University of Toronto Mississauga
An Introduction to Probability and Modelling
STA107H5, Winter 2024

Term Test 1

Duration: 75 Minutes

First name (please write as legibly as possible within the boxes)

Last name

Student ID number

INSTRUCTIONS and POLICIES:

- Please DO NOT rip off any page from this booklet.
- For all questions, complete solutions are required. Show your work to earn full marks and then circle the final answer where appropriate.
- You are allowed to use a non-programmable calculator.
- Simplify final answers and round to 3 decimal places where appropriate.

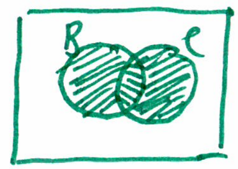
Question 1 (10 points). In a hospital, the probability of a patient receiving a blood pressure check is 70%, a cholesterol level test is 60%, and both screenings during the same visit is 50%. Calculate:

a) the probability that a patient receives at least one of these screenings;

B := event that patient receives a blood pressure check +0.5

C := event that patient receives a cholesterol test +0.5

$$\underbrace{P(B \cup C)}_{+1} = \underbrace{P(B) + P(C) - P(BC)}_{+1}$$



$$= 0.7 + 0.6 - 0.5$$

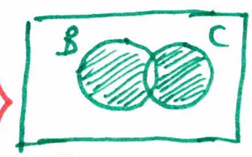
$$= \boxed{0.8}$$

+2

b) the probability that a patient receives exactly one of these screenings.

$$\underbrace{P(BC' \cup B'C)}_{+1} = P(BC') + P(B'C)$$

$$= P(B) - P(BC) + P(C) - P(BC)$$



$$= P(B) + P(C) - 2P(BC)$$

+2

$$= 0.7 + 0.6 - 2 \times 0.5$$

$$= \boxed{0.3}$$

+2

Venn diagrams are for visualization purposes only.

Question 2 (10 points). Simplify the following expression using the properties of operations among events:

$$(A \cup B)\,(A \cup B')\,(A' \cup B)$$

Hint: *You should justify each step of your solution by referring to at least one of the properties $SP1 - SP11$, included within the appendix page of this exam, or any other property such as the De Morgan's Laws.*

$(A \cup B)(A \cup B')(A' \cup B) = [A \cup (BB')](A' \cup B)$ SP6 +2

$= (A \cup \emptyset)(A' \cup B)$ SP7 +2

$= A(A' \cup B)$ SP2 +1

$= AA' \cup AB$ SP6 +2

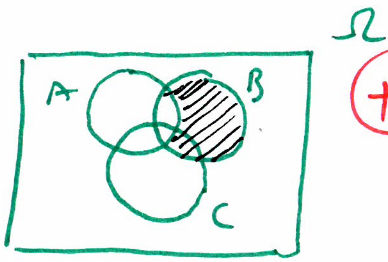
$= \emptyset \cup AB$ SP7 +2

$= \boxed{AB}$ SP2 +1

Question 3 (10 points). Let A, B, and C be three events on a sample space Ω. Use the Venn diagrams to examine whether the following relation is always true:

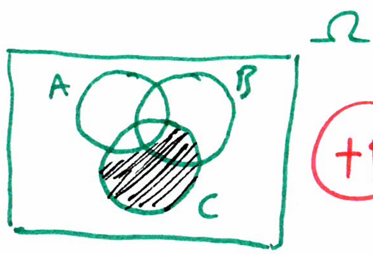
$$(B - A)(C - A) = BC - A.$$

LHS :



