425 Applied 3D Algebra

HW 1

Joey Domino

1. Vector Projection – Found via finding the dot product of vA and vB,
   Dividing it by the length of target vector (vB),
   Multiplying result by target vector (vB)

   Formula for projecting vA onto vB = proj(bA)= (dotProd(vA,vB)/Length(vB))*vB

   a. vA = 5, 0, -9
   vB = 7, 2, 7
   Dot product = vA * vB
   = (5 * 7) + (0 * 2) + (-9 * 7)
   = 35 + 0 - 63
   = -28
   Length of target vector = vB*vB
   = 7*7 + 2*2 + 7*7
   = 49 + 4 + 49
   = 102
   Length of part of vA Parallel to vB (dot/len) = -28/102
   Projection = -28/102 * (7, 2, 7)
   = -14/51 * (7, 2, 7)

   b. vA = 7, 1, 4
   vB = -6, -9, 5
   Dot product = vA * vB
   = (7 * -6) + (1 * -9) + (4 * 5)
   = -42 – 9 + 20
   = -31
   Length of target vector = vB*vB
   = (-6 * -6) + (-9 * -9) + (5 * 5)
   = 36 + 81 + 25
   = 142
   Length of vA Parallel to vB (dot/len) = -31/142
   Projection = -31/142 * (-6, -9, 5)

c. vA = -7, 8, -3

vB = 4, 6, 6

Dot product = vA * vB

= (-7 * 4) + (8 * 6) + (-3 * 6)

= -28 + 48 - 18

= 2

Length of target vector = vB*vB

= (4*4)+(6*6)+(6*6)

= 16 + 36 + 36

= 88

Length of vA Parallel to vB (dot/len) = 2/88

Projection = -2/88 * (4, 6, 6)

= 1/44 * (4, 6, 6)

2. Calculate cross product, then show result is perpendicular to each vertex

a. vA = -4, -6, 4

vB = -4, 5, 6

vW = vA x vB

= ((-6*6) – (5*4)), - ((-4*6) – (-4*4)), ((-4*5)– (-4*-6))

= (-36 – 20), -(-24 + 16), (-20 – 24)

= -56, 8, -44

Cross product is perpendicular to the vector if their dot product is 0.

Dot(vW, vA) = (-56 * -4) + (8 * -6) + (-44 * 4)

= 224 - 48 – 176

= 0

Dot(vW, vB) = (-56 * -4) + (8 * 5) + (-44 * 6)

= 224 + 40 – 264

= 0

b. vA = -10, -4, -7

vB = 6, -9, -9

vW = vA x vB

= ((-4 * -9) – (-9 * -7)), -((-10 * -9) – (6 * -7)), ((-10 * -9) – (6 * -4))

= (36 – 63), -(90 + 42), (90 + 24)

= -27, -132, 114

Cross product is perpendicular to the vector if their dot product is 0.

Dot(vW, vA) = (-10 * -27) + (-4 * -132) + (-7 * 114)

= 270 + 528 – 798

= 0

Dot(vW, vB) = (6 * -27) + (-9 * -132) + (-9 * 114)

= -162 + 1188 – 1026

= 0

c. vA = (6, 7, 6)
   vB = (-3, -1, 7)
   vW = vA x vB
      = (7*7 – (-1*6), -((6*7) – (-3*6)), ((6*-1) – (-3*7))
      = (49 + 6), -(42 + 18), (-6 + 21)
      = 55, -60, 15

   Cross product is perpendicular to the vector if their dot product is 0.

   Dot(vW,vA) = (6 * 55) + (7 * -60) + (6 * 15)
      = 330 – 420 + 90
      = 0

   Dot(vW,vB) = (-3 * 55) + (-1 * -60) + (7 * 15)
      = -165 + 60 + 105
      = 0

3. Points are collinear if their 3 way cross is 0 (P1 – P0) x (p2 – p0) = 0

a. P0 = 9, -9, 8
   P1 = -4, -5, -5
   P2 = -17, -1, -18
   P1 – P0 = -4 – 9, -5 – (-9), -5 – 8
      = -13, 4, -13
   P2 – p0 = -17 – 9, -1 – (-9), -18 – 8
      = -26, 8, -26
   (P1 – P0) x (p2 – p0) = ((4*-26) – (8*-13), -((-13*-26)-(-26*-13)), ((-13*8))-(-26*4))
      = (-104 +104), -(338 – 338), (-104 + 104)
      = 0,0,0

