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A currency trading system based on simplified models using fuzzy multi-criteria hierarchical optimization

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ABSTRACT

This paper is based on some assumptions validated using real data and the most used Forex trading platform Meta Trader 4. First, we assume that any reasonable and relatively simple models, reflecting some trading hypotheses, can be profitable for a certain period. Having a sufficient set of such models optimized for selected currency pairs, we can use the model that provides the greatest profit in the current trading period. The second proposal is to use a fuzzy multiple-criteria approach at the training optimization stage using historical data in order to overcome or reduce the negative effect of overfitting. Here for the first time, the problem of fuzzy multiple-criteria optimization of trading was formulated and solved based on the output parameters of the developed simple single-criteria crisp models. This provides significantly greater profit than that obtained using single-criteria crisp models. The third proposal is to use the hierarchical structure of fuzzy local criteria to solve the multiple-criteria problem. It is shown that this additionally provides a significant increase in profit. The profitability and riskless of the developed trading models are studied based on real quotations of currency pairs USDJPY, EURUSD and AUDUSD using H4 timeframe.

1. Introduction

This article presents the results of improving algorithmic trading systems in the foreign exchange market (Forex). Since in our last article [1], we reviewed the works in this area until 2020, here we present only an analysis of the state of the art in this field based on publications in 2020–2023 years. The dominant area of research during this period was the development of methods for forecasting exchange rates and their trends. Here we analyze the most interesting works devoted to these problems. In [2], the interval time series were used to forecast exchange rates. The synthesis of indicators of fundamental and technical analysis was applied in [3,4] to predict the directional movement and exchange rates. In [5], a new machine learning technique based on technical analysis with the Belief Rule-Based Expert System and Bollinger Band indicator was presented. A promising approach to building deep network-based trade and trend prediction systems was presented in [6]. The so-called ANN-GA hybrid approach, in combination with machine learning methods, was proposed in [7] to analyze trends in the Forex market. It is proved in [8] that an approach based on Elliott wave patterns, supported by the neural network method, can improve the prediction of exchange rates. An incremental type-2 fuzzy based classifier for trend forecasting was proposed in [9]. The recent review of the methods for Forex exchange rates forecasting based on machine learning is presented in [10]. It is important to note that in all these works, the goal was to maximize the accuracy of predicting exchange rates and not to develop real systems for algorithmic trading.

We fully recognize the scientific significance and usefulness of research on the possibility of forecasting exchange rates in the development of Forex trading systems. However, without taking into account factors such as spreads, slippages, transaction costs, etc., which form real trading conditions in practice, the trading strategies turn out to be unprofitable. Even a model predicting exchange rates with 100% accuracy will be unprofitable in the case of a sufficiently large spread. Therefore, in studies aimed to get real profitable trading systems, some other specific approaches are usually used. In the article [11], the multiple criteria-based trading system is presented. The technical analysis indicators are assumed to be local fuzzified criteria. A full trading system based on a set of crisp trading rules and technical analysis indicators is designed and implemented on Forex MetaTrader 5 trading platform in [12]. The authors' conclusion is very important: "The

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results provide a clear answer to the main question that guided our research: Can Deep Learning improve technical analysis of Forex data to predict future price movements? At present, the answer is negative. In general, neural network-based approaches are most often evaluated by considering only their prediction error, without taking into account the overall impact on a broader trading system where the decision to open a position and invest is as important as the decision to close the position".

An approach to designing trading systems based on multi-agent methods of deep learning with reinforcement was presented in [13]. Unfortunately, the assumptions used are far from market practice. The method for trading system development based on the combination of the ensemble multi-class support vector machine (EmcSVM) and fuzzy NSGA-II algorithm is represented in [14]. The EmcSVM is used to predict market trends and the NSGA-II optimizes the hyperparameters of the proposed fuzzy trading system represented by the set of technical analysis rules. The assembly-based method is used to develop an ultra short-term trading system in [15]. The assembly consists of technical indicators called actors. They can be considered as local criteria. Thus, assembly-based approaches seem to be some poorly formulated tasks of multi-criteria optimization camouflaged by modern terminology. A trading system presented in [16] is based on deep learning and data clustering. A dataset was obtained and used for training the deep learning model to obtain the best action (Buy, Sell, or Hold). The work [17] confirms the possibility of obtaining good Forex trading results using pattern-based approaches.

The crucial problem of any trading system is the overfitting. In a nutshell, this means that apart to the excellent results (large profits) obtained at the optimization stage, we often get negative results (losses) during the subsequent testing period. Probably the article [18] was the first to focus directly on overfitting problem in Forex trading. Overfitting seems to be an essential feature of the trading systems and can be considered rather as a norm than an anomaly. It is declared in [18] that the proposed log-distance path loss method solves the overfitting problem. Nevertheless, this is not proved or sufficiently illustrated in [18]. The overfitting problem is very important for the practice of trading. Therefore we will consider it below in more detail.

In our last article [1], continuing our studies presented in [19–22], we extended the set of methods for uncertainty modeling that can be successfully used in the development of Forex trading systems. When analyzing the above short review and relevant reviews in [1,10,19], we come to the conclusion that at now deep learning-based approaches jointly define the mainstream in stock prices and exchange rates forecasting. Meanwhile, the development of trading systems needs, in addition, the other specific methods to be used. Therefore we can observe the tendency to build transparent trading models using mathematical representations of the real technical analysis rules based on common sense adapting their parameters to historical data using neural networks if needed. Also, we can see that in the traders' community, as well in textbooks, dominates a common opinion that a well-designed trading model must be universal, i.e., profitable for different currency pairs and timeframes. Obviously, such a model must be very complex in order to sufficiently reflect the specifics of the currency pairs and timeframes under consideration. Thus, the models become too complex, including many uncertain parameters, the values of which are determined during model training (optimization) based on historical data. In addition, along with the rising of uncertain parameters number, the probability of overfitting usually increases as well. Therefore, to get reliable estimations of these parameter's values, unacceptably large volumes of historical data should be used. In addition, such models provide acceptable results only during some limited periods. Then the models fail, despite careful training, due to qualitative changes in the market activity. According to the observations of the members of the MQL5 community the models offered on the open market, provide acceptable results for about six months after installation. Then they need a serious redesign, which usually consumes too much time. It is not difficult to give other methodological arguments against the use of too complex models when working in conditions of uncertainty of the problem under consideration. We will present them in the next section. Here we only want to state that the use of such models in practice is very difficult. We have sufficient experience in developing such complex models (see [1,19–22]) to notice that in practice only their authors can use and improve them correctly. For example, in order to successfully operate with our latest model [1], the user must be fluent in the modern theory and practice of modeling and multi-criteria optimization of trading processes in conditions of uncertainties of various nature.

We do not intend to underestimate here the scientific importance of solving the problems of designing complex models. However, we believe that such studies are currently of purely scientific rather than practical interest. Therefore, here we propose the simple multi-model approach to design trading systems in the Forex market. It is based on some basic propositions, which practical validation is made in this article. The first proposition comes from the observation that Forex consists of different financial streams depending on the market makers and traders community members trading strategies. Thus, the basic challenge of this article is to substantiate the assumption that any simple model based on a rational trading idea or several ideas can well reflect some market stream and, therefore, will be profitable during some small or not so small period. Therefore, if we have developed a set of such simple models and optimized them for different currency pairs and timeframes, then in each trading period we can use the best model for a particular currency pair to maximize the overall profit. The second proposition is to use the fuzzy multiple-criteria approach in the training (optimization) stage using historical data. Formally, such an approach was already used in [1,15,19], but only to aggregate the Buy, Sell and Hold signals provided by different fuzzified technical analysis indices considered directly or implicitly as local criteria. Nevertheless, only the profit optimization criterion was used. Therefore we can say that in [1,15,19] the multiple-criteria approach plays a collateral role being used to aggregate only input parameters, while not taking into account possible important local criteria represented by the output parameters of trading systems.

However, it is not by chance that all well-known trading platforms such as MetaTrader 4 or MetaTrader 5, in addition to the resulting profit, present the obtained values of parameters such as Profit factor, Relative drawdown, %Winning trades, etc., as optimization results. These parameters together determine the reliability, the degree of risk-free results and the comfort of trading, which in practice are no less important than maximizing profits. Therefore, the motivation of our studies was the revealed need for the formulation and solution of the multiple-criteria trading optimization task based on the local criteria defined by the models' output parameters. Below we will show that this motivation is quite natural and sufficiently justified. Of course, a multiple-criteria solution, which is a compromise, will give a lower optimal profit than in the case of a solution based on one criterion (profit maximization). On the other hand, it is known that a large optimal profit does not always provide a large profit during the test period. On the contrary, it often leads to a loss of money mainly due to the greater likelihood of the overfitting effect. Meanwhile, a compromise multiple-criteria solution provides a smaller but more reliable profit. Therefore, we can assume that this reliability can be extended over testing periods, providing a sufficiently larger resulting profit in the testing period than in the case of single-criteria optimization. The validity of this assumption is proved in the current article. The analysis of the selected local criteria allows us to judge that they form a kind of hierarchical structure that is extended by using several relevant aggregation methods and aggregation of aggregating methods. Optimization based on such a hierarchy of criteria allows us to get an even more reliable solution and is implemented with significantly greater profit at the testing stage than in the case of multiple-criteria optimization without taking into account the hierarchy of criteria. This thesis is also substantiated in this article.

The main contributions of this paper are as follows:

1. Development of a set of simple, but profitable enough on the Forex market trading models-serving as a basis for their fuzzy multiple-criteria and hierarchical fuzzy multiple-criteria extensions.
2. Development of fuzzy multiple-criteria extensions of basic models based on the output trading parameters, justification of their validity using their applications to the selected currency pairs trading optimization.
3. Designing the hierarchical structure of fuzzy local criteria including Forex trading optimization based on output trading parameters and the hierarchical structure with the aggregation of aggregating modes and the substantiation of the developed approaches through practical advantages.
4. The concept of positive and negative overfittings is introduced. The concept of positive and negative overfitting is introduced. It is shown that the use of the developed hierarchical fuzzy multiple-criteria approach significantly increases the positive and reduces the negative effects of overfitting, providing significantly more profitable and reliable trading models.

The rest of the paper is organized as follows. In Section 2, we discuss some methodological aspects concerned with the justification of the use of a set of simple models instead of complex ones because of the stochastic nature of the Forex market. New proposed mathematical tools of technical analysis, such as new indicators and trading rules, are presented as well as the methods for the development of a set of profitable simple models and related problems. Some aspects of the overfitting problem are analyzed and a new concept of positive and negative overfitting is proposed and applied for the analysis. Two simple trading models implemented on the three currency pairs were used to validate the concept of trading on the set of simple models. Such a small set of profitable simple models and currency pairs occurred to be very convenient for exposing the features and problems of the hierarchical fuzzy multiple-criteria extensions of the developed simple models. To implement the simple models' concept, a new practical method called "leader corrections" was proposed and illustrated. This method allows us to partially offset the revealed negative overfitting effect with the positive overfitting one, sometimes even with considerable resulting profits. Section 3 presents the developed multiple-criteria fuzzy hierarchical approach to the design of trading models. Sets of profit dependent, risk minimization and trade reliability fuzzy local criteria are introduced and justified. The methods of aggregation of local criteria used are analyzed and the necessity of aggregation modes is justified. It is shown that the introduced sets of fuzzy local criteria together represent a natural three-level hierarchical structure. The methods for solving the corresponding hierarchical problems of fuzzy multiple-criteria optimization are presented in a general form. Section 4 presents and analyzes the results obtained in comparison with those provided using the basic single-criteria approach. It is shown that a new approach, proposed in this article, provides significantly more profitable and reliable results than those obtained using the basic approach. Section 5 concludes the article.

2. A set of profitable simple trading models development

To increase the number of potential readers of this article, we should choose properly the tools for implementation of developed trading systems. Therefore, we chose the MetaTrader 4 platform because today it is the most popular Forex trading platform in the world [23], although the MetaTrader 5 has a set of additional advantages. Therefore, we will sometimes use the terminology of MetaTrader 4 and fragments of programs written in MQL4 programming language, which can be used to design trading systems. Then as a source of data, we chose the IG broker. According to all ratings, it is the best Forex broker in the USA and the most trusted one, although its spreads do not always seem to be low.

