Loan Analysis with Excel

Video Lecture VL02

Valuation for Financial Engineers

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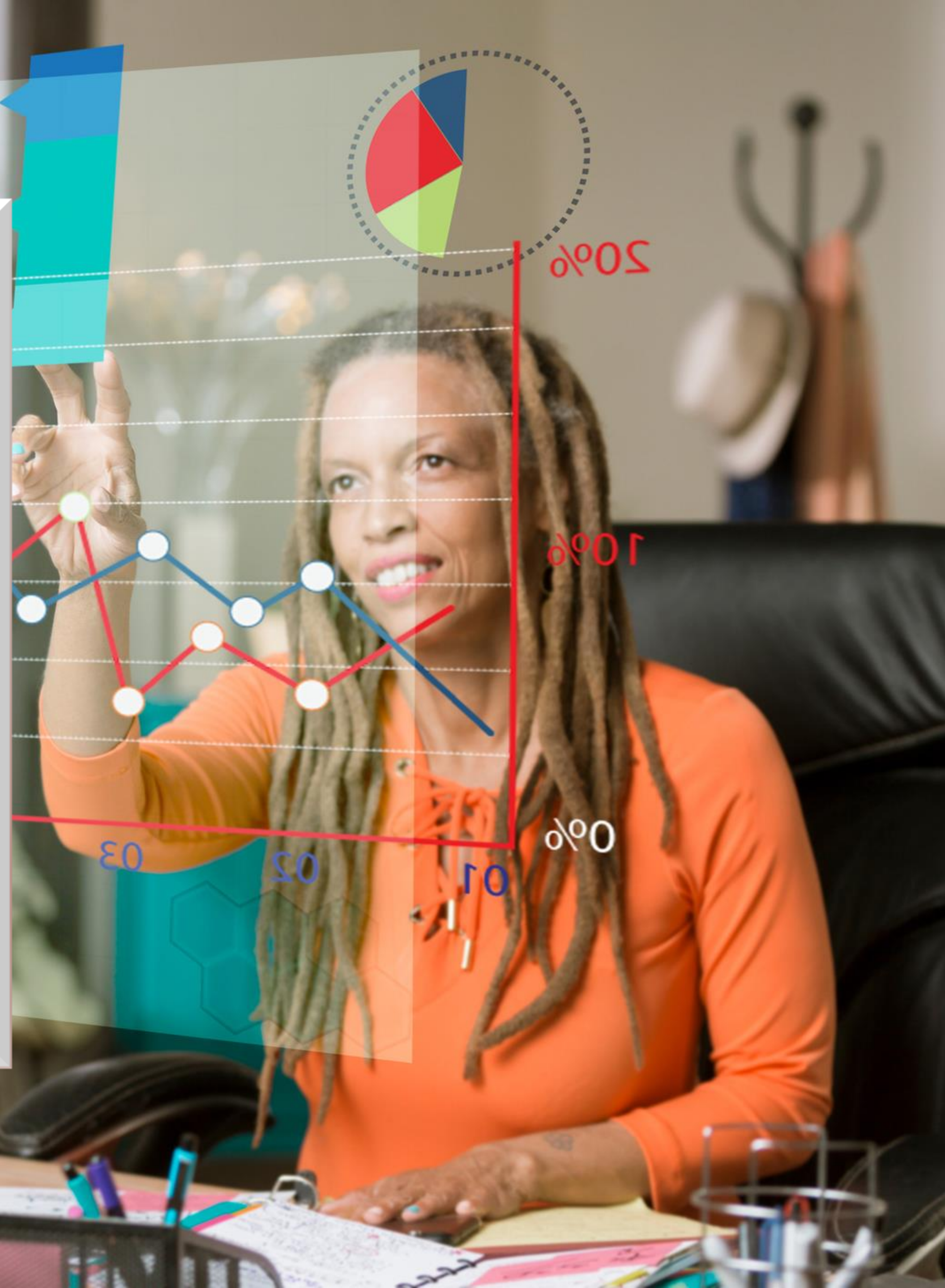
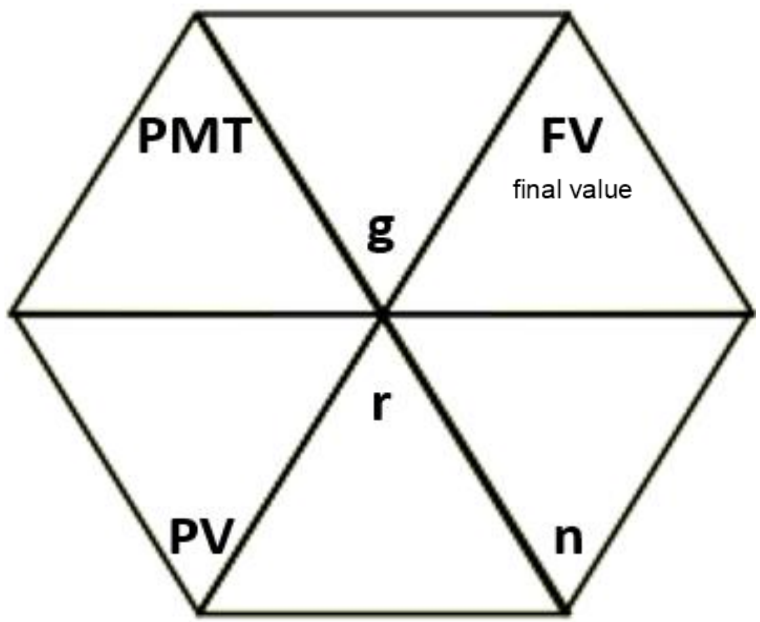


