Pricing and Forwarding Games for Inter-domain Routing

Full version

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***Abstract*—We address the question of strategic pricing of inter-domain traffic forwarding services provided by ISPs, which is also closely coupled with the question of how ISPs route their traffic towards their neighboring ISPs. Posing this question as a non-cooperative game between neighboring ISPs, we study the properties of this pricing game in terms of the existence, computability, and efficiency of the equilibrium. We observe that for "well-provisioned" ISPs, Nash equilibrium prices exist and they result in flows that maximize the overall network utility (generalized end-to-end throughput). For general ISP topologies, equilibrium prices may not exist; however, simulations on a large number of realistic topologies show that best-response based simple price update solutions converge to stable and efficient prices and flows for most topologies. Since our model follows a destination-specific, next-hop routing framework, these solutions can be useful for pricing-based inter-domain traffic engineering, requiring modest changes to existing path-vector inter-domain routing protocols that are used in the current Internet.**

I. Introduction

Over the last few decades, the Internet has penetrated ubiquitously into nearly all aspects of our life. However, while the number and variety of applications that run over the Internet have grown tremendously, its underlying architectural components and protocols have not changed significantly. Reliance on simple traffic routing algorithms like shortest path implies that network resources are not optimally used. There has been much research activity on intra-domain traffic engineering in the recent past [18]–[20], which are likely to gradually make their way into practice. In contrast, inter-domain routing/traffic engineering practices have not changed much, and the topic also remains less researched. One of the difficulties in doing inter-domain traffic engineering is that it requires involvement of (agreement by) multiple Internet Service Providers (ISPs) that are interested solely in maximizing their own profits, and their individual objectives are often in conflict with each other. Moreover, inter-domain traffic engineering issues are also closely related to how traffic forwarding services are priced at the inter-domain level. The current inter-domain traffic routing and pricing strategies are typically policy driven, and do not in general optimize the overall use of Internet resources, or maximize ISP profits. Inter-domain routing follows the BGP standard [11] that only attempts to minimize the number of ISP hops on the path of a flow (in addition to following local policy considerations and certain heuristic rules). Traffic exchange contracts between ISPs are still negotiated manually and span months if not years. These contracts when finally resolved are often simplistic in nature, and charges are typically based on the total traffic offered by the customer ISP irrespective of their destination (point-to-anywhere pricing) [16]. It has been argued correctly that point-to-point or destination-based pricing can result in better efficiency of network resource usage [15]. Moreover, recent measurements also show fast changing patterns for inter-domain traffic [17]; inter-domain service pricing and traffic engineering solutions need to be flexible enough to adapt to such dynamically changing traffic patterns, and variations in customer requirements and willingness-to-pay values.

A. Focus of this Work

The broad question we address in this paper involves determining how inter-domain traffic forwarding services should be priced. More specifically, this involves finding the "right" price that ISPs should charge for the traffic forwarding services they provide to their upstream neighbors, typically done through SLA agreements (short- or long-term) formed between neighboring ISPs or Autonomous Systems (ASes) in the Internet. We study these questions in the framework of destination-based, next-hop routing, which is well aligned with the inter-domain routing strategies as they exist in the current Internet. In other words, each ISP individually determines which next ISP hop it would forward its traffic to (possibly splitting its traffic across multiple downstream ISPs), and the forwarding as well as the pricing decisions are destination-specific. Note that an ISP obtains payments from its upstream neighbors for accepting their traffic for forwarding, and must in turn pay its downstream neighbors for receiving its traffic (i.e., the traffic it has accepted to forward, plus the traffic that originates within its own network). As ISPs are generally interested in maximizing their individual profits, this question is naturally studied in the framework of non-cooperative game theory, as we do in this work. Note that the pricing solution is also closely related to the forwarding paths of the flows, since in order to maximize their profits, ISPs will choose to forward their flows towards the next-hop ISPs that offer to send the traffic to the destination at the cheapest price. This results in an interesting dependency between that of setting the prices for the upstream neighboring ISPs, and the forwarding of traffic to the downstream neighboring ISPs. This interplay between the pricing and forwarding issues is nicely captured in the strategic pricing question for ISPs that we consider in this paper. The issues that are of interest in this context are whether there exist prices that constitute an *equilibrium*, i.e. prices which the ISPs (acting out of of self-interest) are not likely to deviate from, and whether such prices can be computed efficiently and result in efficient use of the network. Another issue worth investigating is whether stable and efficient prices can be obtained through distributed pricing negotiations and traffic forwarding policies, that can be implemented with modest changes to existing inter-domain routing policies and message exchange protocols. These are the questions that we study in this paper.

B. Contribution and Significance

We introduce and analyze (both theoretically and in simulation) a model of dynamic strategic pricing between ISPs, which is also closely integrated to the question of inter-domain traffic engineering. We model the Internet as an interconnected system of ISPs.[1] We represent an ISP as a *cloud*, that is composed of a set of nodes (routers) and capacitated links connected into an arbitrary topology; a subset of these nodes represents the ISP's ingress and egress nodes. At an ingress node, an ISP offers to its neighboring (upstream) ISP a price for forwarding traffic to a destination, and accepts the traffic to be forwarded. The ISP also derives utility for forwarding the traffic generated within its own network, and must decide how to forward (split) its traffic (that is offered to it by upstream neighbors, or generated inside its network) among its downstream neighbors (egress points). ISPs are assumed non-cooperative in nature, and pricing and traffic forwarding decisions are made such that they maximize the individual ISP's profit.

For this strategic pricing game for inter-domain traffic forwarding, we obtain the following theoretical results in this paper. First, we show that breaking ties appropriately, when identical prices are offered by multiple neighboring ISPs, is crucial: if done correctly this leads to the existence of good Nash equilibria, but if done incorrectly no equilibrium may exist. We further give a partial characterization of Nash equilibria with nice structure (which we call *uniform* Nash equilibria), and use the results from [1] to show how to compute (and the existence of) a Nash equilibrium as good as the optimum centralized solution for the case when the network is *well-provisioned*. By *well-provisioned*, we mean that bottlenecks only occur on inter-ISP links, and thus the inside of every cloud has enough capacity to support whatever traffic may be required (see Section III for a precise definition).

While most of our theoretical results are for well-provisioned networks, our simulations show that our model still behaves extremely well even without this assumption. We observe that simple price dynamics converge to stable solutions in an overwhelming majority of our simulations. Moreover, the solutions that price dynamics converge to are not only stable, but have excellent social welfare (overall utility), often within a few percent of the social optimum.

Our theoretical results on the pricing question for inter-domain forwarding are quite significant, as they show that (at least for well provisioned ISPs) there exists a a pricing strategy for inter-domain forwarding that is both *stable* and *efficient*. Adoption of such a pricing strategy would imply that ISPs acting in self-interest will not unilaterally deviate from such pricing. Additionally, such a pricing strategy maximizes aggregate end-to-end throughput (utility), or in other words, makes optimal use of global network resources. Thus use of such a pricing strategy leads to a desirable operating point for the Internet. Our dynamic pricing update policies, that have been observed to quickly converge and result in near-optimal flows (for most topologies), could be used to obtain these prices dynamically in a fully distributed manner. The destination-specific, next-hop nature of our pricing and forwarding framework and policies imply that these can be implemented with minor changes to the path-vector routing framework that constitute the current Internet inter-domain routing practices. Used in this manner, the proposed pricing and flow routing policies could be used for pricing based dynamic inter-domain traffic engineering in the Internet. Our model and results make very loose assumptions on the *intra*-domain routing policy used, and allows a wide variety of intra-domain, packet routing (traffic engineering) policies to be used, which may even differ across domains (ASes or ISPs).

C. Paper Organization

The rest of the paper is structured as follows. In Section III, we describe the system model, define the strategic pricing-forwarding question for ISPs as a non-cooperative game, and include the assumptions that are broadly needed to make the game meaningful. In Section IV, we state and prove the main results from our equilibrium analysis of this game. In Section V, we study through simulations the convergence and flow optimality of some simple price negotiation/update strategies. We conclude in Section VI.

II. Related Work

Over the last two decades, there has been a significant amount of research in traffic engineering issues. However, most of this work is on optimizing routes and traffic flows within a *single* domain (see [18]–[20] for some of the prominent solutions approaches), towards attaining high throughput or utilization in the network and/or minimize congestion in the most overloaded links. For inter-domain routing/traffic engineering, much literature has been devoted to instability

[1] Throughout the paper, we use the terms "domain", "AS" and "ISP" interchangeably. While the terms domain and ISP are often used loosely, these terms are not strictly equivalent. For example, an ISP can control multiple ASes, while there can be customer ASes that are not a part of any ISP [41]. In our framework/model, these terms represent a generic network "cloud" that is under a single administrative control, and the basic unit over which routing/traffic engineering and forwarding pricing decisions are taken.

issues of BGP, and the choice of stable routes for path-vector routing based protocols like BGP (see [21]–[23], [28] for examples). Research in inter-domain traffic engineering has gained momentum over the last decade, although the bulk of this work has been devoted to inter-domain traffic engineering solutions in the context of BGP, through intelligent use of some of its parameters/flexibilities. [24], [25] discuss methods for doing inter-domain traffic engineering by careful control of BGP route advertisements, while [30] discuss how the BGP AS-Path attribute can be manipulated for that purpose. General guidelines for traffic engineering within the context of BGP are discussed in [26]. [31] discusses an extension to BGP to enable multipath inter-domain routing. A cooperative optimization-based approach to inter-domain traffic engineering based on dual decomposition and Nash bargaining is described in [27]. A symbiotic optimization based inter-domain traffic engineering approach is discussed in [29]. Unlike our work, the above line of work on inter-domain traffic engineering does not take into account strategic pricing by different ISPs/domains or study the inter-domain traffic engineering question from a game-theoretic perspective. A related work [2] studies flow equilibrium efficiency with optimal routing at the intra-domain level and selfish routing across domains; the pricing competition question across domains (ISPs) is however not considered in that work.