Before the presentation of the developed models,some general methodological features of our approach should be clarified. Based on traders' observation, very often a great profit at the optimization stage is accompanied by a low profit or even losses at the testing stage. This is commonly called the "overfitting effect". Often we can meet the following assertion: An excellent great optimized profit provided is often accompanied by an extremely low profit at the testing stage; the good optimized profit provides the acceptable testing results (low but positive profit) and the satisfactory optimized profit may result in the satisfactory practically useful testing profits. Clearly, less than satisfactory optimized profit can result in too low profit at the testing stage. Of course, this statement is not always true since the word "always" in the Forex context should be treated as "all is possible". The only relatively reliable recommendation for designing practically useful trading systems (TS), supported by the traders' practice is: to get permissible results of testing, sufficiently long optimization periods should be applied. Unfortunately, this recommendation does not really help to solve the problem under consideration and is further divided into two generated practical problems: Which optimization period is long enough? and Why should we qualify optimized profits as excellent, good or satisfactory? We did not find in the literature and in the "traders community wisdom", represented in the related chats, useful practical propositions to solve these problems. Therefore, the propositions to apply long training periods can be treated only in the context of the concrete actual TS designing task at hand, not as some generalized problem. In addition, during rapid and irregular changes in the market's economic and political environment, the use of long optimization periods largely loses its meaning because, in different periods close to each other, we are actually dealing with different markets. Therefore, the TS optimization for long periods involving several such implicit markets turns out to be useless.

Taking into account the above, our experience, as well as the observations of many practicing traders, here we have proposed an approach that can greatly facilitate the solution of the above problems, based on the following methodological principles. Since Forex is a market with a high degree of stochasticity and uncertainty, in our study we will follow a well-known general methodological principle: the more uncertain the analyzed process is, the simpler its accepted model should be. The next one is the so-called inconsistency principle: "The real complexity of the system and the accuracy of its description are approximately inversely proportional". Compliance with the above principles allows us to develop an approach to design practically useful trading strategies. Then, at the first stage of our research, the simplest crisp models are developed and tested based on the minimum required number of trading instruments, using selected sets of currency pairs and timeframes. Only an optimization based on one criterion can be used, for example, profit maximization. At this stage, the models are consistently extended by including more rules and parameters. In this process, the detail of the models increases, and at some point, the complexity of the models begins to conflict with the uncertainty of the simulated trading process. Then the prediction accuracy of the models cannot grow and decreases along with the increasing complexity of the models. Following these principles, a set of simple trading models of various details has been developed in this work, based on some assumptions related to the real trading process. By doing this, we adhered to the ancient wisdom of Occam: "Entities should not be multiplied unnecessarily" and to the Pareto principle that states that for many outcomes, roughly 80% of consequences come from 20% of causes (the "vital few") [24]. Obviously, the obtained trading models provide acceptable, at least, positive profits in the testing periods. When developing trading models, we used only technical analysis tools, because, according to many experienced traders, fundamental data cannot be a source of reliable market information at the present turbulent time. To implement the above methodological principles, we have developed a set of new technical analysis tools presented in the following subsection.

2.1. New tools of the technical analysis

First, let us consider proposed new indicators. Let us analyze the classical approach to trading based on the intersection of the fast and slow Moving Average (MA) indicators. It is known that often its Buy and Sell signals are considerably delayed. In many ways, this is a consequence of the double transformation of the source data, since two Moving Averages are calculated. To reduce this effect, we propose a new indicator that is an adaptation of the indicator presented earlier in [1].

$$dSMA = (Ask + Bid) * 0.5 - SMA(nn\_parm), \tag{1}$$

where SMA is the Simple Moving Average indicator, nn_parm is the optimized period of this indicator, Ask and Bid - see Appendix. This indicator seems to be primitive in comparison with many classical ones, but it generates trading signals earlier than classical indicators since only one MA is used in the $dSMA$. Clearly, for $dSMA > 0$ we get a Buy signal, else ($dSMA < 0$) a Sell signal.

The disadvantage of classical indicators is that they are based only on closing prices, whereas important information about the price range in the Candles under consideration is lost. In addition, since trading signals are aimed at maximizing profits, it is desirable to have an indicator based directly on the current potential profit. Therefore, let us consider $dd[i] = Close[i] - Open[i]$. If $dd[i] > 0$, we can consider $dd[i]$ as a possible profit on the Long position ($PL[i]$), else ($dd[i] < 0$) the value of $abs(dd[i])$ we can treat as a possible profit on the Short position ($PS[i]$). In this case, we take into account a whole body of a Candle and the $PL[i]$, $PS[i]$ are formulated as different kinds of profit. The averaged on the nn_parm (optimized parameter) values of PL and PS are calculated by the use of the CountPLPS function (Listing 1).

Listing 1: CountPLPS function code

```
void CountPLPS(int n, double &PL, double &PS)
{   double d1, ddP;
    PL=0;   PS=0;
    ddP=Close[1]−Open[1];
    for(int i=1; i<=n; i++)
    {   d1=Close[i]−Open[i];
        if(d1>0)
        {   PL = PL + d1;
        }
        else
        {   if(d1<0)
              PS = PS + MathAbs(d1);
        }
    }
    PL=PL*MathPow(n,−1);
    PS=PS*MathPow(n,−1);
  }
```

The $PL - PS$ indicator generates trading signals as follows: *if* $PL > PS$, then this means that the average possible profit on Long positions is greater than on Short ones and therefore the Buy signal is generated. Opposite, *if* $PL < PS$, then Sell signal is generated.

Let us consider Fig. 1. We can see that $Close_2 - Close_1 = -2$. Then indicators based on Close prices (like $dSMA$) will provide the Sell signal. The $PL - PS$ indicator will generate Buy signal since PL= 5 and PS= 2. The Buy signal is more reasonable due to the fact that second Candle seems to be a usual correction of the trend observed in the first Candle. Therefore we can say that the $PP - PL$ indicator is more transparent, logically justified, and based on more information than classical indicators based only on Close prices.

We carried out special studies based on real data to compare the $dSMA$ and $PL - PS$ indicators. To do this, we have developed the simplest model, which opens Long or Short position based on the $dSMA$ or $PL - PS$ at 8 o'clock and closes it at 20 o'clock without Stop Loss (SL) and Take Profit (TP) restrictions. In all cases, the resulting profits provided by the $PL - PS$ indicator solely were significantly greater than those received using the $dSMA$ indicator. Nevertheless, in particular cases, these indicators working together produce greater profits than those we obtained when using the compared indicators separately.

An important advantage of the $PL - PS$ indicator is that it is not affected by gaps in the notations since it is based on $Close[i] - Open[i]$ differences, and not on $Close[i]$ notations only. Since the standard technical analysis indicators, as well as the new $dSMA$ indicator, are based only on closing prices, they are highly dependent on gaps that may generate incorrect trading signals.

Let us consider introduced trading rules and trading signals filters. First consider the rules for positions opening. Taking into account the above consideration, we start from the rules based on the $PL-PS$ indicator. Their general structure can be presented as follows:

$$\textbf{IF } PL > PS \textit{ and "all filters are greater of zero" } \textbf{THEN } \textit{Buy order} \tag{2}$$

$$\textbf{IF } PL < PS \textit{ and "all filters are greater of zero" } \textbf{THEN } \textit{Sell order}. \tag{3}$$

Below is a detailed description of this structure. The first filter $F1$ represents the relative strength of Buy/Sell signal:

$$F1 = Abs(PL - PS)/Max(PL, PS) - dV2, \tag{4}$$

where $dV2$ is an optimized parameter. The second filter is as follows:

$$F2 = VATR - dATR, \tag{5}$$

where $VATR$ is the simplest form of volatility measure calculated as follows:

$$\begin{aligned} &VATR = 0; \\ &\textbf{for}(int\ i = 1;\ i <= n\_ATR\_n;\ i++) \\ &\quad VATR = VATR + (High[i] - Low[i]); \\ &VATR = VATR/n\_ATR, \end{aligned} \tag{6}$$

n_ATR is the optimized period, $dATR$ is the optimized lower threshold level of volatility, so only if $F2 > 0$ the opening of Buy/Sell positions is allowed.

Unlike the stock market, Forex volumes represent only the number of trades in a Candle without information about their size. Nevertheless, when volumes grow, at some period, we can consider this as a growing market activity that strengthens Buy or Sell signals. Therefore, we propose here a market activity

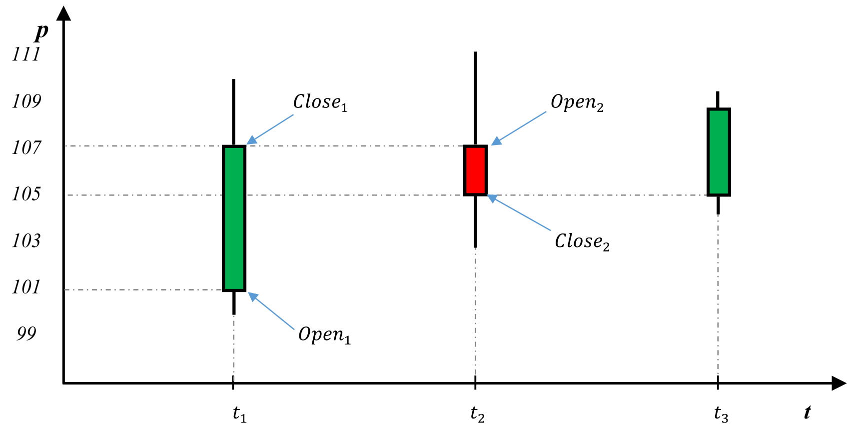

Fig. 1. The case when *dSMA* and $PL - PS$ indicators generate opposite signals.

indicator based on volumes difference between the neighboring Candles. This indicator generates the parameters *VP* and *VN* representing the averaged on the last *nV_parm* Candles positive (*VP*) and negative (*VN*) differences $Volume[i] - Volume[i-1]$ (*nV_parm* is an optimized parameter). This indicator, called $VP - VN$, is calculated using the CountVPVN function (Listing 2).

Listing 2: CountVPVN function code

```
void CountVPVN(int n, double &VP, double &VN)
{   double d1;   VP=0;   VN=0;
    for(int i=2; i<=n+1; i++)
    {   d1=Volume[i-1]-Volume[i];
        if(d1>0)
        {   VP = VP + d1;   }
        else
        {   if(d1<0)
               VN = VN + MathAbs(d1);
        }
    }
    VP=VP*MathPow(n,-1);   VN=VN*MathPow(n,-1);
}
```

Then the strength of market activity growing can be presented as $MAG = Abs(VP - VN)/Max(VP, VN)$.

The corresponding filter is as follows:

$$F3 = MAG - dV3, \tag{7}$$

where $dV3$ is the lower threshold of *MAG* to be optimized. The market signals Buy or Sell can be activated only if $F3 > 0$.

The filters $F1$ and $F3$ can be treated as local criteria supporting Buying/Selling decisions. Since their values lay in the unit interval [0,1], we can aggregate them into generalized trading signals support criterion $GSC = w1 * F1 + w3 * F3$, where $w1$, $w3$ are the local criteria weights to be optimized, taking into account that $w1 + w3 = 1$. Therefore we have introduced the filter

$$F4 = DSC - dV4, \tag{8}$$

where $dV4$ is the lover optimized threshold.

In our studies, we used only weighted sum aggregation of local criteria *GSC*, although it can provide an illogical solution where there is no evident tradeoff between local criteria [25,26]. The considered local criteria can be treated as the membership functions of fuzzy subsets, and the powerful mathematical tools of the fuzzy sets theory can be used. We did that in our papers [1,19–22], but here we try to use only minimal tools to avoid redundant complications and make the article more readable.