Video Lecture VL02 Coverage

- A paradigm for solving present value problems
- Using Excel’s financial functions
- Data Tables, Graphs and “What-if” analysis
- Working with growing annuities
- Loan amortization
- Analysis of bank loans
  - The bank’s perspective
  - Modeling credit losses

A Simple Excel Paradigm for Common Cash Flow Patterns

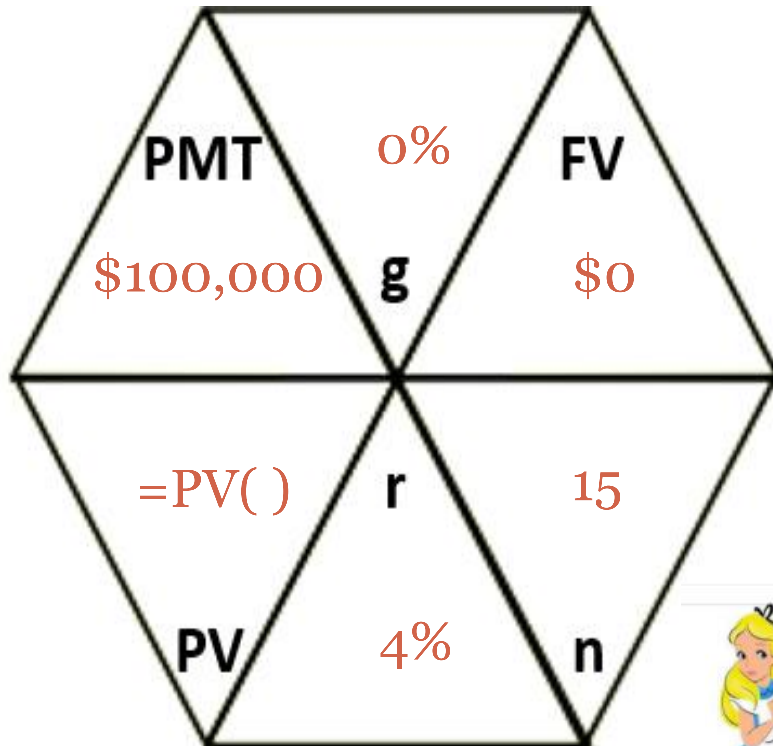


Excel uses the negative PV convention, and decimal interest rates. Also, Excel users can choose the payment timing by selecting [Type].

- Many cash flow patterns can be modeled as a fixed or growing annuity with a final payment
- Once you know 5 of the 6 inputs, the missing value can be determined
- Some details:
  - Make sure the period is consistent with the parameter (n, r)
  - <u>The first payment occurs in one period</u>
  - The PV is often expressed as a negative number, as if it is an investment (outflow) that receives positive cash flows
  - The FV is the amount received *in addition to* the final payment. It is a *final value*, not a *future value*
  - g is the percentage or exponential growth rate in the cash flow per period. Excel does not specifically accommodate growth.

Example 1: Alice

- In these examples, provide known elements of the hexagon leaving only one triangle blank. Do not calculate the missing values yet, as we will do it with Excel.
- Alice has been awarded an insurance settlement of $100,000 per year for 15 years. The first payment is in one year. What is the value of the settlement today if the discount rate is 4% per year?

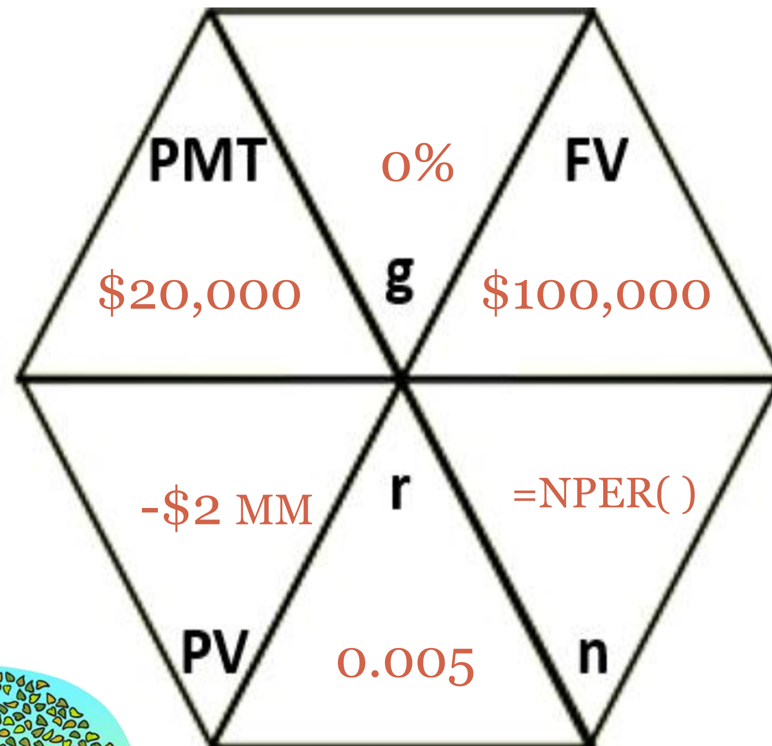


Example 2: Bodhi

- Bodhi is retiring with $2,000,000. If he takes out $20,000 a month, starting in one month, and the monthly interest rate is 50 basis points, how many months will his money last, if he wants to keep $100,000 in the account at the end?

