425 Applied 3D Algebra

HW 3

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1.

a. We are finding an arbitrary axis rotation. We are rotating left around the up axis by pi/5. This means our rotation, or theta, will be negative.

Find new forward vector r^

R = (-3, 3, 2) <- forward vector

V = (9, 13, -6) <- up vector

θ = -PI/5 <- negative rotation

First, normalize forward vector

|r| = sqrt(9 + 9 + 4)

|r| = 4.6904

r^ = r/|r|

= (-3/4.6904, 3/4.6904, 2/4.6904)

= (-0.6396, 0.6396, 0.4264)

<u>Applying Rodriguez formula</u>

$$R_{\hat{\mathbf{r}},\theta}(\mathbf{v}) = \mathbf{v}\cos\theta + [1-\cos\theta](\mathbf{v}\cdot\hat{\mathbf{r}})\,\hat{\mathbf{r}} + [\hat{\mathbf{r}}\times\mathbf{v}]\sin\theta$$

Don't forget to set your calculator to rad!

cos θ = 0.809

1 - cos θ = 0.191

sin θ = -0.5878

v.dot(r^)= (9* -0.6396) + (13 * 0.6396) + (-6 * 0.4264)

= -5.7564 + 8.3148 – 2.5584

= 0

(1-cos θ)(v.dot(r^) = 0

r^.cross(v) = ((0.6396 * -6 – 0.4264 * 13), (0.4264 * 9 - -0.6396 * -6), (-0.6396 * 13 – 0.6396 * 9))

= (-3.8376 – 5.5432), (3.8376 – 3.8376), (-8.3148 – 5.7564)

= (-9.3808, 0, -14.0712)

Now let's break the equation into its pieces

v(cos θ) = (9*0.809, 13*0.809, -6 * 0.809)

= (7.281, 10.517, -4.854)

This is multiplying by 0, so we get an empty vector
(1- cos θ)(v.dot(r^))r^ = (0, 0, 0)

r^.cross(v)(sinθ) = (-9.3808 * -0.5878 , 0 * -0.5878 , -14.0712 * -0.5878)
= (5.514, 0, 8.2711)

Now lets put it together

Rr^,theta(v) = (7.281, 10.517, -4.854) + (0, 0, 0) + (5.514, 0, 8.2711)
= (1.767, 10.517, 3.4171)

We can check if this is right by getting the inverse rotation. This should be equal to the negative version of our solution.

OR we can set R^ to any of the base axes to find that is equals the previous matrix r^(x,theta), r^(y,theta), r^(z,theta)

I'll do this later if I have time.

b.

To find the left axis, we must find the cross product of v and r

R = (-3, 3, 2) <- forward vector
V = (9, 13, -6) <- up vector

left axis (newR) = v x r
= ((13*2 - -6*3), (-6*-3 – 9*2), (9*3 – 13*-3))
newR = (44, 0, 66) <- forward vector

R = (44, 0, 66) <- forward vector
V = (9, 13, -6) <- up vector
θ = PI/8 <- positive rotation

First, normalize forward vector
|r| = sqrt(1936 + 0 + 4356)
|r| = 79.322
r^ = r/|r|
= (44/79.322, 0, 66/79.322)
= (0.5547, 0, 0.8321)

Applying Rodriguez formula

$$R_{\hat{\mathbf{r}},\theta}(\mathbf{v}) = \mathbf{v}\cos\theta + [1-\cos\theta](\mathbf{v}\cdot\hat{\mathbf{r}})\,\hat{\mathbf{r}} + [\hat{\mathbf{r}}\times\mathbf{v}]\sin\theta$$

Don't forget to set your calculator to rad!

cos θ = 0.9240
1 - cos θ = 0.0760
sin θ = 0.3825
v.dot(r^)= (9*0.5547) + (13 * 0) + (-6 *0.8321)
= 4.9923 + 0 – 4.9926
= -.0003
(1-cos θ)(v.dot(r^) = .0000228 (I stop at 4 decimal places. This is 0)
= 0
r^.cross(v) = ((0 * -6 – 0.8321 * 13), -(0.5547 * -6 - 0.8321 * 9),
(0.5547 * 13 – 0 * 9))

= (-10.8173), -(3.3282 – 7.4889), (7.2111)
= (-10.8173, 10.8171, 7.2111)

