Finkont0

- Monte Carlo og optionsprisning -

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- Hvad er fordelingen af log-returns under $\mathbb{P}$ i Black-Scholes?
- 1.2 h)
    1. Hvad siger Delta-metoden?

Monte Carlo

- Monte Carlo simulation is a powerful tool to estimate the expectation of a random variable X or of a stochastic process at a specific time T, X_T.
- Given a stock price process, $(S_t)_{t\geq 0}$, imagine that we are interested in the expected stock price at time T, i.e. $\theta = E(S_T)$. The Monte Carlo approach would then be

1. Simulate n iid realisations of S_T, that is $S_{T_1}, \dots, S_{T_n}$.
2. Compute the sample mean

$$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} S_{T_i} \tag{1}$$

3. By the law of large numbers

$$\hat{\theta} \xrightarrow{a.s.} E(S_T) = \theta \quad \text{for} \quad n \to \infty \tag{2}$$

- $\hat{\theta}$ is called the Monte Carlo estimator, and is in itself a random variable.

Monte Carlo

- For a financial asset which pays $g(S_T)$ at some fixed maturity T, we have already established (or told you) that the time t arbitrage-free price is

$$\Pi(t) = E^Q\left[e^{-\int_t^T r du} g(S(T)) \mid \mathscr{F}_t\right] \tag{3}$$

$$= e^{-r(T-t)} E_t^Q[g(S(T))] \tag{4}$$

- Monte Carlo approach to approximate $\Pi(t)$
  1. Simulate n iid realisations of S_T under $\mathbb{Q}$, that is $S_{T_1}, \ldots, S_{T_n}$.
  2. Compute the sample mean

$$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} g(S_{T_i}) \tag{5}$$

  3. By the law of large numbers

$$e^{-r(T-t)}\hat{\theta} \xrightarrow{a.s.} e^{-r(T-t)} E_t^Q[g(S(T))] = e^{-r(T-t)}\theta \quad \text{for} \quad n \to \infty \tag{6}$$

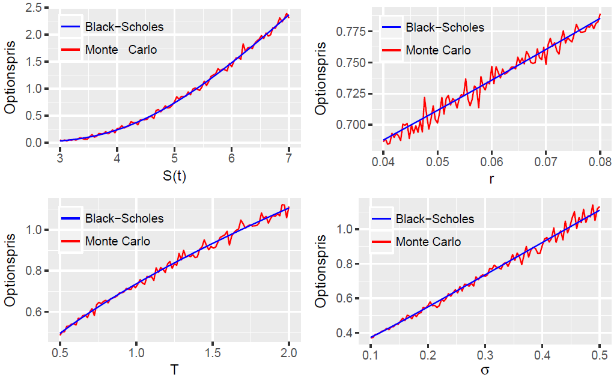
The options

- **Call:** The right to purchase the stock at a fixed point in time for a predetermined amount
- **Put:** The right to sell the stock at a fixed point in time for a predetermined amount

In the Black-Scholes model, the price of a European call is given by:

$$C(t,S(t),K,r,\sigma) = S(t)\phi\left(d_1\right) - e^{-r(T-t)}K\phi\left(d_2\right) \tag{7}$$

Black-Scholes formel



Implicit volatilitet

Implied volatility is the choice of model volatility s.t.

$$C^{obs}(K,T) = C^{BS}(K,T,r,\sigma) \tag{8}$$