Finkont0

- Introduktion til git og $\mathbb{Q}$-measure -

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Siden sidst

- Derive the distribution of $\log\left(\frac{S_n}{S_{n-1}}\right)$ $\left(\text{not } \frac{\log(S_n)}{\log(S_{n-1})}\right)$ under $\mathbb{P}$.
- When deriving closed-form solutions for the MLE's it will be even easier if you find the distribution for $\log\left(\frac{S_n}{S_{n-\Delta t}}\right)$. This is very easy if you have already done it with $\Delta t = 1$

$\mathbb{Q}$-målet - intuition

Example: If I offer you a lottery ticket, which in 50% of the cases yields a return of 100 DKK, and in the other 50% gives no return at all, what price would you be willing to pay for this lottery ticket?

1. < 50?
2. $= 50$?
3. > 50?

How to determine the price?

The price of an asset depends on the risk of the asset and the risk appetite of the investors. Usually, one would require more profit (lower price) for more risky assets. If investors are risk-averse, their reservation price is less than the future expected value of any risky asset.

Hence, to price an asset in a financial market, one will have to adjust the expected value under $\mathbb{P}$ with the risk preferences of all investors in the market. Clearly, that is not feasible.

$\mathbb{Q}$-målet - intuition

This is where the $\mathbb{Q}$-measure comes in handy

Definition 10.4 with 10.11

A probability measure $\mathbb{Q}$ on $\mathbb{F}_T$ is an equivalent martingale measure if

- $\mathbb{Q} \sim \mathbb{P}$ *on* F_T
- *All discounted price processes* $(\frac{S_t}{B_t})$ *are martingales under* $\mathbb{Q}$

The reason being, that if we assume that the market is free of arbitrage and complete (two very important concepts, that every wise student would discuss in their thesis), the first + second fundamental pricing theorem ensures that there exists (arbitrage free market) a unique (complete markets) equivalent martingale measure (the $\mathbb{Q}$-measure)

First + Second Fundamental Pricing Theorem (10.22 + 10.23)

There exists a unique equivalent martingale measure iff the model is arbitrage free and complete

$\mathbb{Q}$-målet - intuition

Hence, every asset can be priced as expected discounted $\mathbb{Q}$-expectations. Thus for a financial asset which pays $g(S(T))$ at some fixed maturity T one arrives at the time t arbitrage-free price, $\Pi(t)$, given by

$$\begin{aligned} \Pi(t) &= E^Q\left[e^{-\int_t^T r du}\Pi(T) \mid \mathscr{F}_t\right] \\ &= E^Q\left[e^{-\int_t^T r du} g(S(T)) \mid \mathscr{F}_t\right] \end{aligned} \tag{1}$$

which is coined the risk-neutral valuation formula.

In particular for $S(t)$

$$S(t) = E^Q\left[e^{-\int_t^T r du} S(T) \mid \mathscr{F}_t\right] \tag{2}$$

Hvad er det, og hvorfor er det nice?

- Software til versionsstyring - særligt brugt af udviklere
- Gør det muligt for flere at udvikle på det samme projekt(branches)
- Løsninger med tilhørende brugerflade kan være med til at skabe overblik i kodeprojekter
- Gør det lettere at dele kode

Workflow