425 Applied 3D Algebra

HW 5

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1. RIGHT HAND ORTHO CAMERA

   Camera position and orientation:

   Position = Cpos (-8, 9, 6)

   Target = Tpos = (-566,-425, 378)

   Up vector (0, 1, 0)

   Camera Orthographic Projection:

Left = -500	Near clip Plane: 1
Right = 500	Far clip Plane : 8000
Top = 264	NDC [-1, 1] on all 3 axes
Bottom = -264	

   Screen details

   (0,0) at top left corner.

   2048 x 1080 pixels

   Z-depth range [0,1]

   A.

   View-to-World Matrix

   1. We need to find the fwd and side vectors

      Normalize "up" vector

      ||v'up'|| = sqrt(1)

      ||v'up'|| = 1

      'up' is already normalized

      V^'up' = (0, 1, 0)

      Get fwd = -(Tpos – Cpos) = -dir

      Fwd is negated before we are inverting our z axis due to our right handedness.

      fwd = -(-558, -434, 372)

      fwd = (558, 434, -372)

      Normalize the fwd vector

      ||vfwd|| = sqrt(558^2+434^2+-372^2) = sqrt(638104)

      ||vfwd|| = 798.8141211

      V^fwd = (558, 434, -372) /798.8141211

      V^fwd = (0.698535, 0.543305, -0.46569)

side = ‘up’ x v^fwd
(0, 1, 0)
(0.698535, 0.543305, -0.46569)

Side = (-0.46569, 0, -0.698535)

Now normalize our new side
||side|| = sqrt(-0.46569^2 + 0 + -0.698535^2) = 0.83953
V^side = (-0.46569, 0, -0.698535) / 0.83953
<u>V^side = (-0.5547, 0, -0.83205)</u>

V^up = v^fwd x v^side

(0.698535, 0.543305, -0.46569)	(0.543305*-0.83205 - -0.46569*0,
(-0.5547, 0, -0.83205)	-(0.698535*-0.83205 - -0.46569*-0.5547),
	0.698535*0 – 0.543305*-0.5547)

<u>V^up = (-0.452057, 0.83953, 0.30137)</u>

Cview-to-world =
$$\begin{pmatrix} \mathbf{-0.5547} & \mathbf{-0.452057} & \mathbf{0.698535} & \mathbf{-8} \\ \mathbf{0} & \mathbf{0.83953} & \mathbf{0.543305} & \mathbf{9} \\ \mathbf{-0.83205} & \mathbf{0.30137} & \mathbf{-0.46569} & \mathbf{6} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

B.

Find C World-to-view

Cworld-to-view = Cview-to-world^-1 = $\begin{pmatrix} \text{R^t} & \text{-(R^t*Cpos)} \\ 0 & 1 \end{pmatrix}$

$$\begin{array}{cc} \text{R^t} & \text{-Cpos} \\ \begin{pmatrix} \mathbf{-0.5547} & \mathbf{0} & \mathbf{-0.83205} \\ \mathbf{-0.452057} & \mathbf{0.83953} & \mathbf{0.30137} \\ \mathbf{0.698535} & \mathbf{0.543305} & \mathbf{-0.46569} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} & \begin{pmatrix} \mathbf{8} \\ \mathbf{-9} \\ \mathbf{-6} \\ \mathbf{1} \end{pmatrix} \end{array}$$

-(R^t*Cpos) = (0.5547, -12.980446, 3.492675)

Cworld-to-view
$$\begin{pmatrix} \mathbf{-0.5547} & \mathbf{0} & \mathbf{-0.83205} & \mathbf{0.5547} \\ \mathbf{-0.452057} & \mathbf{0.83953} & \mathbf{0.30137} & \mathbf{-12.980446} \\ \mathbf{0.698535} & \mathbf{0.543305} & \mathbf{-0.46569} & \mathbf{3.492675} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

C.

