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# Exercises from Sec.8.1 - 8.3

## Table of Contents

|                                    |    |
|------------------------------------|----|
| Exercises from Sec.8.1 - 8.2 ..... | 1  |
| 1.1 .....                          | 1  |
| 1.2 .....                          | 2  |
| 2.1 .....                          | 3  |
| 2.2 .....                          | 4  |
| 3.1 .....                          | 5  |
| 3.2 .....                          | 6  |
| Exercises from Sec.8.3 .....       | 8  |
| 1.1 .....                          | 8  |
| 1.2 .....                          | 10 |
| 1.3 .....                          | 12 |
| 1.4 .....                          | 14 |

## Exercises from Sec.8.1 - 8.2

### 1.1

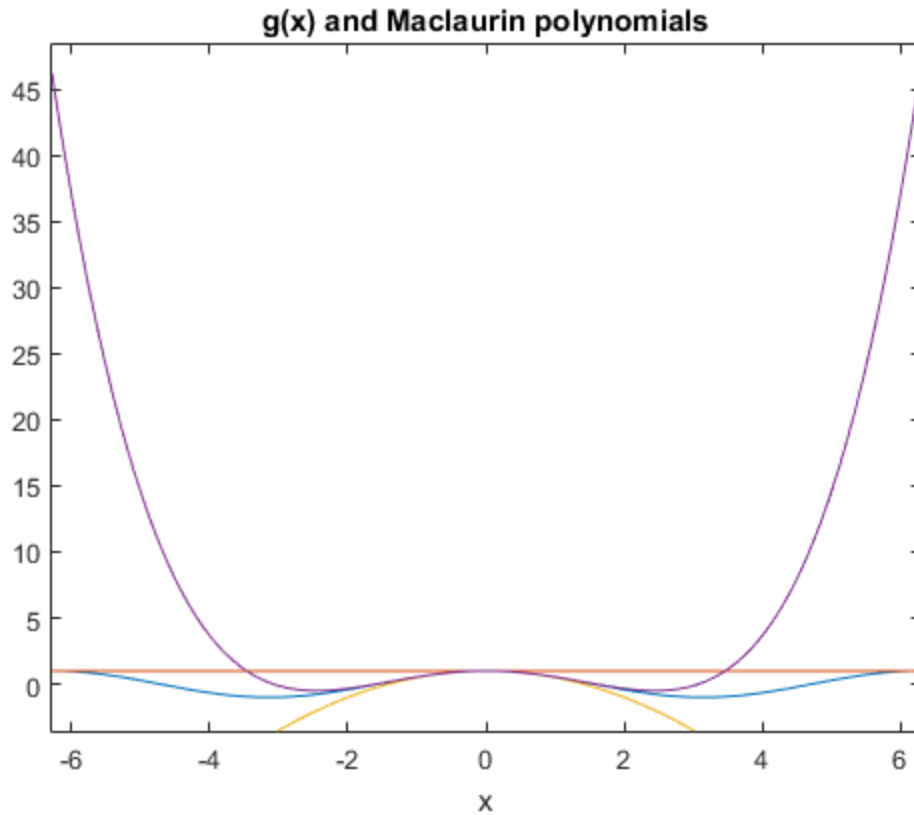
```
clear
clc

syms x
g = @(x) cos(x);

ezplot(g);
hold on

for i = 2 : 2 : 6
    taylorPoly = taylor(g, x, 0, 'Order', i);
    ezplot(taylorPoly);
end

title('g(x) and Maclaurin polynomials');
hold off
```



