# NBA Sleep Tracking Data Imputation

by

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Submitted to the Department of Engineering in Electrical Engineering and Computer Science in partial fulfillment of the requirements for the degree of

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## ABSTRACT

This thesis investigates imputation methods for nights of missing sleep wearable data from NBA Academy athletes. Sparsity in sleep tracking data arises as a result of behavioral non-compliance or device malfunction, hindering the NBA Academy's ability to provide actionable insights that improve player sleep, a crucial component for player development. Motivated by existing work on time series data imputation, four main techniques are evaluated: K-Nearest Neighbors Regression, Linear Interpolation, Linear Regression, and Quadratic Regression. Each technique is applied and evaluated on key sleep metrics such as sleep duration, rMSSD (Root Mean Square of the Successive Differences between Heartbeats), and average heart rate. Results indicate K-Nearest Neighbors Regression and Linear Interpolation, with access to data in the past and future (offline imputation), as the best-performing sleep imputation methods. Furthermore, this thesis utilizes the NBA Academy's shooting and jumping datasets in conjunction with the sleep dataset to explore a relationship between sleep and athletic performance, finding a generally weak correlation between sleep and athletic performance data, regardless of the time lag. This research has applications in all areas of sport and performance as well as in domains where data sparsity is problematic.

Thesis supervisor: Anette Hosoi
Title: Neil and Jane Pappalardo Professor of Mechanical Engineering

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I would like to thank Andrew Murray and the NBA for being excellent project partners. This thesis has allowed me to fulfill a childhood dream of being involved with the NBA.

Mom, Dad, Ashley, and Amanda: thank you for being the most loving and supportive family I could ever hope for. There is no world where I get to where I am today without all of your guidance along the way.

# Biographical Sketch

Joseph David Licht was born in Denver, Colorado, on September 3, 2000. He received his S.B degree in Computer Science and Engineering in 2023 and his Master of Engineering in Electrical Engineering and Computer Science in 2024. During his academic tenure, he served as a Teaching Assistant for Sports Technology & Innovation in 2022 and for Computation Structures in 2023 and 2024.

In the realm of professional experience, Joseph has been an active member of Brass Rat Investments, a student-run hedge fund. Joseph is set to begin work as an analyst at Point72, where he aims to leverage his technical background and research acumen in the finance industry.

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# Chapter 1

# Introduction

## 1.1 Background

The National Basketball Association (NBA) invests heavily in expanding its international presence to improve on-court talent and grow its fan base. This international investment has paid dividends, with international players winning the NBA MVP award, basketball's most prestigious award, each of the last 5 years (Giannis Antetokounmpo 2x, Nikola Jokić 2x, and Joel Embiid 1x). This compares to only 6 international MVPs in the previous 63 years (Hakeem Olajuwon 1x, Steve Nash 2x, Tim Duncan 2x, Dirk Nowitzki 1x).

One of the NBA's more notable recent international talent development initiatives is its international Academies, such as the NBA Academy Africa in Senegal (established 2017) and the NBA Academy Latin America in Mexico (established 2021). These elite training centers offer a rigorous basketball development program to top young talent from their respective regions.

An important area of NBA Academy player development is in the realm of sleep, for which there is an established link between sleep quality and athletic performance [1]. Considering the relationship between sleep and athletic performance, the NBA has provided their Academy players with sleep-tracking wearables since November 2020. This has allowed players to improve their sleep and contextualize their on-court performance, thus enabling the players to best realize their potential. As of the writing of this thesis, the NBA has curated a dataset of 88 athletes' sleep-tracking statistics, amounting to over 5,500 logged nights of sleep.

Despite the relative ease of use of non-invasive sleep-tracking wearables, NBA Academy players only log sleep 42% of nights on average for reasons such as player behavioral compliance or wearable device error. These unlogged nights hinder the resolution of the tracking dataset, thereby limiting the NBA's ability to provide sleep-related actionable insights, such as blocking blue light, regularizing sleep-wake patterns, and mindfulness [2]–[4].

In addition to amassing sleep data, the NBA Academies routinely collect athletic testing data specific to player performance, with quantifiable metrics pertaining to agility, shooting, and jumping. The better a player's agility, shooting, and jumping, the better they perform on the court. Tracking these metrics over time allows the Academies to ensure player improvement while also flagging injury risk [5].

This thesis explores K-Nearest Neighbors Regression, Linear Interpolation, Linear Regression, and Quadratic Regression as methods to impute missing nights of sleep data [6]–[8]. Additionally, this thesis analyzes the relationship between sleep and the NBA's proprietary athletic testing data set using a Linear Regression Coefficient of Determination [9].

With over 33% of Americans having tried Sleep Trackers as of 2023, the expected contribution of this research extends beyond the realm of sports [10]. Providing reliable estimations for missing/unlogged nights can have profound impacts on people's overall well-being.

## 1.2 Paper Structure

In Chapter 1, I introduce the NBA Academies and their research initiatives to develop athletes. I highlight their sleep-tracking investments and outline my research initiative to garner insights from their sleep and athletic performance datasets.

In Chapter 2, I review the existing literature on time series data imputation and literature that establishes a relationship between sleep and athletic performance.

Chapter 3 provides an overview of the NBA Academy's sleep and athletic performance datasets, which are used extensively in this research.

Chapter 4 describes my approach to sleep imputation, describing the model infrastructure, model implementation, and results in detail.

In Chapter 5, I detail my research to establish a relationship between sleep and athletic performance for NBA Academy players.

Chapter 6 contains a discussion of my results as well as recommendations for future work.

Chapter 7 provides a concluding summary of the work in this thesis.

# Chapter 2

# Related Work

## 2.1 Time Series Data Imputation

This chapter details a variety of existing time series data imputation methods with relevance to estimating missing sleep-tracking data. This thesis is primarily motivated by the univariate methods discussed in Section 2.1.1. The additional enumerated methods present avenues to extend the work in this thesis.

### 2.1.1 Air Quality Time Series Imputation

Junninen et al. explore a variety of time series models in the context of air quality datasets [11]. These models have relevance in the domain of sleep data imputation:

- **Univariate Methods**
  - **Univariate Nearest Neighbor**: Imputes missing data using the nearest observed data point. Specifically, given endpoints, $(x_1, y_1)$ and $(x_2, y_2)$ and point $x$, such that $x_1 < x < x_2$, the model estimates $y = \begin{cases} y_1 & \text{if } x \leq x_1 + (x_2 - x_1)/2 \\ y_2 & \text{if } x > x_1 + (x_2 - x_1)/2 \end{cases}$

- **Linear Interpolation (LIN)**: Fits a line between the start and end points of a gap. Specifically, given endpoints, $(x_1, y_1)$ and $(x_2, y_2)$ and point $x$, such that $x_1 < x < x_2$, LIN estimates $y = y_1 + k(x + x_1)$ where $k = \frac{y_2 - y_1}{x_2 - x_1}$
- **Cubic Spline Imputation**: Interpolates a polynomial curve through a set of observed data points $(x_i, y_i)$, where a cubic spline has the following equation: $y = f(x) = a_i + b_i x + c_i x^2 + d_i x^3$ if $x_i \leq x < x_{i+1}$

• **Multivariate Methods**

- **Linear Regression using Expectation–Maximization Algorithm (REGEM)**: The Expectation–Maximization (EM) model is used to find the maximum likelihood estimate of parameters. Using the EM model, this method iteratively estimates linear regression models between missing data and available data.
- **Multivariate Nearest Neighbor**: Each row of N columns is a coordinate in N-dimensional space. The algorithm imputes missing row values using the nearest complete row as defined by the distance matrix computed with the following formulas:

$$\text{dist}_i^{ab} = \sqrt{r_a(i) - r_b(i)}$$

$$\text{dist}^{ab} = \frac{N}{N - N^{md}} (\sum_{i=1}^{N} (\text{dist}_i^{ab})^2)$$

where $r_a(i)$ and $r_b(i)$ are the $i$-th elements of rows $a$ and $b$.

- **Self-Organizing Map (SOM)**: Mapping from input space $R^d$ to a lower dimensional array of neurons/map units (typically two dimensional).
- **Multi-Layer Perceptron (MLP)**: Feed forward neural network with multiple layers. The training uses error back-propagation.

• **Hybrid Model**

- A procedure that conditionally combines multiple methods. For example, a hybrid model that uses a univariate method for small gaps and a multivariate method for larger gaps.

### 2.1.2 Bi-directional Recurrent Neural Network (bRNN)

We now explore the space of deep learning approaches. Consider sequential data with input vectors $x_1^T = \{x_1, x_2, ..., x_T\}$ and output vectors $y_1^T = \{y_1, y_2, ..., y_T\}$. Traditional Recurrent Neural Networks (RNN) manage this sequential data by reading data points in order, making use of all seen data points in prediction. RNNs are proficient in processing sequential data, maintaining a hidden state $h_t$ for each timestep $t$. This hidden state acts as a memory, encapsulating information from previous inputs in the sequence. Mathematically, for each timestep $t$, the hidden state $h_t$ is updated based on the current input $x_t$ and the previous hidden state $h_{t-1}$, as shown in the equation $h_t = f(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$, where $W_{hh}$ and $W_{xh}$ are weight matrices, $b_h$ is a bias term, and $f$ is a non-linear activation function. This mechanism allows the RNN to capture temporal dependencies within the sequence.

Bi-directional RNNs (bRNN) differ from traditional RNNs in that each training sequence is presented with information pertaining to all points before and after [12].

In the context of health wearable data imputations, bRNNs suffer from a lack of generalization due to the large gaps of missing data seen in practice. These large gaps represent areas with very limited sequential information, which is necessary for bRNN performance.

### 2.1.3 HeartImp

The HeartImp framework is designed to address the challenge of imputing missing values in heart rate time series data collected from wearables [7]. It first encodes day-long time series segments using a convolutional encoder with a gating mechanism to manage missing gaps. Then, it uses an "imputation with a reference set" module, which fuses information from both the current observations and the user's historical heart rate records. This approach ensures the imputation is coherent with human health conditions like heart rate fluctuations.