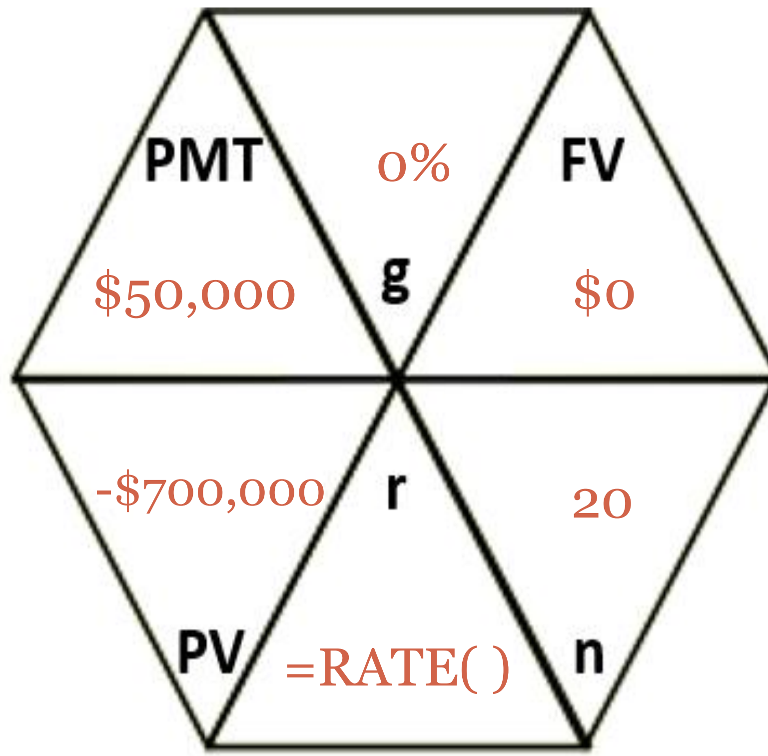
- *Note: 1 basis point = 0.01% = 0.0001*

"One bip"



Example 3: Cong Ming

- Cong Ming won the lottery! Although the prize was advertised to be $1 million, it was actually $50,000 per year starting in one year for 20 years. J.G. Wentworth offered to purchase his payments for $700,000. What rate of return will J.G. Wentworth earn on their investment?



聪明

Excel Financial Functions

	Financial calculation	Excel syntax
Annuity	Present value of annuity including a final payment of FV	=pv(rate, nper, pmt, [fv], [type])
Annuity	Payment that makes present value equal to target value	=pmt(rate, nper, pv, [fv], [type])
Annuity	Number of payments needed to match present value to target value	=nper(rate, pmt, pv, [fv], [type])
Annuity	Interest rate per period that matches present value and target value	=rate(nper, pmt, pv, [fv], [type])
Annuity	Final payment that matches present value and target value	=fv(rate, nper, pmt, [fv], [type])
Other	Present value of a range of cash flows starting in one period	=npv(rate, range of values)
Other	Rate of return that makes the present value of a stream of payments zero (the first payment is immediate)	=irr(range of values, [guess])

PV formula using Excel definitions

$$-PV = \left[\frac{PMT}{RATE}\left(1 - \frac{1}{(1+RATE)^{NPER}}\right) + \frac{FV}{(1+RATE)^{NPER}}\right](1 + RATE)^{TYPE}$$

PV:	Present value
PMT:	Payment
NPER:	Number of periods
RATE:	Interest rate
FV:	Final value
TYPE:	0=End of period payment, 1=Beginning of period
g:	Excel does not have a growth option

Excel Solutions

	A	B	C	D	E
1	Item	Function	Alice	Bodhi	Cong Ming
2	Payment	PMT	100,000	20,000	50,000
3	Final value	FV	0	100,000	0
4	Number of payments	NPER	15	$\boxed{133.90}$	20
5	Interest rate	RATE	4.00%	0.50%	$\boxed{3.67\%}$
6	Present value	PV	$\boxed{-1,111,839}$	-2,000,000	-700,000

	Financial calculation	Excel syntax
Annuity	Present value of annuity including a final payment of FV	=pv(rate, nper, pmt, [fv], [type])
	Payment that makes present value equal to target value	=pmt(rate, nper, pv, [fv], [type])
	Number of payments needed to match present value to target value	=nper(rate, pmt, pv, [fv], [type])
	Interest rate per period that matches present value and target value	=rate(nper, pmt, pv, [fv], [type])
	Final payment that matches present value and target value	=fv(rate, nper, pmt, [fv], [type])

Bodhi's Decision

- In the previous example, Bodhi chose to receive $20,000 per month, and would be able to have his savings last for 134 months.

- If he chose a smaller or larger payment, how many months would his money last?