## 1.2

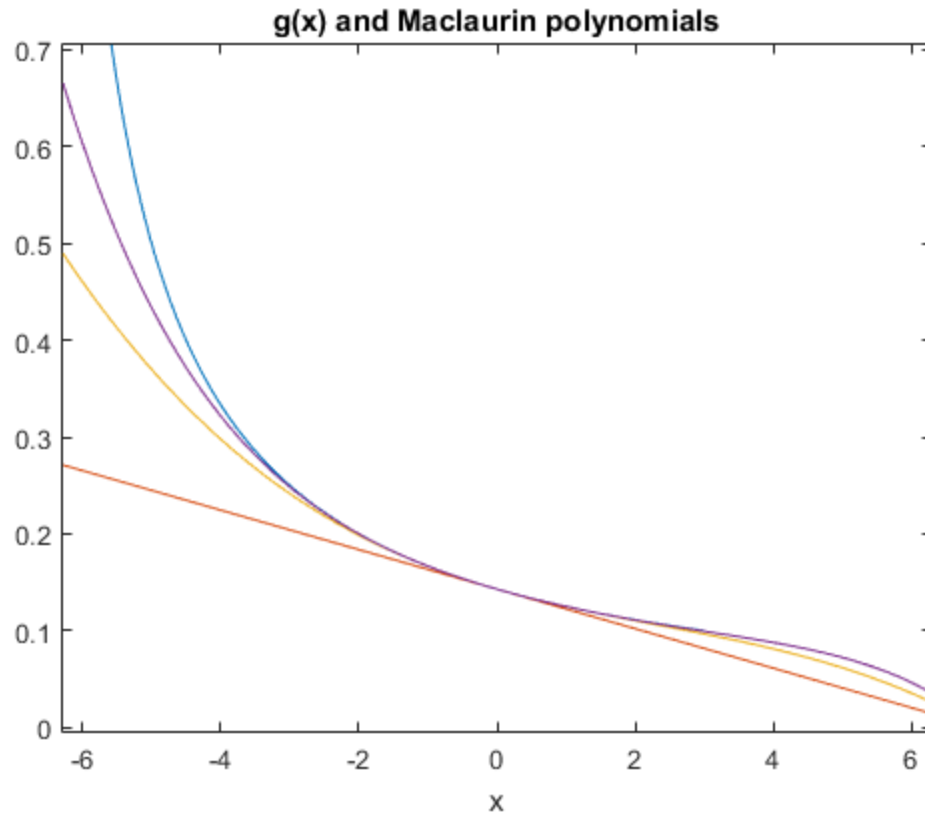
```
clear

syms x
g = @(x) 1./(x+7);

ezplot(g);
hold on

for i = 2 : 2 : 6
    taylorPoly = taylor(g, x, 0, 'Order', i);
    ezplot(taylorPoly);
end

title('g(x) and Maclaurin polynomials');
hold off
```



## 2.1

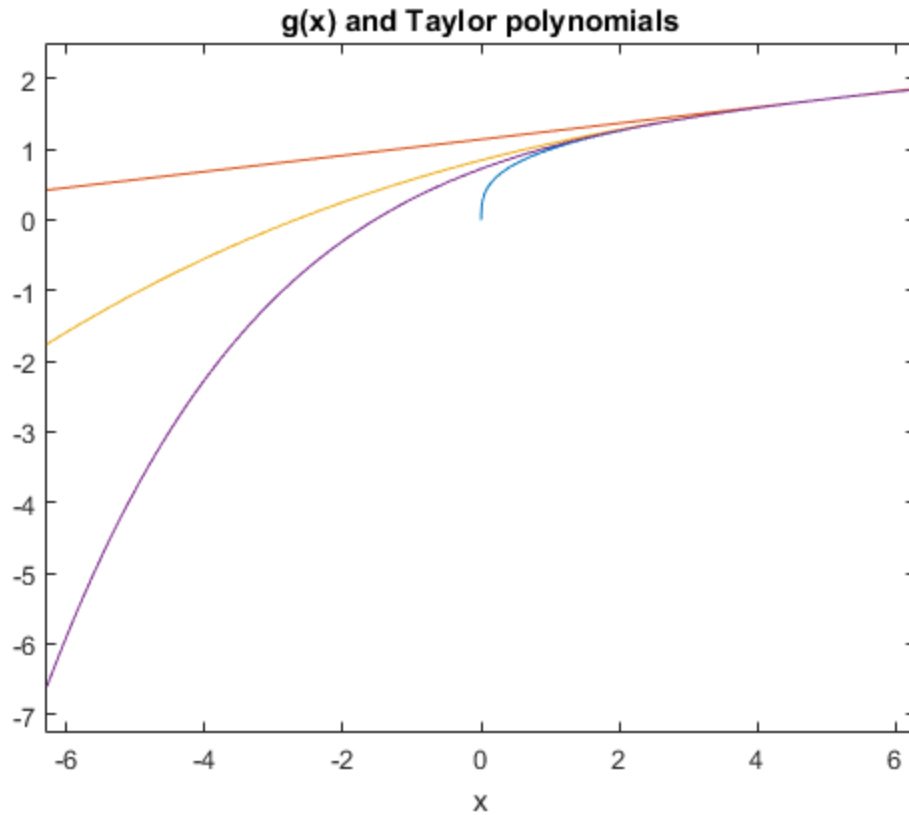
```
clear

syms x
g = @(x) x.^(1/3);

ezplot(g);
hold on

for i = 2 : 2 : 6
    taylorPoly = taylor(g, x, 5, 'Order', i);
    ezplot(taylorPoly);
end

title('g(x) and Taylor polynomials');
hold off
```



