UNIVERSITY OF COPENHAGEN
DEPARTMENT OF ECONOMICS

Exam | The Fiscal Multiplier in HANK-SAM

Advanced Macroeconomics: Heterogeneous Agent Models

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1 Solution method

a) Could the number of inputs to the hh-block be reduced?

The inputs to the household block currently are wages, w, interest rates, r, tax rate, τ, dividends, div, and transfers, $transfers$ and additionally to the transition of households the direct inputs are the unemployment transition probability, δ, and the job-finding rate, $\lambda^{u,s}$.

Of those, the wage input is defined to be in its steady state, w_{ss}, in the model and could thus be left out and instead just implemented as an exogenous parameter. I see this in the production block in block.py, line 11.

```
w[:] = ss.w
```

Here, the wage is clearly defined as just being its steady-state value and could, thus, be excluded as an input and changed into a parameter.

Additionally, the unemployment transition probability, δ_t, is not defined as being in a steady state in the model. However, in the current setup, δ is not set to change and thus is in the production block in line 12, also defined as just having a constant value

```
delta[:] = ss.delta
```

Thus, this could also be removed from the direct inputs to the household transition block and changed into a parameter.

b) Could the number of unknowns be reduced?

Working from equations (15), (17), and (18), I can expand u_t to

$$
\begin{aligned}
u_t &= u_{t-1} + \delta_t(1 - u_{t-1}) - \lambda_t^{u,s} S_t \\
&= u_{t-1} + \delta_t(1 - u_{t-1}) - A\theta_t^{1-\alpha} S_t \\
&= u_{t-1} + \delta_t(1 - u_{t-1}) - A\left(\frac{v_t}{S_t}\right)^{1-\alpha} S_t \\
&= u_{t-1} + \delta_t(1 - u_{t-1}) - A v_t^{1-\alpha} S_t^{\alpha}
\end{aligned}
\tag{1}
$$

As already pointed out in a), for this specific model $\delta_t = \delta_{ss}\ \forall\, t$. Thus, u_t is only dependent on its previous value and the two unknowns v_t and S_t. u_t is decided upon in the labour market block together with both v_t and S_t

```
@nb.njit
def labor_market(par,ini,ss,v,S,delta,u,theta,lambda_v,lambda_u_s,errors_u):

theta[:] = v/S
```

```
lambda_v[:] = par.A*theta**(-par.alpha)
lambda_u_s[:] = par.A*theta**(1-par.alpha)

u_lag = lag(ini.u,u)
errors_u[:] = u - (u_lag-S*lambda_u_s+delta*(1-u_lag))
```

From here, u_t could be left out and instead just be calculated directly from v_t, S_t, and $ini.S$ as $ini.S = ini.u$ thus, also removing the $errors\_u[:]$. u_t would subsequently also have to be removed from the blocks price_setters, dividends, government, and market_clearing and replaced by v_t and S_t with the equation for u_t

c) IR to fiscal spending shock to G_t with persistence

To see the dynamic of the impulse responses, I have plotted the impulse response functions for different shocks of G_t

