

# The Skeptik Proof Compression System

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**Abstract.** This paper describes *Skeptik*: a system for compressing and improving proofs obtained by automated reasoning tools.

## 1 Introduction

## 2 User Interface

## 3 Supported Proof Formats

### 3.1 TraceCheck Format

### 3.2 The veriT Proof Format

### 3.3 The Skeptik Proof Format

## 4 Available Proof Compression Algorithms

### 4.1 RecyclePivots and RecyclePivotsWithIntersection

In [9], Tseitin outlines a first kind of proof redundancy with the concept of *irregular proof*. A proof is irregular if some pivot is resolved more than once on some path from the root to a leaf. But Goerdt [7] proved that the corresponding regular proof of an irregular proof may be exponentially bigger than the original. Therefore, only *partial* regularisation algorithm can compress a proof.

*RecyclePivots* (RP) [1, 2] and *RecyclePivotsWithIntersection* (RPI) [5] are such partial regularisation algorithms. They both compute for each node a set of *safe literals* and then delete a premise iff its auxiliary literal is safe. The set of safe literals is constructed recursively from the root of the proof by adding the dual of the auxiliary literal. The two algorithms only differs for nodes which are premises of more than one resolution. In that case, RP resets the safe literals to the empty set whereas RPI computes the intersection of the safe literals corresponding to each incoming edge.

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## 4.2 LowerUnits

Another kind of proof redundancy happens when a node  $\eta$  appears as premise of many resolutions on the same pivot  $p$ . In that case, it would be better to resolve  $\eta$  on  $p$  only once. The **LowerUnits** (LU) algorithm [5] reduces this kind of redundancy by lowering *units* down to the root of the proof. Units are subproofs whose conclusion consists in a single literal. Such a subproof can always be lowered down the proof.

**LowerUnits** is of linear time complexity in the length of the proof. It is a very fast algorithm which perform good compression ratio.

## 4.3 LowerUnivalents

The **LowerUnivalents** algorithm [3] (LUniv) generalizes **LowerUnits** by exploiting the information of the already lowered subproofs and their pivots. Then, it becomes possible to lower a non-unit node if all its conclusion's literals but one can be deleted by the already lowered subproofs. Moreover, the lowered pivots are safe literals and thus some partial regularization can be achieved simultaneously.

In **Skeptik**, LUniv has been implemented as a replacement for the **fixProof** function used by some algorithms to reconstruct a proof after deletions. Therefore, non-sequential combinations of those algorithms with LUniv are easy to implement. For instance, **LUnivRPI** is a non-sequential combination of LUniv after RPI. This latter algorithm is currently one the best trade-off between compression time and compression ratio [3].

## 4.4 RPI[3]LU and RPI[3]LUniv

RPI[3]LU and RPI[3]LUniv are non-sequential combinations of RPI after LU and LUniv. They consist in three traversals. The first traversal collects subproofs to be lowered down the proof. The second traversal computes the sets of safe literals for each node, taking into account the subproofs marked for being lowered. The last traversal actually compress the proof by removing redundant branches and lowering subproofs.

These algorithm are optimizations of the corresponding sequential compositions, achieving the same compression ratio in less time.

## 4.5 ReduceAndReconstruct

The **ReduceAndReconstruct** (RedRec) approach consists in applying local transformation rules to each node. The set of local rules presented in [8] are sufficient to emulate any other compression algorithm. Unfortunately, the proposed heuristic only consists in trying to apply each rule (with given priorities) to each node in a top-down traversal. For this heuristic to be efficient, the process has to be repeated many times. The resulting algorithm can achieve very good compression ratio if run long enough.

As this algorithm allows experimentations in the local transformation rules, the heuristic to apply them and the termination conditions, its implementation in *Skeptik* is very modular. Each component is defined independently and a convenient framework allows to combine them as desired. A handful of alternative local transformation rules and termination conditions have been implemented too.

