A Method of Computing Equilibrium for the Partnership Formation

Junwu Zhu, Heng Song
School of Information Engineering
YangZhou University
YangZhou, China
Email: jwzhu@yzu.edu.cn, song_heng@foxmail.com

Fang Liu
School of Management
YangZhou Polytechnic College
YangZhou, China
Email: fangliu_yz@hotmail.com

Guocheng Yin
School of Guangling
YangZhou University
YangZhou, China
Email: gcyin@yzu.edu.cn

***Abstract*—In the model of partnership formation, every agent is in hot pursuit of maximizing his own utility. Doft Talman and Zaifu Yang have revealed the necessary condition of the existence of partnership equilibrium, however, the methods of how to compute the equilibrium are not given. The paper presents a method to compute the partnership utility equilibrium based on socially optimal assignment. Firstly, the partnership groups with social optimality are exhausted in the all possible cooperative pairs, and then under the condition of the existence of equilibrium, an algorithm is given to get the Equilibrium Utility Vector so as to approach the equilibrium status after multiple iterations. This solution to computing partnership payoff equilibrium can be obtained in $O(n^2)$ time. At last, to verify the method given by this paper, an example of municipal water supply planning is given to illustrate the correctness and effectiveness.**

***Keywords*-Partnership, Equilibrium, Socially optimal, Utility.**

I. Introduction

There are a lot of resource competitions in the environment of cloud computing and multi-agent system [1], [2]. In order to promote his competitive power and gain a greater utility, every agent tends to seek a partner and form a coalition. If an agent takes part in a partnership and can be distributed an extra utility, he will inevitably join the coalition to maximize his own utility[3], [4]. In real life, cooperation is also one of the most common social relations. For example, when the national table tennis team participates in an international competition, every athlete can act along or seek a partner to join the competition. Therefore, team members will contemplate whether to act along and take as utility his value or to seek a partner and share with his partner the joint utility, the key of decision is the utility(Including ranking and prizes) he can get as high as possible[5]. In the literature [6],Talman and Zaifu presented a model of binary partnership formation and revealed the necessary condition for the existence of equilibrium. On the equilibrium status, the total utility of multi-agent system is maximum, and the partnership is stable and rational. So, they have no motivation to break the status.

In practice, not only the existence of partnership equilibrium should be judged firstly, but also how to group agents and distribute the joint utility on equilibrium status should also be taken into consideration. Because the total utility in equilibrium must be maximum, we can get the grouping of agents by calculating the maximum total utility. In addition, from the individual point of view, the fact that the sum of utility obtained from cooperative agents must be equal or greater than their joint value,which can make the coalition stable. So, getting the individual utility on the equilibrium status has great necessity and practical significance.

This paper presents a method of finding utility equilibrium allocation for the partnership formation problem, including the grouping algorithm based the maximum total utility and the joint utility sharing algorithm. The structure of this paper are organized as following. In section2,we introduce the related work about forming partnership or joining coalition and joint utility sharing. In section 3, the formal definitions about problem model and related concept are given; The grouping algorithm based on the maximum total utility and the joint utility sharing algorithm are designed in section 4 and section 5 respectively. In section 6, we take municipal water supply planning for example to verify the method suggested by this paper. Finally we conclude this paper and look forward to the future research work.

II. Related work

Getting the utility equilibrium for partnership formation is a classic problem about utility sharing in cooperative game theory, and the final utility distribution is one of the key tasks. Because of the particularity of utility sharing problem, it is always studied as a separate section in cooperative game theory[7].

In a transferable utility cooperative game[8],when many agents form partnership, the joint utility sharing which obtained by these agents is one of the most fundamental problem which must be solved. The Core concept is first introduced by D.B.Gililes as a tool to study stable coalitions, all sharing sets which can make partnership stable are called Core[9].In fact,Core is a sharing scheme that any sharing in Core don't cause the participants out of their coalition because forming a new partnership can't make participants combinations obtain higher utility. This idea was first introduced in 1881 by the Edgeworth[10] as economics research,

978-1-4799-2002-0/15/$31 ©2015 IEEE

he supposes that the concept can replace Walras[11] competitive equilibrium. Debrea and Scrafy [12] have proved two conclusions about exchange economy: (1) the competitive equilibrium configuration set is included in Core, which shows that the coalition is stable in the competitive equilibrium configuration. (2) the Core is convergent and it shows that the Core will converge to the competitive equilibrium configuration set with the enlargement of economic scale. Both conclusions uncover a intimate relationship between competitive equilibrium and Core: in a large scale economy, Walras competitive equilibrium is Core.

