

# 425 Applied 3D Algebra

HW 6

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1.

Do these objects intersect?

$$C1 = (-0.80, 0.60, 0.20)$$

$$R1 = 3$$

$$W1 = \begin{pmatrix} -1.414 & 0.8165 & -1.155 & 3 \\ 0 & 1.633 & 1.1547 & 1 \\ 1.4142 & 0.8165 & -1.155 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$C2 = (1.00, 1.00, 0.20)$$

$$R2 = 6$$

$$W2 = \begin{pmatrix} 0 & -1.664 & 2.4962 & -3 \\ 0 & 2.4962 & 1.6641 & 5 \\ -3 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Remember that points have a w of 1!

$$\text{Find } C1' = W1 * C1$$

$$C1' = (4.3901, 2.21074, -0.87246)$$

$$\text{Find } a1 = ||V1_{side}|| = \sqrt{(-1.414)^2 + 0 + 1.4142^2}$$

$$A1 = 1.9998$$

$$\text{Find } r1' = r1 * a1 = 3 * 1.99984 = 5.99952$$

$$\text{Find } C2' = W2 * C2$$

$$C2' = (-4.16476, 7.82902, -1)$$

$$\text{Find } a2 = ||V2_{side}|| = \sqrt{(-3)^2}$$

$$a2 = 3$$

$$\text{Find } r2' = r2 * a2 = 6 * 3 = 18$$

$$C1' - C2' = (8.55486, -5.61828, 0.12754)$$

$$||C1' - C2'|| = \sqrt{(8.55486)^2 + (-5.61828)^2 + (0.12754)^2}$$

$$||C1' - C2'|| = 10.23557$$

$$r1' + r2' = 19.9998$$

Check if: if true, we are intersecting.

$$||C1' - C2'|| < r1' + r2'$$

$$10.23557 < 19.9998$$

**C1 and C2 ARE intersecting**

2.

Do these two objects intersect?

Object 1

Object2

AABB: Min = (-150, 90, 60)

C2` = WC = (-156, 85, 56)

Max = (-147, 95, 61)

r2` = sr = 6

X min/max = (-150, -147)

Y min/max = (90, 95)

Z ,min/max = (60, 61)

Find C1` by checking xyz min/max against C2` xyz positions. Follow these rules

C2` element < C1` element min

If false, then -> C1` element max < C2` element

If false, then -> Take C2 element

C1` = (-150, 90, 60)

Check if clamped point is inside bsphere

C1` - C2` = (6, 5, 4)

$||C1` - C2`|| \geq r2`$  If true, we aren't intersecting

$||C1` - C2`|| = \sqrt{36 + 25 + 16} = 8.774964$

**8.774964 > 6**

**Object 1 and 2 ARE NOT intersecting**

3.

Do these two objects intersect?

Object 1

Object 2

OBB: Min = (-1, 3, -4)

C2` = WC = (30.0, 38.0, -40.0)

Max =(2, 5, 2)

r2` = sr = 4

$$W1 = \begin{pmatrix} 3.5355 & 0 & -3.536 & 40 \\ 0 & 5 & 0 & 40 \\ 3.5355 & 0 & 3.5355 & -40 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Computer Bsphere in OBB local space.

Inverse OBB World

$$W1^{-1} = \begin{pmatrix} 0.14141 & 0 & 0.14143 & 0.0008 \\ 0 & 0.2 & 0 & -8 \\ -0.14141 & 0 & 0.14141 & 11.31302 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$C1' = W1^{-1} * C2' = (-1.4141, -0.4, 1.4141)$$

Clamp  $C1'$  along each axis. Follow these rules:

For each option, if true, take OBB element being tested

$C1'$  element < OBB element min

If false, then  $\rightarrow$  OBB element max <  $C1'$  element

If false, then  $\rightarrow$  Take  $C1$  element

$$\text{Clamped} = (-1, 3, 1.414)$$

Computer clamped  $C'$  to world space

$$W1 = \begin{pmatrix} 3.5355 & 0 & -3.536 & 40 \\ 0 & 5 & 0 & 40 \\ 3.5355 & 0 & 3.5355 & -40 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \\ 1.414 \\ 1 \end{pmatrix}$$

$$Q' = (31.464596, 55, -38.536303)$$

Now to see if OBB is within the Bsphere

$$Q' - C2 = (31.464596, 55, -38.536303) - (30.0, 38.0, -40.0)$$

$$Q' - C2 = (1.464596, 17, 1.463697)$$

$$|| Q' - C2 || = \sqrt{1.464596^2 + 17^2 + 1.463697^2} = 17.12564$$

$$17.12564 \geq 4$$

**Object 1 and 2 ARE NOT intersecting**

4.

Do these two intersect?