Now let's break the equation into its pieces

v(cos θ) = (9*0.9240, 13*0.9240, -6 * 0. 9240)
= (8.316, 12.012, -5.544)

This is multiplying by 0, so we get an empty vector
(1-cos θ)(v.dot(r^))r^ = (0, 0, 0)

r^.cross(v)(sinθ) = (-10.8173 * 0.3825, 10.8171 * 0.3825, 7.2111 * 0.3825)
= (-4.1376, 4.1375, 2.7582)

Now lets put it together

Rr^,theta(v) = (8.316, 12.012, -5.544) + (0, 0, 0) + (-4.1376, 4.1375, 2.7582)
= (4.1784, 16.1495, 2.7858)

We can check if this is right by getting the inverse rotation. This should be equal to the negative version of our solution.

OR we can set R^ to any of the base axes to find that is equals the previous matrix
r^(x,theta), r^(y,theta), r^(z,theta)
I'll do this later if I have time.

2.

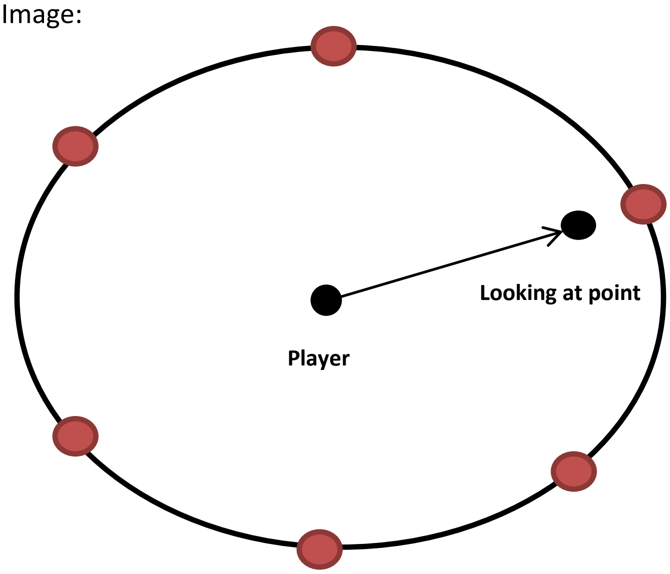
a.

Spawn 6 enemies on the ground (y=0) evenly distributed around a circle of 11 radius centered on the player. First enemy spawns in front of player

Player position = (-4, 0, 3)
Player facing = (4, 0, 5)
We need to do 3 things to fully complete this task

We want 6 enemies spawned evenly around the player, that means we need

Image:



Lets first get the vector from the player to the looking point

AB = (4+4, 0-0, 5-3)
(8,0,2)
With our vector found, lets translate this in a matrix

T= (8, 0, 2)
P= (-4, 0, 3)

$$P`=TtP = \begin{pmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -4 \\ 0 \\ 3 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 0 \\ 5 \\ 1 \end{pmatrix}$$

Now lets normalize this translated vector

Sqrt(16 + 25)
6.4031

V^ = 4/6.4031, 0, 5/6.4031
V^ = .6247, 0, .7809

Now let's find the first enemy spawn by multiplying the normalized vector by the radius

Enemy1 = (11*.6247, 0, 11*.7809)
= (6.8717, 0, 8.5899)

Now let's find the vector from the player to the new enemy

PE = (6.8717+4, 0, 8.5899-3)
PE = (10.8717, 0, 5.5899)

This will act as our baseline for all other enemy spawns

From here we can divide up the circle around the player and move to each spot to place a new enemy by rotating about the y axis. This will evenly distribute the remaining 5 enemies around the player

We will use the same θ and the newest created enemy to process the next spawn to save on processing time

<u>Enemy2</u>
θ = 1pi/3 = 1.0472
cos(θ) = 0.5
sin(θ) = 0.866

$$Ry\ \theta = \begin{pmatrix} 0.5 & 0 & 0.866 & 0 \\ 0 & 1 & 0 & 0 \\ -0.866 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 10.8717 \\ 0 \\ 5.5899 \\ 0 \end{pmatrix}$$

Enemy2 = (10.2767, 0, -6.6202, 0)

