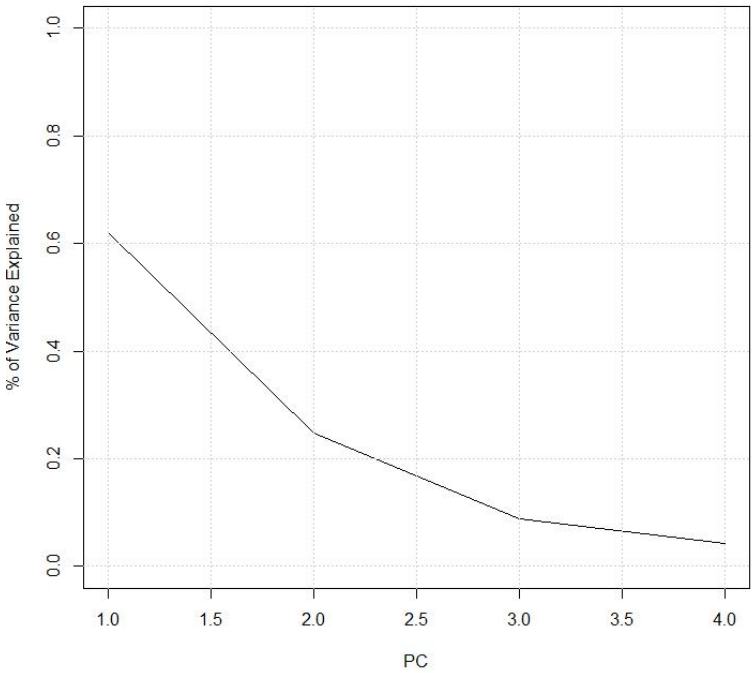
```
# pca.r        James p401

d1=USArrests
dim(d1)                        # [1] 50    4
head(d1)

# a) Compare eigenvalues & variances
#============================================================
summary(d1)
#           Murder                Assault               UrbanPop                 Rape
#    Min.      : 0.800      Min.      : 45.0      Min.      :32.00      Min.      : 7.30
#    1st Qu.: 4.075      1st Qu.:109.0      1st Qu.:54.50      1st Qu.:15.07
#    Median : 7.250      Median :159.0      Median :66.00      Median :20.10
#    Mean      : 7.788      Mean      :170.8      Mean      :65.54      Mean      :21.23
#    3rd Qu.:11.250      3rd Qu.:249.0      3rd Qu.:77.75      3rd Qu.:26.18
#    Max.      :17.400      Max.      :337.0      Max.      :91.00      Max.      :46.00

# covariance matrix
var(d1)
#                        Murder      Assault      UrbanPop          Rape
# Murder        18.970465    291.0624      4.386204    22.99141
# Assault    291.062367 6945.1657 312.275102 519.26906
# UrbanPop      4.386204    312.2751 209.518776    55.76808
# Rape           22.991412    519.2691    55.768082    87.72916
apply(d1,2,var)
#         Murder       Assault      UrbanPop           Rape
#      18.97047 6945.16571    209.51878      87.72916
apply(d1,2,sd)
#       Murder      Assault    UrbanPop          Rape
#    4.355510 83.337661 14.474763    9.366385

eigen(var(d1))
# $values
# [1] 7011.114851    201.992366      42.112651        6.164246
# $vectors
#                        [,1]              [,2]              [,3]              [,4]
# [1,] -0.04170432    0.04482166    0.07989066    0.99492173
# [2,] -0.99522128    0.05876003 -0.06756974 -0.03893830
# [3,] -0.04633575 -0.97685748 -0.20054629    0.05816914
# [4,] -0.07515550 -0.20071807    0.97408059 -0.07232502
```

```
sum(eigen(var(d1))$values)    #[1] 7261.384
sum(diag(var(d1)))                #[1] 7261.384
# sum of eigenvalues = sum variances
# means and variances very differents, need to standardize

# b) Find principal components (scaled data)
#============================================================

m1=prcomp(d1, scale=T)
names(m1)
# [1] "sdev"          "rotation" "center"      "scale"        "x"

# mean and sd of d1 (by default prcomp assumes center=T)

m1$center
#    Murder    Assault UrbanPop          Rape
#      7.788    170.760      65.540      21.232
m1$scale
#      Murder      Assault    UrbanPop           Rape
# 4.355510 83.337661 14.474763    9.366385

# PC loading (eigen) vectors in the rotation matrix

m1$rotation
#                              PC1              PC2              PC3               PC4
# Murder      -0.5358995    0.4181809 -0.3412327    0.64922780
# Assault    -0.5831836    0.1879856 -0.2681484 -0.74340748
# UrbanPop -0.2781909 -0.8728062 -0.3780158    0.13387773
# Rape          -0.5434321 -0.1673186    0.8177779    0.08902432

# sqrt(eigenvalues)
m1$sdev
# 1.5748783 0.9948694 0.5971291 0.4164494

# score vectors
d2 = m1$x
apply(d2,2,sd)
#             PC1             PC2             PC3             PC4
# 1.5748783 0.9948694 0.5971291 0.4164494

# eigen() function
```

