Hybrid Sensor and Mesh Networks: Paradigms for Fair and Energy Efficient Communication

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Abstract— **The issues pertaining to capacity, fairness, and energy efficiency are central to all wireless networks, including wireless sensor and mesh networks. In this paper, we consider hybrid sensor and mesh networks as paradigms for fair and energy efficient communication. A hybrid network consists of sensor or mesh nodes with wireless communication capability and some infrastructure in the form of cables with wireless transceivers attached to their ends. We investigate the trade-offs that exist between the amount of infrastructure and the improvement in fairness and energy efficiency over a pure wireless network. In particular, we show that a limited amount of infrastructure is enough to significantly improve fairness as well as energy efficiency. Moreover, in many scenarios the aggregate capacity of the network can also be substantially improved with the help of a limited infrastructure.**

I. Introduction

Wireless technologies have existed and been utilized for over a hundred years. However, it is only recently that wireless communications has seen a revolution, as people are rediscovering its uses and advantages. Many different types of wireless systems have emerged to serve different application requirements. Wireless sensor and mesh networks are two important forms of wireless systems that are expected to define the future of wireless communications.

A wireless sensor network is a collection of sensors working in collaboration with each other to perform some common task. There is a plethora of applications for these networks such as habitat monitoring, soil quality monitoring, detection of hazardous chemicals and forest fires, military surveillance, and monitoring seismic activity. The sensor nodes often run on batteries that are irreplaceable and it is therefore of paramount importance to efficiently utilize the available energy resources in sensor networks.

Wireless mesh networks (WMNs) are a collection of mesh clients and mesh nodes (routers), with mesh nodes forming the backbone of the network and providing connection to the Internet and other networks. The interest in mesh networks is primarily due to the possibility of community networking and extending the reach of existing wired networks. Achieving high network capacity and ensuring *fairness* are two key objectives in mesh networks.

The features common to both wireless sensor networks and mesh networks are: (i) multi-hop communication and (ii) many-to-few or few-to-many communication paradigm. In the case of a wireless sensor network, a large number of sensor nodes often need to report some data to one or more *sink nodes*. At times, sink nodes may also need to disseminate some important information to all the sensor nodes in the network. Since the wireless communication range of each node is often very small in comparison to the dimension of the network, a data packet typically travels over multiple hops before reaching its destination. A similar scenario exists in a wireless mesh network, where a large group of mesh nodes transmit and receive data from at most a few Internet hubs or gateways. Fig.1 depicts the communication scenario in wireless sensor and mesh networks along with the generic network model for such networks.

An undesired consequence of many-to-few or few-to-many communication scenarios and multi-hop communication is that the network nodes closer to the sink node (see the generic network model of Fig.1) do far more relaying than network nodes that are further away from the sink node. As a result, network nodes closer to the sink node drain their batteries much quicker and die much sooner than other network nodes, resulting in a network partition. Moreover, severe congestion problems

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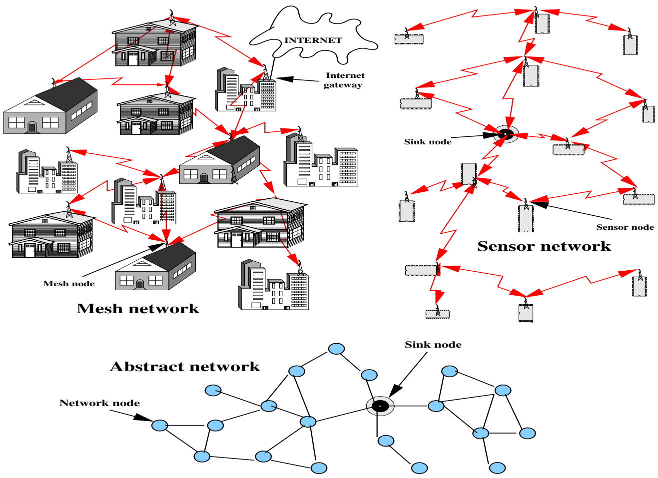

Fig. 1. The many-to-few or few-to-many communication scenario in case of wireless sensor and mesh networks is depicted. A generic network model for such networks is also shown. The term "sink node" is used to denote the base station or sink (Internet gateway) in a sensor (mesh) network. The term "network node" is used for the sensor (mesh) nodes in a sensor (mesh) network.

can occur at these network nodes resulting in large delays.

In this paper, we consider hybrid sensor and mesh networks from the purpose of achieving *fair* and energy efficient communication. The notion of fairness captures the non-uniformity in the relaying burden across the nodes. It is strongly linked to many performance measures of interest including network capacity and lifetime, which are discussed in the subsequent sections. A hybrid network consists of sensor or mesh nodes with wireless communication capability and some infrastructure in the form of cables with wireless transceivers attached to their ends. These wires help reduce the average hop count* of the network, resulting in reduced energy dissipation in the network nodes. Moreover, the wires can result in a reduction in the variation of the relaying burden across the network nodes, thereby improving fairness.

The outline for the rest of the paper is as follows. The system model and related works are discussed in Section II. The trade-offs between the cost of infrastructure, average hop count, and fairness are discussed in Sections III and IV. A deterministic wire placement scheme that achieves the optimal trade-off is also developed. The implications of placing the wires in some random fashion are discussed in Section V. Some concluding remarks are presented in Section VI.

*Average number of hops required by a data packet to reach its destination.

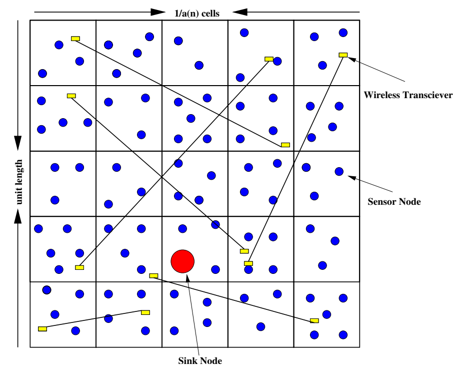

Fig. 2. A hybrid network comprising of the network nodes, wires, and a sink node.

