**FRE6411-Final**

Zeyu Yue

**Problem1:**

**Consider the above 4 period Economy, where there are two traded assets in each state,a risk-free asset and 4 period zero coupon bond .Assume the actual probability of moving from state to state , is 0.5.**

**Assume the following six bonds with maturity and coupon rate of 𝑇𝑖 𝑎𝑛𝑑 𝐶𝑖 , respectively are actively traded in this economy.**

**文本

描述已自动生成**

**For each of the above bonds, calculated the time 0, price, duration and convexity.**

ANS:

First, construct money market value tree using：

and .

Then calculate the Pseudo-Probabilities:

So we can calculate the period 0 price using:

With period 0 price and IRR formula, we can calculate the ytm(y) of each bond.

With y and period 0 price, we can calculate the duration, modified duration and convexity of the bonds.

The results are as follows:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| T | C | P(0) | y | D | MD | Cx |
| 4 | 5% | 1.0003 | 0.0499 | 3.7233 | 3.5463 | 8.2384 |
| 4 | 2.50% | 0.9116 | 0.0499 | 3.8482 | 3.6652 | 8.6146 |
| 4 | 7.50% | 1.0890 | 0.0499 | 3.6187 | 3.4467 | 7.9234 |
| 3 | 0% | 0.8639 | 0.0500 | 3.0000 | 2.8573 | 5.4426 |
| 3 | 1.50% | 0.9048 | 0.0500 | 2.9534 | 2.8129 | 5.3301 |
| 3 | 5.50% | 1.0137 | 0.0500 | 2.8474 | 2.7119 | 5.0743 |

**Consider Future Contract, expiring on period 2, written ONLY on Bond 1 (𝑇1 = 4 𝐶1 = 5%) .**

**Find Future Price and cash flow of this contract in periods 0,1 and 2.**

**Find the Actual and risk adjusted discounted expected value of the above cash flows :**

Calculate the period 2 (5% T=4) bond’s prices in every state in excel, we have:

So at time 0, the price of the future on the (5% T=4) bond maturing at time 2 is:

And

,

|  |  |  |
| --- | --- | --- |
| S(t) | F | CF |
| 0 | 1.0002 | 0 |
| u | 1.0140 | 0.0137 |
| d | 0.9865 | -0.0137 |
| uu | 1.0208 | 0.0068 |
| ud | 0.9982 | -0.0158 |
| du | 1.0018 | 0.0153 |
| dd | 0.9799 | -0.0066 |

The Actual and risk adjusted discounted expected value of the above cash flows are:

|  |  |
| --- | --- |
| E0 | -7.641E-05 |
| E`0 | 2.4937E-16 |

**Bond Future Contract with Delivery (BF):**

**Consider a Future Contract, expiring on period 2. Any Bond with maturity equal or greater than 3 (𝑇𝑖≥ 3) is deliverable against this Futures contract. Let Bond 1 (𝑇1 = 4 𝐶1 = 5%) be the reference Bond. (conversion factor of 1, 𝜂1 = 1.)**

**Find the conversion factor for all bonds deliverable against this contract 𝜂𝑖**

**Find Future Price and cash flow of this contract in periods 0,1 and 2, and identify which bond will be deliver in period 2 in all state**

**Find the Actual and risk adjusted discounted expected value of the above cash flows**

Using the formula:

As we can see in the excel:

Then we can get:

For future price and cash flows, the calculation are shown in the excel and the result are as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| S(t) | F(St) | CF(St) | Bi(deliver) |
| 0 | 0.9961 | 0.0000 |  |
| u | 1.0063 | 0.0102 |  |
| d | 0.9858 | -0.0102 |  |
| uu | 1.0098 | 0.0035 | B4,B5,B6 |
| ud | 0.9982 | -0.0081 | B2 |
| du | 1.0002 | 0.0144 | B4,B5,B6 |
| dd | 0.9797 | -0.0062 | B2 |

The Actual and risk adjusted discounted expected value of the above cash flows are:

|  |  |
| --- | --- |
| E0 | 0.00079132 |
| E`0 | 9.541E-18 |

**Using the Bond Future Contract with Delivery (BF) and Money Market Account, find a mimicking portfolio for a 2 period 6% coupon bond in period 0,1,2 in every state.**

Solve the equation:

We have:

Solve the equation:

We have:

So the value of the portfolio at time 0 is

**Problem2:**