Problem Set #1

Maxwell Petersen

September 22, 2015

Problem 1

Because there is a DFA $M = (Q, \Sigma, \delta, q_0, F)$ that recognizes A, building an NFA $N = (Q', \Sigma, \delta', q_0', F')$ that recognizes $A^{\mathcal{R}}$ proves that $A^{\mathcal{R}}$ is regular. The NFA $N = (Q', \Sigma, \delta', q_0', F')$ has δ' as follows:

$$\begin{aligned}
\delta'(q_0', \epsilon) &= F \\
\delta'(q_0', a) &= \varnothing && \forall a \in \Sigma \\
\delta'(p, a) &= \{q | \delta(q, a) = p\} && \forall q \in Q, a \in \Sigma
\end{aligned}$$

Problem 2

$\textsc{BackwardsAndForwards}(A) = \{w \in A \mid w \in A \text{ and } w^{\mathcal{R}} \in A\}$ is defined by the closure property of union because the language A and the language $A^{\mathcal{R}}$ are both regular and union has been proven to keep sets closed.

Problem 3

a. $A \setminus B = \{w \in A \mid w \notin B\}$ is a regular and closed language because it is the same thing as $A \cap \sim B$ having intersection being proven to keep sets closed if A and B are both regular.

b. $A \triangle B = \{w \mid \text{either } w \in A \text{ or } w \in B \text{ but not both}\}$ is a regular closed set because it has a regular expression $(A \cup B) \cap \sim (A \cap B)$ with union, intersection, and compliment keeping the set closed.

Problem 4

Let there be an alphabet $\Sigma' = \{a' | a \in \Sigma\}$. Let there be homomorphisms un : $(\Sigma \cup \Sigma')^* \to \Sigma^*$ and rem : $(\Sigma \cup \Sigma')^* \to \Sigma^*$ such that $\text{un}(a') = a$ for $a' \in \Sigma'$, $\text{un}(a) = a$ for $a \in \Sigma'$, $\text{rem}(a') = \epsilon \in \Sigma'$, and $\text{rem}(a) = a$ for $a \in \Sigma$. Let L_1 be a language defined as $\text{un}^{-1}(L)$ making it regular because languages have been proven to stay closed under inverse homomorphisms, L_1 is the subset of strings that may have been marked with an apostrophe. Let there be an $L_2 = L_1 \cap \Sigma'^*\Sigma^*$, L_2 is the subset of strings where the first few characters are marked with an apostrophe. L_2 Is regular and closed because sets are kept closed and regular with intersection. Because L_2 is the same thing as SUFFIX then SUFFIX is seen to be closed and regular as well.

Problem 5

$A \oslash B = \{w \in A \mid w \text{ does not contain any string in } B \text{ as a substring}\}$ is a closed set because it has the regular expression of $A \cup (\sim B)$ with union and inverse being proven to keep the set closed assuming that A and B are regular.

Problem 6

Let there be a DFA $M = (Q, \Sigma, \delta, F)$ which recognizes A. Let there also be a NFA $N = (Q', \Gamma, \delta', F')$ which recognizes $f(A)$. The delta transition of δ' is as follows:

$$\begin{aligned} \delta'(q_0', f(\epsilon)) &= \varnothing & \\ \delta'(q_0', f(a)) &= \delta(q_0, a) & \forall a \in \Gamma \\ \delta'(p, f(a)) &= \{q | \delta(q, a) = p\} & \forall q \in Q', a \in \Gamma \end{aligned}$$

Problem 7

Let there be a DFA $M = (Q, \Sigma, \delta, F)$ which recognizes A. Let there also be a NFA $N = (Q', \Gamma*, \delta', F')$ which recognizes $f(A)$. With $f()$ being defined as a homomorphism. The delta transition δ' is as follows:

$$\begin{aligned} \delta'(q_0', f(\epsilon)) &= \epsilon & \\ \delta'(q_0', f(a)) &= \delta(q_0, f(a)) & \forall a \in \Gamma^* \\ \delta'(p, f(a)) &= \{q | \delta(q, f(a)) = p\} & \forall q \in Q', a \in \Gamma^* \end{aligned}$$