The framework employs adversarial training to enhance the learning process and ensure the imputed heart rate series are reasonable.

### 2.1.4 PulseImpute

The transformer model is a deep learning framework that relies on attention mechanisms, eliminating the need for recurrence [13]. It employs a stack of encoder and decoder layers, each containing two sub-layers: a multi-head self-attention mechanism and a position-wise fully connected feed-forward network. The self-attention mechanism allows the model to process different positions of the input sequence simultaneously, facilitating parallel computation and capturing complex dependencies. This architecture significantly improves performance in tasks like machine translation, showcasing its effectiveness in handling sequential data.

In the context of time series data from health wearables, the transformer model has shown promise, as evidenced by the strong performance of transformer-based models in the PulseImpute Challenge, a novel benchmark task aimed at addressing missing data issues in mobile health (mHealth) applications [8].

### 2.1.5 Non-Negative Matrix Factorization

Non-negative matrix Factorization differs from the previously detailed methods in that the previous imputation methods predict missing data only based on each individual user, whereas matrix factorization relies on data across multiple users. This approach allows the exploitation of any relationships between individuals with similar sleep characteristics [6].

Non-Negative Matrix Factorization involves the decomposition of a non-negative matrix $V$ into two non-negative matrix factors $W$ and $H$, such that $V \approx WH$ [14].

## 2.2 Sleep and Athletic Performance

Sleep is a crucial factor in optimizing athletic performance. The relationship between sleep duration and quality and its impact on athletes' physical and mental capabilities has been increasingly recognized in sports science. Mah et al. aimed to assess how increased sleep duration over several weeks affects various performance metrics in collegiate basketball players [1]. The study involved a two-phase approach: an initial baseline period followed by a sleep extension period, with athletes encouraged to achieve over 10 hours of sleep each night. The study's results were significant, demonstrating noticeable improvements in athletic performance following the sleep extension phase. Key improvements included:

- Enhanced sprint times: Sprint times improved, with a reduction from 16.2 seconds at baseline to 15.5 seconds after the sleep extension period.
- Increased shooting accuracy: Both free throw percentage and 3-point field goal percentage increased by 9% and 9.2%, respectively.
- Improved practice and game ratings

# Chapter 3

# Data

## 3.1 NBA Academy Datasets

The three most important datasets used in this thesis are the Sleep Tracking Dataset, the Shooting Dataset, and the Jump Dataset. This chapter provides an overview of each.

### 3.1.1 Sleep Tracking Dataset Overview

Health wearable devices (health wearables) track physiological features, such as heart rate or temperature, using sensors built into devices such as wristwatches, chest straps, and rings [15]. The NBA outfitted their Academy players with a popular health wearable device, providing the data for the NBA's sleep tracking database. This database has 170 fields (columns) for each of the 5,500 nights logged (row entries) by 88 unique NBA Academy athletes.

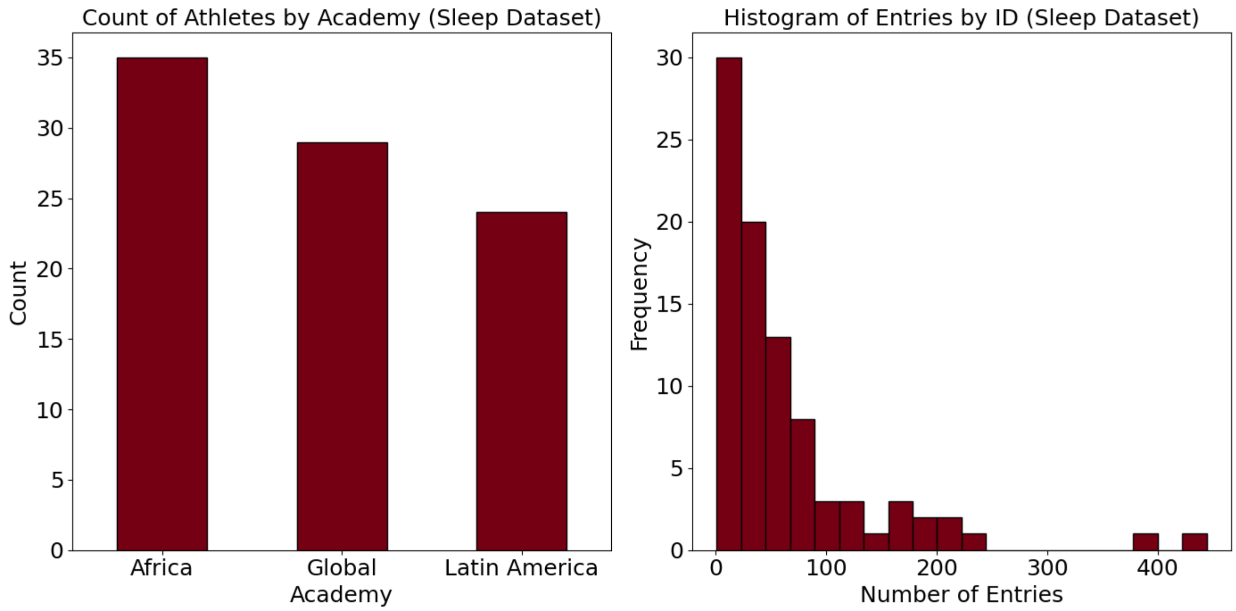


Figure 3.1: NBA Academy Sleep Dataset

### Feature Importance

The most important features in the NBA sleep tracking database as it pertains to this thesis are rMSSD (heart rate variability), average breath rate, duration of sleep, average heart rate, and low heart rate. These 5 fields are demarcated as most interesting for imputation based on discussions with the NBA as well as consulting sleep quality literature such as Paul et al., Sadeghi et al., and Mendonça et al. [16]–[18].

| **Field** | **Definition** |
|---|---|
| rMSSD | Root Mean Square of the Successive Differences between heartbeats. rMSSd is averaged across an entire night of sleep. |
| Breath Average | The average breathing rate (breaths per minute) throughout a single night of sleep. |
| Duration | The total time spent in light sleep, deep sleep, or REM sleep during the course of one night. |
| Heart Rate Average | The average heart rate measured over a single night's sleep. |
| Heart Rate Low | The lowest recorded heart rate during a single night's sleep. |

Table 3.1: Definitions of 5 Most Important Sleep Fields

### 3.1.2 Shooting Dataset

The NBA conducts routine testing of academy players' shooting ability. These tests determine how well players can make a basket from different static positions on the court with no defenders present. This dataset contains 69 unique athletes, representing a subset of the players in the Sleep Tracking dataset.

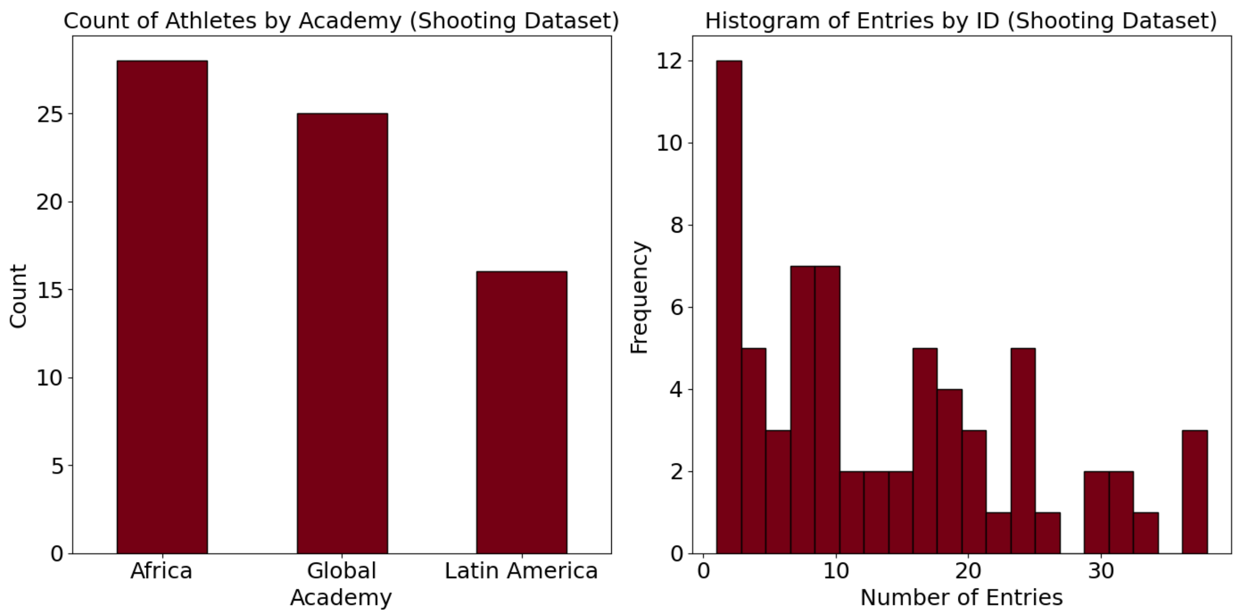


Figure 3.2: NBA Academy Shooting Dataset

#### Feature Importance

In order to understand how well a player performs on a single day and to understand the change in performance across time, I analyze the percentage of made shots across a range of locations on the court. The locations vary by Euclidean distance from the basket (i.e., Corner 3 is a greater distance from the basket than Corner 2) and angle from the basket (i.e., 'Corner Left' vs 'Wing Left'). I also analyze shooting percentages in drills such as Star 15, which span across multiple pre-defined locations. These shooting features are analyzed in the context of sleep's impact on athletic performance in Chapter 5.