Compute Orthographic Projection Matrix
This is when we start using the other fields

Left = -500	Top = 264	Near clip Plane: 1
Right = 500	Bottom = -264	Far clip Plane : 8000

NDC [-1, 1] on all 3 axes

Ortho matrix xyz

0x = 2/(-500 - 500) y = 2/(264- -264) z = 2/(1 – 8000)

x = 0.002 y = 0.0038 z = 0

Ortho Matrix translation xyz

X = -(Right-Left/Right + Left)

X = 0

Y = -(Top-Bottom/Top+Bottom) <- divide by 0. It's 0.

Y = 0

Z = -(Near - Far plane/Near + Far plane)

Z =-(-0.99975)

Mortho

$$\begin{pmatrix} \mathbf{0.002} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.0038} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0.00025} & \mathbf{-1.00025} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

D.

NDC to screen matrix

3x3 xyz

X = Width/2 = 2048/2 Y = -(Height/2) = -1080/2 Z = z-depth range/2

X = 1024 Y = -540 Z = 0.5

NDC-to-Screen

$$\begin{pmatrix} \mathbf{1024} & \mathbf{0} & \mathbf{0} & \mathbf{1024} \\ \mathbf{0} & \mathbf{-540} & \mathbf{0} & \mathbf{540} \\ \mathbf{0} & \mathbf{0} & \mathbf{0.5} & \mathbf{0.5} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

E.

P1: (523, -637, 302) P2: (530, -642, 283) P3: (538, -623, 304)

Steps to complete for each point

- Position in camera space

  Cworld-to-view * P123 <- put Points 1/2/3 together into their own matrix

$$\begin{pmatrix} -0.5547 & 0 & -0.83205 & 0.5547 \\ -0.452057 & 0.83953 & 0.30137 & -12.980446 \\ 0.698535 & 0.543305 & -0.46569 & 3.492675 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

*

$$\begin{pmatrix} -1318 & -1315 & -1311 \\ -804 & -824 & -791 \\ 530 & 510 & 555 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{array}{ccc} \mathbf{P1`} & \mathbf{P2`} & \mathbf{P3`} \end{array}$$
$$\begin{pmatrix} \mathbf{290.6628} & \mathbf{305.6397} & \mathbf{265.97865} \\ \mathbf{67.57466} & \mathbf{43.400489} & \mathbf{82.858401} \\ \mathbf{-1600.809375} & \mathbf{-1600.26607} & \mathbf{-1600.498915} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$

- Is position in the NDC?

  Points are in scene if within NDC range. In this case, [-1, 1]

Mortho * Cworld-to-view[P1,P2,P3]

$$\begin{pmatrix} 0.002 & 0 & 0 & 0 \\ 0 & 0.0038 & 0 & 0 \\ 0 & 0 & 0.00025 & -1.00025 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

*

$$\begin{pmatrix} 290.6628 & 305.6397 & 265.97865 \\ 67.57466 & 43.400489 & 82.858401 \\ -1600.809375 & -1600.26607 & -1600.498915 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{array}{ccc} \textbf{P1''} & \textbf{P2''} & \textbf{P3''} \end{array}$$

$$\begin{pmatrix} \mathbf{0.5813256} & \mathbf{0.6112794} & \mathbf{0.5319573} \\ \mathbf{0.256783708} & \mathbf{0.1649218582} & \mathbf{0.3148619238} \\ \mathbf{-1.40045234375} & \mathbf{-1.4003165175} & \mathbf{-1.40037472875} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$

- Its screen (x,y) coordinates

NDC-to-Screen

$$\begin{pmatrix} 1024 & 0 & 0 & 1024 \\ 0 & -540 & 0 & 540 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 0.5813256 & 0.6112794 & 0.5319573 \\ 0.256783708 & 0.1649218582 & 0.3148619238 \\ -1.40045234375 & -1.4003165175 & -1.40037472875 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1619.2774144 & 1649.9501056 & 1568.7242752 \\ 401.33679768 & 450.942196572 & 369.974561148 \\ -0.200226171875 & -0.20015825875 & -0.200187364375 \\ 1 & 1 & 1 \end{pmatrix}$$

P1 = (1619.2774144, 401.33679768)

P2 = (1649.9501056, 450.942196572)

P3 = (-0.200226171875, -0.20015825875)

F.

