

# Scotogenic Model

## Lagrangian, Rotations and Interactions for eigenstates 'EWSB'

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# 1 Fields

## 1.1 Gauge Fields

Name	$SU(N)$	Coupling	Name
$B$	$U(1)$	$g_1$	hypercharge
$W$	$SU(2)$	$g_2$	left
$g$	$SU(3)$	$g_3$	color

## 1.2 Matter Superfields

Name	Spin	Generations	$(U(1) \otimes SU(2) \otimes SU(3))$
$H$	0	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
Et	0	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$q$	$\frac{1}{2}$	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$l$	$\frac{1}{2}$	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$d$	$\frac{1}{2}$	3	$(\frac{1}{3}, \mathbf{1}, \bar{\mathbf{3}})$
$u$	$\frac{1}{2}$	3	$(-\frac{2}{3}, \mathbf{1}, \bar{\mathbf{3}})$
$e$	$\frac{1}{2}$	3	$(1, \mathbf{1}, \mathbf{1})$
$n$	$\frac{1}{2}$	3	$(0, \mathbf{1}, \mathbf{1})$

# 2 Lagrangian

## 2.1 Input Lagrangian for Eigenstates GaugeES

$$\begin{aligned}
L = & -m_\eta^2 |\eta^0|^2 - m_\eta^2 |\eta^+|^2 + m_H^2 |H^0|^2 + m_H^2 |H^+|^2 - \frac{1}{2} \lambda_2 |\eta^0|^4 - \frac{1}{2} \lambda_2 |\eta^+|^4 - \frac{1}{2} \lambda_1 |H^0|^4 - \frac{1}{2} \lambda_1 |H^+|^4 \\
& - \frac{1}{2} H^{0,2} \lambda_5 \eta^{0,*2} - \eta^+ \lambda_2 |\eta^0|^2 \eta^{+,*} - H^0 H^+ \lambda_5 \eta^{0,*} \eta^{+,*} - \frac{1}{2} H^{+,2} \lambda_5 \eta^{+,*2} - H^0 \lambda_3 |\eta^0|^2 H^{0,*} \\
& - H^0 \lambda_4 |\eta^0|^2 H^{0,*} - H^0 \lambda_3 |\eta^+|^2 H^{0,*} - \eta^0 H^+ \lambda_4 \eta^{+,*} H^{0,*} - \frac{1}{2} \eta^{0,2} \lambda_5 H^{0,*2} - H^+ \lambda_3 |\eta^0|^2 H^- \\
& - H^+ \lambda_3 |\eta^+|^2 H^- - H^+ \lambda_4 |\eta^+|^2 H^- - H^+ \lambda_1 |H^0|^2 H^- - \eta^+ H^0 \lambda_4 \eta^{0,*} H^- \\
& - \eta^0 \eta^+ \lambda_5 H^{0,*} H^- - \frac{1}{2} \eta^{+,2} \lambda_5 H^{-,2} + H^0 d_{L,k\gamma}^* Y_{d,jk}^* \delta_{\beta\gamma} d_{R,j\beta} + H^+ u_{L,k\gamma}^* Y_{d,jk}^* \delta_{\beta\gamma} d_{R,j\beta} \\
& + H^0 e_{L,k}^* Y_{e,jk}^* e_{R,j} + H^+ \nu_{L,k}^* Y_{e,jk}^* e_{R,j} + \frac{1}{2} \text{conj} \left( \text{nR} \left( \{ \text{gt1} \} \right) \left( 2 \right) \right) \text{conj} \left( \text{nR} \left( \{ \text{gt2} \} \right) \left( 1 \right) \right) M_{N,ij} + \frac{1}{2} \text{conj} \left( \text{nR} \left( \{ \text{gt1} \} \right) \left( 1 \right) \right) \text{conj} \left( \text{nR} \left( \{ \text{gt2} \} \right) \left( 1 \right) \right) \\
& - \eta^{0,*} \nu_{L,k}^* Y_{N,jk}^* \nu_{R,j} + H^- d_{L,k\gamma}^* Y_{u,jk}^* \delta_{\beta\gamma} u_{R,j\beta} - H^{0,*} u_{L,k\gamma}^* Y_{u,jk}^* \delta_{\beta\gamma} u_{R,j\beta} \\
& + H^{0,*} d_{R,j\beta}^* \delta_{\beta\gamma} d_{L,k\gamma} Y_{d,jk} + H^- d_{R,j\beta}^* \delta_{\beta\gamma} u_{L,k\gamma} Y_{d,jk} + H^{0,*} e_{R,j}^* e_{L,k} Y_{e,jk} + H^- e_{R,j}^* \nu_{L,k} Y_{e,jk} \\
& + \eta^+ \nu_{R,j}^* e_{L,k} Y_{N,jk} - \eta^0 \nu_{R,j}^* \nu_{L,k} Y_{N,jk} + H^+ u_{R,j\beta}^* \delta_{\beta\gamma} d_{L,k\gamma} Y_{u,jk} - H^0 u_{R,j\beta}^* \delta_{\beta\gamma} u_{L,k\gamma} Y_{u,jk}
\end{aligned}$$

$$+ \frac{1}{2} M_{N,ij}^* \text{nR}(\{\text{gt}1\}) (2) \text{nR}(\{\text{gt}2\}) (1) + \frac{1}{2} M_{N,ij}^* \text{nR}(\{\text{gt}1\}) (1) \text{nR}(\{\text{gt}2\}) (2) \quad (1)$$