## 2.2

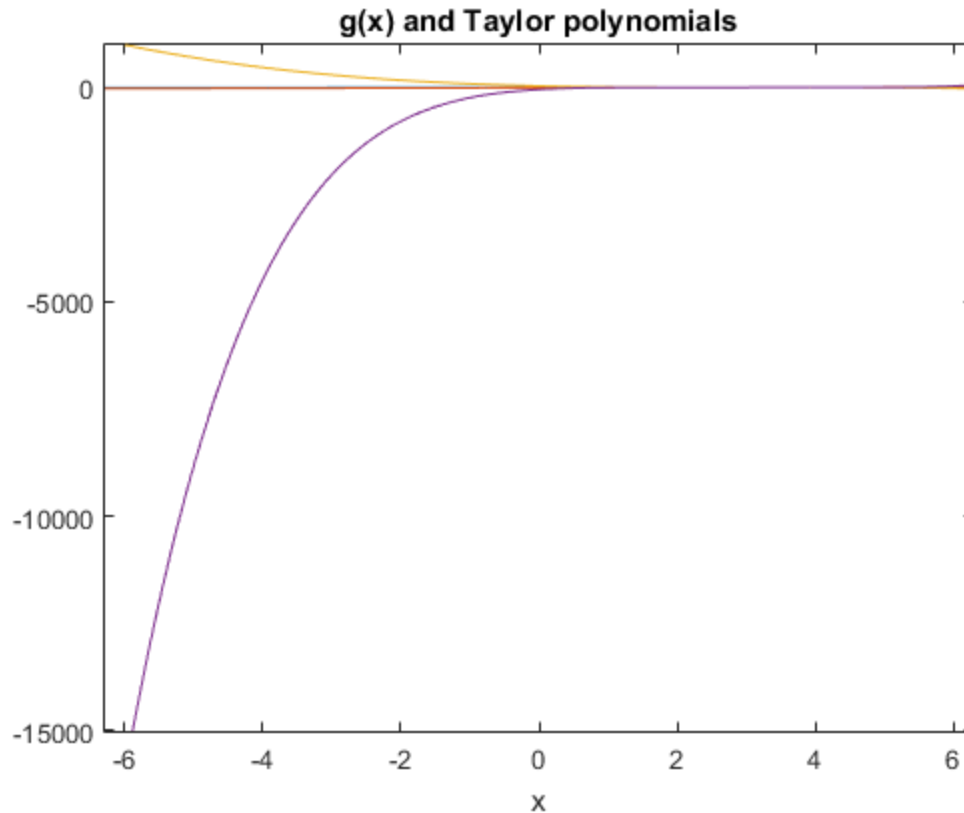
```
clear

syms x
g = @(x) sin(2*x) + x;

ezplot(g);
hold on

for i = 2 : 2 : 6
    taylorPoly = taylor(g, x, pi, 'Order', i);
    ezplot(taylorPoly);
end

title('g(x) and Taylor polynomials');
hold off
```



## 3.1

```
clear
syms fEuler1 fEuler2 dfdt1 dfdt2 x

tI = 0;
tF = pi/2;
dt = pi/8;
fInitialValue = 1;
times = tI:dt:tF;

fEuler1(1) = fInitialValue; % first-order Euler method
fEuler2(1) = fInitialValue; % second-order Euler method

for i = 1:1:(length(times)-1)
    dfdt1(i) = sin(times(i));
    dfdt2(i) = cos(times(i));
    fEuler1(i+1) = fEuler1(i) + dfdt1(i) * dt;
    fEuler2(i+1) = fEuler2(i) + dfdt1(i) * dt + (1/2)*dfdt2(i)*(dt^2);
end

double(fEuler1(length(times))) % first order approximation of f(pi/2)
double(fEuler2(length(times))) % second order approximation of
    f(pi/2)
int('sin(x)',x,0,pi/2) + fInitialValue % exact value of f(pi/2)
```

```
plot(times,fEuler1)
hold on
plot(times,fEuler2)
hold off
```

```
ans =
```