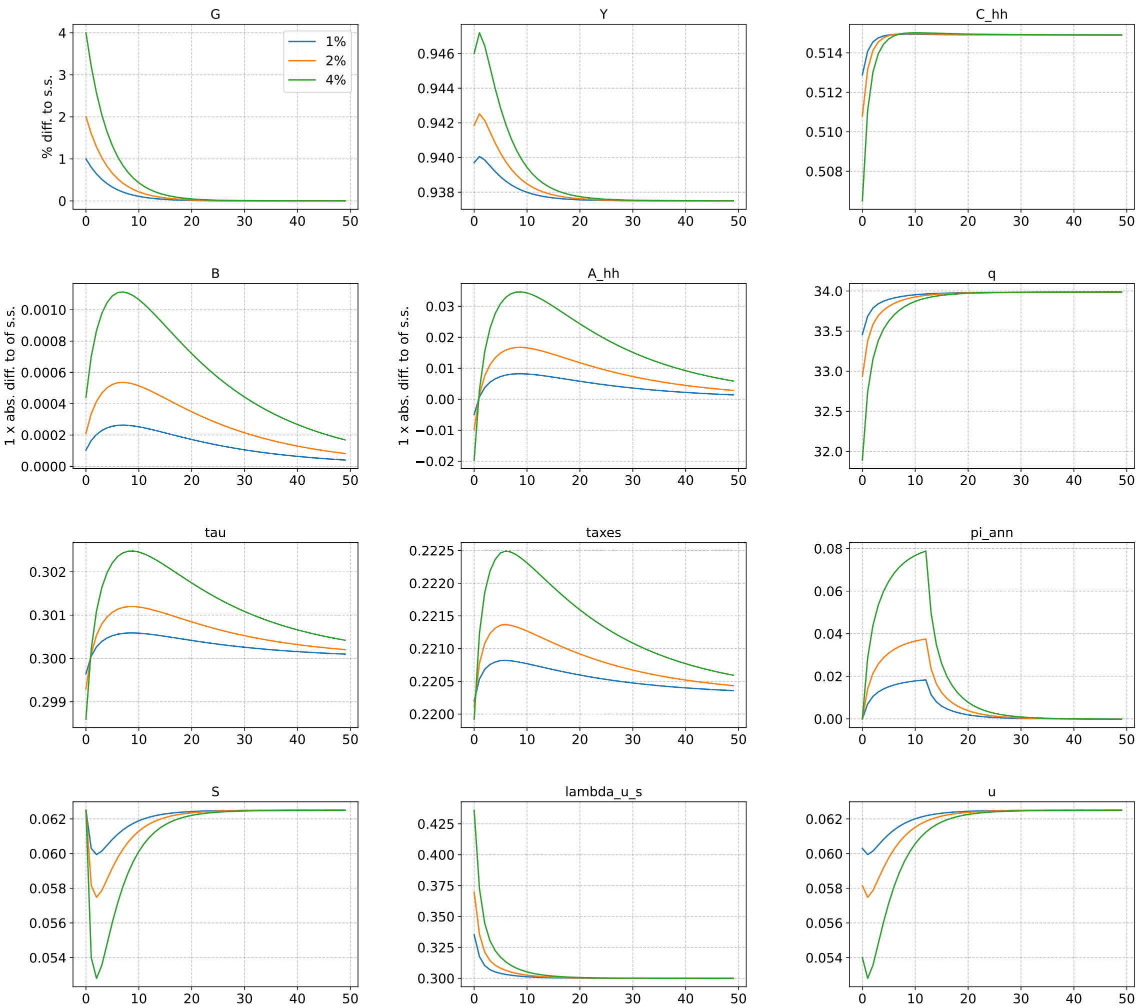

Figure 1: Impulse Response Functions for selected variables for different shocks to G_t

In figure 1, it is apparent that for shocks to government spending of 1%, 2%, and 4%, the on-impact and maximum impact responses in the other variables are approximately linear, e.g. the response in Y, doubles as the shock to G doubles. This is also what I find when calculating the area under paths' difference to steady state.

%-shock	G	Y	C^{hh}	B	A^{hh}
1%	0.0211298	0.0180226	-0.0031072	0.00789759	0.244605
2%	0.0422596	0.0356108	-0.00664882	0.0160731	0.497344
4%	0.0845193	0.0694794	-0.0150399	0.0332797	1.02826

Table 1: Sum of differences from the steady state path

The shock, however, is not somewhat linear as the effect of the increases in taxation is not 1:1 but dampened slightly as ω restricts the change in taxation. This is seen especially for the shock of 4%.

This happens as the government spending goes directly into Y_t, and the government must also issue bonds to cover the cost of the extra spending. The government, in turn, drives up prices on final goods – i.e., increased inflation – and thus, consumption falls for the private households that reduce their assets as the incentive to hold cash drops and to maintain some level of consumption for the non-hand-to-mouth households. The increased inflation then makes the central bank react to increasing interest rates, making it more attractive to save and making real interest rates drop as inflation maintains above steady state.

2 Fiscal multiplier

a) What is the fiscal multiplier?

The fiscal multiplier, as defined in the exam question, measures the discounted change in GDP relative to the discounted changes in the collected taxes.

$$\mathcal{M} = \frac{\sum_t^\infty \frac{Y_t - Y_{ss}}{(1+r)^t}}{\sum_t^\infty \frac{\text{taxes}_t - \text{taxes}_{ss}}{(1+r)^t}} = 1.2709 \tag{2}$$

I find that this fiscal multiplier is $\mathcal{M} = 1.27$, indicating that the discounted increase in GDP is greater than the discounted collected taxes. This, however, is not to say that there is crowding-in in the model. As clearly visible from figure 1, I find that on the aggregate level, there is a clear drop in consumption. When calculating the effect on consumption, it can be seen from table 1 that I find a negative effect.

Breaking down to the individual agents in the economy, I find different responses than seen in figure 1

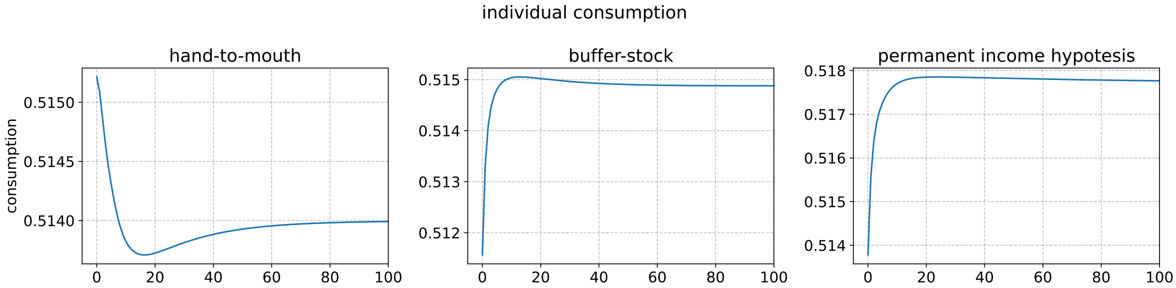

Figure 2: Impulse responses in consumption for a 1% shock to G_t broken down for each type of household

From figure 2, it is apparent that hand-to-mouth agents increase their consumption in the first periods but drop below the s.s. value after 5 periods, whilst for buffer stock and permanent income hypothesis agents, their consumption falls below their steady-state values. Thus, there is an immediate crowding-in effect in the consumption of hand-to-mouth agents; however, their total consumption in the first 480 periods is still below the steady consumption, and thus, there is crowding-out for all household types.

b) How do results change in when ω is increased

When changing ω, I find that the fiscal multiplier falls. This happens as the government adjust their taxes to bring down the debt faster.