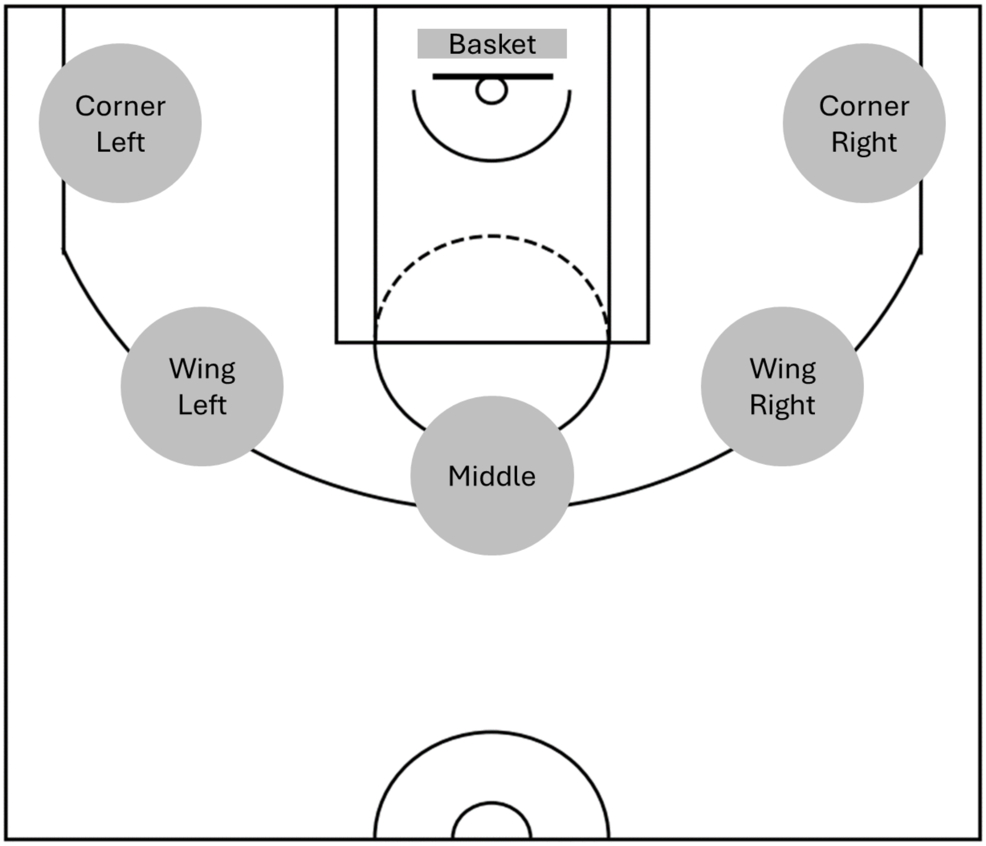


Figure 3.3: Map of Shooting Locations

### 3.1.3 Jump Dataset

The NBA academies routinely collect metrics for a jump test called the counter-movement jump (CMJ). In a CMJ test, athletes stand on a force plate with their arms on their hips. They are instructed to squat downward and explode up, jumping as high as possible while maintaining their hands on the hip the entire time. The CMJ test assesses athlete explosiveness, muscular fatigue, and inter-limb asymmetries. The NBA's jump dataset contains 46 players that are a subset of the players in the Sleep Tracking dataset.

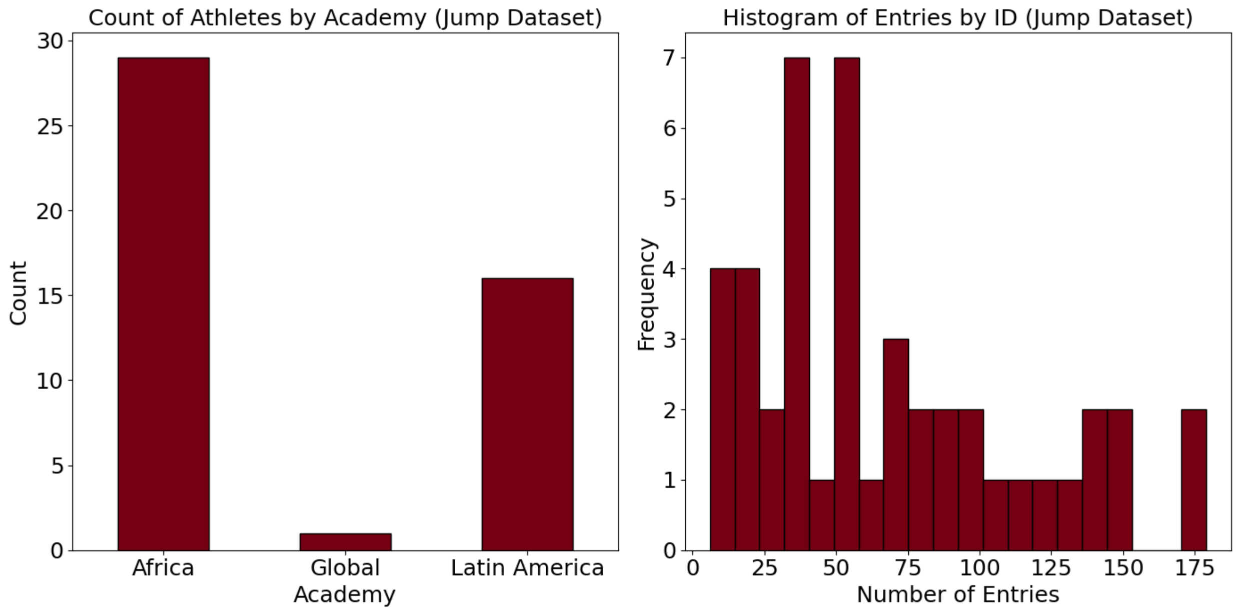


Figure 3.4: NBA Academy Jump Dataset

### Feature Importance

To understand athlete explosiveness, I study the player's Peak Propulsive Force (N), Jump Height (m), and Peak Velocity (m/s). To study muscular fatigue, balance, and injury mitigation I study L|R Avg. Braking Force (%), Braking Impulse (N.s), Stiffness (N/m), and Avg. Braking Force (N). These features are analyzed in the context of sleep's impact on athletic performance in Chapter 5.

# Chapter 4

# Sleep Imputation

Unlogged nights of sleep tracking data represent lost insights for NBA Academy players. Inconsistent sleep tracking hinders the NBA's ability to work with players to improve sleep hygiene with targeted interventions such as blocking blue light, regularizing sleep-wake patterns, and mindfulness [2]–[4].

The NBA sleep tracking dataset is sparsely populated as there are more unlogged than logged nights of sleep. For each player $i$, whose date of their first logged night of sleep is $first\_night_i$ and last logged night of sleep is $last\_night_i$ with $first\_night_i \leq last\_night_i$, I define an unlogged night as any date falling in the interval $[first\_night_i, last\_night_i]$ such that there doesn't exist a valid sleep tracking entry on that night. A valid sleep tracking entry is a night such that each of the five metrics of interest (rMSSD, average breath rate, duration of sleep, and average and low heart rate) are properly recorded. $\text{logged\_nights}_i$ is equal to the set of valid nights for player $i$. $\text{feasible\_nights}_i$ is the set of nights between (inclusive) $first\_night_i$ and $last\_night_i$. Using this notation, I define the proportion of logged nights for each individual player, $i$, as

$$\text{proportion\_logged}_i = \frac{\text{size}(\text{logged\_nights}_i)}{\text{size}(\text{feasible\_nights}_i)}$$

Figure 4.2 plots the histogram of $\text{proportion\_logged}_i$ for all players in the NBA sleep tracking dataset. Here we see the mean of logged nights is 0.42, thus motivating the importance of quality sleep imputation methods. As such, the NBA would like to explore methods that can be applied retroactively (offline) to its existing sleep dataset as well as to real-time sleep data (online).

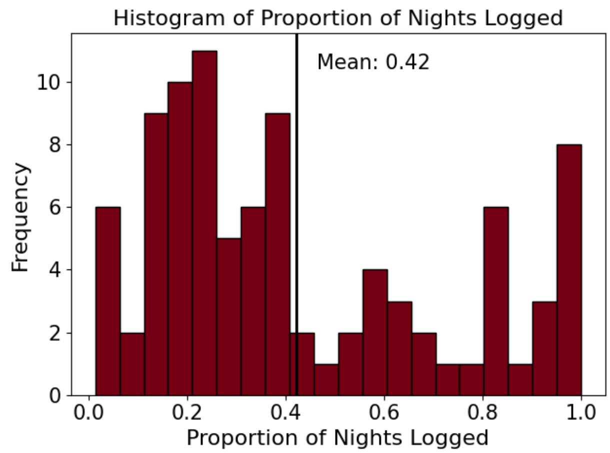


Figure 4.1: Histogram of Athlete's Nights Logged

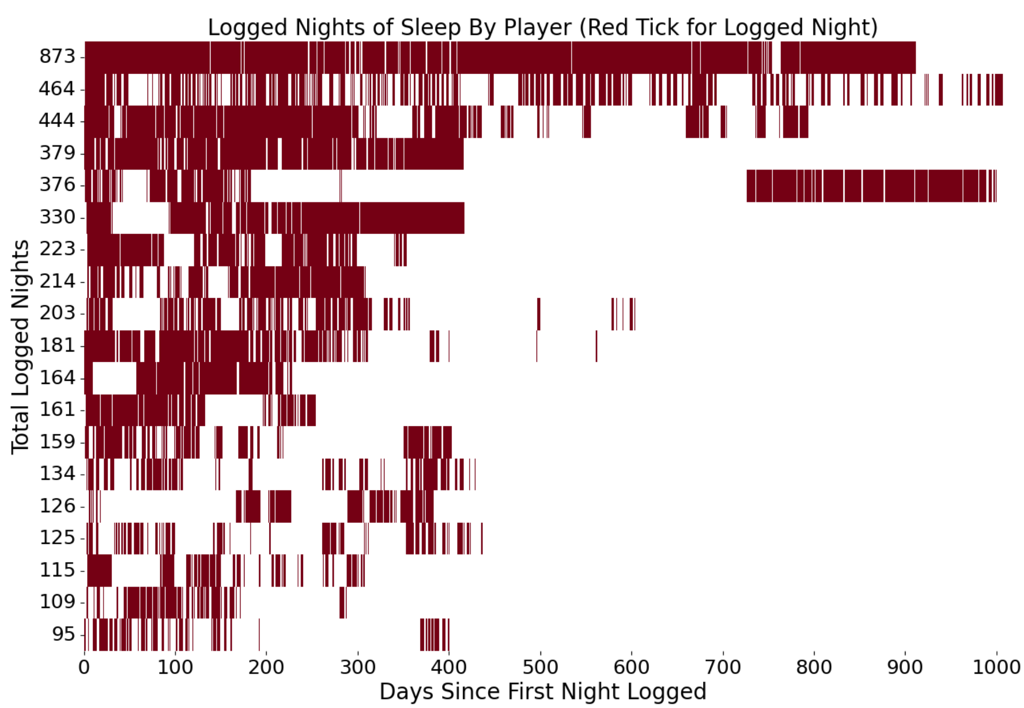


Figure 4.2: Sleep Tracking of Players in Top 20 of Total Logged Nights

## 4.1 Problem Definition

The five sleep metrics we would like to impute are rMSSD (Heart Rate Variability), average breath rate, duration of sleep, and heart rate (average and low). The imputation task is for any night $n$ such that $first\_night_i \leq n \leq last\_night_i$ and $n \notin \text{logged\_nights}_i$, predict the value of $metric_{i,n}^{m}$ for each metric $m$ in the set $M$ ={rMSSD, Breath Average, Duration Integer hr, HR Average, HR Lowest}.