## 2.2 Gauge fixing terms

### 2.2.1 Gauge fixing terms for eigenstates 'GaugeES'

$$L_{GF} = -\frac{1}{2} |\partial_\mu B|^2 \xi_B^{-1} - \frac{1}{2} |\partial_\mu g|^2 \xi_g^{-1} - \frac{1}{2} |\partial_\mu W|^2 \xi_W^{-1} \quad (2)$$

### 2.2.2 Gauge fixing terms for eigenstates 'EWSB'

$$\begin{aligned} L_{GF} = & -\frac{1}{2} |\partial_\mu g|^2 \xi_g^{-1} - \frac{1}{2} |\partial_\mu \gamma|^2 \xi_\gamma^{-1} - \left| -\frac{i}{2} g_2 H^+ v \xi_{W^+} + \partial_\mu W^+ \right|^2 \xi_{W^+}^{-1} \\ & - \frac{1}{2} \left| -\frac{1}{2} A^0 v \xi_Z \left( g_1 \sin \Theta_W + g_2 \cos \Theta_W \right) + \partial_\mu Z \right|^2 \xi_Z^{-1} \end{aligned} \quad (3)$$

## 2.3 Fields integrated out

None

## 3 Field Rotations

### 3.1 Rotations in gauge sector for eigenstates 'EWSB'

$$\begin{pmatrix} B_\rho \\ W_{3\rho} \end{pmatrix} = Z^{\gamma Z} \begin{pmatrix} \gamma_\rho \\ Z_\rho \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} W_{1\rho} \\ W_{2\rho} \end{pmatrix} = Z^W \begin{pmatrix} W_\rho^+ \\ W_\rho^+ \end{pmatrix} \quad (5)$$

$$(6)$$

The mixing matrices are parametrized by

$$Z^{\gamma Z} = \begin{pmatrix} \cos \Theta_W & -\sin \Theta_W \\ \sin \Theta_W & \cos \Theta_W \end{pmatrix} \quad (7)$$

$$Z^W = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ i \frac{1}{\sqrt{2}} & -i \frac{1}{\sqrt{2}} \end{pmatrix} \quad (8)$$

$$(9)$$

### 3.2 Rotations in Mass sector for eigenstates 'EWSB'

#### 3.2.1 Mass Matrices for Scalars

- No Scalar Mixings

### 3.2.2 Mass Matrices for Fermions

- **Mass matrix for Singlet Fermions**, Basis:  $(\nu_R^*), (\nu_R^*)$

$$m_N = \begin{pmatrix} -M_N \end{pmatrix} \quad (10)$$

This matrix is diagonalized by  $Z^{\chi^0}$ :

$$Z^{\chi^0,*} m_N Z^{\chi^0,\dagger} = m_N^{dia} \quad (11)$$

with

$$\nu_{R,i} = \sum_j Z_{ji}^{\chi^0} X_j^{0,*} \quad (12)$$

- **Mass matrix for Neutrinos**, Basis:  $(\nu_L), (\nu_L)$

$$m_\nu = \begin{pmatrix} 0 \end{pmatrix} \quad (13)$$

This matrix is diagonalized by  $U^V$ :

$$U^{V,*} m_\nu U^{V,\dagger} = m_\nu^{dia} \quad (14)$$

with

$$\nu_{L,i} = \sum_j U_{ji}^{V,*} \nu_{L,j} \quad (15)$$

- **Mass matrix for Down-Quarks**, Basis:  $(d_{L,\alpha_1}), (d_{R,\beta_1}^*)$

$$m_d = \begin{pmatrix} -\frac{1}{\sqrt{2}} v \delta_{\alpha_1 \beta_1} Y_d^T \end{pmatrix} \quad (16)$$

This matrix is diagonalized by  $U_L^d$  and  $U_R^d$

$$U_L^{d,*} m_d U_R^{d,\dagger} = m_d^{dia} \quad (17)$$

with

$$d_{L,i\alpha} = \sum_{t_2} U_{L,ji}^{d,*} D_{L,j\alpha} \quad (18)$$

$$d_{R,i\alpha} = \sum_{t_2} U_{R,ij}^d D_{R,j\alpha}^* \quad (19)$$

- **Mass matrix for Up-Quarks**, Basis:  $(u_{L,\alpha_1}), (u_{R,\beta_1}^*)$

$$m_u = \begin{pmatrix} \frac{1}{\sqrt{2}} v \delta_{\alpha_1 \beta_1} Y_u^T \end{pmatrix} \quad (20)$$

