

# Sample size matters when calculating Pillai scores

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### Pillai Scores (Pillai 1955)

What is a Pillai score?

- Output of a MANOVA model ►
- 0 suggests complete overlap
- 1 suggests complete separation

```
# R code
m <- manova(cbind(F1, F2) ~ vowel)
summary <- summary(m)
stats <- summary$stats
stats[1, "Pillai"]
```

How are they used in sociophonetics?

- Quantify mergers (Hay, Warren, & Drager 2006, Fridland et al. 2014, Amengual & Chamorro 2015, Nadeu & Renwick 2016, Tse 2018, Gonzales & Starr 2020, Kettig 2021, etc.)
- Quantify splits (Babel et al. 2013, Fisher et al. 2015, Berry & Ernestus 2018)

Are they good measures?

- Often supported in meta-analyses (Nycz & Hall-Lew 2013, Kelley & Tucker 2020)
  - Appear to be better than Euclidean distance, mixed-effects regression, spectral overlap, SOAM, VOACH, APP, etc.
- However, Bhattacharyya's Affinity may be better suited for vowel data since it's robust to skewed data. (Johnson 2015)

### Issues

#### 1. What is considered merged?

- $p$ -values from MANOVAs inconsistently reported.
  - (Especially in American English-based studies...)
- Ad hoc thresholds are inconsistent across studies.

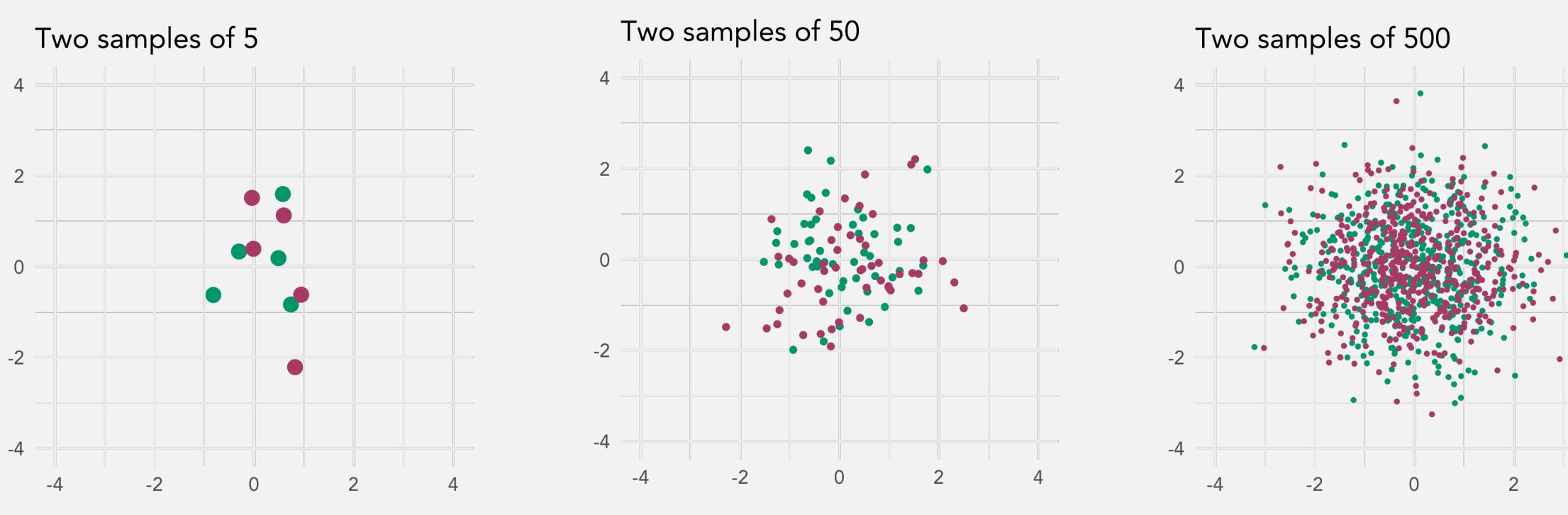
#### 2. Sample size

- Typically not considered when comparing Pillai scores between studies or between individuals in the same study.

