Functional Programming 1: Recursion and Immutable Data

CS 350

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Overview

- Topic: Functional Programming in Racket and plait

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- Required Reading:

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  - Plait videos, HtDP videos

Programming in CS 350

All coding for this class uses:

- The Racket Programming Language

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- The `plait` library for Racket

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- The Dr. Racket editor

Racket

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  - `((((((((Parentheses))))))))`

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  - `((((((((Parentheses))))))))`
- Language for making languages

What is Dr. Racket?

- IDE for Racket

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  - see `https://docs.racket-lang.org/guide/other-editors.html`

Plait

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  - Has what you need to write programming languages
  - Not much else
  - You can do a lot with very little

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- Algebraic Data Types

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  - Racket writes `(f x)`, not `f(x)`
- `x` is not the same as `(x)`
  - `x` gets the value of the variable `x`
  - `(x)` is calling a function named `x` with zero arguments

Numbers

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```
(+ 2 7)
```

Numbers

```
(+ 2 7)
```

Numbers

```
(+ 2 7)
(- 10 0.5)
```

9

Numbers

```
(+ 2 7)
(- 10 0.5)
```

```
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9.5
```

Numbers

```
(+ 2 7)
(- 10 0.5)
(* 1/3 2/3)
```

```
9
9.5
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```
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9.5
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Numbers

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(+ 2 7)
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(* 1/3 2/3)
(/ 1 1000000000000.0)
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9.5
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(+ 2 7)
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(max 10 20)
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```

Booleans

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```
(= (+ 2 3) 5)
```

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```
(= (+ 2 3) 5)
(> (/ 0 1) 1)
```

```
#t
```

Booleans

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(= (+ 2 3) 5)
(> (/ 0 1) 1)
```

```
#t
#f
```

Booleans

```scheme
(= (+ 2 3) 5)
(> (/ 0 1) 1)
(zero? (- (+ 1 2) (+ 3 0)))
```

```scheme
#t
#f
```

Booleans

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```
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Booleans

```
(= (+ 2 3) 5)
(> (/ 0 1) 1)
(zero? (- (+ 1 2) (+ 3 0)))
(and (< 1 2) (> 1 0))
```

```
#t
#f
#t
```

Booleans

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(> (/ 0 1) 1)
(zero? (- (+ 1 2) (+ 3 0)))
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```

```
#t
#f
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```

Booleans

```
(= (+ 2 3) 5)
(> (/ 0 1) 1)
(zero? (- (+ 1 2) (+ 3 0)))
(and (< 1 2) (> 1 0))
(or (zero? 1) (even? 3))
```

```
#t
#f
#t
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```
#t
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(+ 3
  (if (= 2 (+ 1 1))
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```
"hello"
6
```

Functions

- Calling a function replaces variable with concrete argument

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```
(define (addOne [x : Number]) : Number
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(addOne 10)
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11
```

Functions

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```
(define (isRemainder [x : Number]
                     [y : Number]
                     [remainder : Number])
        : Boolean
  (= remainder (modulo x y)))
(isRemainder 10 3 1)
(isRemainder 10 4 1)
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```
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```

Functions (ctd.)

- General form:

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```
(define (functionName
         [argName : argType]
         ...
         [argNameN : argTypeN]) : returnType
  functionBody)
```

- Later in the course we'll see another way of defining functions

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- Only relevant operation is comparison
  - `(symbol=?  'a 'b)`
  - Compares pointers, so very fast

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  (let ([xy (+ x y)])
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    - We'll see this later when we learn about lambdas

Functional Thinking and Recursion

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    - Fast, memory efficient

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- Farther from what the CPU is actually doing
- Some algorithms are more concise with mutation
    - But lots aren't

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Further reference:
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- Takes in a (natural) number, outputs a number

Factorial - Representation

- $n! = 1 \times 2 \times \ldots n$
- Takes in a (natural) number, outputs a number

```
(define (factorial [n : Number]) : Number
  (error 'factorial "TODO"))
```

Factorial - Examples

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```
(test (factorial 0) 1 )
```

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(test (factorial 0) 1 )
(test (factorial 1) 1 )
(test (factorial 2) 2 )
(test (factorial 3) 6 )
```

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Factorial - Examples

```
(test (factorial 0) 1 )
(test (factorial 1) 1 )
(test (factorial 2) 2 )
(test (factorial 3) 6 )
(test (factorial 4) 24 )
```

Factorial - Examples

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(test (factorial 1) 1 )
(test (factorial 2) 2 )
(test (factorial 3) 6 )
(test (factorial 4) 24 )
```

Factorial - Examples

```
(test (factorial 0) 1 )
(test (factorial 1) 1 )
(test (factorial 2) 2 )
(test (factorial 3) 6 )
(test (factorial 4) 24 )
(test (factorial 5) 120 )
```

- Notice the pattern?

Factorial - Template

- A natural number is either

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  - Zero

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```
(define (factorial [n : Number]) : Number
  (if (zero? n)
      (error 'zero "TODO")
      (let ([n-1 (- n 1)])
        (error 'suc "TODO"))
      ))
```

