A CPS Transformation for Gradual Programs with Evidence

CPSC 539B Final Project

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Source Language

Term Syntax

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e	$::=$		
	$\mid$	x	Variables
	$\mid$	b	Booleans
	$\mid$	n	Natural Numbers
	$\mid$	$\lambda x : T . e$	Functions
	$\mid$	$e_1 \ e_2$	Function Application
	$\mid$	$e_1 + e_2$	Addition
	$\mid$	$e_1 \stackrel{?}{=} e_2$	Number Equality Test
	$\mid$	**if** e_1 **then** e_2 **else** e_3	Conditionals
	$\mid$	$\langle e_1 , e_2 \rangle$	Tuples
	$\mid$	$\pi_1 e$	Tuple First Projection
	$\mid$	$\pi_2 e$	Tuple Second Projection
	$\mid$	$\varepsilon \, e$	Evidence Ascription
	$\mid$	**error**	Runtime Type Error

Type Syntax

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$$
\begin{array}{lll}
T & ::= & \text{Types} \\
 & \mid & \mathsf{Nat} \\
 & \mid & \mathsf{Bool} \\
 & \mid & T_1 \to T_2 \\
 & \mid & T_1 \times T_2 \\
 & \mid & ?
\end{array}
$$

$$
\begin{array}{lll}
\varepsilon & ::= & \\
 & \mid & \{T\}
\end{array}
$$

Type Rules

$$\textsc{HastypeAscr}\ \frac{\Gamma \vdash e : T_2 \quad \varepsilon \vdash T_1 \cong T_2}{\Gamma \vdash \varepsilon\, e : T_1}$$

$$\textsc{ConsistentEv}\ \frac{T_3 \sqcap T_1 = T_3 \quad T_3 \sqcap T_2 = T_3}{\{T_3\} \vdash T_1 \cong T_2}$$

Combining Evidence

$\boxed{T_1 \sqcap T_2 = T_3}$ *(Precision Meet)*

MeetDynL

$$\frac{}{? \sqcap T = T}$$

MeetDynR

$$\frac{}{T \sqcap ? = T}$$

MeetRefl

$$\frac{}{T \sqcap T = T}$$

MeetFun

$$\frac{T_1 \sqcap T_1' = T_1'' \qquad T_2 \sqcap T_2' = T_2''}{T_1 \to T_2 \sqcap T_1' \to T_2' = T_1'' \to T_2''}$$

MeetProd

$$\frac{T_1 \sqcap T_1' = T_1'' \qquad T_2 \sqcap T_2' = T_2''}{T_1 \times T_2 \sqcap T_1' \times T_2' = T_1'' \times T_2''}$$

RedAscr

$$\frac{}{\varepsilon_1\,(\varepsilon_2\,e) \longrightarrow (\varepsilon_1 \sqcap \varepsilon_2)\,e}$$

RedAscrFail

$$\frac{\varepsilon_1 \sqcap \varepsilon_2\ \textbf{undefined}}{\varepsilon_1\,(\varepsilon_2\,e) \longrightarrow \textbf{error}}$$

RedAppEv

$$\frac{}{(\varepsilon_1\,(\lambda x : T.\,e))\,(\varepsilon_2\,r) \longrightarrow (\textbf{cod}\,\varepsilon_2)\,([(\textbf{dom}\,\varepsilon_1 \sqcap \varepsilon_2)\,r/x]e)}$$

RedAppEvFail

$$\frac{\textbf{dom}\,\varepsilon_1 \sqcap \varepsilon_2\ \textbf{undefined}}{(\varepsilon_1\,(\lambda x : T.\,e))\,(\varepsilon_2\,r) \longrightarrow \textbf{error}}$$

Examples

`<<no parses (char 4):  <<I***nt>>(<<Bool>>true) + 0 >>` typechecks!

$\{\text{Bool}\} \vdash \text{Bool} \cong ?$ and

`<<no parses (char 3):  <<I***nt>> |- ?  =  Bool >>`

But: fails at runtime!

`<<no parses (char 5):  Int /***\ Bool >>` undefined