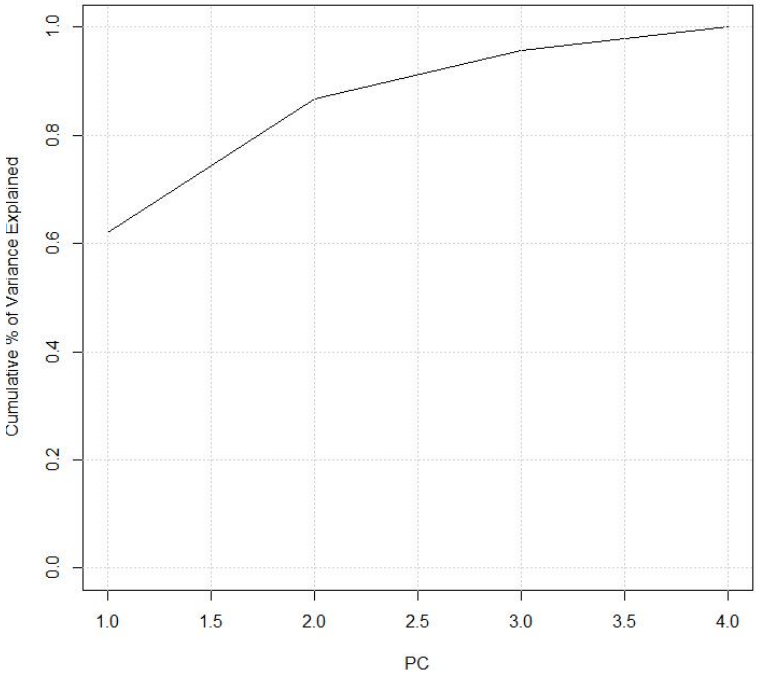
```
#============================================================

cova=var(scale(d1))
#                    Murder      Assault      UrbanPop          Rape
# Murder      1.00000000 0.8018733 0.06957262 0.5635788
# Assault    0.80187331 1.0000000 0.25887170 0.6652412
# UrbanPop 0.06957262 0.2588717 1.00000000 0.4113412
# Rape          0.56357883 0.6652412 0.41134124 1.0000000

m2 = eigen(cova)
#$values
#[1] 2.4802416 0.9897652 0.3565632 0.1734301
#$vectors
#                [,1]            [,2]            [,3]              [,4]
#[1,] 0.5358995    0.4181809 -0.3412327    0.64922780
#[2,] 0.5831836    0.1879856 -0.2681484 -0.74340748
#[3,] 0.2781909 -0.8728062 -0.3780158    0.13387773
#[4,] 0.5434321 -0.1673186    0.8177779    0.08902432

# covariance matrix of transformed data
var(d2)
#                        PC1                  PC2                  PC3                  PC4
# PC1    2.480242e+00    6.706371e-17    4.573978e-17 -3.198568e-16
# PC2    6.706371e-17    9.897652e-01 -9.581526e-17 -1.516830e-16
# PC3    4.573978e-17 -9.581526e-17    3.565632e-01    5.281033e-17
# PC4 -3.198568e-16 -1.516830e-16    5.281033e-17    1.734301e-01

# Big lambda diagonal matrix (eigenvalues in diagonal)
# eigenvalues in main diagonal
sum(diag(var(d2)))          # 4
# covariances (off diagonal) all equal to 0    (PCs uncorrelated)
# PC1 with largest variance across states

# Use eigenvectors to define the PC variables.
#====================================================================

m1$rotation
#                            PC1             PC2             PC3               PC4
# Murder      -0.5358995    0.4181809 -0.3412327    0.64922780
# Assault    -0.5831836    0.1879856 -0.2681484 -0.74340748
# UrbanPop -0.2781909 -0.8728062 -0.3780158    0.13387773
```

```
# Rape          -0.5434321 -0.1673186     0.8177779     0.08902432

# PC1 = 0.536 Murder + 0.58Assault + 0.28 UrbanPop + 0.543 Rape
# A weighted average of crime rates (almost exclude UrbanPop)

# PC2 = 0.4 Murder - 0.87 UrbanPop
# Weighted average of Urban Pop and Murder


# transformed variables in the principal components space.
#============================================================
