425 Applied 3D Algebra

HW 4

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1.

a. World space = (3,-1,3)

b.

Scale x = Length(colx)

Sqrt(-1.94029^2 + 0 + -0.48507^2)

2

Scale y = Len(coly)

Sqrt(0.14292^2 + 2.42956^2 + -0.57166^2)

2.5

Scale z = Len(colz)

Sqrt(0.707107^2 + -0.70711^2 + -2.82843^2)

3

c.

Left (x) unit vector = (-1.94029/2, 0, -0.48507/2)

(-0.970145, 0, -0.242535)

Up (y) unit vector = (0.14292/2.5, 2.42956/2.5, -0.57166/2.5)

(0.057168, 0.971824, -0.228664)

Forward (z)unit vector = (0.707107/3, -0.70711/3, -2.82843/3)

(0.2357023, -0.235703, -0.94281)

d.

Let's first get the Translation, Rotation, and Scalar matrices or TRS

T – world space given in step a

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R – Plug in our unit vectors created in step c

$$\begin{pmatrix} -0.970145 & 0.057168 & 0.2357023 & 0 \\ 0 & 0.971824 & -0.235703 & 0 \\ -0.242535 & -0.228664 & -0.94281 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

S – The scale factors we found during step b

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2.5 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Det(W) = Det(T) * Det(R) * Det(S)

Matrix T is an upper triangle matrix due to the lower half being all 0's. This means we can multiply the main diagonal together to get the determinant. T's determinant is 1.

Det(T) = 1

Matrix R does not affect the length of our world matrix. Its determinant is 1.

Det(R) = 1

Matrix S affects the world matrix, so we will calculate that determinant. Since Matrix S is a lower and upper triangle matrix, we only need to multiply the main diagonal together to get the det.

Det(S) = 2*2.5*3*1
Det(S) = 15

Det(W) = 1 * 1 * 15
Det(W) = 15

e.

To find the second position in terms of the first's local space, we must first invert the original's world space. Then multiply that inverse by the 2^{nd}'s world space.

WA^1 = T^-1 R^-1 S^-1

T^-1 – We need to negate our translation values to get back to 0,0,0

$$\begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R^-1 – Rotation is the translation. AKA: Col 1/2/3 becomes Row 1/2/3

Our rotation matrix

$$\begin{pmatrix} -0.970145 & 0.057168 & 0.2357023 & 0 \\ 0 & 0.971824 & -0.235703 & 0 \\ -0.242535 & -0.228664 & -0.94281 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

R^-1 - Move col 1/2/3 to row 1/2/3

$$\begin{pmatrix} -0.970145 & 0 & -0.242535 & 0 \\ 0.057168 & 0.971824 & -0.228664 & 0 \\ 0.2357023 & -0.235703 & -0.94281 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

S^-1 – 1/our scale factors

$$\begin{pmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1/2.5 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

W^-1 = T^-1R^-1S^-1

Remember to multiply TRS together from RIGHT TO LEFT

$$\begin{pmatrix} -0.48507 & 0 & -0.12127 & 1.81902 \\ 0.02287 & 0.38873 & -0.09147 & 0.59452 \\ 0.07857 & -0.07857 & -0.31427 & 0.62854 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

WB – We aren't given a scale or rotation for the second position, so its world is just the translation

$$\begin{pmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

WA^-1 * WB – second positions location in terms of first object's location and rotation

Second object position relative to first object = (0, 3.88057, 1.37197, 0.47139)

f.

Same flow as e. WA^-1 * Vc   Vector position relative to first position.
NOTE Vectors have a 0 position in 4th column!

WA^-1

$$\begin{pmatrix} -0.48507 & 0 & -0.12127 & 1.81902 \\ 0.02287 & 0.38873 & -0.09147 & 0.59452 \\ 0.07857 & -0.07857 & -0.31427 & 0.62854 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad X$$

$$Vc \quad \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Third vector position relative to first object position
WA^-1 * Vc = (0, -1.69775, -1.28052, -0.15712)

2.

a.

1- Normalize axis r
||r|| = sqrt(16 + 25 + 1)
||r|| = 6.4807
r^ = -4/6.4807, -5/6.4807, 1/6.4807
r^ = (-0.61722, -0.77152, 0.1543)

2- Computer half angle of sin/cos values
$\theta/2$ = 0.5236

3- Build out quaternion q = cos($\theta/2$), (r^ * sin($\theta/2$)
q = (0.866, -0.30861, -0.3858, 0.07715)

b.

