FIXED INCOME SECURITIES

FRE : 6411

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4 Period Tree With Two Assets

0	1	2	3	4
				1
			0.99184	
			1.035	1
		0.926356		
		1.04		1
			0.956938	
			1.045	1
	0.880426			
	1.045			1
			0.956938	
			1.045	1
		0.905322		
		1.05		1
			0.947867	
0.822983			1.055	1
1.05				
				1
			0.956938	
			1.045	1
		0.908778		
		1.05		1
			0.947867	
			1.055	1
	0.847838			
	1.055			1
			0.947867	
			1.055	1
		0.888337		
		1.06		1
			0.938967	
			1.065	1

Risk Neutral Pricing

- Consider the above 4 period Economy, where there are two traded assets in each state s_t , a risk free asset $r_f(s_t)$ and 4 period zero coupon bond $p(t, 4, s_t)$. Assume the actual probability of moving from state s_t to state $s_t u$, is 0.5
- $\pi_{s_t}(s_t u) = \pi_{s_t}(s_t d) = \frac{1}{2}$

Risk Neutral Pricing

- Assume the following six bonds with maturity and coupon rate of $T_i \; and \; C_i$, respectively are actively traded in this economy.
  - 1) $T_1 = 4 \; C_1 = 5\%$
  - 2) $T_2 = 4 \; C_2 = 2.5\%$
  - 3) $T_3 = 4 \; C_3 = 7.5\%$
  - 4) $T_4 = 3 \; C_4 = 0\%$
  - 5) $T_5 = 3 \; C_5 = 1.5\%$
  - 6) $T_6 = 3 \; C_6 = 5.5\%$
- For each of the above bonds, calculated the time 0, price, duration and convexity.

Risk Neutral Pricing

Bond	T_i	C_i	$P(0,T_i,C_i)$	D_i	MD_i	C_{xi}
1						
2						
3						
4						
5						
6						

Single Bond Future (SBF)

- Single Bond Future Contract (SBF):
- Consider Future Contract, expiring on period 2, written ONLY on Bond 1 ($T_1 = 4\ C_1 = 5\%$) . Find Future Price and cash flow of this contract in periods 0,1 and 2.

s_t	$F_{SBF}(s_t)$	$CF_{SBF}(s_t)$
0		
U		
D		
UU		
UD		
DU		
DD		

Single Bond Future (SBF)

- Find the Actual and risk adjusted *discounted* expected value of the above cash flows :
- $E_0\left[\sum_{i=1}^{2}\frac{\boldsymbol{CF_{SBF}(s_i)}}{B(\boldsymbol{s_i},\boldsymbol{s_{i-1}})}\right]$ and $\tilde{E}_0\left[\sum_{i=1}^{2}\frac{\boldsymbol{CF_{SBF}(s_i)}}{B(\boldsymbol{s_i},\boldsymbol{s_{i-1}})}\right]$

Future Contract w Delivery Option

- Bond Future Contract with Delivery (BF):
- Consider a Future Contract, expiring on period 2. Any Bond with maturity equal or greater than 3 $(T_i \geq 3)$ is deliverable against this Futures contract.
- Let Bond 1 ($T_1 = 4$ $C_1 = 5\%$) be the reference Bond. (conversion factor of 1, $\eta_1 = 1$.)
- Find the conversion factor for all bonds deliverable against this contract η_i.

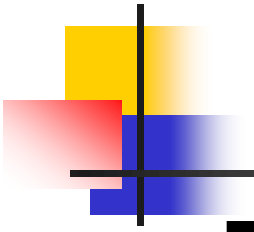
Future Contract w Delivery Option

- Find Future Price and cash flow of this contract in periods 0,1 and 2, and identify which bond will be deliver in period 2 in all state

s_t	$F_{BF}(s_t)$	$CF_{BF}(s_t)$	$B_i Deliver$
0			
U			
D			
UU			
UD			
DU			
DD			

Future Contract w Delivery Option

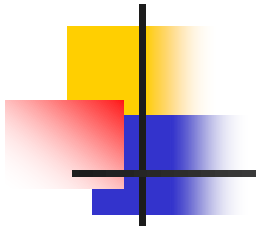
- Find the Actual and risk adjusted *discounted* expected value of the above cash flows :
- $E_0\left[\sum_{i=1}^{2}\frac{\boldsymbol{CF_{BF}(s_i)}}{B(\boldsymbol{s_i},\boldsymbol{s_{i-1}})}\right]$ and $\tilde{E}_0\left[\sum_{i=1}^{2}\frac{\boldsymbol{CF_{BF}(s_i)}}{B(\boldsymbol{s_i},\boldsymbol{s_{i-1}})}\right]$
- Fill the table in the next page
- Describe a brief step by step methodology that you followed.



s_t	$F_{BF}(s_t)$	$F_{SBF}(s_t)$	$CF_{BF}(s_t)$	$CF_{SBF}(s_t)$	$B_i Deliver$
0					
U					
D					
UU					
UD					
DU					
DD					

Risk Neutral Pricing Example

- To summarize , define $B(t+1, s_t)$ as the value of MM account.
  - $B(t+1, s_t) = B(t, s_{t-1})\, r_f(s_t)$
- Define :
  - $\frac{P(t,s_t)}{B(t,s_{t-1})} = A(t, s_t)$
- Then :
  - $\frac{P(t,s_t)}{B(t,s_{t-1})} = \tilde{E}_{t,s_t}\left[\frac{P(t+1,s_{t+1})}{B(t+1,s_t)}\right] = A(t, s_t) = \tilde{E}_{t,s_t}[A(t+1, s_{t+1})]$
- Or $A(t, s_t)$ is a Martingale :
  - $A(0) = \tilde{E}_0[A(1, s_1)] = \tilde{E}_0[A(2, s_2)] = \ldots = \tilde{E}_0[A(t, s_t)]$
  - $A(t, s_t) = \tilde{E}_{t,s_t}[A(t+1, s_{t+1})] = \tilde{E}_{t,s_t}[A(t+n, s_{t+n})]$



- Using the Bond Future Contract with Delivery (BF) and Money Market Account, find a mimicking portfolio for a 2 period 6% coupon bond in period 0,1,2 in every state.