- Excel provides "What-if" analysis in the DATA menu
  - "Data Table" option
  - Select the entire table range as indicated in the red box
  - Call the function
  - For "column input cell" select Bodhi's payment in cell D2 (20,000)

Selected range

=D4
Reference to NPER calculation

	G	H	I
1	Monthly Withdrawal Analysis		
2	Pmt (000s)	134	
3	12	351	
4	14	244	
5	16	190	
6	18	157	
7	20	134	
8	22	117	
9	24	104	
10	26	93	
11	28	85	
12	30	78	
13	32	72	

Monthly payment possible values

NPER calculation For each payment value

Bodhi's Decision with Graph

	G	H
1	Monthly Withdrawal Analysis	
2	Pmt (000s)	134
3	12	351
4	14	244
5	16	190
6	18	157
7	20	134
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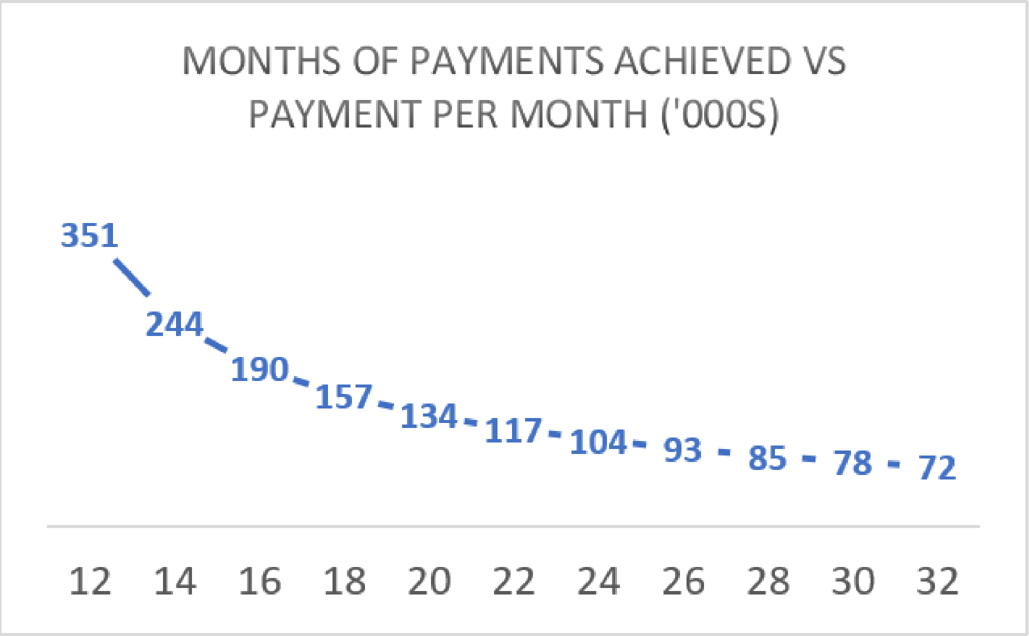
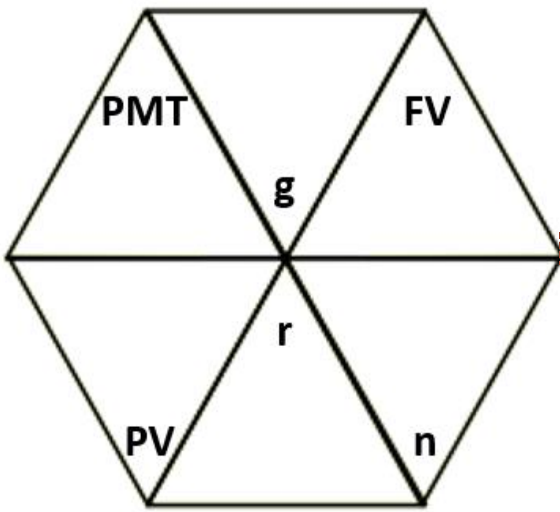
13	32	72



Converting annuities with exponentially growing payments to annuities with flat payments – this conversion is needed in Excel

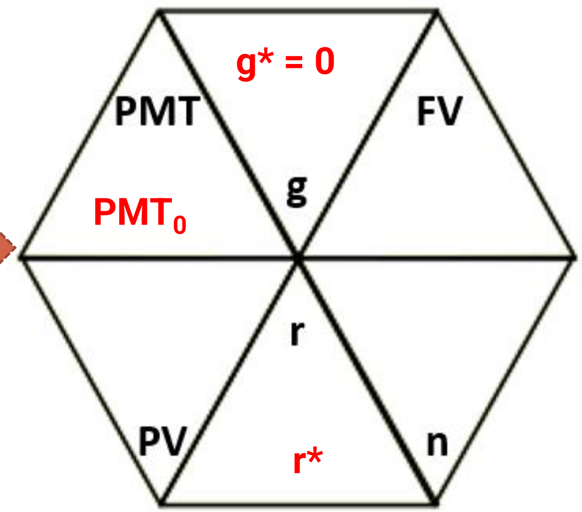
Original, g ≠ 0



$$PMT = PMT_0 = \frac{PMT_1}{1+g}$$

$$r^* = \frac{r-g}{1+g}$$

Zero Growth

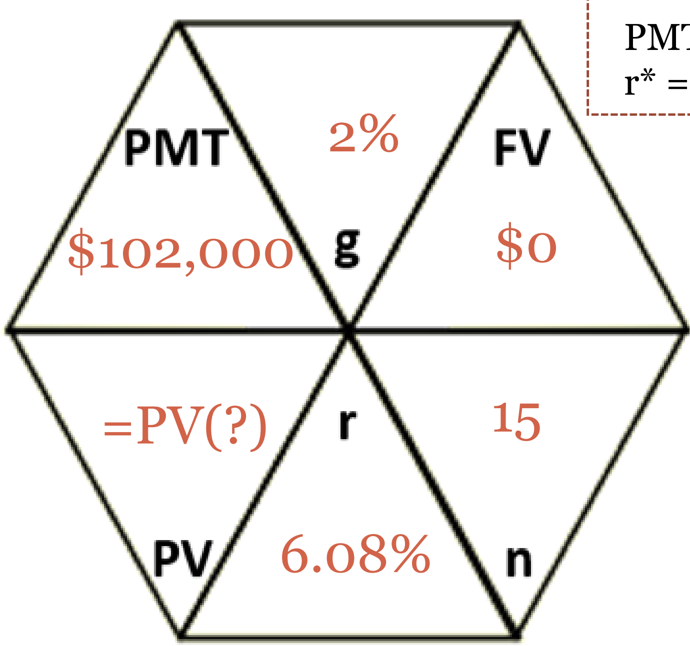


***Note: This trick works for annuities, i.e. FV=0. It also works if the final payment grows at the same rate as the payments (g) from a fixed starting point FV at time 0, such as an inflation adjustment.*