No fancy work. World is 0 because we're a vector, V is our vector. So…
p = (0, -1, 3, 0)

c.

Multiply the two quarternions together:
qp = wq*wp – vq.dot(vp), wq*vp + wp*vq + vq.cross(vp)

wq*wp = 0
vq.dot(vp) = -0.30861*-1 -0.3858*3 + 0
= -0.84879
Wq*vp = (-0.866, 2.598, 0)

wp*vq = (0, 0, 0)

vq.cross(vp) = -0.3858*0 – 0.07715*3,
-0.30861*0 - 0.07715*-1,
-0.30861*3 - -0.3858*-1

(-0.23145, -0.07715, -1.31163)

qp = (0.84879, -1.09745, 2.52085, -1.31163)

d.

Inverse Quaternion
q^-1 = (qw,-qv)
q^-1 = (0.866, 0.30861, 0.3858, -0.07715)

e.

qp = wq*qp – vq.dot(vp), wq*vp + wp*vq + vq.cross(vp)

Wqp * Wq^-1 = 0.84879*0.866
= 0.73505

Vqp.dot(Vq^-1) = (-1.09745, 2.52085, -1.31163) *
(0.30861, 0.3858, -0.07715)
= -0.33868 + 0.97254 + 0.10119
= 0.73505

Wqp * Vq^-1 = 0.84879 * (0.30861, 0.3858, -0.07715)
= (0.26195, 0.32746, -0.06548)
Wq^-1 * Vqp = 0.866 * (-1.09745, 2.52085, -1.31163)
= (-0.95039, 2.18306, -1.13587)

Vqp x Vq^-1 = (2.52085*-0.07715 - -1.31163*0.3858,
-(-1.09745*-0.07715 - -1.31163*0.30861),
-1.09745 * 0.3858 – 2.52085*0.30861)

= (0.31154, -0.48945, -1.20136)

(qp)q^-1 = 0.73505 – 0.73505,
(0.25981, 0.32746, -0.06548) +
(-0.95039, 2.18306, -1.13587) +
(0.31154, -0.48945, -1.20136)

= (0, -0.37904, 2.02101, -2.40271)

f.

$v\hat{}' = \mathbf{p}\cos\theta + [1-\cos\theta](\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} + [\hat{\mathbf{r}}\times\mathbf{p}]\sin\theta$

p = (0, -1, 3, 0)
$\cos\theta = 0.5$
$\sin\theta = 0.86603$
$1-\cos\theta = 0.5$
$\mathbf{p}\cdot\hat{\mathbf{r}}$ = -1.69734
$\mathbf{r}$^ = (-0.61722, -0.77152, 0.1543)
$\hat{\mathbf{r}}\times\mathbf{p}$ = (-0.77152*0 – 0.1543*3,
-(-0.61722*0 – 0.1543*-1),
-0.61722*3 - -0.77152*-1)

= (-0.4629, -0.1543, -2.62318)

v^ = 0.5 * (-1, 3, 0) +
0.5 * -1.69734 * (-0.61722, -0.77152, 0.1543) +
(-0.4629, -0.1543, -2.62318) * 0.86603)

(-0.5, 1.5, 0) +
(0.52382, 0.65477, -0.13095) +
(-0.40089, -0.13363, -2.32422)
(-0.37707, 2.02114, -2.45517)

3.

a.

$$R = \begin{pmatrix} 0.114409 & -0.72235 & -0.682 \\ -0.07215 & 0.67865 & -0.73091 \\ 0.99081 & 0.13283 & 0.025524 \end{pmatrix}$$

degrees = rad * 180/pi

θy = Sin^-1(-0.682)

Option 1:
θy = -0.75049 radians
-42.99991 degrees
Option 2:
θy = 2.3911 radians
137.00009 degrees

-0.73091 = -sin(θx)*cos(Sin^-1(-0.682)
-0.73091/cos(-0.75049) = -sin(θx)
-sin^-1(-0.73091/cos(-0.75049)) = θx
-sin^-1(-0.73091/0.731355) = θx
-sin^-1(0.9993915404) = θx

Option 1:
θx = 1.536 radians
88.0063 degrees
Option 2:
θx = -1.6056 radians
-91.9937 degrees

Cos(θy)*cos(θz) = 0.114409
Cos(θz) = 0.114409/cos(θy)
θz = acos(0.114409/cos(-0.75049)

θz = acos(-0.15643434)

Option 1:
θz = 1.4137 radians
80.99904 degrees
Option 2:
θz = -1.72789 radians
-99.0096 degrees

b.

$$\mathbf{R} = \mathbf{R}_x\mathbf{R}_y\mathbf{R}_z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{bmatrix} \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{bmatrix} \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We want RyRzRx

$$\begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{bmatrix} \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{bmatrix}$$

B1.