For offline prediction, we seek to predict $metric_{i,n}^{m}$ with access to any $metric_{i,j}^{m}$ where $j \neq n$ and $j \in \text{logged\_nights}_i$. For online prediction, we seek to predict $metric_{i,n}^{m}$ with access to any $metric_{i,j}^{m}$ where $j < n$ and $j \in \text{logged\_nights}_i$ [19].

## 4.2 Implementation Infrastructure

### 4.2.1 Dataset for Imputation

Sleep imputation is performed on the NBA sleep tracking dataset, which has 170 sleep tracking parameters for each of the 5,500 nights logged by 88 unique NBA Academy athletes. Additional information pertaining to this dataset is provided in the data chapter.

### 4.2.2 Train-Test Split

To properly train and evaluate model performance, it is important to split available data, *all_data*, into a train set and a test set. The train set is used to tune model parameters, and the test set is used to understand how well the model generalizes to unseen data. Test performance allows for a deeper understanding of how the model will perform when used in production.

Typically, train and test sets are determined by first defining the size of the test set as a proportion, *proportion_test*, such that $0 \leq proportion\_test \leq 1$ and the size of the training set as $proportion\_train = 1 - proportion\_test$. A random split of $all\_data$ is performed to create the sets $test\_set\_random$ and $train\_set\_random$ such that

$$\text{size}(test\_set\_random) \approx \text{size}(all\_data) * proportion\_test$$

$$\text{size}(train\_set\_random) \approx \text{size}(all\_data) * proportion\_train.$$

However, for the imputation algorithms explored in this thesis, this typical method of creating the train and test sets by random pre-defined splits is not applicable because the selection of hyperparameters and model architectures can render certain test points invalid for computation, thus not allowing for a consistent train and test comparison across all explored hyperparameters and models. Take, for example, the *window* hyperparameter, which defines how many nights the algorithm is able to process to produce a prediction. I define $test\_set\_valid_n$ to be the subset of points in $test\_set\_random$ such that each point has $window = n$ of contiguous nights logged in $train\_set\_random$. Notice how for all integer *window* sizes $i$, $j$ such that $i < j$, $\text{size}(test\_set\_valid_i) \leq \text{size}(test\_set\_valid_j)$. This inconsistency in test set sizes grows as I introduce more hyperparameters and model architectures, thus making consistent performance comparisons across methods impossible under the naive random train-test split method.

Therefore, a train test split must be carefully constructed to allow for valid comparisons of sleep imputation performance across all methods and hyperparameters. As such, the test set used in this thesis, $test\_set\_thesis$, is created by searching through the space of algorithms and hyperparameters to find all points in $all\_data$ that remain as valid testing points for each algorithm and hyperparameter. The points that are not valid testing points are thereby placed into the train set, $train\_set\_theis$. To ensure a large enough test set, this thesis only analyzes the individual athletes who are $test\_set\_theis$ satisfies

$$\text{size}(test\_set\_thesis) \geq \text{size}(all\_data) * min\_proportion\_test$$

$$\text{size}(test\_set\_thesis) \approx min\_test\_points.$$

where $min\_proportion\_test = 0.2$ and $min\_test\_points = 100$.

### 4.2.3 Normalization

For each individual athlete, $i$, and night, $n$ in $\text{logged\_nights}_i$ I normalized for each metric $m$ in the set $M$ ={rMSSD, Breath Average, Duration Integer hr, HR Average, HR Lowest} such that:

$$\text{normalized\_metric}^m_{i,n} = \frac{\text{metric}^m_{i,n}}{\frac{1}{|\text{logged\_nights}_i|}\sum_{n \in \text{logged\_nights}_i} \text{metric}^m_{i,n}}$$

This normalization allows for a comparison of model performance across each of the 5 sleep metrics in set $M$.

## 4.3 Sleep Imputation Models

This thesis explores 4 methods for inputting the NBA sleep tracking dataset: K-Nearest Neighbors Regression (KNN Regression), Linear Interpolation (LIN), Linear Regression, and Quadratic Regression. This section provides the motivation for and implementation of each model.

### 4.3.1 K-Nearest Neighbors Regression (KNN Regression)

K-Nearest Neighbors Regression (KNN Regression) is a supervised machine learning model. It is an instance-based model, meaning it produces outputs based on input training data rather than training a generalizable internal model. This is an important property in that training multiple athletes does not alter the model performance for any individual athlete, making it an entirely individualized model. At a high level, KNN Regression performs imputation by selecting nearby "neighboring" points to average together for an imputation value.

**KNN Regression Mechanics**

Say we are given an NBA academy athlete $i$ with normalized training data set $normalized\_train\_data_i$ such that each point in $normalized\_train\_data_i$ is 6 dimensional: date and the 5 metrics in the set $M$ ={rMSSD, Breath Average, Duration Integer hr, HR Average, HR Lowest} (see Train-Test Split and Normalization for training set construction). KNN Regression seeks to predict the metrics in set $M$ for a target night, $target\_date_i$ in the following steps:

1. Select $target\_date_i$ as the date of imputation interest for player $i$.

2. Create the set $within\_window_i$ by filtering $normalized\_train\_data_i$ to only include points such that the point's date falls within:

   - Range $[target\_date_i - window, target\_date_i + window]$ if $backward$ is true
   - Range $[target\_date_i - window, target\_date_i - 1]$ if $backward$ is false

3. Create the set $nearest\_neighbors_i$ by filtering $within\_window_i$ to only include the $number\_neighbors$ nearest neighbors as defined by the distance function between two dates, $j$ and $k$:

$$distance(j,k) = |days\_difference(j,k)|$$

4. For each metric $m$ in set $M$, compute the average of the points in set $within\_window_i$. If hyperparameter $weights$ is equal to 'uniform', the average is unweighted. If $weights$ is equal to 'distance', then the average is a weighted average.

$$uniform\_average^m_{i,target\_date_i} = \frac{\sum_{x \in within\_window\_normalized\_train\_data_i} x^m}{|within\_window\_normalized\_train\_data_i|}$$

$$distance\_average^m_{i,target\_date_i} = \frac{\sum_{x \in nearest\_neighbors_i} x^m \cdot \frac{1}{distance(target\_date_i, date(x))}}{\sum_{x \in nearest\_neighbors_i} \frac{1}{distance(target\_date_i, date(x))}}$$

### KNN Regression Hyperparameter Search Space

The four hyperparameters in KNN Regression are *window*, *number_neighbors*, *weights*, and *backward*. The *window* and *number_neighbors* search space was selected to study how sleep anywhere from 1 night to 10 nights apart from $target\_date_i$ impacts and helps predict sleep metrics on $target\_date_i$. *weights* allows for the study of the importance of sleep nearest to $target\_date_i$. *backward* is used to study the model performance during online and offline use cases. Table 4.1 provides the definitions and search space for each hyperparameter.

| **Hyperparameter** | **Description** | **Parameter Search Space** |
|---|---|---|
| *window* | The number of consecutive nights on either side of the target night that are candidate inputs for imputation | [2, 10] |
| *number_neighbors* | The number of nearest neighbors to use for imputation | [2, 10] |
| *weights* | Function to weight input nights. 'uniform' means all neighboring points contribute equally to the prediction. 'distance' favors the nights closest (by nights apart from target night) to the target night by using the inverse of their distance as prediction weight. | {uniform, distance} |
| *backward* | Flag indicating whether the model should exclusively utilize data from previous nights | {True, False} |

Table 4.1: KNN Regression Hyperparameter Search Space

### KNN Regression Results

For the athletes that satisfy the individual data conditions specified in the Train-Test Split section, I plot the average RMSE by the *window*. I include a separate panel for each of the 5 metrics in set $M$. Within each panel, I have distinct RMSE curves based on the *weight* and *backward* hyperparameter values. Notice how the Duration Integer hr and rMSSD prove to be the most difficult metrics to impute. Breath Average is the easiest to impute. We see that imputing with access to data in the future and the past (green and yellow curves) yields the best results.