Technically, our work is related to the growing body of literature on strategic pricing of bandwidth/flow/services over the Internet, although most of the existing work either uses restrictive network topology models, and/or focuses on "source routing" and pricing game models [9]. In these models, the source node of the traffic determines its entire route from the source to the destination. The efficiency of selfish flow routing by users is considered in [12]. The flow and price control leader-follower game between a set of users (follower) and an ISP (leader) – which controls a link, or a set of tandem links – is studied in [34], [35]. Flow and pricing competition in single-source single-sink parallel-serial networks, where the network consists of a set of parallel links (paths) controlled by profit-maximizing ISP(s), and selfish users control the flows on these paths in response to the path prices, is studied in [6], [32], [33]. This problem is studied for general network topologies and multiple sources in [4]. [36], [37] studies the question of pricing/tolling of network edges (links) towards realizing efficient Nash flow equilibria, but does not address the issue of price competition between providers. For a single profit-maximizing ISP, equilibrium efficiency questions are studied in [38]. The destination-based next-hop model that we consider is in sharp contrast with the source routing and pricing models considered in this prior literature, and provides a better representation of the inter-domain routing protocols and pricing practices in the current Internet. Source routing requires knowledge of the entire path at the source node, and this practical limitation has restricted the use of source routing in the Internet, while next-hop routing involves decision making by agents that is much more local and distributed. Even though BGP determines this next hop based on information on the entire AS-level path, the benefits of making it strictly next-hop have been argued recently [13]. Furthermore, traffic flow and service pricing negotiations in the Internet occur at the inter-domain level, between an ISP and its neighboring ISPs, and have a customer/provider/peering relationship [7]. Our next-hop routing and pricing model also closely captures the Path Vector Contract Switching framework proposed for inter-domain routing in the future Internet [15], where neighboring ISPs establish contracts (on the amount of flow and its pricing) at their ingress/egress nodes towards forwarding traffic for a specific destination.

Next-hop routing and pricing models have been considered recently in [10] and [14]. However these papers only consider a single-source single-sink network, and associate latency functions with links instead of link capacities. [5] considers next-hop inter-domain routing as in BGP, but focuses on the issue of incentive-compatibility; [8] looks at the incentive-compatibility of best-response dynamics in BGP routing. [3] considers a somewhat general routing game that can also include next-hop routing as a special case, but uses a very different pricing mechanism from the one considered here. Specifically, in our paper the ISP can change the price that it is asking to forward traffic unilaterally, while in [3] a price is considered to be a bilateral contract, so that it takes both participants (the payer and the payee) in order to change the asking price. We also specifically consider the pricing game in the context of inter-domain routing, modeling each ISP/domain as a network cloud (an arbitrary topology capacitated network), with a certain set of its routers acting as ingress/egress nodes through which it peers with its neighbors; this is an aspect in which our model differs from all existing models. Unlike the above-mentioned work on network pricing games, which are only focused on properties of equilibrium flows and prices, we also study best-response based simple price update policies by ISPs, whether and how they converge to stable solutions, and the optimality of the resulting flows.

Finally, our model is a generalization of our model in [1]. In [1], we considered only the much simpler case where each ISP subnetwork consists of a single node. For this case, we showed the existence of a Nash equilibrium solution, how to compute it efficiently, and that the price of stability is 1. In our current paper, we extend many of these results to "well-provisioned" networks (Theorem 3), as well as prove new results about the structure, existence and efficiency of Nash equilibrium (Theorems 1, 2 and 4).

III. Network Model and Pricing Game Formulation

Our routing is destination-based, and therefore we consider routing of flows to a specific destination node (edge network), denoted by t, where the flow can originate at multiple source nodes (edge networks). The network used to forward flows towards the destination under consideration is modeled as a directed acyclic graph $G = (V, E)$ containing a special sink node t, and edge capacities c_e. The multi-provider structure of the Internet is captured by partitioning the nodes of this graph into sets $S_1, S_2, \ldots, S_k$, such that the set of nodes S_i, as well as all the links between nodes of S_i, are owned by ISP i. Destination node t is assumed to belong to an ISP of itself. All ISPs, except for the one to which the destination belongs, act as "players" in the pricing game, choosing the forwarding paths and prices in order to serve their own interests. Throughout the paper, therefore, the term "player" refers to one of these non-destination ISPs. Thus, in our model, the graph G is actually a "network-of-networks", where each subnetwork S_i is a "cloud" (or "domain") owned and operated by a separate player (ISP), and edges connecting S_i with S_j are inter-domain links where pricing negotiations between these players (ISPs) occur. It is also through these inter-domain links that ISPs direct flows toward each other.

To arrive at a meaningful model of price competition, we assume that each subnetwork S_i has enough capacity to forward any amount of traffic that enters it. More precisely, we assume that, if all the links entering S_i are saturated, then there is enough capacity on links inside S_i and on links leaving S_i to be able to forward all of this incoming traffic. This condition would arise naturally if there is a large penalty for not forwarding flow that is sent to an ISP i, giving incentive to build more internal infrastructure or reduce incoming capacity. Moreover, as is common in these types of pricing models [4], [10], [14], we make a *non-monopolistic* assumption that, even if we remove a subnetwork S_j from the graph, there would still be enough capacity on links leaving S_i to be able to forward all of its incoming traffic. As discussed below, this prevents ISP j from charging exorbitant prices to ISP i for forwarding i's traffic, which i would have to pay because without S_j, there would not be enough capacity available. Although the non-monopolistic assumption is necessary for our theoretical results to hold, as we show in Section V, price dynamics still behave extremely well even without this assumption.

Other than forwarding each other's traffic, ISPs also generate and forward traffic of their own, which originate from the end-users that the ISP directly serve. In our model, every subnetwork S_i contains a special node w_i where the traffic of ISP i originates. This node represents the ISPs own customer network, that serves the end-users that get direct service from the ISP. Additionally, every ISP i has an associated *source utility* λ_i. This means that if a ISP i sends f_i amount of its own flow (flow originating form vertex w_i) to the sink t, then the ISP will obtain a utility of $\lambda_i \cdot f_i$.

ISP Traffic Pricing and Forwarding Strategy

The core behavior of our model is that ISP i can charge a price on every edge entering S_i. For example, for an edge $e = (u, v)$ with $u \in S_j$ and $v \in S_i$, player i can set some price p_e. In this case, if ISP j sends flow f_e on this edge, then it has to pay $f_e \cdot p_e$ to ISP i. (Our theoretical results also hold for the case where the price per unit flow forwarded changes with the amount of flow being sent on the edge, but for simplicity we will consider such linear pricing.) Since the destination ISP is not a player, it does not set prices for edges that are incident on it, and we assume those prices are fixed at 0. Figure 1 overviews the pricing and forwarding scheme from the perspective of ISP i.

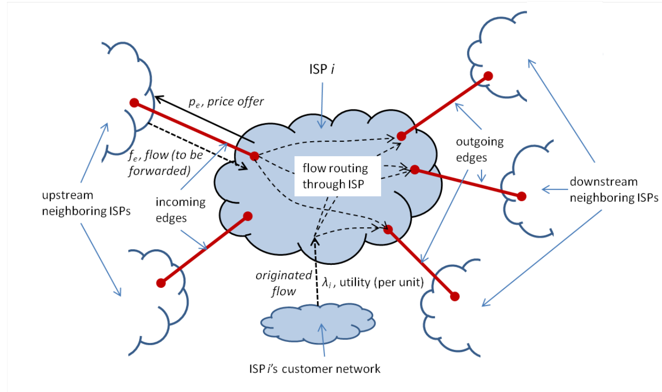

Fig. 1. Price setting, flow generation and forwarding by ISP i

So apart from gaining utility by sending their own flow, ISPs also gain utility for receiving flows from neighboring ISPs and lose utility by paying the next-hop ISPs that receive their flow. Also, observe that for the flow to reach the destination, it is essential that intermediate players forward the flow reliably. Hence every player is required to forward all incoming flow (alternatively, we can think of there being a very large penalty for accepting payment for incoming flow that the player has no intention of forwarding). The forwarding of all flow is always possible due to our assumptions about graph capacities.

