UNIVERSITY OF COPENHAGEN
DEPARTMENT OF ECONOMICS

Assignment II | HANC Welfare Model

Advanced Macroeconomics: Heterogeneous Agent Models

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a) Stationary Equilibrium without a Government

The model is set with 2 parameters that can be shocked, τ and χ, and with the following initial parameter values

Preferences	Income Process	Production	Government
$\beta = 0.96$	$\rho_z = 0.96$	$\Gamma^Y = 1.00$	$\tau = 0.00$
$\sigma = 2.00$	$\sigma_\psi = 0.15$	$\Gamma^G \quad 1.00$	$\chi = 0.00$
$\omega = 2.00$		$\alpha_{ss} = 0.30$	
$\varphi = 1.00$		$\delta = 0.10$	
$\nu = 1.00$			
$\underline{S} = 10^{-8}$			

Table 1: Baseline parameter values

The model's interaction can be seen from the DAG, where we find 5 blocks: production, government, households, mutual funds, and market clearing.

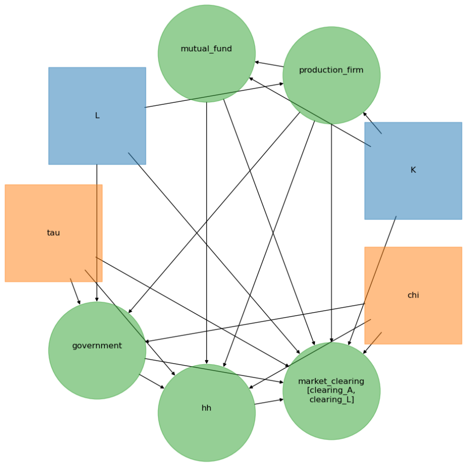

Figure 1: DAG of the model

Solving the model with $\tau = \chi = 0.00$, implying no government, gives the steady state values

	ss
K_{ss}	3.389
Y_{ss}	1.363
C^{hh}_{ss}	1.024
G_{ss}	0.000
I_{ss}	0.339
L^{hh}_{ss}	0.923
L^{Y}_{ss}	0.923
L^{G}_{ss}	0.000
r_{ss}	0.021
w_{ss}	1.034

Table 2: Steady state values for the model with no government

Summing the expected discounted utility for the first 500 periods

$$\bar{v} = \sum_{t=0}^{500} \beta^t u(c_{it}, S_t, \ell_{it}) d\mathbf{D}_{t+k} = \sum_{t=0}^{500} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \frac{(S_t + \underline{S})^{1-\omega}}{1-\omega} - \varphi \frac{\ell_{it}^{1+\nu}}{1+\nu} d\mathbf{D}_{t+k} \tag{1}$$

Yields an expected discounted utility of $\bar{v} = -2.5 \cdot 10^9$

b) Optimal Welfare Policies with Income Tax

Allowing for $\tau \in (0,1)$, the government can now start to produce a flow of services. Utilising a scalar minimizer, I find the tax rate on labour income that maximises expected as $\tau = 65.51\%$. In the new optimum, the steady-state values are

	ss
K_{ss}	2.897
Y_{ss}	1.221
C_{ss}^{hh}	0.517
G_{ss}	0.415
I_{ss}	0.290
L_{ss}^{hh}	1.258
L_{ss}^{Y}	0.843
L_{ss}^{G}	0.415
r_{ss}	0.026
w_{ss}	1.014

Table 3: Steady state values for model with $\tau = 65.51\%$ and $\chi = 0$

We see that as households value consumption and government services equally ($\sigma = \omega$), the optimum is that the tax level is rather high, resulting in an expected discounted utility of -138.97. The capital government services output ratio is therefore $G_{ss}/Y_{ss} = 0.3398$. This ratio of around 1/3 makes sense as the government has to employ labour and buy goods, and thus, government services come at twice the cost of regular consumption – following the optimality constraint of $G_t = \Gamma^G L_t^G$. Investments cause this to be not complete 1/3, which is also why, as capital – and the need for investments – increase the government services output ratio should fall.

c) Optimal Welfare Policies with Income Tax and Transfers

Now allowing for χ different from 0, I utilise a numerical minimizer to find the optimal tax rate and government transfer. I find that the optimal tax rate on labour income decreases to $\tau = 47.71\%$, whilst $\chi = -0.2181$, providing an expected discounted utility of -136.89, i.e. it is optimal to have a lump sum tax. As households see this as exogenous they choose to work more when the $\chi < 0$, and they, therefore, supply more labour in the steady state, whilst the lump sum tax ensures a minimum level of government services.