<u>Enemy3</u>

θ = 1pi/3 = 1.0472

cos(θ) = 0.5

sin(θ) = 0.866

$$Ry\ \theta = \begin{pmatrix} 0.5 & 0 & 0.866 & 0 \\ 0 & 1 & 0 & 0 \\ -0.866 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 10.2767 \\ 0 \\ -6.6202 \\ 0 \end{pmatrix}$$

Enemy3 = (-0.5949, 0, -12.21, 0)

<u>Enemy4</u>

θ = 1pi/3 = 1.0472

cos(θ) = 0.5

sin(θ) = 0.866

$$Ry\ \theta = \begin{pmatrix} 0.5 & 0 & 0.866 & 0 \\ 0 & 1 & 0 & 0 \\ -0.866 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -0.5949 \\ 0 \\ -12.21 \\ 0 \end{pmatrix}$$

Enemy4 = (-10.872, 0, -5.5898, 0)

Enemy5

θ = 1pi/3 = 1.0472

cos(θ) = 0.5

sin(θ) = 0.866

$$Ry\ \theta = \begin{pmatrix} 0.5 & 0 & 0.866 & 0 \\ 0 & 1 & 0 & 0 \\ -0.866 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -10.872 \\ 0 \\ -5.5898 \\ 0 \end{pmatrix}$$

Enemy5= (-10.277, 0, 6.62053, 0)

Enemy6

θ = 1pi/3 = 1.0472

cos(θ) = 0.5

sin(θ) = 0.866

$$Ry\ \theta = \begin{pmatrix} 0.5 & 0 & 0.866 & 0 \\ 0 & 1 & 0 & 0 \\ -0.866 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -10.277 \\ 0 \\ 6.62053 \\ 0 \end{pmatrix}$$

Enemy6= (0.59505, 0, 12.2104, 0)

3.

Given a triangle game object (TGO)

P1 = (-1, 0, -1)

P2 = (0, 0, 2)

P3 = (1, 0, -1)

Starting at world space, compute new values for each point of the TGO after each movement

