Environments, Binding, and Scope

CS 350

Dr. Joseph Eremondi

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Overview

The Road to Midterm

- Today: Environments in Curly

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- Tues: Lambda and First-class Functions in Plait

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Everything up to and including Closures may appear on the midterm

Environments

Functions Review

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- Substitution is forgetful
  - Just replaces function variable with expression
  - Not very useful for debugging

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    - Means reference to undefined variable

The Environment Data Structure: Bindings

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;; Lets us write Env instead of (Listof Binding)
;; But it's not defining a new type,
;; just a new name for the same type.
(define-type-alias Env (Listof Binding))
;; Environment is either empty or extended env
(define emptyEnv : Env
  empty)
(define (extendEnv [bnd : Binding]
                   [env : Env])
        : Env
  (cons bnd env))

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(extendEnv (bind 'x 3) (extendEnv (bind 'y 4) empty))
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(list (bind 'x 3) (bind 'y 4))
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```
(define (lookup [n : Symbol] [env : Env]) : Number
  (type-case (Listof Binding) env
    ;; Can't find a variable in an empty env
    [empty (error 'lookup "undefined variable")]
    ;; Cons: check if the first binding is the var
    ;; we're looking for.
    ;; Return its value  if it is, otherwise
    ;; keep looking in the rest of the list
    [(cons b rst-env) (cond
                        [(symbol=? n (bind-name b))
                         (bind-val b)]
                        [else (lookup n rst-env)])]))
```

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(define (interp [env : Env]
                [defs : (Listof FunDef)]
                [e : Expr] ) : Number
  (type-case Expr e
             ....))
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```
;; {+ e1 e2} evaluates e1 and e2, then adds the results together
    [(Plus l r)
     (+ (interp env defs l) (interp env defs r))]
```

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[(Var x)
      (lookup x env)]
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```
[(Call funName argExpr)
     (let* ([argVal (interp env defs argExpr)]
            [def (get-fundef funName defs)]
            [argVar (mkFunDef-arg def)]
            [funBody (mkFunDef-body def)])
       (interp (extendEnv (bind argVar argVal) emptyEnv) ;;<------
               defs
               funBody))]
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- We call this *static scoping*
- If we extend the environment from the call site, we get *dynamic scoping*

Static Scoping Exapmple

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{define {f x} {+ x y}}
{define {g y} {f y}}
{g 3}
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- Dynamic scoping is WRONG
  - You should understand it, but know that static scoping is what we want

Implementing Let

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- We'll implement with both substitution and environments

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(type-def Expr
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    - See `Curly-Let.rkt`

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(define (interp [defs : (Listof FunDef)] ;;NEW
                [e : Expr] ) : Number
  (type-case Expr e
        ;; ....
     [(Letvar x xexp body)
        (interp defs (subst x (interp defs xexp) body))])
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      (let ([xval (interp env defs xexp)])
        (interp (extendEnv (bind x xval) env)
                defs body))]))
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  - e.g. in Plait, `let` and `let*` have different rules for what's in scope

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  - Instead, call `interp` recursively to add to the *Plait* call stack

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  - Actually implemented for most languages
  - Every time we call a function or define a variable we push onto the call stack
- Curly-Let and others have an *implicit* call stack
  - We don't keep the data structure ourselves
  - Instead, call `interp` recursively to add to the *Plait* call stack
  - When finished eval, plait returns is to part waiting for the result