Linear Models

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Overview

- Many models involves
  - resources and limits
  - choices in production/transport
  - costs
- Constraints of this nature are often expressed as
  - linear constraints
- Solving technology for linear models is highly effective

Linear Constraints

- A linear expression is of the form
  - $\sum_{i=1..n} a_i x_i$
  - where a_i are constants and x_i are variables
- A linear inequality has the form
  - $\sum_{i=1..n} a_i x_i \leq a_0$
  - where a_i are constants and x_i are variables
- A linear equation has the form
  - $\sum_{i=1..n} a_i x_i = a_0$
  - where a_i are constants and x_i are variables
- Linear constraints are either
  - linear inequalities, or linear equations

Linear Models

- A linear model consists of
  - linear constraints, and
  - a linear objective
    - minimize <linear expression>, or
    - maximize <linear expression>
- Linear models are solvable using
  - linear programming (reals), and
  - (mixed) integer programming (integers)
- These solver technologies scale to
  - 100000 variables
  - 100000 constraints
  - and sometimes more

Shipping Problem

- A shipping company has to transport bags of cement to W warehouses from F factories daily. Each warehouse has a daily demand, and each factory a daily output. The cost of shipping one bag is given by a table, e.g.

cost	**w1**	**w2**	**w3**	**w4**
f1	6	5	7	9
f2	3	2	4	1
f3	7	3	9	5

- Find the minimal shipping costs

Shipping Problem: Data and Decisions

- Data

```
int: W; % number of Warehouses
set of int: WARE = 1..W;
int: F; % number of Factories
set of int: FACT = 1..F;
array[WARE] of int: demand;
array[FACT] of int: production;
array[FACT,WARE] of int: cost;
```

- Decisions

```
array[FACT,WARE] of var int: ship;
```

Shipping Problem: Constraints

- Only ship positive amounts

```
forall(f in FACT, w in WARE)
      (ship[f,w] >= 0);
```

- Ship to each warehouse its demand

```
forall(w in WARE)
      (sum(f in FACT)(ship[f,w])
       >= demand[w]);
```

- Dont ship more from each factory than it produces

```
forall(f in FACT)
      (sum(w in WARE)(ship[f,w])
       <= production[f]);
```

Shipping Problem: Objective

- Minimize total shipping costs

```
solve minimize
      sum(f in FACT, w in WARE)
         (cost[f,w]*ship[f,w]);
```

- ...

A Linear Model

- Each constraint is a linear constraint
- The objective is a linear term

- Solving with default solver
  - many solutions found, 43.246s

- Solving with MIP solver
  - one optimal solution found, 0.061s

Improving the Model

- Decisions

```
array[FACT,WARE] of var int: ship;
```

- Unbounded integers are bad for many solvers
  - can even make the problem intractable

- Limit the size of the variable!

```
int: m = max(production);
array[FACT,WARE] of var 0..m: ship;
```

- Remove the first set of constraints!
- But in this case makes no difference!

Overview

- Linear constraints are a major component of many models

- If we can build a linear model
  - or almost linear model
- Then we can solve it very efficiently

- Get used to modeling with linear constraints

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