Consider now the incentives of an ISP i once the prices p_e on inter-domain links have been set. Let E_i^{in} and E_i^{out} be the set of incoming and outgoing edges for ISP i's subnetwork respectively, and let f_i be the amount of traffic that i sends, i.e., traffic bound for t originating at node w_i. Then, ISP i's utility is

$$\sum_{e \in E_i^{in}} f_e \cdot p_e - \sum_{e \in E_i^{out}} f_e \cdot p_e + \lambda_i \cdot f_i. \tag{1}$$

Thus, once the prices are set, and ISPs upstream of ISP i choose how much traffic to send on edges E_i^{in}, ISP i's goal is simply to choose how to forward incoming traffic through its subnetwork S_i (as well as how much of its own traffic to send), in order to maximize the above expression. To do this, the ISP may have some internal traffic engineering policy (e.g., min-hop routing). We assume that all ISPs forward traffic, and decide how much of their own traffic to send, in the optimal way; in other words, they choose the alternative which maximizes expression (1), while following the constraints of their internal routing policy.

There is one detail to add to our model definition: what if several possible flows yield the same highest utility for an ISP i? Which of these alternatives would ISP i choose, i.e., what is the tie-breaking rule? In our model, we leave this up to the player, instead of forcing some tie-breaking rule. In other words, the strategy of ISP i is (p_i^{in}, γ_i), where p_i^{in} is a vector of prices p_e for each edge in E_i^{in}, and γ_i is an arbitrary tie-breaking rule that decides between several possible flows of the same quality. In other words, ISP i's first priority when deciding where to forward traffic and how much of its own traffic to send, is to maximize its utility. If there are several flows that maximize its utility, it breaks ties according to the tie-breaking rule γ_i. We make the following reasonable assumption about the behavior of internal routing policies and tie-breaking rules: *If the price on an edge (u, v) with $u \in S_i$ remains constant, while the prices on other edges of E_i^{out} increase, then the flow on edge (u, v) does not decrease.* In other words, my competitors charging higher prices only causes me to get more flow, not less.

We denote the collective strategy of all ISPs by $\{P, \gamma\}$. Now that we defined the ISP strategies, consider the outcome of the game, i.e., the flow of traffic that would be generated given ISP choices $\{P, \gamma\}$. To compute this flow, simply iterate over S_i in topologically sorted order (recall that our graph is a DAG). For each i, the incoming and outgoing prices are known, as well as the incoming flow on edges E_i^{in}. Thus, we assume that ISP i solves the above optimization problem (using γ_i to break ties in case several flows are optimal for it), and thus generates the flow inside the subnetwork S_i, as well as on the edges E_i^{out}. This process generates a unique flow; we denote the resulting flow by $f(P, \gamma)$, as this is the outcome of the strategy $\{P, \gamma\}$. The final utility of an ISP i given the choices of all ISPs, $\{P, \gamma\}$, is

$$\textit{utility}_i(P, \gamma) = \sum_{e \in E_i^{in}} f_e \cdot p_e - \sum_{e \in E_i^{out}} f_e \cdot p_e + \lambda_i \cdot f_i,$$

where f_e is the flow on edge e in $f(P, \gamma)$, and f_i is the flow originating at node w_i in $f(P, \gamma)$.

Consider an ISP i who is computing its best response to a strategy $\{P, \gamma\}$. Notice that by changing its prices p_i^{in}, the resulting flow f_i^{in} may become completely different from $f(P, \gamma)$. If this were not the case, then i could always raise its incoming price, knowing that this would increase its utility since the flow would remain the same. In essence, players in this game (ISPs) anticipate changes in flow that result from price changes, but myopically assume that the prices of all other players remain the same when computing their own best response. Such behavior is reasonable in ISP routing settings, for example, since price setting takes place on a much slower time scale than routing.

Fixed Tie-Breaking Rules

When defining our model, we leave the tie-breaking rules γ_i as a player (ISP) choice. As we discuss in Section IV, our model has several nice properties, including existence of Nash equilibrium that is as good as a centrally optimal solution. Instead, however, we could have simply assumed a reasonable tie breaking rule for all ISPs. Unfortunately, as the following theorem shows, this lack of choice of the ISPs on the tie-breaking rules leads to instability.

Theorem 1: If tie-breaking rules γ_i were assumed to be fixed, instead of part of a ISP's strategy, then there are networks where all Nash equilibria are arbitrarily bad compared to the socially optimal solution.

The proof of Theorem 1 can be found in the Appendix.

Well-Provisioned Networks

For some of our theoretical results, we need the assumption that internal subnetworks (individual ISP networks) are *well-provisioned*. To be well-provisioned, a network S_i has to be such that, given an incoming flow f_i^{in} and outgoing prices p_i^{out}, the flow that would maximize i's utility would simply be to forward flow on the cheapest outgoing edges, and to send its own flow on outgoing edges with left-over capacity as long as their price p_e is less than λ_i. In other words, a network is well-provisioned when the solution to each ISP's flow optimization problem (i.e., how the ISP should split its traffic among its downstream neighbors) does not need to take the internal capacity into account, since it is large enough to handle the traffic. For example, if each internal edge of S_i had capacity at least as large as the total capacity of E_i^{out}, then S_i is automatically well-provisioned.

While we will assume that the ISP networks are well-provisioned in order to prove good theoretical properties, our simulations show that, even without this assumption, the price dynamics behave extremely well, and converge to solutions with good social welfare.

IV. Price-Flow Equilibrium Analysis: Efficiency and Characterization

In this section, we first show that for general networks, Nash equilibrium may not exist. We then consider the special case when all ISP networks are well-provisioned. We show that in this case, good Nash equilibria always exist, and give a partial characterization of all Nash equilibria with a particularly nice structure.

Theorem 2: For our pricing game, there are networks which admit no Nash equilibrium.

See the Appendix for proof of Theorem 2.

Due to the above result, in the rest of this section we assume that subnetworks S_i are well-provisioned. For this case, it is easy to extend the results from our recent work [1], since when a subnetwork is well-provisioned, a player i's incentives are equivalent to the case when the entire subnetwork S_i consists of a single node. The latter case is the one considered in [1]. Thus, we immediately have the following theorem.

Theorem 3 ([1]): When networks are well-provisioned, there always exists a Nash equilibrium which maximizes social welfare. Moreover, this equilibrium can be computed efficiently.

Other results from [1], such as bounds on price of anarchy, also extend to the well-provisioned model. [1] not only showed that a good Nash equilibrium always exists, but also that it satisfies the following nice property.

Definition 1: Define a uniform Nash equilibrium to be any Nash equilibrium strategy (P, γ) such that for every player i, the price on all edges of E_i^{out} (except edges to t if they exist) is the same.

The above is a reasonable property to expect from a Nash equilibrium, as neighboring ISPs at the outgoing edges of a ISP i are essentially competing for ISP i's traffic. Thus, if some edges of E_i^{out} are cheaper than the others, then they may raise their prices, expecting that the incoming flow from S_i would not change. On the other hand, neighboring ISPs at more expensive edges of E_i^{out} may lower their prices in order to obtain more traffic from S_i and thus make a larger profit.

In this paper, we extend the results of [1] by partially characterizing the structure of all uniform Nash equilibria in well-provisioned networks. Specifically, we give almost matching sets of necessary and sufficient conditions for the existence of such equilibrium.

Theorem 4: If all ISPs are well-provisioned, then flow $f(P, \gamma)$ and prices P that obey properties (a)-(d) below form a uniform Nash equilibrium. Conversely, every uniform Nash equilibrium must satisfy properties (a)-(c).

(a) For every player i, the price on all edges of E_i^{out}, except edges to t if they exist, is the same. Let this price be denoted by y_i.
(b) If $f_i > 0$ then $y_i = \lambda_i$; if $f_i = 0$ then $y_i \geq \lambda_i$.
(c) For all edges (u, v) with $u \in S_j$ and $v \in S_i$, if (u, v) has a positive flow on it, then $y_j \geq y_i$.
(d) For all edges (u, v) with $u \in S_j$ and $v \in S_i$, if (u, v) is unsaturated ($f_e < c_e$), then $y_j \leq y_i$.

Proof: The fact that the conditions of Theorem 4 are sufficient for the solution to be a Nash equilibrium follows easily from a similar theorem in [1]. Thus, all we need to show is that every uniform Nash equilibrium strategy satisfies conditions (a)-(c).

Consider a uniform Nash equilibrium strategy $\{P, \gamma\}$; it satisfies condition (a) by definition. For the following discussion we will consider ISPs i and j and edge $e = (u, v)$ where $u \in S_i$ and $v \in S_j$.