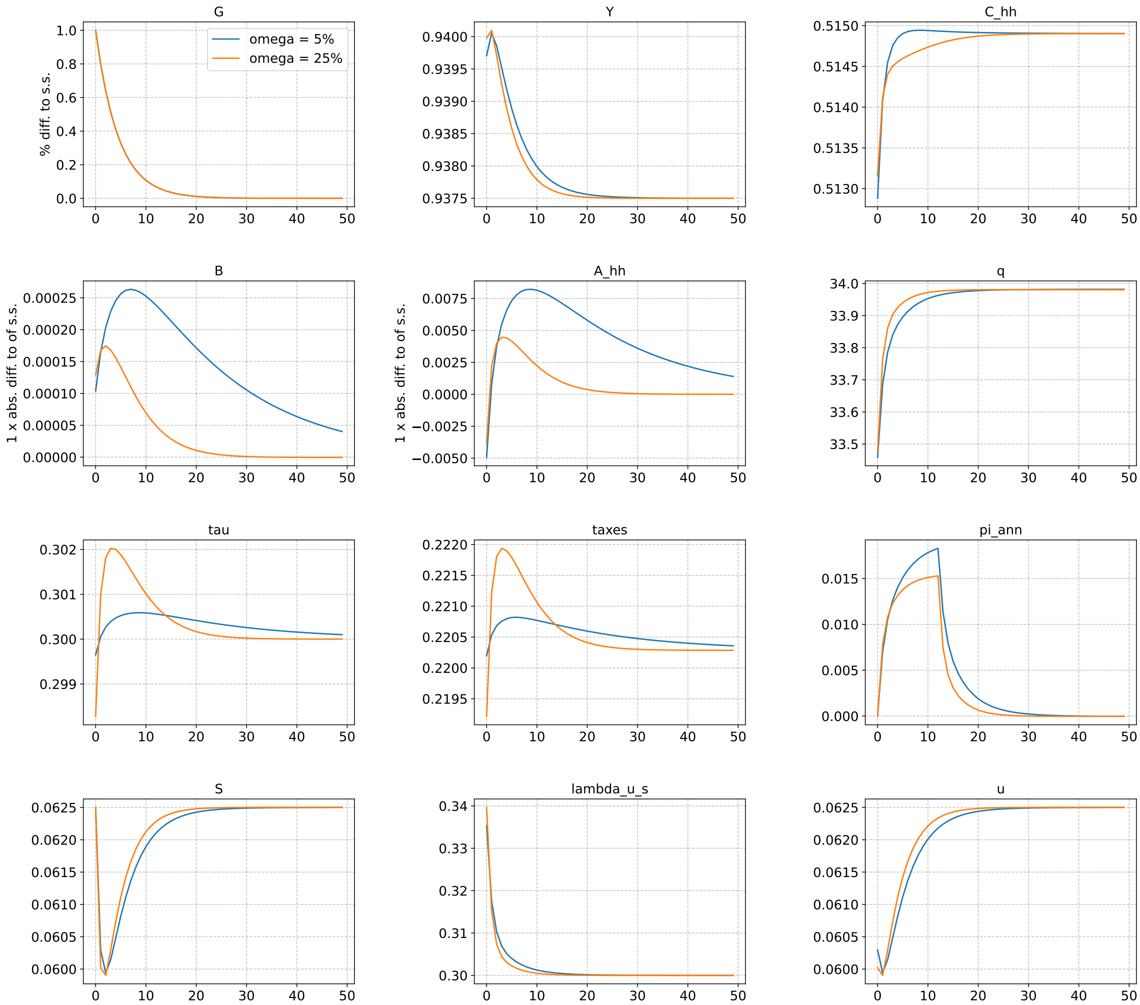

Figure 3: Impulse Response Functions for selected variables for different sizes of ω

This is also seen from the IRFs in figure 3, where the tax rate increases rapidly, bringing down government debt and returning to the steady state in the capital markets more rapidly. As the shock to G_t is identical, the effect on Y_t is similar – though falling slightly faster as households are taxed more harshly. The taxes collected to bring down the government are discounted less, and the overall fiscal multiplier drops.

$$\mathcal{M}_{\omega=25\%} = 0.9776 \tag{3}$$

Thus, for $\omega = 25\%$, the fiscal multiplier drops below 1.

c) How do results change when the share of HTM hh is increased

When the share of hand-to-mouth households increases in the economy, I expect the fiscal multiplier to increase. This follows the intuition from figure 2 where hand-to-mouth agents are seen to increase their spending in the first period as tax rates are lower, and their disposable income, therefore, is greater.

