To: **Computational Finance Course Participants**
Fr: **Jesper Andreasen**
St: **Week 8**
Dt: **16 Jan 2023**

Agenda

No class this week.

Hand in your report for the assignment by EOD today 16 Jan 2023.

Rolf will release the schedule and a few words about the form of the oral exam on 26-27 Jan 2023.

Below we consider the convergence of a Bermudan to an American option.

Bermudan and American Options

Consider an American option with time t intrinsic value of $I(t)$. Let τ be the optimal stopping time and r the interest rate, then the value of the American option can be written as

$$C^A = E[e^{-\int_0^\tau r(u)du} I(\tau)]$$

The Bermudan version of the same option can be exercised at the times $t_h = h \cdot \Delta t$ with $t_m = T$. The value of this option, C^B, is *smaller* than the American option but *bigger* than an option where premature exercise is forced at $\tau + \Delta t$ after any early exercise $\tau < T$. We get

$$\begin{aligned} 0 &\le C^A - C^B \\ &\le E[e^{-\int_0^\tau r(u)du}(I(\tau) - e^{-\int_\tau^{\tau+\Delta t} r(u)du} I(\tau - \Delta t))1_{\tau<T}] \\ &= E[e^{-\int_0^\tau r(u)du} E_\tau[(I(\tau) - e^{-\int_\tau^{\tau+\Delta t} r(u)du} I(\tau - \Delta t))]1_{\tau<T}] \end{aligned}$$

For the Black-Scholes case with constant parameters

$$\frac{dS}{S} = (r-q)dt + \sigma dW$$

we have for the American call $I(t) = S(t) - K$

$$\begin{aligned} &E_\tau[(S(\tau) - K) - e^{-r\Delta t}(S(\tau + \Delta t) - K)] \\ &= S(\tau)(1 - e^{-q\Delta t}) - K(1 - e^{-r\Delta t}) \\ &= (S(\tau)q - Kr)\Delta t + O(\Delta t^2) \\ &= O(\Delta t) \end{aligned}$$

We thus conclude that

$$0 \le C^A - C^B \le O(\Delta t)$$

Strictly speaking this only proves that the convergence of Bermudan to American is $O(\Delta t)$ or higher order. A rigorous proof of $C^A - C^B = O(\Delta t)$ can be established using the analysis in Kim (1990) but this is beyond the scope of this course.

References

Kim, J (1990): "The Analytic Valuation of American Options." *The Review of Financial Studies* 3, 547-572.