This matrix is diagonalized by  $U_L^u$  and  $U_R^u$

$$U_L^{u,*} m_u U_R^{u,\dagger} = m_u^{dia} \quad (21)$$

with

$$u_{L,i\alpha} = \sum_{t_2} U_{L,ji}^{u,*} U_{L,j\alpha} \quad (22)$$

$$u_{R,i\alpha} = \sum_{t_2} U_{R,ij}^u U_{R,j\alpha}^* \quad (23)$$

- **Mass matrix for Leptons**, Basis:  $(e_L), (e_R^*)$

$$m_e = \left( -\frac{1}{\sqrt{2}} v Y_e^T \right) \quad (24)$$

This matrix is diagonalized by  $U_L^e$  and  $U_R^e$

$$U_L^{e,*} m_e U_R^{e,\dagger} = m_e^{dia} \quad (25)$$

with

$$e_{L,i} = \sum_{t_2} U_{L,ji}^{e,*} E_{L,j} \quad (26)$$

$$e_{R,i} = \sum_{t_2} U_{R,ij}^e E_{R,j}^* \quad (27)$$

## 4 Vacuum Expectation Values

$$H^0 = \frac{1}{\sqrt{2}} h + \frac{1}{\sqrt{2}} v + i \frac{1}{\sqrt{2}} A^0 \quad (28)$$

$$\eta^0 = \frac{1}{\sqrt{2}} \eta_R + i \frac{1}{\sqrt{2}} \eta_I \quad (29)$$

## 5 Tadpole Equations

$$\frac{\partial V}{\partial h} = \frac{1}{2} \lambda_1 v^3 - m_H^2 v \quad (30)$$

$$\frac{\partial V}{\partial \eta_R} = 0 \quad (31)$$

## 6 Particle content for eigenstates 'EWSB'

Name	Type	complex/real	Generations	Indices
------	------	--------------	-------------	---------

$H^+$	Scalar	complex	1	
$\eta^+$	Scalar	complex	1	
$A^0$	Scalar	real	1	
$h$	Scalar	real	1	
$\eta_I$	Scalar	real	1	
$\eta_R$	Scalar	real	1	
$N$	Fermion	Majorana	3	generation, 3
$\nu$	Fermion	Majorana	3	generation, 3
$d$	Fermion	Dirac	3	generation, 3, color, 3
$u$	Fermion	Dirac	3	generation, 3, color, 3
$e$	Fermion	Dirac	3	generation, 3
$g$	Vector	real	1	color, 8, lorentz, 4
$\gamma$	Vector	real	1	lorentz, 4
$Z$	Vector	real	1	lorentz, 4
$W^+$	Vector	complex	1	lorentz, 4
$\eta^G$	Ghost	real	1	color, 8
$\eta^\gamma$	Ghost	real	1	
$\eta^Z$	Ghost	real	1	
$\eta^+$	Ghost	complex	1	
$\eta^-$	Ghost	complex	1	

## 7 Interactions for eigenstates 'EWSB'

### 7.1 Three Scalar-Interaction

$$-i\lambda_1 v \tag{32}$$

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$$-i\lambda_5 v \tag{33}$$


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$$-i\left(-\lambda_5 + \lambda_3 + \lambda_4\right)v \tag{34}$$


---

$$-\frac{1}{2}\left(-\lambda_5 + \lambda_4\right)v \tag{35}$$


---



$$\frac{1}{2}(-\lambda_5 + \lambda_4)v \tag{36}$$


---

$$-\frac{i}{2}(\lambda_4 + \lambda_5)v \tag{37}$$


---

$$-i\lambda_3v \tag{38}$$


---

$$-i\left(\lambda_3 + \lambda_4 + \lambda_5\right)v \tag{39}$$


---

$$-\frac{i}{2}\left(\lambda_4 + \lambda_5\right)v \tag{40}$$


---

$$-3i\lambda_1v \tag{41}$$


---

$$-i\lambda_1 v \tag{42}$$


---

## 7.2 Two Scalar-One Vector Boson-Interaction

$$\frac{1}{2} \left( -g_1 \sin \Theta_W - g_2 \cos \Theta_W \right) \left( -p_\mu^h + p_\mu^{A^0} \right) \tag{43}$$


---

$$\frac{1}{2} g_2 \left( -p_\mu^{H^+} + p_\mu^{A^0} \right) \tag{44}$$


---

$$\frac{1}{2}g_2\left(-p_\mu^{H^-}+p_\mu^{A^0}\right) \tag{45}$$


---

$$\frac{1}{2}g_2\left(-p_\mu^{\eta^+}+p_\mu^{\eta_I}\right) \tag{46}$$


---

$$\frac{1}{2}\left(-g_1\sin\Theta_W-g_2\cos\Theta_W\right)\left(-p_\mu^{\eta_R}+p_\mu^{\eta_I}\right) \tag{47}$$


---

$$\frac{1}{2}g_2\left(-p_\mu^{\eta^{+,*}}+p_\mu^{\eta^+}\right) \tag{48}$$


---

$$-\frac{i}{2}g_2\left(-p_\mu^{\eta_R}+p_\mu^{\eta^+}\right) \tag{49}$$


---

$$-\frac{i}{2}\left(g_1\cos\Theta_W+g_2\sin\Theta_W\right)\left(-p_\mu^{\eta^{+,*}}+p_\mu^{\eta^+}\right) \tag{50}$$


---

$$-\frac{i}{2}\left(-g_1\sin\Theta_W+g_2\cos\Theta_W\right)\left(-p_\mu^{\eta^{+,*}}+p_\mu^{\eta^+}\right) \quad (51)$$


---

$$-\frac{i}{2}g_2\left(-p_\mu^{\eta^{+,*}}+p_\mu^{\eta_R}\right) \quad (52)$$


---

$$\frac{i}{2}g_2\left(-p_\mu^{H^+}+p_\mu^h\right) \quad (53)$$


---

$$-\frac{i}{2}g_2\left(-p_\mu^{H^-}+p_\mu^h\right) \tag{54}$$


---

$$-\frac{i}{2}\left(g_1\cos\Theta_W+g_2\sin\Theta_W\right)\left(-p_\mu^{H^-}+p_\mu^{H^+}\right) \tag{55}$$


