SAMPLE OPTION PRICING

Valuation 6103

Prof David Shimko

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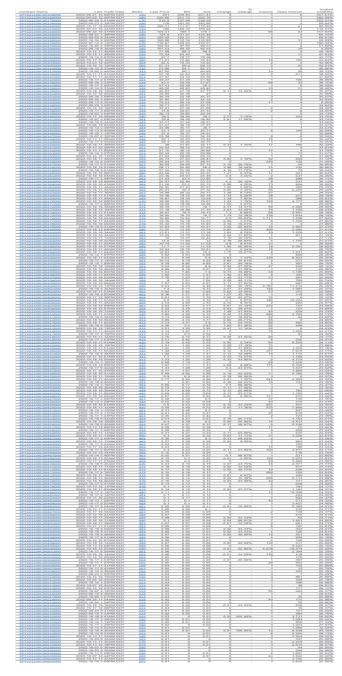
SPY: OCT 17. 2022 EOD

ETF for S&P500

- Option expiry 12.30.22
  - 230 Call options expiring that day
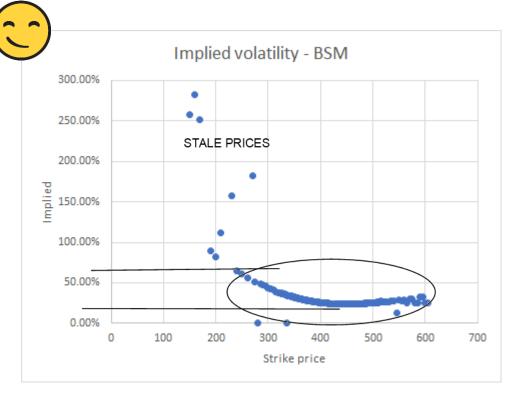
same maturity options

Computed from BSM

Solve for sigma that gives market value of call

If BSM were true, every call would produce the SAME implied volatility.

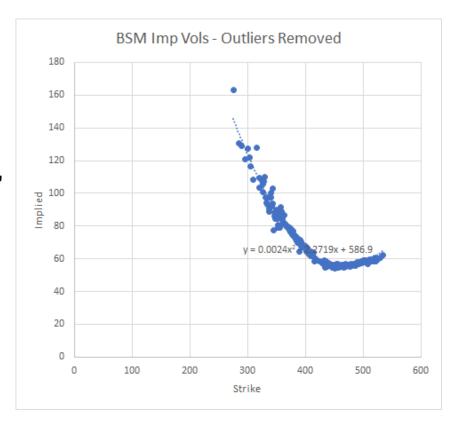| Contract Name	Last Trade Date	Strike	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
SPY221230C00150000	2022-06-27 12:02PM EDT	150	241	249.83	251.47	0	-	1	6	257.61%
SPY221230C00160000	2022-06-29 12:26PM EDT	160	220.66	251.73	252.76	0	-	1	3	282.99%
SPY221230C00170000	2022-06-29 9:41AM EDT	170	210.9	237.06	238.58	0	-	1	1	252.01%
SPY221230C00190000	2022-09-23 2:39PM EDT	190	176	181.05	182.66	0	-	1	2	89.64%
SPY221230C00200000	2022-10-17 10:26AM EDT	200	168.31	171.01	172.46	0	-	2	41	82.65%
SPY221230C00210000	2022-05-23 10:44AM EDT	210	185	168.34	169.22	0	-	1	0	112.21%
SPY221230C00230000	2022-06-22 10:47AM EDT	230	150.41	169.1	170.7	0	-	66	43	157.93%
SPY221230C00240000	2022-09-27 1:38PM EDT	240	124.6	131.91	132.49	0	-	1	2	64.12%
SPY221230C00250000	2022-09-22 9:34AM EDT	250	129.02	122.21	122.76	0	-	2	3	60.61%
SPY221230C00260000	2022-10-14 2:26PM EDT	260	100.86	112.34	112.89	0	-	1	31	56.29%
SPY221230C00270000	2022-07-14 2:13PM EDT	270	110.87	158.24	159.01	0	-	3	4	182.34%
SPY221230C00275000	2022-10-05 2:39PM EDT	275	104.94	97.94	98.53	0	-	1	2	51.62%
SPY221230C00280000	2022-09-16 9:38AM EDT	280	107.54	80.42	82.52	0	-	1	16	0.00%
SPY221230C00285000	2022-10-12 11:30AM EDT	285	77.98	88.3	88.9	0	-	1	0	49.16%
SPY221230C00290000	2022-10-10 11:48AM EDT	290	75.24	83.42	84	0	-	1	17	47.03%
SPY221230C00295000	2022-10-14 2:27PM EDT	295	67.7	78.77	79.38	0	-	1	2	45.77%
SPY221230C00300000	2022-10-17 10:12AM EDT	300	71.01	73.85	74.49	0	-	10	125	43.63%
SPY221230C00304000	2022-10-03 12:32PM EDT	304	65.5	70.33	70.95	0	-	2	2	42.96%
SPY221230C00305000	2022-10-10 12:12PM EDT	305	61.31	69.2	69.79	0	-	2	42	42.01%
SPY221230C00310000	2022-10-12 9:33AM EDT	310	53.66	64.7	65.33	0	-	1	0	40.96%
SPY221230C00315000	2022-10-05 2:45PM EDT	315	67.62	60.1	60.69	0	-	10	20	39.33%
SPY221230C00320000	2022-10-17 11:27AM EDT	320	52.16	55.63	56.22	0	-	2	211	38.02%
SPY221230C00321000	2022-10-11 11:40AM EDT	321	47.07	54.74	55.36	0	-	1	1	37.82%

