After developping we find that we need to solve the system : (to go one step backward in time)

$$\underbrace{M_{(\theta-1)} f^{n+1}}_{b} = \underbrace{M_\theta f^{n}}_{AX}$$

$$\iff AX = b$$

We start with $M_{(\theta-1)} f^{T} = M_\theta f^{T-1}$
↳ vector of payoffs at maturity

and go backward to find f^{0} = vector of current premiums.

$$M_\theta = \begin{pmatrix} \beta_\theta & \gamma_\theta & & & 0 \\ \alpha_\theta & \ddots & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \gamma_\theta \\ 0 & & & \alpha_\theta & \beta_\theta \end{pmatrix}$$ is a tridiagonal matrix

with
$$\begin{cases} \alpha_\theta = -\frac{1}{2}\theta\, dt \left[\frac{\sigma^2}{dx^2} + \frac{\sigma^2 - r}{2dx}\right] \\ \beta_\theta = 1 + \theta dt \left[\frac{\sigma^2}{dx^2} + r\right] \\ \gamma_\theta = \frac{1}{2}\theta\, dt \left[-\frac{\sigma^2}{dx} + \frac{\sigma^2 \cdot r}{2dx}\right] \end{cases}$$