Exercises #3

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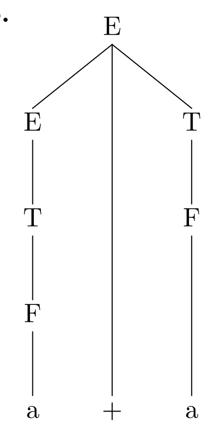
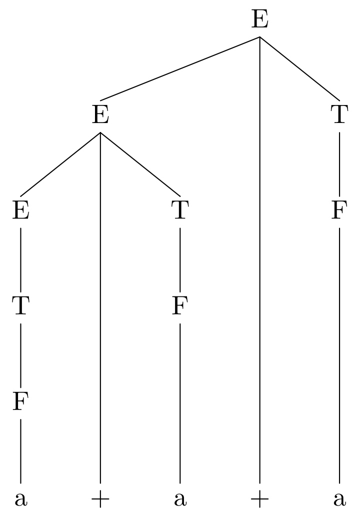
Exercise 2.1

a.

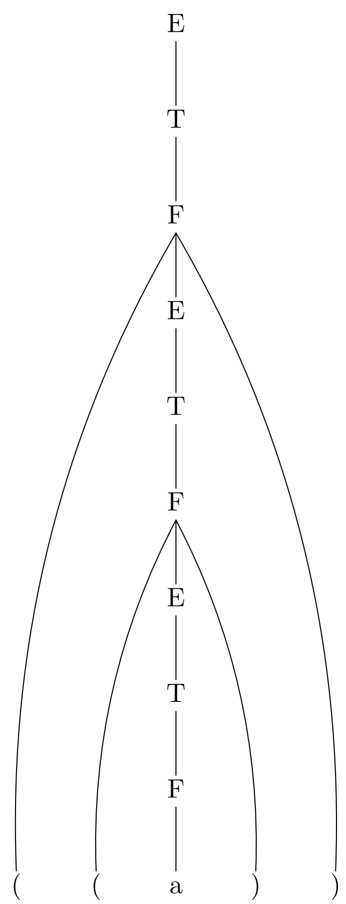
b.



c.



d.



Exercise 2.3

a. What are the variables of G?
$\{R, S, T, X\}$

b. What are the terminals of G?
$\{\mathsf{a}, \mathsf{b}\}$

c. Which is the start variable of G?
R

d. Give three strings in $L(G)$.
aaba, ab, bbaba

e. Give three strings not in $L(G)$.
aaa, bbb, ϵ

f. True or False: $T \Rightarrow \mathsf{aba}$.
False

g. True or False: $T \overset{*}{\Rightarrow} \mathsf{aba}$.
True

h. True or False: $T \Rightarrow T$
False

i. True or False: $T \overset{*}{\Rightarrow} T$
True

j. True or False: $XXX \overset{*}{\Rightarrow} \mathsf{aba}$
True

k. True or False: $X \overset{*}{\Rightarrow} \mathsf{aba}$
False

l. True or False: $T \overset{*}{\Rightarrow} XX$
True

m. True or False: $T \overset{*}{\Rightarrow} XXX$
True

n. True or False: $S \overset{*}{\Rightarrow} \epsilon$
False

o. Give a description in English of $L(G)$.
G is the set of every string that is not a palindrome

Exercise 2.4

a. $\{w \mid w \text{ contains at least three 1s}\}$
$S \rightarrow T1T1T1T$
$T \rightarrow T1 \mid T0 \mid 1T \mid 0T \mid \epsilon$

b. $\{w \mid w \text{ starts with and ends with the same symbol}\}$
$S \rightarrow 1T1 \mid 0T0$
$T \rightarrow T1 \mid T0 \mid 1T \mid 0T \mid \epsilon$

c. $\{w \mid \text{the length of } w \text{ is odd}\}$
$S \rightarrow TST \mid T$
$T \rightarrow 1 \mid 0$

d. $\{w \mid \text{the length of } w \text{ is odd adn the middle value is a } 0\}$
$S \rightarrow 0 \mid 0S0 \mid 0S1 \mid 1S0 \mid 1S1$

e. $\{w \mid w = w^{\mathcal{R}}, \text{ that is, } w \text{ is a palindrome}\}$
$S \rightarrow \mathsf{a}S\mathsf{a} \mid \mathsf{b}S\mathsf{b} \mid \mathsf{a} \mid \mathsf{b} \mid \epsilon$

f. The empty set
$S \rightarrow \emptyset$

Exercise 2.6

a. The set of string over the alphabet $\{\mathsf{a}, \mathsf{b}\}$ where there are more a's than b's.
$S \to T\mathsf{a}T$
$T \to TT \mid \mathsf{a}T\mathsf{b} \mid \mathsf{b}T\mathsf{a} \mid \mathsf{a} \mid \epsilon$