### Methods

#### Data Generation

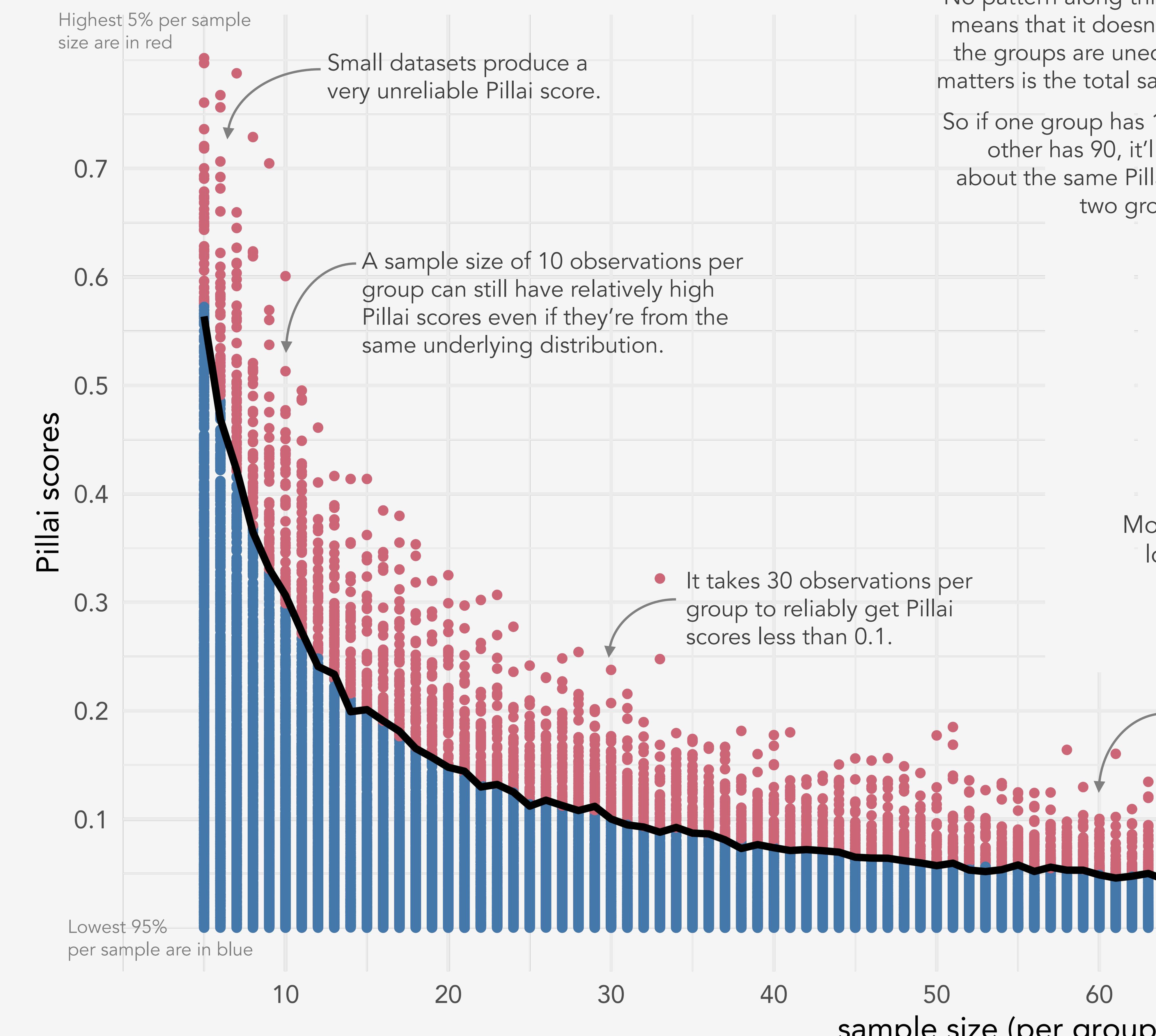
- Two samples drawn from the same bivariate normal distribution.
- In theory, the Pillai score should be 0 because they're pulled from literally the same underlying distribution.



#### Two Experiments

- Simulated sample sizes from 5 to 100
  - Experiment 1: two distributions were equally sized
  - Experiment 2: two distributions were unequally sized
- Repeated this 1000 times for each sample size.
  - Produced 921,600 pairs of distributions total.
  - Calculated the Pillai scores of each pair.

