Taxi Routing Optimization

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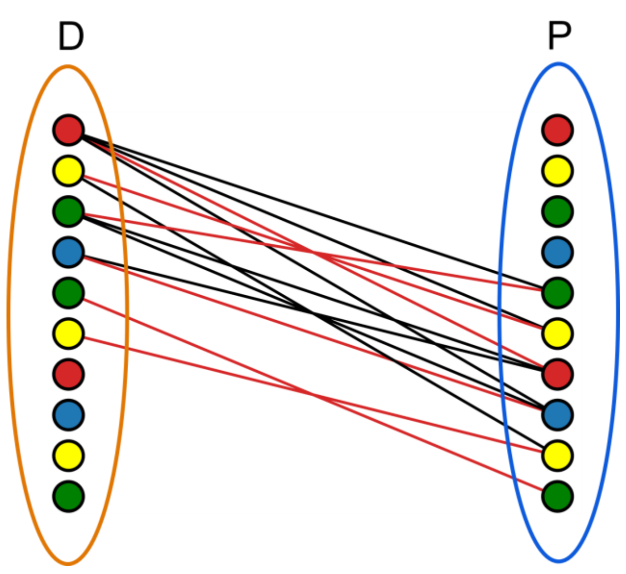
INTRODUCTION

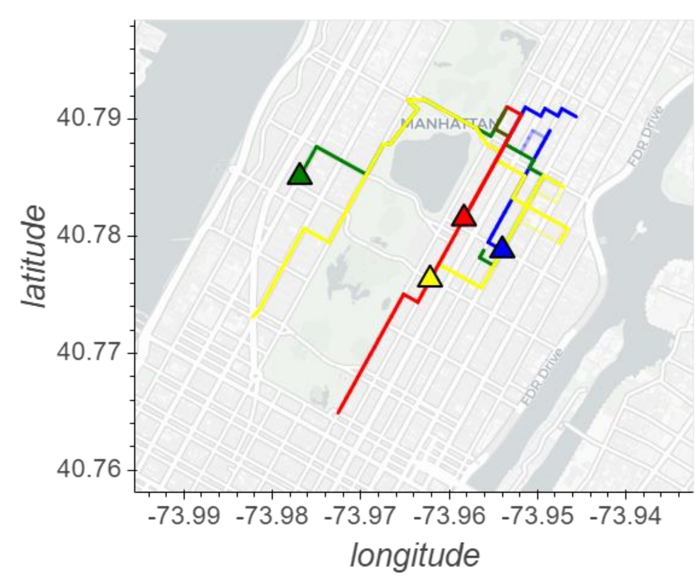
Bipartite Graph for Taxi Routing

- Consider a bipartite graph $G = (D, P, E)$
- Let $D = \{(d_1, T_1^d), \cdots, (d_n, T_n^d)\}$ denote the set of drop-off nodes, $P = \{(p_1, T_1^p), \cdots, (p_n, T_n^p)\}$ denote the set of pickup nodes, where there are n taxi trips to be covered, $N = \{(p_1, T_1^p, d_1, T_1^d), \cdots, (p_n, T_n^p, d_n, T_n^d)\}$; i.e., the i^{th} trip picks up a passenger at time T_i^p at point p_i and drops off the fare at time T_i^d at point d_i, $i \in \{1, \cdots, n\}$
- $E = \left\{\left\{(d_i, T_i^d), (p_j, T_j^p)\right\} : T_j^p - T_i^d - \delta \le time(d_i, p_j) \le T_j^p - T_i^d\right\}$
- Elapsed time between the drop-off time for d_i and the pickup time for p_j, that is, $T_j^p - T_i^d$, is at least the time needed to travel between these two points $time(d_i, p_j)$ and at most $time(d_i, p_j) + \delta$; hence, the taxi can reach the new pickup in time, and does not need to wait more than δ minutes for the pickup to be ready

Lemma Given any matching of size m, there exists a covering of all n trips with $n - m$ taxis