b. The compliment of the language $\{a^n b^n \mid n \geq 0\}$
$S \to T \mid V \mid X\mathsf{b}X\mathsf{a}X$
$T \to U\mathsf{a}U$
$U \to UU \mid \mathsf{a}U\mathsf{b} \mid \mathsf{b}U\mathsf{a} \mid \mathsf{a} \mid \epsilon$
$V \to W\mathsf{b}W$
$W \to WW \mid \mathsf{a}W\mathsf{b} \mid \mathsf{b}W\mathsf{a} \mid \mathsf{b} \mid \epsilon$
$X \to X\mathsf{a} \mid X\mathsf{b} \mid \epsilon$

c. $\{w\#x \mid w^{\mathcal{R}} \text{ is a substring of } x \text{ for } w, x \in \{\mathsf{1}, \mathsf{0}\}^*\}$
$S \to TX$
$T \to \mathsf{0}T\mathsf{0} \mid \mathsf{1}T\mathsf{1} \mid \#X$
$X \to \mathsf{0}X \mid \mathsf{1}X \mid \epsilon$

d. $\{x_1\#x_2\#\cdots x_k \mid k \geq 1, \text{ each } x_i \in \{\mathsf{a}, \mathsf{b}\}^*, \text{ and for some } i \text{ and } j,\ x_i = x_j^{\mathcal{R}}\}$
$S \to \mathsf{a}T\mathsf{a}U \mid \mathsf{b}T\mathsf{b}U \mid \mathsf{a} \mid \mathsf{b}$
$T \to \epsilon \mid \#S \mid S \mid \#S\#$
$U \to \epsilon \mid \#S$

Exercise 2.13

Let $G = (V, \Sigma, R, S)$ be the following rammer. $V = \{S, T, U\}; \Sigma = \{\mathsf{0}, \#\}$; and R is the set of rules:

$S \to TT \mid U$
$T \to \mathsf{0}T \mid T\mathsf{0} \mid \#$
$U \to \mathsf{0}U\mathsf{00} \mid \#$

a. Describe $L(G)$ in English.
$L(G)$ is the set of all strings where there is some string of 0's a least one # then some more 0's then repeated with # in between each repeat guaranteeing that the string is going to be either two #'s or the first and last character is going to be a 0.

b. Prove that $L(G)$ is not regular.
Through exhaustive search a regular expression cannot be made for these rules. Without a regular expression the grammar produces a non regular language.

Exercise 2.14

Initial:
$A \to BAB \mid B \mid \epsilon$
$B \to \mathsf{00} \mid \epsilon$
Adding $A_0 \to A$:
$A_0 \to A$
$A \to BAB \mid B \mid \epsilon$
$B \to \mathsf{00} \mid \epsilon$
Converting rules to those that have at most length two:
$A_0 \to A$
$A \to BC \mid B \mid \epsilon$
$B \to \mathsf{00} \mid \epsilon$
$C \to AB$
Removing $B \to \epsilon$:
$A_0 \to A$
$A \to BC \mid B \mid C \mid \epsilon$
$B \to \mathsf{00}$
$C \to AB \mid A$
Removing $A \to \epsilon$:
$A_0 \to A \mid \epsilon$
$A \to BC \mid B \mid C$
$B \to \mathsf{00}$
$C \to AB \mid A \mid B \mid \epsilon$
Removing $C \to \epsilon$
$A_0 \to A \mid \epsilon$
$A \to BC \mid B \mid C$
$B \to \mathsf{00}$
$C \to AB \mid A \mid B$
Removing unit variables:
$A_0 \to BC \mid \mathsf{00} \mid AB \mid BC \mid \mathsf{00} \mid \epsilon$
$A \to BC \mid \mathsf{00} \mid AB \mid BC \mid \mathsf{00}$
$B \to \mathsf{00}$
$C \to AB \mid BC \mid \mathsf{00} \mid C$
No terminals paired with variables so the CFG is in CNF

Exercise 2.17

Let there be some regular expression R. For every union in R create a new variable S with rule $S \to \{\text{expression}\} \mid \{\text{expression}\}$, i.e $\mathsf{a} \cup \mathsf{b} \Rightarrow S \to \mathsf{a} \mid \mathsf{b}$. For every concatenation in R create a new variable S with rule $S \to \{\text{expression}\}\{\text{expression}\}$, i.e $\mathsf{a} \circ \mathsf{b} \Rightarrow S \to \mathsf{ab}$. For every Kleene star in R create a new variable S with rule $S \to \{\text{expression}\}S$, i.e $\mathsf{a}^* \Rightarrow S \to \mathsf{a}S$.