#### 4.6 Split

The Split [4] algorithm is a technique to lower pivot variables in a proof.

From a proof with conclusion  $C$ , two proofs with conclusion  $v \vee C$  and  $\neg v \vee C$  are constructed, where the variable  $v$  is chosen heuristically.

In a first step the positive/negative premises of resolvents with pivot  $v$  are removed from the proof. Afterwards the proof is fixed, by traversing it top-down and fixing each proof node. A proof node is fixed by either replacing it by one of or resolving the fixed premises.

The roots of the resulting proofs are resolved, using  $v$  as pivot, to obtain a new proof of  $C$ .

The time-complexity of this algorithm is linear in the proof length and it suits very well for repeated application. This can be done iteratively or recursively. Also multiple variables can be chosen in advance. All these variants are implemented into *Skeptik*.

#### 4.7 TautologyElimination

#### 4.8 StructuralHashing

#### 4.9 DAGification

#### 4.10 Subsumption

Subsumption based algorithms use, as the name implies, the subsumption relation on clauses for compressing a proof. A clause  $C_1$  subsumes a clause  $C_2$  *iff* every literal occurring in  $C_1$  also occurs in  $C_2$ . The general goal is to replace a clause  $C$  by another clause  $D$ , used elsewhere in the proof, that are such that  $C$  subsumes  $D$ . There are three subsumption based compression algorithms implemented into *Skeptik*.

**Top-down Subsumption** searches for subsumed clauses among all clauses visited earlier in a top-down traversal. The time-complexity of this algorithm is worst case quadratic in the number of proof nodes.

**Bottom-up Subsumption** searches for subsumed clauses among all clauses visited earlier in a bottom-up traversal. A subsumed clause  $D$  can only replace a clause  $C$ , if  $D$  is not an ancestor of  $C$  in the graph representing the proof. Bottom-up Subsumption has the same time-complexity as Top-down Subsumption, but the additional ancestor-check makes it slower in practise.

**RecycleUnits** [2] is a special case of Bottom-up Subsumption that only searches for subsumed clauses that are unit (i.e. contain only one literal). This algorithm has a worst case time-complexity that is quadratic in the number of unit clauses.

#### 4.11 Pebbling

Pebbling algorithms compress proofs in the **space measure**, as opposed to the usual length compression. The space measure of a proof indicates how many proof nodes maximally have to be kept in memory, while reading the proof, simultaneously.

This measure is closely related to the **Black Pebbling Game** [6], which lends its name to the algorithms described here. A strategy for this game directly corresponds to a topological node ordering, i.e. an ordering of nodes such that every premise of each node is lower than the node itself, combined with deletion information, i.e. extra lines in the proof output indicating that a node can be deleted from memory. An optimal strategy for the Black Pebbling Game would therefore result in optimal space compression of the proof. However, as shown in [6], finding the optimal solution is PSPACE-complete and therefore not a feasible approach for this program.

To obtain algorithms that have an acceptable runtime greedy heuristics for finding a good node ordering are used.

**Top-down pebbling** directly corresponds to playing the game with limited information on how the proof looks. At every point there are nodes which can be pebbled (initially these are only the axioms). Using information from these nodes and their children nodes, it is decided which node to pebble next, which can make other nodes pebbleable. This is done until the root node is pebbled. Unfortunately the lack of knowledge about the structure of the proof often results in bad space compression.

**Bottom-up pebbling** constructs a node ordering by visiting proof nodes and their premises recursively, starting from the root node. At every node it is chosen heuristically in what order its premises are visited. After all premises are visited, the node is added to the order.

## 5 Implementation Details

ToDo: statistics about the code

### 5.1 Organization of the Code

ToDo: brief description of the package structure

### 5.2 Data Structures for Formulas

### 5.3 Data Structures for Proofs

## 6 Conclusions and Future Work

ToDo: limitation: memory consumption in Scala underlying symbols are strings

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