Example 4: Alice revisited

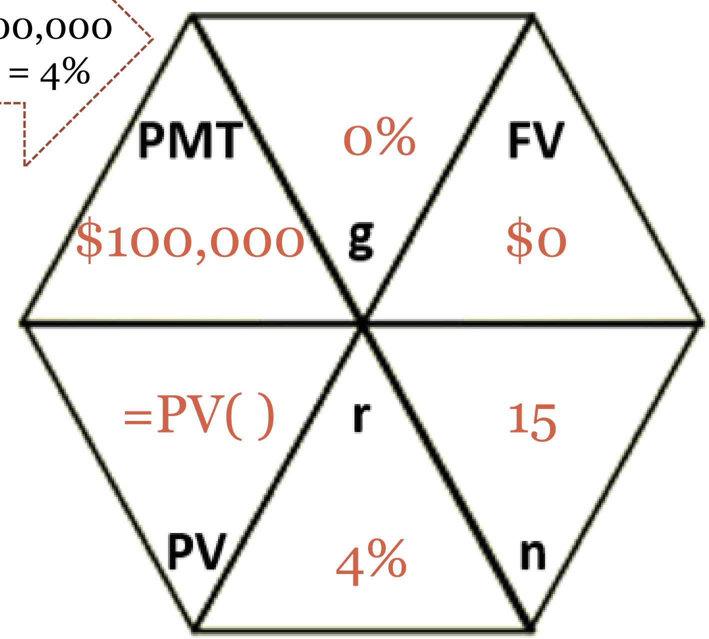
- Alice has been awarded an insurance settlement of $102,000 next year, increasing 2% per year for a total of 15 years. What is the value of the settlement today if the discount rate is 6.08% per year?

Stated problem $g \neq 0$



$PMT_0 = 102{,}000/1.02 = 100{,}000$
$r^* = (0.0608 - 0.02)/1.02 = 4\%$

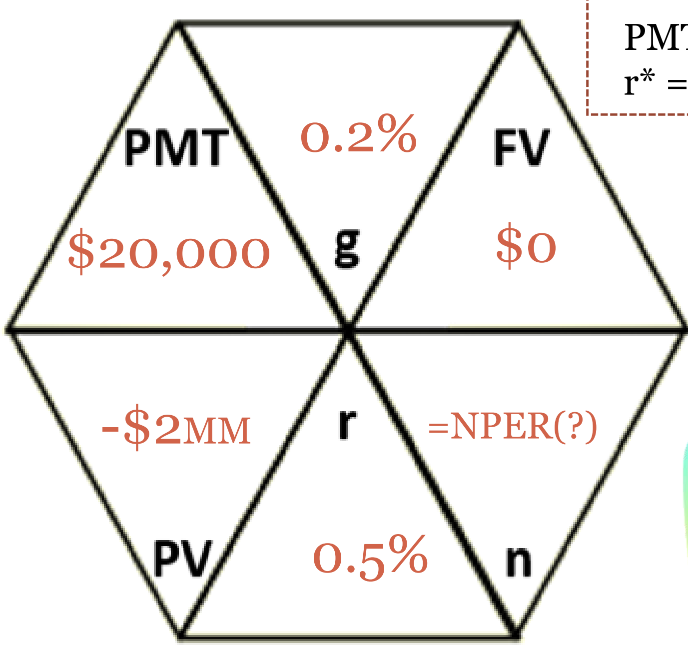
Converted to $g=0$



Example 5: Bodhi revisited

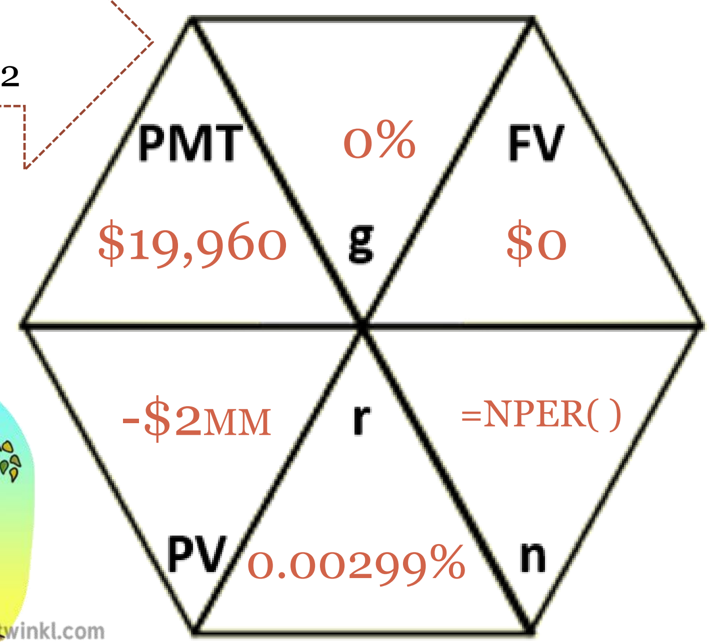
- Bodhi is retiring with \$2,000,000. If he takes out \$20,000 a month, starting in one month growing at 20 basis points per month, and the monthly interest rate is 50 basis points, how many months will his money last? Assume no final payment in this example.