# eigenvectors span a new p-dimensional space
# score vectors are the transformed observations in this new space
d2 = m1$x
head(d2)
#                              PC1               PC2                PC3                 PC4
# Alabama        -0.9756604     1.1220012 -0.43980366     0.154696581
# Alaska          -1.9305379     1.0624269     2.01950027 -0.434175454
# Arizona         -1.7454429 -0.7384595     0.05423025 -0.826264240
# Arkansas         0.1399989     1.1085423     0.11342217 -0.180973554
# California -2.4986128 -1.5274267     0.59254100 -0.338559240
# Colorado       -1.4993407 -0.9776297     1.08400162     0.001450164
tail(m1$x)
#                                  PC1              PC2               PC3              PC4
# Vermont             2.7732561     1.3881944     0.83280797 -0.1434337
# Virginia           0.0953667     0.1977278     0.01159482     0.2092464
# Washington         0.2147234 -0.9603739     0.61859067 -0.2186282
# West Virginia 2.0873931     1.4105263     0.10372163     0.1305831
# Wisconsin          2.0588120 -0.6051251 -0.13746933     0.1822534
# Wyoming             0.6231006     0.3177866 -0.23824049 -0.1649769


# Variance of the PCs are the eigenvalues
#============================================================

apply(d2,2,var)
#            PC1            PC2            PC3            PC4
# 2.4802416 0.9897652 0.3565632 0.1734301

# proportion of variance explained (PVE) by each PC
#============================================================
```

```
# variance of PCs
aux=m1$sdev^2
#    2.4802416 0.9897652 0.3565632 0.1734301

sum(aux)      # 4
pve=aux/sum(aux)
# [1] 0.62006039 0.24744129 0.08914080 0.04335752

m2$values/4
# [1] 0.62006039 0.24744129 0.08914080 0.04335752
# each eigenvalue divided by 4

cumsum(pve) # [1] 0.6200604 0.8675017 0.9566425 1.0000000
#    87% variability in the dataset explained by PC1 & PC2

# plots
plot(pve, xlab="PC", ylab="% of Variance Explained", ylim=c(0,1),type='l')
grid()
```




```
plot(cumsum(pve), xlab="PC", ylab="Cumulative % of Variance Explained", ylim=c(0,1),type='l')
grid()
```