II. System Model and Related Work

A. System Model

We consider a generic network formed by n network nodes uniformly distributed within a square $\mathcal{R}$ of area $\mathcal{A}(n)$ (see Figure 2). The network nodes are equipped with wireless transceivers for communication purposes. There is a single sink node located at some arbitrary location inside $\mathcal{R}$.

The infrastructural support is available in the form of wires with wireless transceivers at their ends. The wires can be placed either in a random or deterministic fashion. The wire placement schemes are considered in subsequent sections.

Time is divided into slots and each packet transmission occupies one slot. The success or failure of a wireless transmission depends on the SINR at the receiving node. We refer the readers to our earlier work [13] for a detailed description of the transmission model. The network is divided into square cells of size $a(n) = \sqrt{\frac{32\mathcal{A}(n)\log n}{n}}$. Thus, there are a total of $\mathcal{N}(n) = \mathcal{A}(n)/a^2(n)$ cells in the network†. Following the line of analysis in [13], it can be shown that each cell almost surely contains $\Theta(\log n)$ nodes.

All transmissions are carried out at the same power level and between *adjacent* cells, where by *adjacent cells* we mean the cells that are horizontal or vertical neighbors of a given cell. The packets are routed along a path that has the smallest hop count. For the sake of concreteness, we assume that each packet is first routed horizontally until it reaches the column of cells containing the sink node, and then routed vertically to

†The round-off errors do not effect the nature of our results and will be ignored throughout the paper.

the cell containing the sink node. A scheduling scheme that enables the above communication scenario while ensuring that each cell gets its turn to transmit at least once in a fixed, independent of n, number of slots is presented in [13].

B. Related Work

Watts and Strogatz [14] were among the first to show that the addition of a few random connections (or rewiring of a few local connections[‡]) to a ring lattice substantially can reduce its diameter substantially while preserving local clustering, thereby leading to a small world effect. A significant amount of work on small world networks has followed the same line of reasoning. In fact, similar ideas have been used in many different contexts including efficient resource querying in wireless ad hoc networks [8].

A fundamental difference between our work and the previous works on small world networks is that we exploit the small world phenomenon to design efficient communication networks, whereas, previous works on small world networks focus mainly on determining the reason for small world behavior of natural networks. In particular, unlike previous works on small world networks, we do not limit our study to random placement of short cuts (wires). Furthermore, the research on small world networks has mainly focused on a regime where the number of short cuts is roughly the same as the number of nodes in the network, whereas, due to practical considerations, we allow the number of short cuts to be much smaller than the number of nodes in the network.

In [2], the authors propose the addition of wired infrastructure to a sensor network for reducing the average energy dissipation across the nodes. They demonstrate through simulations that significant energy savings can be achieved by adding a few wires to a pure wireless network in a strategic manner. However, their wire placement algorithm can achieve at most a constant factor improvement in the energy dissipation for all values of n. In [5], the authors study how to exploit various forms of heterogeneity in sensor networks. Most of their results are based on simulations.

In our previous work [13], we studied the average energy dissipation and the disparity in energy dissipation across the nodes in a sensor network. We also studied the trade-offs between the cost of infrastructure and the above performance measures in a hybrid network. We also proposed wire placement and routing schemes to optimally trade-off the cost with the above performance measures.

A major difference between [13] and this paper is in the model for the cost of infrastructure. In [13], the cost of infrastructure was assumed to be proportional to the number of wires deployed. In this paper, however, we consider the cost of infrastructure to be proportional to the total length of the wires deployed. One would expect the cost of wiring to be much more than the cost of the wireless transceivers, and the latter is proportional to the total length of the wires deployed. As shown in the sequel, this change in the cost model fundamentally impacts the nature of the trade-offs as well as the wire placement schemes required for achieving the trade-offs.

Hybrid ad hoc networks have been studied in [12], [10]. Hybrid sensor and mesh networks differ from hybrid ad hoc networks in that the communication scenario in hybrid sensor and mesh networks is many-to-few (or few-to-many), rather than the many-to-many communication scenario, which is typical of hybrid ad hoc networks. In spite of this, most results in this paper regarding the average hop count are also applicable to hybrid ad hoc networks. We will discuss this issue further in Section VI.

Several energy efficient routing protocols for wireless sensor and mesh networks have been proposed in the literature. Most of these protocols [6], [7], [9] are based on data centric routing and require application awareness. The energy efficiency in these protocols is achieved by means of data aggregation, caching, and elimination of redundant packet transfers by application specific naming of data (meta-data). Such protocols can be used even in hybrid networks to reduce the energy dissipation in the network nodes even further.

Networked Info-mechanical Systems (NIMS) are currently being developed at UCLA and partner universities [3], [4]. These are particular instances of the kind of scenario we envision in this paper. NIMS infrastructure consists of two hierarchies of sensor nodes. The first type of nodes are cheap, immobile, run on irreplaceable batteries, and have very limited capabilities. The second type of nodes are much more sophisticated and are placed on a collection of steel cables, where they can move and recharge their batteries. An efficient use of the infrastructural support along with the capability to harvest solar energy can possibly eliminate the need for replacement/maintenance of the unsophisticated sensor nodes in NIMS, and other such hybrid sensor networks.

[‡]Note that the rewiring can be problematic as it can result in loss of connectivity.

III. Average Hop Count and Fairness in Wireless Networks

In this section, we will define the notions of average hop count and fairness index and study them in the context of wireless networks. We start with their definitions.

Definition 1 (Average Hop Count): Let h_i be the length of the shortest (in terms of hops) path from the cell containing the network node i to the cell containing the sink node. The average hop count $\mathcal{H}$ of the network is given by $\mathcal{H} := \frac{1}{n}\sum h_i$.

Definition 2 (Faines Index): Let r_i be the number of cells whose packets pass through cell i in order to reach the sink node. The fairness index $\mathcal{F}$ is given by $\mathcal{F} := \min_{i=1}^{n} r_i / \max_{i=1}^{n} r_i$.