The introduced filters F1–F4 can be used in the rules (2) and (3) in different combinations generating a set of simple models. Consider the rules based on the joint use of the *dSMA* and $PL - PS$ indicators. To formulate them, we will use the following basic rules:

$$\begin{aligned}&\textbf{IF } dSMA > 0 \textbf{ THEN } Buy;\\&\textbf{IF } PL > PS \textbf{ THEN } Buy;\\&\textbf{IF } dSMA < 0 \textbf{ THEN } Sell;\\&\textbf{IF } PL < PS \textbf{ THEN } Sell.\end{aligned} \tag{9}$$

The most cautious rules based on the two indicators under consideration can be represented by a consensus of the basic rules:

$$\begin{aligned}&\textbf{IF } dSMA > 0 \textit{ and } PP > PL \textbf{ THEN } Buy;\\&\textbf{IF } dSMA < 0 \textit{ and } PP < PL \textbf{ THEN } Sell.\end{aligned} \tag{10}$$

In practice, we often observe conflicting situations when $dSMA > 0$ and $PL < PS$, or $dSMA < 0$ and $PL > PS$. Of course, this can be treated as the absence of signal, but a more promising approach is based on the comparison of actual strengths of the basic rules (9). In this case, no signal is generated, but only in the case when the difference between the strengths of $dSMA < 0$ and $PP > PL$ is greater than some minimal optimized value $d1$. The relative strengths of $dSMA\langle\rangle 0$ (*dDD*) and $PL\langle\rangle PS$ (*dPP*) and their difference (*dd*) are calculated as follows. Taking into account the expression (1), we obtain:

$$\begin{aligned}a &= (Ask + Bid) * 0.5;\\ b &= SMA;\\ dDD &= Abs(a - b)/Max(a, b);\\ dPP &= Abs(PL - PS)/Max(PL, PS);\\ dd &= Abs(dPP - dDD).\end{aligned} \tag{11}$$

Then we get the following rules:

$$\textbf{IF } (dSMA > 0 \textit{ and } PL > PS) \textit{ and “all filters are greater of zero” } \textbf{THEN } Buy, \tag{12}$$

$$\textbf{IF } (dSMA < 0 \textit{ and } PL < PS) \textit{ and “all filters are greater of zero” } \textbf{THEN } Sell, \tag{13}$$

$$\textbf{IF } (dSMA > 0 \textit{ and } PL < PS \textit{ and } dDD > dPP \textit{ and } dd > d1) \textit{ and “all filters are greater of zero” } \textbf{THEN } Buy, \tag{14}$$

$$\textbf{IF } (dSMA > 0 \textit{ and } PL < PS \textit{ and } dDD < dPP \textit{ and } dd > d1) \textit{ and “all filters are greater of zero” } \textbf{THEN } Sell, \tag{15}$$

$$\textbf{IF}\ (dSMA < 0\ and\ PL > PS\ and\ dDD < dPP\ and\ dd > d1)\ and\ \text{“}all\ filters\ are\ greater\ of\ zero\text{”}\ \textbf{THEN}\ Buy, \tag{16}$$

$$\textbf{IF}\ (dSMA < 0\ and\ PL > PS\ and\ dDD > dPP\ and\ dd > d1)\ and\ \text{“}all\ filters\ are\ greater\ of\ zero\text{”}\ \textbf{THEN}\ Sell. \tag{17}$$

In the rules (12)–(17), the filters $F1$–$F4$ can be used in all possible combinations. Consider the tools used to open positions. The main function OrderSend is used in MetaQuotes 4 to open position and simultaneously establish the Stop Loss (ST) and Take Profit (TP) limits:

$$\begin{aligned}&int\ \textbf{OrderSend}\,(string\ symbol,\ int\ cmd,\ double\ volume,\\&\quad double\ price,\ int\ slippage,\\&\quad double\ stoploss,\ double\ takeprofit,\ string\ comment = NULL,\\&\quad int\ magic = 0,\\&\quad datetime\ expiration = 0,\ color\ arrow\_color = clrNONE);\end{aligned} \tag{18}$$

In our applications, this function takes the forms:

$$\begin{aligned}&\textbf{IF}\ (OrderSend(Symbol(),\ OP\_BUY,\ 1,\ Ask,\ 20,\ Bid - dSL,\ Ask\\&\quad +dTP,\ NULL,\ MAGICMA,\ 0,\ Green) < 0)\\&\textbf{THEN}\ Buy\ signal\end{aligned} \tag{19}$$

$$\begin{aligned}&\textbf{IF}\ (OrderSend(Symbol(),\ OP\_SELL,\ 1,\ Bid,\ 20,\ Ask + dSL,\ Bid - dTP,\\&\quad NULL,\ MAGICMA,\ 0,\ Red) < 0)\\&\textbf{THEN}\ Sell\ signal\end{aligned} \tag{20}$$

It is seen that the volume is equal to 1 because we used only one lot in trading and the slippage is equal to 20 that means the acceptance of difficult trading conditions. The SL and TP levels are calculated as follows: $SL = Bid - dSL$, $TP = Ask + dTP$ for the Long position; $SL = Ask + dSL$, $TP = Bid - dTP$ for the Short position. The values of dSL and dTP are assumed to be dependent on the volatility. Therefore they were calculated as follows:

$$\begin{aligned}&avgHL = 0;\\&\textbf{for}(int\ i = 1;\ i <= Nv;\ i++)\\&\quad avgHL = avgHL + (High[i] - Low[i]);\\&dSL = avgHL * wl\_SL;\\&dTP = avgHL * wp\_TP,\end{aligned} \tag{21}$$

where wl_SL, wp_TP and Nv are optimized parameters. The ST and TP are corrected at each new Candle if a considerable relative price movement takes place:

$$dP = Close[i] - Open[i]; \tag{22}$$

$$\begin{aligned}&\textbf{IF}\ \text{“}position\ is\ Long\text{”}\ and\ dP > 0\ and\ dP/Close[i]\ >\ d22\\&\textbf{THEN}\quad SL = SL + ChangingPoints;\quad TP = TP + ChangingPoints,\end{aligned} \tag{23}$$

$$\begin{aligned}&\textbf{IF}\ \text{“}position\ is\ Short\text{”}\ and\ dP < 0\ and\ abs(dP)/Close[i]\ >\ d22\\&\textbf{THEN}\quad SL = SL - ChangingPoints;\quad TP = TP - ChangingPoints,\end{aligned} \tag{24}$$

where $d22$ and $ChangingPoints$ are optimized parameters. Finally, the SL and TP correction is provided by the OrderModify function, and the OrderClose function closes open positions if necessary. In practice, we often see opportunities to close losing position early than SL level is reached. Therefore, we call such situations Fast Stop Loss (FSL) and the corresponding filter is calculated as follows:

$$F5\ =\ Abs(Close[i] - Open[i])\,/\,VATR - d3, \tag{25}$$

where $VATR$ is the volatility measure (see (6)) and $d3$ is the optimized threshold parameter. We can see that this filter is based on the relative strength of the current price changing (rising or falling) in the direction opposite to the current open position.

Let us assume that we have a Long position and, suddenly, there is a local trend in the Short direction. If it is small, we can consider it as a local trend correction, but if it is already large enough, we can consider it a trend reversal. So, we have to close a Long position. Similarly, in the case of an open Short position. The relevant rules are as follows:

$$\begin{aligned}&\textbf{IF}\ \text{“}we\ have\ open\ Long\ position\text{”}\ and\ Close[i] - Open[i] < 0\ and\\&\quad F5 > 0\\&\textbf{THEN}\quad Close\ position,\end{aligned} \tag{26}$$

$$\begin{aligned}&\textbf{IF}\ \text{“}we\ have\ open\ Short\ position\text{”}\ and\ Close[i] - Open[i] > 0\ and\\&\quad F5 > 0\\&\textbf{THEN}\quad Close\ position,\end{aligned} \tag{27}$$

The developed set of technical analysis tools, including new indicators, thresholds and rules, allows us to build a relatively large set of simple profitable models for trading on the Forex market based on their reasonable combinations. We have no intention of presenting and validating them all, taking into account the goals of this article — to develop a hierarchical multiple-criteria fuzzy extension of the basic single-criteria crisp models and prove its practical effectiveness. Therefore, we will consider only two simple models with three currency pairs, characterized by a wide variety of profits provided during a year.

2.2. Validation of developed simple models

For our analysis, we chose three symbols USDJPY, AUDUSD and EURUSD, since the broker chosen (IG broker) provides almost the same typical spread for them: 9 p. for EURUSD and 10 p. for USDJPY and AUDUSD. This greatly facilitates the analysis of the results.

In the course of our preliminary research using the developed simple models, we found that the annual profit received by the USDJPY symbol is about twice as much as from the AUDUSD symbol, which, in turn, is twice as profitable as the EURUSD symbol. This is very important as we will study the effects of multiple-criteria fuzzy extensions of the simple models of low, moderate, and significant initial profitability. We will use only the H4 timeframe, since it is preferred by professional traders and it is less sensitive to the influence of uncertain factors such as slippage, swaps, etc. In our models, we start trading at 8 o'clock and close all open positions at 20 o'clock to avoid swaps and uncontrolled surprises at night when we sleep. We will use only two models named $M1$ and $M2$, providing the greatest annual profits for the selected symbols $USDJPY$, $AUDUSD$ and $EURUSD$ in 2022. The $M1$ model is the simplest, capable of making a noticeable profit. It consists of rules (2)–(3) without filters, rules (19)–(20) used to open positions and set SL and TP levels, and rules (23)–(24) for modifying SL and TP levels. The $M2$ model is an extension of $M1$ combining the rules (2)–(3) with the filter $F1$, the rules (19)–(20), (23)–(24) and, in addition, the rules (26)–(27) were used to close unprofitable positions earlier than SL when that is reasonable.

The set of model $M1$ optimized parameters is $ParM1 = (n\_parm,\ wl\_SL,\ wp\_TP,\ Nv,\ ChangingPoints)$. Therefore, the corresponding optimization problem can be formulated as

$$ParM1_{opt} = Arg(Max\ Profit_{M1}(ParM1)), \tag{28}$$

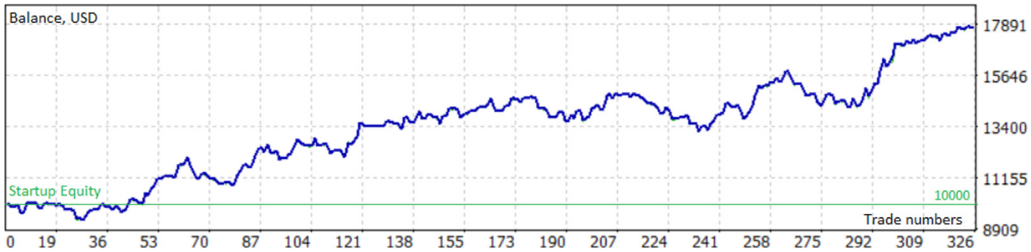

Fig. 2. The balance curve: Model $M1$, *USDJPY*, $H4$, optimization period 2021.01.01–2022.01.01. Total net profit= \$7730, Profit factor = 1.39, Relative drawdown = 13%, Total trades = 325, Profit trades = 46%.

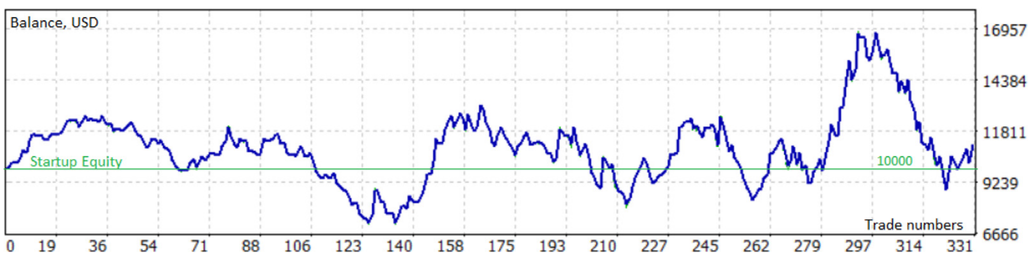

Fig. 3. The balance curve: Model $M1$, *USDJPY*, $H4$, testing period 2022.01.01–2023.01.01. Total net profit = \$1125, Profit factor = 1.02, Relative drawdown = 48%, Total trades = 330, Profit trades = 47%.

where $Profit_{M1}$ is the profit provided by the model $M1$. Similarly, for the model $M2$ we get

$$ParM2_{opt} = Arg(Max\ Profit_{M2}(ParM2)), \tag{29}$$

where $ParM2 == (n\_parm,\ wl\_SL,\ wp\_TP,\ Nv,\ ChangingPoints,\ n\_ATR,\ dV2,\ d3)$, $Profit_{M2}$ is the profit provided by the model $M2$. These and all subsequent optimization tasks were solved using the genetic algorithm built into the MetaTrader 4 platform.

In the considered models $M1$ and $M2$, we do not use the *dSMA* indicator in the rules (2)–(3) because, according to our research, in general, it brings significantly smaller profits than the $PL - PS$ indicator if it is used exclusively. We obtained noticeable profits on the *GBPUSD* and *USDCHF* symbols, which are characterized by high volatility, only when *dSMA* and $PL - PS$ indicators were used jointly in the rules (12)–(17) with the filter $F3$. Consider the results obtained using the model $M1$ for the symbol *USDJPY* and the timeframe $H4$ on the optimization period 2021.01.01–2022.01.01 and the testing period 2022.01.01–2023.01.01, presented in Figs. 2–3. These charts and all subsequent ones are parts of MetaTrader 4 platform reports, where the abscissa axis represents Candle numbers, the ordinate axis represents the Balance (Balance = Profit + Equity). In all our studies, we assumed Equity= \$10 000.

