425 Applied 3D Algebra

HW 7

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1.

Line S defined by P0 = (4, 237, -3) and P1 = (-4, 243, 3)
For each point below, computer nearest point on the segment

$S(t)$=$P0$+$t(P1-P0)$" for " 0≤t≤1

Q = the point we are testing against
V = The vector of the line = P1 – P0 = (-8, 6, 6)
v.dot(v) = 136

"If t < 0 then return " Q^′=$S(0)$=P_0
"If t > 1 then return " Q^′=$S(1)$=P_1
"Otherwise return " Q^′=$S(t)$

A. Qa = (5, 228, -12)
   Q-P0 = (5, 228, -12) - (4, 237, -3) = (1, -9, -9)
   v.dot(Q-P) = -116
   t = v.dot(Q-P)/v.dot(v) = -116/136 = -0.8529411765

   t is < 0. Q` = s(0) = P0
   <u>Q` = (4, 237, -3)</u>

B. Qb = (-16, 240, 15)
   Q-P0 = (-16, 240, 15) - (4, 237, -3) = (-20, 3, 18)
   v.dot(Q-P) = 286
   t = v.dot(Q-P)/v.dot(v) = 286/136 = 2.102941176

   t > 1. Q` = P1
   <u>Q` = (-4, 243, 3)</u>

C. Qc = (-13, 231, -6)
   Q-P0 = (-13, 231, -6) - (4, 237, -3) = (-17,-6, -3)
   v.dot(Q-P) = (-8, 6, 6).dot((-17,-6, -3)) = 82
   t = v.dot(Q-P)/v.dot(v) = 82/136 = 0.6029411765

   t !> 1 && t !< 0. Q` = $P0+t(P1-P0)$
   Q` = (4,237,-3) + (-4.823529412, 3.617647059, 3.617647059)
   <u>Q` = (-0.82353, 240.61765, 0.61765)</u>

D. Qd = (-4, 237, 6)

Q-P0 = (-4, 237, 6) - (4, 237, -3) = (-8, 0, 9)

v.dot(Q-P) = 118

t =v.dot(Q-P)/v.dot(v) = 118/136 = 0.8676470588

t !> 1 && t !< 0. Q` = $P0+t(P1-P0)$

Q` = (4,237,-3) + (-6.94117647, 5.205882353, 5.205882353)

Q` = (-2.94118, 242.20588, 2.20588)

2.

AABB defined by

Min = (4, 13, 16)

Max = (16, 17 24)

Determine whether each line intersects with AABB

How to test (do for x y and z separately):

If Vx = 0

if P0x < Minx OR P0x > Max x

return false

else [Sx, tx] = [-inf, inf] <- means all axis are covered

else

a = (Min x – P0x)/Vx

b = (Max x – P0x)/Vx

sx = smallest of a or b

tx = largest of a or b

Set min/max table with what was found

if $[s_x, t_x]$ overlaps $[s_y, t_y]$ AND

$[s_y, t_y]$ overlaps $[s_z, t_z]$ AND

$[s_z, t_z]$ overlaps $[s_x, t_x]$ then return true

Test As such (MaxA < MinB || MinA > MaxB)

If true, return false

P V

a. L(t) = (-8, 14, 15) + t(5, 0, 1)

Sx = 2.4 tx = 4.8

Sy = -inf ty = inf

Sz = 1 tz = 9

Min	Max		Min	Max		
2.4	4.8		-1E+100	1E+100	->	TRUE
-1E+100	1E+100	VS	1	9	->	TRUE
1	9		2.4	4.8	->	TRUE

LINE AND AABB INTERSECT

P V

b. L(t) = (8,- 29, -4) + t(0, -7, 1)

Sx = -inf tx = inf

Sy = -6 ty = -6.571428571

Sz = 20 tz = 28

Min	Max		Min	Max		
-1E+100	1E+100		-6.5714	-6	->	TRUE
-6.5714	-6	VS	20	28	->	FALSE
20	28		-1E+100	1E+100	->	TRUE

LINE AND AABB DO NOT INTERSECT

3.

