Problem Set #3

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November 16, 2015

Problem 1

Define a TM D of the following form
D = ”

a. Move all symbols of the input tape to the left one spot.

b. In the new free first spot place a unique symbol.

c. Place transitions to prevent the TM from moving left when the unique symbol it under the tape head.

”

Problem 2

Define a TM D and E in the following form:
D = ”

a. Generate a DFA M’ such that M’ accepts $w^{\mathcal{R}}$ for all $w \in$ M.

”

and E = ”

a. Run $EQ_{DFA}\langle M,M'\rangle$.

b. E accepts if EQ_{DFA} accepts otherwise reject.

”

E halts on all input either win an accept or reject thus E is a decider for L.

Problem 3

For a language $L^C$to be decidable there is some TM C that can act as a decider upon it. L will also have a TM D in a form what when D receives some string w that it will run w on C and preform the opposite state that C returns with, i.e. when C accepts D rejects and vice versa. For all input w there is a direct relationship for all decidable languages and their compliments. To further illustrate this point if L^C is undecidable there is no way to determine with consistency that L is also decidable with the aforementioned relation of L and L^C.

Problem 4

Assume there is some decider R for HW. Let I be the code for a Python program that infinitely counts. Let D be a TM that decides A_{TM} D is defined as follows:
D = "

a. If input M is seen as a python program proceed to step 2, else skip to step 4.

b. Run R$\langle$M,$w\rangle$ for some input string w.

c. If R accepts D accepts, otherwise reject

d. Run $A_{TM}\langle$M,$w\rangle$

e. If A_{TM} accepts then D accepts otherwise reject.

" The decider D defined as so when run D$\langle$I,$w\rangle$ for any string w D will stall. Proving contradiction on R meaning HW is not decidable.

Problem 5

M =$\{\langle$I,$w\rangle$ | I moves head left one left most cell on input$w\}$
Let A be a TM which is defined as follows:
A = "

a. Check that the last input symbol is the largest symbol in Σ based on converting the symbols in to their ASCII hex code.

b. Then check that the first symbol on the tape is the lowest symbol in Σ.

" Run M$\langle$D$\langle$A$\rangle$,$w\rangle\rangle$ with D being defined in Problem 1. Because the first symbol is not in Σ nor can be accessed because there are rules in place to force the read head to the right upon reading the unique symbol. Those rules will conflict creating an infinite loop preventing M from being a decider.

Problem 6

As proven in the text if a TM is recognizable and its compliment is also recognizable then it is a decider otherwise the compliment may not be recognizable by a TM.

Problem 7

L is not decidable, define L as follows
L = "

- **a.** Run M on input$w$.
- **b.** For each state transition check for state q
- **c.** If the current state is q then reject.
- **d.** When the input string finishes regardless of M accepting or rejecting the string if L hasn't rejected by this point then accept.

" If the input TM is not a decidable TM then the input M may loop making L loop making L non decidable.