

Exponential function

Johan Kølsten de Wit

March 2021

Abstract

This short rapport analyses whether a Taylor expansion terminated at 10th order of the exponential function is a valid approximation.

Introduction

The exponential function is an important function in modern physics, and is present in a variety of essential equations, which today are both carried out analytically and numerically. For computing the exponential value of a certain value, x , numerically, one has to utilize approximations.

In the rapport, I will approximate the function by a Taylor expansion of the 10th order and test whether or not this implementation is equal to function implemented in the *math.h* library of *c*. The function implemented is as follows:

$$\exp x \simeq \sum_{n=0}^{10} \frac{x^n}{n!} \quad (1)$$

Implementation in c

The function which computes $\exp(x)$ has the a *double* value, *x*, as input and then proceed to ensure the input is valid before calculating the Taylor expansion. This is done by the two following if-statements:

1. The first one being if $x < 0$ the $\exp(x)$ will then return the value of $1/\exp(-x)$.
This is made to ensure correct numerical values of $x < 0$, by using the following relation

$$1/\exp(x) = \exp(-x) \quad (2)$$

This condition ensures that values of $x < 0$ converge to zero instead of increasing values for large $x < 0$.

2. The next condition checks if $x > 1./8$ the function will return $\exp(x/2)^2$ which again is a rewriting of the $\exp x$

$$(\exp(x/2))^2 = \exp(x) \quad (3)$$

Excluding the condition will cause the Taylor expansion to return lower values than anticipated.

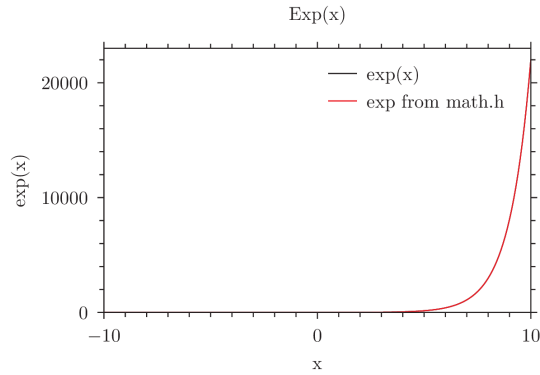


Figure 1: A plot of the Taylor expansion of $\exp x$ of x ranging from -10 to 10 with stepsize of $1/10$. The value of $\exp(x)$ from `math.h` is plotted aswell.

In figure 1 the value of $\exp x$ from the Taylor expansion is plotted along with the computed value from `math.h` with $x \in [-10, 10]$. The two lines are equal to each other, suggesting that the Taylor expansion implemented works.

By removing the conditions written above the following plot is made As seen in figure 2 the Taylor

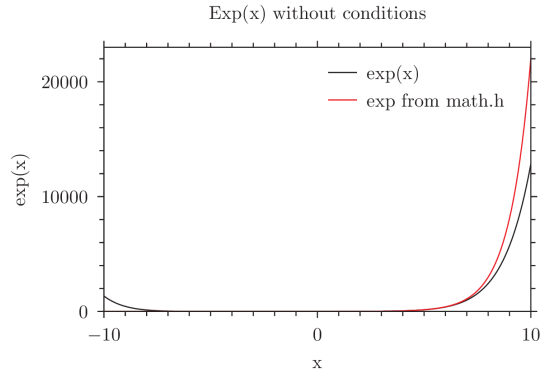


Figure 2: A plot of $\exp x$ from the Taylor expansion (with conditions excluded) and the `math.h` lib.

expansion deviates from the calculated values of `math.h` with the lower returned values for $x > 1.8$ and the incorrect increasing values for $x < 0$.

Conclusion

The implementation of the exponential function proved succesful when comparing it to the values given by `math.h`. The correct values computed is caused by the two conditions and the high order of the Taylor expansion. This suggests that higher orders merely would increase the computation time.