Compute distance between two lines

Get a = v1.dot(v1) b = v1.dot(v2) c = v2.dot(v2) d = (P1- P2).dot(v1) e = (P1 – P2).dot(v2)

If b^2 – ac = 0

Lines are parallel

Use P1 and L2(P2,V2) to find nearest point

C1 = P1

C2 = P2 + (e/c)v2

Else

C1 = P1 + (cd – be/b^2 – ac)*v1

C2 = P2 + (db – ae/b^2 – ac)*v2

Distance = ||C1 – C2|| = sqrt(C1x*C2x – C1y*C2y – C1z*C2z)

L1(t) =(1, -17, -20) + t (1, 5, 1) L2(t) =(8, -29, -4) + t (0, -7, 1)

P1 – P2 = (-7, 12, -16)

a = 27 b = -34 c = 50 d = 37 e = -100

b^2 – ac = -194 <- not parallel

cd – be = -1550

db – ae = 1442

cd-be/b^2-ac = 7.989690722

db-ae/b^2-ac = -7.432989691

(cd-be/b^2-ac) *v1 = (7.989690722, 39.94845361, 7.989690722)

(db-ae/b^2-ac) *v2 = (0, 52.03092784, -7.432989691)

C1 = P1 + (cd-be/b^2-ac) *v1 = (8.989690722, 22.97875364, -12.01030928)
C2 = P2 + (db-ae/b^2-ac) *v2 = (8, 23.03092784, -11.43298969)

C1 – C2 = (0.989690722, -0.0521742, -0.57731959)
Distance = ||C1 – C2|| = sqrt(0.9794877252 + 0.002722147 + 0.333297909)
= sqrt(1.305507781)
Distance = 1.146955876 <- using smaller decimal places will change this value slightly

4.

Given Plane defined by:
P0 = (12, -12, -15) n = (0, 3, -4)
Determine if the following two spheres:
Intersect with the plane
If their center is above or below the plane

Distance from C to plane
d = abs(((C-P0).dot(n))/||n||)

If d < r` they intersect

||n|| = sqrt(0^2 + 3^2 + -4^2) = 5

Sphere above/below plane Check
(C-P0).dot(n)
Pos = above Neg = Below

1. C` = (9, -7, -22) r` = 8
   a. Check intersection
      C – P0 = (-3, 5, -7)
      C – P0.dot(n) = 43
      d = C – P0.dot(n)/||n|| = 43/5 = 8.6
      d < r`?
      8.6 < 8 FALSE
      <u>SPHERE AND PLANE DO NOT INTERSECT</u>
   b. Above or below plane?
      C – P0.dot(n) = 43
      <u>SPHERE IS ABOVE PLANE</u>

2. C` = (9, -15, -10) r` = 12
   a. Check intersection
      C-P0 = (-3, - 3, 5)
      C-P0.dot(n) = -29
      D = C-P0.dot(n)/||n|| = -29/5 = -5.8
      d < `r?
      -5.8 < 12 TRUE
      <u>SPHERE AND PLANE INTERSECT</u>
   b. Above or below plane?
      C-P0.dot(n) = -29
      <u>SPHERE IS BELOW PLANE</u>

5.

Consider plane defined by:

P0 = (-3, -3, -6)　　　n = (0, -1, 3)

Determine if the segments intersect the plane

If t`<0 or >1

NO INTERSECTION

else

Intersection line = $(t')=S1+t`\ (S2-S1)$

$t'=((P0-S1\ )\cdot\mathbf{n})/((S2-S1)\cdot\mathbf{n})$

<u>Segment 1:</u>　S1 = (10, -15, -7)　　S2 = (9, -14, -12)

P0 – S1 = (-13, 12, 1)　　<u>t` is not < 0 or > 1</u>

S2 – S1 = (-1, 1, -5)　　<u>Continuing</u>

(P0 – S1).dot(n) = -9　　t`(S2-S1) = (-0.5625, 0.5625, -2.8125)

(S2 – S1).dot(n) = -16　　S1 + t`(S2-S1) = (9.4375, -14.4375, -9.8125)

`t = -9/-16 = 0.5625

(9.4375, -14.4375, -9.8125)

<u>Intersection point</u>

<u>Segment 2:</u>　S1 = (10, -14, -9)　　S2 = (8, -16, -10)

P0 – S1 = (-13, 11, 3)　　**<u>t` is < 0 or > 1</u>**

S2 – S1 = (-2, -2, -1)　　**<u>SEGMENT AND PLANE DO NOT INTERSECT</u>**

(P0 – S1).dot(n) = -2

(S2 – S1).dot(n) = -1

t` = -2/-1 = 2

6.