INFORMATION IN THE "SMILE"

- Implied volatility
  - Using the market price of the option, what volatility level causes the BSM price of the option to equal the market price?
  - This is the implied vol

- Implied vol is to options what YTM is to bonds
  - It is a pricing tool, but some of the meaning is lost since the implied vol differs for every strike

- Often called a "vol smile"
  - Sometimes a smirk!

- The equity option smirk suggests a negatively-skewed risk-neutral distribution of the stock price at expiration

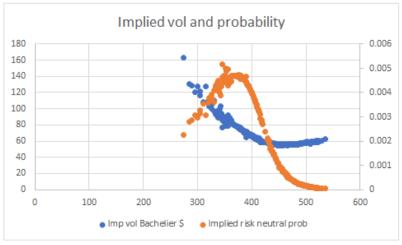Proof that BSM is mis-specified.oo
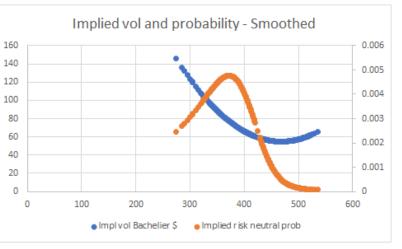
PROBLEMS WITH BSM & EQUITY OPTIONS

- BSM assumes future stock price distributions are lognormal, therefore positively skewed
  - Empirically, these distributions are negatively skewed

- The lognormal distribution has relatively thin tails
  - In practice, the kurtosis of the market returns is greater, implying thicker tails

- BSM implicitly assumes stock prices are positive
  - However, prices can reach zero in practice

- If BSM were true, then implied vol would be the same for all strikes
  - The smile indicates that the model is incorrectly specified

BSM Imp Vols - Outliers Removed

$y = 0.0024x^2 - 2.2719x + 586.9$

WHAT IF RETURNS ARE NORMAL?

- Use the Bachelier model to compute implied dollar volatilities

- Reduces the skew problem
  - But not really enough

- Does not affect the kurtosis problem

- Best to use a negatively skewed distribution

CORRECTING FOR SKEWNESS & KURTOSIS

- Use the skew-normal distribution to add skewness and kurtosis

- Skewness parameter
  - Lambda = -0.93

- Kurtosis parameter
  - Gamma = 0.19

- Implications
  - Two extra parameters
  - Constant across strikes
  - Better specification
  - Better hedge ratios?

GENERAL EUROPEAN OPTION PRICING FORMULA

- Determine the appropriate risk-neutral probability distribution of future underlying values
  - Substitute the risk-neutral mean for the natural mean
  - This is SD/B, where D is a dividend factor and B is a discount factor

- Calculate or simulate expected option payout at maturity using this probability distribution

- Discount at the risk-free interest rate to find the present value

$$M \longleftarrow M^* = S(1+r)$$

RISK NEUTRAL PROBABILITY

$$C_0 = e^{-rT} E(C_T | S_0)$$

$$C_0 = e^{-rT} \int_{-\infty}^{\infty} max(S_T - X, 0) f^*(S_T) dS_T$$

You can value European call for any probability distribution associated with the underlying stock