

Environments, Binding, and Scope

CS 350

Dr. Joseph Eremondi

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Overview

- Functional Programming (Plait Tutorial)

The Road So Far

- Functional Programming (Plait Tutorial)
- Parsing (PLAI ch2)

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- Interpreters (PLAI ch3)

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- Desugaring (PLAI ch4)

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- Interpreters (PLAI ch3)
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- Functions (PLAI ch5)

The Road to Midterm

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Everything up to and including Closures may appear on the midterm

Environments

- Evaluate their argument

Functions Review

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- Evaluate the result
- If we ever interpret a variable, raise an error

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- Substitution is forgetful
 - Just replaces function variable with expression
 - Not very useful for debugging

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 - If there's an entry for x in the environment, return that
 - Error otherwise
 - Means reference to undefined variable

The Environment Data Structure: Bindings

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```
'x  
3
```

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;; Lets us write Env instead of (Listof Binding)  
;; But it's not defining a new type,  
;; just a new name for the same type.  
(define-type-alias Env (Listof Binding))  
;; Environment is either empty or extended env  
(define emptyEnv : Env  
  empty)  
(define (extendEnv [bnd : Binding]  
               [env : Env])  
  : Env  
  (cons bnd env))  
  
emptyEnv  
(extendEnv (bind 'x 3) (extendEnv (bind 'y 4) empty))
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```
'()  
(list (bind 'x 3) (bind 'y 4))
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Looking up variables

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```
(define (lookup [n : Symbol] [env : Env]) : Number
  (type-case (Listof Binding) env
    ;; Can't find a variable in an empty env
    [empty (error 'lookup "undefined variable")]
    ;; Cons: check if the first binding is the var
    ;; we're looking for.
    ;; Return its value if it is, otherwise
    ;; keep looking in the rest of the list
    [(cons b rst-env) (cond
                        [(symbol=? n (bind-name b))
                         (bind-val b)]
                        [else (lookup n rst-env)]))])
```

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(define (interp [env : Env]
               [defs : (Listof FunDef)]
               [e : Expr] ) : Number
  (type-case Expr e
    ....))
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```
;; {+ e1 e2} evaluates e1 and e2, then adds the results together  
[(Plus l r)  
 (+ (interp env defs l) (interp env defs r))]
```

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[(Var x)  
 (lookup x env)]
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```
[(Call funName argExpr)
  (let* ([argVal (interp env defs argExpr)]
        [def (get-fundef funName defs)]
        [argVar (mkFunDef-arg def)]
        [funBody (mkFunDef-body def)])
    (interp (extendEnv (bind argVar argVal) emptyEnv) ;;<-----
             defs
             funBody)))]
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- If we extend the environment from the call site, we get *dynamic scoping*

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- A language has *dynamic scoping* if undefined variables get their values from the point where the function is *called*

Static Scoping Exapmple

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{define {f x} {+ x y}}  
{define {g y} {f y}}  
{g 3}
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- Dynamic scoping is **WRONG**
 - You should understand it, but know that static scoping is what we want

Implementing Let

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- We'll implement with both substitution and environments


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(type-def Expr
  ....
  [(Letvar [x : Symbol]
           [xval : Expr]
           [body : Expr]])
)
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```
(define (interp [defs : (Listof FunDef)] ;;NEW
          [e : Expr] ) : Number
  (type-case Expr e
    ;; ....
    [(LetVar x xexp body)
     (interp defs (subst x (NumLit (interp defs xexp)) body))])])
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(define (subst [toReplace : Symbol]
               [replacedBy : Expr]
               [replaceIn : Expr]) : Expr
  (type-case Expr replaceIn
    [(LetVar x xexp body)
     (LetVar x
              (subst toReplace replacedBy xexp)
              (if (symbol=? x toReplace)
                  body
                  (subst toReplace replacedBy body))))])
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    [(Letvar x xexp body)
     (let ([xval (interp env defs xexp)])
       (interp (extendEnv (bind x xval) env)
                defs body))])])
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 - e.g. in Plait, `let` and `let*` have different rules for what's in scope

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 - Actually implemented for most languages

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