CS 301: Languages and Automata **Fall 2015**

Practice Exam #1

September 23, 2015

Problem 1 True/false. For each of the claims below, say whether it is true or false. If the claim is false, *give a counterexample*; if the claim is true, briefly *explain why it is true.*

a. If A contains a finite number of strings, then $A^{\mathcal{C}}$, the complement of A, is regular.

b. The intersection of the language of a NFA and the language of a regular expression is regular.

c. A NFA can have zero states.

d. Let f be a homomorphism. If $f(A)$ is regular, then A is regular.

e. If A is a language and M is a DFA such that M accepts every string in A, then A is regular.

Problem 2 A string is of *even length* if the number of characters in the string is even. For example, ϵ, "`abcd`", and "`010101`" are all of even length, whereas "`a`" and "`246`" are not.

For a language L, define

$$\textsc{Evens}(L) = \{w \in L \mid w \text{ is of even length}\}.$$

For example, for $L = \{\epsilon, \texttt{0}, \texttt{00}, \texttt{000}\}$, $\textsc{Evens}(L) = \{\epsilon, \texttt{00}\}$.

a. Using the notation above, show that if L is a regular language (over any alphabet Σ), $\textsc{Evens}(L)$ is also a regular language.

b. Show that there exists a language L_{b} (over the alphabet of your choice) such that L_{b} is not regular but $\textsc{Evens}(L_{\mathrm{b}})$ *is* regular.

c. Show that there exists a language L_{c} (over the alphabet of your choice) such that L_{c} is not regular and $\textsc{Evens}(L_{\mathrm{c}})$ is also *not* regular.

Problem 3 Closure properties.

a. Give an example of a nonregular language L such that L^* *is* regular, or prove that this is impossible. [*Hint: Think about* Σ^*.]

b. Prove that the class of nonregular languages is closed under complement or give a counterexample.

c. Prove that the language $\{w \mid w \text{ has the same number of } \texttt{a}\text{s as } \texttt{b}\text{s}\}$ is not regular using closure properties of regular languages.

Problem 4 Double. For a language A, define $\textsc{Double}(A) = \{ww \mid w \in A\}$. Show that regular languages are *not* closed under $\textsc{Double}$. That is, give a regular language A such that $\textsc{Double}(A)$ is not regular.

Problem 5 Half. For a language A, define $A_{\frac{1}{2}-} = \{x \mid \exists y \text{ such that } |x| = |y| \text{ and } xy \in A\}$. Prove that if A is regular, then $A_{\frac{1}{2}-}$ is regular. This problem is very hard! Don't bother thinking about it until you understand the others. [*Hint: Let* $M = (Q, \Sigma, \delta, q_0, F)$ *be a DFA recognizing* A *and for each* $q \in Q$*, let* $N_q = (Q, \Sigma, \delta', q, F)$ *be an NFA whose start state is* q *and whose transition function is* $\delta'(r, a) = \{s \in Q \mid \delta(r, b) = s \text{ for some } b \in \Sigma\}$*. (Note that* $\delta'(r, a)$ *doesn't depend on* a*.) Now perform a cartesian product–like construction to build an NFA* N *from* M *and the* N_q *whose set of states is* $Q' = Q \times \prod_{q \in Q} Q = Q^{|Q|+1}$*. That is, if* M *has* $n = |Q|$ *states, then* N *will have* n^{n+1} *states, each of which is a* $(n+1)$*-tuple. Consider what the language of each* N_q *is.*]

Problem 6 Constructions. *This will be updated with a problem in a day or so.*