The definition of Core shows that Core may not exist and it will rupture the formed partnership; It also shows that Core may contain the only solution and it is valuable in practice. But, in more general case, it may have lots of solutions and these solutions form a set called "solution set". The "solution set" can be figured out by Linear programming, but Core with many solutions have inevitable defects: it is unfavourable for handling the conflict and dispute because each agent has his own reasonable solution. Therefore, according to the practical background of the game, disposing the "solution set" properly has a good practical significance.

Shapley Value[13] proposed by Shapley in 1953 solved this problem well, it is a solution with great practical value. The main particularity is to share utility according to the marginal contribution and the utility which the agent could get is equal to the excepted value of the marginal contributions made by the agent to the joined coalition[14]. In game (N, v), suppose $v(\varnothing) = 0$, the number of the agents in coalition S is s, according to Shapley Value,the joint utility $v(N)$ will be shared as the following formula: $\Phi_i(v) = \sum_{s \subseteq N \backslash i} \frac{s!(n-s-1)!}{n!}(v(S \cup \{i\}) - v(S))$. The idea of Shapley Value can be understood as probability: Assume that the agents form a coalition in random sequence,then there will be $n!$ sequences for agent set N. Agent i can form coalition S with the other $(|S| - 1)$ agents, the marginal contribution made by the agent to his coalition is $v(\{S\}) - v(\{S \backslash \{i\}\})$. Because there are $(|S| - 1)!(n - |S|)!$ sequences for agent set $S \backslash i$ and$N \backslash S$[15],the excepted value for marginal contribution made by agent i to coalition S is $\sum_{s \subseteq N \backslash i} \frac{(s-1)!(n-s)!}{n!}(v(S \cup \{i\}) - v(S)) = \sum_{s \subseteq N \backslash i} \frac{s!(n-s-1)!}{n!}(v(S \cup \{i\}) - v(S))$.

Although Core and Shapley Value play a vital role in cooperative game theory, both of them have their own shortcomings. For example, because of the problem which Core is over "static" and the fact that bargaining is a dynamic process is not considered, Aumann and Maschler[16] introduce bargaining set in 1964. In addition, Schmeidler[17] proposed the nucleolus in 1969 and the main idea includes three properties: Meeting the rationality of individual and the coalition; Every game problem has only one nucleolus; If there is a core, nucleolus is a part of the core; Nucleolus reflects the egalitarian idea, Shapley Value follows the principle of utilitarian [18].

Therefore, for the game model which the number of cooperative agents at most is 2, this paper presents an effective equilibrium utility sharing algorithm based on the socially optimal assignment. Under the premise of having calculated the maximum total utility, differs from Shapley Value, a relatively rational equilibrium utility vector in the cooperative game is found from Core which with lots of solutions by using this algorithm and the game reached a stable equilibrium status .

III. THE MODEL

Suppose there are n agents and let $N = \{1, 2, , n\}$ denotes the set of agents. Assume that every agent can act alone or cooperate with only one partner to obtain utility. If agents i cooperates with $j(i, j \in N, i \neq j)$, they will obtain the joint utility $v(\{i, j\})$. If agent i act alone($i \in N$), the utility $v(\{i\})$ will be gotten. Let utility vector $r = (r_1, r_2, ..., r_n)$, where r_i is the utility of agent i gets in the cooperative game. Let $I_k = \{1, ..., k\}, k > 0$ and k denotes the number of a partition on N. A partition $P = \{U_1, ..., U_k\}$ on N is an assignment of N satisfying that $|U_h| \leq 2$ for every $h \in I_k$.