Will update if I have time.

G.

User clicks = (497, 530)

Compute world-space points on the near and far planes.

Set click to near/far

$$\begin{array}{cc} \text{Near} & \text{Far} \\ \begin{pmatrix} 497 \\ 530 \\ 0 \\ 1 \end{pmatrix} & \begin{pmatrix} 497 \\ 530 \\ 1 \\ 1 \end{pmatrix} \end{array}$$

Screen to NDC matrix

Inverse NDC-to-Screen

X = 2/Width = 2/2048	Y = -(2/Height) = -2/1080	Z = 2/z-depth range
X = 0.000977	Y = -0.001852	Z = 2

Screen-to-NDC

$$\begin{pmatrix} 0.000977 & 0 & 0 & -1 \\ 0 & -0.001852 & 0 & 1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

*

$$\begin{pmatrix} 497 & 497 \\ 530 & 530 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$$

Screen-to-NDC positions

$$\begin{array}{cc} \text{Near} & \text{Far} \\ \end{array}$$

$$\begin{pmatrix} -0.514431 & -0.514431 \\ 0.01844 & 0.01844 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}$$

Invert ortho projection and multiply it against the NDC positions

Mortho

$$\begin{pmatrix} 0.002 & 0 & 0 & 0 \\ 0 & 0.0038 & 0 & 0 \\ 0 & 0 & 0.00025 & -1.00025 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$
M_{ortho}^{-1} = \begin{pmatrix} 500 & 0 & 0 & 0 \\ 0 & 263.1578947 & 0 & 0 \\ 0 & 0 & 4000 & 4001 \\ 0 & 0 & 0 & 1 \end{pmatrix}
*
\begin{pmatrix} -0.514431 & -0.514431 \\ 0.01844 & 0.01844 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}
$$

$$
\begin{array}{cc} \text{Near} & \text{Far} \end{array}
$$

$$
\begin{pmatrix} -257.2155 & -257.2155 \\ 4.852631578268 & 4.852631578268 \\ 1 & 8001 \\ 1 & 1 \end{pmatrix}
$$

Now take the Projection Near/Far and multiply the world-to-view matrix by it

$$
C_{view\text{-}to\text{-}world} = \begin{pmatrix} -0.5547 & -0.452057 & 0.698535 & -8 \\ 0 & 0.83953 & 0.543305 & 9 \\ -0.83205 & 0.30137 & -0.46569 & 6 \\ 0 & 0 & 0 & 1 \end{pmatrix}
*
\begin{pmatrix} -257.2155 & -257.2155 \\ 4.852631578268 & 4.852631578268 \\ 1 & 8001 \\ 1 & 1 \end{pmatrix}
$$

$$
\textbf{Near} \begin{pmatrix} \mathbf{133.182307} \\ \mathbf{13.6172348} \\ \mathbf{221.012904} \\ \mathbf{1} \end{pmatrix}
\qquad
\textbf{Far} \begin{pmatrix} \mathbf{5721.462307} \\ \mathbf{4360.057235} \\ \mathbf{-3504.5071} \\ \mathbf{1} \end{pmatrix}
$$

2.

Camera position and orientation
Position = Cpos = (5, 3, 2)
Target = Tpos = (434, -480, 270)
'Up' vector = (0, 1, 0)

Z-depth [0,1]

Camera Perspective
Field of View = PI/6
Near clip plane: 1
Far clip plane: 7000
Aspect ratio: Same as screen (1280x720)
NDC [-1,1] on all 3 axes

a.

Compute the view to world matrix
Done the same way as otho.