Stated problem $g \neq 0$



PMT_0 = 20,000/1.002
r^* = (0.005 – 0.002)/1.002

Converted to $g=0$



Application: Consumer Loans

- Let's say you need to borrow \$5000
- Your bank will allow you to borrow for 24 months at an annual interest rate of 6%
- The loan will be fully amortized (nothing due at the end)
- You have a lot of questions:
  - What are the monthly payments?
  - What is my total cost of borrowing \$5000?
  - What do I owe if I pay off early?

Step 1: Payment & first month amortization

- Which of the following best represents the cash flow pattern?
  - Single payment, annuity, perpetuity, growing annuity, or combination?
- How can we calculate the payment?
  - In Excel, $221.60 $\boxed{\text{=PMT(0.06/12,24,-5000)}}$
- What happens in the first month?
  - At the end of the first month (5000 × 0.005 = $25) is due in interest
  - The payment of $221.60 is used to pay the interest first
  - The remainder (221.60 – 25 =) $196.60 is used to reduce the principal
  - The ending balance of the first month becomes the beginning balance of the second month

AMORTIZATION TABLE

Month	Begin	Payment	Interest	Prin Redox	End
1	$5,000.00	221.60	25.00	196.60	4,803.40
2	4,803.40				
	V_0	C	rV_0	V_1-V_0	V_1

Step 2: Complete the amortization table

Try this at home!

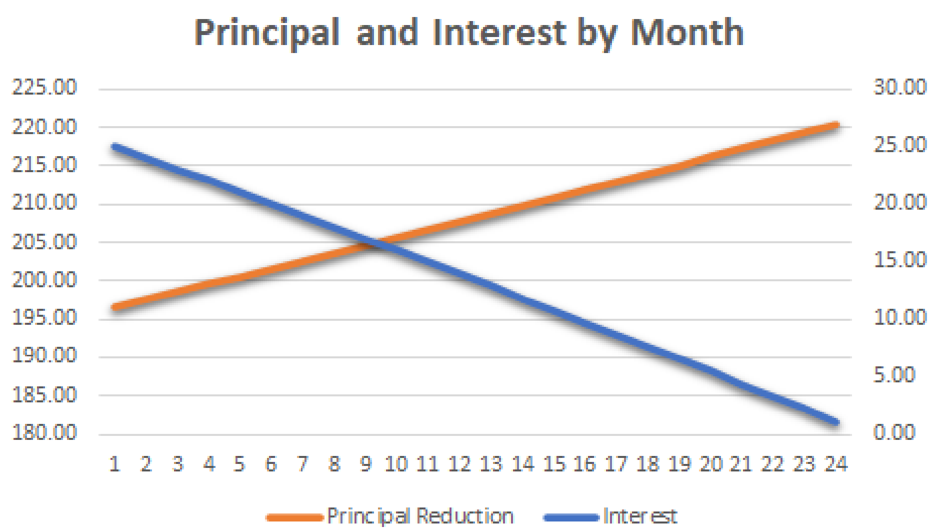
- Month 1 is simple to copy to the other months if you use absolute and relative addressing in Excel
- If your table is completed correctly, the final balance after month 24 is *exactly zero*
- The "End" column shows the balance of the loan if it is prepaid in any month
  - e.g. If paid off in month 15, balance is $1945.47
- The ending column always equals the present value of the remaining payments
  - e.g. At end of month 15, 9 payments remain, so
  - 1945.47 = -PV(0.005,9,221.60)
- Do this in Excel to get the exact values!

AMORTIZATION TABLE

Month	Begin	Payment	Interest	Prin Redox	End
1	$5,000.00	221.60	25.00	196.60	4,803.40
2	4,803.40	221.60	24.02	197.59	4,605.81
3	4,605.81	221.60	23.03	198.57	4,407.24
4	4,407.24	221.60	22.04	199.57	4,207.67
5	4,207.67	221.60	21.04	200.56	4,007.11
6	4,007.11	221.60	20.04	201.57	3,805.54
7	3,805.54	221.60	19.03	202.58	3,602.96
8	3,602.96	221.60	18.01	203.59	3,399.37
9	3,399.37	221.60	17.00	204.61	3,194.77
10	3,194.77	221.60	15.97	205.63	2,989.14
11	2,989.14	221.60	14.95	206.66	2,782.48
12	2,782.48	221.60	13.91	207.69	2,574.79
13	2,574.79	221.60	12.87	208.73	2,366.06
14	2,366.06	221.60	11.83	209.77	2,156.29
15	2,156.29	221.60	10.78	210.82	1,945.47
16	1,945.47	221.60	9.73	211.88	1,733.59
17	1,733.59	221.60	8.67	212.94	1,520.66
18	1,520.66	221.60	7.60	214.00	1,306.66
19	1,306.66	221.60	6.53	215.07	1,091.59
20	1,091.59	221.60	5.46	216.15	875.44
21	875.44	221.60	4.38	217.23	658.22
22	658.22	221.60	3.29	218.31	439.90
23	439.90	221.60	2.20	219.40	220.50
24	220.50	221.60	1.10	220.50	0.00

Step 3: Interest and principal over time

- The sum of interest and principal reduction always equals the payment
- The interest payments fall at an increasing rate as principal reductions reduce future interest charges



A quick way to compute total interest paid:
Total interest = Payment × Months − Loan amount

Not advised for formal analysis - combining cash flows at different time periods without discounting

The GVE and Loan Amortization

- From the bank's point of view, the loan is an asset
  - For the borrower, it's a liability

- At inception, the asset is worth V_0 = \$5000.
  - What is it worth at the end, i.e. what is V_1?