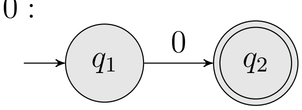
Problem 8

$$A = \texttt{1}(\texttt{1} \cup \texttt{0})^*\texttt{0}$$
$$B = \texttt{0}^*\texttt{1}\texttt{0}^*\texttt{1}\texttt{0}^*\texttt{1}\texttt{0}^*$$
$$C = (\texttt{1} \cup \texttt{0})^*\texttt{0101}(\texttt{1} \cup \texttt{0})^*$$
$$D = (\texttt{1} \cup \texttt{0})(\texttt{1} \cup \texttt{0})\texttt{0}(\texttt{1} \cup \texttt{0})^*$$
$$E = (\texttt{0}((\texttt{1} \cup \texttt{0})(\texttt{1} \cup \texttt{0}))^*)(\texttt{1}(\texttt{1} \cup \texttt{0}))\quad((\texttt{1} \cup \texttt{0})\quad(\texttt{1} \cup \texttt{0}))^*)$$
$$F = \sim \texttt{110}$$
$$G = (\texttt{1} \cup \texttt{0})(\texttt{1} \cup \texttt{0})(\texttt{1} \cup \texttt{0})(\texttt{1} \cup \texttt{0})(\texttt{1} \cup \texttt{0})$$
$$H = (\sim\texttt{11}) \cup (\sim\texttt{111})$$
$$I = (\texttt{1}(\texttt{1} \cup \texttt{0}))^*$$
$$J = \texttt{0}^*\texttt{1}\texttt{0}^*$$
$$K = \texttt{0}^*$$
$$L = (\texttt{0}\quad\texttt{1}^*\quad\texttt{0}^*)^* \cup (\texttt{0}^*\quad\texttt{1}\quad\texttt{0}^*\quad\texttt{1}\quad\texttt{0}^*)$$
$$M = \texttt{1} \cap \sim \texttt{1}$$
$$N = (\texttt{0} \cap \texttt{1})^*$$

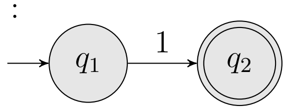
Problem 9

Finding the NFA of : $(0 \cup 11)^*01(00 \cup 1)^*$

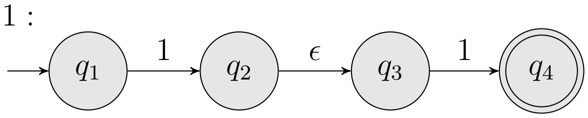
0 :



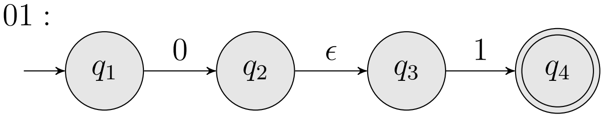
1 :



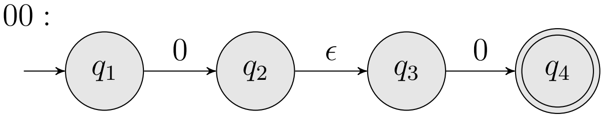
11 :



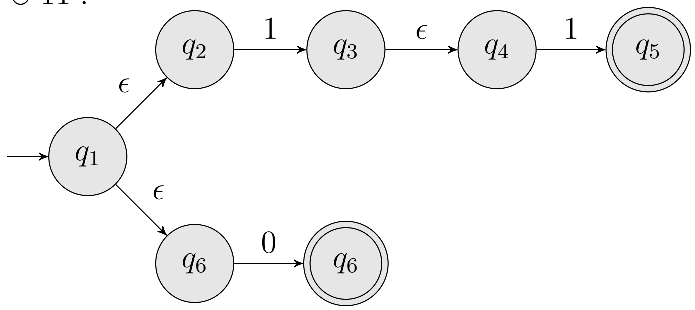
01 :



