

Global Constraints

Peter Stuckey

Global Constraints

- ▶ Technically any constraint which can take an unbounded number of variables as input
 - so linear constraints are “global”
- ▶ Global constraints are
 - constraints that arise in many problems
- ▶ Global constraints make
 - models smaller
 - solving easier (since solvers can use the information of the structure)

alldifferent

- ▶ The alldifferent constraint
 - `alldifferent([x1, x2, ..., xn])`
 - enforces that $x_i \neq x_j$, for each $i \neq j$
- ▶ Probably the most common global constraint
- ▶ `alldifferent([7,3,2,5,1,6])` holds
- ▶ `alldifferent([5,3,2,7,4,3])` does not hold

- ▶ The lexicographic less than constraint
 - `lex_less([x1, x2, ..., xn], [y1, y2, ... yn])`
 - requires that the `[x1, x2, ..., xn]` is **lexicographically smaller** than `[y1, y2, ..., yn]`
 - that is
 - $x_1 < y_1$ or (
 - $x_1 = y_1$ and ($x_2 < y_2$ or
 - $x_2 = y_2$ and ($x_3 < y_3$ or
 -
 - $x_n < y_n$) ...)))
- ▶ Useful for symmetry breaking
- ▶ `lex_less([7,3,5,4,2], [7,3,5,7,2])` holds

table

- ▶ The table constraint encodes arbitrary relations

- `table([x1, x2, ..., xn], T)`

- requires that $[x1, \dots, xn]$ take value from one row in the 2d array T

- ▶ `table([x1,x2,x3], [| 3, 4, 5 | 5, 12, 13 | 6, 8, 10 |])`

- holds when $[x1,x2,x3] = [5,12,13]$

- doesn't hold when $x1 = 4$

circuit

- ▶ The circuit constraint encodes Hamiltonian circuits, a single loop that visits each node in a graph exactly once

- `circuit([x1, ..., xn])`

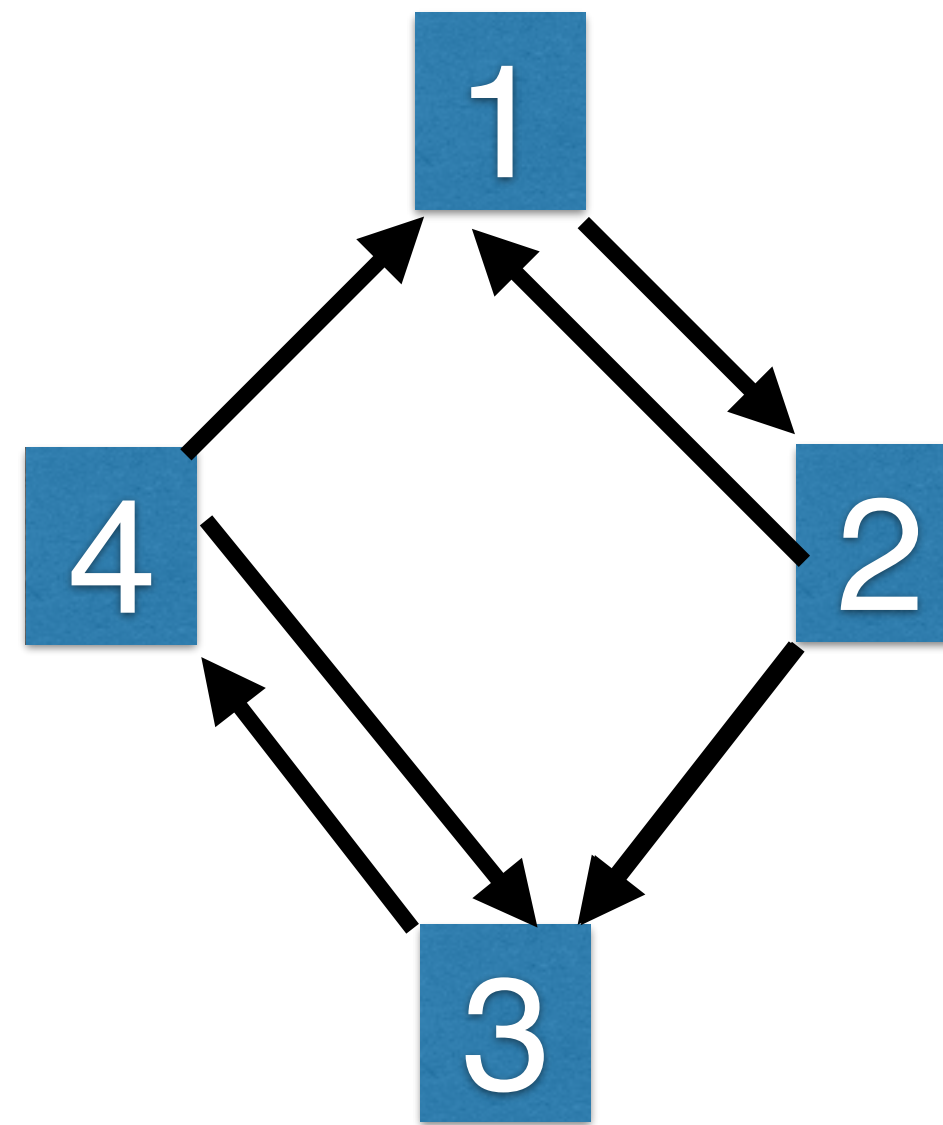
- $x_i = j$ means visit node j after node i