We now prove that condition (b) must hold in every uniform Nash equilibrium. First, it is clear that if $y_i < \lambda_i$, then i would saturate all outgoing edges with its own flow to maximize its utility, and thus f_i would be greater than 0. Thus, $f_i = 0$ implies that $y_i \geq \lambda_i$ in a uniform Nash equilibrium. Similarly, it is clear that if $f_i > 0$, then $y_i \leq \lambda_i$, since otherwise ISP i would not send its own flow, and thereby improve its utility. All that is left to show is that when $f_i > 0$, we cannot have that $y_i < \lambda_i$. If $y_i < \lambda_i$ then consider edge $e = (u, v)$ such that $p_e = y_i$. Here, $f_e = c_e$ since i will saturate this edge with its own flow. Note that such an edge does exist since $f_i > 0$. In this case ISP j can increase the payment it receives from i by increasing the price of edge (u, v) to be arbitrarily close to λ_i. This deviation will not affect the flow on outgoing edges of i since all outgoing prices equal $y_i < \lambda_i$, and so all outgoing edges of i would still be saturated. This means the flow in the network remains the same and utility of ISP j strictly increases. Hence strategy $\{P, \gamma\}$ would not be a Nash equilibrium unless condition (b) is satisfied.

Now let us suppose to the contrary that there exists a uniform Nash equilibrium strategy $\{P, \gamma\}$ that does not satisfy condition (c). Let j be the first ISP in topological sort order for whom $y_i < y_j$ and there exists a flow on edge (u, v). Now consider a deviation for ISP j where it increases price of all incoming edges to y_j if they were lower earlier. After deviation, no matter if the flow on edge (u, v) increases or decreases, ISP j will be better off since it was losing money on the flow before. Since the price of (u, v) increased while the prices of all other edges remained constant, then due to our assumption about tie-breaking rules, we know that no outgoing edge, other than (u, v), will experience a decrease in flow (although they may experience an increase in flow). We need to examine how this affects the utility of ISP j. W.l.o.g., consider a path $p = \{u, w_1, w_2, \ldots, w_k, v\}$ which experiences an increase in flow after deviation. Note that after deviation, all incoming prices of ISP j are $\geq y_j$. Combined with the fact that condition (b) implies $y_j \geq \lambda_j$, it means, even in the worst case, j will not lose money for receiving flow from path p. Thus the deviation will be profitable for ISP j. And since such deviations can not exist in a Nash equilibrium solution, all uniform Nash equilibrium solutions satisfy condition (c).

So far it has been shown that a uniform Nash equilibrium strategy satisfies conditions (a)-(c). We now show an example of a uniform Nash equilibrium which satisfies conditions (a)-(c) but does not satisfy condition (d).

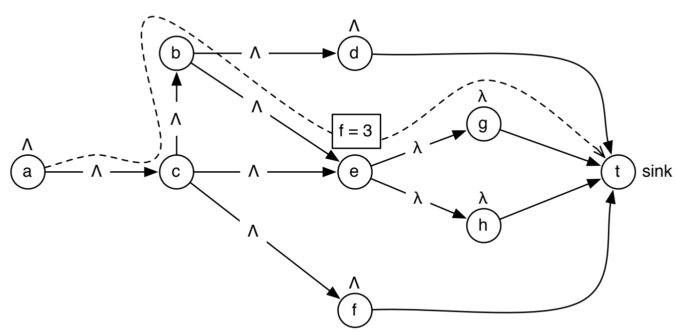

Fig. 2. Example before deviation

Consider the example given in figure 2. Every node is a separate ISP. Edge (c, e) has capacity 1, edges (e, g), (e, h), (g, t) and (h, t) have capacity 4 while the remaining edges have capacity 3. ISPs a, d and f have source utility Λ and ISPs g and h have source utility λ where $\Lambda \gg \lambda$. Now consider a pricing strategy as given in Figure 2. Let us assume that the tie-breaking rules induce a flow of size 3 as shown in the figure, while other ISPs with positive source utilities saturate their outgoing edges. It is easy to verify that this strategy satisfies conditions (a)-(c) but edge (c, e) does not satisfy condition (d). Here $y_c = \Lambda > y_e = \lambda$ even though edge (c, e) is unsaturated. The obvious choice for ISP e is to lower the price on edge (c, e) in order to gain flow on the edge. If ISP e reduces the price of the edge (c, e) by a small amount, then the tie-breaking rule for c may result in a very different flow as shown in Figure 3, since how the flow is apportioned among the outgoing edges of c with prices Λ is allowed to depend on whether c is forwarding 3 units of flow on these edges, or only 2 units of flow.

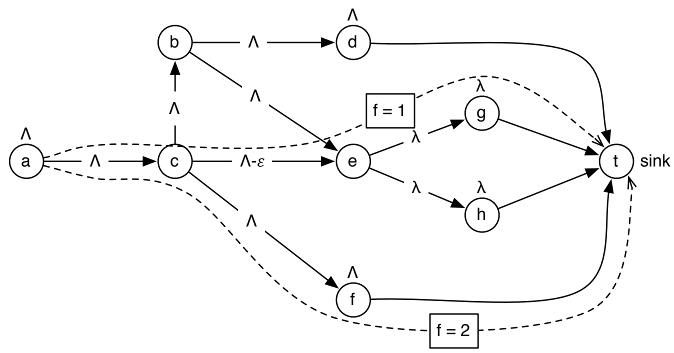

Fig. 3. Example after deviation

Player e ends up losing the profitable flow from edge (b, e) while receiving smaller amount of flow on edge (c, e). This is clearly not profitable for player e, and so is not an improving deviation. Through examination of all other players, it is not difficult to see that the original strategy is in Nash equilibrium, even though condition (d) is not satisfied. ■

The above theorem implies that the conditions of Theorem 4 essentially characterize all uniform Nash equilibria for the ISP pricing and forwarding game. Unfortunately, as we showed in the proof of Theorem 4, there exist uniform Nash equilibria that do not obey property (d), and so this property remains sufficient but not necessary for the existence of uniform Nash equilibrium.

We showed above that for well-provisioned networks, good Nash equilibria exist and can be computed efficiently. Moreover, we partially characterized the set of possible uniform Nash equilibria using a few simple properties. In the following section, we will show that, even if the network is not well-provisioned (and even if monopolies exist), simple price dynamics behave extremely well on average. Specifically, our extensive simulations show that price dynamics in our setting converge quickly, and to solutions that achieve good social welfare.

V. Simulation Study: Convergence of Dynamic Pricing Updates

In this section, we present results from our simulation study that explores the convergence of a simple, best-response based incremental price update policy, and shows that for a large fraction of randomly generated network topologies, it converges quickly to a solution that yields near-optimal network utility. We first perform the study on well-provisioned ISPs, which allows us to simulate networks of up to 500 ISPs. Subsequently, we study networks where ISPs have general topologies (i.e., they are not necessary well-provisioned); for this case, we study networks with up to 50 ISPs, and each ISP comprising of up to 50 nodes (routers). Note that our analytical model and results in Sections III and IV assumed a "non-monopolistic" network topology, which ensures that there is enough competition in the network, and thus no single ISP can charge exorbitantly high prices. In our experimental study, we simulated networks where this assumption is made to hold (by choosing link/edge capacities appropriately), as well as networks where the non-monopolistic assumption does not hold. Our extensive simulation experiments show that the nature of the convergence and performance results do not differ significantly across these two kinds of networks. As expected, however, the convergence and performance results for monopolistic network topologies are slightly worse (on average) than that for non-monopolistic networks with similar parameters. In the following, therefore, we only present the simulation study on networks where the non-monopolistic assumption may not hold, which yields more conservative performance/convergence results, and has wider applicability in practice.

A. Network Generation, Dynamic Price Update Policy, and Convergence Criteria

For well-provisioned networks, we experimented with two different models of inter-ISP connectivity: (i) *Uniformly-random connectivity*, (ii) *BRITE-generated connectivity*. Since we consider a single, specified destination node/ISP, the forwarding network must constitute a directed, acyclic graph (DAG), which is created in the two cases as follows. For uniformly-random connectivity, the specified number of ISPs is created and numbered uniquely (numbers representing the topological order). Then each ISP is given a random number of outgoing edges, chosen uniformly at random between 2 and 6 (inclusive). The endpoints are randomly chosen from the subset of ISPs further down in the topological order. If the chosen number of outgoing edges is greater than or equal to the number of possible endpoints (i.e., number of ISPs further down in the topological order), then only one edge is created per endpoint (note that this ensures connectivity to the destination ISP). Note that the number of incoming edges of ISPs get determined automatically through the above procedure, and can be greater than 6 (no constraint is placed on them). For BRITE-generated connectivity, we use the BRITE simulator [39] to generate the initial network (with average node degree set to 4), from which the destination-specific directed acyclic network is created. Note that the BRITE simulator uses the Barabasi-Albert model [40] to generate scale-free networks that are more representative of router and ISP connectivity in the Internet. In this case, the generated graph is given direction by first choosing a sink (destination) randomly from the ISPs, and then doing a breadth-first search from the sink, thus dividing the ISPs into layers, where all ISPs in the same layer have the same shortest distance to the sink. The edges inside each layer are directed similarly to the uniformly-random connectivity case: by creating a random ordering of ISPs and directing edges from lower-numbered to higher-numbered ISPs. An edge between layers is directed from the ISP that is farther away from the sink to the ISP that is closer to the sink.

