Quantitative Options Strategies

Introduction to Options

Plain Vanilla Options

- An option is a contract giving the buyer the right, but not the obligation, to buy (in the case of a call) or sell (in the case of a put) the underlying asset at a specific price on or before a certain date.

- People use options for income, to speculate, and to hedge risk.

- Options are known as derivatives because they derive their value from an underlying asset.

- A stock option contract typically represents 100 shares of the underlying stock, but options may be written on any sort of underlying asset from bonds to currencies to commodities.

European Options

“European” - refers to exercise type not to geographic region

“European” option can be exercised only at expiry.

European Call Option ---> payoff at expiry = Max(S-K, 0)

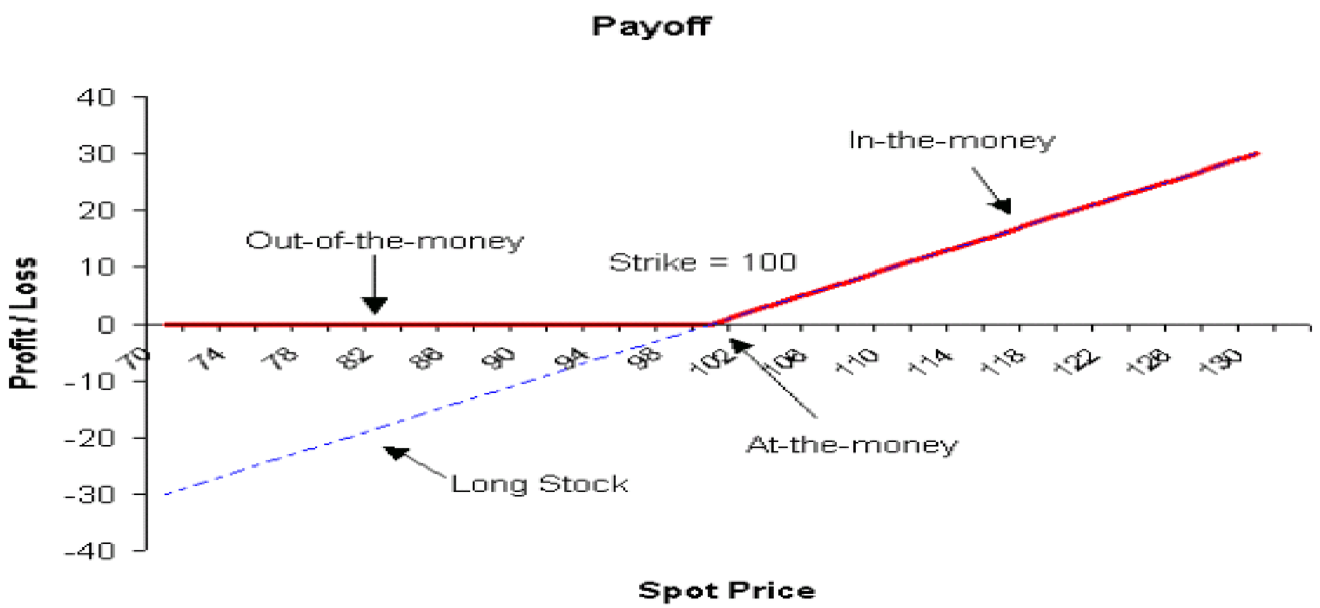
European Put Option ---> payoff at expiry = Max(K-S, 0)