- The GVE suggests (since r includes the risk premium):
  - $rV_0 = (V_1 - V_0) + C$
  - Monthly Interest = (End balance – Begin balance) + Payment, or
  - End balance = Begin balance + Monthly Interest – Payment
    - Also, Payment – Monthly Interest = Principal Reduction

The bank's analysis of your loan

- Assumptions
  - The bank's cost of capital is 1.2% per year (monthly rate 0.1%)
  - Borrowers in this loan cohort have an average default rate of 20 bppm (basis points per month)
  - On default in any given month, no further payments are received

- Analysis
  - The bank has been promised 24 payments of 221.60
  - The probability of default in month 1 is 20 bp, and no payments are ever received in this case
  - The probability of default in month 2 is 20 bp of 99.8%, or (0.002×0.998), since a borrower has to survive month 1 to default in month 2
  - The present value of the loan is the present value of the payments in each default month, taken at the bank's cost of capital.
  - The overall loan value is the probability-weighted PV of the possible default outcomes

The bank's credit spreadsheet

Present value of payments received by default month

Payments received given default month

Probability of default in given month

Probability-weighted PV = Expected PV = $5,123.73

=sumproduct(prob, pv receipts)

The no-default case

Month	Prob	Payments received	PV receipts
1	0.2000%		0
2	0.1996%	1	$221.38
3	0.1992%	1 1	$442.54
4	0.1988%	1 1 1	$663.48
5	0.1984%	1 1 1 1	$884.20
6	0.1980%	1 1 1 1 1	$1,104.70
7	0.1976%	1 1 1 1 1 1	$1,324.98
8	0.1972%	1 1 1 1 1 1 1	$1,545.04
9	0.1968%	1 1 1 1 1 1 1 1	$1,764.87
10	0.1964%	1 1 1 1 1 1 1 1 1	$1,984.49
11	0.1960%	1 1 1 1 1 1 1 1 1 1	$2,203.89
12	0.1956%	1 1 1 1 1 1 1 1 1 1 1	$2,423.07
13	0.1953%	1 1 1 1 1 1 1 1 1 1 1 1	$2,642.03
14	0.1949%	1 1 1 1 1 1 1 1 1 1 1 1 1	$2,860.77
15	0.1945%	1 1 1 1 1 1 1 1 1 1 1 1 1 1	$3,079.30
16	0.1941%	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	$3,297.60
17	0.1937%	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	$3,515.69
18	0.1933%	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	$3,733.56
19	0.1929%	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	$3,951.21
20	0.1925%	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	$4,168.65
21	0.1922%	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	$4,385.86
22	0.1918%	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	$4,602.86
23	0.1914%	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	$4,819.65
24	0.1910%	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	$5,036.21
25	95.3088%	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	$5,252.56

Defaults

No default

The bank's business analysis

Expected yield - considers defaults
YTM = maximum yield

- The profit on this loan is $123.73 in present value terms
- The credit spread is the difference between the YTM (based on the promised payments) and the bank's cost of capital, i.e. 20 bppm.
  - Monthly YTM is the implied rate of return earned on the annuity valued at 5123.73
  - Annual YTM = Monthly YTM × 12 (no compounding)
  - This (no compounding) standard applies to all loans and bonds
- Is it a coincidence that the credit spread approximately equals the default rate?

	Monthly	Annual
Bank's cost of capital	0.10%	1.20%
Default rate	0.20%	2.40%
Expected loan value	$5,123.73	
Yield to maturity (promised)	0.300601%	3.61%
Credit spread	0.200601%	2.41%
Loan amount	$5,000.00	
Net present value	$123.73	

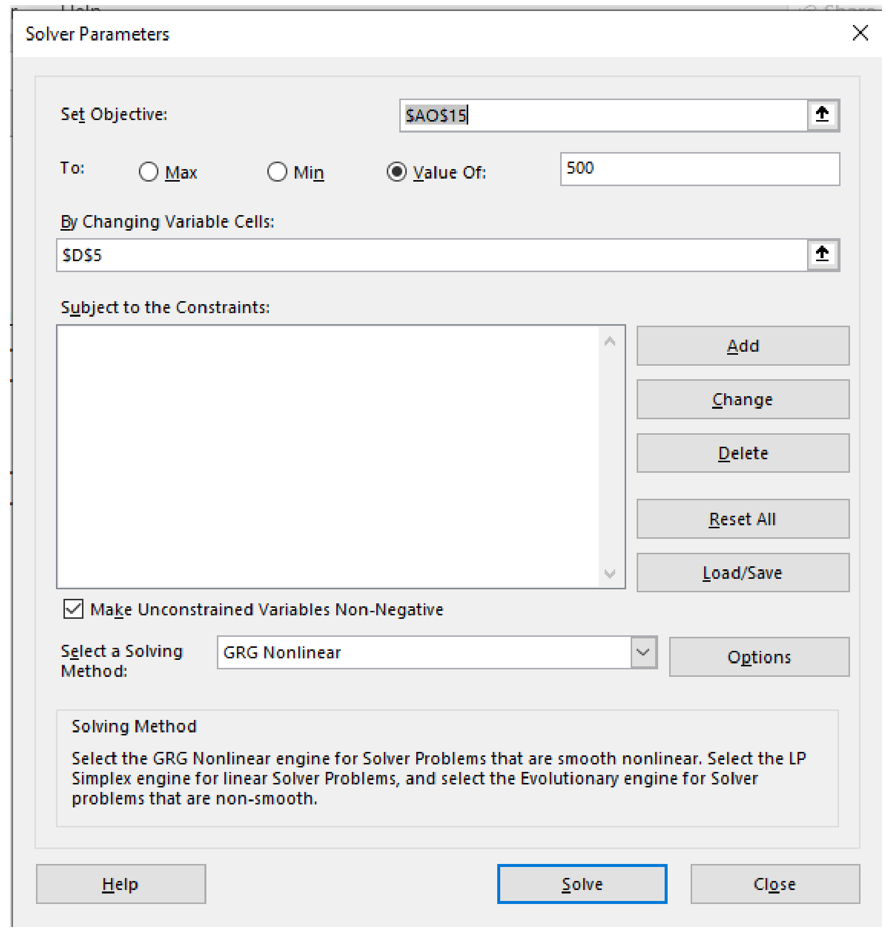
YTM - default rate approx = expected return