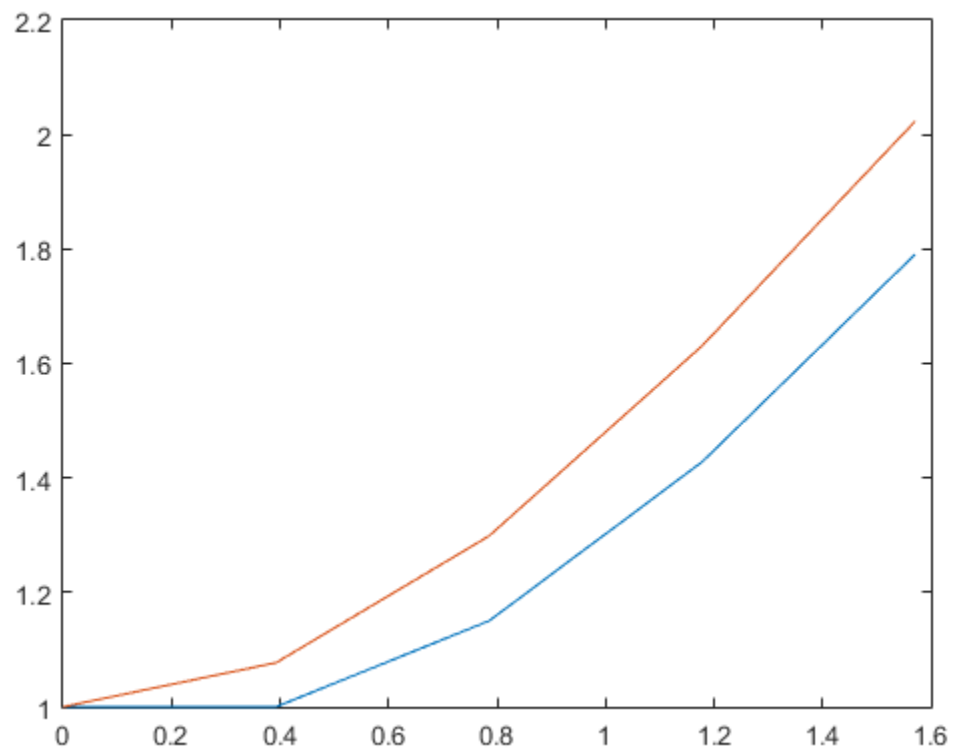
```
1.7908
```

```
ans =
```

```
2.0231
```

```
ans =
```

```
2
```



## 3.2

```
clear
syms fEuler1 fEuler2 dfdt1 dfdt2 x
```

```
tI = 0;
tF = 16;
dt = 4;
fInitialValue = 1;
times = tI:dt:tF;

fEuler1(1) = fInitialValue; % first-order Euler method
fEuler2(1) = fInitialValue; % second-order Euler method

for i = 1:1:(length(times)-1)
    dfdt1(i) = 1/(1+times(i));
    dfdt2(i) = -1/(1+times(i))^2 ;
    fEuler1(i+1) = fEuler1(i) + dfdt1(i) * dt;
    fEuler2(i+1) = fEuler2(i) + dfdt1(i) * dt + (1/2)*dfdt2(i)*(dt^2);
end

double(fEuler1(length(times))) % first order approximation of f(16)
double(fEuler2(length(times))) % second order approximation of f(16)
double(int('1/(1+x)',x,0,16) + fInitialValue) % exact value of f(16)

plot(times,fEuler1)
hold on
plot(times,fEuler2)
hold off

ans =

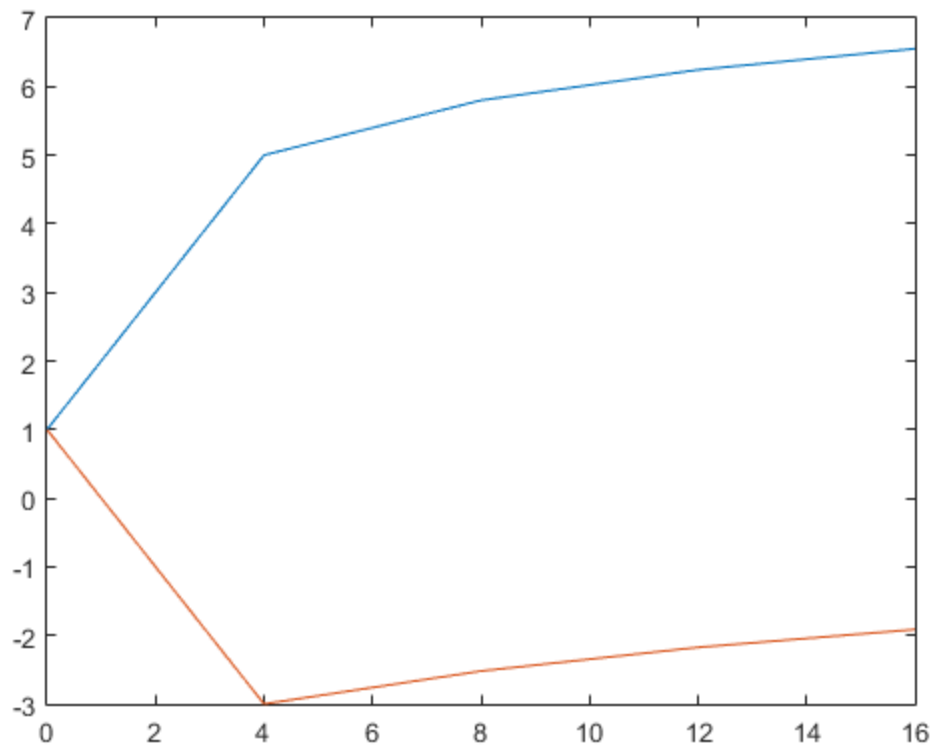
    6.5521

ans =

   -1.9140

ans =

    3.8332
```