Once the connectivity between ISPs is determined (as described above), each ISP is first given a total outgoing capacity equal to its total incoming capacity plus a random amount chosen uniformly from $[0, 1]$ (so ISPs must be traversed in topological order). This outgoing capacity is then divided randomly between the ISP's outgoing edges. If the ISPs are not well-provisioned, the intra-ISP network topologies are also chosen in a similar manner; we discuss this in Section V-C when we discuss the simulation results for general-topology ISP networks. Finally, the utility values (per unit flow) λ_i are chosen uniformly at random as an integer on $[0, 30]$.

For any given set of prices, the flows are determined by traversing the network of ISPs in the topological order. Given the incoming flows (which is already determined when the ISP is considered), the ISP decides how much of its own flow to generate and how to direct all the flow (that it receives on its incoming edges, as well as what it generates) by solving a linear program, to maximize its overall utility given current prices and incoming flow.

The incremental price update policy used by the ISPs on its incoming edges (i.e., edges in the network that connect an ISP to its upstream neighbors), works as follows. Given the current set of prices, an ISP determines the estimated flow (calculated the same way as described above) on the edge if (i) the price on this edge was reduced by unit amount, and (ii) if the price on this edge was increased by unit amount. This is further used to determine the expected change in utility in both cases (i) and (ii). The ISP then increases or decreases the edge price by one if it is expected to increase the ISP's utility; the price remains the same if no gain in utility can be made. Two other simple and intuitive methods of changing price were tested, and did not provide any observable benefits; thus we only present the results for the price update policy as described above. The pricing update procedure was run for 300 cycles (iterations), where a cycle means that each node is given the opportunity to alter its prices (the order in which they are given this opportunity is a random permutation which remains consistent between cycles). We say that convergence is attained at time (cycle) t (¡ 200) if the following criteria holds between t and $\max(300, t+100)$: (i) the minimum total utility in the network must be at least 0.9 times the maximum total utility, and (ii) the slope of the linear-least-squares-regression line on total utility can be at most 0.00002 times the maximum total utility. (The total utility, and its maximum and minimum, are based on the measured values at the end of each cycle of the simulation.) In other words, if (i) and (ii) does not hold for at least 100 consecutive cycles (of the 300 cycles of simulation time), we assume that the utility/prices have not converged. All average results are calculated by averaging over 200 randomly generated networks (uniformly-random or BRITE-generated), with similar parameters.

B. Networks of Well-Provisioned ISPs

We now present the simulation results for the case of well-provisioned networks, for both uniformly-random and BRITE-generated connectivity.

Frequency of Convergence: We first study how frequently our pricing update policy converged to result in stable total utility/prices/flow values. Figures 4 and 5 plot the ratio of networks of a given size in which the total utility values converged, to the total number of networks tested (we simulated on 200 different networks for each size, as noted earlier). We observe that the total utility values converged in a majority of the networks, and the convergence frequency is much better (almost 100%) for BRITE-generated topologies. This is particularly significant since BRITE-generated topologies are closely representative of inter-ISP connectivity topologies in the Internet. Recall that the networks simulated do not in general satisfy the non-monopolistic assumption under which the equilibrium existence and efficiency results have been shown (in Section IV). Therefore, these simulation results show that despite the monopolistic nature of some (possibly many) ISPs, simple best-response based incremental pricing update policies can result in stable prices and flows. (We will see shortly that the resulting stable flows are "efficient" as well.) Note that our convergence criteria ((i) and (ii), as described above) are quite strict and the simulation time is fairly short (300 cycles of simulation time, out of which the total utility must not vary significantly over a 100-cycle period, for convergence). With less strict convergence criteria and longer simulation period, the frequency of convergence is likely to be higher. Also note that frequency of convergence goes down as the number of ISPs increase; this is clearly observed in the case of uniformly-random connectivity between ISPs (Figure 4). This is partly due to the fact that the convergence time increases with increasing number of ISPs, as we intuitively expect (we support this observation below when we show convergence time results). This naturally implies that convergence frequency as measured within a fixed time-limit of 300 cycles is likely to reduce as the number of ISPs become larger.

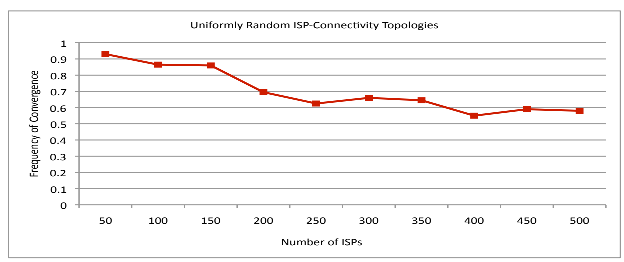

Fig. 4. Frequency of Convergence (Uniformly-random connectivity)

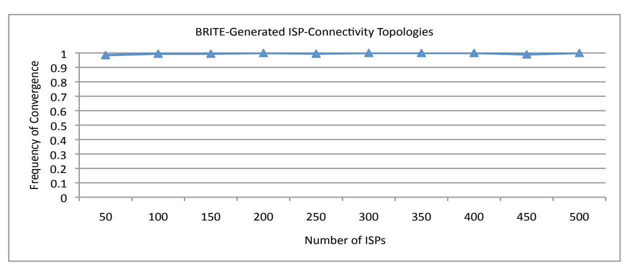

Fig. 5. Frequency of Convergence (BRITE-generated connectivity)

Time of Convergence: Next we study the average convergence time, counting only the networks for which the pricing update policy converged, as discussed above. The results are shown in Figures 6 and 7. We observe that the average convergence time is quite short: for networks with 500 ISPs, it is about 80 cycles for uniformly-random connectivity, and less than 30 cycles for BRITE-generated connectivity. Note that each step (iteration) of the pricing update procedure requires local information exchange between ISPs (an ISP exchanges price and flow information with neighboring ISPs); so the message exchange and computation required in each cycle can be done quickly if executed in a distributed manner. The short convergence time for BRITE networks is also part of the reason why we see a very high convergence rate (frequency) in Figure 5.

The convergence time increases with increasing number of ISPs, as we intuitive expect. However the corresponding rate of increment (slope) is fairly modest, particularly for BRITE-generated topologies. The number of ISPs (ASes) in the Internet runs into thousands (tens of thousands) [41]; yet, our results (we did extensive simulations for up to 500 ISPs) hint that stable and efficient prices (and the corresponding flows) should be computable across all ISPs/ASes in the Internet in a reasonable amount of time.

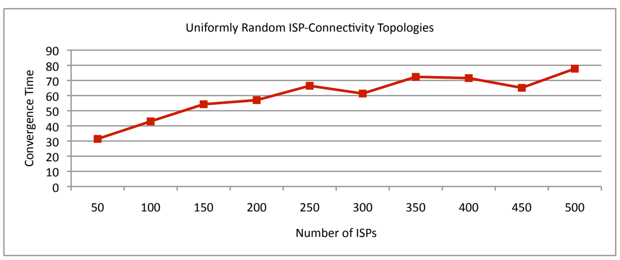

Fig. 6. Convergence Time (Uniformly-random connectivity)

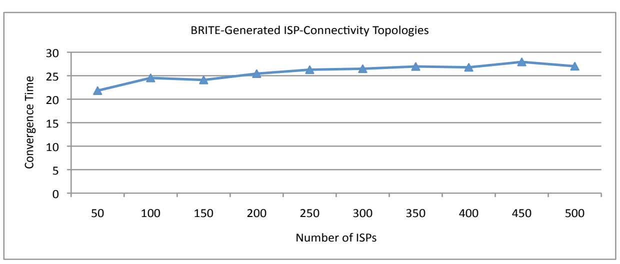

Fig. 7. Convergence Time (BRITE-generated connectivity)

Utility upon Convergence vs Optimum Utility: Next we study the network utility attained on convergence, and discuss how it compares with the optimal utility. Note that the network utility represents the aggregate weighted source-to-destination flow value, where the traffic originating at ISP i is weighted by its utility value (per unit flow) λ_i. Thus this network utility can be viewed as generalized end-to-end throughput attained in the network.

Figures 8 and 9 show the network utility attained on convergence, as a fraction of the optimum utility. Note that the optimum utility is calculated by solving a weighted maxflow problem in the entire network (from all ISP sources to the single destination ISP). The network utility attained on convergence corresponds to the utility values averaged over the last 100 cycles.

For both types of networks, we observe that the attained utility remains quite close to the optimum, on the average, for networks up to 500 ISPs. While the ratio of attained utility and optimal utility reduces slightly as the number of ISPs increase, this reduction is quite modest – it reduces from 90% to 80% as we increase the number of ISPs from 50 to 500. Note that since the non-monopolistic assumption required for Theorem 3 may not hold in the simulated networks, the price of stability need not be unity. However, our simulations show that there exist stable price-flow solutions that attain high utility in networks on the average, many of which may not satisfy the non-monopolistic assumption.

Note that all the results reported so far all correspond to average statistics; we will look into detailed statistics on the attained utility (for smaller number of ISPs) in Section V-C.