Of course, the result presented in Fig. 3 is generally unacceptable. However, we are not obligated to use such long testing periods. Let us remember the folk wisdom "if something moves in a certain direction, it will most likely continue this movement for some time" and assume that this time is 1.5 months from the start of the testing period. Then we get an excellent result presented in Fig. 4. Therefore, in our case, the following procedure seems to be very profitable: optimize the model for the last 12 months, then trade for 1.5 months and withdraw the profit of \$2232; then re-optimize the model for the last 12 months, including the past testing period (trading) and trade again for 1.5 months, etc. This procedure of sequential re-optimization is called the window moving method and requires setting the duration of optimization and testing periods.

According to our research, when using our models and timeframes $H1$ and $H4$, the greatest profits were obtained with the use of one year optimization period and the testing period of one month. The results obtained for 1 Lot trading with the Leverage 1:100 on the selected symbols using two selected models are presented in Table 1. The table shows the profits received on the annual optimization periods and monthly profits on the following testing periods. We can see that the annual profit from the symbol *USDJPY* is more than twice as high as from the *AUDUSD* symbol, which in turn is more than twice as high as from the symbol *EURUSD* with quite comparable risk-free degrees estimated as a percentage of profitable months. The developed models $M1$ and $M2$ can be used in real trading. Nevertheless, for all the symbols considered, we see significant local losses in the testing periods, despite promising results in the optimization ones, especially for the symbol *AUDUSD* in July and September 2022. Obviously, in order to get rid of this undesirable phenomenon or at least reduce its impact, it is necessary to understand its essence.

At first glance, in its external form, this is a typical overfitting effect. Overfitting is a problem of general importance, therefore, many of its different definitions can be found in the literature [27–31]. However, almost all of them somehow correlate with the inconsistency principle discussed at the beginning of this Section.

In the stock market and Forex market research, overfitting is usually considered as a problem of forecasting stock prices and exchange rates. Some methods have been proposed to reduce the negative impact of the overfitting effect in financial market research. They are based on the so-called regularization of L1/L2 to the loss function [32,33] or on noise in the initial data at the optimization stage [34]. A fairly broad overview of such methods was made in [18]. It is important that, according to the authors' conclusion, all the analyzed methods still need manual work for constant adjustment of hyperparameters. This greatly hinders their use in practice.

Since we are dealing with the design of real trading systems in the Forex market, we mainly strive for direct forecasting of profits obtained, taking into account market conditions, including spreads, slippage, transaction costs, etc. Therefore, a trading strategy based only on forecasting exchange rates is usually unprofitable, even if it is 100% accurate. In addition, the desire to maximize the accuracy of the exchange rates prediction often turns out to be in contradiction with market practice, since the most profitable strategies often generate no more than 30% or

Table 1
The profits (*USD*) obtained using the single criterion approach (*Profit* → *max*).

Period	*USDJPY* single criterion *Profit* → *max* model *M*1		*EURUSD* single criterion *Profit* → *max* model *M*2		*AUDUSD* single criterion *Profit* → *max* model *M*1	
	Optimized annual profit	Test monthly profit	Optimized annual profit	Test monthly profit	Optimized annual profit	Test monthly profit
01.2022	7384	1826	10 835	3331	10 418	3720
02.2022	9046	−860	16 768	−149	14 389	526
03.2022	7386	706	13 189	887	11 784	−1491
04.2022	7312	−1283	11 765	−2576	15 445	893
05.2022	6770	536	10 692	2561	15 118	3790
06.2022	8286	2992	14 051	601	16 888	571
07.2022	13 355	421	14 063	−1643	15 687	−3154
08.2022	12 632	−1228	14 324	519	16 418	3083
09.2022	12 916	−199	11 608	191	17 808	−2685
10.2022	12 693	−1781	11 365	2413	13 802	1162
11.2022	13 677	6245	13 982	−1653	17 438	2305
12.2022	21 037	7632	10 572	−1374	18 132	−1636
Sum		15 007		3108		7084
Winning months		58%		58%		67%
Average monthly profit		1250		257		590

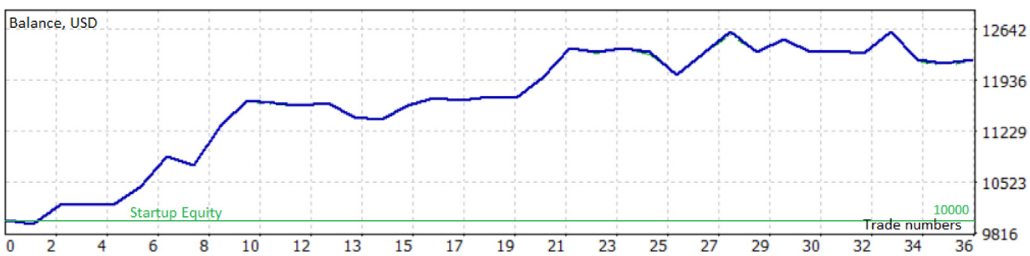

Fig. 4. The balance curve: Model *M*1, *USDJPY*, *H*4, testing period 2022.01.01–2022.02.15. Total net profit = $2232, Profit factor = 2.02, Relative drawdown = 6.5%, Total trades = 36, Profit trades = 53%.

Fig. 5. The balance curve: *USDJPY*, model *M*1, *H*4, optimization period 2021.11.01–2022.11.01. Total net profit = $15 883, Profit factor = 1.37, Relative drawdown = 14%, Total trades = 437, Profit trades = 40%.

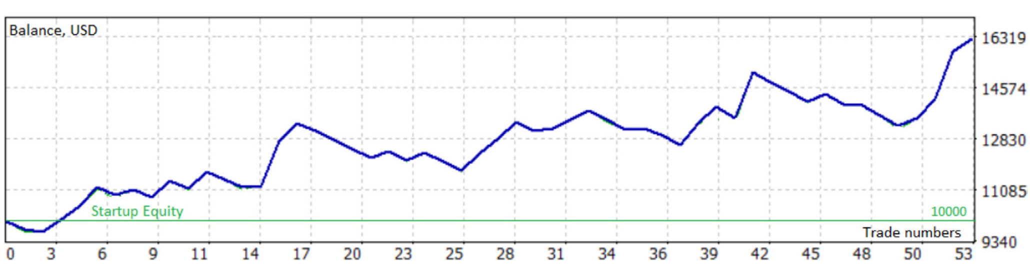

Fig. 6. The balance curve: *USDJPY*, model *M*1, *H*4, testing period 2022.11.01–2022.12.15. Total net profit = $6269, Profit factor = 1.82, Relative drawdown = 17%, Total trades = 53, Profit trades = 49%.

less profitable trades. Let us consider the balance curve on the year optimization period 2021.11.01–2022.11.01 (Fig. 5) and on the following testing 1.5 month period 2022.11.01–2022.12.15 (Fig. 6).

To compare the results obtained during the optimization and testing periods, we will use their average monthly profits, which are 1319 and 4179, respectively. Thus, we can say that, in this case, the trading during the testing period is 3.2 times more profitable than during the optimization period. In addition, such important parameters of trading quality as the Profit factor and the Profit trades are significantly higher in the testing period. The exception is the parameter Relative drawdown, which is

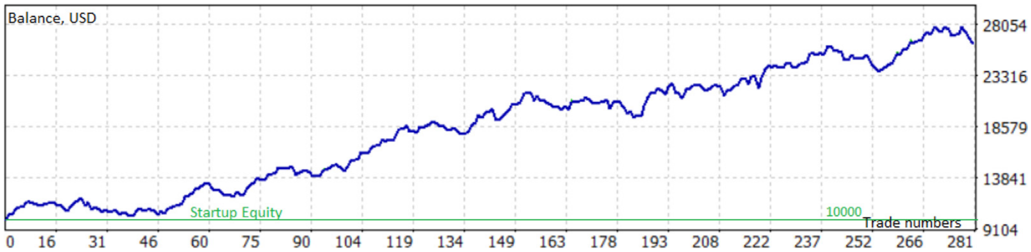

Fig. 7. The balance curve: *AUDUSD*, model $M1$, $H4$, optimization period 2021.09.01–2022.09.01. Total net profit = \$16 367, Profit factor = 1.51, Relative drawdown = 16%, Total trades = 280, Profit trades = 58%.

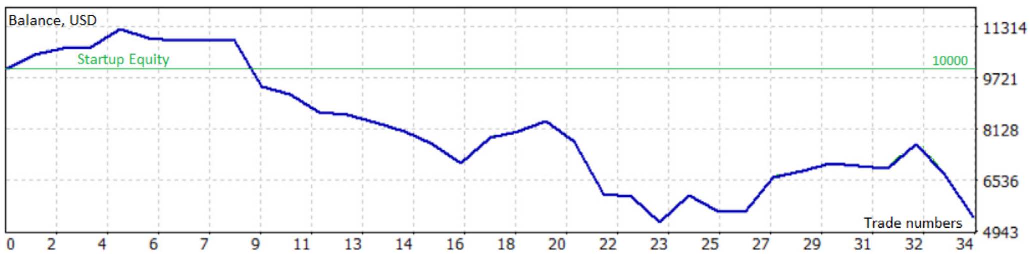

Fig. 8. The balance curve: *AUDUSD*, model $M1$, $H4$, testing period 2022.09.01–2022.10.15. Total net profit = -\$4586, Profit factor = 0.55, Relative drawdown = 59%, Total trades = 34, Profit trades = 42%.

slightly larger in the test period, but this is generally undesirable. However, in the case under consideration, this does not matter, since the value of the parameter Relative drawdown = 17% is less than the generally accepted upper limit of the acceptable values of this parameter. Thus, we can confirm the overwhelming advantage of trading in the test period in this case. From a strictly scientific point of view, the case considered is an example of pronounced significant overfitting, since in the case of zero overfitting, we would see zero difference between the monthly profits for the optimization and testing periods. To simplify the further analysis, we will call this phenomenon "positive overfitting". The example considered is not, to some extent, accidental. In Table 1, we can see 22 cases of profitable months, most of which are cases of overfitting. The standard treatments of overfitting are too universal and, therefore, not useful to operate with the positive overfitting effect. We cannot explain the nature of positive overfitting by a sudden change in market behavior, because in many cases we do not observe such changes. In addition, this explanation is so general that with its help, we can explain all the mysterious cases in the Forex market. The fashionable theory of "black swans" [35] is also not suitable for explanation, since we can see too many such swans even in Table 1. Thus, we often encounter situations of strong positive overfitting, but in practice we can only thank them for the large profit they provide. In any case, we do not intend to fight them. This means that we should not abandon its use in practical trading, although we do not have a clear explanation of the positive overfitting effect. Another unexpected logically sound conclusion is that, based on the above analysis, the use of known methods of reducing overfitting can lead to eliminating of positive overfitting and, consequently, to a decrease in profits.

Let us consider Figs. 7 and 8. We will call the observed phenomenon "negative overfitting", the essence of which practically corresponds to the definitions proposed in the traders' community. They can be summarized as follows: if we get a large profit at the optimization stage and significant losses at the testing stage, then we have an overfitting problem.

In other words, positive overfitting takes place when the strength of the uptrend of profit at the testing stage is noticeably more significant than at the optimization stage, negative overfitting is a situation of a profits' downtrend at the testing stage, and the profits' side trend means relatively small fluctuations in profit around some not so great value at the testing stage. The side trend rarely occurs (see Table 1) and does not play a noticeable role in the trading. Therefore the final results (profits or losses) are defined by competition of positive and negative overfittings. In Table 1, we can see that such a struggle can ultimately bring large profits, accompanied by relatively low risks, defined as the relative number of unprofitable transactions. The following question arises: how to organize trading based on such information as presented in Table 1?