Note that the average hop count is related to the energy dissipation in the network nodes. Indeed, if each network node generates data at the same rate then the total energy dissipation in the network is proportional to the average hop count. We would therefore like the average hop count to be as small as possible.

The fairness index is related to the network capacity and lifetime. A small value of fairness index indicates an imbalance in the relaying burden across the cells, which leads to an imbalance in the amount of energy dissipation in the network nodes inside different cells. This imbalance in energy dissipation results in some network nodes dying much earlier than the others, which can, at times, render the network useless. Moreover, the cells with a larger relaying burden can also become the bottlenecks, reducing the aggregate network capacity. We would therefore like the fairness index to be as large as possible.

We now study the average hop count and fairness index in wireless networks. Recall the system model in Section II-A. The packets are forwarded to the sink node in a greedy fashion using adjacent cell communication scheme. It is straightforward to show that the average hop count scales as $\Theta\left(\sqrt{n/\log n}\right)$ for all possible locations of the sink node (see [13]).

Consider now cells located at the four corners of the network. These cells relay only their own packets. Whereas, all packets must eventually pass through one of the (at most four) cells that are adjacent to the sink node. Noting that there are $\Theta(n/\log n)$ cells in the network, it follows that the fairness index is $\Theta(\log n/n)$.

From the above discussion, it is clear that wireless networks exhibit a large average hop count and small fairness index. In subsequent sections, we will discuss how to improve this situation with the help of a limited amount of infrastructure in a cost-effective manner.

IV. Hybrid Networks

In this section, we study the average hop count and fairness index in hybrid networks. The goal is to decrease the average hop count and increase the fairness index as much as possible with the help of a limited amount of infrastructure. In our previous work [13], we considered a setting in which the cost of the infrastructure is proportional to the number of wires deployed. In this paper, however, we look at a more realistic cost model, to be described next. We must point out that this difference in modeling the cost of infrastructure leads to some fundamental differences in the nature of the involved trade-offs as well as the proposed solutions.

In this paper, we assume that the cost of infrastructure is proportional to the length of the wiring deployed[§]. The goal is to improve the average hop count and fairness index as much as possible under a given cost budget. To be more precise, we assume that the cost budget is such that the total length of wire deployed can be no more than $\mathcal{N}(n)a(n)l(n)$, where $\Theta(\sqrt{\log n/n}) \leq l(n) \leq 1$.

Determining the placement of wires that minimizes the average hop count under a given budget constraint is a very difficult problem in its own right; leave aside the added objective of trying to maximize the fairness index. Many similar (in fact, seemingly less difficult) problems have been shown to be NP-Complete in the computer science literature (see, for example, [11], [1] and the references therein). In this paper, we focus on the design of wire placement and routing schemes under which the average hop count and fairness index are at most a constant factor from the optimal for all possible network sizes.

A. An Upper Bound on the Improvement

In this section, we lower bound the average hop count and upper bound the fairness index under all possible wire placement and routing schemes that use wiring of length no more than $\mathcal{N}(n)a(n)l(n)$.

Proposition 1: If the length of wiring is no more than $\mathcal{N}(n)a(n)l(n)$, then $\mathcal{H}$ and $\mathcal{F}$ must scale as $\Omega(1/l(n))$ and $O(l(n))$, respectively, under all possible wiring and routing schemes.

Proof: First, let us look at the average hop count. Since the average hop count cannot be smaller than 1, the result trivially holds for $l(n) = \Theta(1)$. Assume, therefore, that $l(n) = o(1)$. Now consider n to be large enough,

[§]Note that the digging costs are usually in proportion to the length of the wiring.

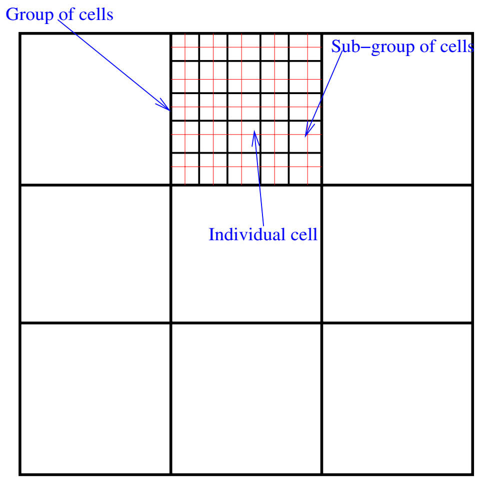

Fig. 3. A division of cells into groups and sub-groups for $l(n) = 1/80$ and $\mathcal{N}(n) = 720$ as specified in the proof of Proposition 1.

and divide the network into groups of cells with each group consisting of $1/8l(n) \times 1/8l(n)$ cells. Observe that there are greater than $63\mathcal{N}(n)l^2(n)$ such groups of cells in the network, excluding the one containing the sink node. Since the total length of wire deployed is no more than $\mathcal{N}(n)a(n)l(n)$, at most $48\mathcal{N}(n)l^2(n)$ groups can contain wiring of length more than $a(n)/48l(n)$. Thus, there are at least $15\mathcal{N}(n)l^2(n)$ groups containing wiring of length no more than $a(n)/48l(n)$. Now, divide each such group of cells into 25 sub-groups (see Fig. 3). Clearly, all cells in the middle sub-group require at least $1/16l(n) - 1/48l(n) - 1/40l(n) = 1/60l(n)$ hops in order to reach the sink node. Thus, the average hop count $\mathcal{H}$ is at least

$$15\mathcal{N}(n)l^2(n) \times \frac{1}{64l^2(n)} \times \frac{1}{60l(n)} \times \frac{1}{\mathcal{N}(n)} \geq \frac{1}{256l(n)}.$$

Next, let us look at the fairness index. Clearly, the four corner cells in the network will not be involved in relaying any packets for any other cell in the network irrespective of how the wires are placed in the network. Also, since the average hop count scales as $\Omega(1/l(n))$, there must exist at least one cell in the network that relays packets for $\Omega(1/l(n))$ other cells. The fairness index $\mathcal{F}$ must therefore scale as $O(l(n))$. ■