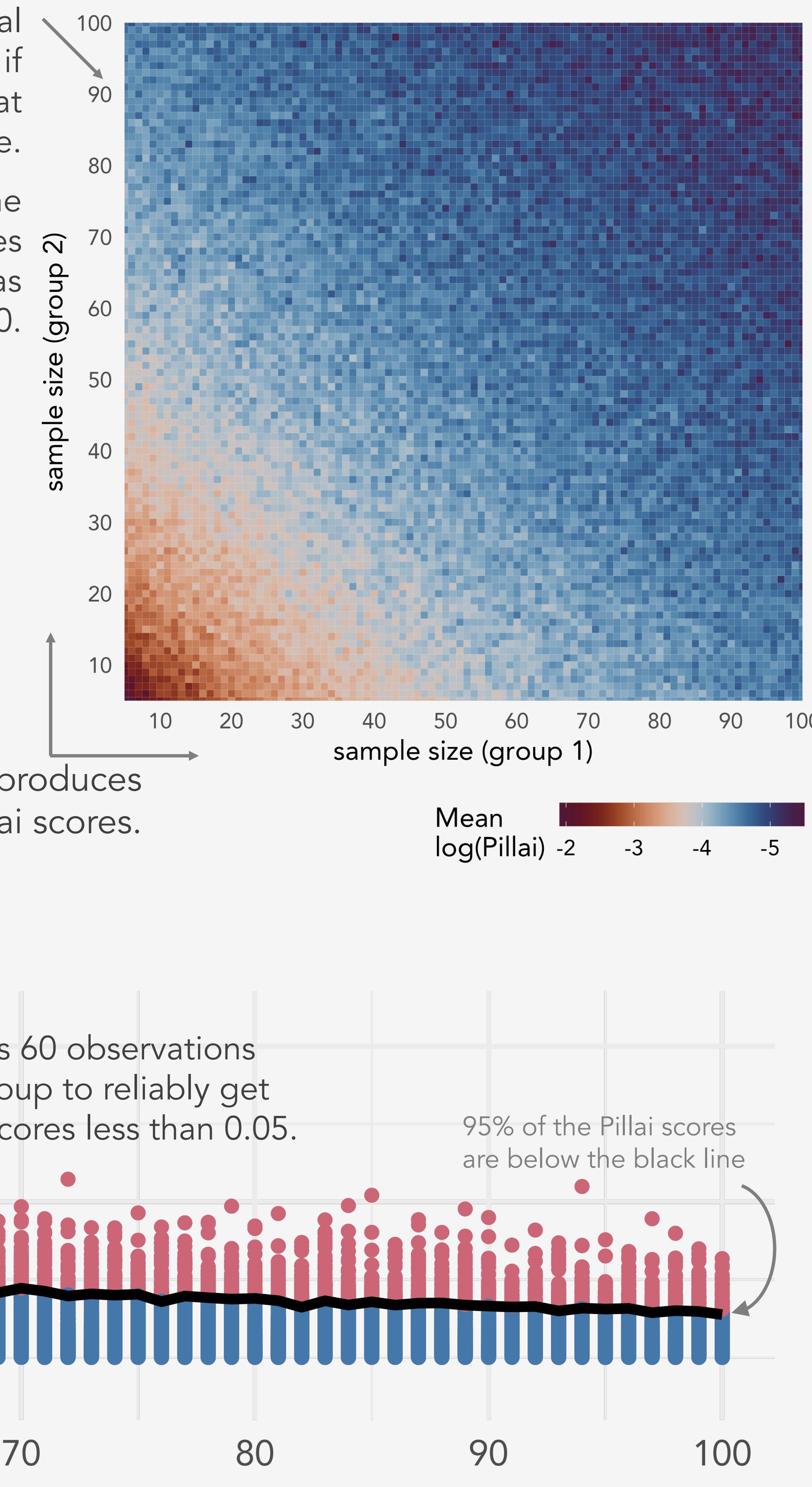
### Experiment 1: Equal Sample Sizes



### Experiment 2: Unequal Sample Sizes

No pattern along this diagonal means that it doesn't matter if the groups are unequal. What matters is the total sample size.

So if one group has 10 and the other has 90, it'll produce about the same Pillai score as two groups of 50.



More data produces lower Pillai scores.

95% of the Pillai scores are below the black line

### Implications

#### Mergers are probably underreported and separation is probably overreported

- It takes a lot of data (more than many studies use) to get reliably low Pillai scores
- Mergers may be more common previously reported

#### Statistical significance should be reported

- Reporting  $p$ -values from MANOVA tests removes the need for ad hoc thresholds, puts less weight on Pillai scores, and makes interpretation more objective.

#### Comparison across studies

- Analyses of speakers with less data will look *less merged*
- Analysis of speakers with more data will look *more merged*

#### Comparison within studies

- Speakers with less data will have inflated Pillai scores.
- Reading tasks will have higher scores than conversational data (which will be interpreted as style differences)

### References

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