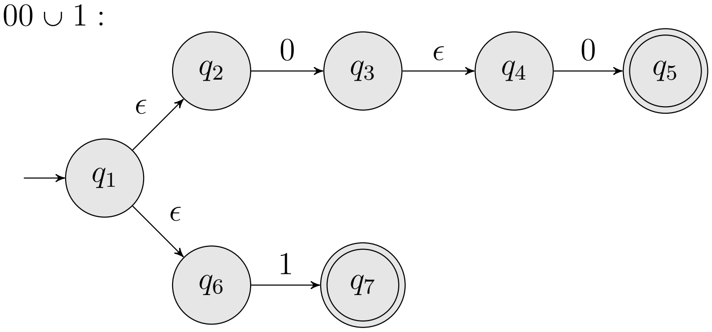
00 :



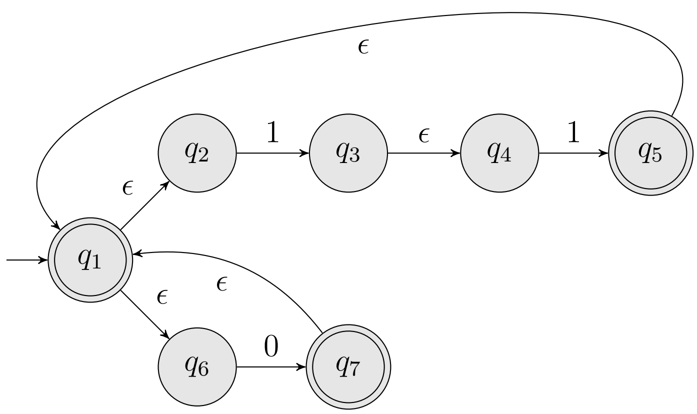
$0 \cup 11$:



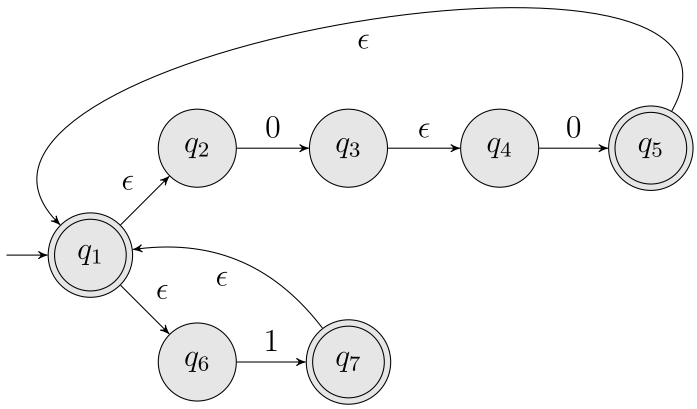
$00 \cup 1$:



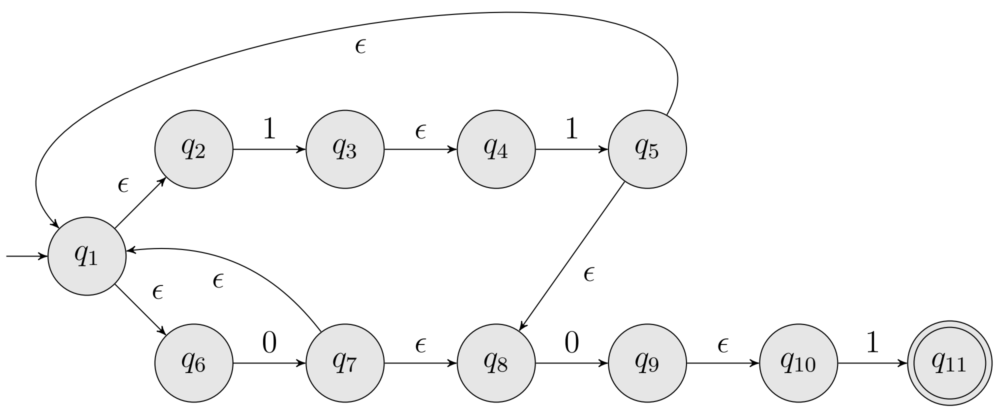
$(0 \cup 11)^*$:



$(00 \cup 1)^*$:



$(0 \cup 11)^*01$



$(0 \cup 11)^*01(00 \cup 1)^*$