425 Applied 3D Algebra

HW 2

Joey Domino

1. Matrix Multiplication

$$\begin{pmatrix} x1, x2 \\ x3, x4 \end{pmatrix} * \begin{pmatrix} a1, a2 \\ a3, a4 \end{pmatrix} = \begin{matrix} x1*a1+x2*a3 & x1*a2+x2*a4 \\ x3*a1+x4*a3 & x3*a2+x4*a4 \end{matrix}$$

a.

$$\begin{pmatrix} -3 & -1 & 3 \\ -2 & -4 & -1 \\ 4 & -1 & -2 \end{pmatrix} \bullet \begin{pmatrix} 4 & 1 & 1 \\ 3 & -1 & 2 \\ 2 & -5 & 4 \end{pmatrix}$$

$$\begin{pmatrix} -12-3+6 & -3+1-15 & -3-2+12 \\ -8-12-2 & -2+4+5 & -2-8-4 \\ 16-3-4 & 4+1+10 & 4-2-8 \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{-9} & \mathbf{-17} & \mathbf{7} \\ \mathbf{-22} & \mathbf{7} & \mathbf{-14} \\ \mathbf{9} & \mathbf{15} & \mathbf{-6} \end{pmatrix}$$

b.

$$\begin{pmatrix} -4 & -1 & 3 \\ -2 & 1 & -4 \\ -5 & -5 & -5 \end{pmatrix} \bullet \begin{pmatrix} 1 & -4 & 2 \\ -2 & 3 & 4 \\ -2 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} -4+2-6 & 16-3+0 & -8-4-3 \\ -2-1+8 & 8+3+0 & -4+4+4 \\ -5+10+10 & 20-15+0 & -10-20+5 \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{-8} & \mathbf{13} & \mathbf{-15} \\ \mathbf{5} & \mathbf{11} & \mathbf{4} \\ \mathbf{15} & \mathbf{5} & \mathbf{-25} \end{pmatrix}$$

c.

$$\begin{pmatrix} -1 & 0 & 4 \\ 2 & -1 & 1 \\ 2 & -4 & -2 \end{pmatrix} \bullet \begin{pmatrix} -5 & 3 & -1 \\ 3 & 2 & -4 \\ 1 & 2 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 5+6+2 & 0-2-8 & 1+0+12 \\ -10-3+1 & 6-2+2 & -2+4+3 \\ -10-12-2 & 6-8-4 & -2+16-6 \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{13} & \mathbf{-10} & \mathbf{13} \\ \mathbf{-12} & \mathbf{6} & \mathbf{5} \\ \mathbf{-24} & \mathbf{-6} & \mathbf{8} \end{pmatrix}$$

d.

$$\begin{pmatrix} -4 & 2 & -2 \\ -5 & 2 & -4 \\ -3 & -3 & -4 \end{pmatrix} \bullet \begin{pmatrix} -4 & -3 & -5 \\ -4 & 3 & -2 \\ 4 & 4 & -5 \end{pmatrix}$$

$$\begin{pmatrix} 16-8-8 & 12+6-8 & 20-4+10 \\ 20-8-16 & 15+6-16 & 25-4+20 \\ 12+12-16 & 9-9-16 & 15+6+20 \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{0} & \mathbf{10} & \mathbf{26} \\ \mathbf{-4} & \mathbf{-5} & \mathbf{41} \\ \mathbf{8} & \mathbf{-16} & \mathbf{41} \end{pmatrix}$$

2. Compare Order of Operations
   a. A took me about 8-10 minutes to write out.

$$A = \begin{pmatrix} -4 & -4 & -4 \\ -2 & 4 & -4 \\ -5 & -4 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 3 & -5 & -1 \\ -2 & -4 & -3 \\ 4 & 4 & -5 \end{pmatrix} \quad V = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$$