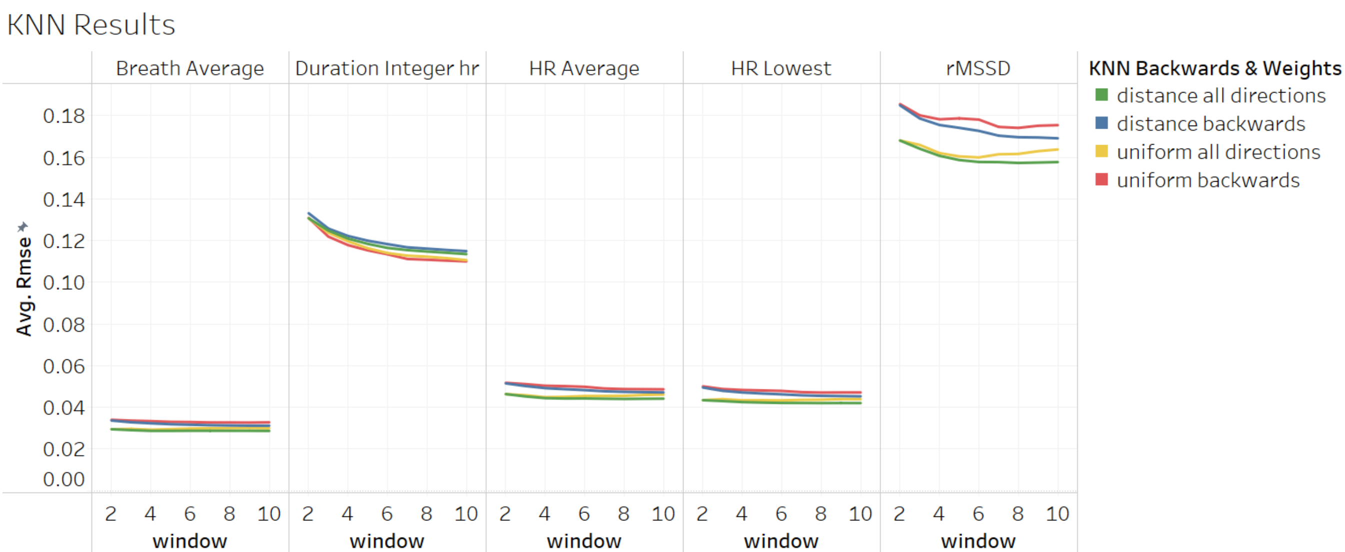


Figure 4.3: KNN Results by *window*

### 4.3.2 Linear Interpolation (LIN)

Linear Interpolation is a simple method that finds a point along a straight line between endpoints [11]. This method leverages the insight that nearby nights of sleep in the past and future are good predictors of missing nights of sleep.

#### LIN Mechanics

Say we are given an NBA academy athlete $i$ with normalized training data set $normalized\_train\_data_i$ such that each point in $normalized\_train\_data_i$ is 6 dimensional: date and the 5 metrics in the set $M =${rMSSD, Breath Average, Duration Integer hr, HR Average, HR Lowest} (see Train-Test Split and Normalization for training set construction). LIN seeks to predict the metrics in set $M$ for a target night, $target\_date_i$ using the following steps:

1. Select $target\_date_i$ as the date of imputation interest for player $i$.

2. Create the set $within\_window_i$ by filtering $normalized\_train\_data_i$ to only include points such that the point's date falls within: $[target\_date_i - window, target\_date_i + window]$

3. Select point $nearest\_less_{target\_date_i}$ from $within\_window_i$ such that $nearest\_less_{target\_date_i}$ is defined as:

$$nearest\_less_{target\_date_i} = \underset{x \in within\_window_i, date(x) < target\_date_i}{\arg\min} distance(target\_date_i, date(x))$$

For notational simplicity, I define $(date(nearest\_less_{target\_date_i}), nearest\_less^m_{target\_date_i})$ $= (x^m_{i,1}, y^m_{i,1})$

4. Select point $nearest\_greater_{target\_date_i}$ from $within\_window_i$ such that $nearest\_greater_{target\_date_i}$ is defined as:

$$nearest\_greater_{target\_date_i} = \underset{x \in within\_window_i, date(x) > target\_date_i}{\arg\min} distance(target\_date_i, date(x))$$

For notational simplicity, I define $(date(nearest\_greatertarget\_date_i)$, $nearest\_greater^m_{target\_date_i}) = (x^m_{i,2}, y^m_{i,2})$.

5. For each metric $m$ in $M$, compute

$$target\_date^m_i = y^m_{i,1} + k(target\_date_i + x^m_{i,1})$$

where

$$k = \frac{y^m_{i,2} - y^m_{i,1}}{x^m_{i,2} - x^m_{i,1}}$$

**LIN Hyperparameter Search Space**

The only hyperparameter in LIN is $window$ where the search space selected to study how sleep anywhere from 1 night to 10 nights apart from $target\_date_i$ impacts and helps predict sleep metrics on $target\_date_i$ Table 4.2 outlines the *window* hyperparameter definition and search space.

| Hyperparameter | Description | Parameter Search Space |
|---|---|---|
| *window* | The number of consecutive nights on either side of the target night that are candidate inputs for imputation | [1, 10] |

Table 4.2: LIN Hyperparameter Search Space

### LIN Results

For the athletes that satisfy the individual data conditions specified in the Train-Test Split section, I plot the average RMSE for each of the 5 metrics in set $M$. Notice how Duration Integer hr and rMSSD prove to be the most difficult metrics to impute. Breath Average is the easiest to impute.

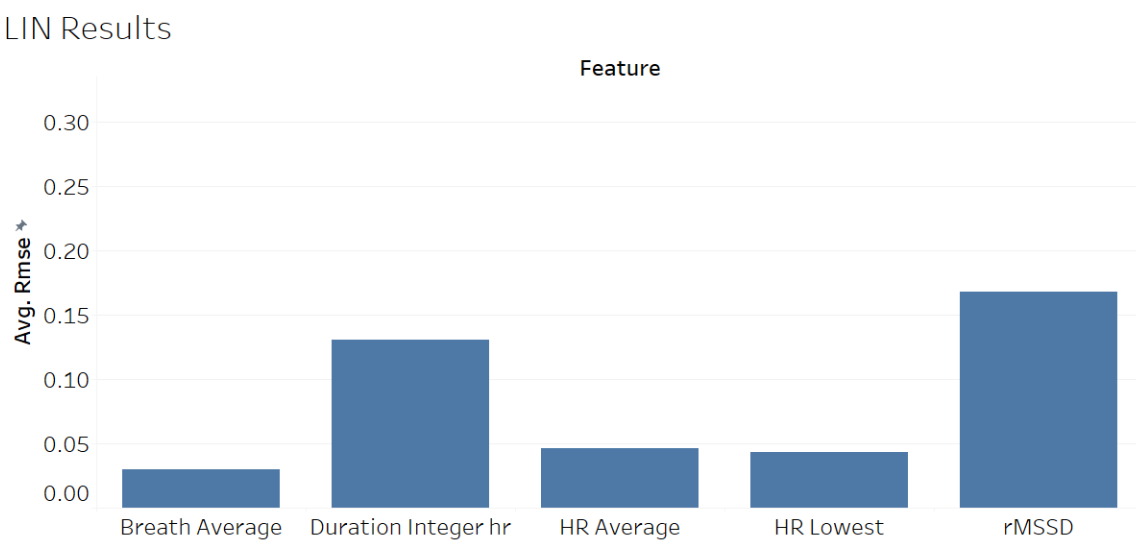


Figure 4.4: LIN Results by Sleep Feature

## 4.3.3 Linear & Quadratic Regression

Regression is a method for fitting a set of input points to a line (linear regression) or a curve (polynomial). The fitted line or curve is used to impute the target point.

### Linear & Quadratic Regression

Say we are given an NBA academy athlete $i$ with normalized training data set $normalized\_train\_data_i$ such that each point in $normalized\_train\_data_i$ is 6 dimensional: date and the 5 metrics in the set $M =${rMSSD, Breath Average, Duration Integer hr, HR Average, HR Lowest} (see Train-Test Split and Normalization for training set construction). Linear Regression and Quadratic Regression seeks to predict the metrics in set $M$ for a target night, $target\_date_i$ using the following steps:

1. Select $target\_date_i$ as the date of imputation interest for player $i$.

2. Create the set $within\_window_i$ by filtering $normalized\_train\_data_i$ to only include points such that the point's date falls within: $[target\_date_i - window, target\_date_i - 1]$

3. Fit the function $p(x)$ to the data in $within\_window_i$ by minimizing the squared error, $error = \sum_{j \in within\_window_i} (metric(j) - p(date(j)))^2$

   - Linear Regression ($degree = 1$): $p(x) = \beta_0 + \beta_1 x$

   - Quadratic Regression ($degree = 2$): $p(x) = \beta_0 + \beta_1 x + \beta_2 x^2$

4. Predict $target\_date_i^m = p(target\_date_i)$

### Linear & Quadratic Regression Hyperparameter Search Space

The two hyperparameters in Linear & Quadratic Regression are *window* and *degree*. The *window* search space was selected to study how to sleep anywhere from 1 night to 15 nights prior to $target\_date_i$ impacts and helps predict sleep metrics on $target\_date_i$. This window is set such that at least 3 nights must be present in order to allow for a valid quadratic fit. *degree* is set to 1 for a linear fit and 2 for a quadratic fit. Table 4.3 outlines the definitions and search space for each hyperparameter.

| Hyperparameter | Description | Parameter Search Space |
|---|---|---|
| *window* | The number of consecutive nights required to be present before the target imputation night | [3, 15] |
| *degree* | 1 for Linear Regression, 2 for Quadratic Regression | [1, 2] |

Table 4.3: Linear & Quadratic Regression Hyperparameter Search Space

### Linear & Quadratic Results

For the athletes that satisfy the individual data conditions specified in the Train-Test Split section, I plot the average RMSE by *window*. I include a separate panel for each of the 5 metrics in set $M$. Within each panel, I have separate RMSE curves based on the *weight* and *backward* hyperparameter values. Notice how the Duration Integer hr and rMSSD prove to be the most difficult metrics to impute. Breath Average is the easiest to impute. Notice that both models, especially Quadratic Regression, experience significant performance improvements as their window increases.

