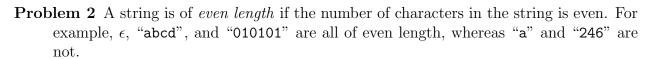
CS 301: Languages and Automata

Fall 2015

## Practice Exam #1

September 23, 2015

<ul> <li>Problem 1 True/false. For each of the claims below, say whether it is true or false. If the claim is false, give a counterexample; if the claim is true, briefly explain why it is true.</li> <li>a. If A contains a finite number of strings, then A<sup>C</sup>, the complement of A, is regular.</li> </ul>
<b>b.</b> The intersection of the language of a NFA and the language of a regular expression
is regular.
c. A NFA can have zero states.
<b>d.</b> Let $f$ be a homomorphism. If $f(A)$ is regular, then $A$ is regular.
<b>e.</b> If $A$ is a language and $M$ is a DFA such that $M$ accepts every string in $A$ , then $A$ is regular.



For a language L, define

EVENS
$$(L) = \{ w \in L \mid w \text{ is of even length} \}.$$

For example, for  $L = \{\epsilon, 0, 00, 000\}$ , EVENS $(L) = \{\epsilon, 00\}$ .

**a.** Using the notation above, show that if L is a regular language (over any alphabet  $\Sigma$ ), EVENS(L) is also a regular language.

**b.** Show that there exists a language  $L_{\rm b}$  (over the alphabet of your choice) such that  $L_{\rm b}$  is not regular but EVENS( $L_{\rm b}$ ) is regular.

c. Show that there exists a language  $L_c$  (over the alphabet of your choice) such that  $L_c$  is not regular and EVENS( $L_c$ ) is also *not* regular.

## Problem 3 Closure properties.

**a.** Give an example of a nonregular language L such that  $L^*$  is regular, or prove that this is impossible. [Hint: Think about  $\Sigma^*$ .]

**b.** Prove that the class of nonregular languages is closed under complement or give a counterexample.

**c.** Prove that the language  $\{w \mid w \text{ has the same number of as as bs}\}$  is not regular using closure properties of regular languages.

**Problem 4** Double. For a language A, define Double(A) =  $\{ww \mid w \in A\}$ . Show that regular languages are *not* closed under Double. That is, give a regular language A such that Double(A) is not regular.

Problem 5 Half. For a language A, define  $A_{\frac{1}{2}-}=\{x\mid\exists y\text{ such that }|x|=|y|\text{ and }xy\in A\}.$  Prove that if A is regular, then  $A_{\frac{1}{2}-}$  is regular. This problem is very hard! Don't bother thinking about it until you understand the others. [Hint: Let  $M=(Q,\Sigma,\delta,q_0,F)$  be a DFA recognizing A and for each  $q\in Q$ , let  $N_q=(Q,\Sigma,\delta',q,F)$  be an NFA whose start state is q and whose transition function is  $\delta'(r,a)=\{s\in Q\mid\delta(r,b)=s\text{ for some }b\in\Sigma\}.$  (Note that  $\delta'(r,a)$  doesn't depend on a.) Now perform a cartesian product-like construction to build an NFA N from M and the  $N_q$  whose set of states is  $Q'=Q\times\prod_{q\in Q}Q=Q^{|Q|+1}$ . That is, if M has n=|Q| states, then N will have  $n^{n+1}$  states, each of which is a (n+1)-tuple. Consider what the language of each  $N_q$  is.]

**Problem 6** Constructions. This will be updated with a problem in a day or so.