STA107 Final Exam Review

Question 1

The event that exactly one of the three events $A, B,$ and C defined on a sample space Ω occurs is $(AB'C') \cup (A'BC') \cup (A'B'C)$.

A. True

B. False

Question 2

If for the events A and B we know that $P(A) - P(B) = 1/3$, then $P(A - B) \leq 1/3$.

A. True

B. False

Question 3

The coefficient of x^2 in the binomial expansion of $(x + 3)^6$ is 15.

A. True

B. False

*Courtesy of Minh Trinh for preparing the review sheet.

Question 4

For the events A and B of a sample space Ω, it is known that $P(A) \neq 0$ and $P(B) \neq 0$. If A and B are independent, then they are also disjoint (mutually exclusive).

A. True

B. False

Question 5

Let A_1 and A_2 be two disjoint events defined on a sample space Ω and assume that B is another event on this sample space with $P(B) > 0$. Then,

$$P(A_1 \cup A_2 | B) = P(A_1|B) + P(A_2|B)$$

A. True

B. False

Question 6

Suppose we throw a fair die repeatedly, and let Y be a random variable denoting the number of throws until 5 appears for the first time. Then, the range of values for Y is $R_Y = \{0, 1, 2, 3, 4, 5, 6\}$

A. True

B. False

Question 7

Let X be a random variable with a geometric distribution. Then, $P(X > k + r | X > k) = P(X > r)$ for any nonnegative integers k and r.

A. True

B. False

Question 8

If the cumulative distribution function F of a random variable X has a jump at the point a, then we have $F(a) = P(X = a)$.

A. True

B. False

Question 9

A box contains n_1 Red and n_2 Black balls. Suppose we select a ball randomly, note its colour but do not return it back to the urn, and repeat this experiment n times (where $n \leq min\{n_1, n_2\}$). If X is a random variable denoting the number of Red balls selected, then X has a hypergeometric distribution.

A. True

B. False

Question 10

We throw a fair die repeatedly until an outcome of 4 or greater appears for the fifth time. If X is a random variable representing the number of trials until this happens, then X has a negative binomial distribution with parameters $r = 5, p = 2/3$.

A. True

B. False

Question 11

We throw a fair die repeatedly 5 times. If X is a random variable representing the number of times that even numbers appear, then X has a negative binomial distribution with parameters $r = 5, p = 1/2$.

A. True

B. False

Question 12

We throw a fair die repeatedly until an outcome of 4 or greater appears for the first time. If X is a random variable representing the number of trials until this happens, then X has a geometric distribution with parameter $p = 1/3$.

A. True

B. False

Question 13

We throw a fair die repeatedly until an outcome of 4 or greater appears for the first time. If X is a random variable representing the number of trials until this happens, then X has a negative binomial distribution with parameter $r = 1, p = 1/2$

A. True

B. False

Question 14

A box contains n_1 Red and n_2 Black balls. Suppose we select a ball randomly, note its colour. If X is a random variable denoting the number of Red balls selected, then X has a Bernoulli distribution.

A. True

B. False

Question 15

A box contains n_1 Red and n_2 Black balls. Suppose we select a ball randomly, note its colour. If X is a random variable denoting the number of Red balls selected, then X has a Binomial distribution.

A. True

B. False

Question 16

A box contains n_1 Red and n_2 Black balls. Suppose we select a ball randomly, note its colour and return it back to the urn, and repeat this experiment n times (where $n \leq min\{n_1, n_2\}$). If X is a random variable denoting the number of Red balls selected, then X has a Binomial distribution.

A. True

B. False

Question 17

The expression $ABC(A \cup B \cup C')$ is equal to

A. ABC

B. $\emptyset$

C. ABC'

D. $ABCC'$

Question 18

The expression $[ABC(A \cup B \cup C')] \cup (A \cup B')$ is equal to

A. ABC

B. $\emptyset$

C. $A \cup B'$

D. $A \cap B'$

The next 6 questions will guide you to explain why

$$\sum_{k=1}^{n} k\binom{n}{k} = n \cdot 2^{n-1}$$

Question 19

Suppose we have n people, and we would form a committee of any size, but it must have a chairperson. How many ways are there to select a chairperson from a set of n people?

A. k

B. n

C. $n!$

D. $n-1$

Question 20

This is the continue of Question 19. After choosing the chairperson for the committee, we need to decide among the remaining $n-1$ people, who will be in the committee. How many ways are there this selection can be done? Keep in mind that we would like to form a committee of any size.

A. $(n-1)!$

B. $n-1$

C. 2^n

D. 2^{n-1}

Question 21

Combining the information from Question 19 and Question 20, how many ways are there to form a committee of any size with one chairperson from n people?

A. $k \cdot (n-1)!$

B. k^{n-1}

C. $2^{n-1} + n$

D. $2^{n-1} \cdot n$

Question 22

Suppose we have n people, and we would form a committee of any size, but it must have a chairperson. What are the possible size of the committee?