Definition 1 (Equilibrium Allocation): If an allocation (P^*, r^*) on N satisfying that $r_i^* \geq v(\{i\})$ for every $i \in N$ and $r_j^* + r_h^* \geq v(\{j, h\})$ for every $j, h \in N, j \neq h$, then the allocation is called equilibrium allocation.

any agent i in the game can obtain the utility r_i^* at least equal the utility he gets whether act along or cooperative with other agents.Therefore, one agent in equilibrium status cannot obtain higher utility by revising partnership and he has no subjective intention to leave the current allocation.

Definition 2 (Socially Optimal Assignment): An assignment $\bar{P} = \{\bar{U}_1, ., \bar{U}_k\}$ is socially optimal assignment if $\sum_{i=1}^{k} v(\bar{U}_i) \geq \sum_{j=1}^{l} v(\bar{U}_j)$for any allocation $P = \{U_1, ..., U_l\}$ on N. For a socially optimal assignment $\bar{P} = \{\bar{U}_1, ., \bar{U}_k\}$, the value $\sum_{i=1}^{k} v(\bar{U}_i)$ is called the social value or socially maximum total utility.

Socially optimal assignment is an assignment with maximum total utility in all assignments of a game. There may be one or more socially optimal assignments in a game.

Definition 3 (Reasonable Partners): In a game, if the utility that agent i and j obtain satisfies that $v(\{i, j\}) \geq v(\{i\}) + v(\{j\})$, we call agent i and j reasonable partners.

The reasonable partners set is denoted by $RP = \{\{i, j\} | v(\{i, j\}) \geq v(\{i\}) + v(\{j\})\}$

Definition 4 (Marginal Value): The marginal value of agent i and j is defined as: $m(\{i, j\}) = v(\{i, j\}) - v(\{i\}) - v(\{j\})$.Reasonable partnership can make cooperative agents get positive marginal value.

In a game(N, v) with an allocation(P, r), there is a mapping f among agents in N, let $f(i) = i$ if agent i acts along, $f(j) = h$ if agent h cooperate with j in P. Let $I = \{i|f(i) = i\}, C = \{h|f(j) = h, j \neq h\}$, the set A denotes that the set of agents who act alone, the set B denotes that the set of agents who form partnership.

Definition 5 (Game Allocation): The game allocation is a triple and we define game allocation by $ICV =< I, C, V >$, where $I = \{i|f(i) = i\}, C = \{(j,h)|f(j) = h, j \neq h\}, V = \sum_{i \in I} v(i) + \sum_{\{j,h\} \in C)} v(\{j,h\})$.V represents the total utility of the allocation in the game.

Let $uICV =< N, \varnothing, uV >$ and $uV = \sum_{i \in N} v(i)$.The set of all possible game allocations in a game is defined as $ICVS$ and $ICV_i \subseteq ICVS, 1 \leq i \leq t, t$ is the number of all possible allocation in the game.

Definition 6 (Utility Vector): Let $r^N = (v(\{1\}), ...,$ v({n})) denote the vector of all agents independent values. A utility vector r is rational utility vector if $r \geq r^N$.

For any rational utility vector r, the net utility function of agent $i \subseteq N$ is defined by $L^i(r) = max\{r_i, v(\{i,h\}) - r_h, h \neq i\}$.

Proposition 1: If an allocation (P^*, r^*) is an equilibrium allocation in a game, then the assignment P^* is socially optimal assignment.

Proof: Proof Let $P^* = \{U_1^*, ..., U_k^*\}$ and take any assignment $P = U_1, ..., U_l$ in N. Let $I = \{i|f(i) = i\}, C = \{h|f(j) = h, j \neq h\}$. Since (P^*, r^*) is an allocation for (N, v), then $\sum_{i=1}^{k} v(U_i^*) = \sum_{i \subseteq N} r_i^*$, from the definition of equilibrium allocation, we have

$$\begin{aligned}\sum_{i=1}^{k} v(U_i^*) &= \sum_{i \in N} r_i^* \\ &= \sum_{i \in I} r_i^* + \sum_{\{j,h\} \subseteq C} (r_j^* + r_h^*) \\ &\geq \sum_{i \in I} r_i^* + \sum_{\{j,h\} \in C} v(\{j,h\}) = \sum_{j=1}^{l} v(U_j)\end{aligned}$$

This shows that the assignment P^* is socially optimal assignment. ■

From the proof of proposition 1, we know that if an allocation(P^*, r^*) is equilibrium allocation, $\bar{P}$ is a socially optimal assignment, then $(\bar{P}, r^*)$ is socially optimal allocation,so the equilibrium assignment can be gotten by searching socially optimal assignment.