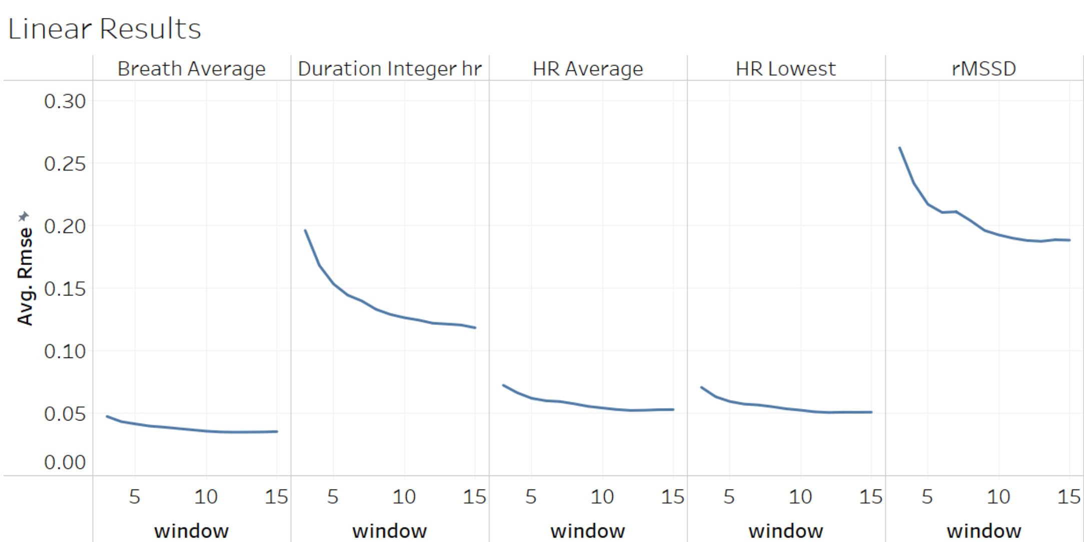


Figure 4.5: Linear Results by by *window*

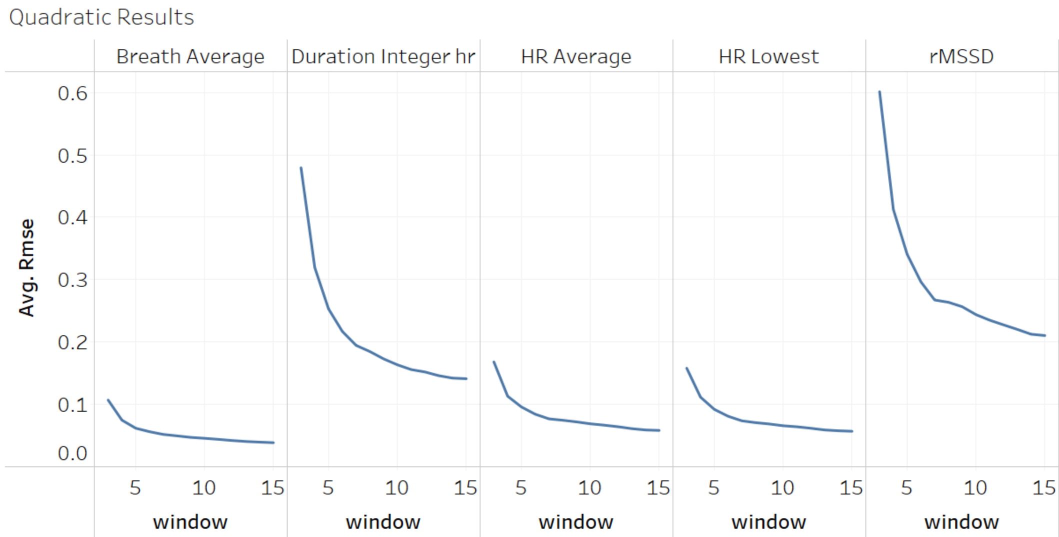


Figure 4.6: Quadratic Results by *window*

## 4.4 Summary Results

I provide 3 types of summary results. The first, Figure 4.7, plots the average RMSE value by method and feature as we vary *window*. The second, Figure 4.8 plots the average RMSE by method and feature. The third, Figure 4.9 plots the maximum and minimum RMSE by method and feature.

These summary figures provide the following insights:

- KNN Regression and LIN tend to outperform Linear and Quadratic Regression.
- Linear Regression and Quadratic Regression see the largest performance gains from increases in *window* size
- Duration Integer hr and rMSSD are the most difficult metrics to impute. These two metrics also experience the largest variability between their minimum and maximum RMSE.