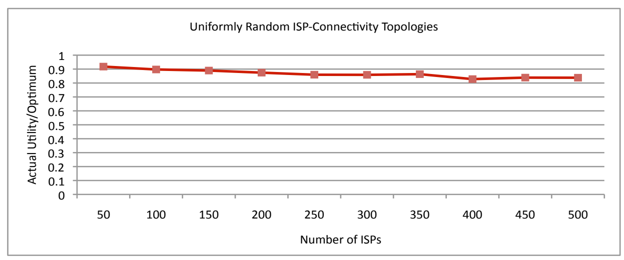

Fig. 8. Ratio of Average Utility attained on convergence and Optimal Utility (Uniformly-random connectivity)

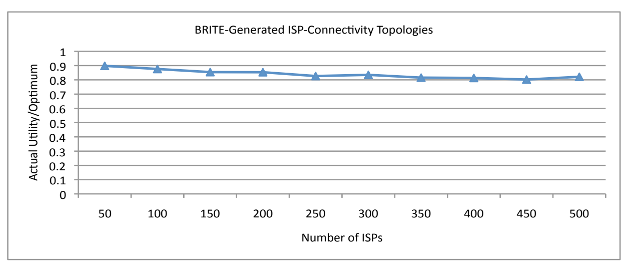

Fig. 9. Ratio of Average Utility attained on convergence and Optimal Utility (BRITE-generated connectivity)

Network Utility and Price Evolutions with Time: We now look at how the network utility and edge prices evolve over time in a sample (typical) network, to gain better understanding of the convergence process. For this set of simulations, we consider a single network of 1000 ISPs generated randomly according to the uniformly-random connectivity model. After each cycle, the total utility in the network and the prices of the edges were recorded.

Figure 10 shows how the total utility value evolves over time. The figure shows that the network utility reaches a more-or-less stable value fairly quickly (in only 60-70 cycles, which is quite significant, considering the network has 1000 ISPs, and our price update procedure is decentralized); but exhibits small fluctuations around that value for the rest of the simulation time. Thus although convergence occurs fast, it is not perfectly smooth convergence, as some small fluctuations in utility values remain - this behavior was observed to be typical, in our simulation experiments. These fluctuations in network utility are a result of small fluctuations in the price and flow values, and is better understood by looking at sample price evolution charts, as we show next.

Figure 11 shows how the incoming edge prices of an ISP (the prices that are set by the ISP) evolve over time, for two different ISPs. Note that each curve in the figure corresponds to a different edge (price variable). In both cases, the price convergence process is quite fast, i.e., convergence occurs within 60 cycles. The price convergence process for ISP 771 is "smooth", i.e., once the prices converge to their stable values, they do not deviate from those values. We have observed that this kind of behavior is typical for most ISPs. The price convergence process for ISP 997 is however quite different; even though the prices seem to "converge" to some steady-state values, they occasionally fluctuate around those values, and these fluctuations do not die down over time. This kind of fluctuation in price occur for a small fraction of ISPs, in many of our simulations. A closer inspection reveals that the range of these fluctuations is (+/-)1, which is also the step size of the pricing updates. This implies that these fluctuations can be reduced by making the step size of pricing updates smaller, although typically at the cost of having longer convergence times.

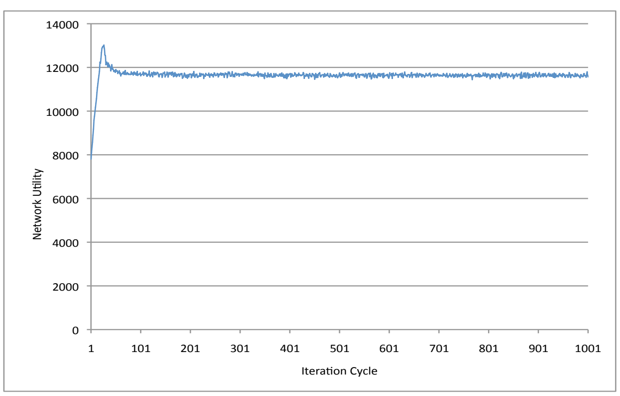

Fig. 10. Time graph showing the Convergence of the Network Utility for a sample network

C. Networks of ISPs with General Topologies

Preliminaries: Next we perform our simulation study on networks where ISPs have general topologies, and therefore may not be well-provisioned. We have experimented with both uniformly-random and BRITE-generated connectivity models at both the intra-ISP and inter-ISP levels. Below, we only present results for the case where both intra- and inter-ISP topologies are determined by BRITE; our findings for the other cases are similar. In this case, BRITE is first used to create a network of ISPs - in the same way as described in Section V-A. Subsequently, BRITE is used to create a network of router nodes for each of these ISPs. If two ISPs are connected in the upper level (inter-ISP) network, an edge will be put between two routers belonging to the two ISPs (one from each ISP). These "gateway" routers are chosen by finding the lowest degree non-leaf router node in each of the two ISPs. Once the network has been created, the edges are given direction and capacity as follows. The edges between ISPs are assumed directional, and are directed based on the breadth-first-search from the sink ISP (as in Section V-B). Edges within an ISP are bidirectional (can carry flow in both directions) and have capacity one. The capacities of the inter-ISP edges are determined in an iterative manner, traversing the ISPs in the reverse topological order. When considering a specific ISP, a maxflow through (and including) that ISP to the destination is computed, assuming all incoming edges of the ISP have infinite capacity (note that since we are traversing the ISPs in the reverse topological order, the capacities of all edges from that ISP to the sink are already determined by then). Then, for each incoming edge, a random number r, $0.9 \leq r \leq 1.0$, is created, and the edge is given capacity $r * f$, where f is the amount of flow on this edge in the previously found maxflow. This ensures that every ISP can always forward all traffic that enters its subnetwork on ingress links.

Additionally, for each ISP, a "source node" is chosen at random from all the nodes in the ISP. All flow generated inside an ISP is assumed to originate at that source node; the corresponding λ value is chosen as before - uniformly at random as an integer on $[0, 30]$. For a given set of prices, the flow finding algorithm works as in Section V-A, except that the intra-ISP topology (the intra-ISP network connectivity and link capacities) must be additionally considered in the linear program that finds the flow maximizing the ISP's utility. Prices are changed in the network as before: there is a consistent random permutation of the ISPs, this permutation is iterated through and each ISP is given the opportunity to raise or lower the price on each of its incoming edges based on estimated change in its own utility.

In the general-topology ISP model that we consider here, the intra-ISP topologies need to be considered in our simulations and numerical computations; this practically limits us from simulating networks with very large number of ISPs as before, or having a large number of nodes (routers) per ISP. Below, we show the results for networks up to 50 ISPs with up to 50 nodes per ISP.

The method of determining convergence is the same as that discussed in Section V-A, except that the simulation time is 200 cycles (instead of 300) and convergence is determined based on whether conditions (i) and (ii) holds for 75 consecutive cycles (instead of 100).

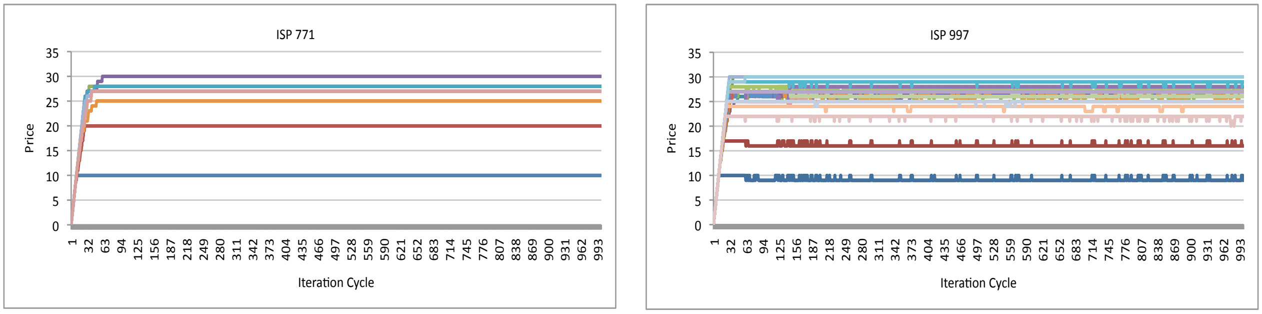

Fig. 11. Time graph of the price changes on the incoming edges of two ISPs in the sample network (the ISP number refers to its position in the topological order)

Utility upon Convergence vs Optimum Utility: Figures 12 and 13 show the histograms of network utilities as observed over simulations over 200 network topologies, for each setting of parameters. The attained utility shown is calculated by averaging over the last 75 cycles in the 200-cycle simulation period. The average utility was observed to be close to 90% of the optimum in all cases, which is consistent with our observations for the case of well-provisioned ISPs (Section V-B). The figures show that for an overwhelming majority of the ISPs, the utility attained on convergence was 80% or more (of the optimum). From the figures, we also observe that, interestingly, the tail seems to improve as the number of ISPs in the network, or the number of nodes (routers) per ISP, increases. For example, for networks of 50 ISPs and 50 nodes per ISP, all of the 200 networks simulated resulted in a (attained utility / optimal utility) ratio of 0.5 or more, on convergence. This implies that the utility ratio distribution attained on convergence of our pricing update policy is likely to improve (i.e., the utility ratio is likely to be more consistently high) as we scale up in terms of number of ISPs, or number of nodes per ISP.