For convenience, here we introduce an object that we call a trading tool (TT), consisting of three components: TT = {model, timeframe, symbol}. Then $TT1$ = {$M1$, $H4$, $USDJPY$}, $TT2$ = {$M2$, $H4$, $EURUSD$} and $TT3$ = {$M1$, $H4$, $AUDUSD$}. Due to the complete dominance of trading with the $TT1$, the use of portfolio approaches is impractical; therefore it makes sense to trade mainly with the $TT1$, using the other two TTs as an auxiliary if it is reasonable. Then we can simulate the trading as follows. Imagine that we start trading $TT1$ from the beginning of 2022. Then in January we get a big profit (positive overfitting), in February — small allowable losses, and in March — a small profit (see Table 1). The first real problem we meet at the beginning of April, when we observe systematic losses during the first 14 candles (see Fig. 9). Suppose we cannot tolerate these losses any longer, especially considering that they are similar to negative overfitting. Then we can look for a suitable TT which is characterized by a strong and steady growth of profit since the beginning of April and has a profitable history. We do not see such a TT in Table 1, however, we found a suitable one ($TT4$ = {$M2$, $H4$, $USDCHF$}) in our depository (see Fig. 10). Then, if we switch trading from $TT1$ to $TT4$ on the Candle at number 14, we will not only compensate for the losses of $TT1$, but also, thanks to the large positive overfitting of $TT4$, we will get a significant final profit. Therefore we called this practical method "leader corrections".

Let us turn to the overfitting problem. In the Forex literature, the following "practical advice" to reduce negative overfitting often can be found. It is based on the observations that overfitting is often accompanied by too great optimal profits. Then some suboptimal solutions with a lower optimal profit, but simultaneously with acceptable positive profits at the testing stage, can be found. This assumption is implicitly supported by the

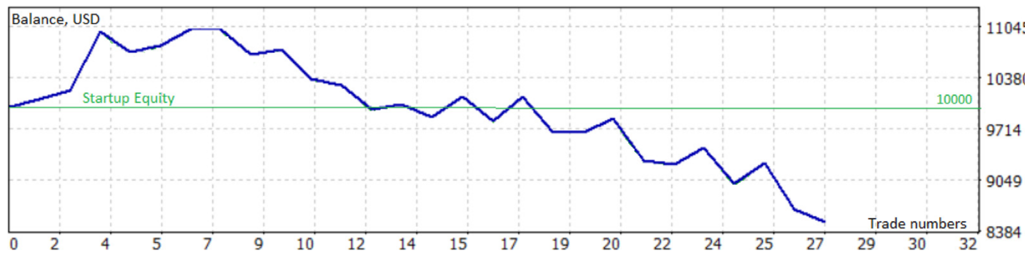

Fig. 9. The balance curve: $TT1$, testing period 2022.04.01–2022.05.01. Total net profit = −$1490, Profit factor = 0.64, Relative drawdown = 25%, Total trades = 27, Profit trades = 48%.

Fig. 10. The balance curve: $TT4$, testing period 2022.04.01–2022.05.01. Total net profit = $2696, Profit factor = 2.6, Relative drawdown = 12%, Total trades = 23, Profit trades = 61%.

Table 2
The suboptimal results for $TT1$, year optimal (2022.08.01–2022.08.01) and monthly testing periods.

$Profit_{opt}$	14 264	10 500	7061	4000	1000
$Profit_{test}$	−457	−1182	−3938	−3742	−2344

MetaTrader 4 platform, which, as a result of the optimization stage, provides the table with rows containing the parameters of a large number of suboptimal solutions allocated from the solution with the highest profit ($max\ Profit_{opt}$) to the solution with the lowest profit ($min\ Profit_{opt}$). It is usually recommended to use a suboptimal solution with a profit near the middle of the interval [$min\ Profit_{opt}$, $max\ Profit_{opt}$] with good other properties that are not clearly defined. Perhaps sometimes this gives acceptable results, especially thanks to MetaTrader 4, which provides the appropriate tools to help do this manually. We carefully studied this problem and came to the conclusion that the described approach can overcome the negative overfitting problem only randomly.

The typical result is presented in Table 2. We can see that the worst suboptimal solution (with the greatest negative overfitting) was obtained for the $Profit_{opt}$ set in the middle of the interval [$min\ Profit_{opt}$, $max\ Profit_{opt}$]. The best result was obtained for the $Profit_{opt} = max\ Profit_{opt}$. All this is in complete contradiction with the recommendations described above based on the sentiments of the traders' community.

Nevertheless, in these vague formulations, a completely rational and feasible proposal is hidden or implicitly presented: in order to overcome or reduce the negative impact of overfitting, it is necessary to solve the problem of multiple-criteria optimization that provides a compromise of the local criteria of trading quality, in which the profit maximization would be most important, but nevertheless one of the local criteria. In the next section, we will show that such an approach, in many cases, completely eliminates or considerably reduces the negative overfitting effect while enhancing the positive overfitting one, and all this is collectively expressed in a significant increase in profit. At last, we do not advise rushing with the use of known methods of combating the overfitting, since they do not take into account and apparently destroy the positive overfitting effect identified in this work, which often plays a decisive role in the formation of profit as it will be shown in the next section.

3. A multiple-criteria fuzzy hierarchical trading model

A multiple-criteria approach is not something exotic in the application to the designing of trading models, because it reflects the thinking of the trader in the decision-making process. To illustrate, consider the case of comparing two obtained profits:

- Profit1 = GrossProfit1 - GrossLoss1 = $100 - $92 = $8,
- Profit2 = GrossProfit2 - GrossLoss2 = $15 - $11 = $4.

At first glance, we should choose Profit1, which is twice as much as Profit2. Let us introduce the Relative Profit (RP), RP = (GrossProfit - GrossLoss)/GrossProfit. This parameter is a measure of the reliability of profit, since with large Grossprofit and Grossloss, a relatively small difference between them (profit) may be statistically insignificant, since small fluctuations in the values of Grossprofit and Grossloss may lead to losses instead of profit. In our case, we have $RP1 = 0.08$, $RP2 = 0.27$ and $RP2/RP1 = 3.3$. Thus, the first profit is 2 times greater than the second one, but 3.3 times less reliable. The conflict between informally defined local criteria of profitability and reliability is obvious, and a compromise solution is needed. Since all local criteria of trade quality, based on the output parameters of trade, are in conflict, the use of a multiple-criteria approach seems to be best choice. A multiple-criteria solution at the optimization stage, which is a compromise of local criteria of profit maximization, reliability and riskless, should propagate this compromise to the testing stage through optimized parameters. As a result, we will get a model that is resistant to random fluctuations in the market and, at the same time, focused on maximizing profit. This should eliminate, or at least, reduce the negative overfitting effect and, as a result, increase the strength of the positive overfitting effect. All this leads to an increase in profits.

3.1. Formulation of fuzzy local criteria

Using the tools of fuzzy set theory, in particular membership functions, to formalize local criteria seems quite natural.

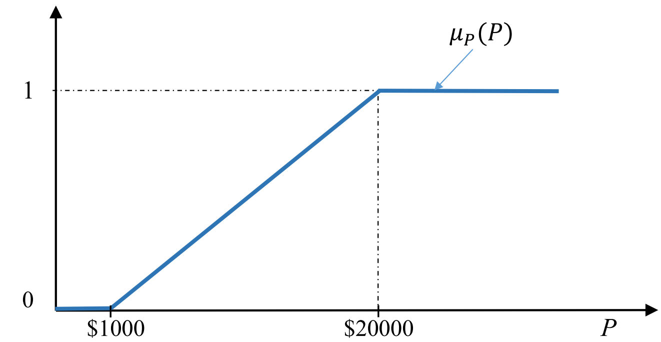

Fig. 11. The membership function $\mu_P(P)$ of the profit maximization criterion.

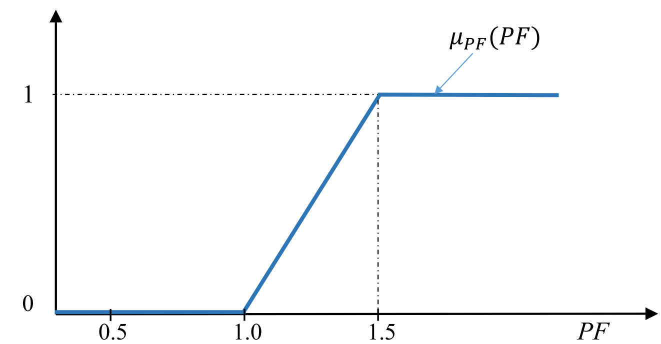

Fig. 12. The membership function $\mu_{PF}(PF)$ of the Profit factor maximization criterion.

Meanwhile, in the community of traders, we cannot find consensus on the values of the parameters used to develop these membership functions. Therefore, in order to take into account the observed diversity of traders' opinions, it would be possible to use more complex mathematical tools, such as fuzzy sets of type 2 or interval fuzzy sets of type 2. This approach has been successfully applied in other applications [36,37]. However, here we will consider this approach only as a direction for future work. Therefore, in order to exclude a possible discussion related to the establishment of boundaries of supports for membership functions representing local criteria, here we will use extremely broad supports that we have found in the literature or based on our own experience.

Consider the profit-dependent local criteria (PC)

The first and most important is the profit maximization criterion represented by the membership function $\mu_P(P)$ (see Fig. 11) rising from zero to 1 in the interval [1000,20 000], which is seems to be quite suitable in our conditions (one year optimization period, Equity = \$10 000, Leverage 1:100 and one Lot trading).

The second criterion is based on the Profit Factor, defined as the Gross profit divided by the Gross loss (including commissions) for the entire trading period. Therefore the corresponding membership function $\mu_{PF}(PF)$ (see Fig. 12) rises from zero to 1 in the interval [1, 1.5]. This is based on the common traders' opinion.

These local criteria should be aggregated into the general profit dependent criterion APC taking into account their weights (relative importance) W_P, W_{PF} such that $W_P + W_{PF} = 1$:

$$APC = Agg(W_P, \mu_P(P), W_{PF}, \mu_{PF}(PF)), \tag{30}$$

where Agg is the selected aggregating operator. In our opinion, we can assume $W_P = 0.7$ and $W_{PF} = 0.3$.

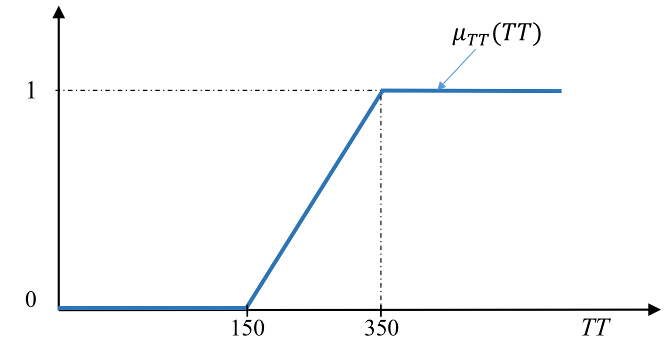

Fig. 13. The membership function $\mu_{TT}(TT)$ of the Total Trades number maximization criterion.

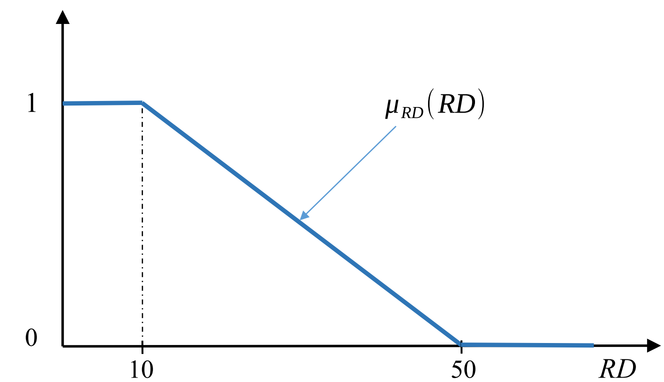

Fig. 14. The membership function $\mu_{RD}(RD)$ of the Relative drawdown minimization criterion.

Risk minimization local criteria (RC)

The first criterion depends on the Total Trades number (TT). This criterion reflects the well-known fact of an increase in the probability of negative overfitting, i.e., an increase in losses along with a decrease in TT. Therefore, this is a risk criterion increasing from 0 to 1 in the range [150, 350] (in our conditions). Its membership function $\mu_{TT}(TT)$ is presented in Fig. 13.

The next local risk criterion is the Relative drawdown minimization. Relative drawdown (RD) is the percentage of the difference between the maximum of equity and the subsequent minimum of equity to the maximum of equity. Thus, if RD increases, the risk of losses also increases. Consequently, this criterion can be represented by the membership function $\mu_{RD}(RD)$ (see Fig. 14), decreasing from 1 to 0 in the range [10%,50%], based on the diversity of traders' opinions.