2. We need to find the fwd and side vectors

   We are in a right handed area, negate the fwd direction.
   Get fwd = -(Tpos – Cpos) = -dir
   Fwd = (-429, 483, -268)

   Normalize the fwd vector
   ||fwd|| = sqrt(429^2+-483^2+268^2) = 699.3954532
   v^fwd = (-429, 483, -268)/ 699.3954532
   <u>v^fwd = (-0.613387, 0.690596, -0.3831881)</u>

   NOTE: up is not normalized, so neither will side. NORMALIZE IT

   side = 'up' x v^fwd
   'up' = (0, 1, 0)
   V^fwd = (-0.613387, 0.690596, -0.3831881)

   Side = (-0.3831881, 0, 0.613387)
   Now normalize our new side
   ||side|| = sqrt(0.3831881^2 + 0 + -0.613387^2) = 0.7232404
   V^side = (-0.3831881, 0, 0.613387) / 0.7232404
   <u>V^side = (-0.529821, 0, 0.8481093)</u>

   Now for the actual Up vector
   V^up = v^fwd x v^side
   (-0.613387, 0.690596, -0.3831881)
   (-0.529821, 0, 0.8481093)
   <u>V^up = (0.58570, 0.72324, 0.36589)</u>

Cview-to-world =

$$\begin{pmatrix} -0.529821 & 0.58570 & -0.613387 & 5 \\ 0 & 0.72324 & 0.690596 & 3 \\ 0.848109 & 0.36589 & -0.3831881 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b.

Find C World-to-view

Cworld-to-view = Cview-to-world^-1 = $\begin{pmatrix} R^\wedge t & -(R^\wedge t^*Cpos) \\ 0 & 1 \end{pmatrix}$

R^tman -Cpos

$$\begin{pmatrix} -0.529821 & 0 & 0.848109 \\ 0.58570 & 0.72324 & 0.36589 \\ -0.613387 & 0.690596 & -0.3831881 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -5 \\ -3 \\ -2 \\ 1 \end{pmatrix}$$

-(R^t*Cpos) = (0.952887, -5.83, 1.7615232)

Cworld-to-view

$$\begin{pmatrix} -0.529821 & 0 & 0.848109 & 0.952887 \\ 0.58570 & 0.72324 & 0.36589 & -5.8299 \\ -0.613387 & 0.690596 & -0.3831881 & 1.7615232 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

c.

Field of View = PI/6

Near clip plane: 1

Far clip plane: 7000

Aspect ratio: Same as screen = 1280x720

NDC [-1,1] on all 3 axes

Z-depth [0,1]

Perspective Matrix

d = cot(Theta/2) = 1/Tan((PI/6)/2) = 3.73205

a = width/height = 1280/720 = 1.77778

X = d/a Y = d Z = -(f+n)/(f-n) Tz = (2*n*f)/(n-f)

X = 2.09928 Y = 3.73205 Z = -1.00029 Tz = -2.00029

Mpersp

$$\begin{pmatrix} 2.09928 & 0 & 0 & 0 \\ 0 & 3.73205 & 0 & 0 \\ 0 & 0 & -1.0029 & -2.00029 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

d.

NDC to screen matrix

3x3 xyz

X = Width/2 = 1280/2	Y = -(Height/2) = 720/2	Z = z-depth range/2
X = 640	Y = -360	Z = 0.5

NDC-to-Screen

$$\begin{pmatrix} \mathbf{640} & \mathbf{0} & \mathbf{0} & \mathbf{640} \\ \mathbf{0} & \mathbf{-360} & \mathbf{0} & \mathbf{360} \\ \mathbf{0} & \mathbf{0} & \mathbf{0.5} & \mathbf{0.5} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

e.

$$\begin{pmatrix} P1 \\ 523 \\ -637 \\ 302 \\ 1 \end{pmatrix} \quad \begin{pmatrix} P2 \\ 530 \\ -642 \\ 283 \\ 1 \end{pmatrix} \quad \begin{pmatrix} P3 \\ 538 \\ -623 \\ 304 \\ 1 \end{pmatrix}$$