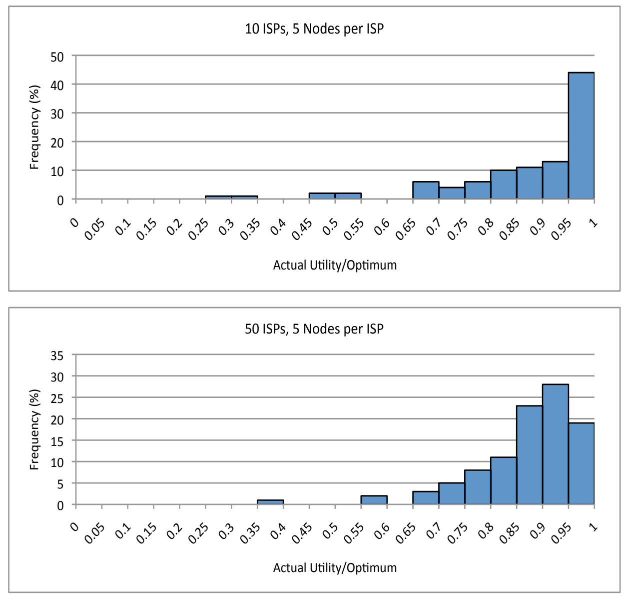

Fig. 12. Histogram of Network Utility, for different number of ISPs (2 cases)

Convergence Time and Frequency: Finally, we look at a representative result of the convergence time and frequency by looking at random BRITE networks with 50 ISPs and 50 nodes per ISP. The result is shown in Figure 14. Note that the bar corresponding to the negative value of the convergence time in the figure (in red) corresponds to the frequency (percentage) of non-convergence within the simulation time of 125 cycles. We notice here that for a significant majority of the networks, the pricing update procedure converges very quickly, say within 30 cycles. However, a fairly long tail for the convergence time is observed here - sometimes the pricing updates take fairly long to converge, but this happens for a small fraction of networks. This also explains the high rate of non-convergence observed here (about 15%), which is also partly a result of the short simulation time (125 cycles); the rate of non-convergence would reduce as the total simulation time (the time limit for convergence) is increased. We also observe that the convergence time tail becomes longer (and the non-convergence frequency becomes higher) with increases in the number of ISPs or the number of nodes per ISP; the growth rates are however fairly modest. Note that these facts are consistent with our observations in Section V-B for well-provisioned ISP networks.

VI. Conclusion

In this paper, we have considered the question of strategic pricing of forwarding services provided by ISPs, which is also closely coupled with the issue of inter-domain traffic engineering. For "well-provisioned" ISPs, we argue that there exists an efficient and easily-computable equilibrium to the pricing game between ISPs. We also give almost matching sets of necessary and sufficient conditions towards characterizing all "uniform" Nash equilibria of the game. We also observe the importance of breaking ties appropriately, for good equilibrium to exist. While we show that equilibrium may not exist in general when ISPs have arbitrary topologies (not necessarily well-provisioned), extensive simulation studies demonstrate that in a vast majority of network topologies, a simple best-response based incremental pricing update procedure converges quickly

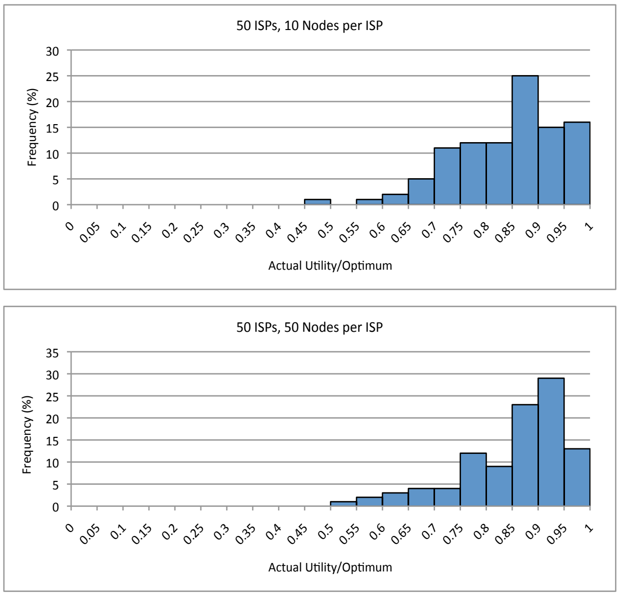

Fig. 13. Histogram of Network Utility, for different ISP sizes (2 cases)

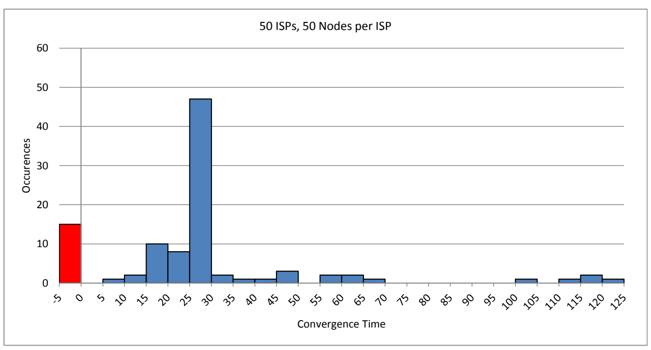

Fig. 14. Histogram of Convergence Time

to stable prices and flows. Furthermore, the overall utility attained on convergence is quite close to (typically 80-90% of) the optimal network-wide utility (generalized end-to-end throughput).

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APPENDIX

Proof of Theorem 1

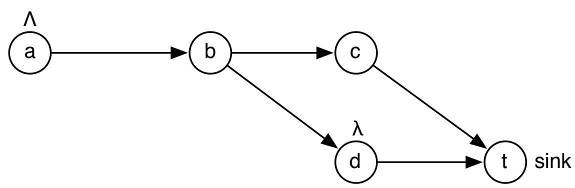

Fig. 15. Example showing arbitrarily large price of stability for fixed tie-breaking rules.

We show that the example in Figure 15 has arbitrarily high price of stability when tie-breaking rules γ_i are assumed to be fixed.

In the example, all edges have capacity 1. Every node is a separate player, i.e., the size of each set S_i equals 1. Thus, for the purposes of this example, we can refer to nodes instead of players. Node a has source utility of Λ and d has source utility of λ. The other nodes have source utilities of 0. It is easy to verify that the optimal solution for this example consists of flow of size 1 on path $a \to b \to c \to t$ and a flow of size 1 on edge (d, t). This optimal solution gives a social welfare (i.e., sum of player utilities) of $(\Lambda + \lambda)$. Node b has a fixed tie-breaking rule γ_b where it prefers sending to player d over player c, whenever it has several ways of forwarding flow that results in the same utility for b.

Let any Nash equilibrium solution for this instance in which node a sends some of its own flow be called a *good* equilibrium. We first show that this instance does not admit a good equilibrium. In order to prove this by contradiction, suppose that such an equilibrium does exist and $f(a, b) > 0$. We denote price of edge (b, c) as p_c and that of edge (b, d) as p_d.

If $p_d = 0$ then the preference of edge (b, d) over edge (b, c) ensures that $f(b, d) > 0$. But now node d has a profitable deviation where it can increase p_d to λ thereby receiving more utility by either sending its own flow or forwarding flow by charging a higher price. Hence $p_d > 0$. Now consider the values that p_c can take.

- $p_c < p_d$: We have already shown that $p_d > 0$. Let $p_c = p_d - \delta$ where $\delta < p_d$. In this case, node b will forward flow $f(a, b)$ on edge (b, c). Now consider the deviation for node c where it increases p_c to $p_d - \delta/2$. Node b will still forward the flow $f(a, b)$ on edge (b, c), which implies utility of node c would increase, making it a profitable deviation.
- $p_c \geq p_d$: Again, the preference of edge (b, d) over edge (b, c) ensures that node b will forward flow $f(a, b)$ on that edge (b, d). This means utility of node $c = 0$. Now consider the deviation for node c where it decreases p_c to $p_d - \epsilon$ where $\epsilon < p_d$. Now flow $f(a, b)$ will be routed to edge (b, c) giving a utility of $f(a, b) \cdot p_c$ to node c. Since $f(a, b)$, $p_c > 0$, this is a profitable deviation for node c.

Therefore there does not exist a stable price for edge (b, c) when $f(a, b) > 0$, and hence a good equilibrium does not exist for this instance. Now consider a strategy ψ, where all edge prices are $\geq \Lambda$ (except for edges to t, which always have price 0 since t is not a player). In this solution node d sends its own flow and the social utility is λ. It is easy to verify that this strategy is in Nash equilibrium. Since good equilibria do not exist and d is the only other node with positive source utility, ψ is a Nash equilibrium strategy with the highest social utility. Hence:

$$\text{Price of Stability} = \frac{\Lambda + \lambda}{\lambda}$$

Thus if $\Lambda \gg \lambda$ then price of stability can be arbitrarily large, and any existing Nash equilibrium is much worse than the social optimum. The above example can be adjusted such that, with fixed tie-breaking rules, no Nash equilibrium exists.