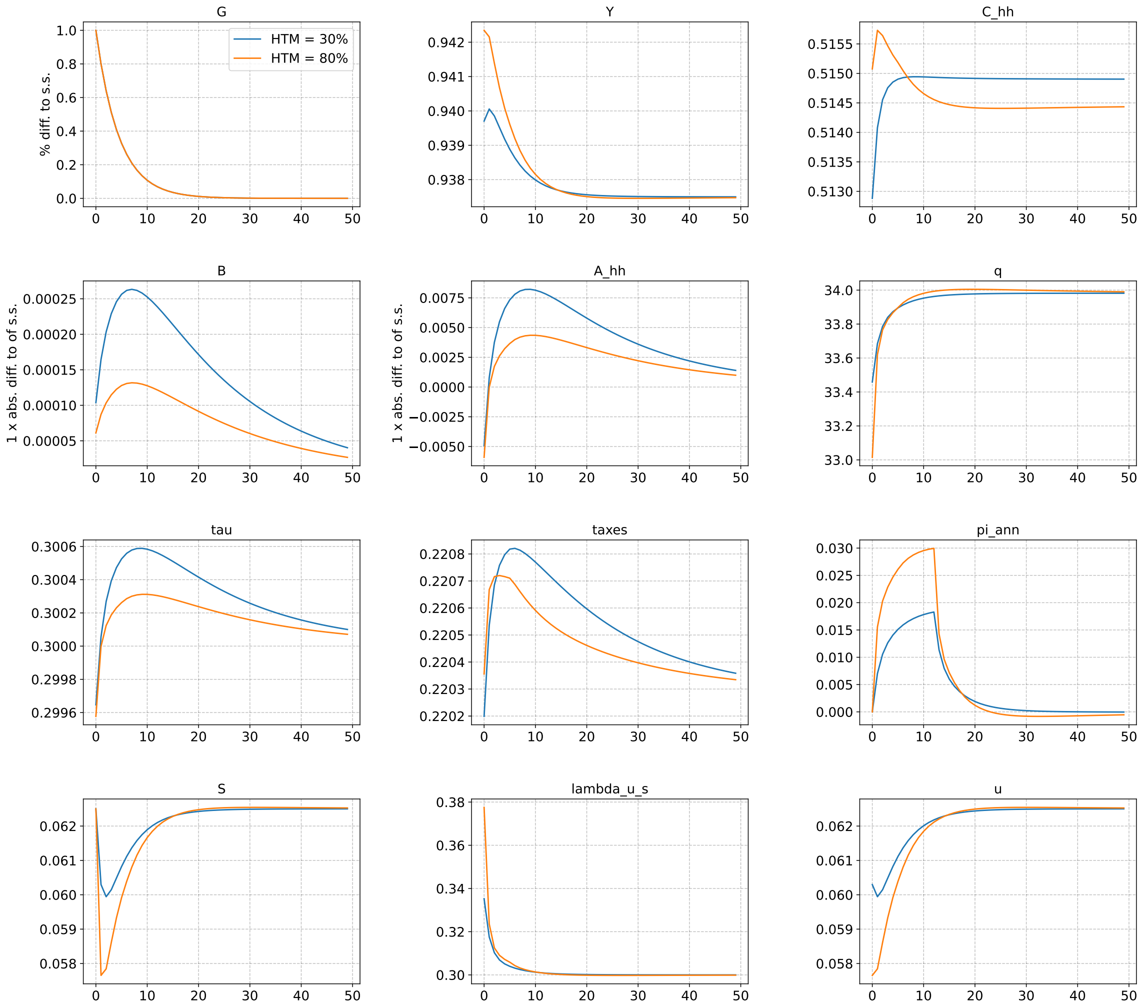

Figure 4: Impulse Response Functions for selected variables for different shares of HTM hh

Looking at figure 4, I find the on-impact shock to Y to be almost twice as high as in the baseline scenario; this is because the household consumption is much greater in the first periods. Where buffer-stock and permanent income hypothesis households lower their consumption as a response. As the hand-to-mouth households reach a sufficiently large share of the population – here 80% – this effect dominates that of the other households, resulting in the crowding in of consumption on an aggregate level. As there is now a greater aggregate demand in society, the value of jobs increases and the risk of unemployment drops while job-finding increases, lowering unemployment. The increase in employment leads to even more hand-to-mouth agents with a job amplifying the effects. However, the aggregate steady-state level of consumption, and thus utility, is lower as there is a lower share of households save to self-insure themselves.

Further, I find that the collected taxes are lower as the number of bonds issued is much lower. This happens as the steady-state debt level is around half of the baseline – $B_{ss|HTM=30\%} = 0.0161$ and $B_{ss|HTM=80\%} = 0.0082$ – the price of refinancing existing debt is lower as q drops on impact. Though the number of bonds needed to finance the shock is greater, as prices drop, the effect from a lower initial level dominates, making the total number of bonds issued lower. The needed taxes to repay this debt are, therefore, smaller. Thus, the total effect on the financial multiplier must unconditionally lead to an increase.

$$\mathcal{M}_{HTM=80\%} = 2.7391 \tag{4}$$

When computing the financial multiplier, I find exactly this as it rises from $\mathcal{M}_{HTM=30\%} = 1.27$ to $\mathcal{M}_{HTM=80\%} = 2.74$.

d) How do results change when there is a representative hh

To implement the RANK model, I follow a similar fashion to what is found in GEModel-ToolsNotebook under HANK-sticky-prices. As the representative agent discounts with $\beta^{RA} = 1/(1+r_{ss})$ they have a higher discount factor when compared to the buffer-stock and permanent income hypothesis, with yearly discount factors of $\beta^{RA} = 1/(1+0.02) = 0.980$, $\beta^{BS} = 0.940$, and $\beta^{PIH} = 0.975$. Thus, I expect them to focus more on lifetime consumption rather than immediate gains, thus having smaller fluctuations in their behaviour when adjusting to a temporary shock, which in turn would lower the fiscal multiplier.