K - strike

S - underlying price at expiry

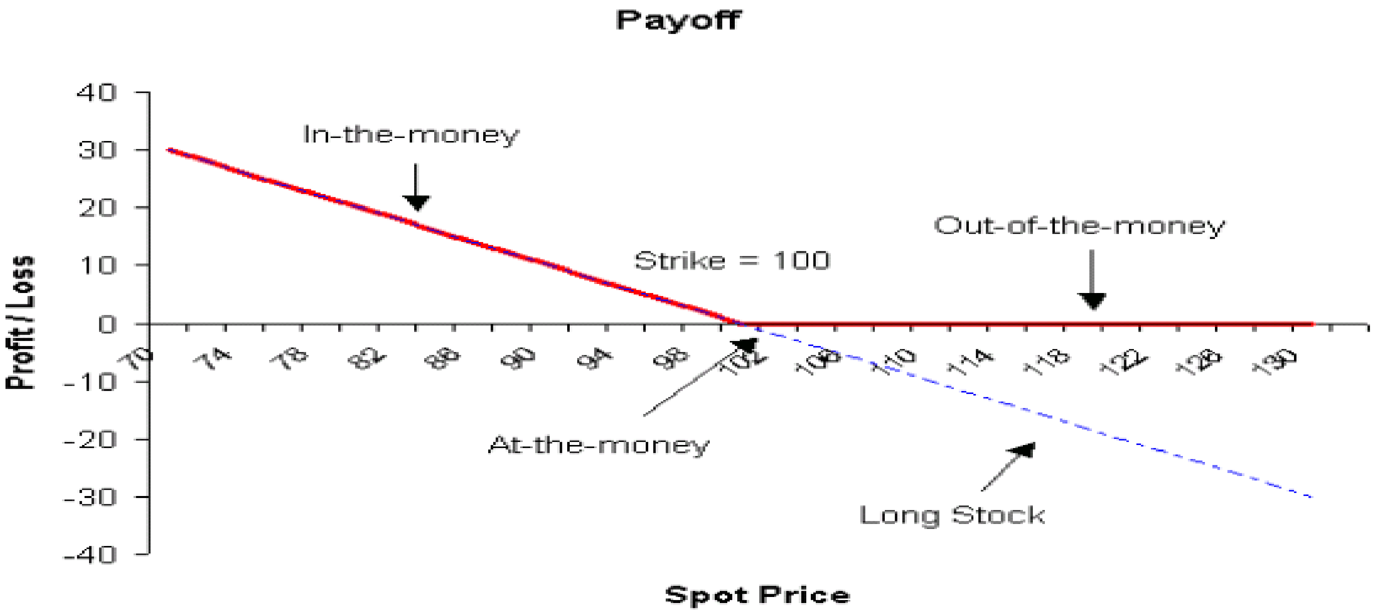
Call Option

This diagram shows the option's payoff as the underlying price changes. Above the strike price of $100, the payoff of the option is $1 for every $1 appreciation of the underlying. If the stock falls below the strike price at expiration, the option expires worthless. Therefore, a call option has unlimited upside potential, but limited downside.



Put Option

A put option is the right, but not the obligation, to sell an asset at a prespecified price on, or before, a prespecified date in the future. The payoff diagram of a put option looks like a mirror image of the call option (along the Y axis). Below the strike price of $100, the put option earns $1 for every $1 depreciation of the underlying. If the stock is above the strike at expiration, the put expires worthless.



Black Scholes Model (BSM)

$$C = S_0\, e^{-qt} * N(d_1) - X\, e^{-rt} * N(d_2)$$

$$P = X\, e^{-rt} * N(-d_2) - S_0\, e^{-qt} * N(-d_1)$$

... where *N(x)* is the standard normal cumulative distribution function.

The formulas for d_1 and d_2 are:

$$d_1 = \frac{ln(\frac{S_0}{X}) + t\,(r - q + \frac{\sigma^2}{2})}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Greeks (definitions)

V - option premium

S - underlying

T - time to expiry

R - risk free rate

Sigma - volatility

dV/dS = Delta

d Delta / dS = Gamma

dV/dT = Theta

dV/dR = Rho

dV/dSigma = Vega

Greeks

Delta

$$Call\ delta = e^{-qt} * N(d_1)$$

$$Put\ delta = e^{-qt} * \ (N(d_1) - 1)$$

Gamma

$$Gamma = \frac{e^{-qt}}{S_0\ \sigma\ \sqrt{t}} * \frac{1}{\sqrt{2\pi}} * e^{\frac{-d_1{}^2}{2}}$$

Greeks (cont1)

Theta

$$Call\ theta =$$

$$= \frac{1}{T}\left(-\left(\frac{S_0\ \sigma\ e^{-qt}}{2\sqrt{t}} * \frac{1}{\sqrt{2\pi}} * e^{\frac{-d_1{}^2}{2}}\right) - r\ X\ e^{-rt}\ N(d_2) + q\ S_0\ e^{-qt}\ N(d_1)\right)$$

$$Put\ theta =$$

$$= \frac{1}{T}\left(-\left(\frac{S_0\ \sigma\ e^{-qt}}{2\sqrt{t}} * \frac{1}{\sqrt{2\pi}} * e^{\frac{-d_1{}^2}{2}}\right) + r\ X\ e^{-rt}\ N(-d_2) - q\ S_0\ e^{-qt}\ N(-d_1)\right)$$

… where T is the number of days per year (calendar or trading days, depending on what you are using).