IV. THE SOCIALLY OPTIMAL ASSIGNMENT SEARCHING ALGORITHM

Suppose that each agent is reasonable in a game, if agent i and j are reasonable partners, the pair of agents will have chance to become cooperative partner, because their utility will grow after they cooperate,it is rational to make the pair of agents with positive marginal value form partnership, and the pair of agents with negative marginal value are ignored because cooperation cant bring extra utility for them. Therefore, to get the socially optimal assignment, we only need to find the allocation with maximum total utility from the allocations where all agent pairs are reasonable partners.

According to the idea above, the tree can be used to visualize the process of searching socially optimal assignment. After the whole tree is produced, the leafs with maximum total utility on the tree denotes socially optimal assignment. Specific process can be seen in the following practical case.

Algorithm 1: Searching Possible Equilibrium Location

Input: the initial ICV $uICV$; reasonable partners;
Output: all possible social optimal allocation $ICVS$;

$first \leftarrow true$,$count \leftarrow 0$;
$Create \quad ICVS \quad NS$;
$Create \quad ICVS \quad US$ and $US \leftarrow \varnothing$;
$US \leftarrow US \cup uICV$;
while *first is true or count* $\neq 0$ **do**
  $first \leftarrow false$,$count \leftarrow 0$;
  foreach $ICV \ in \ US$ **do**
    if $Exist\{i,j\} \in RP \ and \ i,j \in I$
    then $C \leftarrow \{i,j\} \cup C$;
      $I \leftarrow I \backslash \{i,j\}$;
      $V \leftarrow \sum_{i \in I} v(i) + \sum_{\{j,h\} \in C} v(\{j,h\})$;
      $NS \leftarrow NS \cup ICV$;
      $count++$;
    else $NS \leftarrow NS \cup ICV$;
  end
  $US \leftarrow NS$,$NS \leftarrow \varnothing$;
end
return *the ICV that the V is max in ICV of US*

In fact, The algorithm above is a recursion and the ineligible allocation will be reduced in the progress of recursion.The analysis of time complexity is the progress of analyzing the running time that one algorithm spends. The quality of an algorithm depends on the worst-case running time which is the longest running time for the algorithm whatever are input. Suppose there are n agents and the running time of the algorithm is $T(n)$. In the worst-case, any pairs of agents can form reasonable partners and there is an recursive formula:

$$f(n) = \begin{cases} C_n^2 T(n-2) & \text{when } n > 2, \\ 1 & \text{when } x \leqslant 2. \end{cases} \quad (1)$$

then $T(n) = C_n^2 * C_{n-2}^2 * C_{n-4}^2 * \cdots * C_2^2$,so $T(n) = \frac{n(n-1)}{2} * \frac{(n-2)(n-3)}{2} * \frac{(n-4)(n-5))}{2} * \cdots * \frac{2*1}{2} = \frac{n!}{2^{\frac{n}{2}}} = O(n!)$

V. THE EQUILIBRIUM UTILITY SHARING ALGORITHM

From the proof of proposition 1, we know that the equilibrium allocation is the sufficient condition of socially optimal assignment. In a game, if equilibrium allocation exists, then it must be included in socially optimal assignment. Therefore, the equilibrium allocation can be gotten by socially optimal assignments searched. The algorithm then proceeds as follows.

First, in the socially optimal allocations of a game,we initialize the utility vector r^0:

$$r_i^0 = \begin{cases} v(i) & \text{if } i \in I \text{ } or \text{ } i \in C \& f(i) = j \& i \preccurlyeq j. \\ v(\{i,j\}) - v(j) & \text{if } i \in C \& f(i) = j \& i \succcurlyeq j. \end{cases} \tag{2}$$

For the agent i who acts alone, he will obtain the utility $r_i = v(i)$; for the agent j and h who form partnership, the preference of the pair of agents should be determined, we assume that the preference is $j \succcurlyeq h$ and then we have $r_j = v(j,h) - v(h), r_h = v(h)$.