- Its position in camera space

Multiply World-to-view by p1p2p3

$$\begin{pmatrix} -0.529821 & 0 & 0.848109 & 0.952887 \\ 0.58570 & 0.72324 & 0.36589 & -5.83 \\ -0.613387 & 0.690596 & -0.3831881 & 1.7615232 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$*$$

$$\begin{pmatrix} 523 & 530 & 538 \\ -637 & -642 & -623 \\ 302 & 283 & 304 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{array}{ccc} \mathbf{P1`} & \mathbf{P2`} & \mathbf{P3`} \\ \begin{pmatrix} \mathbf{-20.014578} \\ \mathbf{-49.714} \\ \mathbf{-874.672336} \\ \mathbf{1} \end{pmatrix} & \begin{pmatrix} \mathbf{-39.837396} \\ \mathbf{-56.18221} \\ \mathbf{-875.1384511} \\ \mathbf{1} \end{pmatrix} & \begin{pmatrix} \mathbf{-26.265675} \\ \mathbf{-30.07136} \\ \mathbf{-874.9711732} \\ \mathbf{1} \end{pmatrix} \end{array}$$

- Its position in the NDC

Take the perspective matrix and multiply it by P1`P2`P3`

Then divide it by the width

$$\begin{pmatrix} 2.09928 & 0 & 0 & 0 \\ 0 & 3.73205 & 0 & 0 \\ 0 & 0 & -1.0029 & -2.00029 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$*$$

$$\begin{pmatrix} -20.014578 & -39.837396 & -26.265675 \\ -49.714 & -56.18221 & -30.07136 \\ -874.672336 & -875.1384511 & -874.9711732 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -42.01620330384 \\ -185.5351337 \\ 875.2085957744 \\ 874.672336 \end{pmatrix}\begin{pmatrix} -83.62984867488 \\ -209.6748168305 \\ 875.67606260819 \\ 875.1384511 \end{pmatrix} \quad \begin{pmatrix} -55.139006214 \\ -112.227819088 \\ 875.50829960228 \\ 874.9711732 \end{pmatrix}$$

Divide each by their w's to get 1 back in 4th row.

$$\mathbf{P1''} \begin{pmatrix} \mathbf{-0.0480365} \\ \mathbf{-0.2121196} \\ \mathbf{1.0006131} \\ \mathbf{1} \end{pmatrix} \quad \mathbf{P2''} \begin{pmatrix} \mathbf{-0.0955618} \\ \mathbf{-0.2395905} \\ \mathbf{1.0006143} \\ \mathbf{1} \end{pmatrix} \quad \mathbf{P3''} \begin{pmatrix} \mathbf{-0.0630181} \\ \mathbf{-0.1282646} \\ \mathbf{1.0006139} \\ \mathbf{1} \end{pmatrix}$$

- Its screen (x,y) coordinates.

Multiply Px" by NDC-to-Screen to get screen coordinates.

$$\begin{pmatrix} 640 & 0 & 0 & 640 \\ 0 & -360 & 0 & 360 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -0.0480365 & -0.0955618 & -0.0630181 \\ -0.2121196 & -0.2395905 & -0.1282646 \\ 1.0006131 & 1.0006143 & 1.0006139 \\ 1 & 1 & 1 \end{pmatrix}$$

Mndc2scr

$$\begin{array}{ccc} \textbf{P1''} & \textbf{P2''} & \textbf{P3''} \\ \begin{pmatrix} 609.25664 \\ 436.363056 \\ 1.00030655 \\ 1 \end{pmatrix} & \begin{pmatrix} 578.840448 \\ 446.25258 \\ 1.00030715 \\ 1 \end{pmatrix} & \begin{pmatrix} 599.668416 \\ 406.175256 \\ 1.00030695 \\ 1 \end{pmatrix} \end{array}$$

f.

