

Functional Programming 2: Defining Data Types

CS 350

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Warmup: Pairs

Pairs: “AND” for types

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(fst myPair) ;; Number
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2
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Algebraic Data Types

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- Racket lets us define our own types mixing AND and OR

First example

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(define-type Shape
  (Rectangle [length : Number]
             [width : Number])
  (Circle [radius : Number]))

(define tv (Rectangle 16 9))
(define loonie (Circle 1))
```

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- It has two *constructors*, Rectangle and Circle
 - i.e. a Shape is a circle or a rectangle
- Rectangle has two fields with type Number, length and width
- Circle has one field with type Number

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 - Doesn't **do** anything except package the data together
- A Shape either has two numbers OR one number
 - Depending on the tag

Auto-generated Functions

```
(Rectangle? tv)
(Circle? tv)
(Rectangle-length tv)
;; (test/exn (Circle-radius tv) "")
   ;;raises an error, no such field present
```

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```
(define (area [shp : Shape]) : Number
  (type-case Shape shp
    [(Rectangle l w)
     (* l w)]
    [(Circle r)
     (* 3.14 (* r r))]
  )
(area tv)
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144

3.14

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 - See Racket window

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(define-type (Optionof 'a)  
  (none)  
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- Pattern matching guarantees no null pointer errors
 - We'll see a more detailed example

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- You can define all (non-recursive) datatypes with this and pairs
- e.g. Shape is (Either (Number * Number) Number)

Recursive Data

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- This allows us to define **trees**
 - of arbitrary depth
- Data can be traversed using recursion

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(define-type NumList  
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```
(define (sum [xs : NumList])
  (type-case NumList xs
    [(Nil)
     0]
    [(Cons h t)
     (let ([sumRest (sum t)])
       (+ h (sumRest))
      )])
  )
(sum (Cons 100 (Cons 20 (Cons 3 (Nil)))))
```

Example: Filesystem

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```
(define-type Filesystem
  (File [name : String]
        [data : Number])
  (Folder [name : String]
           [contents : (Listof Filesystem)]))
```

Linear Search using Recursion

- Find the first matching file

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```
(define (search [target : String]
               [fs : Filesystem]) : (Optionof Number)
  (type-case Filesystem fs
    [(File name data)
     (if (string=? name target)
          (some data)
          (none))])
    [(Folder _ contents)
     (searchList target contents)]))
```

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```
(define (searchList [target : String]
                   [files : (Listof Filesystem)])
  : (Optionof Number)
  (type-case (Listof Filesystem) files
    [empty (none)]
    [(cons h t)
     (let ([result (search target h)])
       (if (none? result)
           (searchList target t)
           result))
     ]))
```

Testing the search

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```
(define InnerSolarSystem
  (Folder "Sun"
    (list (File "Mercury" 1)
          (File "Venus" 2)
          (Folder "EarthSystem"
            (list (File "Earth" 3)
                  (File "Moon" 3.5)))
          (Folder "MarsSystem"
            (list (File "Mars" 4)
                  (File "Phobos" 4.3)
                  (File "Demos" 4.6))))))
(search "Moon" InnerSolarSystem)
(search "Jupiter" InnerSolarSystem)
```