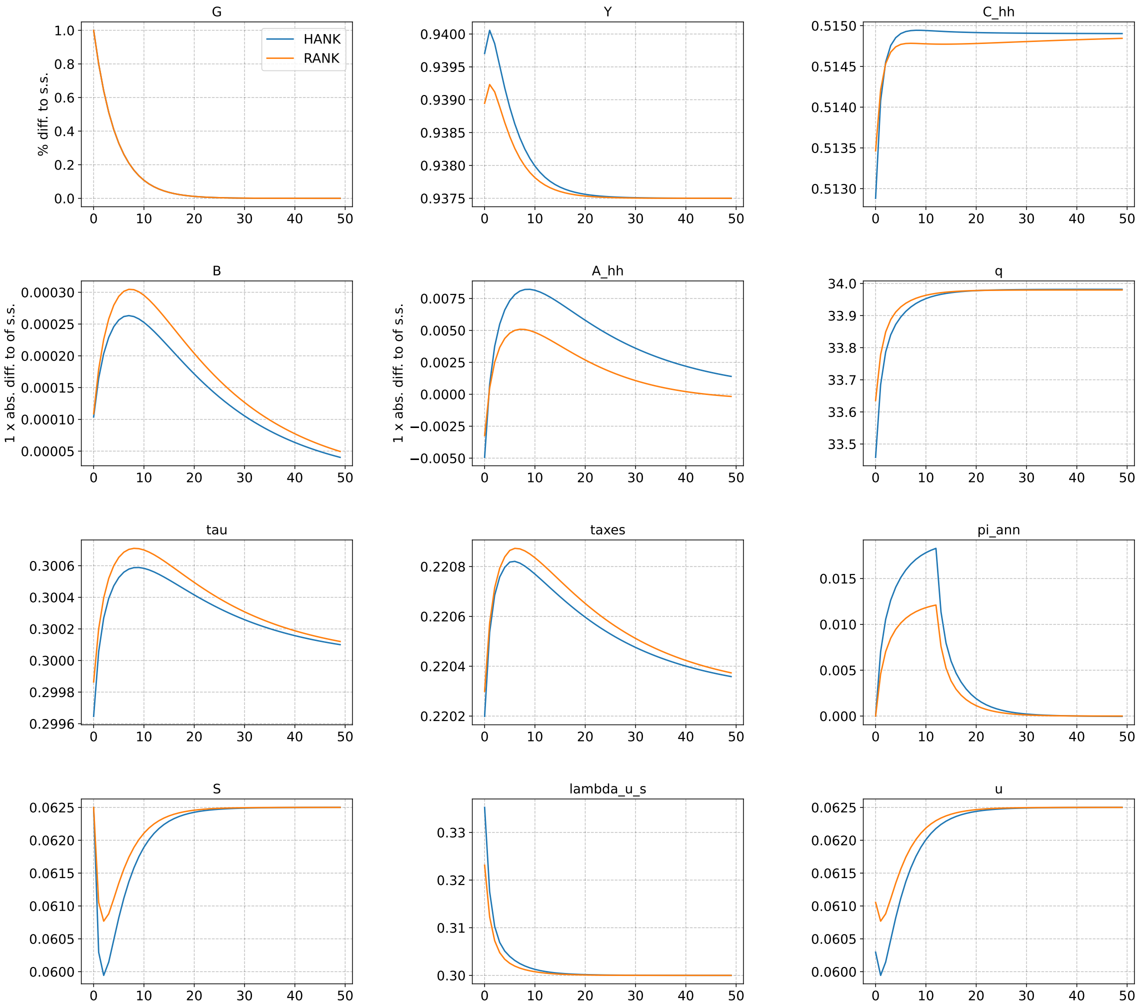

Figure 5: Impulse Response Functions for HANK and RANK models

In figure 5, I find exactly that representative agents have lower fluctuations in both consumption and assets. Furthermore, it is apparent that the total demand does not increase as much; this happens as unemployment stays at a higher level due to a higher unemployment risk. I find that the job-finding rate is relatively similar, and thus, the effect is primarily driven by a higher u_t, than in the HANK model.

As more agents stay unemployed, the costs of financing their unemployment are also higher, and thus, I find higher collected tax from which I can then conclude that the fiscal multiplier must fall.

$$\mathcal{M}_{RA} = 0.7316 \tag{5}$$

When calculating the fiscal multiplier, I find that it is just over half of the HANK fiscal multiplier. As mentioned, this is because the representative agent cares more about their lifetime consumption; thus, they choose to adjust their consumption less in the event of a shock. This, in turn, gives a relatively higher unemployment risk, which in turn makes the representative agent more fearful of the future and, thus, more aware of their current consumption.

3 Unemployment insurance duration

a) Fiscal spending multiplier with permanently longer benefits

In the case of permanently higher unemployment benefits from the cautionary households – buffer-stock and permanent income hypothesis – as the risks associated with unemployment are decreased. This leads to a higher steady-state consumption and lower levels of bond and savings.