Second,for the agents who form partnership,denoting $r_j = L^j(r)$,$r_h = v(j,h) - r_j$,then a new utility vector $r^1 = (r_1^1, \cdots r_n^1)$ is generated .Comparing r_h with $L^h(r^1)$, if $r_h < L^h(r^1)$, by the way of denoting $r_h = L^h(r^1), r_j = v(j,h)Cr_h$ to generate a new utility vector r^2. Repeating step2 until each agent$i$ satisfy $r_i^n \geq L^i(r^{n-1})$.The utility of each agent will converge and shown as $r_i^n = r_i^{n-1}$, since the utility of each agent i satisfying $r_i^n \geq L^i(r^{n-1}) \geq v(\{i,j\}) - r_j^n(i \neq j)$, that is $r_i^n + r_j^n \geq v(\{i,j\})$, so r^n is the equilibrium utility vector r^*.

Algorithm 2: Get Equilibrium Utility Vector

```
Input: all possible social optimal allocation ICVS;
Output: the Equilibrium Utility Vector set ℝ*;
foreach ICV in ICVS do
    foreach i ∈ I do
        r_i ← v({i});
    end
    foreach (j,h) ∈ C do
        r_j ← v({j,h}) − v(h);
        r_h ← v({h});
    end
    while exists(i,j) ∈ C and r_i < L^i(r) do
        r_i ← L^i(r);
        r_j ← v({i,j}) − r_i;
    end
    ℝ* ← ℝ* ∪ r;
end
return ℝ*
```

Suppose there are n agents and the running time of the algorithm is $T(n)$. In the worst case, there are $\frac{n}{2}$ pairs of agents who form partnership and 0 agent act alone, each loop time cost is $\frac{n}{2} * \frac{n}{2} + \frac{n}{2} * \frac{n}{2} = \frac{n^2}{2}$ and after k times while loop, then $T(n) = \frac{1}{2} * kn^2 = O(n^2)$

VI. EXAMPLE

There are 5 adjacent cities named A, B, C, D, E in Province T, and each city need to build a municipal water supply project and they can cooperate with each other. After finishing the project and deducting the cost(Including construction cost and communication cost), the detail fiscal revenue of the cities are as follows:

If each city act alone, A, B, C, D, E will obtain fiscal revenue about $100, 300, 400, 700, 100$, thousand dollars respectively; If A cooperate with one of City B, C, D, E, the joint fiscal revenue about $100, 600, 1100, 700$ thousand dollars will be obtained respectively; If B cooperative with one of C, D, E, the joint fiscal revenue about $1200, 800, 300$ thousand dollars will be obtained respectively; If C cooperative with one of D,E, the joint fiscal revenue about $1600, 1700$ thousand dollars will be obtained respectively; If D cooperative with E, the joint fiscal revenue about 400 thousand dollars will be obtained respectively.

Due to the communication cost increased with the increase of the number of cooperative city, the final profit will be little if the number of cooperative city is more than 2 , each city decides not to cooperate with more than 1 cities. For each city, compared with acting alone, cooperation can bring more fiscal revenue, which makes each city prefer to cooperation. So in the process of building the municipal water supply project, the problem about how to cooperate can make Province T obtains the maximum total fiscal revenue and how to share the joint profit gotten by the cooperative cities should be solved to ensure that the municipal water supply project be built successfully. Modeling the case above as the game (N,v), where:

$$N = \{A, B, C, D, E\};$$

$$v(A) = 100, v(B) = 300, v(C) = 400, V(D) = 700, v(E) = 100;$$

$$v(A,B) = 100, v(A,C) = 600, v(A,D) = 1100, v(A,E) = 700;$$

$$v(B,C) = 1200, v(B,D) = 800, v(B,E) = 300;$$

$$v(C,D) = 1600, v(C,E) = 1700;$$

$$v(D,E) = 400;$$

From the definition of reasonable partners, we know that

$$RP = \{\{A,C\}, \{A,D\}, \{A,E\}, \{B,C\}, \{C,D\}, \{C,E\}\}$$

According to "the socially optimal assignment searching algorithm",the process of this algorithm is shown as table1. From the table,we know that each assignment will have only one independent agent in recursion 2,so the reasonable partners cannot be generated, and the recursion is over. Calculating the utility value of each assignment and the