- Application: *Maximum cardinality matching gives rise to the minimum number of taxis that cover all trips [1][2]*



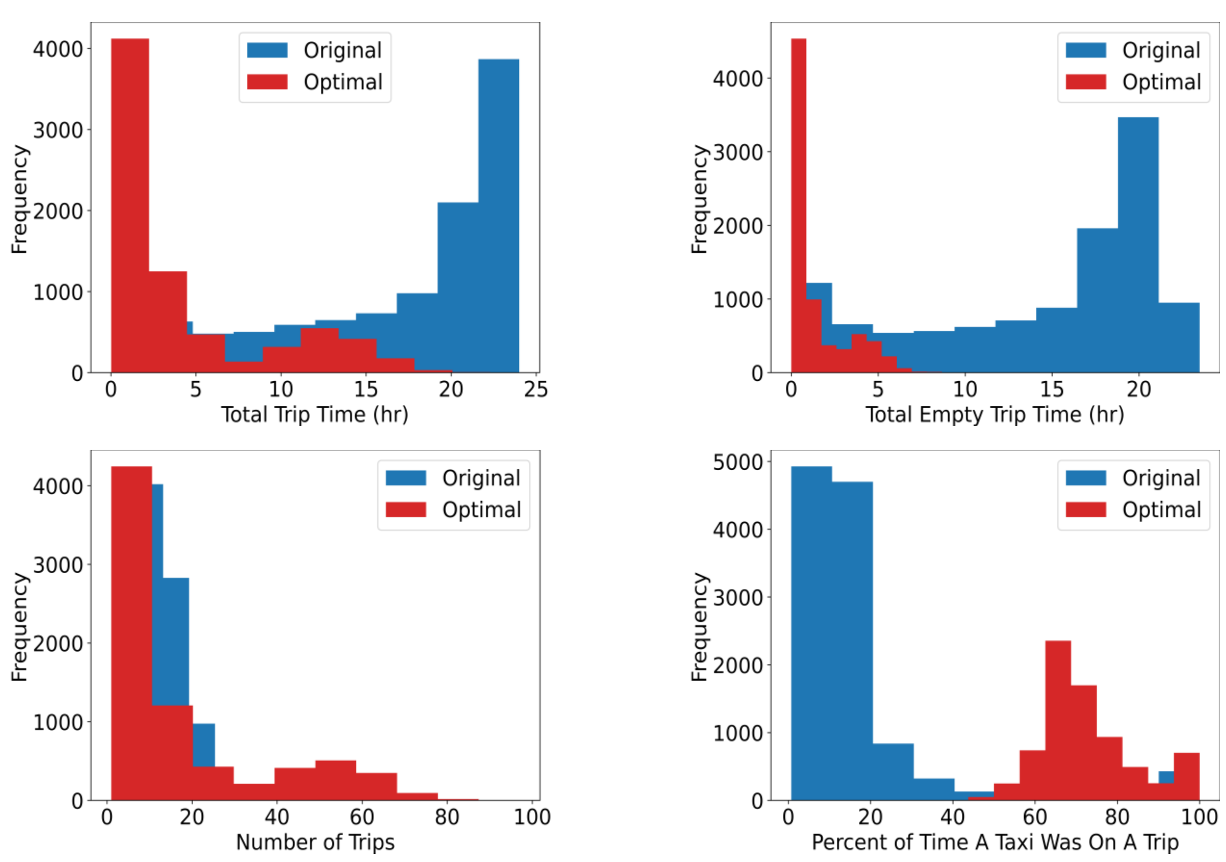


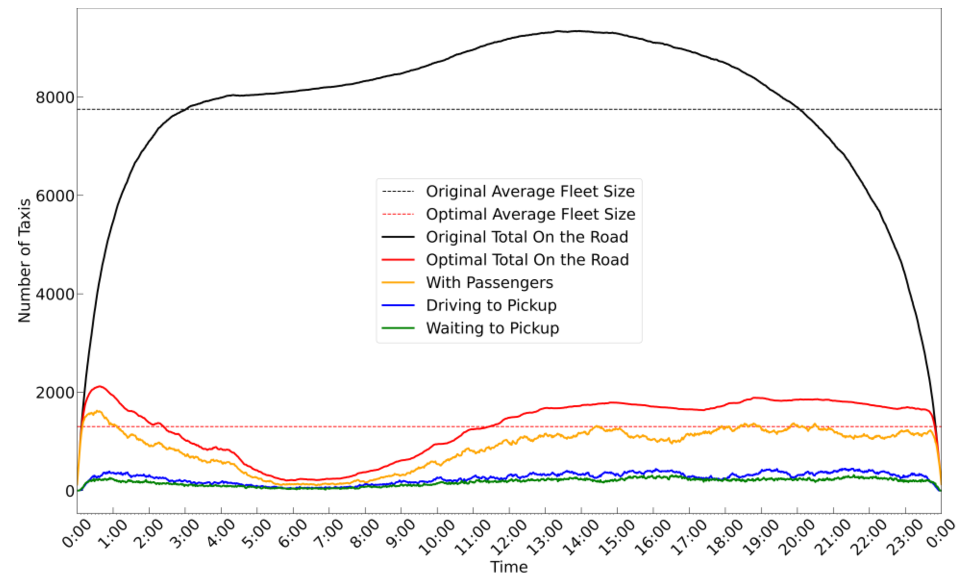
- An example of 10 trips that can be covered by 4 taxis
- Maximum cardinality matching ($m = 6$) is colored in red
- Nodes are colored with respect to their corresponding taxi routes

COMPUTATIONAL RESULTS

The optimal taxi routing (with $\delta = 10$) is compared with the actual one

- Total number of taxis that covers all trips reduces from 11568 to 7475, a *35%* reduction
- Total trip time and empty trip time are greatly reduced, from 16.1 to 4.2 hours and from 14.3 hours to 1.4 hours, respectively, on average