---

$$-\frac{i}{2}\left(-g_1\sin\Theta_W+g_2\cos\Theta_W\right)\left(-p_\mu^{H^-}+p_\mu^{H^+}\right) \tag{56}$$


---

### 7.3 One Scalar-Two Vector Boson-Interaction

$$\frac{i}{2}g_2^2v\left(g_{\sigma\mu}\right) \tag{57}$$

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$$\frac{i}{2}v\left(g_1\sin\Theta_W+g_2\cos\Theta_W\right)^2\left(g_{\sigma\mu}\right) \tag{58}$$

---

$$\frac{i}{2}g_1g_2v\cos\Theta_W\left(g_{\sigma\mu}\right) \tag{59}$$

---



$$-\frac{i}{2}g_1g_2v\sin\Theta_W\left(g_{\sigma\mu}\right) \tag{60}$$


---

$$\frac{i}{2}g_1g_2v\cos\Theta_W\left(g_{\sigma\mu}\right) \tag{61}$$


---

$$-\frac{i}{2}g_1g_2v\sin\Theta_W\left(g_{\sigma\mu}\right) \tag{62}$$


---

## 7.4 Two Fermion-One Vector Boson-Interaction

$$-\frac{i}{2}g_3\delta_{ij}\lambda_{\alpha,\beta}^\gamma\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (63)$$

$$+\frac{i}{2}g_3\delta_{ij}\lambda_{\alpha,\beta}^\gamma\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (64)$$


---

$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(-3g_2\sin\Theta_W+g_1\cos\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (65)$$

$$+\frac{i}{3}g_1\cos\Theta_W\delta_{\alpha\beta}\delta_{ij}\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (66)$$


---

$$-i \frac{1}{\sqrt{2}} g_2 \delta_{\alpha\beta} \sum_{a=1}^3 U_{L,ja}^{d,*} U_{L,ia}^u \left( \gamma_\mu \cdot \frac{1 - \gamma_5}{2} \right) \quad (67)$$


---

$$\frac{i}{6} \delta_{\alpha\beta} \delta_{ij} \left( 3g_2 \cos \Theta_W + g_1 \sin \Theta_W \right) \left( \gamma_\mu \cdot \frac{1 - \gamma_5}{2} \right) \quad (68)$$

$$+ -\frac{i}{3} g_1 \delta_{\alpha\beta} \delta_{ij} \sin \Theta_W \left( \gamma_\mu \cdot \frac{1 + \gamma_5}{2} \right) \quad (69)$$


---

$$-i \frac{1}{\sqrt{2}} g_2 \sum_{a=1}^3 U_{L,ja}^{e,*} U_{ia}^V \left( \gamma_\mu \cdot \frac{1 - \gamma_5}{2} \right) \quad (70)$$


---

$$\frac{i}{2}\delta_{ij}\left(g_1\cos\Theta_W+g_2\sin\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (71)$$

$$+ig_1\cos\Theta_W\delta_{ij}\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (72)$$


---

$$\frac{i}{2}\delta_{ij}\left(-g_1\sin\Theta_W+g_2\cos\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (73)$$

$$+ig_1\delta_{ij}\sin\Theta_W\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (74)$$


---

$$-\frac{i}{2}g_3\delta_{ij}\lambda_{\alpha,\beta}^\gamma\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (75)$$

$$+-\frac{i}{2}g_3\delta_{ij}\lambda_{\alpha,\beta}^\gamma\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (76)$$


---

$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(3g_2\sin\Theta_W+g_1\cos\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (77)$$

$$+\frac{2i}{3}g_1\cos\Theta_W\delta_{\alpha\beta}\delta_{ij}\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (78)$$


---

$$-\frac{i}{6}\delta_{\alpha\beta}\delta_{ij}\left(3g_2\cos\Theta_W-g_1\sin\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (79)$$

$$+\frac{2i}{3}g_1\delta_{\alpha\beta}\delta_{ij}\sin\Theta_W\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (80)$$


---

$$-i\frac{1}{\sqrt{2}}g_2\delta_{\alpha\beta}\sum_{a=1}^3U_{L,ja}^{u,*}U_{L,ia}^d\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (81)$$


---

$$-\frac{i}{2}\delta_{ij}\left(g_1\sin\Theta_W+g_2\cos\Theta_W\right)\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (82)$$

$$+\frac{i}{2}\delta_{ij}\left(g_1\sin\Theta_W+g_2\cos\Theta_W\right)\left(\gamma_\mu\cdot\frac{1+\gamma_5}{2}\right) \quad (83)$$


---

$$-i\frac{1}{\sqrt{2}}g_2\sum_{a=1}^3U_{ja}^{V,*}U_{L,ia}^e\left(\gamma_\mu\cdot\frac{1-\gamma_5}{2}\right) \quad (84)$$


---

## 7.5 Two Fermion-One Scalar Boson-Interaction

$$\frac{1}{\sqrt{2}}\delta_{\alpha\beta}\sum_{b=1}^3U_{L,jb}^{d,*}\sum_{a=1}^3U_{R,ia}^dY_{d,ab}\left(\frac{1-\gamma_5}{2}\right) \quad (85)$$

$$+ -\frac{1}{\sqrt{2}}\delta_{\alpha\beta}\sum_{b=1}^3\sum_{a=1}^3Y_{d,ab}^*U_{R,ja}^dU_{L,ib}^d\left(\frac{1+\gamma_5}{2}\right) \quad (86)$$