Remember W = TRS

World = Translation * rotation * scale

GO RIGHT TO LEFT

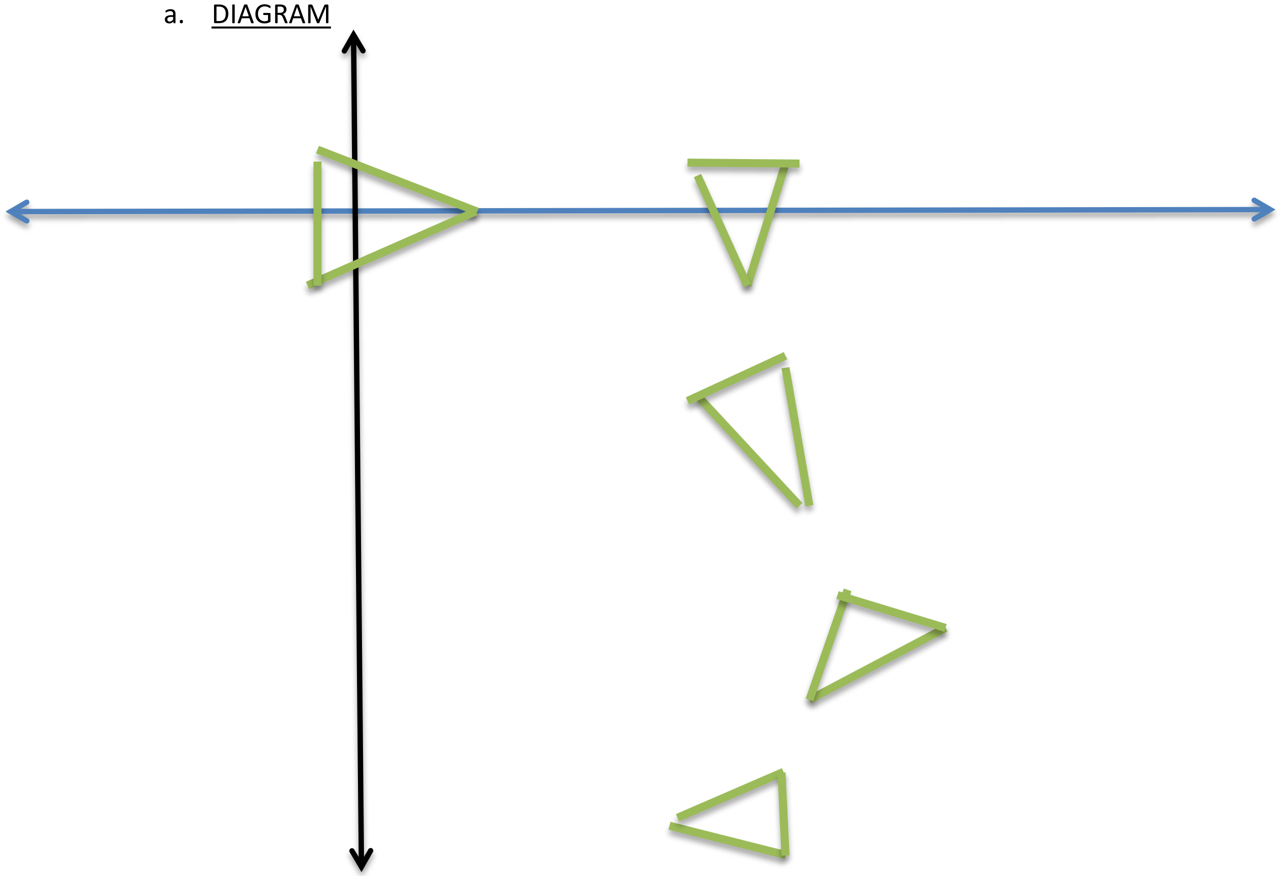
SCALE

ROTATE

TRANSLATE

1. Move forward 8, rotate along y by -2pi/4
2. Move forward 8, rotate along y by 1pi/4
3. Move forward 5, rotate y -3pi/4
4. Move forward 7, rotate y -2pi/4

a. DIAGRAM



b. Matrix calculations

Get our starting position in the world.

Starting World

$$\begin{pmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \end{pmatrix}$$

Initial Position

$$\begin{pmatrix} -1\ 0\ 1 \\ 0\ 0\ 0 \\ -1\ 2\ -1 \\ 1\ 1\ 1 \end{pmatrix}$$

Multiply them together.

We're multiplying by an identity, so initial position does not change

World Starting position

$$\begin{pmatrix} -1\ 0\ 1 \\ 0\ 0\ 0 \\ -1\ 2\ -1 \\ 1\ 1\ 1 \end{pmatrix}$$

Step 1: Move forward 8 and rotate by -2pi/4 radian

θ = -2pi/4 = -1.57

cos(θ) = 0.001

sin(θ) = -1

Translation T = $\begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix}$

We are rotating on the Y axis. We need to negate X's sin call

Rotation = $\begin{pmatrix} \cos(1.57) & 0 & \sin(1.57) \\ 0 & 1 & 0 \\ -\text{Sin}(1.57) & 0 & \cos(1.57 \end{pmatrix}$

Rotation = $\begin{pmatrix} 0.001 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0.001 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Scale = no change

Put them together

$$Wstep = \begin{pmatrix} 0.001 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0.001 & 8 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Reconstruct the world matrix. Multiply them together

$$WiWstep \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.001 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0.001 & 8 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$WiWstep \begin{pmatrix} 0.001 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0.001 & 8 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Apply to all points to get proper transform and rotation

$$\text{New Position} \quad \begin{pmatrix} 0.001 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0.001 & 8 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 2 & -1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1.001 & -2 & 1 \\ 0 & 0 & 0 \\ 7.001 & 8.002 & 8.999 \\ 1 & 1 & 1 \end{pmatrix}$$

I’m going to round these up/down as they’re basically the int value.

$$\begin{pmatrix} \mathbf{1} & \mathbf{-2} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{7} & \mathbf{8} & \mathbf{9} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$

Step 2: Move forward 8, rotate on y by 1pi/4

θ = 1pi/4 = 0.785
cos(θ) = 0.707
sin(θ) = 0.707

Translation T = $\begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix}$

We are rotating on the Y axis. We need to negate X's sin call

Rotation = $\begin{pmatrix} 0.7071 & 0 & 0.7071 \\ 0 & 1 & 0 \\ -0.7071 & 0 & 0.7071 \end{pmatrix}$