- To offset the reductions in trip time, the number of trips and on-trip percentage are largely increased, from 11.3 to 17.5 and from 17.1% to 72.6% respectively, on average





A comparison of the time distribution of the optimal taxi fleets (with breakdown) and the actual taxi fleets

- On average, number of circulating taxis reduces from 7748 to 1300, an *83%* reduction

THEORETICAL RESULTS

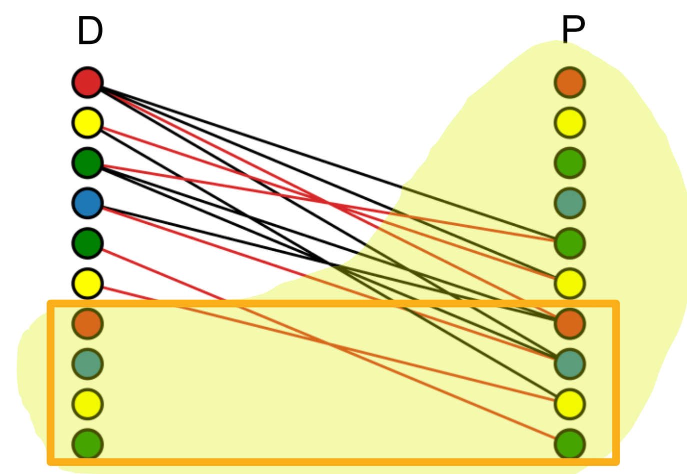
Min-max theorem is an important concept in solvable discrete optimization problems; a min-max theorem is proved in the taxi routing problem

- Consider a simpler setup: $E = \left\{\left\{(d_i, T_i^d), (p_j, T_j^p)\right\} : time(d_i, p_j) \le T_j^p - T_i^d\right\}$; no upper bound ($\delta$) to the waiting time

Definition Any two trips N_i and N_j are considered *compatible* with each other if a taxi can reach from d_i to p_j or from d_j to p_i in time