RyRzRx

Highlighting 5 selections

cos(y)*cos(z)	-cos(y)*sin(z) + sin(y)*sin(x)	**sin(y)*cos(x)**
sin(z)	cos(z)+coz(x)	**-sin(x)**
-sin(y)*cos(z)	sin(y)sin(z)+cos(y)*sin(x)	cos(y)+cos(y)*cos(x)

B2.

R =

0.016065	0.4129	0.910633
-0.39073	0.84092	-0.3744
-0.92036	-0.3498	0.174844

Sin(θz) = -0.39073

θz = arcsine(-0.39073)

θz = -0.4014245

First option:

θz = **-0.40142 radians**

-22.99967 degrees

Second Option:

θz = **2.74017 radians**

157.00033 degrees

cos(θy)*cos(θz) = 0.016065

θy = acos(0.016065/cos(-0.4014245))

θy = acos(0.016065/0.92051)

θy = acos(0.0174523)

First Option:
θy = **1.55334 radians**
88.99983 degrees
Second Option:
θy = **-1.58825 radians**
-91.00017 degree

$\text{Sin}(\theta y)^*\cos(\theta x) = 0.910633$
$\theta x = \text{acos}(0.910633/\sin(1.55334))$
$\theta x = \text{acos}(0.910772)$
$\theta x = 0.42565$

First Option:
θx = **0.42565 radians**
24.38794 degrees
Second Option:
θx = **-2.71594 radians**
-155.61206 degrees

4. Since scale is 1, we can ignore that matrix
   a.
      W = TR

      P1 = position (-2, 0, 4)
      F1 = forward(z) (1, 2, 3)
      U1 = up(y) (-2, 10, -6)

      Find left (x)

      Get the length of fwd and up

      LenUp = sqrt((-2,10,-6)^2)
      LenUp = 11.83216

      LenFwd = sqrt((1,2,3)^2)
      LenFwd = 3.74165

      Now lets get the unit vectors

      Uup= (-0.16903, 0.84515, -0.50709)

Ufwd (0.26726, 0.5345, 0.80179)

To get x, we need the cross product of up and forward unit vectors

Uup x Ufwd = (0.84515*0.80179 - -0.50709*0.5345,
-(-0.16903*0.80179 - -0.50707*0.26726),
-0.16903*0.5345 – 0.84515*0.26726)

(0.94867, 0, -0.31622)

$$W = \begin{pmatrix} \mathbf{0.94867} & \mathbf{-0.16903} & \mathbf{0.26726} & \mathbf{-2} \\ \mathbf{0} & \mathbf{0.84515} & \mathbf{0.5345} & \mathbf{0} \\ \mathbf{-0.31622} & \mathbf{-0.50709} & \mathbf{0.80179} & \mathbf{-4} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

b. Convert W into TR

$$R = \begin{pmatrix} \mathbf{0.94867} & \mathbf{-0.16903} & \mathbf{0.26726} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.84515} & \mathbf{0.5345} & \mathbf{0} \\ \mathbf{-0.31622} & \mathbf{-0.50709} & \mathbf{0.80179} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

$$T = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{-2} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{-4} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

c.

W = TR

P1 = position (-5, 1, -1)
F1 = forward(z) (-2, -5, 3)
U1 = up(y) (-10, 13, 15)

Find left (x)

Get the length of fwd and up

LenUp = 22.2261

LenFwd = 6.16441

Now let's get the unit vectors

Uup= (-0.44992, 0.584898, 0.67488)

Ufwd = (-0.32444, -0.811108, 0.486665)

To get x, we need the cross product of up and forward unit vectors

Uup x Ufwd = (0.584898*0.486665 – 0.67488*-0.811108,
-(-0.44992*0.486665 – 0.67488*-0.32444),
-0.44992*-0.811108 – 0.584898*-0.32444)

(0.83205, 0, 0.554698)

$$\mathbf{W} = \begin{pmatrix} \mathbf{0.83205} & \mathbf{-0.44992} & \mathbf{-0.32444} & \mathbf{-5} \\ \mathbf{0} & \mathbf{0.584898} & \mathbf{-0.811108} & \mathbf{1} \\ \mathbf{0.554698} & \mathbf{0.67488} & \mathbf{0.486665} & \mathbf{-1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

d.