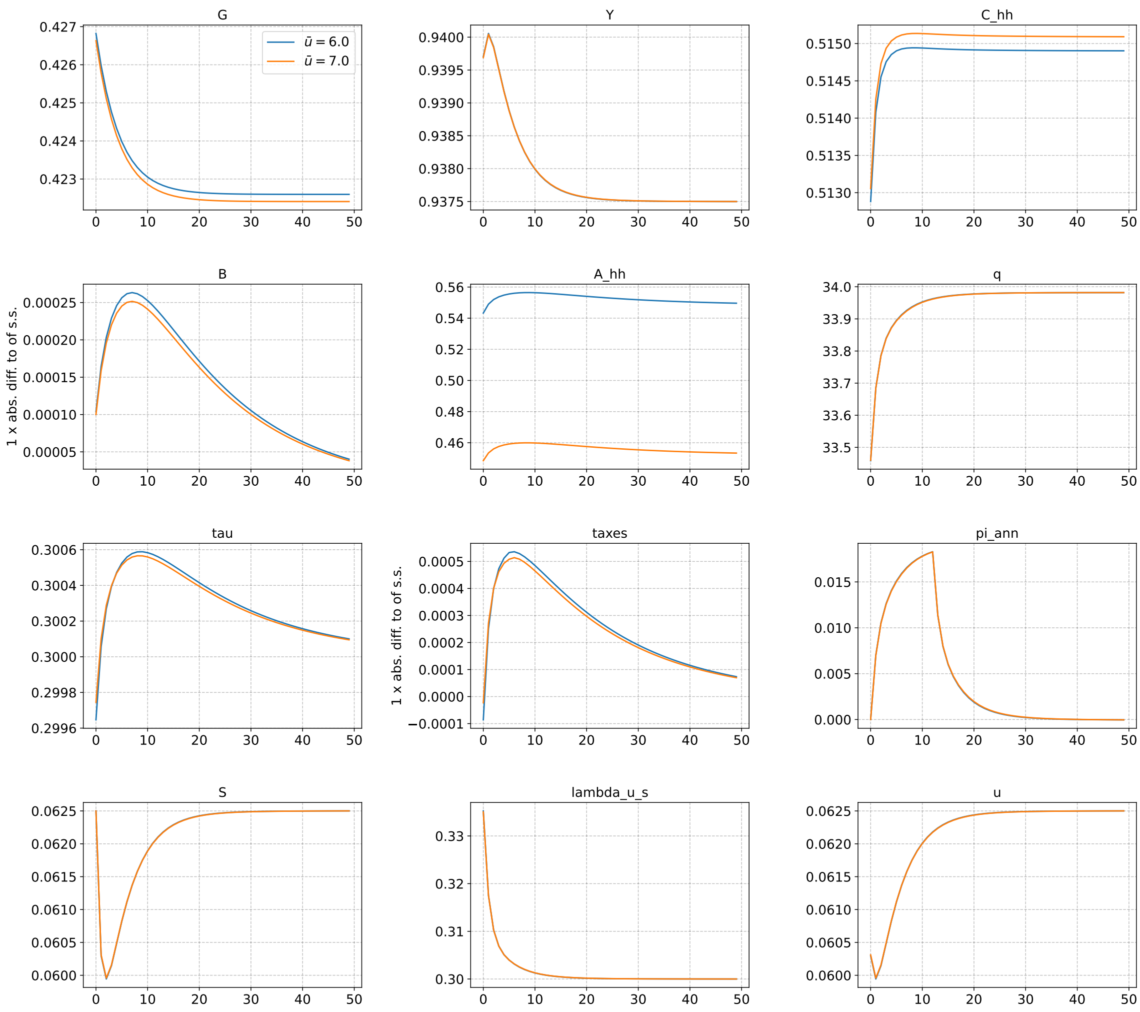

Figure 6: Impulse Response Functions for selected variables with different lengths of high unemployment benefits

I find that there is no net impact on Y compared to the baseline scenario, though C^{hh} is higher G is equally lower. The important difference for the fiscal spending multiplier lies in the fact that G is slightly lower and the steady state bond level is lower, implying that fewer bonds need to be issued, and thus, the collected tax to repay the bonds is smaller.

This leads to a slightly increased fiscal spending multiplier.

$$\mathcal{M}_{\bar{u}=7} = 1.3205 \tag{6}$$

b) Fiscal multiplier of a 1-month extension for 12 months

With a non-permanent extension to the unemployment benefits, households again are expected to decrease their savings initially to consume more, but as the time when the extension is stopped nears, they increase their asset holding again, thus reducing spending, whilst the government increase debt in the period of where they pay out higher benefits.

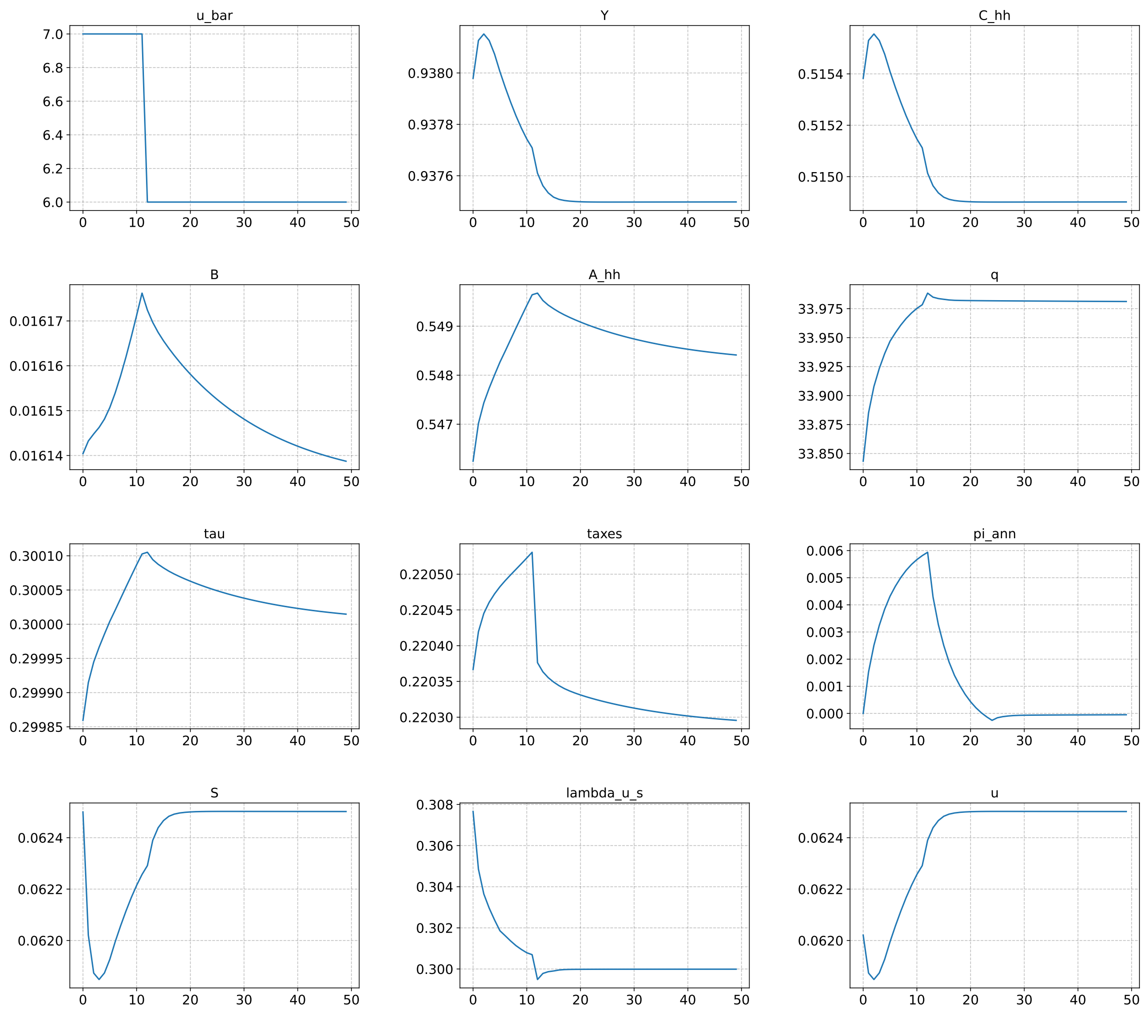

Figure 7: Impulse Response Functions for selected variables with different temporarily extended unemployment benefits