Vega

$$Vega = \frac{1}{100}\ S_0\ e^{-qt}\ \sqrt{t}\ * \frac{1}{\sqrt{2\pi}} * e^{\frac{-d_1{}^2}{2}}$$

Greeks (cont2)

Rho

$$Call\, rho = \frac{1}{100} X\, t\, e^{-rt} * N(d_2)$$

$$Put\, rho = -\frac{1}{100} X\, t\, e^{-rt} * N(-d_2)$$

Delta Hedged Option Position

delta-hedged option position,

$$C - \Delta S_t \tag{1.1}$$

where C is the value of the option, S_t is the underlying price at time, t, and Δ is the number of shares we are short. Over the next time step the underlying changes to S_{t+1}. The change in the value of the portfolio is given by the change in the option and stock positions together with any financing charges we incur by borrowing money to pay for the position.

$$C(S_{t+1}) - C(S_t) - \Delta(S_{t+1} - S_t) - r(C - \Delta S_t) \tag{1.2}$$

To see why the last term is positive we need to consider our cash flows. We bought the option, so we need to finance that cost, but we shorted stock so we receive money for this. Over a single time step we gain $r\Delta S_t$ from this.

Note also that we assume that the time step is small enough that we can take delta to be unchanged.

$$\Delta(S_{t+1} - S_t) + \frac{1}{2}(S_{t+1} - S_t)^2 \frac{\partial^2 C}{\partial S^2} + \theta - \Delta(S_{t+1} - S_t) - r(C - \Delta S_t) \tag{1.3}$$

or

$$\frac{1}{2}(S_{t+1} - S_t)^2 \Gamma + \theta - r(C - \Delta S_t) \tag{1.4}$$

where Γ is the second derivative of the option price with respect to the underlying. Equation 1.4 gives the change in value of the portfolio, or the profit the trader makes when the stock price changes by a small amount. It has three separate components.

1. The first term gives the effect of gamma. Since gamma is positive, the option holder makes money. The return is proportional to half the square of the underlying price change.
2. The second term gives the effect of theta. The option holder loses money due to the passing of time.
3. The third term gives the effect of financing. Holding a hedged long option portfolio is equivalent to lending money.

Further, we see in the next chapter that on average

$$(S_{t+1} - S_t)^2 \cong \sigma^2 S^2$$

where σ is the standard deviation of the underlying's returns, generally known as *volatility*. So we can rewrite Equation 1.4 as

$$\frac{1}{2}\sigma^2 S^2 \Gamma + \theta - r(C - \Delta S_t) \tag{1.5}$$

If we accept that this position should not earn any abnormal profits because it is riskless and financed with borrowed money, the equation can be set equal to zero. Therefore, the equation for the fair value of the option is

$$\frac{1}{2}\sigma^2 S^2 \Gamma + \theta - r(C - \Delta S_t) = 0 \tag{1.6}$$

Before continuing, we need to make explicit some of the assumptions that this informal derivation has hidden.

- To write down Equation 1.1, we needed to assume the existence of a tradable underlying asset. In fact we assumed that it could be shorted and the underlying could be traded in any size necessary without incurring transaction costs.
- Equation 1.2 has assumed that the proceeds from the short sale can be reinvested at the same interest rate at which we have borrowed to finance the purchase of the call. We have also taken this rate to be constant.
- Equation 1.3 has assumed that the underlying changes are continuous and smooth. And as we mentioned earlier, we have considered second order derivatives with respect to price but only first order with respect to time. This is a very limiting assumption and will be returned to in some depth.

1. Using an estimate of the volatility over the life of the option, calculate a theoretical option price.
2. Using the quoted price of the option, calculate the implied standard deviation or volatility.