The aggregation of the above local criteria into the general risk criterion ARC is presented as follows:

$$ARC = Agg(W_{TT}, \mu_{TT}(TT), W_{RD}, \mu_{RD}(RD)), \tag{31}$$

where Agg is the chosen aggregating operator, W_{TT} and W_{RD} are the weights (relative importance) of corresponding local criteria such that $W_{TT} + W_{RD} = 1$. In our opinion, we can assume $W_{TT} = 0.6$, $W_{RD} = 0.4$.

Consider the local criteria of trade reliability (or trade comfort) (CC)

These criteria are not directly related to the financial success of trading, rather they implicitly reflect some of the moods of traders who are worried not only about their money, but also about their mental comfort and reliability of the results obtained at the optimization stage. Among many possible such criteria, we choose the two that, in our opinion, seem the most obvious and are used in practice without any mathematical formalization,

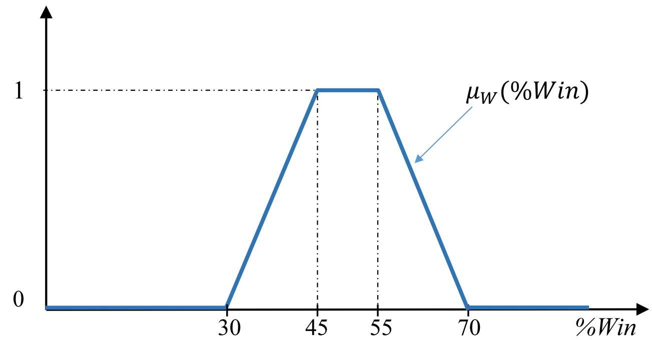

Fig. 15. Winning trades relative number dependent criterion.

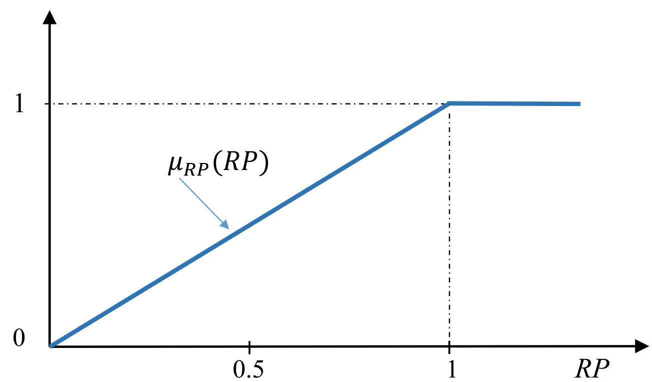

Fig. 16. Relative profit dependent criterion.

based mainly on the trader's experience and intuition. It is known that the high *%Win* generally does not provide a great total profit, and that it is possible to build a profitable strategy with only 20%–30% of winning trades. Nevertheless, we feel very insecure and anxious if our strategy provides less than 30%Win. But we feel great when the *%Win* oscillates in the range of 45%–55%. The greater values of *%Wind* (greater than 55%) involve the suspicion that the strategy overfitting took place. The above consideration allows us to present the local criterion $\mu_W(\%Win)$ based on *%Win* as the trapeze [30%, 45%, 55%, 70%] (see Fig. 15). Of course, the critical points of this trapeze can be treated as optimized parameters if this is sensible.

The second local criterion is based on the following parameter we have named Relative Profit (*RP*), *RP* = (GrossProfit − GrossLoss)/GrossProfit. Often the relatively high net Profit (GrossProfit − GrossLoss) may be obtained as the difference between so high GrossProfit and GrossLoss that the obtained result (net Profit) seems to be unreliable and even doubts about the statistical validity of the result may arise (see more detailed analysis at the beginning of this section). Then taking into account that the greater *RP*, the better the general strategy performance, the corresponding local criterion $\mu_{RP}(RP)$ was designed as the function rising from 0 to 1 when *RP* is increasing in the interval [0,1] (see Fig. 16).

Finally, the general trading comfort (or reliability) criterion can be presented as the aggregation:

$$ACC = Agg(W_W, \mu_W(\%Win), W_{RP}, \mu_{RP}(RP)), \tag{32}$$

where *Agg* is the used aggregating operator, the values of weight W_W and W_{RP} are assigned by the expert (trader) or optimized. At the first stage of studies, we can assume $W_W = 0.6$, $W_{RP} = 0.4$.

Consider the used methods for aggregation of local criteria. Suppose we have n local criteria of varying importance W_i, determined by the corresponding membership functions μ_i. Then, to solve a multiple-criteria optimization problem, they must be combined into some generalized criterion. There are so many aggregation operators proposed in the literature that it seems almost impossible to conduct an exhaustive review of them. Generally, choosing an appropriate method for aggregation is an application-dependent problem [38]. Therefore, here we will limit ourselves to using only three simplest aggregation operators based on the minimal use of basic mathematical operations applied for aggregation:

- weighted sum aggregation

$$Agg_1 = \sum_{i=1}^{n} W_i * \mu_i \tag{33}$$

- geometrical aggregation (Harrington [39], 1965)

$$Agg_2 = \prod_{i=1}^{n} \mu_i^{W_i} \tag{34}$$

- maximal pessimism aggregation (Yager [40],1979)

$$Agg_3 = \min \mu_i^{W_i} \quad for \quad i = 1, \ldots, n. \tag{35}$$

It is worth noting that operation (34) was proposed by Harrington in the context of the desirability function introduced in [39], which in the formulation of the multiple-criteria optimization problem practically coincides with the membership function. It is important that operators (33)–(35) usually serve as atomic ones for designing more complex operating modes. It was shown in [25] that the use of operators (33) and (34) can lead to illogical results, whereas operator (35) is not burdened with such problems. To illustrate, consider the following example. Suppose that we aggregate two defined in the Section 3.1 criteria $\mu_P(P)$ and $\mu_{PF}(PF)$ into the general profit dependent criterion *APC* (28). Then if we have the GrossProfit = \$5 and GrossLoss = \$4, the profit is P = GrossProfit – GrossLoss = \$1. This evidently unacceptable profit generates zero value of corresponding local criterion $\mu_P(P)$ (see Fig. 11). Since PF = GrossProfit/GrossLoss, in this case, we have $\mu_{PF}(PF) = 0.83$ (see Fig. 12). Then, using the weighted sum aggregation (31) and taking into account that $W_{RP} = 0.4$, we get $Agg_1 = 0.33$ that seems to be an illogical result for the profit dependent general criterion in the case of zero profit and $\mu_P(P) = 0$. The reason is that the most commonly used weighted sum aggregation provides reasonable results only if there are significant tradeoffs between local criteria; however, there are no such criteria among those introduced in Section 3.1. The aggregations (34) and (35), in the case of local criteria represented by membership functions, do not need a compromise between the criteria, but (34) is characterized by unpleasant disadvantages concerned with the local criteria weighting, analyzed in [25]. Nevertheless, we will use all three aggregation methods considered because their negative properties can be revealed only in the rare extreme situations at the optimization stage. Therefore, it is not known in advance how trading models optimized with the help of different aggregation operators will manifest themselves at the testing stage. In addition, to obtain more reliable consensus solutions, we will use the aggregation of aggregating operators.

3.2. Hierarchical structure of the problem

Suppose we consider only the first aggregation operator (33). Then, aggregating up the original local criteria, we get:

$$\begin{aligned} APC_1 &= Agg_1(W_P,\ \mu_P(P),\ W_{PF},\ \mu_{PF}(PF)), \\ ARC_1 &= Agg_1(W_{TT},\ \mu_{TT}(TT),\ W_{RD},\ \mu_{RD}(RD)), \\ ACC_1 &= Agg_1(W_W,\ \mu_W(\%Win),\ W_{RP},\ \mu_{RP}(RP)), \end{aligned} \tag{36}$$

where the values of APC_1, ARC_1 and ACC_1 vary in the interval [0,1] and can naturally be considered as criteria depending on the profit, risk and comfort (or reliability) of trading, respectively, obtained using weighted sum aggregation (33). Similarly, the values of APC_2, ARC_2, ACC_2 and APC_3, ARC_3, ACC_3 can be obtained. This indicates the expediency of introducing the next level of aggregation using a generalized local criterion. Then, for the first aggregation operator used, we obtain the generalized criterion:

$$GC_1 = Agg_1(W_{APC}, APC_1, W_{ARC}, ARC_1, W_{ACC}, ACC_1), \tag{37}$$

where W_{APC}, W_{ARC} and W_{ACC} ($W_{APC} + W_{ARC} + W_{ACC} = 1$) are the optimized weights of corresponding criteria. In the initial studies, we assumed $W_{APC} = 0.5$, $W_{ARC} = 0.3$ and $W_{ACC} = 0.2$. Similarly, the generalized local criteria GC_2 and GC_3 based on two other considered aggregation operators were obtained.

It is easy to see that the sequence of aggregations (36) and (37) is an evident two-level hierarchy. Of course, in practice, various combinations of aggregation operations can be used: for example, we can use Agg_2 in (37) and Agg_3 in (36) and vice versa. However, to avoid possible confusion, we will limit the use of such somewhat exotic combinations whenever possible.

By definition, the general criterion GC_1 is a function of models $M1$ or $M2$, optimized parameters $ParM1$ or $ParM2$ (see (28) and (29)), and the set of local criteria weights being the model optimized parameters as well: $CW = (W_P, W_{PF}, W_{TT}, W_{RD}, W_W, W_{RP}, W_{APC}, W_{ARC}, W_{ACC})$. Therefore, the solution of the multiple-criteria fuzzy hierarchical optimization problem for the model $i = 1{,}2$ and the generalized local criterion GC_1 at the current hierarchy level can be performed as:

$$(ParMi,\ CW)_{opt1} = Arg(Max\ GC_{1,Mi}(ParMi,\ CW)). \tag{38}$$

The aggregation GC_1 can be considered as a criterion of generalized trading quality, directly related to the choice of aggregation method. In this way, also the solutions $(ParMi, CW)_{opt2}$ and $(ParMi, CW)_{opt3}$, based on the GC_2 and GC_3, respectively, were obtained. Then the aggregation of GC_1, GC_2 and GC_3 at the third level of the hierarchy seems quite natural to reach a consensus between the aggregation methods used. In turn, they can be aggregated into more general criteria:

$$GGC_i = Agg_i(W_{GC1},\ GC_1,\ W_{GC2},\ GC_2,\ W_{GC3},\ GC_3), \tag{39}$$

where W_{CG1}, W_{GC2} and W_{GC3} are the optimized weights of corresponding local criteria, $i = 1$ to 3 is the number of aggregating operator (see (33)–(35)). It is worth noting that the weights W_{CG1}, W_{GC2} and W_{GC3} implicitly reflect our relative trust in the aggregation operators used.

The solution of the corresponding multiple-criteria fuzzy hierarchical optimization task for the model $i = 1{,}2$ and the generalized local criterion GGC_1 can be presented as the extension of the solution (38):

$$\begin{aligned}(ParMi,\ CW,\ W_{CG1},\ W_{GC2},\ W_{GC3})_{opt1} \\ = Arg(Max\ GGC_{1,Mi}(ParMi,\ CW,\ W_{CG1},\ W_{GC2},\ W_{GC3})).\end{aligned} \tag{40}$$

Similarly, the solutions $(ParMi,\ CW,\ W_{CG1},\ W_{GC2},\ W_{GC3})_{opt2}$ and $(ParMi,\ CW,\ W_{CG1},\ W_{GC2},\ W_{GC3})_{opt3}$ based on the general criteria $GGC2$ and $GGC3$, respectively, can be obtained.

The described above structure of introduced local criteria is presented in Fig. 17.

It is easy to see that this structure is not closed, since we are dealing with an unlimited sequence of hierarchical levels with corresponding generalized local criteria, such as GC, GGC, …, $GGGGC$, etc. However, based on our experience, we can say that the hierarchy presented in Fig. 17 is quite sufficient for practical purposes, since its further expansion no longer improves the results.

4. The results

In this section, we present the results obtained using a fuzzy multiple-criteria hierarchical extension of the single-criteria $M1$ and $M2$ models (see Section 2) performed in the previous section. To make the results comparable, here we will use the values of external parameters, such as spreads, slippage, timeframes, position sizes, optimization and testing periods, etc., the same as in Section 2. The currency pairs studied will also be the same. Thus, the results presented in Table 1 will serve as a basis for comparing single-criteria with multiple-criteria approaches and will be referred to as "basic solutions" in the future.

Solutions (38) and (40) provide us with optimized, in the optimization period, values of weights of local criteria, as well as optimized parameter sets $ParM1$ or $ParM2$ of models $M1$ or $M2$, respectively. Then, using the $M1$ or $M2$ models with their optimized parameters, we get profits and related trading characteristics during testing periods.