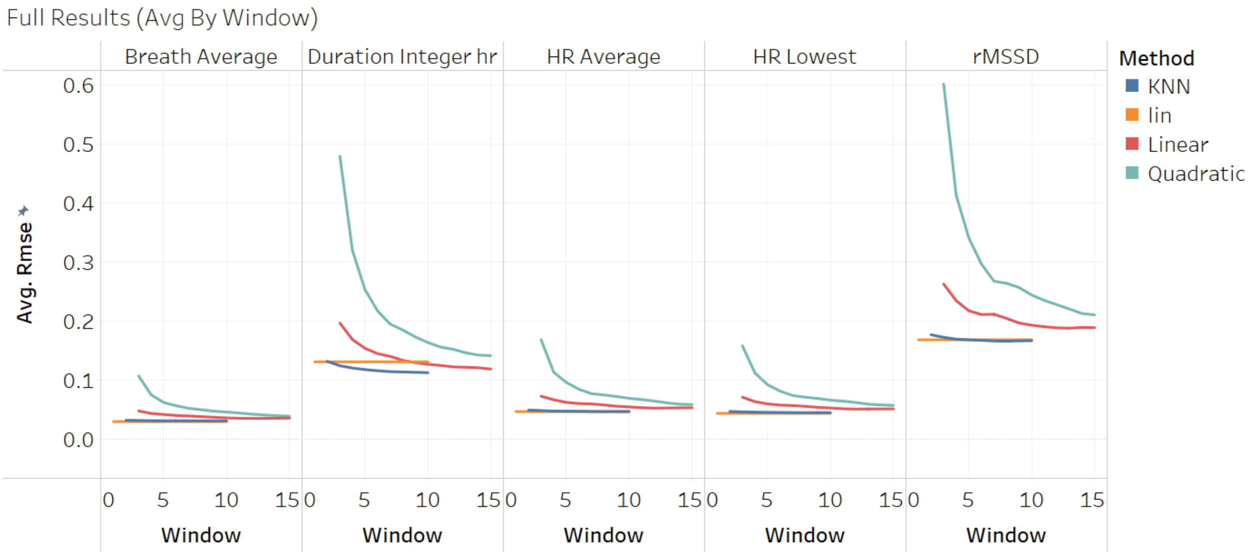


Figure 4.7: Full Results: Average by *window*

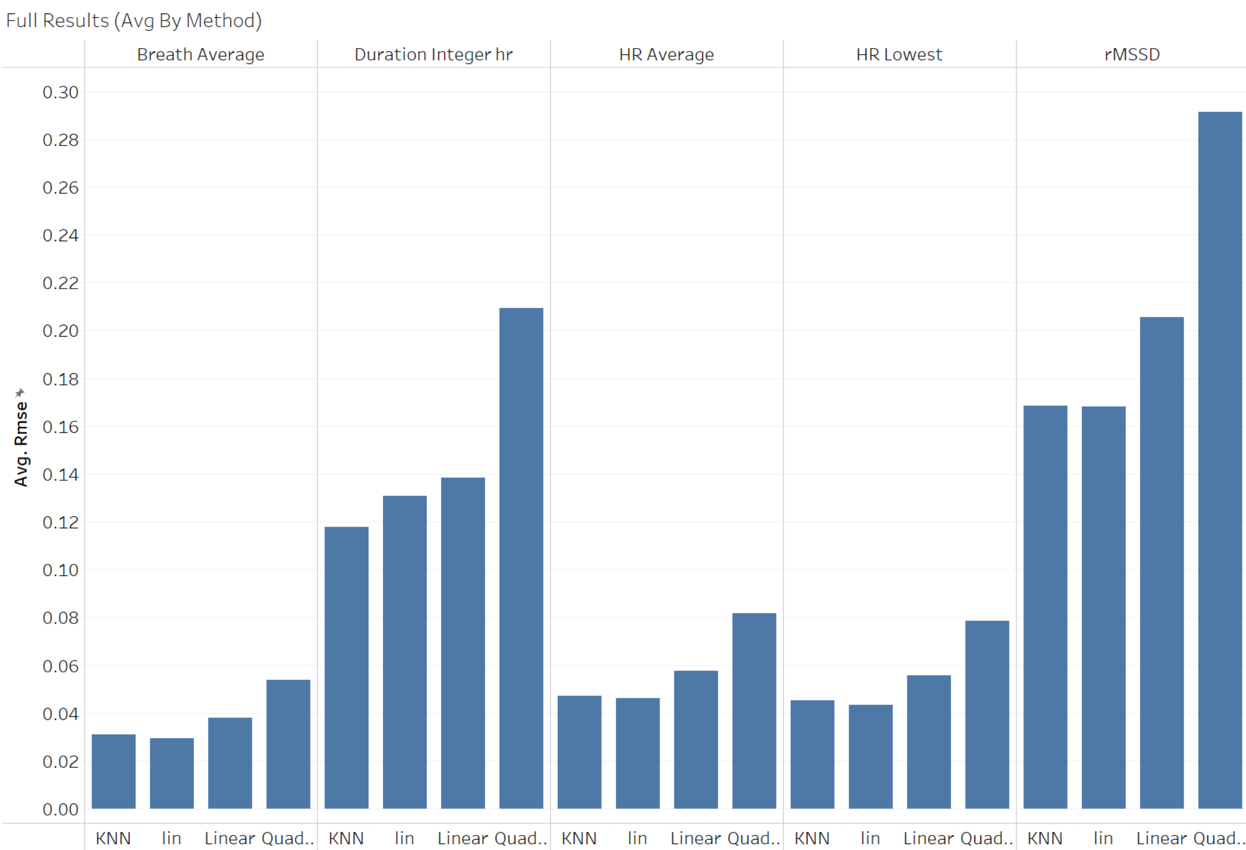


Figure 4.8: Full Results: Average by Method

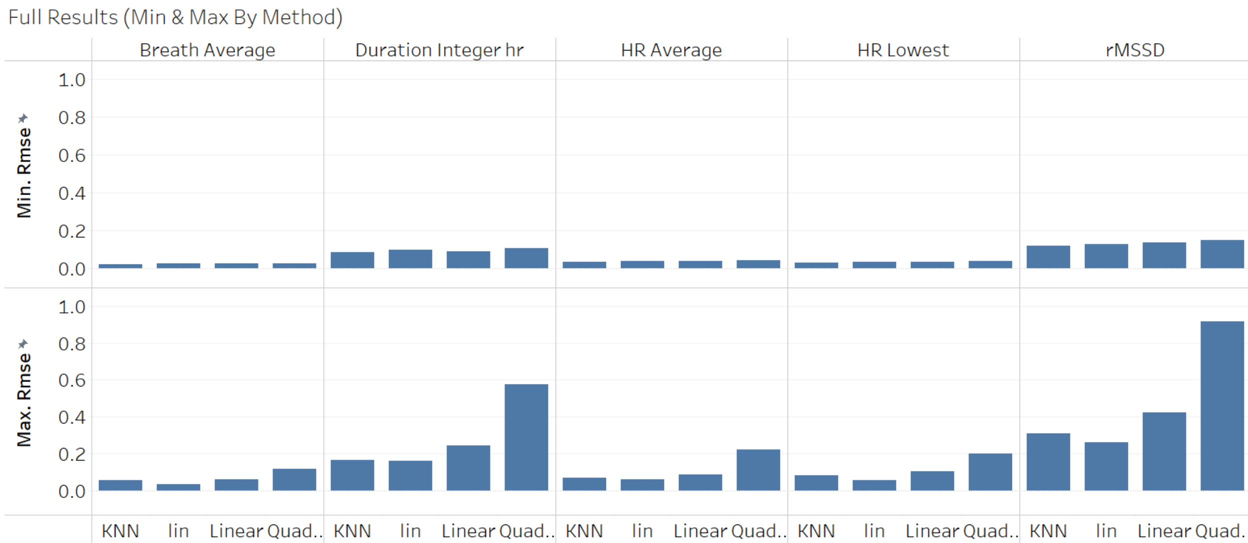


Figure 4.9: Full Results: Minimum and Maximum by Method

# Chapter 5

# From Sleep to Athletic Performance

The NBA Academies are built to enable international players to reach their full athletic potential. The NBA has invested in sleep-tracking wearables to help players improve their on-court performance. Studying the relationship between the NBA Sleep Tracking dataset and their athletic testing datasets (Shooting and Jumping datasets) is paramount to the NBA's mission to develop basketball players within the Academy system.

Linear Regression is a method for fitting a set of input pairs to a line according to the formula, $p(x) = \beta_0 + \beta_1 x$. Once a line has been fit, we can determine the Coefficient of Determination, $R^2$, which explains how much of the variance in the dependent variable (athletic testing data) is predicted by the independent variable (sleep data). $R^2$ ranges from [0, 1], with values closer to 1 indicating a sleep metric that has a large impact on an athletic testing metric, supporting the NBA's investment in sleep-tracking wearables as a means to improve on-court performance. I study sleep from 1 to 7 nights (single night and average across all nights in a window) prior to a testing date to capture any lingering effects of prior nights of sleep.

## 5.1 Implementation

I define the sleep set $M$, the shooting set $S$, and the jump set $J$ as follows:

$$M = \{rMSSD, \text{Breath Average}, \text{Duration Integer hr}, \text{HR Average}, \text{HR Lowest}\}$$

$$\begin{aligned} S = \{&\text{Corner L\%}, \text{Corner R\%}, \text{Wing L\%}, \text{Wing R\%}, \text{Mid\%}, \\ &\text{Overall\%}, \text{Corner 1\%}, \text{Corner 2\%}, \text{Corner 3\%}, \text{Corner 4\%}, \\ &\text{Star 15\%}, \text{SMS\%}\} \end{aligned}$$

$$\begin{aligned} J = \{&\text{Peak Propulsive Force (N)}, \text{Avg. Braking Force (N)}, \text{Braking Impulse (N.s)}, \\ &\text{Jump Height (m)}, \text{L}|\text{R Avg. Braking Force (\%)}, \text{Peak Landing Force (N)}, \\ &\text{Peak Velocity (m/s)}, \text{Stiffness (N/m)}\} \end{aligned}$$

I perform Linear Regression and determine $R^2$ on the NBA's sleep and athletic testing datasets using the following steps:

1. Select an athletic testing set $A$ as $A = S$ or $A = J$
2. For each sleep metric $m \in M$, athletic testing metric $a \in A$, nightly lag $n \in [1, 7]$, blended average $b \in \{True, False\}$ create the set $sleep\_to\_athletic\_points^{m,a,n,b}$ such that each pair is $(x^m_{i,d}, y^a_{i,d})$ for all players $i$ $s.t.$ $|A_i| \geq 2$ and for every date $d \in |S_i|$:
    - $x^m_{i,d}$: The sleep metric $m$ on night $d$ for player $i$
    - $y^a_{i,d}$: The athletic testing metric $a$ for player $i$ on night $d - n$ if $b = False$ or the blended average of night metrics on nights $d - n$ to $d - 1$ if $b = True$

To ensure consistency across players, each data point is normalized. Specifically, each

$x_{i,d}^m$ is normalized by dividing by the individual player's mean (for sleep metric $m$) $\overline{x_i^m}$ and each $y_{i,d}^a$ is normalized by dividing by the individual player's mean (for athletic metric $a$) $\overline{y_i^a}$

3. For each pair $(x_{i,d}^m, y_{i,d}^a)$ in the set $sleep\_to\_athletic\_points^{m,a,n,b}$, fit a line according to the formula $p(x) = \beta_0 + \beta_1 x$. The line is fit by minimizing the squared error,

$$error = \sum_{(x_{i,d}^m, y_{i,d}^a) \in sleep\_to\_athletic\_points^{m,a,n,b}} (y_{i,d}^a - p(x_{i,d}^m))^2$$

4. Determine the Coefficient of Determination, $R^2$ as

$$R^2 = 1 - \frac{\sum_{(x_{i,d}^m, y_{i,d}^a) \in sleep\_to\_athletic\_points^{m,a,n,b}} (y_{i,d}^a - p(x_{i,d}^m))^2}{\sum_{(x_{i,d}^m, y_{i,d}^a) \in sleep\_to\_athletic\_points^{m,a,n,b}} (y_{i,d}^a - \overline{y}^a)^2}$$

I perform these steps across the following hyperparameter search space.

| **Hyperparameter** | **Description** | **Parameter Search Space** |
|---|---|---|
| *lag* | The interval of nights prior to the date of the athletic testing | [1, 7] |
| *blended* | If $True$, perform a blended average of all nights of sleep that lie within the range $[testing\_date - lag, testing\_date - 1]$. If $False$ only use a single night's sleep on date $testing\_date - lag$ | {True, False} |

Table 5.1: $R^2$ Hyperparameter Search Space

## 5.2 Results

In Figure 5.1, I plot the maximum $R^2$ values by sleep and shooting feature. In Figure 5.3, I plot the maximum $R^2$ values by sleep and jumping feature. These maximum values are computed across the entire hyperparameter search space. In Figures 5.2 and 5.4, I select the shooting and jump athletic features with the maximum $R^2$ (corner right shooting percentage and jump average breaking force, respectively) and analyze performance across different lag values (colored by blended average).

I find all $R^2$ to be less than 0.18. This means that the strength of the correlation is weak in all instances, regardless of the sleep metric, athletic testing metric, or nightly lag. It is important to note that this is a preliminary analysis, and there may indeed exist strong relationships and correlations in the data that this analysis did not explore. Suggestions for areas of future exploration are provided in Section 6.2.