## Exercises from Sec.8.3

### 1.1

```
clc
clear
syms fEuler1 fEuler2 dfdt1 dfdt2

tI = 0;
tF = 3;
dt = 3/5;
fInitialValue = 1;
times = tI:dt:tF;

fEuler1(1) = fInitialValue; % first-order Euler method
fEuler2(1) = fInitialValue; % second-order Euler method

for i = 1:1:(length(times)-1)
    dfdt1(i) = times(i) + fEuler1(i);
    fEuler1(i+1) = fEuler1(i) + dfdt1(i) * dt;
end

for i = 1:1:(length(times)-1)
    dfdt1(i) = times(i) + fEuler2(i);
    dfdt2(i) = 1 + dfdt1(i);
```



```
fEuler2(i+1) = fEuler2(i) + dfdt1(i) * dt + (1/2)*dfdt2(i)*(dt^2);
end

double(fEuler1(length(times))) % first order approximation of f(3)
double(fEuler2(length(times))) % second order approximation of f(3)
g = dsolve('Df=t+f','f(0)=1'); % solve the diff-equ using dsolve
f = inline(g);
f(3)

plot(times,fEuler1)
hold on
plot(times,fEuler2)
ezplot(f,times)
hold off

ans =

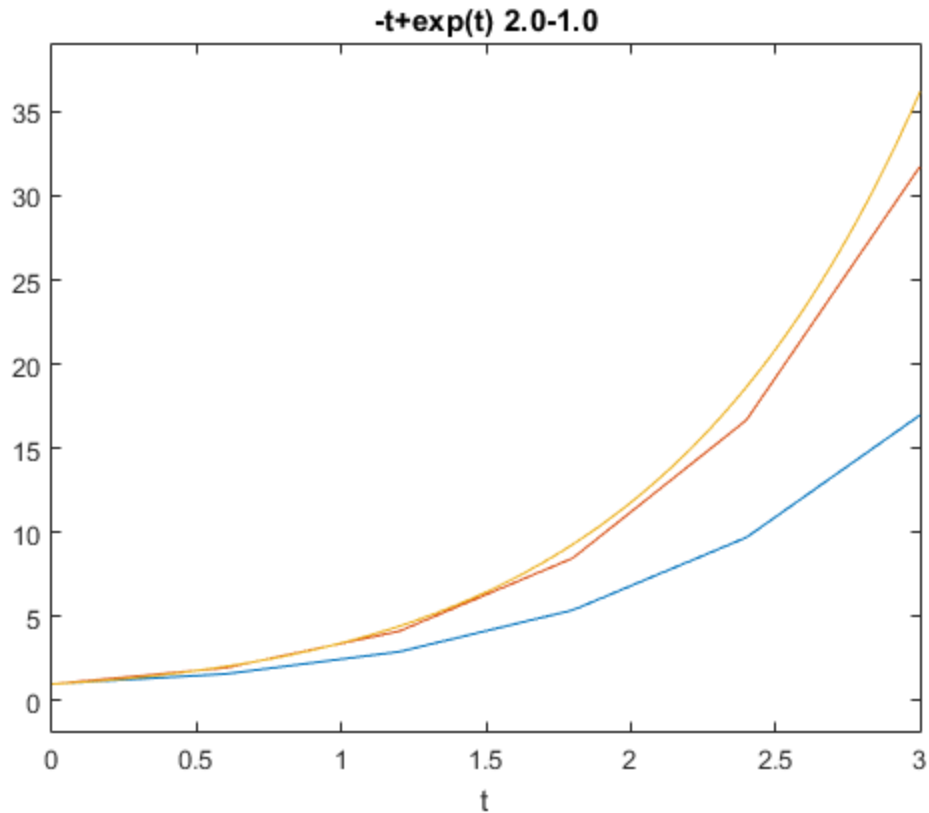
    16.9715

ans =

    31.7380

ans =

    36.1711
```