Factorial - Recursion

- Divide problem into base case and recursive cases

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```
(define (factorial [n : Number]) : Number
  (if (zero? n)
      (error 'zero "TODO")
      (let*
          ([n-1 (- n 1)]
           [fn-1 (factorial n-1)])
          (error 'suc "TODO"))
      ))
```

Factorial - Filling holes

- Example gives the base case for `0`

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```
(define (factorial [n : Number]) : Number
  (if (zero? n)
      1
      (let*
          ([n-1 (- n 1)]
           [fn-1 (factorial n-1)])
          (* n fn-1))
      ))
```

Run Tests

Run Tests

```
(test (factorial 0) 1 )
(test (factorial 5) 120 )
```

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```

```
good (factorial 0) at line 11
  expected: 1
  given: 1

good (factorial 5) at line 12
  expected: 120
  given: 120
```

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  - Aside: I research languages where you *can* express this with types

Another Example: Exponentiation

- Live coding in Dr. Racket

Unbounded Data: Lists

Functional Linked Lists

- Every linked list is one of:

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- We call the operation of appending an element to a list `cons`
  - Historical name, goes back to LISP days
- Cons does **not** change its input
  - Creates a new list whose tail is the old list

Lists in Racket

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- Lots more helper functions, see the documentation

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```
(define (list-template
         [xs : (Listof Number)])
  (if (empty? xs)
      (error 'nil "TODO")
      (let ([h (first xs)]
            [t (rest xs)]
            [tRet (list-template t)])
        (error 'cons "TODO"))
      ))
```

Example: Sum

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```
(define (sum [xs : (Listof Number)])
  : Number
  (if (empty? xs)
      0
      (let* ([h (first xs)]
             [t (rest xs)]
             [tRet (sum t)])
        (+ h tRet))))
 (sum '())
 (sum '(1 2 3))
 (sum '(100 20 3))
```

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(sum '())
(sum '(1 2 3))
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```

```
0
6
123
```

Pattern Matching: Motivation

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- Don't want to apply getters on the wrong data
  - e.g. `first '()` will raise an error

Pattern Matching:

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- e.g. Increment each number in a list

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```
(define (increment [xs : (Listof Number)])
        : (Listof Number)
  (type-case (Listof Number) xs
    [empty
       empty]
    [(cons h t)
       (cons (+ h 1) (increment t))]))
(increment '(2 3 4))
```

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```

```
  '(3 4 5)
```

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    - e.g. `first '(1 2 3)` is a `Number`, but `first '(#t #f #t)` is a Boolean
- Later, this will be very useful for writing generic list operations

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- We can combine two lists into a single list

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  - Works for list with any contents
  - We never do anything with the contents other than copy
  - This function is built into Plait as `append`

```
(define (concat [xs : (Listof 'elem)]
                [ys : (Listof 'elem)])
        : (Listof 'elem)
  (type-case (Listof 'elem) xs
    [empty
       ys]
    [(cons h t)
       (cons h (concat t ys))]))
```

Example: List Concatenation (ctd.)

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Example

```
(concat '(1 2 3) '(4 5 6))
```

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```

Example: List Concatenation (ctd.)

Example

```
(concat '(1 2 3) '(4 5 6))
(concat '("3" "5") '("0"))
```

```
'(1 2 3 4 5 6)
```

Example: List Concatenation (ctd.)

Example

```
(concat '(1 2 3) '(4 5 6))
(concat '("3" "5") '("0"))
```

```
'(1 2 3 4 5 6)
'("3" "5" "0")
```

Example: List Concatenation (ctd.)

Example

```
(concat '(1 2 3) '(4 5 6))
(concat '("3" "5") '("0"))
(concat '() '(#t))
```

```
'(1 2 3 4 5 6)
'("3" "5" "0")
```

Example: List Concatenation (ctd.)

Example

```
(concat '(1 2 3) '(4 5 6))
(concat '("3" "5") '("0"))
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```

```
'(1 2 3 4 5 6)
'("3" "5" "0")
'(#t)
```

Example: List Concatenation (ctd.)

Example

```
(concat '(1 2 3) '(4 5 6))
(concat '("3" "5") '("0"))
(concat '() '(#t))
(concat '(#f) '())
```

Results

```
'(1 2 3 4 5 6)
'("3" "5" "0")
'(#t)
```

Example: List Concatenation (ctd.)

Example

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(concat '(1 2 3) '(4 5 6))
(concat '("3" "5") '("o"))
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```

Results

```
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```

More Examples

- Demo: Dr. Racket (as time permits)

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    - Duplicating each element of a list

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    - Filtering out odd elements of a list