   Colinear

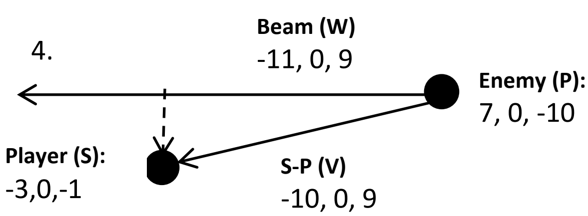
b. P0 =(3, 7, -6)
   P1 = (-9, -3, -3)
   P2 = (39, 37, -15)
   P1 – P0 = -9 – 3, -3 – 7, -3 – (-6)
      = -12, -10, 3
   P2 – p0 = 39 – 3, 37 – 7, -15 – (-6)
      = 36, 30, -9
   (P1 – P0) x (p2 – p0) = ((-10*-9) – (30*3), -((-12*-9)-(36*3)), ((-12*30))-(36*-10))
      = (90 - 90), -(108 – 108), (-360 + 360)
      = 0,0,0

   Colinear

c. P0 = (-10, -6, -8),
P1 = (5, -6, -2)
P2 = (3, 2, 4)
P1 – P0 = 5 + 10, -6 + 6, -2 + 8
= 10, 0, 6
P2 – p0 = 3 + 10, 2 + 6, 4 + 8
= 13, 8, 12
(P1 – P0) x (p2 – p0) = ((0*12) – (8*6), -((10*12)-(13*6)), ((10*8))-(13*0))
= (0 - 48), -(120 - 78), (80 - 0)
= -48, -42, 80
Not Colinear

d. P0 = (-7, 8, 1)
P1 = (-9, -10, -4)
P2 = (-6, -3, 6)
P1 – P0 = -9 + 7, -10 - 8, -4 - 1
= -2, -18, -5
P2 – p0 = -6 + 7, -3 - 8, 6 -1
= 1, -11, 5
(P1 – P0) x (p2 – p0) = ((-18*5) – (-11*-5), -((-2*5)-(1*-5)), ((-2*-11))-(1*-18))
= (-90 - 55), -(-10 + 5), (22 + 18)
= -35, 5, 40
Not colinear

4.



a. Distance to enemy = length of S-P
= Sqrt(-10*-10 + 0*0 + 9*9)
= sqrt(100 + 0 + 81)
= sqrt(181) < can stop here to avoid floats
= 13.4536

b. If the enemy fired, would it hit you? If not, what distance will it miss?
Find Length of perpendicular vector from the player to the beam.
This is the calculation we'll be using.
d = V – ((V*W)/(W*W))*W

-Let's break it down into each of its parts for easier digestion.
V = -10, 0, 9
W = -11, 0, 9

-Get the length of W
W*W = -11*-11 + 0*0 + 9*9
= 121 +0 + 81
= 202

-Now the dot product of vV and vW
V * W = -10*-11 + 0*0 + 9*9
= 110 + 81
= 191
-Divide the two in preparation for the projection calculation
V*W/W*W = 191/202
= 0,9455
-Get our projection
Projection vW = (V*W/W*W)*W
= (-11 *191)/202, (0*191)/202, (9*191)/202
= -2101/202, 0, 1719/202
= -10.4, 0, 8.5099

-Find the perpendicular vector based on where the projection ends.
PerpvW = V-(V*W/W*W)*W
= -10 + (2101/202), 0+0, 9 – (1719/202)
= -10 + 10.4, 0-0, 9 – 8.5099
= 0.401, 0, 0.4901

-The length of the perpendicular vector is our distance from the beam. Anything over a 0 is a miss!
Distance from beam = length of perp vW
= sqrt(perpvW) < can stop here to avoid floats.
= sqrt(0.401*0.401 + 0*0 + 0.4901*0.4901)
= sqrt(0.1608 + 0 + 0.2402
= sqrt(.401)
= 0.6332

c. How to determine if laser is pointing toward or away from player
The projection of the beam vector will be positive if heading towards the player.

5. Find if the following points are left or right of line L(t) on a plane P
Plane has a normal n = (-1, 161, 43)
Plane has a point P0 = (9, 0, -2)
Line vector v = (11, 1, -4)
Line L(t) = P0 + tv

To find out what side of the line we're on relative to the plane, we'll need to first find a left vector based off of the normal vector and the vector creating the line. We'll do this by crossing the two.

vLeft = n x v
vLeft= ((161*-4)-(1*43)), -((-1*-4)-(43*11)), ((-1*1)-(161*11))
vLeft = -687, 469, 1771

Now we can calculate what side of L(t) the following vectors are on. Find the dot product of vLeft *w. If it's positive, the vector is on the left and on the right if negative.

