Arithmetic: Our First Interpreter

CS 350

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Interpreters: Overview

The Church Turing Thesis

Alonzo Church

(lambda calculus)

Kurt Gödel

(general recursive functions)

Alan Turing

(Turing machines)

Turing Completeness

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The Church Turing Thesis

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All Turing Complete Languages can simulate each other

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- The features of a language you're interpreting are *completely unrelated* to the features of the language the interpreter is written in
  - Sometimes you can piggyback on the implementation language features, but that's a matter what's *convenient*, not what's *possible*

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- The implementation language is NOT the language you're interpreting
- In this class, the implementation language is Racket/plait
- We'll write interpreters for a bunch of small languages
  - We'll call them "Curly" because we write them with curly brackets
  - Write Curly programs in Racket files using quotation

General Form of an Interpreter

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(define (interp [e : Expr]
                [x : SomeContext]
                ...
                [y : OtherContext])
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  ....)
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- What the context arguments and `Value` datatype are depend on the language
    - Initially we have no context arguments, and `Value` is very simple
    - Will get more complicated as we go through the course

Our First Interpreter

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```
(define-type Expr
  (NumLit [n : Number])
  (Plus [left : Expr]
        [right : Expr])
  (Times [left : Expr]
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    - Base cases are literals, translate directly into values
    - Recursive cases are operations
        - Interpret sub-expressions recursively
        - Combine according to value version of the operation

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(define (interp [e : Expr] ) : Number
  (type-case Expr e
             [(NumLit n) n]
             [(Plus l r)
                (+ (interp l) (interp r))]
             [(Times l r)
                (* (interp l) (interp r))]))
(define (eval s-exp) (interp (parse s-exp)))
(eval `3)
(eval `{+ 2 5})
(eval `{+ {* 1 {+ 2 1}} {+ {* 3 4} {* 0 1000000}}})
```

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```
3
7
15
```

Adding features

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  - Evaluates to `x` if `cond` evaluates to `0`

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  - AST definition
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- Example: `{if0 cond x y}`
  - Evaluates to `x` if `cond` evaluates to `0`
  - Evaluates to `y` if `cond` evaluates to anything else

Updating the dataype

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```
(define-type Expr
  (NumLit [n : Number])
  (Plus [left : Expr]
        [right : Expr])
  (Times [left : Expr]
         [right : Expr]))
  ;;NEW
  (Ifo [test : Expr]
       [thenBranch : Expr]
       [elseBranch : Expr])
```

Updating the parser

Updating the parser

```
    [(s-exp-match? `{if0 ANY ANY ANY} s)
     (If0 (parse (second (s-exp->list s)))
            (parse (third (s-exp->list s))
            (parse (fourth (s-exp->list s))]
```

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             [(Plus l r)
                (+ (interp l) (interp r))]
             [(Times l r)
                (* (interp l) (interp r))])
             ;; NEW
             [(If0 test thenBranch elseBranch)
               (if (= 0 (interp test))
                 (interp thenBranch)
                 (interp elseBranch)
               )])
```