Oh no, not the GVE again!

- The expected cash flows are declining at a rate of 20 bppm
- Use the growing annuity formula
  - g = -0.002
  - r* = (0.001 + 0.002)/(1 − 0.002) = $\boxed{0.00300601}$
- The YTM exactly equals r*
  - The credit spread is r* − r = π(1 + r)/(1 - π) ≡ p ≈ π
- The time 0 payment is the mortgage payment amount of $221.60.
- The loan value is $5123.73 (exactly the same)
- Since the expected loss every month is 0.002 times the loan value, the charge is proportional
- In this special case, the GVE includes an expected default loss as part of the capital gain:
- $rV_0 = \left(-\frac{C}{(1+r)^n} - pV_0\right) + C \quad \longrightarrow \quad r^* V_0 = -\frac{C}{(1+r)^n} + C$

Bonus Excel Question

- As the bank manager, you've been directed to make the profit on each loan equal to 10% of loan value
- With all other assumptions the same, what would the contracted loan rate have to equal?
- HINT: Use Excel Solver
  - Install the "Solver" add-in in Excel
  - Also install the Data Analysis Toolpak for statistical analysis, you'll need it later
  - Call these from your DATA tab
- Use Excel to find the value of the contract rate that makes the profit $500 for the $5000 loan
- Answer: 13.07%



Further complexities in loan modeling

- Consumers often prepay their loans
- Default rates are not constant and are not easy to predict
  - Machine learning models are becoming more popular in retail credit underwriting
- Recoveries on some loans are possible
- Shifts in macroeconomic variables cause changes in fraud rates, default rates, prepayments and recoveries (think COVID)
- This challenging space provides job opportunities for aspiring financial engineers

Summary

- Excel provides an easy paradigm and a popular standard for performing present value calculations and solving for missing parameters
- Excel also provides What-if analysis, graphics, and goal-seeking capabilities (among other things we shall discover)
- Consumer loans provide not only a present value application, but a chance to do more advanced financial analysis and business decision-making, such as credit underwriting
- Every student of finance must understand amortization tables and their uses
- These simple analyses form the basis for more advanced problem solving, such as default assessment and credit pricing.

Review Questions for Recitation

1. What are the 5 parameters of the standard Excel present value paradigm?
2. What parameter is not included in Excel?
3. How does one compute an amortization table?
4. What are two ways to find a loan balance after a given number of months?
5. What is the easiest way to compute the total interest paid over the life of the loan?
6. How does one incorporate default probabilities into the analysis of a loan amortization?
7. If a loan has a constant default probability per period π, and the risk-free rate is r, what is the loan's yield to maturity? What is its credit spread?
8. Name three factors that make analysis of retail loans difficult.

Loan terminology

- Contract rate
- Amortize/amortization
- Principal
- Interest
- Payment
- Principal reduction
- Asset
- Liability
- Cost of capital
- bppm
- Cohort
- Underwriting
- Default rate
- Prepayment
- YTM
- Loss given default (LGD)
- Recovery

Key formulas

- Amortization table formulas (monthly loans)
  - Payment = PMT(Rate/12,nper,loan amt)
  - Interest = Begin × Rate/12
  - Principal reduction = Payment − Interest
  - End = Begin − Principal reduction, or
  - End = Begin + Interest - Payment
  - Total interest paid = Payment × # of payments − Loan amount
- IRR on a loan with constant conditional full default rates (no recovery) and no additional risk premium
  - $r^* = \frac{r+\pi}{1-\pi}$
- Credit spread on that loan
  - $CS = r^* - r$

Financial calculation	Excel syntax
Present value of annuity including a final payment of FV	=pv(rate, nper, pmt, [fv], [type])
Payment that makes present value equal to target value	=pmt(rate, nper, pv, [fv], [type])
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Interest rate per period that matches present value and target value	=rate(nper, pmt, pv, [fv], [type])
Final payment that matches present value and target value	=fv(rate, nper, pmt, [fv], [type])
Present value of a range of cash flows starting in one period	=npv(rate, range of values)
Rate of return that makes the present value of a stream of payments zero (the first payment is immediate)	=irr(range of values, [guess])