$B - A$

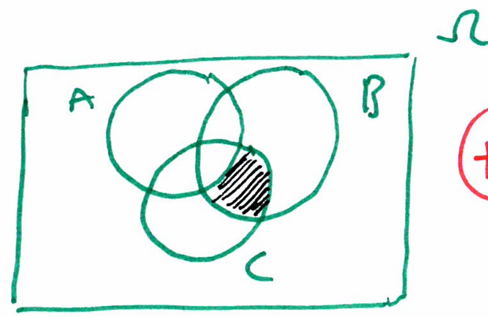
+1



$C - A$

+1

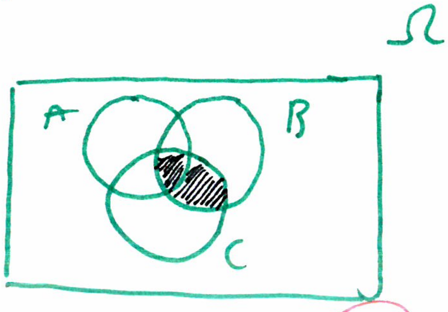
$\Rightarrow$



+2

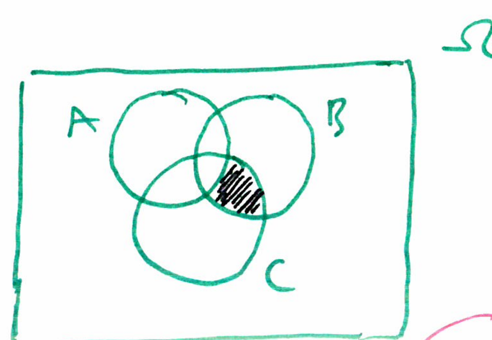
$(B - A)(C - A)$

RHS :



BC +2

$\Rightarrow$



+2

$BC - A$

$\therefore$ Two shaded areas coincide, so the relation holds true (or it is an identity). +2

Question 4 (10 points). A consultant travels frequently between three cities: Toronto, Montreal, and Vancouver. Let T, M, and V be the events that she travels to Toronto, Montreal, and Vancouver, respectively. It is known that:

i) the probability that she travels to Toronto is twice the probability of travelling to Montreal;

ii) the probability of travelling to Vancouver is three times the probability of travelling to Montreal; and

iii) the sum of the probabilities of travelling to Toronto and Montreal is 0.5.

Find the probabilities that she travels to each of these three cities; i.e., find $P(T)$, $P(M)$ and $P(V)$.

$$P(T) + P(M) = 0.5 \Rightarrow \quad +2$$

$$2P(M) + P(M) = 0.5 \Rightarrow \quad +1$$

$$\left.\begin{array}{l} 3P(M) = 0.5 \Rightarrow \\ P(M) = \frac{1}{6} \approx \boxed{0.107} \end{array}\right\} +1$$

$$P(T) = 2P(M) \Rightarrow \quad +2$$

$$P(T) = 2 \times \frac{1}{6} = \frac{1}{3} \approx \boxed{0.333} \quad +1$$

$$P(V) = 3P(M) \Rightarrow \quad +2$$

$$P(V) = 3 \times \frac{1}{6} = \frac{1}{2} = \boxed{0.5} \quad +1$$

APPENDIX

Proposition 1.1 *The following properties hold true for the operations among events:*

*SP*1. $A \cup A = A \qquad AA = A$

*SP*2. $A \cup \emptyset = A \qquad A\emptyset = \emptyset$

*SP*3. $A \cup \Omega = \Omega \qquad A\Omega = A$

*SP*4. $A \cup B = B \cup A \qquad AB = BA$

*SP*5. $A \cup (B \cup C) = (A \cup B) \cup C \qquad A(BC) = (AB)C$

*SP*6. $A \cup (BC) = (A \cup B)(A \cup C) \qquad A(B \cup C) = (AB) \cup (AC)$

*SP*7. $A \cup A' = \Omega \qquad AA' = \emptyset$

*SP*8. $(A')' = A$

*SP*9. *If* $A \subseteq B$ *and* $B \subseteq C$, *then* $A \subseteq C$

*SP*10. *If* $A \subseteq B$, *then* $B' \subseteq A'$ *and vice versa*

*SP*11. *If* $A \subseteq B$, *then* $AB = A$ *and* $A \cup B = B$.