---

$$\frac{1}{\sqrt{2}} \sum_{b=1}^3 U_{L,jb}^{e,*} \sum_{a=1}^3 U_{R,ia}^{e,*} Y_{e,ab} \left( \frac{1-\gamma_5}{2} \right) \quad (87)$$

$$+ -\frac{1}{\sqrt{2}} \sum_{b=1}^3 \sum_{a=1}^3 Y_{e,ab}^* U_{R,ja}^e U_{L,ib}^e \left( \frac{1+\gamma_5}{2} \right) \quad (88)$$


---

$$\frac{1}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^3 U_{L,jb}^{u,*} \sum_{a=1}^3 U_{R,ia}^{u,*} Y_{u,ab} \left( \frac{1-\gamma_5}{2} \right) \quad (89)$$

$$+ -\frac{1}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{u,ab}^* U_{R,ja}^u U_{L,ib}^u \left( \frac{1+\gamma_5}{2} \right) \quad (90)$$


---

$$\frac{1}{\sqrt{2}} \sum_{b=1}^3 U_{jb}^{V,*} \sum_{a=1}^3 Z_{ia}^{\chi^0,*} Y_{N,ab} \left( \frac{1-\gamma_5}{2} \right) \quad (91)$$

$$+ -\frac{1}{\sqrt{2}} \sum_{b=1}^3 \sum_{a=1}^3 Y_{N,ab}^* Z_{ia}^{\chi^0} U_{jb}^V \left( \frac{1+\gamma_5}{2} \right) \quad (92)$$


---

$$i \sum_{b=1}^3 U_{L,jb}^{e,*} \sum_{a=1}^3 Z_{ia}^{\chi^0,*} Y_{N,ab} \left( \frac{1-\gamma_5}{2} \right) \quad (93)$$


---

$$-i \frac{1}{\sqrt{2}} \sum_{b=1}^3 U_{jb}^{V,*} \sum_{a=1}^3 Z_{ia}^{\chi^0,*} Y_{N,ab} \left( \frac{1-\gamma_5}{2} \right) \quad (94)$$

$$+ -i \frac{1}{\sqrt{2}} \sum_{b=1}^3 \sum_{a=1}^3 Y_{N,ab}^* Z_{ia}^{\chi^0} U_{jb}^V \left( \frac{1+\gamma_5}{2} \right) \quad (95)$$


---



(96)

$$+ i \sum_{b=1}^3 \sum_{a=1}^3 Y_{N,ab}^* Z_{ja}^{X^0} U_{L,ib}^e \left( \frac{1+\gamma_5}{2} \right) \quad (97)$$


---

$$i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^3 U_{L,jb}^{d,*} \sum_{a=1}^3 U_{R,ia}^{d,*} Y_{d,ab} \left( \frac{1-\gamma_5}{2} \right) \quad (98)$$

$$+ i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{d,ab}^* U_{R,ja}^d U_{L,ib}^d \left( \frac{1+\gamma_5}{2} \right) \quad (99)$$


---

$$i\delta_{\alpha\beta} \sum_{b=1}^3 U_{L,jb}^{d,*} \sum_{a=1}^3 U_{R,ia}^{u,*} Y_{u,ab} \left( \frac{1-\gamma_5}{2} \right) \quad (100)$$

$$+ i\delta_{\alpha\beta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{d,ab}^* U_{R,ja}^d U_{L,ib}^u \left( \frac{1+\gamma_5}{2} \right) \quad (101)$$


---

$$(102)$$

$$+ i \sum_{b=1}^3 \sum_{a=1}^3 Y_{e,ab}^* U_{R,ja}^e U_{ib}^V \left( \frac{1+\gamma_5}{2} \right) \quad (103)$$


---

$$i \frac{1}{\sqrt{2}} \sum_{b=1}^3 U_{L,jb}^{e,*} \sum_{a=1}^3 U_{R,ia}^{e,*} Y_{e,ab} \left( \frac{1-\gamma_5}{2} \right) \quad (104)$$

$$+ i \frac{1}{\sqrt{2}} \sum_{b=1}^3 \sum_{a=1}^3 Y_{e,ab}^* U_{R,ja}^e U_{L,ib}^e \left( \frac{1+\gamma_5}{2} \right) \quad (105)$$


---

$$-i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^3 U_{L,jb}^{u,*} \sum_{a=1}^3 U_{R,ia}^{u,*} Y_{u,ab} \left( \frac{1-\gamma_5}{2} \right) \quad (106)$$

$$+ -i \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{u,ab}^* U_{R,ja}^u U_{L,ib}^u \left( \frac{1+\gamma_5}{2} \right) \quad (107)$$


---

$$i \delta_{\alpha\beta} \sum_{b=1}^3 U_{L,jb}^{u,*} \sum_{a=1}^3 U_{R,ia}^{d,*} Y_{d,ab} \left( \frac{1-\gamma_5}{2} \right) \quad (108)$$

$$+ i \delta_{\alpha\beta} \sum_{b=1}^3 \sum_{a=1}^3 Y_{u,ab}^* U_{R,ja}^u U_{L,ib}^d \left( \frac{1+\gamma_5}{2} \right) \quad (109)$$


