

To: Computational Finance Course Participants
Fr: Frederik Kryger-Baggesen
St: Week 2
Dt: 03 May 2024

Agenda

Class 03 May 2024 13:15-15 in room on 4th floor in Math building

Bring your Windows laptop.

The backward and forward pde and their associated discretized theta scheme.

The need for matrix algebra for banded – and especially tridiagonal – matrices

Implementation of some Numerical Recipes algorithms

To do list

Install latest Visual Studio C++ 2022 community version on your laptop. Include the module ‘Desktop development with C++’.

Download the Week 2 solution from GitHub: <https://github.com/brnohu>.

Verify the solution compiles and that you can access the debugger from Excel.

Implement today’s code in the solution file from week2, i.e. not your own from week1

Implement the `kMatrixAlgebra::banmul()` and `kMatrixAlgebra::tridag()` functions in the `kMatrixAlgebra` namespace using the predefined declarations. The algorithms can be found in the attached from ‘Numerical Recipes’.

Expose both functions into two new Excel function `xBanmul()` and `xTridag()` in the `xlExport.cpp` in either `CompFin_32` or `CompFin_64`.

Create a spreadsheet where you call your exposed implementation. Verify that output is correct and step through your functions in the debugger.

If time allows, implement the functions `kFiniteDifference::dx()`, `kFiniteDifference::dxx()` and `kFd1d::calcAx()`

Notes

Pde

$$0 = \partial_t f + A f, \quad A = -r + \mu f_s + \frac{1}{2} \sigma^2 f_{ss}$$

Backward theta scheme:

$$\begin{aligned}f\left(t_{h+\frac{1}{2}}\right) &= [I + (1 - \theta)\Delta t\bar{A}]f(t_{h+1}) \\[I - \theta\Delta t\bar{A}]f(t_h) &= f\left(t_{h+\frac{1}{2}}\right)\end{aligned}$$

Forward theta scheme

$$\begin{aligned}[I - \theta\Delta t\bar{A}]'p\left(t_{h+\frac{1}{2}}\right) &= p(t_h) \\p(t_{h+1}) &= [I + (1 - \theta)\Delta t\bar{A}]'p\left(t_{h+\frac{1}{2}}\right)\end{aligned}$$

Where ' denotes transpose, I denotes the identity matrix, \bar{A} denotes the discretized approximation to A and $0 \leq \theta \leq 1$ is a constant.

Note that the backward scheme is solved for $f(t_h)$ given the values of $f(t_{h+1})$, while the forward scheme is solved for $p(t_{h+1})$ given the values of $p(t_h)$, i.e. we run backward and forward in time respectively.