## 1.2

```

clc
clear
syms fEuler1 fEuler2 dfdt1 dfdt2

tI = 0;
tF = 3;
dt = 3/5;
fInitialValue = 1;
times = tI:dt:tF;

fEuler1(1) = fInitialValue; % first-order Euler method
fEuler2(1) = fInitialValue; % second-order Euler method

for i = 1:1:(length(times)-1)
    dfdt1(i) = times(i) - fEuler1(i);
    fEuler1(i+1) = fEuler1(i) + dfdt1(i) * dt;
end

for i = 1:1:(length(times)-1)
    dfdt1(i) = times(i) - fEuler2(i);
    dfdt2(i) = 1 - dfdt1(i);
    fEuler2(i+1) = fEuler2(i) + dfdt1(i) * dt + (1/2)*dfdt2(i)*(dt^2);
end

```

```
double(fEuler1(length(times))) % first order approximation of f(3)
double(fEuler2(length(times))) % second order approximation of f(3)
g = dsolve('Df=t-f','f(0)=1'); % solve the diff-equ using dsolve
f = inline(g);
f(3)
```

```
plot(times,fEuler1)
hold on
plot(times,fEuler2)
ezplot(f,times)
hold off
```

```
ans =
```

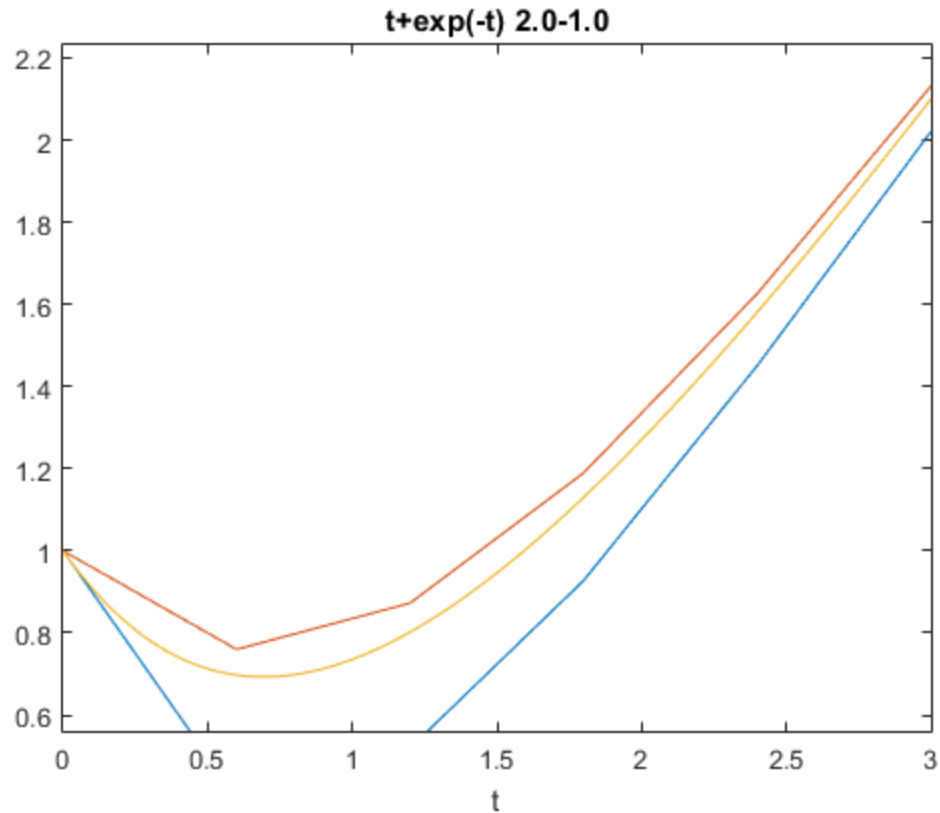
```
2.0205
```

```
ans =
```

```
2.1313
```

```
ans =
```

```
2.0996
```