B. A Near Optimal Wire Placement and Routing Scheme

In this section, we develop a deterministic wire placement and routing scheme, namely HWPR (hierarchical wire placement and routing). The HWPR scheme is order optimal for almost the entire range of cost budgets, i.e., it achieves maximum possible reductions (in order sense) in the average hop count and fairness index under a given cost budget. In order to simplify the notation, we will assume $\log_3(\sqrt{\mathcal{N}(n)}l(n))$ to be an integer and $9/l(n)$ to be an odd integer. The scheme and its analysis can easily be extended to other cases as well. The description of the scheme follows:

HWPR: Hierarchical Wire Placement and Routing

- *Wire Placement*: A hierarchical grouping of the cells is performed as illustrated in Fig.4. First, the network is divided into nine groups of cells, each consisting of $\sqrt{\mathcal{N}(n)}/3 \times \sqrt{\mathcal{N}(n)}/3$ cells. These groups are termed as level-1 groups. Each level-1 group is further divided into nine sub-groups, each consisting of $\sqrt{\mathcal{N}(n)}/9 \times \sqrt{\mathcal{N}(n)}/9$ cells. These groups are termed as level-2 groups. This process is continued, forming level-3, level-4,..., level-($\log_3(\sqrt{\mathcal{N}(n)}l(n)) - 2$) groups. Note that a level-k group consists of $\sqrt{\mathcal{N}(n)}/3^k \times \sqrt{\mathcal{N}(n)}/3^k$ cells. In particular, a level-($\log_3(\sqrt{\mathcal{N}(n)}l(n)) - 2$) group consists of $9/l(n) \times 9/l(n)$ cells. Also, by convention, the entire network is termed as the level-0 group. For each level-k group, $k = 1, 2, ..., \log_3(\sqrt{\mathcal{N}(n)}l(n)) - 2$, the cell in the center of the group is connected with its counterparts in the adjacent level-k groups. These connections will henceforth be referred to as *flat* connections. For each level-k group, $k = 1, ..., \log_3(\sqrt{\mathcal{N}(n)}l(n)) - 2$, the cell in the center of the group is connected with the cell in the center of the level-$(k-1)$ group that contains the given level-k group, provided it is not already connected with it. These connections will henceforth be referred to as *hierarchical* connections. Each cell in the center column of a level-($\log_3(\sqrt{\mathcal{N}(n)}l(n)) - 2$) group is connected with its adjacent cells in the column. Finally, the cell containing the sink node is connected with the cell in the center of its level-($\log_3(\sqrt{\mathcal{N}(n)}l(n)) - 2$) group.
- *Routing*: The routing is also performed in a hierarchical fashion as follows. Each cell v in a level-($\log_3(\sqrt{\mathcal{N}(n)}l(n)) - 2$) group routes all its packets to the cell in the center of the group. The packets are first routed horizontally using wireless communication until they reach a cell in the center column of the network. They are then routed toward the cell at the center of the column using the wired connections. From there, the packet is routed up the hierarchy, until it reaches the cell in the center of the level-k group that contains both cell v and the sink node. The packet is then routed

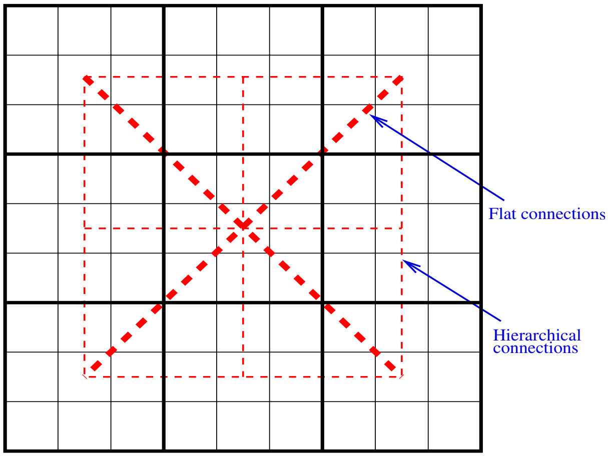

Fig. 4. The flat and hierarchical connections are depicted.

downwards toward the cell in the center of the level-$(\log_3(\sqrt{\mathcal{N}(n)}l(n)) - 2)$ group that contains the sink node. If this cell is not the cell containing the sink node, then the packet is forwarded to the cell containing the sink node using the wired connection between the two cells.

We next analyze the performance of HWPR.

Proposition 2: The HWPR scheme uses a total length of wire no more than $5\mathcal{N}(n)a(n)l(n)/9$ and provides a fairness index of $\Theta(l(n))$. The average hop count under HWPR is $\Theta(1/l(n))$ for $l(n) = O(1/\log n)$ and $\Theta(\log n)$ for $l(n) = o(1/\log n)$.

Proof: First, let us look at the total length of wire used by HWPR. It is easily seen that the length of wire required to connect the level-k groups with each other is no more than $2\sqrt{\mathcal{A}(n)} \times 3^k$. Thus, the length of wire used for flat connections at the same level is no more than

$$2\sqrt{\mathcal{A}(n)} \sum_{k=1}^{\log_3(\sqrt{\mathcal{N}(n)}l(n))-2} 3^k \leq \frac{\sqrt{\mathcal{A}(n)\mathcal{N}(n)}l(n)}{3} = \frac{\mathcal{N}(n)a(n)l(n)}{3}. \quad (1)$$

Now, the length of wire required for connecting the level-k groups with their corresponding level-$(k-1)$ groups is no more than

$$4\sqrt{2} \times \frac{\sqrt{\mathcal{A}(n)}}{3^k} \times 9^{k-2},$$

where the second term is the dimension of a level-k group and the third term is the number of level-$(k-1)$ groups in the network. Thus, the total length of wire used for hierarchical connections is

$$\frac{4\sqrt{2}}{9}\sqrt{\mathcal{A}(n)} \sum_{k=1}^{\log_3(\sqrt{\mathcal{N}(n)}l(n))-2} 3^k \leq \frac{\sqrt{\mathcal{A}(n)\mathcal{N}(n)}l(n)}{3} = \frac{2\sqrt{2}\mathcal{N}(n)a(n)l(n)}{27}. \quad (2)$$