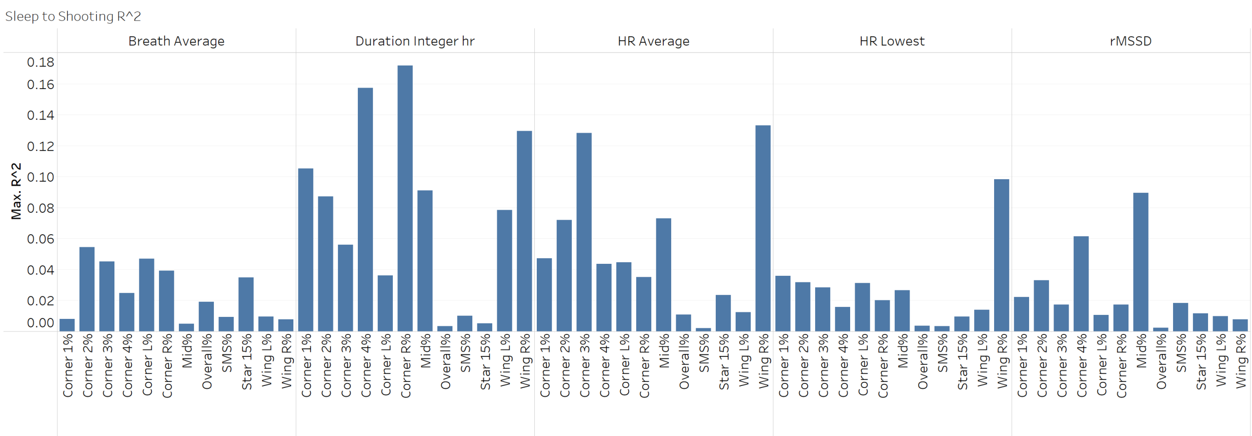


Figure 5.1: Shooting $R^2$ Results

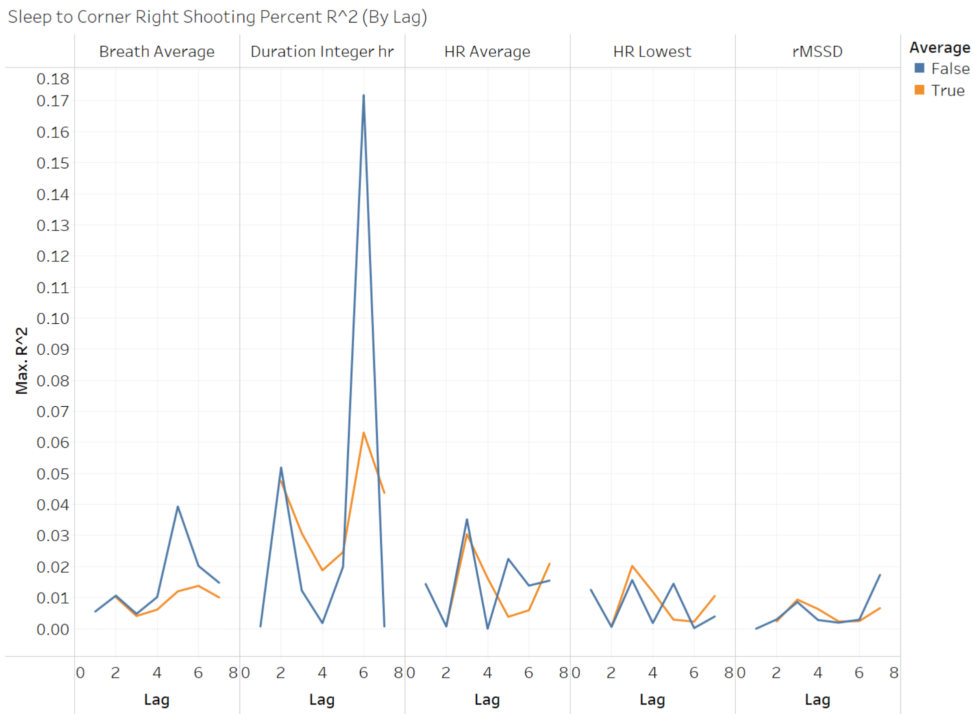


Figure 5.2: Corner Right Shooting Percent $R^2$ By Lag

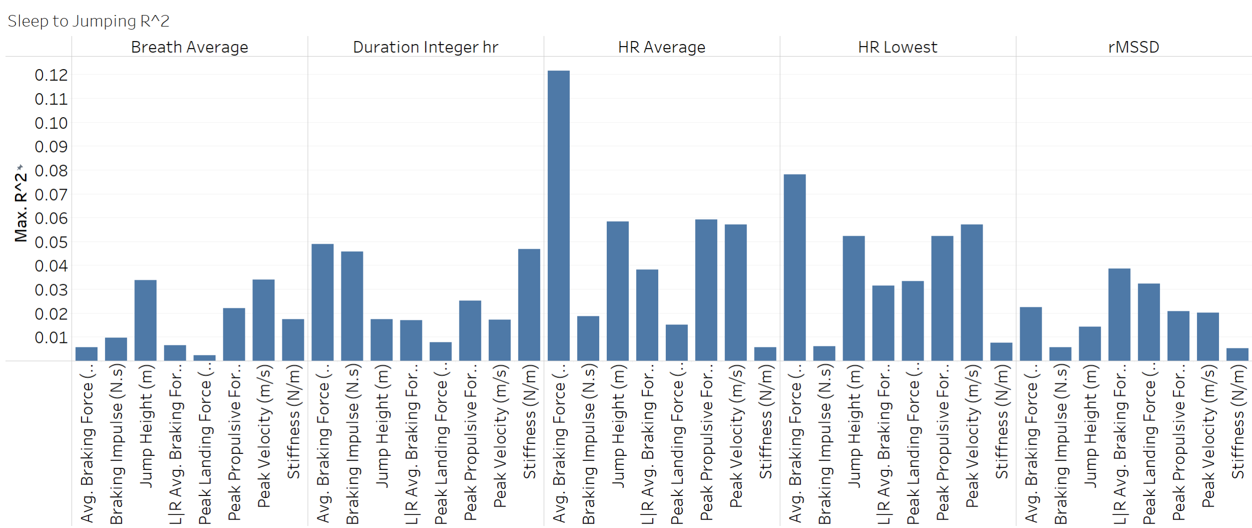


Figure 5.3: Jumping $R^2$ Results

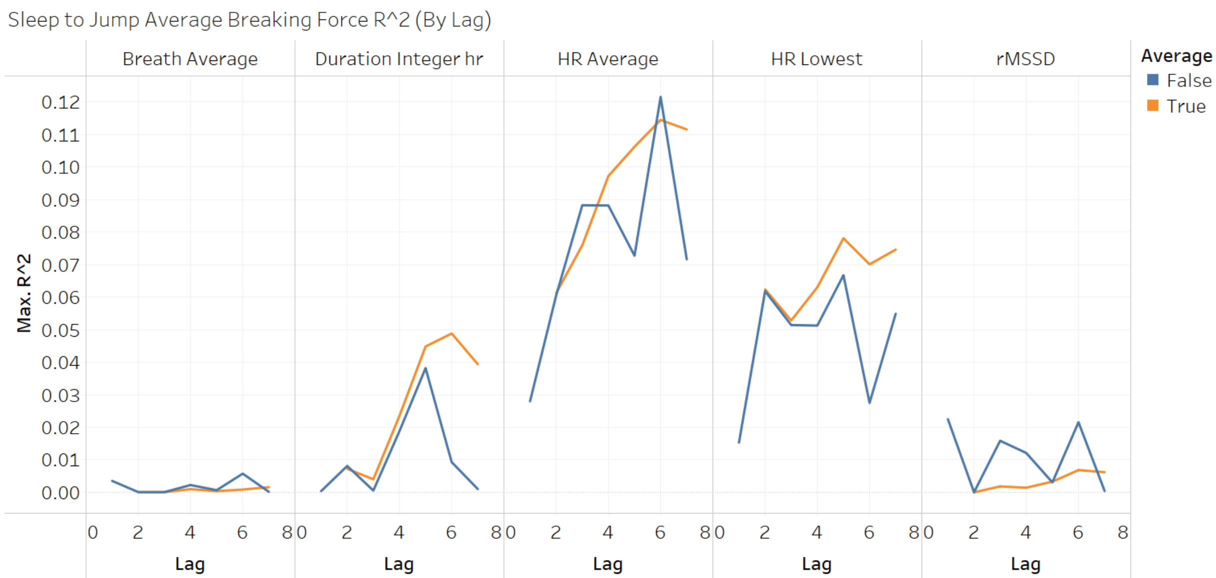


Figure 5.4: Jump Average Breaking Force $R^2$ By Lag

# Chapter 6

# Discussion & Future Work

## 6.1 Discussion

The findings of this thesis highlight the opportunities and challenges of working with sleep and athletic testing data. Among the tested sleep imputation methods, KNN Regression and Linear Interpolation stand out as performing well. They are particularly well-performing relative to Linear Regression and Quadratic Regression on low amounts of input data, such as on two input points. However, all methods struggle to perform well on Duration and rMSSD. This reveals the need to incorporate daytime activity, such as exercise intensity, into imputation models, which has been shown by Brand et al. to have a measurable impact on sleep duration and quality [20]. Additionally, these methods rely on consecutive nights of sleep being logged near the date of the missed night of sleep tracking. This does not generalize well to a real-world scenario where a player may have a long stretch of inconsistent sleep tracking.

The weak $R^2$ values between sleep tracking data and the shooting and jumping athletic testing data highlight the difficulty of performing causal analysis on features that have a complex interplay. However, this weak $R^2$ with existing data motivates alternative method exploration as well as alternative data collection processes (feature type and collection frequency).

## 6.2 Future Work

In the realm of imputation of the NBA Academy Sleep tracking dataset, there are several areas for future research:

1. **Hybrid Models for stretches of extreme sparsity**: The explored models were evaluated on nights where the sleep surrounding the target night is relatively consistent. A hybrid approach could combine the models explored in this thesis with a baseline model that performs well on sparse stretches of data.

2. **Day of Week Consideration**: The models in this thesis do not consider the day of the week, despite the fact that there are natural variations in sleep across different periods in the week (i.e. weekday vs. weekend).

3. **Multivariate Imputation**: Explored methods in this thesis are univariate. Extending into multivariate methods that capture the interplay of multiple sleep metrics is a promising avenue for future research.

4. **Day Time Activity Consideration**: Studying player activity during the day, such as what they ate and how hard they worked out, can add some important insight to predicting missed nights of sleep.

5. **Neural Approach**: Neural approaches (recurrent neural networks) were briefly considered for the sleep imputation task. However, this neural approach was not fully explored due to its apparent infeasibility with the current amount of data. As NBA Academy players log more data over time, neural approaches may prove promising as a means of imputation.

Considering the established relationship between sleep and athletic performance values, the weak $R^2$ values on sleep and athletic data seen in this thesis, I recommend areas for further exploration to formally establish this relationship on NBA Academy Data:

1. **Multivariate Autoregression [21]**: Multivariate autoregression allows for a more nuanced exploration of the interplay between sleep and athletic performance. For example, to predict athletic metric $a$ for player $i$ on night $n$ one would train a model for each sleep metric $m$ and a nightly lag:

$$a_{i,n} = \sum_{m \in m} \sum_{l \in lag} \alpha_{m,l} m_{i,n-l}$$

2. **Game and Injury Data**: Sleep may have a very large impact on a player's overall game performance, as well as any injuries they sustain over the course of games or training.

3. **More Stringent Regression Requirements**: The existing approach performs regression on any player that has at least 2 testing metrics after a night of sleep. As the NBA tracks more training sessions, this minimum requirement for inclusion can be increased to only include players with sufficient sleep and testing data.

# Chapter 7

# Conclusion

In collaboration with the NBA and its international Academy programs, this thesis explored two main questions:

1. What is the best model to impute missing nights of sleep tracking data?

2. What is the relationship between sleep and athletic performance?

To answer question 1, the thesis evaluates 4 main models: KNN Regression, Linear Interpolation, Linear Regression, and Quadratic Regression. These methods were evaluated using a robust implementation framework that allowed for proper model comparisons across a variety of hyperparameters. These hyperparameters explored scenarios of differing numbers and proximity of input nights as well as consideration of online vs. offline model performance. This research finds KNN Regression and Linear Interpolation to be the best-performing methods. These methods perform best with access to data in the past and future (offline imputation).

Question 2 was researched by computing the Coefficient of Determination, $R^2$, across a wide search space of different nightly lags (point and blended) as well as across different sleep and athletic testing (jump and shooting) data. Ultimately, only weak values of $R^2$ were found, highlighting the importance of more nuanced methods such as multivariate autoregression and further data collection to better study any causal effects between sleep and athletic testing amongst NBA academy players.

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