---

$$i \sum_{b=1}^3 U_{jb}^{V,*} \sum_{a=1}^3 U_{R,ia}^{e,*} Y_{e,ab} \left( \frac{1-\gamma_5}{2} \right) \quad (110)$$


---

## 7.6 Three Vector Boson-Interaction

$$g_3 f_{\alpha,\beta,\gamma} \left( g_{\rho\mu} \left( -p_\sigma^{g\gamma\mu} + p_\sigma^{g\alpha\rho} \right) + g_{\rho\sigma} \left( -p_\mu^{g\alpha\rho} + p_\mu^{g\beta\sigma} \right) + g_{\sigma\mu} \left( -p_\rho^{g\beta\sigma} + p_\rho^{g\gamma\mu} \right) \right) \quad (111)$$


---

$$-ig_2 \sin \Theta_W \left( g_{\rho\mu} \left( -p_\sigma^{W_\mu^+} + p_\sigma^{W_\rho^-} \right) + g_{\rho\sigma} \left( -p_\mu^{W_\rho^-} + p_\mu^{\gamma\sigma} \right) + g_{\sigma\mu} \left( -p_\rho^{\gamma\sigma} + p_\rho^{W_\mu^+} \right) \right) \quad (112)$$


---

$$ig_2 \cos \Theta_W \left( g_{\rho\mu} \left( -p_\sigma^{Z_\mu} + p_\sigma^{W_\rho^-} \right) + g_{\rho\sigma} \left( -p_\mu^{W_\rho^-} + p_\mu^{W_\sigma^+} \right) + g_{\sigma\mu} \left( -p_\rho^{W_\sigma^+} + p_\rho^{Z_\mu} \right) \right) \quad (113)$$


---

## 7.7 Four Scalar-Interaction

$$-3i\lambda_1 \tag{114}$$

---

$$i\left(-\lambda_3 - \lambda_4 - \lambda_5\right) \tag{115}$$

---

$$-i\lambda_3 \tag{116}$$

---

$$i\left(-\lambda_3-\lambda_4+\lambda_5\right) \tag{117}$$


---

$$-i\lambda_1 \tag{118}$$


---

$$-i\lambda_1 \tag{119}$$


---

$$\frac{i}{2}(-\lambda_4 - \lambda_5) \tag{120}$$


---

$$-i\lambda_5 \tag{121}$$


---

$$\frac{i}{2}(-\lambda_4 - \lambda_5) \tag{122}$$


---

$$\frac{1}{2} \left( -\lambda_5 + \lambda_4 \right) \tag{123}$$


---

$$\frac{1}{2} \left( -\lambda_4 + \lambda_5 \right) \tag{124}$$


---

$$-3i\lambda_2 \tag{125}$$


---



$$-i\lambda_2 \tag{126}$$


---

$$-i\lambda_2 \tag{127}$$


---

$$i\left(-\lambda_3 - \lambda_4 + \lambda_5\right) \tag{128}$$


---

$$-i\lambda_3 \tag{129}$$


---

$$\frac{1}{2} \left( -\lambda_4 + \lambda_5 \right) \tag{130}$$


---

$$\frac{1}{2} \left( -\lambda_5 + \lambda_4 \right) \tag{131}$$


---

$$-2i\lambda_2 \tag{132}$$


---

$$-2i\lambda_5 \tag{133}$$


---

$$-i\lambda_2 \tag{134}$$


---

$$\frac{i}{2} \left( -\lambda_4 - \lambda_5 \right) \tag{135}$$


---

$$-i\lambda_3 \tag{136}$$


---

$$i \left( -\lambda_3 - \lambda_4 \right) \tag{137}$$


---

$$-3i\lambda_2 \tag{138}$$


---

$$i\left(-\lambda_3-\lambda_4-\lambda_5\right) \tag{139}$$


---

$$-i\lambda_3 \tag{140}$$


---

$$\frac{i}{2}(-\lambda_4 - \lambda_5) \tag{141}$$


---

$$-3i\lambda_1 \tag{142}$$


---

$$-i\lambda_1 \tag{143}$$


---

$$-2i\lambda_5 \tag{144}$$


---

$$-2i\lambda_1 \tag{145}$$


---

## 7.8 Two Scalar-Two Vector Boson-Interaction

$$\frac{i}{2}g_2^2(g_{\mu\nu}) \tag{146}$$


---

$$\left(\frac{i}{2}g_1^2 \sin \Theta_W^2 + \frac{i}{2}g_2^2 \cos \Theta_W^2 + ig_1g_2 \cos \Theta_W \sin \Theta_W\right)(g_{\mu\nu}) \quad (147)$$


---

$$\frac{1}{2}g_1g_2 \cos \Theta_W(g_{\mu\nu}) \quad (148)$$


---

$$-\frac{1}{2}g_1g_2 \sin \Theta_W(g_{\mu\nu}) \quad (149)$$


---



$$-\frac{1}{2}g_1g_2\cos\Theta_W\left(g_{\mu\nu}\right) \tag{150}$$


---

$$\frac{1}{2}g_1g_2\sin\Theta_W\left(g_{\mu\nu}\right) \tag{151}$$


---

$$\frac{i}{2}g_2^2\left(g_{\mu\nu}\right) \tag{152}$$


---

$$\left(\frac{i}{2}g_1^2 \sin \Theta_W^2 + \frac{i}{2}g_2^2 \cos \Theta_W^2 + ig_1g_2 \cos \Theta_W \sin \Theta_W\right)(g_{\mu\nu}) \quad (153)$$