The length of wire required to connect the cells in the center column of level-$(\log_3(\sqrt{\mathcal{N}(n)}l(n)) - 2)$ groups is no more than

$$\frac{9a(n)}{l(n)} \times \frac{\mathcal{N}(n)l^2(n)}{81} = \frac{\mathcal{N}(n)a(n)l(n)}{9}, \quad (3)$$

where the first term is length of wire required per level-$(\log_3(\sqrt{\mathcal{N}(n)}l(n)) - 2)$ group and the second term is the number of level-$(\log_3(\sqrt{\mathcal{N}(n)}l(n)) - 2)$ groups in the network. Combining (1)-(3), and noting that the length of wire required for connecting the cell containing the sink node with the cell in the center of its level-$(\log_3(\sqrt{\mathcal{N}(n)}l(n)) - 2)$ group is $O(\sqrt{\mathcal{A}(n)}l(n))$, we obtain that the total length of wire used by HWPR is no more than $\mathcal{N}(n)a(n)l(n)\left(\frac{1}{3} + \frac{2\sqrt{2}}{27} + \frac{1}{9}\right) + \frac{\mathcal{N}(n)a(n)l(n)}{9} + O(\sqrt{\mathcal{A}(n)}l(n)) \leq 5\mathcal{N}(n)a(n)l(n)/9$.

Now, let us look at the average hop count. A packet originating at some arbitrary cell in the network takes $\Theta(1/l(n))$ hops in order to reach the cell at the center of its level-$(\log_3(\sqrt{\mathcal{N}(n)}l(n)) - 2)$ group and then another $\Theta(\log_3 \sqrt{\mathcal{N}(n)}l(n))$ hops in order to reach the sink node. The total number of steps is therefore $\Theta(1/l(n) + \log_3 \sqrt{\mathcal{N}(n)}l(n))$, that proves our claim regarding the average hop count.

Now, observe that the cell at the center of a level-$(\log_3(\sqrt{\mathcal{N}(n)}l(n)) - 2)$ group relays packets for $\Theta(1/l(n))$ other cells in its group. None of the cells relays packets for $w(l(n))$ cells and the corner cells do not relay packets for any other cell. Thus, the fairness index is $\Theta(l(n))$. ■

The hierarchical routing scheme used in HWPR can be implemented as follows. The sink node can flood its location information to all cells that belong to the same level-$(\log_3(\sqrt{\mathcal{N}(n)}l(n)) - 2)$ group as the cell containing the sink node. The cell at the center of this group can pass the information to the level-$(\log_3(\sqrt{\mathcal{N}(n)}l(n)) - 1)$ cell that it is connected to, and so on. In this manner, the location of the sink node can be propagated up in the hierarchy until it reaches the cell at the center of the network (level-0 group). The overhead associated with flooding the location of the sink node is therefore $\Theta(1/l(n))$.

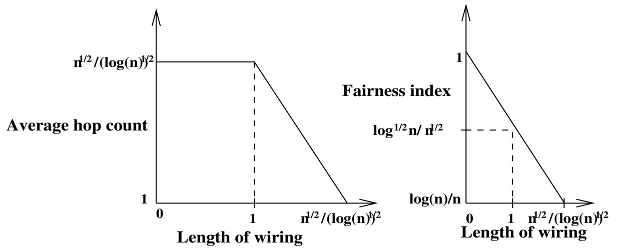

Fig. 5. The infrastructure cost versus the average hop count and fairness index.

C. Discussion of the Trade-offs

The results in the previous two subsections characterize the trade-offs between the infrastructure cost versus the average hop count and fairness index (see Fig.5). For example, with a wire length budget of $\Theta(\sqrt{n/\log n})$ ($l(n) = \Theta(1)$), one can make the average hop count scale as $\Theta(\log n)$ and fairness index scale as $\Theta(1)$, which is very close to their optimal values (in order sense). The other extreme case is to have a wire length budget of $\Theta(1)$ ($l(n) = \Theta(\sqrt{\log n/n})$). In such a case, the average hop count stays close to its value under the wireless setting, whereas, the fairness index can be made to scale as $\Theta(\sqrt{\log n/n})$, which is better than $\Theta(\log n/n)$ - its value under the wireless setting. A regime not considered in this work is where $\log n/n \leq l(n) \leq \sqrt{\log n/n}$. It can be shown that the average hop count cannot be improved in such a regime, however, the fairness index can be improved. In particular, the fairness index can be made to scale as $\Theta(l(n))$.

V. The Price of Lack of Planning

The HWPR scheme requires strategic wire placement and sophisticated routing of packets in order to achieve near optimal trade-offs between the infrastructure cost versus the average hop count and fairness index. A natural question to ask is: Can similar trade-offs be obtained by using simpler wire placement and routing schemes? In order to answer this question, we look at a class of random wire placement schemes and analyze their performance under the greedy geographic routing scheme.

The class of schemes we consider is characterized by the parameter $\alpha \geq 0$. For a given α, the corresponding scheme places a wire between two cells that are at a hop distance of d from each other with a probability proportional to $d^{-\alpha}$. More precisely, the probability $\mathbb{P}(u,v)$ that cell u is connected to cell v with a wire is given by

$$\mathbb{P}(u,v) = C(n)d(u,v)^{-\alpha},$$

where $d(u,v)$ is the shortest hop distance between cells u and v, and $C(n)$ is a normalization constant. For technical convenience, we consider wired connections to be unidirectional in this section. Also, the cost budget is modeled by imposing the constraint that the expected length of wire deployed should be smaller than $\mathcal{N}(n)a(n)l(n)$. These modifications have no impact on the order of the results that follow.