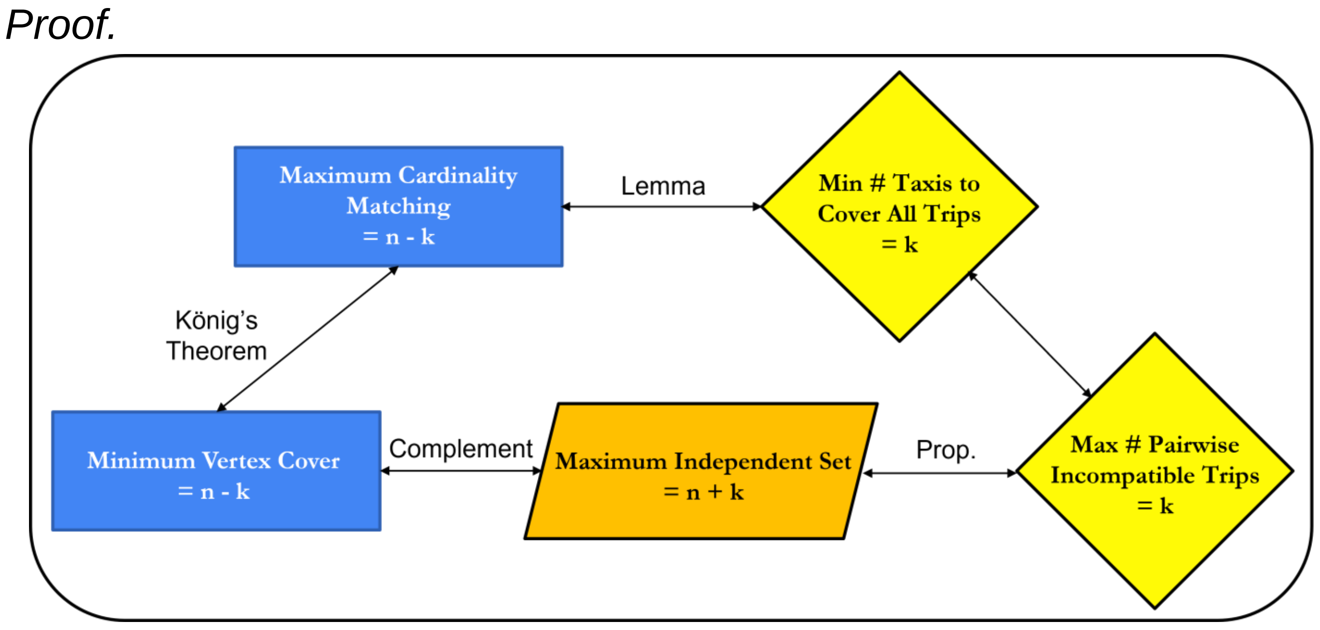
Proposition Consider two disjoint set $P = \{p_1, \cdots, p_n\}$ and $D = \{d_1, \cdots, d_n\}$ and a subset $I \subseteq P \cup D$, where $|I| = n + k$, for some $k \ge 0$; then there exists a subset $K \subseteq \{1, \cdots, n\}$ where $|K| = k$ and $S = \bigcup_{i \in K} \{p_i, d_i\} \subseteq I$

- Application: An independent set of size $n + k$ (nodes in shaded region) leads to k trips that are *pairwise incompatible* (orange box)



Theorem
The *maximum size of a set of trips that are pairwise incompatible* is equal to the minimum number of taxis needed to cover all trips

Proof.



FUTURE WORK

- The dual object when there's an upper bound (δ) to the waiting time for the new pickup?
- An optimal taxi routing that accounts for drivers' rest times

REFERENCES

[1] Vazifeh, Mohammad M., et al. "Addressing the minimum fleet problem in on-demand urban mobility." Nature 557.7706 (2018): 534-538.

[2] Zhan, Xianyuan, et al. "A graph-based approach to measuring the efficiency of an urban taxi service system." IEEE Transactions on Intelligent Transportation Systems 17.9 (2016): 2479-2489.