IPV(initialization)	IPV(recursion 1)	IPV(recursion 2)	
Independent={ A, B, C, D, E} Partner = ∅ Value = 1600	Independent={B,D,E} Partner={(A,C)} value=1700		Finishing Recursion
	Independent={B,C,E} Partner={(A,D)} Value = 1900	Independent={E} Partner={(A,D),(B,C)} Value = 2400	
		Independent={B} Partner={(A,D),(C,E)} Value = 3100	
	Independent= {A, D, E} Partner={(B, C)} Value=2100	Independent={E} Partner={(A,D),(B,C)} Value = 2400	
		Independent={D} Partner={(B,C),(A,E)} Value = 2500	
	Independent={B, C, D} Partner={(A, E)}, Value = 2100	Independent={B} Partner={(A,E),(D,C)} Value = 2600	
		Independent={D} Partner={(A,E),(B,C)} Value = 2400	
	Independent={B, A, E} Partner={(D, C)} Value = 2100	Independent={B} Partner={(A,E),(D,C)} Value = 2600	

Figure 1. Searching Socially Optimal Assignment

assignment with maximum utility value is socially optimal assignment.

In this case,the assignment $\bar{P} = \{\{A, D\}, \{B\}, \{C, E\}\}$ with maximum value 3100 ,so the assignment $\bar{P}$ is socially optimal assignment.

For the case above, Spanning tree can be also used to demonstrate the running process of the socially optimal assignment searching algorithm , as shown in Figure 2:

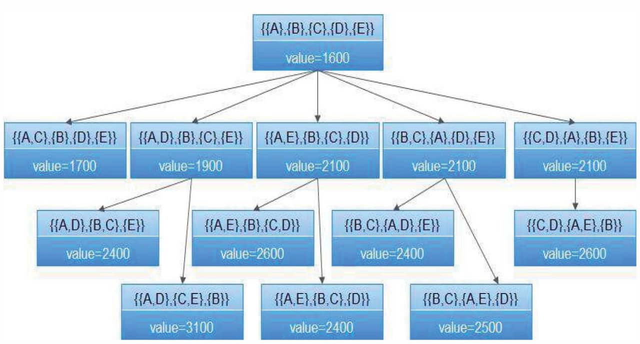

Figure 2. The spanning tree for searching socially optimal assignment

We know that the socially optimal assignment is $\{\{A, D\}, \{B\}, \{C, E\}\}$ and the following problem is sharing the joint utility and it is the most critical,the joint utility sharing will affect that whether the partnership in a game can be formed or not. The equilibrium state in a game represents that each agent will not out of the current partnership.according to "the equilibrium utility sharing algorithm" initializing the utility vector $r^0 = (100, 300, 400, 1000, 1300)$; In the socially optimal assignment, A cooperates with City D,while C cooperates with E,and B acts alone,then the utility of B is $r_B = v(\{B\}) = 300; r_A^1 = L^A(r) = 200, r_D^1 = v(\{A, D\}) - r_A^1 = 900, r_c^1 = L^c(r) = 900; r_E^1 = v(\{C, E\}) - r_c^1 = 800;$

So we get $r^1 = (200, 300, 900, 900, 800)$,then $r_D^2 = L^D(r_1) = 900; r_E^2 = L^E(r^1) = 800$,then $r^2 = (200, 300, 900, 900, 800)$.

Therefore, the utility vector $r^* = r^2 = (200, 300, 900, 900, 800)$ is the equilibrium utility vector in this case.

VII. CONCLUSION AND FUTURE WORK

Modeling the game where agents form the coalition, for the property that equilibrium allocation must be the socially optimal assignment, this paper presents two algorithm to confirm the equilibrium utility vector in a game and the example of municipal water supply planning shows that the confirmed utility vector is stable.