4.1. Solution at the second level of the hierarchy with fixed values of local criteria weights

Consider the case when all local criteria weights are fixed and equal to the values assigned by experts based on their intuition and affections. These values are presented in the previous Section. In Tables 3–5, we present the results obtained for the three considered currency pairs at the second level of the hierarchy (solution (38)) with the fixed local criteria weights. We can see that for the *USDJPY* pair, the annual profits obtained using the weighted sum and minimum aggregation type are greater than the base one, whereas geometric aggregation provided significantly lower profit (see Table 3). It is important that the use of the weighted sum aggregation reduces the number of losing months from 5 to 2, compared to the base result, and minimal type aggregation reduces this number from 5 to 4. Thus, we can say that profit growth is accompanied by a decrease in risk, determined by the number of losing months.

For the *EURUSD* pair, only when using geometrical aggregation, the annual profit greater than the base one was obtained, but it remains very insignificant as before (see Table 4).

For the *AUDUSD* pair, we did not receive an annual profit exceeding the basic one (see Table 5).

Thus, we can say that an approach based on assigning the weights of local criteria directly by experts may slightly improve the results for some currency pairs and worsen them for other pairs. On the other hand, the of experts' opinions, concerning the values of weights, reflect only their intentions to achieve the best compromise of local criteria at the optimization stage and are not sufficiently justified from the perspective of the testing period. In such circumstances, the optimization of weights as part of the solution (38) seems quite natural.

4.2. Solution at the second level of hierarchy with optimized values of local criteria weights

The solutions (38) for the considered currency pairs in the case of optimized local criteria weights are presented in Tables 6–8.

We can see that for the AUDUSD pair (see Table 8), the profit obtained using the three aggregation methods considered significantly exceeds the base profit (single-criteria approach *Profit* → *max*), for the USDJPY pair (see Table 6), such profits are 30%–40% higher than the base one, and even for the EURUSD pair (see Table 7), using the min- type aggregation, an annual profit was obtained that is almost three times higher basic. The results obtained are good enough to stop the analysis, since everyone who has received similar results in practical trading will consider them as excellent. Nevertheless, we will consider one more level of the local criteria hierarchy.

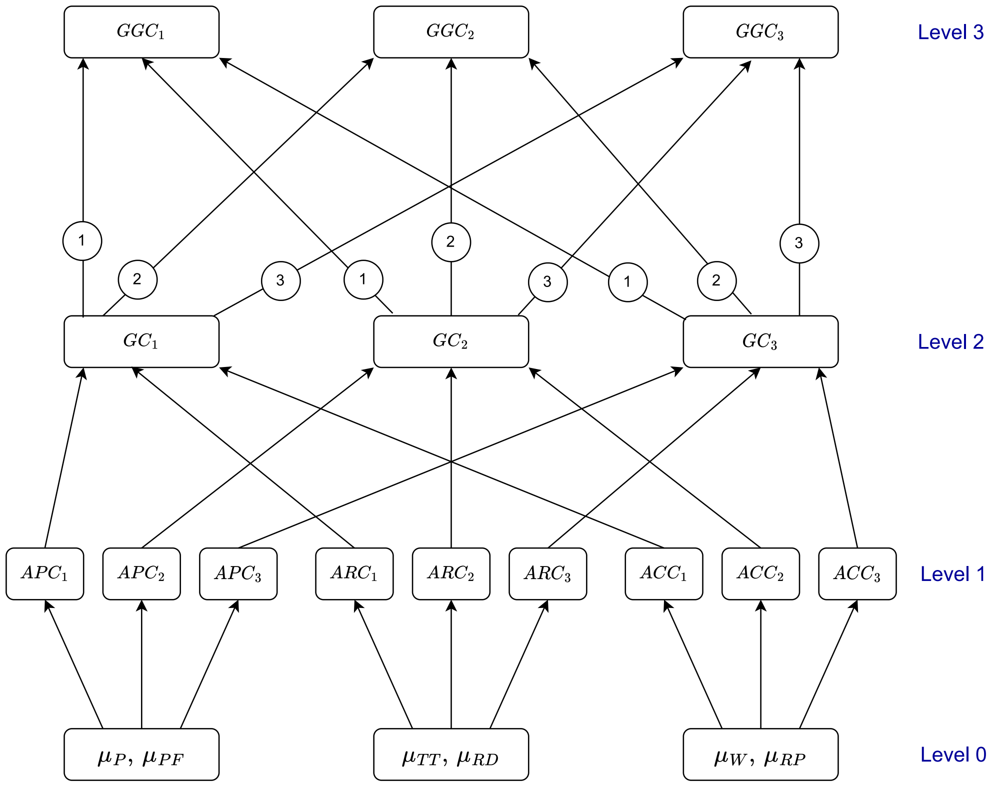

Fig. 17. Hierarchical structure of introduced local criteria: the numbers on the arrows indicate aggregation methods.

Table 3
USDJPY, *M*1, *H*4, fixed local criteria weights, level 2 in the hierarchy.

Period	Single criterion *Profit* → *max*		Weighted sum aggregation (33)		Geometrical aggregation (34)		Min. type aggregation (35)	
	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit
01.2022	7384	1826	6193	1032	5129	1789	6075	1393
02.2022	9046	−860	7707	−1144	7083	−1246	8186	−1124
03.2022	7386	706	5531	43	4992	692	6110	176
04.2022	7312	−1283	6268	152	4400	−2166	5270	−1173
05.2022	6770	536	7752	461	3653	417	2650	957
06.2022	8286	2992	9262	3713	6601	3350	9272	4335
07.2022	13 355	421	12 894	528	12 122	−363	14 732	607
08.2022	12 632	−1228	11 794	−988	9013	−2856	11 792	−300
09.2022	12 916	−199	14 824	97	12 694	−1729	13 933	−1293
10.2022	12 693	−1781	12 236	1211	11 799	−625	12 523	4091
11.2022	13 677	6245	14 049	4820	10 075	4439	15 125	3207
12.2022	21 037	7632	20 136	6584	18 130	6568	21 731	6695
Sum		15 007		16 510		8250		17 571

4.3. Solution at the third level of the hierarchy: aggregation of aggregation modes

At this level of hierarchy, we deal with the aggregation (39) of general criteria GC_1, GC_2 and GC_3, which implicitly represent the effectiveness of the aggregation operations used. However, we do not have any reasonable assumptions regarding the values of their weights. Therefore, at this stage we will consider them equally important and will not assign them weights.

As a result, a slightly modified model identical in shape to (40) was used at the optimization stage. The results for the different aggregation methods used are presented in Tables 9–11, where "M-C monthly profit" means the first monthly profit on the testing period, based on the solution of a fuzzy multi-criteria hierarchical optimization task during the optimization period.

4.4. Analysis of the results

We can see that for the pair *USDJPY*, the maximal annual profit at the third level of the hierarchy ($25 977, see Table 11) is 14% greater than that at the second hierarchy level ($22 689, see Table 6); for the pair *AUDUSD*, the maximal annual profit at the third level of hierarchy ($18 457, see Table 9) is 122% greater than that at the second hierarchy level ($8311, see Table 8), while the maximal profit from the *EURUSD* pair at the third level of hierarchy ($6515, see Table 9) is 28% lesser than that at the second level ($9105, see Table 7). Thus, if we prefer to trade *USDJPY* pair, we should use the full three hierarchy levels model and the min-type aggregation of local criteria. To trade the *AUDUSD* pair we also should use this full model but using the weighted sum aggregation. To get the best results trading the *EURUSD* pair, we should use the two-level model with the min-type aggregation.

Table 4
EURUSD, M2, H4, fixed local criteria weights, level 2 in the hierarchy.

Period	Single criterion *Profit* → *max*		Weighted sum aggregation (33)		Geometrical aggregation (34)		Min. type aggregation (35)	
	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit
01.2022	10 835	3331	11 797	3243	7988	3502	7723	3820
02.2022	16 768	−149	16 077	−450	13 519	−890	12 257	−1272
03.2022	13 189	887	14 007	−87	12 154	907	10 312	537
04.2022	11 765	−2576	11 813	−2142	9557	−18	10 968	−551
05.2022	10 692	2561	11 917	−318	10 819	2909	10 956	2263
06.2022	14 051	601	14 526	733	11 573	1604	13 458	938
07.2022	14 063	−1643	15 175	−1892	13 939	−2115	13 315	−1820
08.2022	14 324	519	14 747	1310	8935	1198	15 101	1312
09.2022	11 608	191	14 563	−3296	8161	−3398	14 751	−3333
10.2022	11 365	2413	11 430	−736	3949	791	8822	301
11.2022	13 982	−1653	11 811	−317	6499	981	8783	942
12.2022	10 572	−1374	13 573	−1709	9467	−1283	4547	−620
Sum		3108		−5664		4187		2495

Table 5
AUDUSD, *M*1, *H*4, fixed local criteria weights, level 2 in the hierarchy.

Period	Single criterion *Profit* → *max*		Weighted sum aggregation (33)		Geometrical aggregation (34)		Min. type aggregation (35)	
	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit
01.2022	10 418	3720	9901	2256	8807	3045	10 922	2192
02.2022	14 389	526	14 282	1587	10 637	361	15 096	1154
03.2022	11 784	−1491	12 342	−834	12 109	−207	12 827	−134
04.2022	15 445	893	15 962	−713	13 559	−812	15 259	−962
05.2022	15 118	3790	14 467	1140	12 248	1716	13 727	1487
06.2022	16 888	571	13 737	2496	11 819	2672	13 595	2962
07.2022	15 687	−3154	15 881	−2330	15 712	−1420	16 348	−1201
08.2022	16 418	3083	14 971	769	12 903	1233	15 057	1474
09.2022	17 808	−2685	14 981	−2055	12 517	−979	15 198	−2611
10.2022	13 802	1162	12 124	1047	11 019	162	11 444	15
11.2022	17 438	2305	14 635	666	12 684	1825	15 264	876
12.2022	18 132	−1636	16 581	324	13 505	−1180	16 113	−1482
Sum		7084		4353		6416		3770

Table 6
USDJPY, M1, H4, optimized local criteria weights, level 2 in the hierarchy.

Period	Single criterion *Profit* → *max*		Weighted sum aggregation (33)		Geometrical aggregation (34)		Min. type aggregation (35)	
	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit
01.2022	7384	1826	5198	1643	3219	691	5546	1542
02.2022	9046	−860	7727	−1127	6907	−1105	3928	−1148
03.2022	7386	706	4362	130	4599	550	4810	573
04.2022	7312	−1283	5076	−1092	5749	877	5728	807
05.2022	6770	536	5910	1145	5396	1878	5402	1453
06.2022	8286	2992	7064	4224	7047	3593	8985	5437
07.2022	13 355	421	11 188	437	5779	−54	13 652	616
08.2022	12 632	−1228	9192	250	5236	190	6027	1733
09.2022	12 916	−199	8520	−844	4168	866	10 483	−71
10.2022	12 693	−1781	10 134	−118	6773	1240	11 608	−541
11.2022	13 677	6245	11 312	6177	11 923	5769	10 971	4943
12.2022	21 037	7632	14 599	7647	18 020	6484	17 079	6155
Sum		15 007		19 384		22 689		21 500

These recommendations should not be understood as practical advice, since, according to our experience, they vary greatly depending on the broker providing the data, the time zone in which we are located, the version of the MetaTrader 4 platform used, etc.

The main purpose of this article is not to present the good results obtained, but to substantiate the newly developed approaches for obtaining such results. Let us look at the results obtained from the perspective of the overfitting problem. First, let us introduce some parameters that are useful for our analysis. The case of positive overfitting (*PO*) will be indicated if the profit in the first month of testing following the optimization period exceeds the average monthly profit for the optimization period. The negative overfitting (*NO*) will always be indicated in the case of a negative profit on the first monthly testing period. The Total *PO* is the sum of monthly overfitting profits on the

Table 7
EURUSD, M2, H4, optimized local criteria weights, level 2 in the hierarchy.