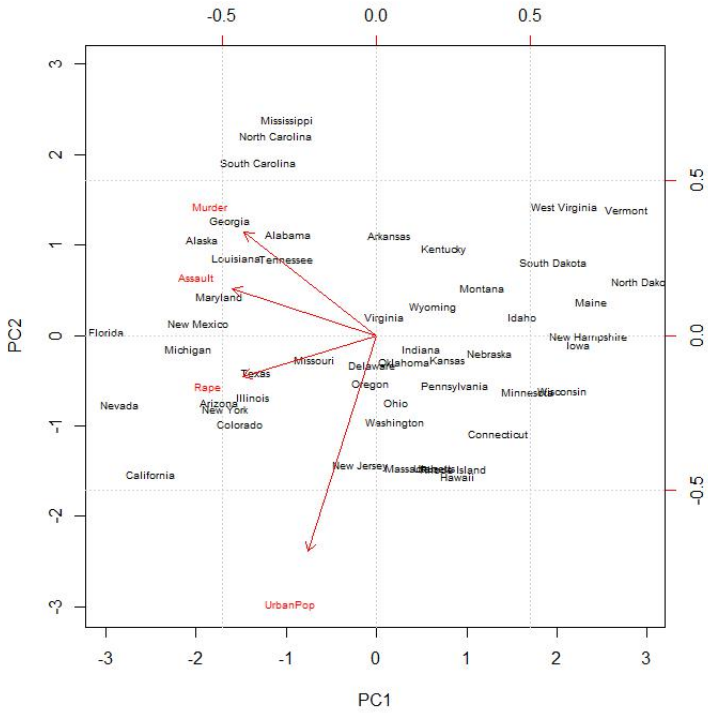

# biplots

#============================================================

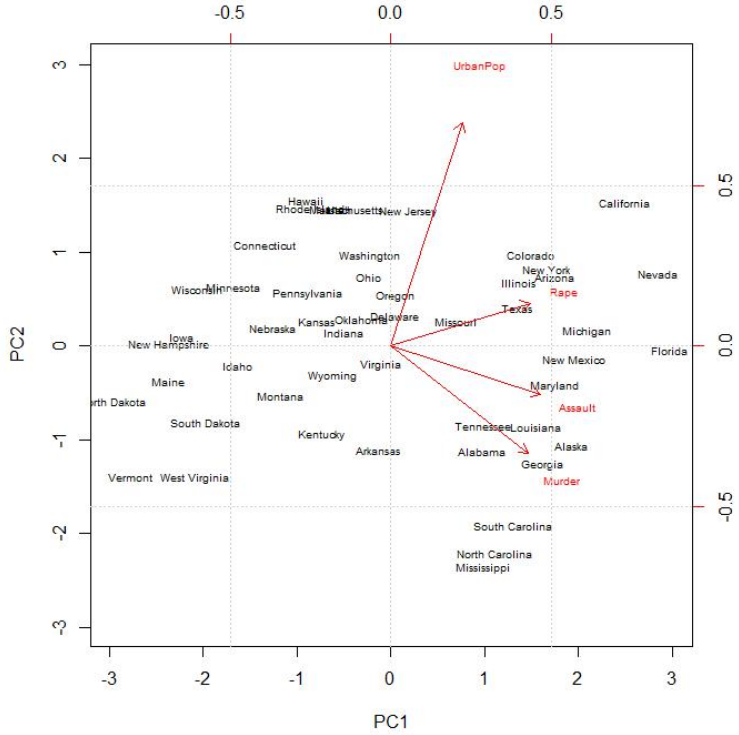
biplot(m1, scale=0)

biplot(m1, scale=0,cex=0.6)