In a game, if there are many potential partnership, the socially optimal assignment searching algorithm still need high time cost. The idea of Edmonds Blossom-Contraction Algorithm can help us find a socially optimal assignment in a shorter polynomial time, but it is not applicable to the game with more than one socially optimal assignments. The future work is designing an efficient approximate algorithm to find all socially optimal assignment in a game. In addition, designing a more suitable utility sharing algorithm is one of our future work.

VIII. ACKNOWLEDGEMENTS

Project supported by the National Nature Science Foundation of China (No.61170201),Innovation Foundation for Graduate Students of Jiangsu Province(No.CXLX12 0916),National Undergraduate Innovative Training Program of China(No.KYLX_1349), Nature Science Funding of Jiangsu Provincial Department of Education(14KJB520041).

REFERENCES

[1] Yang Liu,Yasser Mohamed. Modelling industrial construction operations using a multi-agent resource allocation framework[J]. Engineering, Construction and Architectural Management,2012, 19(4):406-427.

[2] Moore,J.C,Raghav Rao,H.Multi-agent resource allocation: an incomplete information perspective[J]. Projects, Man and Cybernetics, IEEE Transactions on, 1994,2(8):1208-1219.

[3] Iturralde M, Wei Anne,Ali Yahiya,et al. Resource allocation for real time services using cooperative game theory and a virtual token mechanism in LTE networks[C]. Consumer Communications and Networking Conference (CCNC).Las Vegas: IEEE.2012: 879-883.

[4] Suris, Virginia Tech, Blacksburg, et al. Cooperative Game Theory for Distributed Spectrum Sharing[C].Communications,2007.ICC '07. IEEE InternationalConference.Glasgow:IEEE.2007:5282 -5287.

[5] Theo Driessen,Dongshuang Hou.Interaction between Dutch Soccer Teams and Fans:A Mathematical Analysis through Cooperative Game Theory[J].Applied Mathematics,2012,03(01):86-91.

[6] Dolf Talman,Zaifu Yang.A model of partnership formation[J]. Journal of Mathematical Economics, 2011,47(2):206-212.

[7] Shih-Chieh Hsieh,I-ShouUniv,Kaohsiung,et al.Allocation of transmission losses based on cooperative game theory and current injection models[C]. Industrial Technology, 2002. IEEE ICIT '02. 2002 IEEE International Conference.2002: 850-853.

[8] Liu Bingfeng. Partners Selecting Research of Knowledge Sharing in Industry-University Cooperation Based on the Cooperative Game Theory[C]. Intelligent Projects and Applications (ISA), Wuhan:ISA.2011:1-4.

[9] Serrano,R. Cooperative Game Core and Shapely Value[C].Encyclopedia of Complexity and Project Science.Berlin:Springer-Verlag,2007.

[10] Edgeworth F Y. Mathematical psychics:An essay on the application of mathematics to the moral sciences[M].London: Kegan Paul.1881.

[11] John Q.Cheng, Michael P. Wellman. The WALRAS Algorithm: A Convergent Distributed Implementation of General Equilibrium Outcomes[J],Computer Economics Report.1998,12(1):1-24.

[12] Debreu G,Scarf H.The limit theorem on the Core of an economy[J].International Economic Review, 1963,4:235-246.

[13] Shapley L S.A Value for n-Person Games[G].Kuhu H W,Tucker A W.Contributions to the Theory of Games II.Annals of Mathematics Studies,1953:307-317.

[14] U. Faigle,W. Kern.The Shapley value for cooperative games under precedence constraints[J]. International Journal of Game Theory, 1992,21(3):249-266.

[15] Xiaohui Yu,Qiang ZhangProfit.allocation in production cooperative game based on interval Shapley value[J].Trans. Beijing Inst. Technol.,2008,28 (7):655-658.

[16] Aumann R J, Maschler M.The bargaining set for cooperative game[G].Advances in Game Theory. Dresher L, Shapley L S, Tucker A. Princeton:Princeton University Press,1964.

[17] Schmeidler D.The Necleolus of A Characteristic Function Game[J], SIAM Journal on Applied Mathematics,1969,17(6):1163-1170.

[18] Jeroen Kuipers,Manuel A,Mosquera,et al.Zarzuelo.Sharing costs in highways: A game theoretic approach[J].European Journal of Operational Research,2013,228(1):158-168.