Period	Single criterion *Profit* → *max*		Weighted sum aggregation (33)		Geometrical aggregation (34)		Min. type aggregation (35)	
	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit
01.2022	10 835	3331	8140	3777	7136	3515	7433	3899
02.2022	16 768	−149	13 112	356	14 994	−28	11 020	1367
03.2022	13 189	887	9401	1377	9568	1596	6625	1339
04.2022	11 765	−2576	9060	615	6733	−3114	10 162	−613
05.2022	10 692	2561	9219	2629	11 254	2641	6558	2860
06.2022	14 051	601	9884	401	11 476	2093	9885	2280
07.2022	14 063	−1643	13 012	−2333	15 485	−2513	14 538	−1902
08.2022	14 324	519	6328	246	13 976	1121	14 692	967
09.2022	11 608	191	9726	−2754	5444	−1761	14 182	−2217
10.2022	11 365	2413	10 164	−1412	2245	−53	7123	−147
11.2022	13 982	−1653	9407	−1989	7032	−235	2401	1871
12.2022	10 572	−1374	12 666	−1513	9367	−1470	7756	−598
Sum		3108		−1402		1764		9105

Table 8
AUDUSD, M1, H4, optimized local criteria weights, level 2 in the hierarchy.

Period	Single criterion *Profit* → *max*		Weighted sum aggregation (33)		Geometrical aggregation (34)		Min. type aggregation (35)	
	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit	Opt. year profit	Test monthly profit
01.2022	10 418	3720	7112	3813	4519	4389	4140	3289
02.2022	14 389	526	10 040	1516	9281	1025	11 431	1711
03.2022	11 784	−1491	10 429	−1226	10 272	−1365	10 188	−962
04.2022	15 445	893	10 680	−1111	12 006	184	9686	−717
05.2022	15 118	3790	11 477	2209	8117	1186	10 975	1392
06.2022	16 888	571	11 704	3253	12 317	3017	8820	2300
07.2022	15 687	−3154	14 868	−2015	15 322	−1482	13 084	−1924
08.2022	16 418	3083	12 897	1454	13 450	1747	11 080	1601
09.2022	17 808	−2685	9974	−375	14 411	−2144	11 431	−2019
10.2022	13 802	1162	9716	755	11 312	922	4000	2665
11.2022	17 438	2305	9099	1961	13 401	790	10 485	1098
12.2022	18 132	−1636	12 737	−732	13 133	−203	12 160	−223
Sum		7084		7612		8066		8311

Table 9
Results based on the aggregation: $GGC_1 = \frac{1}{3}(GC_1 + GC_2 + GC_3)$.

Period	USDJPY, M1, H4		EURUSD, M2, H4		AUDUSD, M1, H4	
	Profit → *max* monthly profit	M-C monthly profit	*Profit* → *max* monthly profit	M-C monthly profit	*Profit* → *max* monthly profit	M-C monthly profit
01.2022	1826	1724	3331	3465	3720	4087
02.2022	−860	−872	−149	255	526	871
03.2022	706	623	887	4370	−1491	−313
04.2022	−1283	−1083	−2576	−134	893	412
05.2022	536	326	2561	2934	3790	2904
06.2022	2992	3377	601	294	571	1776
07.2022	421	70	−1643	−133	−3154	−1034
08.2022	−1228	−740	519	1024	3083	3252
09.2022	−199	498	191	−2453	−2685	−668
10.2022	−1781	1900	2413	−1314	1162	2235
11.2022	6245	4504	−1653	−315	2305	3971
12.2022	7632	6998	−1374	−1477	−1636	664
Sum	15 007	17 325	3108	6515	7084	**18 457**

testing periods during a year considered. Similarly, the Total *NO* is defined. Then the Relative *PO* = Total *PO*/Total Profit and the Relative *NO* = Total *NO*/Total Profit, where the Total Profit is the sum of all (positive and negative) profits on the testing periods during a year. It is easy to see that the Relative *PO* and Relative *NO* represent the relative contributions of the *PO* and *NO*, respectively in the Total Profit. Some illustrative examples are presented in Table 12, where "M-C" means the first monthly profit on the

Table 10
Results based on the aggregation: $GGC_2 = (GC_1 * GC_2 * GC_3)$.

Period	USDJPY, M1, H4		EURUSD, M2, H4		AUDUSD, M1, H4	
	Profit → max monthly profit	M-C monthly profit	*Profit → max* monthly profit	M-C monthly profit	*Profit → max* monthly profit	M-C monthly profit
01.2022	1826	1312	3331	2956	3720	2078
02.2022	−860	−452	−149	535	526	954
03.2022	706	963	887	1448	−1491	−492
04.2022	−1283	−98	−2576	−1633	893	388
05.2022	536	1292	2561	2371	3790	2722
06.2022	2992	4597	601	−1806	571	3114
07.2022	421	415	−1643	−6213	−3154	−512
08.2022	−1228	−199	519	2110	3083	3024
09.2022	−199	−391	191	−3576	−2685	−802
10.2022	−1781	3390	2413	1667	1162	1645
11.2022	6245	6391	−1653	−3988	2305	3937
12.2022	7632	5506	−1374	−2823	−1636	−931
Sum	15 007	**22 726**	3108	−8952	7084	18 150

Table 11
Results based on the aggregation: $GGC_3 = \min(GC_1, GC_2, GC_3)$.

Period	USDJPY, M1, H4		EURUSD, M2, H4		AUDUSD, M1, H4	
	Profit → max monthly profit	M-C monthly profit	*Profit → max* monthly profit	M-C monthly profit	*Profit → max* monthly profit	M-C monthly profit
01.2022	1826	1567	3331	3235	3720	4022
02.2022	−860	−1243	−149	−290	526	1725
03.2022	706	789	887	924	−1491	−332
04.2022	−1283	−1731	−2576	−1648	893	365
05.2022	536	1574	2561	1722	3790	3591
06.2022	2992	5115	601	−847	571	1732
07.2022	421	1224	−1643	−3020	−3154	−1220
08.2022	−1228	100	519	2662	3083	3197
09.2022	−199	−126	191	−1717	−2685	−719
10.2022	−1781	3251	2413	2967	1162	1434
11.2022	6245	6433	−1653	568	2305	3255
12.2022	7632	8406	−1374	−2759	−1636	392
Sum	15 007	**25 977**	3108	2630	7084	17 442

Table 12
The comparison of the best results obtained.

	USDJPY, M1, H4		*EURUSD, M2, H4*		*AUDUSD, M1, H4*	
	Profit → max	M-C	*Profit → max*	M-C	*Profit → max*	M-C
Monthly average profit	1250	2165	259	759	590	1538
Total PO	18 695	23 205	10 312	14 583	14 953	18 255
Total NO	5351	3100	7305	5477	8976	2015
Relative PO	1.25	0.89	3.32	1.6	2.11	0.99
Relative NO	0.36	0.12	2.35	0.6	1.27	0.11

testing period, based on the solution of a fuzzy multi-criteria hierarchical (*FMCH*) optimization task during the optimization period.

We can see that for all three currency pairs considered, the use of the *FMCH* method provides significantly larger profits (presented in Table 12 by average monthly profits) than in the case of a single-criteria approach (*Profit → max*). In all cases, this profit growth is accompanied by a significant increase in total *PO*, while reducing total *NO*. All this testifies to the crucial role of *PO* and *NO* in profit formation. We can also see that the *FMCH* method provides Relative *PO*s 1.4–2.2 times smaller than in the case of the single-criteria approach, while simultaneously reducing the Relative *NO*s by 3.0–11.5 times. Thus, we can say that the increase in the Total Profit is due to the overwhelming decrease in Relative *NO* compared to the decrease in Relative *PO*. In other words, this positive effect (an increase in profit) of the developed *FMCH* method is manifested primarily by the suppression of the negative overfitting by the partial replacement of losing periods with winning ones, which leads to an increase in the Total Profit even with a relatively small increase in the positive overfitting effect. In turn, the growth of the Total Profit causes a certain decrease in the Relative *PO* against the background of a significant increase in profit. Summarizing, we can say that the aims of this paper, declared in Introduction, have been reached.

5. Conclusion

Following the well-known methodological principles and using the proposed new technical analysis tools, a set of relatively simple, at the same time quite profitable and risk-free models for trading on the Forex market was developed.

The effect of overfitting has been declared the most important problem of algorithmic trading. This problem has been studied, and new concepts of positive and negative overfitting have been proposed to analyze the results of trading.

For the implementation of a general approach to model a real trading process, a multi-model strategy called "the leader correction approach ", was proposed. It is shown that this strategy makes it possible to compensate for a visible negative overfitting effect with a positive overfitting one, even with considerable final profits.

To implement a general approach to modeling the real trading process, a multi-model strategy was proposed, called the "leader correction approach". It is shown that this strategy makes it possible to compensate for the visible negative effect of overfitting on any currency pair with the positive effect of overfitting observed on another pair, sometimes with a significant final profit.

The developed simple trading models were extended using a fuzzy multiple-criteria approach at the optimization stage aimed at eliminating or reducing the negative overfitting effect. To do this, for the first time in this article, it is proposed to aggregate fuzzy local criteria determined by the output parameters of the model, such as Profit, Profit Factor, Relative Drawdown, Winning Trades, Total Trades, etc. It is shown that such an approach provides resulting profits that significantly exceed the ones provided by the basic simple single-criteria crisp models. In our research, we found that the local criteria used form a logically justified hierarchical structure that served as the basis for further extension of the models.

It is shown that the use of the revealed hierarchical structure of fuzzy local criteria for solving the fuzzy multiple-criteria hierarchical problem provides an significant additional increase in profit when compared with the results of a fuzzy multiple-criteria model without hierarchy.

It is shown that the positive effect (an increase in profit) of the developed fuzzy multiple-criteria hierarchical approach method is manifested primarily by the suppression of the negative overfitting by the partial replacement of losing periods with winning ones, which leads to a significant increase in the Total Profit even with a relatively small increase in the positive overfitting effect.

The profitability and riskless of the developed trading models were studied based on real quotations of currency pairs *USDJPY*, *EURUSD* and *AUDUSDT* provided by the *IG*-Broker, using the *H*4 timeframe and the MetaTrader 4 platform.

CRediT authorship contribution statement

Pavel Sevastjanov: Conceptualization, Formal analysis, Investigation, Methodology, Resources, Supervision, Writing – original draft, Writing – review & editing. **Krzysztof Kaczmarek:** Formal analysis, Methodology, Resources, Software, Writing – original draft, Writing – review & editing. **Leszek Rutkowski:** Formal analysis, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix

List of terms

Term	Description
Ask	The price at which the market is prepared to sell a product.
Balance	The current balance of the account (investment account) after taking into account all closed positions, given in the base currency.
Balance curve	The curve presenting the current account values (balance), resulting from the closing of successive market positions (balance(position number)).
Bid	The price at which the market is prepared to buy a product.
BUY	Taking a long position on a product.
Candle	Graphical representation of the price volatility (OHLC - Open, High, Low, Close) of a given financial instrument in a given period, e.g. 1H (1 h).
Equity	The current value of the current account balance (balance) plus the profit/minus the loss of the currently open positions.
Gross Loss	Total loss from all closed losing positions.
Gross Profit	Total profit from all closed profitable positions.
Long position	A position that appreciates in value if market price increases.
Loss trades	Number of all losing trades.
Lot	A unit to measure the amount of the deal e.g. EURUSD 1 lot = 100k. (e.g. value of the first transaction rate in a given period - Open, value of the last transaction rate - Close, etc.).
Profit	It defines the value of profit/loss obtained from a given market position. In practice, it is the result of the product of the position size (volume) and the difference between the cost price and the selling price, minus the transaction costs.
Profit Factor	The ratio between gross profits and gross losses. Profit Factor below 1.0 means that the trading system is loss-making.
Profit trades	Number of all profitable trades.
Relative drawdown	The percentage between the difference of the maximum equity high and the subsequent equity low to the maximum equity high.
SELL	Taking a short position in expectation that the market is going to go down.
Short position	An investment position that benefits from a decline in market price.
Spread	The difference between the bid and the ask price.
Startup Equity	The value of the initial balance of the investment account in the base currency (deposit).
Stop Loss	It defines the maximum level of acceptable losses, in practice it is the value of the exchange rate at which the order to automatically close a loss-making position will be activated.
Take Profit	It defines the target level of profit, in practice it is the value of the rate at which the order to automatically close the position bringing profit at the expected level will be activated.

Timeframe	Time frame (e.g. 1H - 1 h, 4H - 4 h) within which data coming from the market are aggregated.
Total net profit	The total profit/loss obtained from all closed trades.
Total trades (% of total)	Total number of closed market positions/trades (percentage of winning positions).
Volume (one position)	The size of a given market position expressed in a lot.
Volume (one timeframe)	The total volume of all transactions executed during the period (timeframe).

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