Implementing Lambdas with Substitution and Dynamic Typing

CS 350

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Broad Strategy

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  - Functions are not numbers
  - Need to implement **dynamic typing**

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```
(define (parse s-expr)
  (cond
    ....
    [(s-exp-match? `{fun {SYMBOL} ANY} s)
     (SurfFun (s-exp->symbol
               (first (s-exp->list
                       (second (s-exp->list s)))))
              (parse (third (s-exp->list s))))]))
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  - Function stores the info we need to call it

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  - We'll see more of this for `interp`

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(define (value->expr [v : Value]) : Expr
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  - Want `(interp (value->expr v))` to produce `v`
- Not actually doing any computation, just changing the constructors so the type checker is happy
- In a more sophisticated implementation language, we could make values a *subtype* of expressions, but that's beyond this course

Substitution for Calls

- Needs to substitute in function and body

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```
(define (subst [toReplace : Symbol]
               [replacedBy : Expr]
               [replaceIn : Expr]) : Expr
  (type-case Expr replaceIn
  ....
  [(Fun x body)
     ;; Don't substitute if variable is shadowed
     (if (symbol=? x toReplace)
         (Fun x body)
         (Fun x (subst toReplace replacedBy body)))]
  ))
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  - So that we can produce functions as the result of expressions
- Our previous interpreter assumed that `interp` always returned a number
- We need to introduce **dynamic type checking** in Curly-Fun

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    - Wrap numeric results in `NumV`
    - Perform dynamic type checks to extract fields

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- *Dynamic* because we check *while the program is running*
  - If we checked before it ran, it would be *static type checking*

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  - Good practice for programming in less safe languages

Defining some helper functions

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```
(define (checkAndGetNum [v : Value]) : Number
  (type-case Value v
     [(NumV n) n]
     [else
     (error 'curlyTypeError
            (string-append "Expected Number, got function:"
                           (to-string v)))]))
(define (checkAndGetFun [v : Value]) : (Symbol * Expr)
  (type-case Value v
     [(FunV x body)
      (pair x body)]
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  - Better error-message than e.g. `NumV-num` gives

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  - Just package up the data in the Value type

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(define (interp expr)
  (type-case Expr interp
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    [(Call funExpr argExpr)
      (let* ([argVal (interp argExpr)]
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             [funParam (fst funVal)]
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  - Do a dynamic type check to make sure the result is a function, not a number

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  - Substitution lets us build new functions at run time based on values to other functions

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(define (interp expr)
  (type-case Expr expr
        [(If0 test thn els)
           (let ([testNum (checkAndGetNum (interp test))])
             (if (= 0 testNum)
                 (interp thn)
                 (interp els)))]))
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                  [y : Value]) : Value
  (let ([nx (checkAndGetNum x)]
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(define (interp expr)
  (type-case Expr expr
     [(Plus l r)
       (liftVal2 + (interp l) (interp r))]
     [(Times l r)
       (liftVal2 * (interp l) (interp r))]))
```

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- `interp` runs forever

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```
;; BAD! Don't do this
(define (interp expr
  (type-case Expr expr
      [(Ifo test thn els)
      (let ([thenVal (interp thn)]
            [elseVal (interl els)])
        (if (= 0 (checkAndGetNum (interp test)))
            thenVal
            elseVal))])))
(run `{ifo 0
           {+ 1 1}
           {letvar f {fun x {x x}} {f f}}})
```

- Loops because it evaluates the untaken branch