---

$$\frac{1}{2}g_1g_2 \cos \Theta_W(g_{\mu\nu}) \quad (154)$$


---

$$-\frac{1}{2}g_1g_2 \sin \Theta_W(g_{\mu\nu}) \quad (155)$$


---

$$-\frac{1}{2}g_1g_2\cos\Theta_W\left(g_{\mu\nu}\right) \tag{156}$$


---

$$\frac{1}{2}g_1g_2\sin\Theta_W\left(g_{\mu\nu}\right) \tag{157}$$


---

$$\frac{i}{2}g_1g_2\cos\Theta_W\left(g_{\mu\nu}\right) \tag{158}$$


---

$$-\frac{i}{2}g_1g_2\sin\Theta_W\left(g_{\mu\nu}\right) \tag{159}$$


---

$$\left(\frac{i}{2}g_1^2\cos\Theta_W^2+\frac{i}{2}g_2^2\sin\Theta_W^2+ig_1g_2\cos\Theta_W\sin\Theta_W\right)\left(g_{\mu\nu}\right) \tag{160}$$


---

$$\left(\frac{i}{2}g_1g_2\cos2\Theta_W-\frac{i}{4}g_1^2\sin2\Theta_W+\frac{i}{4}g_2^2\sin2\Theta_W\right)\left(g_{\mu\nu}\right) \tag{161}$$


---

$$\frac{i}{2}g_2^2(g_{\mu\nu}) \tag{162}$$


---

$$\left(\frac{i}{2}g_1^2\sin\Theta_W^2+\frac{i}{2}g_2^2\cos\Theta_W^2-ig_1g_2\cos\Theta_W\sin\Theta_W\right)(g_{\mu\nu}) \tag{163}$$


---

$$\frac{i}{2}g_2^2(g_{\mu\nu}) \tag{164}$$


---

$$\left(\frac{i}{2}g_1^2 \sin \Theta_W^2 + \frac{i}{2}g_2^2 \cos \Theta_W^2 + ig_1g_2 \cos \Theta_W \sin \Theta_W\right)(g_{\mu\nu}) \quad (165)$$


---

$$\frac{i}{2}g_1g_2 \cos \Theta_W(g_{\mu\nu}) \quad (166)$$


---

$$-\frac{i}{2}g_1g_2 \sin \Theta_W(g_{\mu\nu}) \quad (167)$$


---

$$\frac{i}{2}g_2^2(g_{\mu\nu}) \tag{168}$$


---

$$\left(\frac{i}{2}g_1^2\sin\Theta_W^2+\frac{i}{2}g_2^2\cos\Theta_W^2+ig_1g_2\cos\Theta_W\sin\Theta_W\right)(g_{\mu\nu}) \tag{169}$$


---

$$\frac{i}{2}g_1g_2\cos\Theta_W(g_{\mu\nu}) \tag{170}$$


---

$$-\frac{i}{2}g_1g_2\sin\Theta_W\left(g_{\mu\nu}\right) \tag{171}$$


---

$$\frac{i}{2}g_1g_2\cos\Theta_W\left(g_{\mu\nu}\right) \tag{172}$$


---

$$-\frac{i}{2}g_1g_2\sin\Theta_W\left(g_{\mu\nu}\right) \tag{173}$$


---



$$\left(\frac{i}{2}g_1^2 \cos \Theta_W^2 + \frac{i}{2}g_2^2 \sin \Theta_W^2 + ig_1g_2 \cos \Theta_W \sin \Theta_W\right)(g_{\mu\nu}) \quad (174)$$


---

$$\left(\frac{i}{2}g_1g_2 \cos 2\Theta_W - \frac{i}{4}g_1^2 \sin 2\Theta_W + \frac{i}{4}g_2^2 \sin 2\Theta_W\right)(g_{\mu\nu}) \quad (175)$$


---

$$\frac{i}{2}g_2^2(g_{\mu\nu}) \quad (176)$$


---

$$\left(\frac{i}{2}g_1^2 \sin \Theta_W^2 + \frac{i}{2}g_2^2 \cos \Theta_W^2 - ig_1g_2 \cos \Theta_W \sin \Theta_W\right)(g_{\mu\nu}) \quad (177)$$


---

## 7.9 Four Vector Boson-Interaction

$$ig_3^2 \left( - \sum_{a=1}^8 f_{\alpha,\delta,a} f_{\beta,\gamma,a} - \sum_{a=1}^8 f_{\alpha,\gamma,a} f_{\beta,\delta,a} \right) (g_{\rho\sigma} g_{\mu\nu}) \quad (178)$$

$$+ ig_3^2 \left( - \sum_{a=1}^8 f_{\alpha,\beta,a} f_{\gamma,\delta,a} + \sum_{a=1}^8 f_{\alpha,\delta,a} f_{\beta,\gamma,a} \right) (g_{\rho\mu} g_{\sigma\nu}) \quad (179)$$

$$+ ig_3^2 \left( \sum_{a=1}^8 f_{\alpha,\gamma,a} f_{\beta,\delta,a} + \sum_{a=1}^8 f_{\alpha,\beta,a} f_{\gamma,\delta,a} \right) (g_{\rho\nu} g_{\sigma\mu}) \quad (180)$$