- ▶ For example

- `circuit([2,3,4,1])` holds

- `circuit([2,1,4,3])` doesn't hold

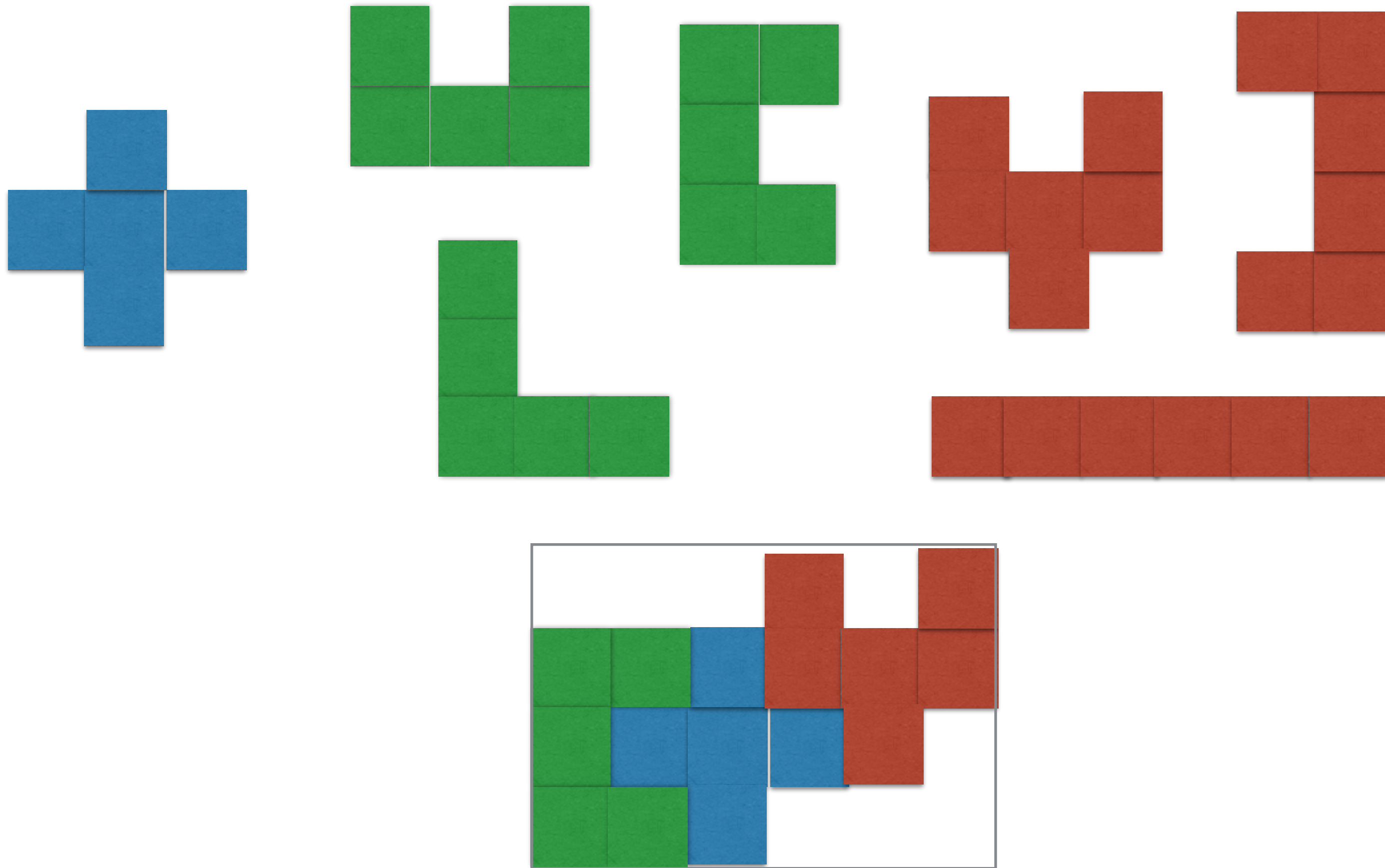
- `circuit([2,3,4,3])` doesn't hold



regular

- ▶ The regular constraint encodes that a sequence of values is part of a regular language
 - $\text{regular}([x_1, \dots, x_n], Q, S, d, q_0, F)$
 - the sequence $x_1 x_2 \dots x_n$ is a member of the regular language defined by DFA (Q, S, d, q_0, F)
- ▶ Useful for encoding complex state transitions, e.g. DFA for $1^*((01)^+1)^*$
 - $\text{regular}([1, 0, 1, 1, 0, 1, 0, 1, 1], \dots)$ holds
 - $\text{regular}([1, 1, 1, 1, 0, 1, 1], \dots)$ holds
 - $\text{regular}([1, 1, 1, 0, 1, 1, 1], \dots)$ doesn't hold

- Pack k dimensional objects with possibly different configurations so they don't overlap



Global Constraint Library

- ▶ MiniZinc includes a library of global constraints
 - Alldifferent and related constraints
 - Lexicographic constraints
 - Sorting constraints
 - Channeling constraints
 - Counting constraints
 - Scheduling constraints
 - Packing constraints
 - Extensional constraints (table, regular etc.)

Overview

- ▶ Global constraints are
 - important for making concise efficient models
- ▶ We will introduce more global constraints as their need arrives
- ▶ There are many global constraints
 - 100+ in MiniZinc
 - 300+ in the Global Constraint Catalog

EOF