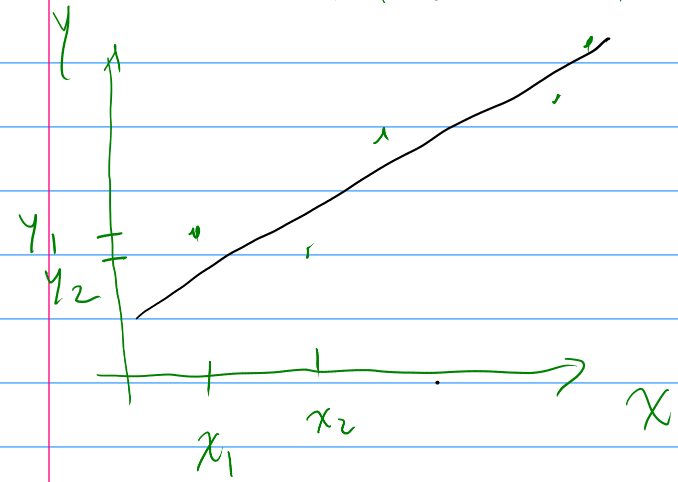
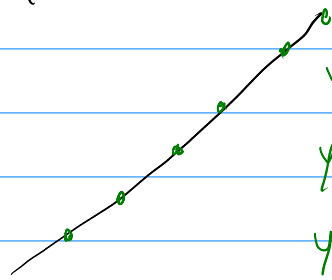
Metodo Matricial de Regresión.



$y = mx + b$



$y_1 = mx_1 + b$

$y_2 = mx_2 + b$

$y_3 = m x_3 + b$

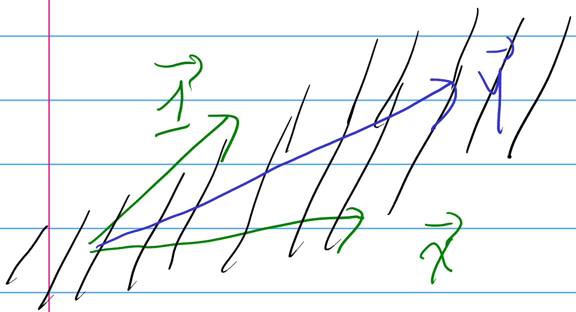
$\vdots$

$y_n = mx_n + b$

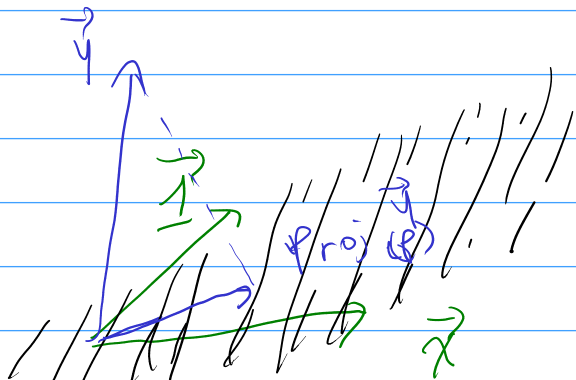
$$\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_3 \\ \vdots & \\ & x_n \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix}$$

$$P \vec{v} = y$$

$$C(P) = \{ z : z = a\vec{1} + b\vec{x},\ a, b \in \mathbb{R} \} \quad \vec{1} \in \mathbb{R}^n,\ \vec{x} \in \mathbb{R}^n$$

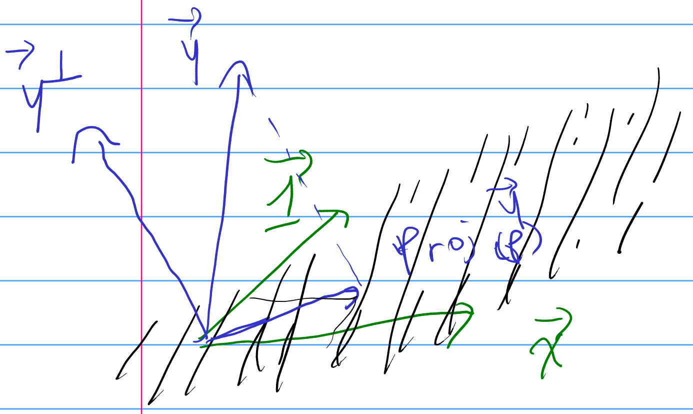


$$\vec{y} = b\vec{1} + m\vec{x}$$



$$\text{Proj}_{C(P)}\vec{y} = m^*\vec{x} + b^*\vec{1}$$

$$\begin{bmatrix} | & | \\ \vec{1} & \vec{x} \\ | & | \end{bmatrix} \underbrace{\begin{bmatrix} b^* \\ m^* \end{bmatrix}}_{\vec{v}^*} = \text{Proj}\,\vec{y}$$



$$\vec{y}^{\perp} = \vec{y} - \text{Proj}_{C(P)}\vec{y}$$

$$\vec{y}^{\perp} \cdot \vec{1} = 0$$

$$\vec{y}^{\perp} \cdot \vec{x} = 0$$

$$\begin{bmatrix} — & \vec{1} & — \\ — & \vec{x} & — \end{bmatrix} \begin{bmatrix} | \\ \vec{y}^{\perp} \\ | \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$P^T \vec{y}^{\perp} = \vec{0}$$

$$P^T\left(\vec{y} - \text{Proj}_{C(P)}\vec{y}\right) = \vec{0}$$

$$\text{Proj}_{C(P)}\vec{y} = P\vec{v}^*$$

$$P^T(\vec{y} - P\vec{v}^*) = \vec{0}$$

$$P^T\vec{y} - P^T P v^* = 0$$

$$P^T P \vec{v}^* = P^T\vec{y}$$

$$\boxed{\vec{v}^* = (P^T P)^{-1} P^T \vec{y}}$$