Note that, if instead of having fixed orderings, b were able to pick its ordering γ_b to prefer node c over d, then prices of Λ on (a, b) and λ on (b, c) and (b, d) result in a Nash equilibrium with the same quality as the social optimum.

Proof of Theorem 2

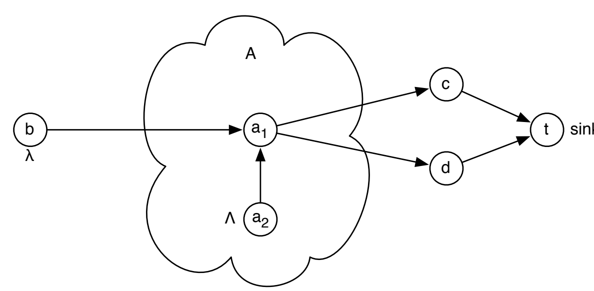

Fig. 16. Example showing that Nash Equilibrium may not exist in the cloud model.

In the example illustrated in Figure 16, player A controls nodes a_1 and a_2; the rest of the nodes are each controlled by a separate player. Node a_2 has a source utility of Λ. Player b has source of utility of λ, where $\lambda < \Lambda$. Players c and d do not have any source utility. All edge capacities are 1.

We will use the following notation throughout. Price of edge (b, a_1) will be denoted by p_b and the quantity of flow on the edge will be denoted by f_b. Similarly, the price of edges (a_1, c) and (a_1, d) will be denoted by p_c and p_d respectively whereas the quantity of flow will be denoted by f_c and f_d. We will mark the difference in the values of these quantities after a strategy deviation by using $'$. For example the flow on edge (a_1, c) after deviation will be denoted by f'_c. Utility of a player x will be denoted by $u(x)$.

Unless otherwise mentioned, the quantity ϵ is an infinitesimally small amount greater than 0. This also implies $(\Lambda - \lambda) \gg \epsilon$. The value ϵ is typically used to avoid invoking the tie-breaking rule. For example, if player A wants to draw a flow of size 1 from player b, the maximum price that can set on edge (b, a_1) is $p_b = \lambda$. At this price though the tie breaking rule comes into play that may result in $f_b < 1$. In such a case, player A sets p_b to $\lambda - \epsilon$ which guarantees flow of size 1.

Now let us assume that this example accepts a Nash equilibrium solution. Observe that only nodes a_2 and b send their own flow. Consider the following cases for the flow generated in the Nash equilibrium solution:

a) Case 1:: **Neither a_2 nor b send their own flow**

Since a_2 is not sending its own flow, it must be true that $p_c, p_d \geq \Lambda$. In this situation player c can reduce price on edge (a_1, c) so that $p'_c = \Lambda - \epsilon$. This would prompt player A would send its own flow form node a_2 on edge (a_1, c). This would result in player c's utility increasing from 0 to $\Lambda - \epsilon$ making it a profitable deviation.

Hence there does not exist a Nash equilibrium strategy where neither a_2 no b send their own flow.

b) Case 2:: **b sends own flow but a_2 does not**

Since b is sending its own flow, $p_b \leq \lambda$. Utility of player A is as follows:

$$u(A) = f_b \cdot p_b - f_c \cdot p_c - f_d \cdot p_d$$

Now consider a deviation for player A where she increases price on edge (b, a_1) such that $p'_b > \lambda$. This would result in reducing f'_b to 0. Player A then replaces the lost flow from b by sending its own flow from node a_2 and sends the same quantity of flow to nodes c and d. Player A's new utility is:

$$\begin{aligned} u(A)' &= f_b \cdot \Lambda - f_c \cdot p_c - f_d \cdot p_d \\ &> f_b \cdot \lambda - f_c \cdot p_c - f_d \cdot p_d \\ &\geq f_b \cdot p_b - f_c \cdot p_c - f_d \cdot p_d \\ &= u(A) \end{aligned}$$

This is a valid deviation for player A, hence there does not exist a Nash equilibrium strategy where b sends own flow but a_2 does not.

c) Case 3:: **a_2 sends own flow but b does not**

We claim that in this case $p_c = p_d = 0$. Note that the maximum flow that a_2 can send is of size 1. This implies that if $f_d > 0$ then $f_c < 1$ and vice versa. Let us assume that $f_d > 0$. This also means that $p_d \leq p_c$. Now consider the following implications of this assumption:

1) If $p_d > 0$ and $p_c > p_d$ then $u(c) = 0$ as it will not receive any flow. But player c can change price of edge (a_1, c) to $p'_c = p_d - \epsilon$, thereby obtaining flow of size f_d and utility $(p_d - \epsilon) \cdot f_d$.
2) If $p_d > 0$ and $p_c = p_d$ then utility of player c is $u(c) = p_c \cdot f_c = p_d \cdot f_c$. Now player c can change price of edge (a_1, c) to $p'_c = p_d - \epsilon$ for $0 < \epsilon < \frac{p_d \cdot f_d}{f_c + f_d}$. This would imply that flow f_d would now be routed to c and its new utility will be:

$$\begin{aligned} u(c)' &= (p_d - \epsilon) \cdot (f_d + f_c) \\ &> \left(p_d - \frac{p_d \cdot f_d}{f_c + f_d}\right)(f_d + f_c) \\ &= p_d \cdot f_c \\ &= u(c) \end{aligned}$$

3) If $p_d = 0$ and $p_c > p_d$ then d would increase price on edge (a_1, d) to $p'_d = \epsilon$ such that $0 < \epsilon < p_c$ and gain utility of value $\epsilon \cdot f_d$ which is more than her previous utility: 0.
4) Since all the above cases have valid deviations, Nash equilibrium can exist only if $p_c = p_d = 0$.

The above argument holds symmetrically for the case where $f_c > 0$. This proves our claim that $p_c = p_d = 0$.

Consider the utility of player A: $u(A) = \Lambda(f_c + f_d)$. Since b is not sending any of its own flow, $p_b \geq \lambda$. We have already shown that $p_c = p_d = 0$ and $f_c + f_d \leq 1$. This means player A can increase its utility by sending the complete 1 unit of flow from node a_2 and setting $p'_b = \lambda - \epsilon$. This will result in b sending its flow f_b of size 1. The resulting utility of player A: $u(A)' = \Lambda \cdot 1 + (\lambda - \epsilon) > u(A)$. Which means this would be a valid deviation for player A.

Hence there does not exist a Nash equilibrium strategy where a_2 sends own flow but b does not.

d) Case 4:: **Both a_2 and b send their own flow**

We first look at the case where $f_c + f_d < 2$. Again, we claim that in this case $p_c = p_d = 0$. A similar analysis as the one in **Case 3** can be carried out. It is easy to see that if $f_c < 1$ then $p_c \geq p_d$ and vice versa. Let us assume that $f_c < 1$. Now consider the following implications of this assumption:

1) Consider the case where $p_d > 0$ and $p_c > p_d$. If $f_c = 0$ then the same argument as given in **Case 3.1** holds.
   If $f_c > 0$ then $f_d = 1$. Let $p_d = p_c - \delta$ where $\delta < p_c$. Now player d can set $p'_d = p_c - \delta/2$, thereby obtaining the same amount of flow at a higher price and hence increase its utility.
2) If $p_d > 0$ and $p_c = p_d$ then the argument of **Case 3.2** holds exactly.
3) If $p_d = 0$ and $p_c > p_d$ then d would increase price on edge (a_1, d) to $p'_d = \epsilon$ such that $0 < \epsilon < p_c$ and gain utility of value $\epsilon \cdot f_d$ which is more than her previous utility: 0.
4) Since all the above cases have valid deviations, Nash equilibrium can exist only if $p_c = p_d = 0$.

Given $p_c = p_d = 0$, player A will send all of its own flow from node a_2 if it previously wasn't and also set price $p'_d = \lambda - \epsilon$ to draw a flow of size 1 from player b. It is easy to see that this is a beneficial deviation for player A. This results in $f'_c + f'_d = 2$.

Finally, we consider the case where $f_c + f_d = 2$. Since player b is sending its own flow, $p_b \leq \lambda$. If $\min(p_c, p_d) > \lambda$, then this cannot be a Nash equilibrium, since A can gain utility by increasing the price p_b, and reducing the flow f_b. This is because if $\min(p_c, p_d) > \lambda$, then A is losing utility from flow received from b. Thus, in every Nash equilibrium, it must be that $\min(p_c, p_d) \leq \lambda$.

W.l.o.g. let $p_c = min(p_c, p_d) \leq \lambda$. Then, player c can gain utility by setting $p'_c = \Lambda - \epsilon$. Player A will still send its own flow of size 1 and is obligated to forward flow of size 1 which it receives from b. This clearly is a beneficial deviation for player c as its utility jumped from being $\leq \lambda$ to $\Lambda - \epsilon$. Thus, no Nash equilibrium exists when $f_c + f_d = 2$.

e) Conclusion: The four considered cases are the only possible configurations of flow. Since a Nash equilibrium strategy cannot exist in any of these configurations, there does not exist one for this example.