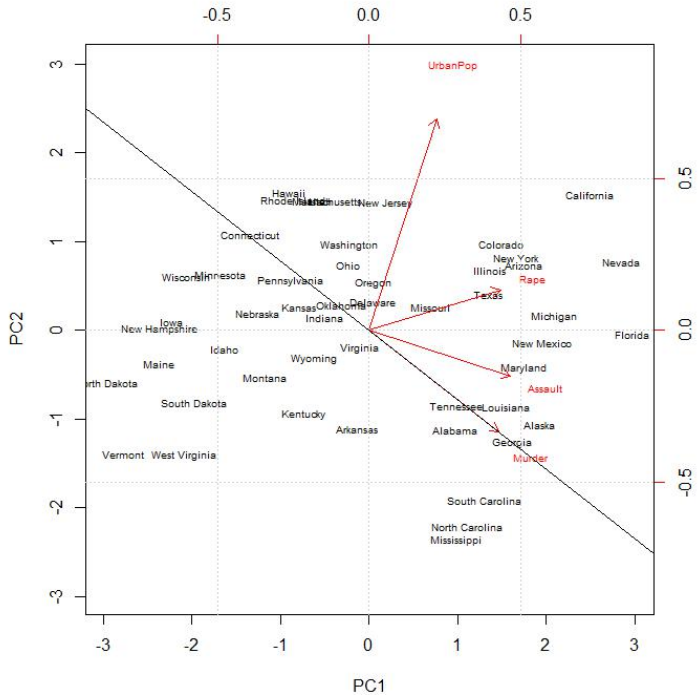
grid()



```
# mirror image
m1$rotation=-m1$rotation
m1$x=-m1$x
biplot(m1, scale=0,cex=0.6)
grid()
```




```
# axis found in 1st two cols of rotation matrix

rot=m1$rotation
#                          PC1          PC2          PC3          PC4
# Murder      0.5358995 -0.4181809  0.3412327 -0.64922780
# Assault     0.5831836 -0.1879856  0.2681484  0.74340748
# UrbanPop 0.2781909  0.8728062  0.3780158 -0.13387773
# Rape          0.5434321  0.1673186 -0.8177779 -0.08902432

# Murder axis
slope1=rot[1,2]/rot[1,1]
slope1      # -0.7803345
abline(0,slope1)
```




# interpret the PCs

#==========

# states are the observations

# states with large values in PC1 have high crime rates

# (PC1 weights -col1- in rotation are 0.5359, 0.5831, 0.5434)

# California, Nevada, Florida vs North Dakota, Vermont

# states with large values in PC2 have high urban areas

# (PC2 largest weight -col2- in rotation is 0.8728)

# California vs Mississippi

# original vs transformed values

#==========

```
d3=data.frame(d1,d2)
head(d3)
#                Murder Assault UrbanPop Rape        PC1        PC2        PC3         PC4
# Alabama          13.2     236       58 21.2 -0.9756604  1.1220012 -0.43980366  0.154696581
# Alaska           10.0     263       48 44.5 -1.9305379  1.0624269  2.01950027 -0.434175454
# Arizona           8.1     294       80 31.0 -1.7454429 -0.7384595  0.05423025 -0.826264240
# Arkansas          8.8     190       50 19.5  0.1399989  1.1085423  0.11342217 -0.180973554
# California        9.0     276       91 40.6 -2.4986128 -1.5274267  0.59254100 -0.338559240
```

```
# Colorado       7.9      204       78 38.7 -1.4993407 -0.9776297  1.08400162  0.001450164
tail(d3)
#                Murder Assault UrbanPop Rape       PC1        PC2        PC3         PC4
# Vermont         2.2      48       32 11.2 2.7732561  1.3881944  0.83280797 -0.1434337
# Virginia        8.5     156       63 20.7 0.0953667  0.1977278  0.01159482  0.2092464
# Washington      4.0     145       73 26.2 0.2147234 -0.9603739  0.61859067 -0.2186282
# West Virginia   5.7      81       39  9.3 2.0873931  1.4105263  0.10372163  0.1305831
# Wisconsin       2.6      53       66 10.8 2.0588120 -0.6051251 -0.13746933  0.1822534
# Wyoming         6.8     161       60 15.6 0.6231006  0.3177866 -0.23824049 -0.1649769
```