Scale = no change
Put them together

Wstep = $\begin{pmatrix} 0.7071 & 0 & 0.7071 & 0 \\ 0 & 1 & 0 & 0 \\ -0.7071 & 0 & 0.7071 & 8 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Reconstruct the world matrix. Multiply them together

WiWstep $\begin{pmatrix} 0.001 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0.001 & 8 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.7071 & 0 & 0.7071 & 0 \\ 0 & 1 & 0 & 0 \\ -0.7071 & 0 & 0.7071 & 8 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

WiWstep $\begin{pmatrix} -.7071 & 0 & -0.7071 & -8 \\ 0 & 1 & 0 & 0 \\ 0.7071 & 0 & 0.7071 & 8 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Multiply against current position to get new position

NewPos $\begin{pmatrix} .7071 & 0 & -0.7071 & -8 \\ 0 & 1 & 0 & 0 \\ 0.7071 & 0 & 0.7071 & 8 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 2 & -1 \\ 1 & 1 & 1 \end{pmatrix}$

$$\begin{pmatrix} -0.7071+0.7071-8 & -1.4142-8 & 0.7071+0.7063-8 \\ 0 & 0 & 0 \\ -0.7071-0.7071+8 & -1.4142+8 & 0.7071-0.7071+8 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{-8} & \mathbf{-9.4142} & \mathbf{-6.5858} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{6.5858} & \mathbf{9.4142} & \mathbf{8} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$

Step 3: Move forward 5, rotate y axis -3pi/4

θ = -3pi/4 = -2.355
cos(θ) = -0.706
sin(θ) = -0.708

Translation T = $\begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$

We are rotating on the Y axis. We need to negate X's sin call

Rotation = $\begin{pmatrix} 0.706 & 0 & -0.708 \\ 0 & 1 & 0 \\ 0.708 & 0 & 0.706 \end{pmatrix}$

Scale = no change

$$\begin{pmatrix} .7071 & 0 & -0.7071 & -8 \\ 0 & 1 & 0 & 0 \\ 0.7071 & 0 & 0.7071 & 8 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.706 & 0 & -0.708 & 0 \\ 0 & 1 & 0 & 0 \\ 0.708 & 0 & 0.706 & 5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0.4992-.5006 & 0 & -.5006-0.4992 & -3.5355-8 \\ 0 & 1 & 0 & 0 \\ .4992+.5006 & 0 & -.5006+.4992 & 3.5355+8 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

New WiWstep. Multiply by original placement

$$\begin{pmatrix} -0.0014 & 0 & -1 & -11.5355 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -.0014 & 11.5355 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 2 & -1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0.0014+1-11.5355 & -2-11.5355 & -0.0014+1-11.5355 \\ 0 & 0 & 0 \\ -1+.0014+11.5355 & -0.0028+11.5355 & 1+.0014+11.5355 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{-10.5341} & \mathbf{-13.5355} & \mathbf{-10.5369} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{10.5369} & \mathbf{11.5327} & \mathbf{12.5369} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$

Step 4: Move forward 7, rotate on y by -2pi/4

θ = -2pi/4 = -1.57

cos(θ) = 0.001
sin(θ) = -1

Translation T = $\begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix}$

We are rotating on the Y axis. We need to negate X's sin call
Rotation = $\begin{pmatrix} 0.001 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0.001 \end{pmatrix}$

Scale = no change

We know the flow by now. Let's compact things.

Get WiWstep

$$\begin{pmatrix} -0.0014 & 0 & -1 & -11.5355 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -0.0014 & 11.5355 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.001 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0.001 & 7 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Get the new position.

$$\begin{pmatrix} -1 & 0 & 0.0004 & -18.5355 \\ 0 & 1 & 0 & 0 \\ 0.0004 & 0 & -1 & 11.5257 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 2 & -1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1-.0004-18.5355 & .0008-18.5355 & -1+.0004-18.5355 \\ 0 & 0 & 0 \\ -.0004+1+11.5257 & -2+11.5257 & .0004+1+11.5257 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{-17.5359} & \mathbf{-18.5347} & \mathbf{-19.5359} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{12.5253} & \mathbf{9.5257} & \mathbf{12.5261} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{pmatrix}$$