## 1.3

```

clc
clear
syms fEuler1 fEuler2 dfdt1 dfdt2

tI = 0;
tF = 3;
dt = 3/5;
fInitialValue = 1;
times = tI:dt:tF;

fEuler1(1) = fInitialValue; % first-order Euler method
fEuler2(1) = fInitialValue; % second-order Euler method

for i = 1:1:(length(times)-1)
    dfdt1(i) = times(i) / fEuler1(i);
    fEuler1(i+1) = fEuler1(i) + dfdt1(i) * dt;
end

for i = 1:1:(length(times)-1)
    dfdt1(i) = times(i) / fEuler2(i);
    dfdt2(i) = (fEuler2(i)-times(i)*dfdt1(i))/(fEuler2(i)^2);
    fEuler2(i+1) = fEuler2(i) + dfdt1(i) * dt + (1/2)*dfdt2(i)*(dt^2);
end

```

```
double(fEuler1(length(times))) % first order approximation of f(3)
double(fEuler2(length(times))) % second order approximation of f(3)
g = dsolve('Df=t/f','f(0)=1'); % solve the diff-equ using dsolve
f = inline(g);
f(3)
```

```
plot(times,fEuler1)
hold on
plot(times,fEuler2)
ezplot(f,times)
hold off
```

```
ans =
```

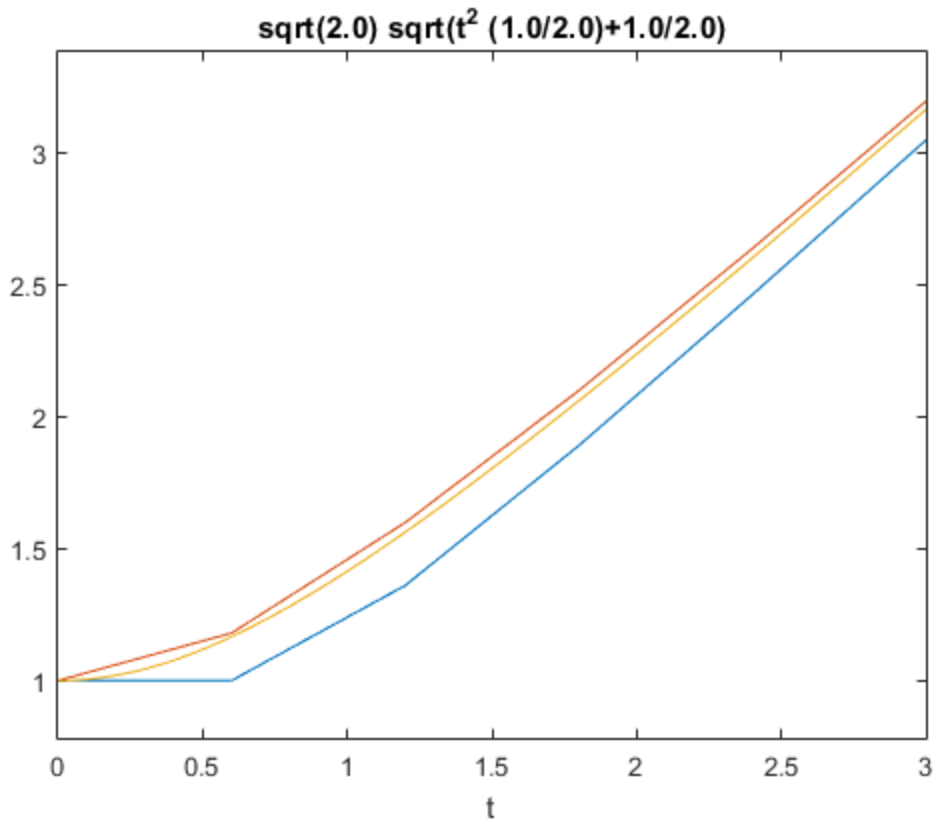
```
3.0461
```

```
ans =
```

```
3.1934
```

```
ans =
```

```
3.1623
```



