ASSIGNMENT

The submission deadline is Wednesday November 19th, at 12:00 hr. The assignment can be done individually or in groups of two students freely chosen. Cooperation between students who are not part of the same group is not allowed. Remember to follow the Generative AI regulations (see *Canvas*); debugging using AI is prohibited as well. Submit through *Wiseflow* a written report as a pdf file with your model formulations and the answers to the questions required in each part. The formulation of your models can be typed in a text editor (e.g. Word, LaTeX), or written by hand and scanned to the report. In addition, submit a zipped folder containing your Python files and any other file used in your calculations.

Part A

The *Northernmost* city is being built and will include 20 distinct neighbourhoods, as shown by the house icons in Figure 1. The Northernmost Council is planning where to locate electrical stations to service the needs of the neighbourhoods. Because laying electrical line to each neighbourhood is expensive, the placement of the stations requires careful consideration. The stations may be located in any cell (including the same cell as a neighbourhood). The number of stations to install is unlimited. However, there is a fixed cost for installing a station. This fixed cost is \$2000 for a station installed at any cell, except for the four central cells (E5, E6, F5, F6) in which the fixed cost is \$2500. Here the symbol \$ represents the very strong currency of Northernmost.

Every neighbourhood must be connected to exactly one station. Connecting a neighbourhood to a station requires a cable whose length is equal to the distance between the station and the neighbourhood. The cost of electrical wiring is \$100 per km. Distances are measured using a direct line between cells, which are each 1km apart. For example, the distance between cell D2 and A1 is $\sqrt{(4-1)^2+(2-1)^2} = 3.16$ km. [1] Each cable connection is exclusively devoted to serve one neighbourhood, even if there are other neighbourhoods along the way (e.g., if a station is located in cell J7, connecting neighbourhoods at H7 and I7 to such station requires two cables, where one goes from H7 to J7 and the other one from I7 to J7).

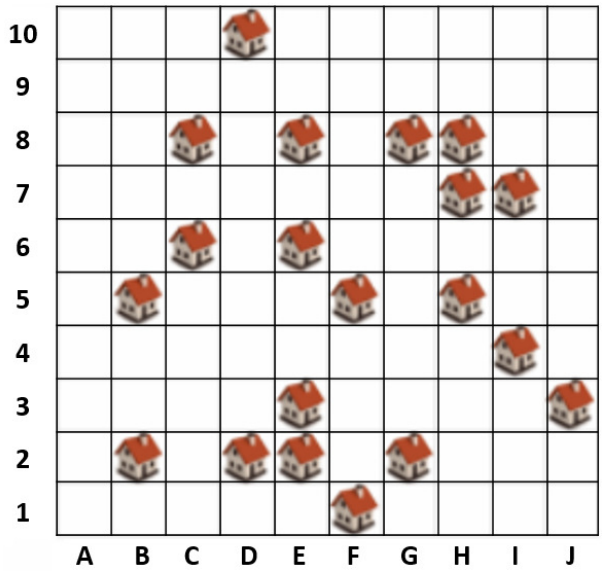

Figure 1: Map of Northernmost.

[1] Recall from basic geometry the distance between two points (x_1, y_1) and (x_2, y_2) can be calculated as $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$. You may compute the distances as you wish, e.g. in Python. For this numerical example, in Python the result is obtained by coding `sqrt`$((4-1)**2+(2-1)**2)$ after the `sqrt`-function is imported from the `math`-module. (To do that, write the line `from math import sqrt` in the beginning of your python file.)

1. As an expert in mathematics for decisions, the Northernmost Council asks you to support the decision on where to locate stations while minimizing the total cost. Formulate an integer linear model for this problem.
2. Implement the model in Python and solve it. Where would you recommend to locate station(s)? How much is the total cost?

The following two tasks introduce some modifications. Each task is independent from each other.

2. Suppose that instead of an unlimited number of stations, the Northernmost Council wants to locate only one station. Formulate an integer linear model for this new situation. Remember to include the model formulation in your report. Implement and solve the model in Python. Where is the station located? How much is the total cost?
3. The Northernmost Council would like to analyse how much would be the cost if instead of all neighbourhoods, only a fraction of these is served. If the requirement is that at least 14 neighbourhoods would be served, how would you modify your integer linear model? Implement and solve the model in Python. Where should the station(s) be located? How much is the optimal cost?
   Repeat the same exercise using 16 and 18 instead of 14 (you do not need to write the model again in your report or to enclose your code again using these two new numbers; you may simply run the model with the corresponding changes and answer the questions about the location of the station(s) and the optimal cost).

Part B

Along with the city of *Northernmost*, several other cities have been built in the area. Now it is time to connect the cities by roads. The Northernmost Council contacted you to help decide which of the roads to build. They provided you the distances of any potential road to build, for any pair of cities where building a road is possible, in the following Table 1. (Since the terrain in the area is quite rough, there

	Northern	Eastern	Easternmost	Southern	Southernmost	Western	Westernmost
Northernmost	24					173	
Northern		100	150	10		60	
Eastern			76	10	41		
Southern					52	21	152
Southernmost						37	
Western							94

Table 1: Distances between the different cities, in km. As all roads go both ways, only one direction is given.

are some pairs of cities that cannot be connected by a direct road.) Building one kilometer of road costs \$500,000. Along with the distance of each potential road, you are provided with an estimate of how much money the council can make by collecting toll fees over the coming 5 years, if that road is built; see the following Table 2. Your task is to find a set of roads to build such that any city can be reached

	Northern	Eastern	Easternmost	Southern	Southernmost	Western	Westernmost
Northernmost	14					35	
Northern		41	78	60		28	
Eastern			35	6	21		
Southern					25	12	12
Southernmost						8	
Western							50

Table 2: Expected toll revenues within five years, in million \$, if a given road is built.

from any other city (possibly by passing through other cities). To protect the pristine nature around Northernmost, environmental restrictions only allow you to build the smallest number of roads necessary to connect the cities. Among all such sets of roads to build, you should find the one that maximizes the net profit.

1. Assuming that you can only rely on toll revenue from the next five years, formulate the above problem as a network optimization problem. Specify the input of the problem and name the computational task.[2]

2. Write a Python program to solve the problem formulated in 1. What are the roads that should be built? What would be total net profit/loss of the project? Repeat this task two more times, once assuming that a road generates toll revenue for 10 years, and then for 20 years. In both cases, you can assume that the revenue scales linearly, meaning the toll revenue you can expect in 10 years is equal to two times the toll revenue you can expect in 5 years, etc.

3. The council decides to build the roads you suggested based on the expected *5-year* toll revenues. The council member you are in touch with says that, for undisclosed reasons, they do not want to build a road between Northern and Southern. They ask you to make a single change in your proposal, which is to replace the road Northern-Southern with a road that ensures that after replacement, each city is again connected to every other city. Again, choose the edge that maximizes the net profit. Which road is it? Justify your choice. (If your proposal does not include the road Northern-Southern, then perform the task for any road of your choice that is part of your proposal.)

[2]To describe the network, both mathematical representations using sets and functions and graphical representations are accepted. In the latter case, clearly name the functions that label nodes and edges (if any).