(AB)

$$\begin{pmatrix} -12+8-16 & 20+16-16 & 4+12+20 \\ -6-8-16 & 10-16-16 & 2-12+20 \\ -15+8+12 & 25+16+12 & 5+12-15 \end{pmatrix}$$

$$\begin{pmatrix} -20 & 20 & 36 \\ -30 & -22 & 10 \\ 5 & 53 & 2 \end{pmatrix}$$

(AB)V

$$\begin{pmatrix} -40+40-72 \\ -60-44-20 \\ 10+106-4 \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{-72} \\ \mathbf{-124} \\ \mathbf{112} \end{pmatrix}$$

b. **B is faster. Took me about 5 minutes to write out**

(Bv)

$$\begin{pmatrix} 6-10+2 \\ -4-8+6 \\ 8+8+10 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ -6 \\ 26 \end{pmatrix}$$

A(Bv)

8+24-104

4-24-104

10+24+78

$$\begin{pmatrix} \mathbf{-72} \\ \mathbf{-124} \\ \mathbf{112} \end{pmatrix}$$

3. Gaussian Elimination to find Inverse Matrix

$$A= \begin{pmatrix} 4 & -5 & -3 \\ 1 & -5 & 3 \\ 2 & 4 & -3 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 4 & -5 & -3 & 1 & 0 & 0 \\ 1 & -5 & 3 & 0 & 1 & 0 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{array}\right)$$ Divide first row by 4 to get 1 in top left

$$\left(\begin{array}{ccc|ccc} 1 & -5/4 & -3/4 & 1/4 & 0 & 0 \\ 1 & -5 & 3 & 0 & -1 & 0 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & -5/4 & -3/4 & 1/4 & 0 & 0 \\ 0 & -5/1 - (-5/4) & 3/1 - (-3/4) & 0 & 1 & 0 \\ 0 & 4/1 - 2(-5/4) & -3/1 - 2(-3/4) & 0 & 0 & 1 \end{array}\right)$$

Use ‘Add multiple rows to another” operation on rows 2 and 3

$$\left(\begin{array}{ccc|ccc} 1 & -5/4 & -3/4 & 1/4 & 0 & 0 \\ 0 & -20/4 - (-5/4) & 12/4 - (-3/4) & -1/4 & 1 & 0 \\ 0 & 16/4 - 2(-5/4) & -12/4 - 2(-3/4) & -1/2 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & -5/4 & -3/4 & 1/4 & 0 & 0 \\ 0 & -15/4 & 15/4 & -1/4 & -1 & 0 \\ 0 & 13/2 & -3/2 & -1/2 & 0 & 1 \end{array}\right)$$

Clean it up

$$\left(\begin{array}{ccc|ccc} 1 & -5/4 & -3/4 & 1/4 & 0 & 0 \\ 0 & -15/4*-4/15 & 15/4*-4/15 & -1/4*-4/15 & 1*-4/15 & 0 \\ 0 & 13/2 & -6/4 & -1/2 & 0 & 1 \end{array}\right)$$

Multiply R2 to get 1 in 2^{nd} position

$$\left(\begin{array}{ccc|ccc} 1 & -5/4 & -3/4 & 1/4 & 0 & 0 \\ 0 & 1 & -1 & 1/15 & -4/15 & 0 \\ 0 & 13/2 & -3/2 & -1/2 & 0 & 1 \end{array}\right)$$

V—Zero out r1 and r2

$$\left(\begin{array}{ccc|ccc} 1 & -5/4+5/4*1 & -3/4-5/4*-1 & 1/4+ (5/4)*(1/15) & 0+(5/4)*(-4/15) & 0 +(5/4)*0 \\ 0 & 1 & -1 & 1/15 & -4/15 & 0 \\ 0 & 13/2-13/2*1 & -3/2+13/2*-1 & -1/2-(13/2)*(1/15) & 0-(13/2*-4/15) & 1 - (-5/4)*0 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1/4+ (5/60) & 0- (-1/1)*(1/3) & 0 \\ 0 & 1 & -1 & 1/15 & -4/15 & 0 \\ 0 & 0 & 5 & -1/2-(13/2)*(2/30) & 0-(13/2*-4/15) & 1 \end{array}\right)$$