Object 1:

OBB Min = (-2.50, -2.00, -1.00)  
Max = (3.50, 2.00, 1.00)

$$W1 = \begin{pmatrix} s1 & u1 & f1 & \\ -0.474 & -0.048 & -0.151 & 0 \\ 0 & 0.4767 & -0.151 & 4 \\ 0.1581 & -0.143 & -0.452 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$W1 S^2 = 0.249906$$

Object 2:

OBB Min = (-4.75, -5.00, -3.50)  
Max = (5.25, 5.00, 4.50)

$$W2 = \begin{pmatrix} s2 & u2 & f2 & \\ 0.3536 & -0.289 & -0.204 & 4 \\ 0 & .02887 & -0.408 & 1 \\ 0.3536 & 0.2887 & 0.2041 & -6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$W2 S^2 = 0.24973681$$

a.

**Object 1's forward axis**

**f1 = (-0.151, -0.151, -0.452)**

**Object 2's up axis**

**u2 = (-0.289, 0.2887, 0.2887)**

**Cross product of both above axes**

**f1 x u2 = (0.0868987, 0.1742217, -0.0872327)**

b.

For each axes, perform SAT overlap test. B and C are answered in the table below.

$$Sf1 = -0.151^2 + -0.151^2 + -0.452^2$$

$$Sf1 = 0.249906$$

$$||f1|| = \sqrt{S1} = \sqrt{0.249906}$$

$$||f1|| = 0.49991$$

$$Su2 = -0.289^2 + 0.2887^2 + 0.2887^2$$

$$Su2 = 0.25022$$

$$||u2|| = 0.50022$$

$$Sfxu = (0.0868987^2 + 0.1742217^2 + -0.0872327^2)$$

$$Sfxu = 0.04551$$

$$||Sfxu|| = \sqrt{0.04551}$$

$$||fxu|| = 0.21334$$

$$\begin{aligned}
C1 &= (P1_{\max} + P1_{\min})/2 = (1, 0, 0)/2 &= (0.5, 0, 0) \\
C2 &= (P2_{\max} + P2_{\min})/2 = (0.5, 0, 1)/2 &= (0.25, 0, 0.5) \\
C2 - C1 &= (-0.25, 0, 0.5)
\end{aligned}$$

$$\begin{aligned}
&\quad \quad \quad a \quad b \quad c \\
D1 &= P1_{\max} - C1 = (3, 2, 1) \\
D2 &= P2_{\max} - C2 = (5, 5, 4)
\end{aligned}$$

$$W1^{-1} = \begin{pmatrix} & s1 & & u1 & & f1 \\ -1.8981 & -0.0008 & 0.6344 & 1.2721 \\ -0.1911 & 1.9066 & -0.5731 & -8.7725 \\ -0.6034 & -0.6035 & -1.8092 & -1.2045 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Let's do world 1 together

$$\begin{aligned}
V_{\text{local } 1} (v1 \cdot f1) &= W1^{-1} * f1 = (-0.0000149, 0.0000007, 1.0000003) \\
|V1 \cdot f1 * d1| &= |(-0.0000149, 0.0000007, 1.0000003) \cdot (3, 2, 1)| \\
&= |-0.0000447 + 0.0000014 + 1.0000003| \\
&= 0.999957 \\
\text{Proj\_max1 } f1 &= |v1 \cdot f1 \cdot d1| / ||f1|| = 0.999957 / 0.49991 \\
&= 2.000274049
\end{aligned}$$

$$\begin{aligned}
V_{\text{local } 1} u2 &= W1^{-1} * u2 = (0.73147122, 0.44020935, -0.52216389) \\
|v1 \cdot u2 \cdot d1| &= |(0.73147122, 0.44020935, -0.52216389) \cdot (3, 2, 1)| \\
&= 2.55319346 \\
\text{Proj\_max1 } u2 &= |v1 \cdot u2 \cdot d1| / ||u2|| = 2.55319346 / 0.50022 \\
&= 5.104141098
\end{aligned}$$

$$\begin{aligned}
v_{\text{local } 1} (v1 \cdot f_{xu}) &= W1^{-1} * f_{xu} = (-0.22042222471, 0.36555781202, 0.00024392931) \\
|v1 \cdot f_{xu} \cdot d1| &= |(-0.22042222471, 0.36555781202, 0.00024392931) \cdot (3, 2, 1)| \\
&= |-0.66126667413 + 0.73111562404 + 0.00024392931| \\
&= 0.0700928793 \\
\text{Proj\_max1 } f_{xu} &= |v1 \cdot f_{xu} \cdot d1| / ||f_{xu}|| = 0.0700928793 / 0.21334 \\
&= 0.3285501045
\end{aligned}$$