If our estimate of volatility differs significantly from that implied by the option market then we can trade the option accordingly. If we forecast volatility to be higher than that implied by the option, we would buy the option and hedge in the underlying market. Our expected profit would depend on the difference between implied volatility and realized volatility. Equation 1.6 says that instantaneously this profit would be proportional to

$$\frac{1}{2}S^2\Gamma(\sigma^2 - \sigma^2_{implied}) \tag{1.7}$$

A complementary way to think of the expected profit of a hedged option is by considering vega. Vega is defined as the *partial derivative of the option price with respect to implied volatility*. It is generally expressed as the change in value of an option if implied volatility changes by one point (e.g., from 19 to 18 percent). This means that if we buy an option at $\sigma_{implied}$ and volatility immediately increases to σ we would make a profit of

$$vega(\sigma - \sigma_{implied}) \tag{1.8}$$

The relationship between the instantaneous profit of Equation 1.7 and the total profit of Equation 1.8 could be proved by integrating 1.7 over time and using the relationship between gamma and vega,

$$vega = \sigma T S^2 \Gamma \tag{1.9}$$

Interaction of Options Markets with underlying market

Effects of delta hedging

- Options Market Makers trying to stay delta neutral ---> serious moves in underlying force market makers to hedge triggering positive feedback loop
- Use options data to predict the motion of underlying

GME short & gamma squeeze

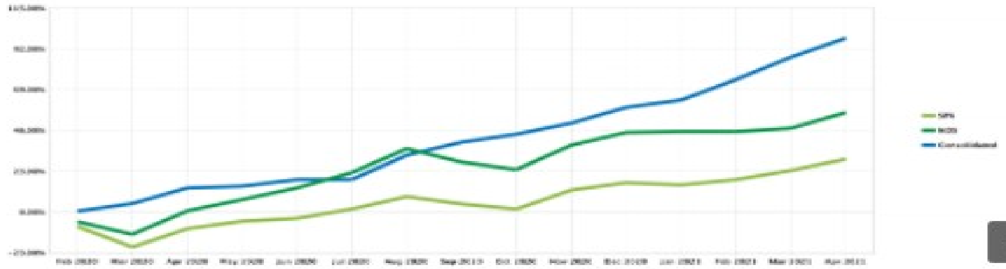
GME gamma squezze

https://spotgamma.com/gme-gamma-squeeze/

Example

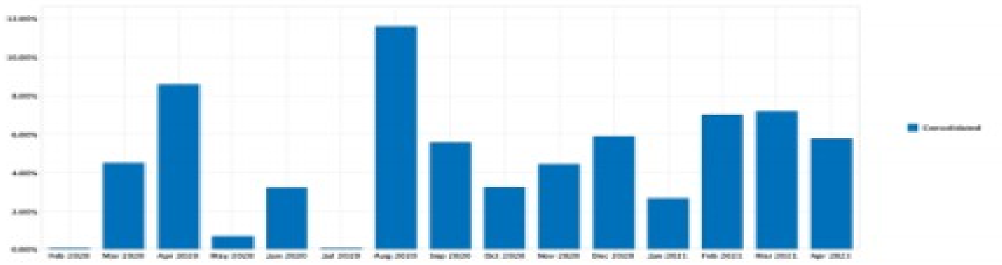
Cumulative Benchmark Comparison

Analysis Period: February 1, 2020 - April 30, 2021



Time Period Performance Statistics

Analysis Period: February 1, 2020 - April 30, 2021



Example (cont)

Risk Measures Benchmark Comparison

Analysis Period: February 1, 2020 - April 30, 2021

Risk Analysis

	SPX	NDX	Consolidated
Ending VAMI	1,296.28	1,557.79	1,977.73
Max Drawdown	19.87%	12.91%	N/A
Peak-To-Valley	Start - Mar 20	Start - Mar 20	N/A
Recovery	4 Months	1 Month	N/A
Sharpe Ratio	1.04	1.68	5.22
Sortino Ratio	1.65	3.73	N/A
Standard Deviation	6.53%	6.62%	3.11%
Downside Deviation	4.10%	2.98%	0.00%
Correlation	0.39	0.35	-
β:	0.19	0.16	-
α:	0.52	0.50	-
Tracking Error	6.04%	6.26%	-
Information Ratio	11.29	6.71	-
Turnover	-	-	1,186.02%
Mean Return	1.96%	3.21%	4.70%
Positive Periods	10 (66.67%)	10 (66.67%)	15 (100.00%)
Negative Periods	5 (33.33%)	5 (33.33%)	0 (0.00%)

Value Added Monthly Index (VAMI)