<-simplify

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1/4+ (1/12) & -1/3 & 0 \\ 0 & 1 & -1 & 1/15 & -4/15 & 0 \\ 0 & 0 & 5 & -1/2-26/60) & 0-(52/30) & 1 \end{array}\right)$$

<-simplify

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 3/12+ 1/12 & -1/3 & 0 \\ 0 & 1 & -1 & 1/15 & -4/15 & 0 \\ 0 & 0 & 5 & -15/30 -13/30 & 0+26/15 & 1 \end{array}\right)$$

<-simplify

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 4/12 & -1/3 & 0 \\ 0 & 1 & -1 & 1/15 & -4/15 & 0 \\ 0 & 0 & 5 & -28/30 & 26/15 & 1 \end{array}\right)$$

<-simplify

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1/3 & -1/3 & 0 \\ 0 & 1 & -1 & 1/15 & -4/15 & 0 \\ 0 & 0 & 5 & -14/15 & 26/15 & 1 \end{array}\right)$$ <-simplify

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1/3 & -1/3 & 0 \\ 0 & 1 & -1 & 1/15 & -4/15 & 0 \\ 0 & 0 & 5/5 & -14/15*1/5 & 26/15*1/5 & 1/5 \end{array}\right)$$ <-Divide r3 by 5

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1/3 & -1/3 & 0 \\ 0 & 1 & -1 & 1/15 & -4/15 & 0 \\ 0 & 0 & 1 & -14/75 & 26/75 & 1/5 \end{array}\right)$$ <-Simplify

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1/3 & -1/3 & 0 \\ 0 & 1 & -1 & 1/15 & -4/15 & 0 \\ 0 & 0 & 1 & -14/75 & 26/75 & 1/5 \end{array}\right)$$ <-Simplify

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -2+2 & 1/3+2*-14/75 & -1/3+2*26/75 & 0+2*1/5 \\ 0 & 1 & -1+1 & 1/15+1*-14/75 & -4/15+1*26/75 & 0+1*1/5 \\ 0 & 0 & 1 & -14/75 & 26/75 & 1/5 \end{array}\right)$$ <-Zero r1r2

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 25/75-28/75 & -25/75+52/75 & 2/5 \\ 0 & 1 & 0 & 5/75-14/75 & -20/75+26/75 & 1/5 \\ 0 & 0 & 1 & -14/75 & 26/75 & 1/5 \end{array}\right)$$ <-Simplify

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3/75 & 27/75 & 2/5 \\ 0 & 1 & 0 & -9/75 & 6/75 & 1/5 \\ 0 & 0 & 1 & -14/75 & 26/75 & 1/5 \end{array}\right)$$ <-Simplify

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/25 & 9/25 & 2/5 \\ 0 & 1 & 0 & -3/25 & 2/25 & 1/5 \\ 0 & 0 & 1 & -14/75 & 26/75 & 1/5 \end{array}\right)$$ <-Inverse!

$$\boldsymbol{\begin{pmatrix} -1/25 & 9/25 & 2/5 \\ -3/25 & 2/25 & 1/5 \\ -14/75 & 26/75 & 1/5 \end{pmatrix}}$$ **<-Inverse!**

PROOF

(A^i)A =A(A^i) = I Multiple matrix by its inverse to get an identity matrix.

$$\text{Inverse} = \begin{pmatrix} -1/25 & 9/25 & 2/5 \\ -3/25 & 2/25 & 1/5 \\ -14/75 & 26/75 & 1/5 \end{pmatrix}$$

$$A = \begin{pmatrix} 4 & -5 & -3 \\ 1 & -5 & 3 \\ 2 & 4 & -3 \end{pmatrix}$$

<u>A*A^I</u>

$$\begin{pmatrix} -4/25+15/25+42/75 & 36/25+-10/25+-78/75 & 8/5+-5/5+-3/5 \\ -1/25+15/25+-42/75 & 9/25+-10/25+78/75 & 2/5+-5/5+3/5 \\ -2/25+-12/25+42/75 & 18/25+8/25-78/75 & 4/5+4/5-3/5 \end{pmatrix}$$

$$\begin{pmatrix} 33/75+42/75 & 78/85-78/75 & 0 \\ 42/75-42/75 & -3/75+78/75 & 0 \\ -42/75+42/75 & 78/75-78/75 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