Now for World 2

$$W2^{-1} = \begin{matrix} & \begin{matrix} s2 & u2 & f2 \end{matrix} \\ \begin{pmatrix} 1.4137 & 0.0007 & 1.4144 & 2.8311 \\ -1.1541 & 1.1544 & 1.1541 & 10.3867 \\ -0.8166 & -1.6341 & 0.8166 & 9.8006 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

$$V \text{ local } 2 \text{ (v2`f1)} = W2^{-1} * f1 = (-0.8528832, -0.5216985, 0.0009525)$$

$$\begin{aligned} |v2f1.dot(d2)| &= |(-0.8528832, -0.5216985, 0.0009525).dot(5, 5, 4)| \\ &= |-4.264416 + -2.6084925 + 0.00381| \\ &= 6.8690985 \end{aligned}$$

$$\begin{aligned} Proj\_max2 \text{ f1} &= |v2f1.dot(d2)| / ||f1|| = 6.8690985 / 0.49991 \\ &= 13.74067032 \end{aligned}$$

$$v \text{ local } 2 \text{ (v2`u2)} = W2^{-1} * u2 = (-0.00001993, 0.99999885, -0.00001485)$$

$$\begin{aligned} |v2`f1.dot(d2)| &= |(-0.00001993, 0.99999885, -0.00001485).dot(5,5,4)| \\ &= |-0.00009965 + 4.99999425 + -0.0000594| \\ &= 4.9998352 \end{aligned}$$

$$\begin{aligned} Proj\_max2u2 &= |v2`f1.dot(d2)| / ||u2|| = 4.9998352 / 0.50022 \\ &= 9.99527248 \end{aligned}$$

$$v \text{ local } 2 \text{ f xu} = W2^{-1} * fxu = (-0.0004112835, 0.00015648174, -0.42689138121)$$

$$\begin{aligned} |v2`fxu.dot(d2)| &= |(-0.0004112835, 0.00015648174, -0.42689138121).dot(5,5,4)| \\ &= |-0.0020564175 + 0.0007824087 + -1.707565525| \\ &= 1.708839534 \end{aligned}$$

$$\begin{aligned} Proj\_max2fxu &= |v2`fxu.dot(d2)| / ||fxu|| = 1.708839534 / 0.21334 \\ &= 0.3645638262 \end{aligned}$$

Now plug everything into the table!

$$W1 \text{ S}^2 = 0.249906$$

$$W2 \text{ S}^2 = 0.24973681$$

$$C2-C1 = (-0.25, 0, 0.5)$$

$$f1 = (-0.151, -0.151, -0.452)$$

$$||f1|| = 0.49991$$

$$u2 = (-0.289, 0.2887, 0.2887)$$

$$||u2|| = 0.50022$$

$$f1 \times u2 = (0.0868987, 0.1742217, -0.0872327) \quad ||fxu|| = 0.21334$$

$$\begin{aligned} |(C2-C1).dot(vf1)| / ||f1|| &= (-0.25, 0, 0.5) \cdot (-0.151, -0.151, -0.452) / 0.49991 \\ &= 0.18825 / 0.49991 \\ &= 0.3765677822 \end{aligned}$$

$$\begin{aligned}
 |(C2-C1).dot(vu2)|/||u2|| &= (-0.25, 0, 0.5) (-0.289, 0.2887, 0.2887)/ 0.50022 \\
 &= 0.2166/0.50022 \\
 &= 0.4330094758
 \end{aligned}$$

$$\begin{aligned}
 |(C2-C1).dot(vfxu)|/||fxu|| &= (-0.25, 0, 0.5) (0.0868987, 0.1742217, -0.0872327)/ 0.21334 \\
 &= 0.065341025/0.21334 \\
 &= 0.3062764835
 \end{aligned}$$

Axes Label	f1	u2	f1xu2
Axes (v)	-0.151	-0.289	0.0868987
	-0.151	0.2887	0.1742217
	-0.452	0.2887	-0.0872327
Length	0.49991	0.50022	0.21334
v local 1 (W1^-1*v)	-0.237	0.73147122	-0.22042222471
	0	0.44020935	0.36555781202
	0.07905	-0.52216389	0.00024392931
Proj_max1	2.000274049	5.104141098	0.3285501045
Rho1 = proj_max1 * s1^2	0.4998805	1.275555485	0.0821066424
v local 2 (W2^-1*v)	-0.8528832	-0.00001993	-0.0004112835
	-0.5216985	0.99999885	0.00015648174
	0.0009525	-0.00001485	-0.42689138121
Proj_max 2	13.74067032	9.99527248	0.3645638262
Rho = proj_max2 * s2^2	3.431551173	2.496187464	0.091045007
D =  (C2-C1).dot(v) /  v	0.3765677822	0.4330094758	0.3062764835
	3.931431673	3.771742949	0.1731516494
Overlap: D <= rho1 + rho2	CLEAR	CLEAR	OBB'S INTERSECT ALONG F1xU2