The idea behind looking at the above class of schemes is to capture the "unplanned" nature of wire placement in some mathematically precise manner. In particular, $\alpha = 0$ case represents the scenario in which all pair of cells are equally likely to be connected with each other. This can arise as a result of randomly placing a small number of long wires in the network. As α is increased, one captures scenarios in which a larger number of shorter wires are placed randomly in the network.

Let $\beta = \inf\{x : \lim_{n\to\infty} n^{-x}/l(n) = 0\}$. We have the following result concerning the average hop count under the above class of schemes:

Proposition 3: The average hop count $\mathcal{H}$ scales as $\Omega(n^{\delta})$ for all $\delta < \delta_o$, where

1) $\delta_o = 1/2$ for $\alpha \leq 2$;
2) $\delta_o = \min\left\{1/2, \frac{\beta}{\alpha-1} + \frac{1/2}{\alpha-1}\right\}$ for $2 < \alpha \leq 3$; and
3) $\delta_o = \min\left\{1/2, \frac{\beta}{\alpha-1} + \frac{(\alpha-2)}{2(\alpha-1)}\right\}$ for $\alpha > 3$.

In order to prove the above result, we first upper bound the normalization constant $C(n)$. Throughout the rest of this section we assume n to be large.

Lemma 1: The normalization constant $C(n)$ satisfies:

$$C(n) \leq \begin{cases} \frac{2^{9/2-\alpha}(3-\alpha)l(n)}{\mathcal{N}(n)^{\frac{3-\alpha}{2}}}, & \alpha < 3 \\ \frac{4\sqrt{2}l(n)}{\log(\mathcal{N}(n))}, & \alpha = 3 \\ 2\sqrt{2}(\alpha-3)l(n), & \alpha > 3 \end{cases}$$

Proof: Let $\mathcal{C}$ denote the set of cells in the network. Since the expected length of wire deployed must be smaller than $\mathcal{N}(n)a(n)l(n)$, we have

$$\sum_{u\in\mathcal{C}}\sum_{v\in\mathcal{C}\setminus u} C(n)d(u,v)^{-\alpha}\frac{a(n)d(u,v)}{\sqrt{2}} \leq \mathcal{N}(n)a(n)l(n).$$

Note that the length of wire required to connect a pair of cells that are at a lattice distance of k hops from each other is at least $\frac{ka(n)}{\sqrt{2}}$. Now, observing that every cell in the network has at least k cells at a lattice distance of k

from it, $k = 1, 2, ..., \sqrt{\mathcal{N}(n)}/2$, we have

$$C(n) \sum_{k=1}^{\sqrt{\mathcal{N}(n)}/2} k k^{-\alpha} \frac{k a(n)}{\sqrt{2}} \le a(n) l(n). \quad (4)$$

Simplifying (4), we have

$$C(n) \le \frac{\sqrt{2} l(n)}{\sum_{k=1}^{\sqrt{\mathcal{N}(n)}/2} k^{2-\alpha}}. \quad (5)$$

Using straightforward algebraic manipulations, it can be shown that

$$\sum_{k=1}^{\sqrt{\mathcal{N}(n)}/2} k^{2-\alpha} \ge \begin{cases} \frac{\mathcal{N}(n)^{\frac{3-\alpha}{2}}}{2^{4-\alpha}(3-\alpha)}, & \alpha < 3 \\ \frac{1}{4}\log(\mathcal{N}(n)), & \alpha = 3 \\ \frac{1}{2(\alpha-3)}, & \alpha > 3 \end{cases} \quad (6)$$

The result now follows from (5) and (6). ■

Let $N_\delta(u)$ denote the set of cells that are at a lattice distance of n^δ or less from the cell u and let c_s be the cell containing the source node. The next step is to upper bound the probability that the packet reaches some cell in $N_\delta(c_s)$ at any given step.

Lemma 2: Let u be an arbitrary cell in the network. Then, the probability $\mathbb{P}_\delta(u)$ that cell u has a wired connection to one or more cells v in $N_\delta(c_s)$ satisfies:

$$\mathbb{P}_\delta(u) \le \begin{cases} \frac{2^{\frac{15}{2}-\alpha} n^{\delta(2-\alpha)}(3-\alpha) l(n)}{\mathcal{N}(n)^{\frac{3-\alpha}{2}}(2-\alpha)}, & \alpha < 2 \\ \frac{2^{15/2-\alpha}\delta \log n (3-\alpha) l(n)}{\mathcal{N}(n)^{\frac{3-\alpha}{2}}}, & 2 \le \alpha < 3 \\ \frac{32\sqrt{2} l(n)}{(\alpha-2)\log(\mathcal{N}(n))}, & \alpha = 3 \\ \frac{16\sqrt{2}(\alpha-3) l(n)}{\alpha-2}, & \alpha > 3 \end{cases}$$

Proof: First, observe that $\mathbb{P}_\delta(u) \le \mathbb{P}_\delta(c_s)$. Since there are at most $4k$ cells at a lattice distance of k from the cell c_s, we have

$$\mathbb{P}_\delta(c_s) \le C(n) \sum_{k=1}^{\lfloor n^\delta \rfloor} 4k^{1-\alpha}. \quad (7)$$

Using straightforward algebra, it can be shown that

$$\sum_{k=1}^{\lfloor n^\delta \rfloor} 4k^{1-\alpha} \le \begin{cases} \frac{8n^{\delta(2-\alpha)}}{2-\alpha}, & \alpha < 2 \\ 8\delta \log(n), & \alpha = 2 \\ \frac{8}{\alpha-2}, & \alpha > 2 \end{cases} \quad (8)$$

Using (7) and (8), along with Lemma 1, the result follows. ■

The final result that we need to prove Proposition 3 is concerning the probability of having a long range connection.