<u>A^I * A</u>

$$\begin{pmatrix} -4/25+9/25+4/5 & 5/25+-45/25+8/5 & 3/25+27/25+-6/5 \\ -12/25+2/25+2/5 & 15/25+-10/25+4/5 & 9/25+6/25-3/5 \\ -56/75+26/75+2/5 & 70/75+-130/75+4/5 & 42/75+78/75-3/5 \end{pmatrix}$$

$$\begin{pmatrix} 1/5+4/5 & -40/25+40/25 & 30/25-30/25 \\ -2/5+2/5 & 1/5+4/5 & 15/25-15/25 \\ -30/75+30/75 & -60/75+60/75 & 8/5-3/5 \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

4. Determinant (Laplace) <- Check your determinant is correct by finding it again with another row!

|A| = Ai,j(-1)^i+j|~Ai,J|

a.

$$\begin{pmatrix} 1 & -4 & -2 \\ -3 & -2 & 3 \\ 1 & -4 & 2 \end{pmatrix}$$

Top Row

1| -2 3 | - -4 |-3 3 | + -2|-3 -2 |
|-4 2| | 1 2| | 1 -4 |

1 * (-2*2 – 3*-4) - -4(-3*2 – 3*1) + -2(-3*-4 - -2*1)

8 – 36 -28

DET = -56

Bottom Row:

1|-4 -2 | + -4(-1) |1 -2 | + 2|1 -4 |
|-2 3| | -3 3| | -3 -2 |

1*(-12 – 4) + 4(3-6) + 2(-2-12)

-16 – 12 – 28

DET CHECK = -56

b.

$$\begin{pmatrix} -3 & -5 & 2 \\ -1 & 0 & -4 \\ 3 & 1 & 3 \end{pmatrix}$$

bottom Row

3| -5 2 | - 1 |-3 2 | + 3|-3 -5 |
|0 -4| | -1 -4| | -1 0 |

60 – 14 -15

DET = 31

Top row:

-3|0 -4| - -5|-1 2 | + 2|-1 0 |
|1 3| | 3 3| | 3 1 |

-3(0*3 - -4*1) - -5(-1*3 – 2*3) + 2(-1*1 – 0*3)

-12 + 45 -2

DET CHECK = 31

c.

$$\begin{pmatrix} -3 & 3 & 2 & 4 \\ -4 & 0 & 0 & 3 \\ 4 & 3 & -2 & -1 \\ -4 & 0 & 2 & 0 \end{pmatrix}$$

Row 2:

$$-(-4)\begin{vmatrix} 3 & 2 & 4 \\ 3 & -2 & -1 \\ 0 & 2 & 0 \end{vmatrix} + 0\begin{vmatrix} -3 & 2 & 4 \\ 4 & -2 & -1 \\ -4 & 2 & 0 \end{vmatrix} - 0\begin{vmatrix} -3 & 3 & 4 \\ 4 & 3 & -1 \\ -4 & 0 & 0 \end{vmatrix} + 3\begin{vmatrix} -3 & 3 & 2 \\ 4 & 3 & -2 \\ -4 & 0 & 2 \end{vmatrix}$$

First 3x3 Top Row:

$$3\begin{vmatrix} -2 & -1 \\ 2 & 0 \end{vmatrix} - 2\begin{vmatrix} 3 & -1 \\ 0 & 0 \end{vmatrix} + 4\begin{vmatrix} 3 & -2 \\ 0 & 2 \end{vmatrix}$$

$6 – 0 + 24$

30

4th 3x3 Bottom Row:

$$-4\begin{vmatrix} 3 & 2 \\ 3 & -2 \end{vmatrix} - 0\begin{vmatrix} -3 & 2 \\ 4 & -2 \end{vmatrix} + 2\begin{vmatrix} -3 & 3 \\ 4 & 3 \end{vmatrix}$$