I'll do this if I have time.

g.

Get near/far clip planes in screen space

$$\begin{array}{cc} \text{Near} & \text{Far} \\ \begin{pmatrix} 854 \\ 277 \\ 0 \\ 1 \end{pmatrix} & \begin{pmatrix} 854 \\ 277 \\ 1 \\ 1 \end{pmatrix} \end{array}$$

Invert NDC-to-screen matrix

Screen-to-NDC matrix

X = 2/1280 = 0.0015625

Y = -2/720 = -0.00278

Z = 2/dz = 2/1 = 2

$$\begin{pmatrix} 0.0015625 & 0 & 0 & -1 \\ 0 & -0.00278 & 0 & 1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

NDC

$$\begin{array}{cc} \text{Near} & \text{Far} \\ \begin{pmatrix} 0.334375 \\ 0.22994 \\ -1 \\ 1 \end{pmatrix} & \begin{pmatrix} 0.334375 \\ 0.22994 \\ 1 \\ 1 \end{pmatrix} \end{array}$$

Inverse Perspective Matrix

d = cot(Theta/2) = 1/Tan((PI/6)/2) = 3.73205 near plane: 1

a = width/height = 1280/720 = 1.77778 far plane: 7000

X = a/d Y = 1/d Z = 0 Wfwd = -(f-n)/(2fn) Tw = (f+n)/(2fn)

X = 0.476355 Y = 0.26795 Z = 0 Wfwd = -0.49993 Tw= 0.50007

Mpersp^-1

$$\begin{pmatrix} 0.476355 & 0 & 0 & 0 \\ 0 & 0.26795 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -0.49993 & 0.50007 \end{pmatrix} * \begin{pmatrix} 0.334375 & 0.334375 \\ 0.22994 & 0.22994 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}$$

Mpersp^-1 (Hom. Coord)

$$\begin{array}{cc} \text{near} & \text{far} \\ \begin{pmatrix} 0.159281203 \\ 0.061612423 \\ -1 \\ 1 \end{pmatrix} & \begin{pmatrix} 0.159281203 \\ 0.061612423 \\ -1 \\ 0.00014 \end{pmatrix} \end{array}$$

Divide each coordinate by their w row to get value back to 1

Camera Space

$$\begin{array}{cc} \text{near} & \text{far} \\ \begin{pmatrix} 0.159281 \\ 0.061612 \\ -1 \\ 1 \end{pmatrix} & \begin{pmatrix} 1114.968115 \\ 431.2873923 \\ -7000.007 \\ 1 \end{pmatrix} \end{array}$$

Now multiply against the Cview-to-world to get proper cords

$$\text{Cview-to-world} = \begin{pmatrix} -0.529821 & 0.58570 & -0.613387 & 5 \\ 0 & 0.72324 & 0.690596 & 3 \\ 0.848109 & 0.36589 & -0.3831881 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 0.159281 & 1114.968115 \\ 0.061612 & 431.2873923 \\ -1 & -7000.007 \\ 1 & 1 \end{pmatrix}$$

World Space

$$
\begin{array}{cc}
\textbf{Near} & \textbf{far} \\
\begin{pmatrix} 5.5650827 \\ 2.3539643 \\ 2.5408190 \\ 1 \end{pmatrix} &
\begin{pmatrix} 3960.5847977 \\ -4519.252541 \\ 3787.7376193 \\ 1 \end{pmatrix}
\end{array}
$$

3.