---

$$ig_2^2 \sin \Theta_W^2 (g_{\rho\sigma} g_{\mu\nu}) \quad (181)$$

$$+ ig_2^2 \sin \Theta_W^2 (g_{\rho\mu} g_{\sigma\nu}) \quad (182)$$

$$+ -2ig_2^2 \sin \Theta_W^2 (g_{\rho\nu} g_{\sigma\mu}) \quad (183)$$


---

$$\frac{i}{2} g_2^2 \sin 2\Theta_W (g_{\rho\sigma} g_{\mu\nu}) \quad (184)$$

$$+ -ig_2^2 \sin 2\Theta_W (g_{\rho\mu} g_{\sigma\nu}) \quad (185)$$

$$+ \frac{i}{2} g_2^2 \sin 2\Theta_W (g_{\rho\nu} g_{\sigma\mu}) \quad (186)$$


---

$$2ig_2^2 (g_{\rho\sigma} g_{\mu\nu}) \quad (187)$$

$$+ -ig_2^2 (g_{\rho\mu} g_{\sigma\nu}) \quad (188)$$

$$+ -ig_2^2 (g_{\rho\nu} g_{\sigma\mu}) \quad (189)$$


---

$$- 2ig_2^2 \cos \Theta_W^2 \left( g_{\rho\sigma} g_{\mu\nu} \right) \quad (190)$$

$$+ ig_2^2 \cos \Theta_W^2 \left( g_{\rho\mu} g_{\sigma\nu} \right) \quad (191)$$

$$+ ig_2^2 \cos \Theta_W^2 \left( g_{\rho\nu} g_{\sigma\mu} \right) \quad (192)$$


---

## 7.10 Two Ghosts-One Vector Boson-Interaction

$$g_3 f_{\alpha,\beta,\gamma} \left( p_\mu^{\eta_\beta^G} \right) \quad (193)$$


---

$$- ig_2 \sin \Theta_W \left( p_\mu^{\eta_\mu^\gamma} \right) \quad (194)$$


---

$$ig_2 \sin \Theta_W \left( p_\mu^{\eta^\gamma} \right) \tag{195}$$


---

$$ig_2 \sin \Theta_W \left( p_\mu^{\eta^+} \right) \tag{196}$$


---

$$ig_2 \cos \Theta_W \left( p_\mu^{\eta^+} \right) \tag{197}$$


---

$$-ig_2 \sin \Theta_W \left( p_\mu^{\eta^+} \right) \tag{198}$$


---

$$-ig_2 \cos \Theta_W \left( p_\mu^{\eta^+} \right) \tag{199}$$


---

$$-ig_2 \sin \Theta_W \left( p_\mu^{\eta^-} \right) \tag{200}$$


---

$$ig_2 \sin \Theta_W \left( p_\mu^{\eta^-} \right) \tag{201}$$


---

$$ig_2 \cos \Theta_W \left( p_\mu^{\eta^-} \right) \tag{202}$$


---

$$-ig_2 \cos \Theta_W \left( p_\mu^{\eta^-} \right) \tag{203}$$


---

$$-ig_2 \cos \Theta_W \left( p_\mu^{\eta^Z} \right) \tag{204}$$


---

$$ig_2 \cos \Theta_W \left( p_\mu^{\eta^Z} \right) \tag{205}$$


---

## 7.11 Two Ghosts-One Scalar-Interaction

$$\frac{1}{4}g_2^2 v \xi_{W^+} \tag{206}$$


---



$$-\frac{1}{4}g_2^2 v \xi_{W^+} \tag{207}$$


---

$$\frac{i}{8} v \xi_Z \left( 2g_1 g_2 \cos 2\Theta_W + \left( -g_2^2 + g_1^2 \right) \sin 2\Theta_W \right) \tag{208}$$


---

$$-\frac{i}{4} g_2 v \xi_{W^+} \left( g_1 \cos \Theta_W + g_2 \sin \Theta_W \right) \tag{209}$$


---

$$-\frac{i}{4}g_2v\xi_{W^+}\left(g_1\cos\Theta_W+g_2\sin\Theta_W\right) \tag{210}$$


---

$$-\frac{i}{4}g_2^2v\xi_{W^+} \tag{211}$$


---

$$\frac{i}{4}g_2v\xi_Z\left(g_1\sin\Theta_W+g_2\cos\Theta_W\right) \tag{212}$$


---

$$-\frac{i}{4}g_2^2v\xi_{W^+} \tag{213}$$


---

$$\frac{i}{4}g_2v\xi_Z\left(g_1\sin\Theta_W+g_2\cos\Theta_W\right) \tag{214}$$


---

$$-\frac{i}{4}v\xi_Z\left(g_1\sin\Theta_W+g_2\cos\Theta_W\right)^2 \tag{215}$$


---

$$-\frac{i}{4}g_2v\xi_{W^+}\left(-g_1\sin\Theta_W+g_2\cos\Theta_W\right) \tag{216}$$


---

$$-\frac{i}{4}g_2v\xi_{W^+}\left(-g_1\sin\Theta_W+g_2\cos\Theta_W\right) \tag{217}$$


---

## 8 Clebsch-Gordan Coefficients