A. $\{0, 2, 4, ...\}$

B. $\{0, 1, 2, ..., n-1\}$

C. $\{1, 2, 3, ..., n\}$

D. $\{0, 2, 2, ..., n\}$

Question 23

This is the continue of question 4. Let us look into a specific case where we would like to form a committee of size k. In order to form a committe of size k with exactly 1 chairperson, we will first pick k people to be in the committee. After that, we will decide who will be the chairperson. Based on that, from a group of n people, how many ways are there to form a committee of size k with one chairperson?

A. $\binom{n}{k} \cdot k$

B. $n^k \cdot k$

C. $\binom{n}{k}$

D. $k!$

Question 24

Combining the information from Question 22 and Question 23, how many ways are there to form a committee of any size with one chairperson from n people? Keep in mind that in question 5, we dealt with a specific size of committee, but we are actually interested in a committee of any size here!

A. $\binom{n}{k} \cdot k$

B. $\sum_{k=1}^{n} k\binom{n-1}{k}$

C. $\sum_{k=1}^{n} k\binom{n}{k}$

D. $\prod_{k=1}^{n} k\binom{n}{k}$

Question 25

An insurance company deals with three insurance policies: 40% of life insurance, 25% of car insurance, and 35% of health insurance. The probability that a life insurance policyholder will file a claim in a given year is 0.50. The probability that a car insurance policyholder will file a claim in a given year is 0.20. Lastly, the probability that a health insurance policyholder will file a claim in a given year is 0.10. At the course of this year, a policyholder files a claim. Calculate the conditional probability that the claim comes from a car insurance policyholder.

A. 0.1754

B. -0.0001

C. 0.234

D. 1.9

Series Summary

As you might realize, geometric series and Taylor series expansion of e^x showed up few times in this course. Therefore, I would like to summarize the formula here for you!

1. **Finite Geometric Series**

$$S_n = a + ar + ar^2 + ... + ar^{n-1} = \sum_{k=0}^{n-1} ar^k$$

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

2. **Infinite Geometric Series**

$$S = a + ar + ar^2 + ar^3 + ... = \sum_{k=0}^{\infty} ar^k$$

$$S_n = \frac{a}{(1-r)}, \text{for } |r| < 1$$

3. **Taylor Series expansion of e^x**

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Question 26

Find the value of the constant c so that the function given below can be used as a probability mass function of a discrete random variable X on the range R_X:

$$P(X = x) = \frac{c \cdot 4^x}{7^x}, R_X = \{1, 2, ...\}$$

A. $4/3$

B. $3/4$

C. $1/2$

D. 2

Question 3

Find the value of the constant c so that the function given below can be used as a probability mass function of a discrete random variable X on the range R_X:

$$P(X = x) = \frac{c \cdot 4^x}{x!}, R_X = \{0, 1, 2, ...\}$$

A. e^4

B. $\frac{1}{e^{-4}}$

C. $\frac{1}{e}$

D. $\frac{1}{e^4}$

Question 28

In a factory out of 50 machines produced during a day, 8 are defective. An engineer selects six machines at random to examine whether they have this error or not. Let X be a random variable denoting the number of defective machines. Find the probability mass function of X. Then answer what is the probability that at least two of the machines selected are defective?

A. 1

B. $\frac{\binom{8}{2}\binom{42}{4}}{\binom{50}{6}}$

C. $1 - \frac{\binom{8}{1}\binom{42}{5}}{\binom{50}{6}}$

D. $1 - \frac{\binom{8}{1}\binom{42}{5}}{\binom{50}{6}} - \frac{\binom{42}{6}}{\binom{50}{6}}$

Question 32

Janet thinks she might have a fever. Or maybe just a headache or a cold. She is not really sure. She prepares to take her temperature with a digital thermometer which reports Fahrenheit temperature (rounded to the nearest integer) and she thinks she will see one of the values in {98, 99, 100, 101, 102} each with probability 1/5. Let X be number she actually sees. Compute $Var(X)$.

A. 2

B. 100

C. -10

D. 50

Question 33

Use question 32 to compute $E[X^2]$

A. 2

B. 10000

C. 10002

D. 50

Question 34

This is the continue of Question 32. After taking her temperature, regardless of what the thermometer shows, Janet plans to a roll a three-sided die (which takes values in 0, 1, 2 each with equal probability). If N is the number that comes up, Janet will take N ibuprofen tablets. Compute the expectation $E[2N^3 + 3X^2]$

A. 1000

B. 30010

C. 30011

D. 30012

Question 35

In a random group of 40 people, what is the probability that at least two people share the same birthday? Assume a year has 365 days.

A. 0

B. $\frac{365 \cdot 364 \cdot \ldots \cdot 326}{365^{40}} = 10.9\%$

C. $1 - \frac{365 \cdot 364 \cdot \ldots \cdot 326}{365^{40}} = 89.1\%$

D. 1

Question 36

In a random group of 40 people, what is the probability that someone has the same birthday as you? Assume a year has 365 days.

A. 0

B. $1 - (\frac{364}{365})^{40}$

C. $1 - \frac{364 \cdot 363 \cdot \ldots \cdot 325}{365^{40}}$

D. 1

Fun fact: Most people tend to assume that questions (35) and (36) have essentially the same answer. The fact that they do not is called the birthday paradox. The application of this in cryptography is called birthday attack, and it is used to forge signature on a document (just one example).