## 1.4

```

clc
clear
syms fEuler1 fEuler2 dfdt1 dfdt2

tI = 0;
tF = 3;
dt = 3/5;
fInitialValue = 1;
times = tI:dt:tF;

fEuler1(1) = fInitialValue; % first-order Euler method
fEuler2(1) = fInitialValue; % second-order Euler method

for i = 1:1:(length(times)-1)
    dfdt1(i) = fEuler1(i)^(1/2);
    fEuler1(i+1) = fEuler1(i) + dfdt1(i) * dt;
end

for i = 1:1:(length(times)-1)
    dfdt1(i) = fEuler2(i)^(1/2);
    dfdt2(i) = (1/2) * fEuler2(i)^(-1/2) * dfdt1(i);
    fEuler2(i+1) = fEuler2(i) + dfdt1(i) * dt + (1/2)*dfdt2(i)*(dt^2);
end

```

```
double(fEuler1(length(times))) % first order approximation of f(3)
double(fEuler2(length(times))) % second order approximation of f(3)
g = dsolve('Df=f^(1/2)', 'f(0)=1'); % solve the diff-equ using dsolve
f = inline(g(1));
f(3)
```

```
plot(times,fEuler1)
hold on
plot(times,fEuler2)
ezplot(f,times)
hold off
```

```
ans =
```

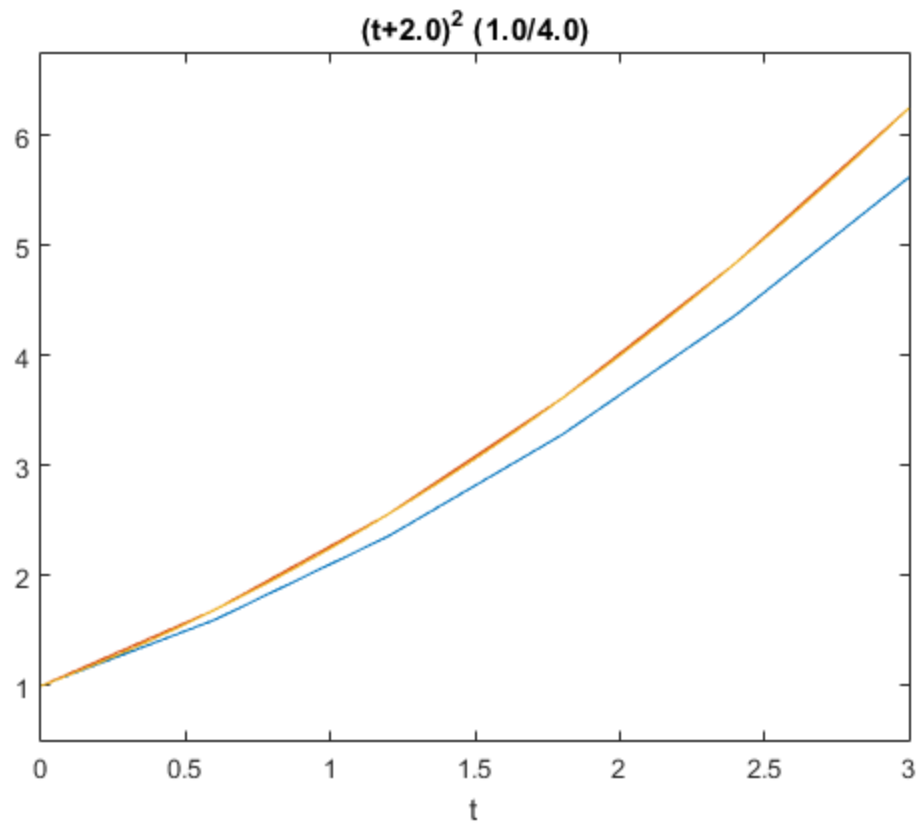
```
5.6211
```

```
ans =
```

```
6.2500
```

```
ans =
```

```
6.2500
```



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