In figure 7, I find that exactly this happens. Additionally, as demand increases, the value of jobs increases, leading to lower unemployment rates and higher job-finding rates, which increase employment, and thus, there is a delayed effect on consumption and demand. As the government takes on debt right away, it must increase taxes; however, those top, together with debt, at a later period than demand, which pushes up the fiscal multiplier.

$$\mathcal{M}_{\bar{u}=7 \text{ for } t\in[0:11]} = 1.4798 \tag{7}$$

I find that the fiscal multiplier of a temporary shock to the unemployment benefits is slightly larger than the previous baseline shock to G.

c) Fiscal multiplier of a 12-month extension next year

In the case of an expected future increase in unemployment, I expect the fiscal multiplier to increase greatly. Buffer-stock and permanent income hypothesis households are expected to start smoothening their consumption as a response already before the shock occurs which would increase demand. The government, however, does not incur extra costs in the first periods, and thus, a significant impact on the fiscal multiplier is expected.

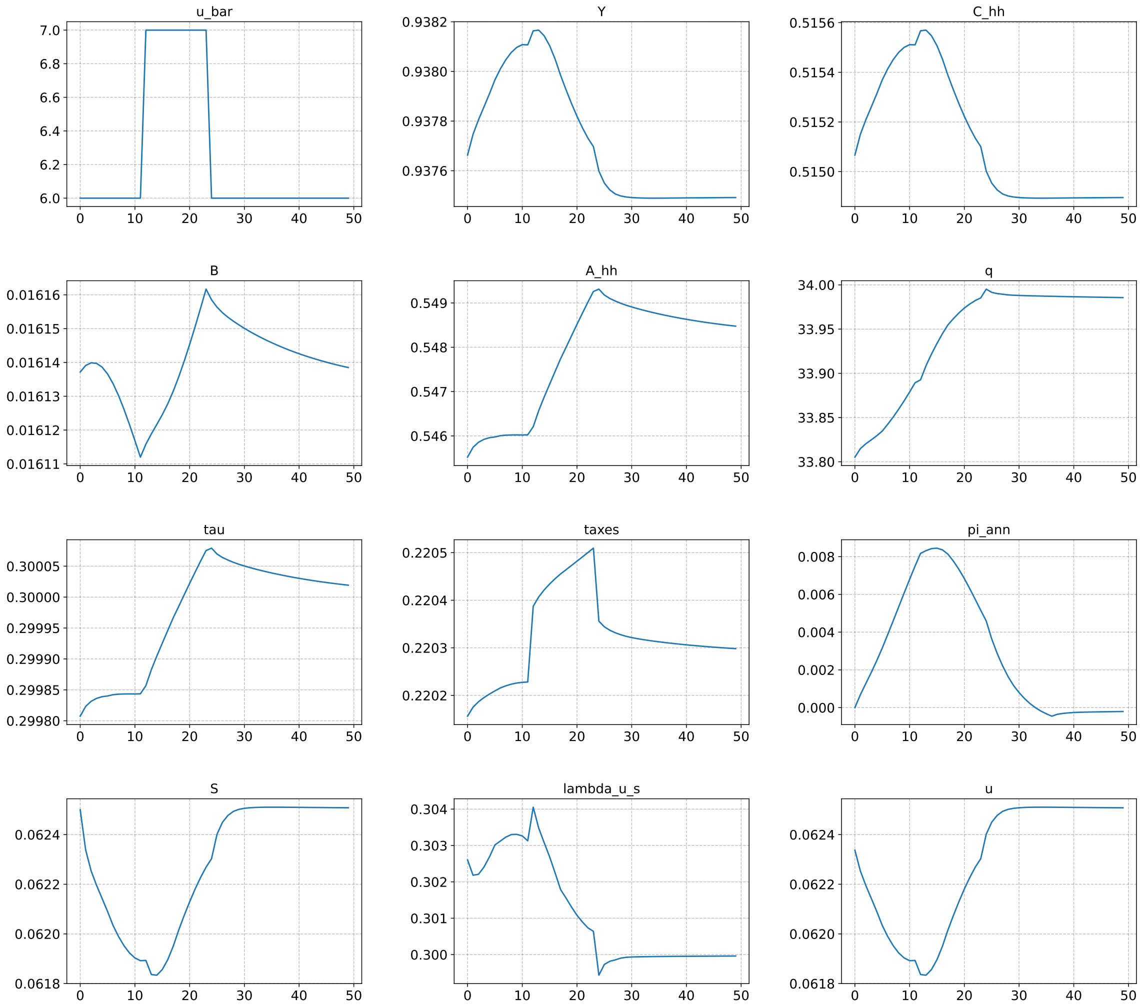

Figure 8: Impulse Response Functions for selected variables with an expected temporary extension of unemployment benefits

From the simulation found in figure 8, it is apparent how the behaviour is similar to what was seen in figure 7. However, households just start changing their behaviour earlier, leading to a greater overall impact on consumption and aggregate demand as unemployment remains lower for a longer period of time. Interestingly, as households shift their choice towards greater consumption, the bond price drops, resulting in cheaper refinancing of debt, and thus, the taxes fall already in period 0 and stay below the steady state level until period 12.

$$\mathcal{M}_{\bar{u}=7 \text{ for } t\in[12:23]} = 5.1859 \tag{8}$$

When calculating the fiscal multiplier, I get $\mathcal{M}_{\bar{u}=7 \text{ for } t\in[12:23]} = 5.1859$, a more than 3-fold increase from a similar shock coming without any warning as seen in b). Thus, in such an economy, it is a much more powerful stimulus when shock is anticipated.

d) Fiscal multiplier of an expected extension which is cancelled

I have calculated the case where the government retracts an anticipated shock equivalent to the one in c) for the first 6 periods – from 0 to and including period 5 – followed by a return to steady state from the state after the end of period 5. In c), I found that taxes initially dropped, and thus, I expect the financial multiplier to increase drastically as the government never has the high cost of increasing the unemployment benefits.