Lemma 3: Consider an arbitrary cell u in the network and suppose $\alpha > 2$. The probability $\mathbb{P}_\gamma(u)$ that cell u is connected to some cell v in the network such that $d(u, v) \ge n^\gamma$ satisfies:

$$P_\gamma(u) \le \begin{cases} \frac{2^{15/2-\alpha}(3-\alpha) n^{\gamma(2-\alpha)} l(n)}{(\alpha-2)\mathcal{N}(n)^{\frac{3-\alpha}{2}}}, & 2 < \alpha < 3 \\ \frac{32\sqrt{2} n^{\gamma(2-\alpha)} l(n)}{\log(\mathcal{N}(n))}, & \alpha = 3 \\ \frac{16\sqrt{2}(\alpha-3) n^{\gamma(2-\alpha)} l(n)}{\alpha-2}, & \alpha > 3 \end{cases}$$

Proof: Once again, noting that there are at most $4k$ cells at a lattice distance of k from the cell u, it follows that

$$\mathbb{P}_\gamma(u) \le C(n) \sum_{\lceil n^\gamma \rceil}^{\infty} 4k^{1-\alpha} \le \frac{8C(n) n^{\gamma(2-\alpha)}}{\alpha - 2}.$$

The result now follows by using Lemma 1. ■

We are now ready to prove Proposition 3.

Proof of Proposition 3: First, let us consider $\alpha \le 2$. Consider a packet originating at some arbitrary source node in the network. Let $\mathcal{G}$ be the event that the packet reaches a cell in $N_\delta(c_s)$ within the first n^δ steps, where δ is a constant to be fixed later. Note that $\mathcal{G} = \bigcup_{i=1}^{\lfloor n^\delta \rfloor} \mathcal{G}_i$, where $\mathcal{G}_i$ is the event that at step i the packet reaches a cell in $N_\delta(c_s)$. Now, we have

$$\mathbb{P}(\mathcal{G}) = \mathbb{P}\left(\bigcup_{i=1}^{\lfloor n^\delta \rfloor} \mathcal{G}_i\right) \le \sum_{i=1}^{\lfloor n^\delta \rfloor} \mathbb{P}(\mathcal{G}_i) \le n^\delta \mathbb{P}_\delta(c_s)$$

Using Lemma 2, it follows that

$$\mathbb{P}(\mathcal{G}) \le \begin{cases} \frac{2^{\frac{15}{2}-\alpha} n^{\delta(3-\alpha)}(3-\alpha) l(n)}{\mathcal{N}(n)^{\frac{3-\alpha}{2}}(2-\alpha)}, & \alpha < 2 \\ \frac{2^{\frac{15}{2}-\alpha}\delta n^\delta \log n (3-\alpha) l(n)}{\mathcal{N}(n)^{\frac{3-\alpha}{2}}}, & \alpha = 2 \end{cases}$$

Note that $\mathbb{P}(\mathcal{G}) = o(1)$ for $\delta < 1/2$ if $\alpha \le 2$. In such a case, consider n to be large enough so that $P(\mathcal{G}) \le 1/4$, and define:

- $\mathcal{T}$ - The cell c_o containing the source node and the cell c_s are at a hop distance of more than $\sqrt{\mathcal{N}(n)}/4$ from each other.
- $\mathcal{D}$ - The packet reaches the sink node within n^δ hops.
- N_o - The random variable representing the number of hops required in order to reach the sink node staring from the source node.

It is straightforward to show that $\mathbb{P}(\mathcal{T}) \ge 3/4$. Thus, we have

$$\mathbb{P}(\mathcal{T} \cap \overline{\mathcal{G}}) \ge 1 - \mathbb{P}(\overline{\mathcal{T}}) - \mathbb{P}(\mathcal{G}) \ge 1/2.$$

Note that if $\mathcal{T}$ occurs and $\mathcal{G}$ does not occur, then $\mathcal{D}$ cannot occur. For suppose it does. Since the cells c_s and c_o are at a lattice distance of more than $\sqrt{\mathcal{N}(n)}/4$ from each other and $\delta < 1/2$, the packet must be forwarded at least once using a wired connection. Moreover, the final time this occurs the wired contact must be in $N_\delta(c_s)$. Contradicting our initial assumption that G does not occur. Since this holds for an arbitrary source node, we have

$$\begin{aligned}\mathbb{E}[\mathcal{H}] &\geq \frac{1}{n}\sum_{o=1}^{n}\mathbb{E}[N_o/\mathcal{T}\cap\overline{\mathcal{G}}]\cdot\mathbb{P}(\mathcal{T}\cap\overline{\mathcal{G}})\\ &\geq \frac{1}{n}\sum_{o=1}^{n}\frac{n^\delta}{2}=\frac{n^\delta}{2},\end{aligned}$$

proving the first claim in Proposition 3.

Next, let us consider $\alpha > 2$. Let $\mathcal{T}, \mathcal{D}$, and N_o be defined as before and redefine the events $\mathcal{G}_i$ and $\mathcal{G}$ as follows:

- $\mathcal{G}_i$ - At step i, the packet is with a cell u that has a wired connection with a cell v such that $d(u,v) \geq n^\gamma$.
- $\mathcal{G} = \cup_{i=1}^{\lfloor n^\delta \rfloor}\mathcal{G}_i$ - During the first $\lfloor n^\delta \rfloor$ steps, the packet reaches a cell u that has a wired connection with a cell v such that $d(u,v) \geq n^\gamma$.

Using Lemma 3 and the definition of $\mathcal{G}$, it is straightforward to show that

$$\mathbb{P}(\mathcal{G}) \leq \begin{cases} \frac{2^{15/2-\alpha}(3-\alpha)n^{\delta+\gamma(2-\alpha)}l(n)}{(\alpha-2)\mathcal{N}(n)^{\frac{3-\alpha}{2}}}, & 2<\alpha<3\\ \frac{32\sqrt{2}n^{\delta+\gamma(2-\alpha)}l(n)}{\log(\mathcal{N}(n))}, & \alpha=3\\ \frac{16\sqrt{2}(\alpha-3)n^{\delta+\gamma(2-\alpha)}l(n)}{\alpha-2}, & \alpha>3\end{cases}$$

Now, for any given $\delta < 1/2$ set $\gamma > 0$ such that $n^{\delta+\gamma} = o(\sqrt{n/\log n})$ and observe that $\mathbb{P}(\mathcal{G}) = o(1)$ for $\delta < \min\left\{1/2, \frac{\beta}{\alpha-1} + \frac{1/2}{\alpha-1}\right\}$ if $2 < \alpha \leq 3$; and $\delta < \min\left\{1/2, \frac{\beta}{\alpha-1} + \frac{(\alpha-2)}{2(\alpha-1)}\right\}$ if $\alpha > 3$.

