOME SECURITIES

---

FRE : 6411

Sassan Alizadeh, PhD

Tandon School of Engineering

NYU

2023



# Complete Market

---

- Consider two period one factor economy.
- In both period 0 and 1 probability of jump to state "u" from any current state is 0.5
  - $\pi(u|0) = 0.5$  ,  $\pi(uu|s_1 = u) = 0.5$  &  $\pi(du|s_1 = d) = 0.5$
- In period 0, there is a risky asset "A" with price  $P_A(0) = 100$  and price evolution of this asset in periods 1 and 2 are:
  - $P_A(1, u) = 110$  &  $P_A(1, d) = 90$
  - $P_A(2, uu) = 120$  &  $P_A(2, ud) = 100$
  - $P_A(2, du) = 100$  &  $P_A(2, dd) = 80$
- In period 0, since asset "A" is the only traded asset, therefore the market is not complete.



# Complete Market

---

- To complete the market the Option Exchange decides to offer a call option on asset "A", expiring in period 1, with strike price of 100.  $C_A(0, T = 1, k = 100)$
- Market traded price of this option in period 0 is  $PC_A(0, T = 1, k =$



# Complete Market

---

- Question 2:
- Using risky asset "A" and the call on "A",  $C_A(0, T = 1, k = 100)$ . Find a mimicking portfolio of an asset that pay \$1 in either state "u" & "d". (i.e. a risky free asset). What is the price of this asset in period 0 ?
- What is the implicit risk free rate in period 0 ?
- Find are the *risk-neutral* probabilities  $\tilde{\pi}_0(u)$  &  $\tilde{\pi}_0(d)$  ?
- What are the actual expected rate of return of asset "A" and and the call on "A",  $C_A(0, T = 1, k = 100)$ 
  - $E_0 \left[ \frac{P_A(s_1)}{P_A(0)} \right] =$   $E_0 \left[ \frac{PC_A(1, T=1, k=100, s_1)}{PC_A(0, T=1, k=100)} \right] =$



# Complete Market

---

- What are the *risk adjusted* or *risk neutral* expected rate of return of asset "A" and the call on "A",  $C_A(0, T = 1, K =$