Sistemas de Controle I

Resolução Lista de Exercícios 01

1) $\ddot{y}(t) + \dot{y}(t) + y(t) = 0$, $y(0) = 1$ e $\dot{y}(0) = 2$

$\ddot{y}(t) = s^2 Y(s) - sY(0) - \dot{y}(0) = \boxed{s^2 Y(s) - s - 2}$

$\dot{y}(t) = sY(s) - y(0) = \boxed{sY(s) - 1}$

$s^2 Y(s) - s - 2 + sY(s) - 1 + Y(s) = 0$

$(s^2 + s + 1) Y(s) = s + 3$

$$\boxed{Y(s) = \frac{s+3}{s^2+s+1}}$$ //

// // //

2) $\ddot{y}(t) + 5\dot{y}(t) + 4y(t) = \dot{x}(t) - x(t)$

a) $\ddot{y}(t) + 5\dot{y}(t) + 4y(t) = sX(s) - X(s)$

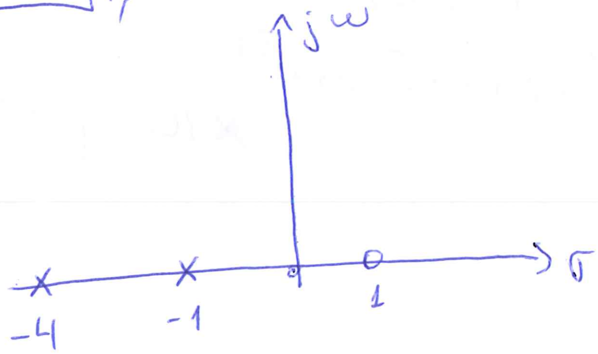
$s^2 Y(s) + 5sY(s) + 4Y(s) = (s-1) X(s)$

$(s^2 + 5s + 4) Y(s)$

$$\boxed{G(s) = \frac{Y(s)}{X(s)} = \frac{s-1}{s^2+5s+4}}$$ //

b) zero = 1

Polos = -1, -4



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3) $G(s) = \dfrac{1}{2s+2}$

a) $\boxed{\text{Polo} = -1}$ //

b) $\lim_{s \to 0} G(s) = \lim_{s \to 0} \dfrac{1}{2s+2} = \dfrac{1}{2}$ //

4) Para $p = 1$ | Para $P = 5$ | Para $p = 10$

$T_s = \frac{4}{1} = \boxed{4s}$ | $T_s = \frac{4}{5} = \boxed{0{,}8s}$ | $T_s = \frac{4}{10} = \boxed{0{,}4s}$ //

- Sistema mais lento para $p = 4$ e mais rápido para $p = 10$

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5) $G(s) = \frac{100}{s^2 + 2s + 25} = \frac{2 w_n^2}{s^2 + 2\xi w_n s + w_n^2}$

a) $w_n^2 = 25 \therefore \boxed{w_n = 5 \text{ rad/s}}$ //

b) $2\xi w_n = 2 \therefore \xi = \frac{2}{2.5} = \boxed{0.2}$ //

c) $w_d = w_n \sqrt{1 - \xi^2} = 5\sqrt{1 - (0{,}2)^2} = \boxed{4{,}9 \text{ rad/s}}$ //

d) $P_{1,2} = -\xi w_n \pm j w_d = -0{,}2 . 5 \pm j 4{,}9 = \boxed{-1 \pm j 4{,}9}$ //

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6)

a) $T_r = \frac{\pi - \cos^{-1} 0{,}2}{4{,}9} = \boxed{0{,}36 s}$ //

b) $T_p = \frac{\pi}{4{,}9} = \boxed{0{,}64 s}$ //

c) $T_s = \frac{4}{0{,}2 . 5} = \boxed{4s}$ //

d) $O_{s\%} = e^{-(0{,}2 . \pi / \sqrt{1 - (0{,}2)^2})} \times 100 = \boxed{52{,}66\%}$ //

7)

a) $$T(s) = \frac{G(s)}{1+G(s)\underbrace{H(s)}_{=1}} = \frac{\dfrac{10}{s(s^2+6s+24)}}{1+\dfrac{10}{s(s^2+6s+24)}}$$

$$T(s) = \frac{\dfrac{10}{s(s^2+6s+24)}}{\dfrac{s(s^2+6s+24)+10}{s(s^2+6s+24)}} = \frac{10}{s(s^2+6s+24)+10}$$

$$\boxed{T(s) = \frac{10}{s^3+6s^2+24s+10}}$$ //

b) $$T_1(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{\dfrac{1}{s(s+1)}}{1+\dfrac{1}{s(s+1)}\cdot s} = \frac{\dfrac{1}{s(s+1)}}{1+\dfrac{1}{s+1}} = \frac{\dfrac{1}{s(s+1)}}{\dfrac{s+1+1}{s+1}}$$

$$T_1(s) = \frac{\frac{1}{s}}{s+2}\,\frac{(s)}{(s)} = \boxed{\frac{1}{s^2+2s}}$$

$$T_2(s) = \frac{160\,T_1(s)}{1+160T_1(s)\underbrace{H(s)}_{=1}} = \frac{\dfrac{160}{s^2+2s}}{1+\dfrac{160}{s^2+2s}} = \frac{\dfrac{160}{s^2+2s}}{\dfrac{s^2+2s+160}{s^2+2s}}$$

$$\boxed{T_2(s) = \frac{160}{s^2+2s+160}}$$ //

c) $T(s) = \dfrac{G(s)}{1 + G(s)\underbrace{H(s)}_{1}} = \dfrac{\dfrac{16(s+2)}{(s+1)^2}}{1 + \dfrac{16(s+2)}{(s+1)^2}} = \dfrac{\dfrac{16(s+2)}{\cancel{(s+1)^2}}}{\dfrac{(s+1)^2 + 16(s+2)}{\cancel{(s+1)^2}}}$