Consider n large enough so that $\mathbb{P}(\mathcal{G}) \leq 1/4$. Once again, we claim that if $\mathcal{T}$ occurs and $\mathcal{G}$ does not occur then $\mathcal{D}$ cannot occur. This is because if $\mathcal{G}$ does not occur, then the distance traveled by the packet in the first n^δ hops is at most $n^{\delta+\gamma} = o(\sqrt{n/\log n}) = o(\sqrt{\mathcal{N}(n)})$ hops. If $\mathcal{T}$ occurs in this case, then the packet clearly cannot reach the sink node in the first n^δ hops, and thus, $\mathcal{D}$ cannot occur. Since

$$\mathbb{P}(\mathcal{T}\cup\overline{\mathcal{G}}) \geq 1-\mathbb{P}(\overline{\mathcal{T}})-P(\mathcal{G}) \geq 1/2,$$

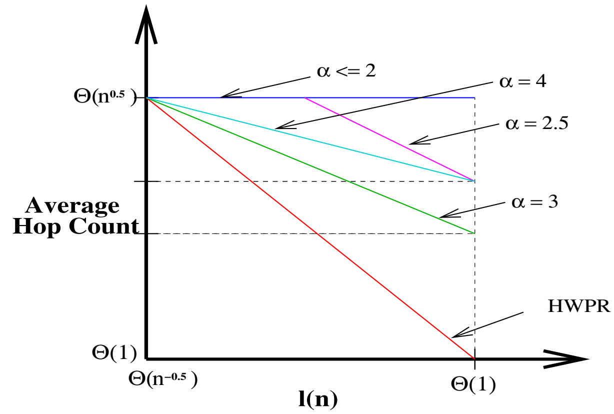

Fig. 6. The infrastructure cost versus the average hop count.

we have

$$\begin{aligned}\mathbb{E}[\mathcal{H}] &\geq \frac{1}{n}\sum_{o=1}^{n}\mathbb{E}[N_o/\mathcal{T}\cap\overline{\mathcal{G}}]\cdot P(\mathcal{T}\cap\overline{\mathcal{G}})\\ &\geq \frac{1}{n}\sum_{o=1}^{n}\frac{n^\delta}{2}=\frac{n^\delta}{2},\end{aligned}$$

proving the rest of the claims in Proposition 3. ■

The above results are illustrated in Fig.6. Next, we summarize the main results:

- For $\alpha \leq 2$, there is no improvement in the average hop count under any value of $l(n)$. Thus, in this case, the wire length budget is simply not enough to result in any improvement in the average hop count.
- For $2 < \alpha \leq 3$, we have an improvement in the hop count for $l(n) = \Omega(n^{\frac{2-\alpha}{2}})$; for $l(n) = o(n^{\frac{2-\alpha}{2}})$ there is no improvement in the average hop count. Also, the average hop count cannot be reduced beyond $\Theta\left(n^{\frac{1}{2(\alpha-1)}}\right)$.
- For $\alpha > 3$, there is an improvement in the hop count for all values of $l(n)$. However, the average hop count cannot be reduced beyond $\Theta\left(n^{\frac{\alpha-2}{2(\alpha-1)}}\right)$.

Clearly, the best value of α is 3. We note that the value of α that yields maximum improvement in the average hop count was shown to be 2 in our previous work [13], when considering a number-of-wires based cost budget. We believe that this difference is due to the fact that the longer connections are more costly and therefore less favorable than shorter connections in the wire-length based cost model considered in this work. This results in the optimal value of α being higher under the cost model considered in this paper as compared to the model considered in our previous work [13]. Observe that the performance of the random wire placement and routing scheme is far from optimal for all values of α. These results suggest that there is a large price to be paid for not using the optimal wire placement and routing scheme. This is unlike the results in [13], where the random wire placement and routing scheme considered above was shown to yield an average hop count within $\Theta(\log^2 n)$ of the optimal for $\alpha = 2$ and all values of $l(n)$. From this, we conclude that the need for having a good wire placement and routing scheme is more so under a wire-length based cost model as compared to a number-of-wires based cost model.

VI. Concluding Remarks

We considered hybrid sensor and mesh networks as paradigms for energy efficient and fair communication. A hybrid network consists of a pure wireless network and some additional infrastructure in the form of wires with wireless transceivers attached to their ends. We considered the problem of cost efficient design of such hybrid networks, where the efficiency is evaluated in terms of the reduction in the average energy dissipation and disparity in energy dissipation across the nodes.

We used the notions of average hop count and fairness index in order to study the average energy dissipation and disparity in energy dissipation across the nodes, respectively. We characterized the trade-offs between the infrastructure cost versus the average hop count and fairness index in hybrid networks. Our results showed that both average hop count and fairness index can be significantly improved under a modest wire length budget, provided a good wire placement and routing scheme is used, like the HWPR scheme proposed in the paper. The HWPR scheme uses intelligent wire placement and routing in order to achieve near optimal improvements in average hop count and fairness index.

In order to understand whether using a good wire placement and routing scheme is crucial to achieving close to optimal improvements in average hop count and fairness index, we studied the performance of a wide class of random wire placement schemes under greedy geographic routing. The idea behind looking at these schemes was to capture the "unplanned" nature of wire placement in some mathematically precise manner. The results showed that all these schemes perform quite poorly in comparison to the optimal scheme, thereby showing that it is essential to use a good wire placement and routing scheme in order to achieve maximum possible improvements in average hop count and fairness index.

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