$-4(-6 – 6) – 0 + 2(-9 – 12)$

$48 – 42$

6

Finish 4x4 calculation

$4*30 +0 -0 + 3*6$

$120 + 18$

DET = 138

Row 4:

```
-(-4)|3   2    4| + 0|-3  2  4 | - 2  |-3  3  4| + 0 |-3  3  2|
      |0    0    3|     |-4    0   3|       |-4  0  3|       |-4  0  0|
      |3   -2  -1|     |4  -2  -1|         |4  3  -1|        |4    3  -2|
```

First 3x3 Middle Row:

```
 -0|2    4|   +  0|3  4 | -  3|3   2|
    |-2  -1|         |3  -1|       |3   -2 |
```

0 – 0 + 36

36

3rd 3x3 Top Row:

```
 -3|0    3|   -  3|-4  3 | +  4|-4   0 |
    |3   -1|         |4  -1|         |4   3 |
```

27 + 24 – 48

3

Bring them back into 4x4 caclulation

-(-4)36 +0 – 2*3 + 0

144 – 6

DET CHECK: 138

5.

n = 1/sqrt(2) (-1, 0, -1)

T(v) = v – (v*n)n

a. Show T is a linear transformation

Will only be called linear if for any u and v vectors and scalar a

T(u +v) = T(u) + T(v)

T(au) = aT(u)

U = (u1, u2, u3)

V = (v1,v2,v3)

T(u+v) = **|(u1+v1) + 2(u2+v2)|**
|(u3+v3) – 3(u2+v2)|

T(u)+T(v) = |u1 + 2u2| + |v1 + 2v2|
|u3 – 3u2| |v3 – 3v3|

|(u1+v1) + 2(u2+v2)|
|(u3 + v3) – 3(u3 + v3)|

T(au) = a(u1,u2,u3)
(au1,au2,au3)

|au1 + 2au2|
|au3 – 3au2|

|a(u1 + 2u2)|
|a(u3 – 3u2)|

$$\mathbf{a \begin{pmatrix} (u1 + 2u2) \\ (u3 - 3u2) \end{pmatrix} \quad = aT(u)}$$

b. Give matrix form M for T. Characterize transformation

Given that we are a linear transformation:

T(e0) = (1,0,0)*n
T(e1) = (0,1,0)*n
T(e2) = (0,0,1)*n

$$M = \begin{pmatrix} T(e00) & T(e10) & T(e20) \\ T(e00) & T(e10) & T(e20) \\ T(e00) & T(e10) & T(e20) \end{pmatrix} \begin{bmatrix} n1 \\ n2 \\ n3 \end{bmatrix}$$

c. Let v = (0, 0, 2). Compute T(v) and Mv. Answers should be the same.

T(v) = (v*n)n
(0,0, 2)(-1,0,-1)
0 +0 -2
-2 * (-1, 0, -1)
(2, 0, 2)

T(e0) = ((1,0,0) * (-1,0,-1))(-1,0,-1)
-1(-1,0,-1)
(1, 0, 1)
T(e1) = ((0,1,0) * (-1,0,-1)) (-1,0,-1)
0(-1,0,-1)
(0, 0, 0)
T(e2) = ((1,0,0) * (-1,0,-1)) (-1,0,-1)
-1(-1,0,-1)
(1, 0, 1)

$$Mv = \begin{pmatrix} 1, & 0, & 1 \\ 0, & 0, & 0 \\ 1, & 0, & 1 \end{pmatrix}$$

$$Mt = \begin{pmatrix} 1, & 0, & 1 \\ 0, & 0, & 0 \\ 1, & 0, & 1 \end{pmatrix} * \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{pmatrix} 1*0 + 0*0 + 1*2 \\ 0*0 + 0*0 + 0*2 \\ 1*0 + 0*0 + 1*2 \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{2} \\ \mathbf{0} \\ \mathbf{2} \end{pmatrix}$$

d. BONUS: what was this calculating?
   Flipped the item around and increased its speed or displaced it further than its old trajectory