P1. (-1, -3, 9)
W = P1 – P0
= 8, -3, 11
k = vLeft * w
k = -687 * 8 + 469 * -3 + 1771 * 11
k = -5496 – 1407 + 19481
k = 12578
P1 is on the LEFT side of L(t)

P2. (39, 9, -35)
W = P2 – P0
= 39 – 9, 9 – 0, -35 + 2
= 30, 9, -33
k = vLeft * w
k = -687 * 30 + 469 * 9 + 1771 * -33
k = -20610 + 4221 - 58443
k = -74832
P2 is on the RIGHT side of L(t)

P3. (-31, -12, 42)

W = P3 – P0

= -31 – 9, -12 – 0, -42 + 2

= -40, -12, -40

k = vLeft * w

k = -687 * -40 + 469 * -12 + 1771 * -40

k = 27480 -5628 - 70840

k = -48988

P3 is on the RIGHT side of L(t)

6. Determine if each line below is parallel to the plane.

   If it is, compute if the line is in the plane or not

   If not, compute the intersection point (with floating points)

P0 = -10, 4,-4

N = 32, -38,- 8

To calculate if line is parallel to plane: dot(v,n) = 0

To calculate if line is in plane = dot((S-P0),n) = 0

To calculate if line intersects plane

Find a point on the line that satisfies plane relation = t` = ((P0 – S) * n)/(dot(v,n)

Find intersection point with t` = L(t`) = S + t`v = S + ((P0-S)*n)/dot(v,n))*v

a. L(t) = (7,8,6) +t(-3,-4,7)

Checking for parallel to plane

Dot(v,n) = -3 * 32 + -4 * -38 + 7 * -8

= -64 + 152 – 56

= 32

Dot is not 0 so we can conclude that:

a) **This line is not parallel to the plane**

Find plane relation

t` = ((P0 – S) * n)/(dot(v,n)

P0-S = (-10 -7), (4 – 8), (-4 – 7)

= (-17, -4, -11)

(P0-S)*n = -17*32 + -4*-38 + -11 *-8

= -544 + 152 + 88

= -304

(P0-S)*n)/dot(v,n) = -304/32

t` = -9.5

Finding intersection point
t` = L(t`) = S + t`v = S + ((P0-S)*n)/dot(v,n))*v

We have everything for the shorter calculation. Let's use that.
t` = (7,8,6) +9.5 (-3,-4,7)
= (7,8,6) + (-28.5, -38, 66.5)
Below is our intersection point on the plane
b) **t` = -21.5, -30, 72.5**

b. L(t) = (-8,-10,8) +t(4,-8,7)
Dot(v,n) = 4 * 32 + -8 * -38 + 7 * -8
= 128 + 304 -56
= 376
Dot is not 0 so we can conclude that:
a) **This line is not parallel to plane**

Find plane relation
t` = ((P0 – S) * n)/(dot(v,n)
P0-S = (-10 + 8), (4 + 10), (-4 – 8)
= (-2, 14, -12)
(P0-S)*n = -2*32 + 14*-38 + -12 *-8
= -64 – 532 + 96
=-500
(P0-S)*n)/dot(v,n) = -500/376
t` = -1.3298

Finding intersection point
t` = L(t`) = S + t`v = S + ((P0-S)*n)/dot(v,n))*v

We have everything for the shorter calculation. Let's use that.
t` = (-8, -10, 8) + -1.3298 (4,-8,7)
= (-8,-10,8) + (-5.3192, 10.6384, -9.3086)
Below is our intersection point on the plane
b) **t` = -13.3192, 0.6384, -1.3086**

c. L(t) = (-7,8,-11) +t(13,12,-5)
Dot(v,n) = 13 * 32 + 12 * -38 + -5 * -8
= 416 – 456 + 40
= 0
a) **This line is parallel to the plane**

Finding if line is in the plane

dot((S-P0),n) = 0

S – P0 = (-7 +10), (8-4), (-11+4)

= 3, 4, -7

Dot((S-P0),n) = (3*32 + 4 * -38 + -7 *-8)

= (64 – 152 + 56)

= -32

The dot product needed to be 0, so we can conclude that:

b) **Line is NOT in the plane**