W

$$
\begin{pmatrix}
-0.44659 & 0.539634 & -0.71369 & 1 \\
-0.6621 & 0.337197 & 0.669271 & 3 \\
0.601815 & 0.771423 & 0.206702 & 15 \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

TGO model

$$
\begin{array}{cccc}
P1 & P2 & P3 & P4 \\
\end{array}
$$

$$
\begin{pmatrix}
0 & 0 & -1 & 1 \\
1 & 0 & 0 & 0 \\
0 & 2 & -1 & -1 \\
1 & 1 & 1 & 1
\end{pmatrix}
$$

Tris(a,b,c)

$$
\begin{pmatrix}
T1 & 1, 2, 4 \\
T2 & 1, 4, 3 \\
T3 & 1, 3, 2 \\
T4 & 2, 3, 4
\end{pmatrix}
$$

Camera Position and orientation

Cpos = (7, -1, 9)

Tpos = TGO at (1, 3, 15)

‘up’ vector = (0, 1, 0)

z-depth range [0,1]

Camera Perspective projection

FOV: PI/6

near clip plane: 1

far clip plane: 1000

Aspect Ratio: Screen: 1280x720

NDC [-1,1] on all 3 axes

Determine which triangles will be back-face culled and which will be visible to camera

Multiply world against TGO model to get vertices in world space

Ra	Rb		Rc	
Ba		Bc	Bb	<-each tri a,b,c
Ga	Gc	Gb		
	Wa	Wb	Wc	

$$
\begin{array}{cccc}
P1 & P2 & P3 & P4
\end{array}
$$

$$
\begin{pmatrix}
1.539634 & -0.42738 & 2.16028 & 1.2671 \\
3.337197 & 4.338542 & 2.992829 & 1.668629 \\
15.771423 & 15.413404 & 14.191483 & 15.395113 \\
1 & 1 & 1 & 1
\end{pmatrix}
$$

Let's go in order

<u>T1 - red</u>

V1 (b-a) = (-1.967014, 1.001345, -0.358019)
V2 (c-a) = (-0.272534, -1.668568, -0.37631)

Get the normal vector by crossing the two we just created.

N = v1 x v2 = (1.001345*-0.37631 - -0.358019*-1.668568
-(-1.967014*-0.37631 - -0.358019*-0.272534),
-1.967014*-1.668568 – 1.001345*-0.272534)

(−0.97420, −0.64263, 3.55500)

Vlos = (a –Cpos) = (-5.460366, 4.337197, 6.771423)

Now get the dot product of the normal and Vlos. If < 0, we are visible to the camera

Vlos.dot(n) = −0.97420 * -5.460366 + −0.64263 * 4.337197 + 3.55500 * 6.771423
26.5708273

<u>Red is back-faced culled</u>

<u>T2 - Blue</u>

V1 (b-a) = (-0.272534, -1.668568, -0.37631)
V2 (c-a) = (0.620646, -0.379141, -1.57994)

Get the normal vector by crossing the two we just created.

N = v1 x v2 = (2.49356, −0.66414, 1.13892)

Vlos = (a –Cpos) = (-5.460366, 4.337197, 6.771423)

Now get the dot product of the normal and Vlos. If < 0, we are visible to the camera

Vlos.dot(n) = -8.78415
<u>Blue is visible from the camera</u>

T3 - Green

V1 (b-a) =(0.620646, -0.379141, -1.57994)

V2 (c-a) =(-1.967014, 1.001345, -0.358019)

Get the normal vector by crossing the two we just created.

N = v1 x v2 =(1.717805, 3.32997, −0.124295)

Vlos = (a –Cpos) = (-5.460366, 4.337197, 6.771423)

Now get the dot product of the normal and Vlos. If < 0, we are visible to the camera

Vlos.dot(n) = 4.221237851

Green is back-culled

T4 - White

V1 (b-a) = (2.58408, -1.345713, -1.221921)

V2 (c-a) = (1.69448, -2.669913, -0.018291)

Get the normal vector by crossing the two we just created.

N = v1 x v2 = (−3.23781, −2.02326, −4.61899)

Vlos = (a –Cpos) = (-7.42738, 5.338542, 6.413404)

Now get the dot product of the normal and Vlos. If < 0, we are visible to the camera

Vlos.dot(n) = -16.37601535

White is visible from the camera