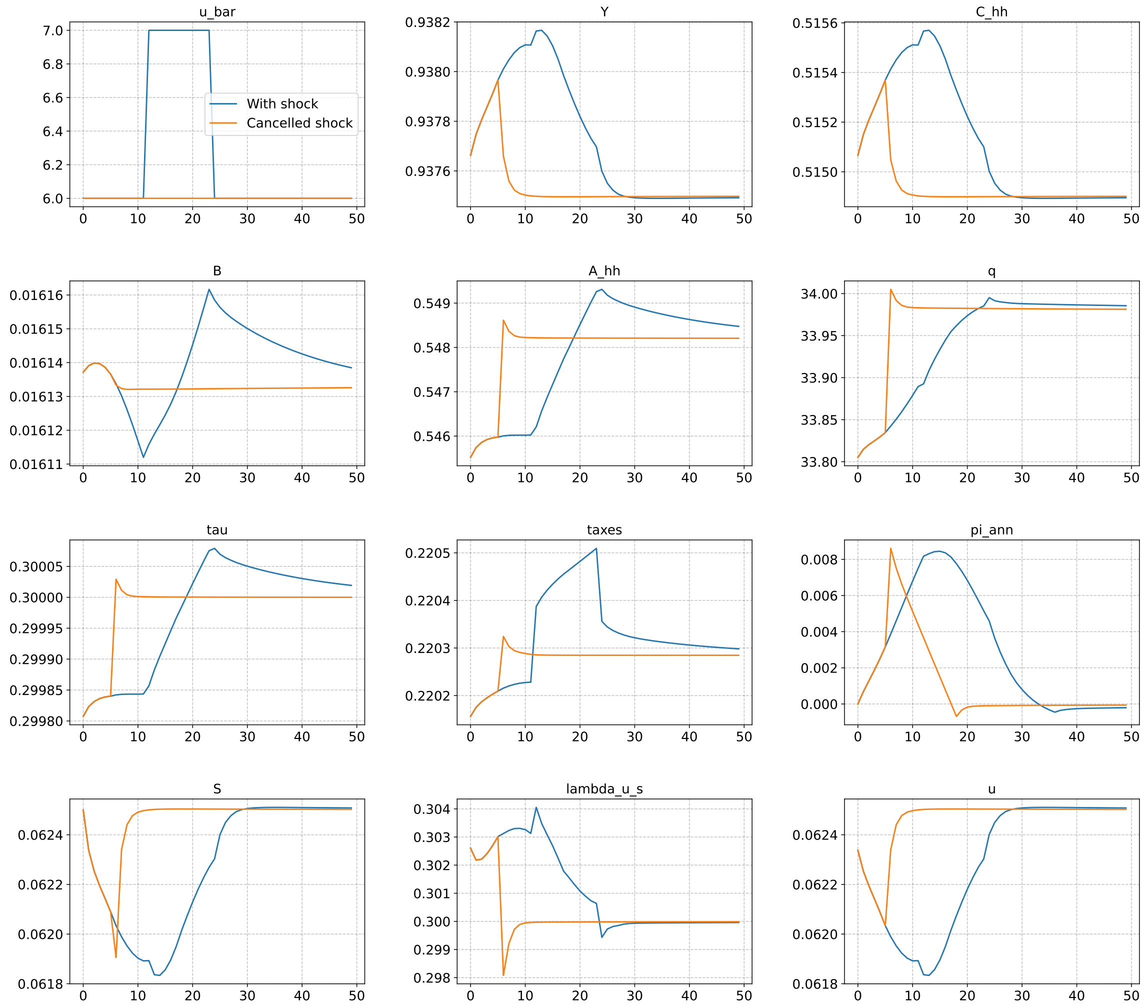

Figure 9: Impulse Response Functions for selected variables with an expected temporary extension of unemployment benefits. In blue, the government sticks to it; in orange, the government cancels the shock after 5 periods

From the simulation shown in figure 9, it is seen how the path follows exactly the one from c) in the first 6 periods; however, when households buffer-stock, and permanent income hypothesis households no longer expect the increase in unemployment benefits and thus quickly adjusts their choice to increase savings up towards the steady-state level. Interestingly, I find that taxes actually do not increase by a huge amount above the steady state level, and thus, taxes collected relative to the steady state are actually negative. This has a huge impact on the fiscal multiplier

$$\mathcal{M}_{\bar{u}=7 \text{ expected } t\in[12:23], \text{ but cancelled}} = \frac{0.00196002}{-0.00052346} = -3.7444 \tag{9}$$

I find a negative fiscal multiplier. However, this is not from a fall in the discounted Y but rather from a drop in the discounted collected taxes that fall below 0. Essentially, this leads to a scenario where households are encouraged to spend more money, which in turn increases Y without experiencing any costs. This leads to what Friedman would call a 'free lunch'. However, this only works under the assumption that households have no effective way of penalising the government for not following through with the announced policies. In a more realistic world, households would stop believing the government, which could lead to a scenario where no previous announcement of policies would have any effect before they are enacted, i.e., the scenario seen in c) would be impossible, and the fiscal multiplier in that scenario would be as in b) where the household would not anticipate the shock.