Inv rotation

$$R^{-1} = \begin{pmatrix} 0.94867 & 0 & -0.31622 & 0 \\ -0.16903 & 0.84515 & -0.50709 & 0 \\ 0.26726 & 0.5345 & 0.80179 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Inv translation

$$T^{-1} = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{4} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

$$W^{-1} = \begin{pmatrix} 0.94867 & 0 & -0.31622 & 0.63246 \\ -0.16903 & 0.84515 & -0.50709 & -2.36642 \\ 0.26726 & 0.5345 & 0.80179 & 3.74168 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

W1->2 = W2 * W1^-1 <- MAKE SURE SECOND MATRIX IS ON LEFT

$$\begin{pmatrix} 0.83205 & -0.44992 & -0.32444 & -5 \\ 0 & 0.584898 & -0.811108 & 1 \\ 0.554698 & 0.67488 & 0.486665 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{W1\text{->}2} = \begin{pmatrix} \mathbf{0.77868} & \mathbf{-0.55366} & \mathbf{-0.29509} & \mathbf{-4.62301} \\ \mathbf{-0.31564} & \mathbf{0.06079} & \mathbf{-0.94693} & \mathbf{-3.41902} \\ \mathbf{0.54221} & \mathbf{0.8305} & \mathbf{-0.12743} & \mathbf{-0.42528} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

e.

Multiply last answer against original first position.

W1->2 *W1 = W2

$$\begin{pmatrix} 0.77868 & -0.55366 & -0.29509 & -4.62301 \\ -0.31564 & 0.06079 & -0.94693 & -3.41902 \\ 0.54221 & 0.8305 & -0.12743 & -0.42528 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad x$$

$$\begin{pmatrix} 0.94867 & -0.16903 & 0.26726 & -2 \\ 0 & 0.84515 & 0.5345 & 0 \\ -0.31622 & -0.50709 & 0.80179 & -4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

<u>W1->2 *W1</u>

$$\begin{pmatrix} \mathbf{0.83202} & \mathbf{-0.44991} & \mathbf{-0.32442} & \mathbf{-5.00001} \\ \mathbf{0} & \mathbf{0.58491} & \mathbf{-0.811105} & \mathbf{0.99998} \\ \mathbf{0.554674} & \mathbf{0.67487} & \mathbf{0.486641} & \mathbf{-0.99998} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

Copying from answer C

$$W2 = \begin{pmatrix} 0.83205 & -0.44992 & -0.32444 & -5 \\ 0 & 0.584898 & -0.811108 & 1 \\ 0.554698 & 0.67488 & 0.486665 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

I can round up most of the answers to be perfect copies of W2. Otherwise, the answer is off but 100 thousandth of a decimal place. A negligible difference.

f.

$$W1\text{->}2 = \begin{pmatrix} 0.77868 & -0.55366 & -0.29509 & -4.62301 \\ -0.31564 & 0.06079 & -0.94693 & -3.41902 \\ 0.54221 & 0.8305 & -0.12743 & -0.42528 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

F1 length from 1a
LenFwd = 3.74165

Multiply W1->2 * f1 vector to get translation

F1` = (-1.21391, -3.03485, 1.82092)

Length F1`	= sqrt((-1.21391, -3.03485, 1.82092)^2)
	= sqrt(13.999641657)
Length of F1`	**= 3.74161**

U1 Length from 1a
LenUp = 11.83216

Multiply W1->2 * u1
U1` = (-5.32342, 6.92076, 7.98516)

Length of u1` = sqrt((-5.32342, 6.92076, 7.98516)^2)
= sqrt(139.9984996996)
Length of u1` = 11.8321

The lengths are the same as the original vectors.

F2 x F1`

F2 = (-2, -5, 3)
F1` = (-1.21391, -3.03485, 1.82092)

(-5*1.82092 – 3*-3.03485,
-2*1.82092 – 3*-1.21391,
-2*-3.03485 - -5*-1.21391)

F2 x F1` = (0, 0, 0)

Cross product is 0, we are aligned.