$T(s) = \dfrac{16(s+2)}{(s+1)^2 + 16(s+2)} = \dfrac{16s + 32}{s^2 + 2s + 1 + 16s + 32} = \boxed{\dfrac{16s+32}{s^2 + 18s + 33}}$

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8) $T(s) = \dfrac{G(s)}{1 + G(s)\underbrace{H(s)}_{1}} = \dfrac{\dfrac{25}{s(s+5)}}{1 + \dfrac{25}{s(s+5)}} = \dfrac{\dfrac{25}{\cancel{s(s+5)}}}{\dfrac{s(s+5) + 25}{\cancel{s(s+5)}}}$

$T(s) = \dfrac{25}{s(s+5) + 25} = \boxed{\dfrac{25}{s^2 + 5s + 25}} = \dfrac{\alpha \omega_n^2}{s^2 + 2\xi\omega_n^2 s + \omega_n^2}$

$\omega_n^2 = 25 \therefore \boxed{\omega_n = 5 \text{ rad/s}}$

$\xi = \dfrac{5}{2 \cdot \omega_n} = \dfrac{5}{2 \cdot 5} = 0,5$ //

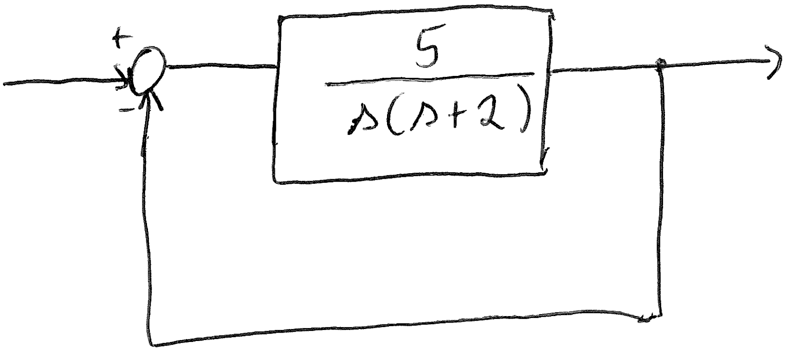
$T_p = \dfrac{\pi}{5\sqrt{1-(0,5)^2}} = \boxed{0,726 \text{ s}}$ //

$O_{s\%} = e^{-0,5 \cdot \pi / \sqrt{1-(0,5)^2}} \times 100 = \boxed{16,3\%}$ //

$T_s = \dfrac{4}{0,5 \cdot 5} = \boxed{1,6 \text{ s}}$ //

9)

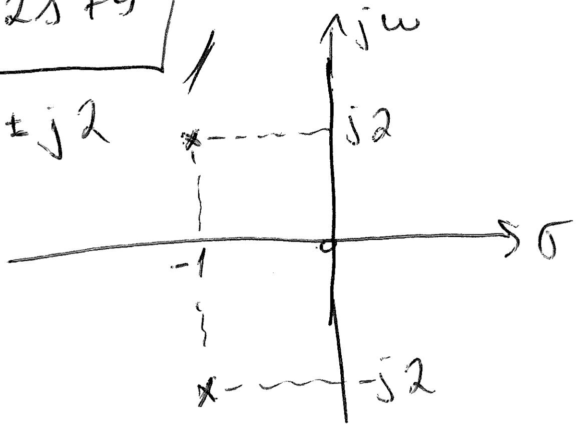
a)



b) $$T(s) = \frac{G(s)}{1+G(s)\underbrace{H(s)}_{\to 1}} = \frac{\frac{5}{s(s+2)}}{1+\frac{5}{s(s+2)}} = \frac{\frac{5}{\cancel{s(s+2)}}}{\frac{s(s+2)+5}{\cancel{s(s+2)}}}$$

$$\boxed{T(s) = \frac{5}{s^2+2s+5}}$$

c) Pólos: $-1 \pm j2$



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10) $$T(s) = \frac{K}{s^2+30s+K} \to s^2 + 2\xi\omega_n s + \omega_n^2$$

$\omega_n = \sqrt{K}$

$2\xi\omega_n = 30 \to \sqrt{K} = \frac{30}{2 \cdot 0{,}59} \to \boxed{K = 646{,}36}$ //

$OS\% = 10\% = 0{,}1$

$$x = \left(\frac{\ln(OS)}{\pi}\right)^2 \qquad \xi = \sqrt{\frac{x}{1+x}} = \sqrt{\frac{0{,}537}{1{,}537}} = 0{,}59$$

$$x = \left(\frac{\ln(0{,}1)}{\pi}\right)^2 = 0{,}537$$