Given OBB and plane:

OBB: Min = (-2.250, -5.500, -4.875) Plane: P0 = (-15, -3, 9)

Max = (1.750, 6.500, 5.125) n = (-3, 1, -3)

$$W = \begin{pmatrix} 1.24808 & -0.6749 & -0.4867 & -13 \\ 0 & 0.87735 & -1.2167 & 4 \\ 0.83205 & 1.01232 & 0.73 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$W\text{^-1} = \begin{pmatrix} 0.55469 & 0.00006 & 0.36981 & 6.8412 \\ -0.29995 & 0.38993 & 0.44992 & -5.90892 \\ -0.21629 & -0.54072 & 0.32443 & -0.97329 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

For OBB:

a. What is the center C in LOCAL SPACE?

C Local = max+min/2 = (-0.25, 0.5, 0.125)

b. What is the half-diagonal vector d = (a, b, c)?

a = (Max x – Min x)/2 = 2

b = (Max y – Min y)/2 = 6

c = (Max z – Min z)/2 = 5

d = (2, 6, 5)

c. What is the (uniform) scale factor used in construction of W? (assume TRS form)

Len x = sqrt(1.24808^2 + 0 + 0.83205^2)
= 1.5

Len y = sqrt(-0.6749^2 + 0.87735^2 + 1.01232^2)
= 1.5

Len z = sqrt(-0.4867^2 + -1.2167^2 + 0.73^2)
= 1.5

$$\mathbf{S} = \begin{pmatrix} \mathbf{1.5} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1.5} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1.5} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

d. What is the center C in <u>WORLD SPACE?</u>

<u>World Space C = W*LocalC = (-13.7103075, 4.2865875, 1.3893975)</u>

e. Determine if OBB intersects the plane using algorithm from class

$$W^{-1} = \begin{pmatrix} 0.55469 & 0.00006 & 0.36981 & 6.8412 \\ -0.29995 & 0.38993 & 0.44992 & -5.90892 \\ -0.21629 & -0.54072 & 0.32443 & -0.97329 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

OBB: Min = (-2.250, -5.500, -4.875) Plane: P0 = (-15, -3, 9)
Max = (1.750, 6.500, 5.125) n = (-3, 1, -3)

a,b,c
d = (2, 6, 5)

n = v

1. Compute v` = W^-1*v = (-2.773511, -0.059996, -0.8651577)
2. Compute ProjMax = (|v`x*a| + |v`y*b| + |v`z*c|)/||v||
   |V`x*a| = 5.547022
   |V`y*b| = 0.359976
   |V`z*c| = 4.3257885
   ||v|| = sqrt(19) = 4.3589

   ProjMax = 10.2327865/4.3589= 2.34756

   s^2 = Wfwd.dot(Wfwd) = 2.25014

   (s^2)projMax = 2.34756 * 2.25014 = 5.28234

   h = (s^2)projMax = 5.28234
3. Compute S1 and S2
   C = (-13.7103075, 4.2865875, 1.3893975)
   h*(n/||n||) = 5.28234*((-3,1,-3)/4.3589) = (-3.635554842, 1.211851614, -3.635554842)
   S1 = C – (h*(n/||n||)) =
   (-13.7103075, 4.2865875, 1.3893975) – (-3.635554842, 1.211851614, -3.635554842)
   = (-10.07475266, 3.074735886, 5.024952342)
   S2 = (C + (h*n))/||v|| =
   (-13.7103075, 4.2865875, 1.3893975) + (-3.635554842, 1.211851614, -3.635554842)
   = (-17.34586234, 5.498439114, -2.246157342)

4. Compute d1 and d2

   d1 = (S1 – P0).dot(n)

   d2 = (S2 – P0).dot(n)

   S1 - P0 = (-10.07475266, 3.074735886, 5.024952342) - (-15, -3, 9)

   = (4.92524734, 6.074735886, -3.975047658)

   S2 – P0 = (-17.34586234, 5.498439114, -2.246157342) - (-15, -3, 9)

   = (-2.34586234, 8.498439114, -11.24615734)

   d1 = (4.92524734, 6.074735886, -3.975047658).dot((-3,1,-3)) = 3.224136836

   d2 = (-2.34586234, 8.498439114, -11.24615734).dot((-3,1,-3)) = 49.27449815

   if d1 > 0, OBB above the plane

   If d1 < 0 AND d2 > 0, OBB intersects the plane

   If d2 < 0, OBB below the plane

   <u>**D1 > 0, OBB ABOVE THE PLANE**</u>