The new steady-state values are

	ss
K_{ss}	3.479
Y_{ss}	1.399
C_{ss}^{hh}	0.606
G_{ss}	0.444
I_{ss}	0.348
L_{ss}^{hh}	1.391
L_{ss}^{Y}	0.946
L_{ss}^{G}	0.444
r_{ss}	0.021
w_{ss}	1.034

Table 4: Steady state values for model with $\tau = 47.71\%$ and $\chi = -0.2181$

In the new steady state, the capital government services output ratio is, therefore, $G_{ss}/Y_{ss} = 0.3177$.

To further understand the difference between using a lump-sum tax and an income tax, I have plotted the household labour supply for fixed government spending but different ways of collecting the necessary taxes.

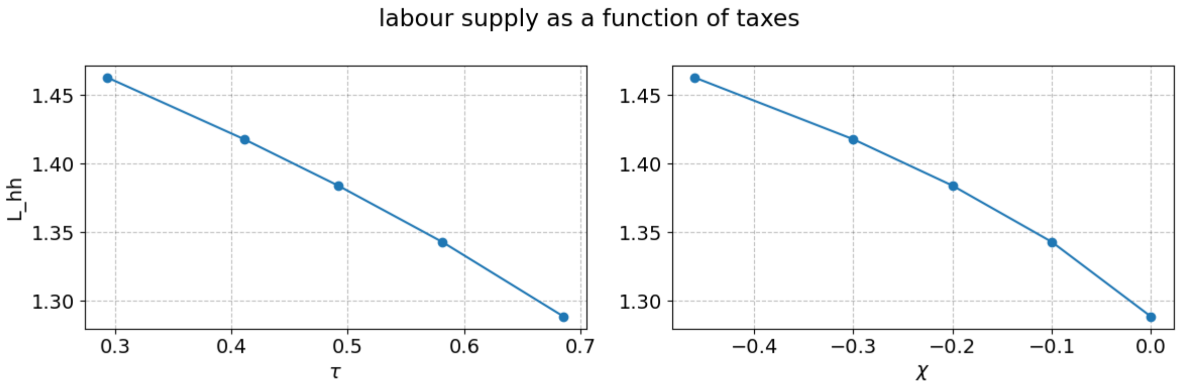

Figure 2: Total labour supply for a fixed $G = G_{ss} = 0.444$

From figure 2, it is apparent that the substitution effect dominates the income effect, and households choose to work less as τ increases and the marginal utility of labour decreases, whilst the opposite is true for χ. This happens as τ that is included in their first order condition of the households where χ is not, and thus there is no substitution effect in χ; only the income effect is present, and households choose to work more as they get poorer. I, therefore, find that the lump-sum tax is less distortionary on labour supply than the income tax, as households do not face lower marginal utility on labour.

Though the labour supply – and thus overall production – is higher, the additional lump-sum tax also comes with a cost. As everyone pays the same with a lump-sum tax, its presence would increase the inequality in society.

Figure 3: CDF of savings for different tax combinations for a fixed $G = G_{ss} = 0.444$

In figure 3, I find that the mass in the CDF is more centred in the middle for $\chi = 0$, whilst as χ decreases, the mass becomes more centred in the ends. Therefore, though the production increases with labour supply as χ decreases, the wealth inequality in society also increases. The solution with a lump-sum tax, however, is still optimal as the social planner in the model only seeks to maximise expected utility and does not care about inequality.

d) Increase in Productivity

I now set the productivity in the private sector,Γ^Y, to 1.1. From here, I solve the model first with the tax rates found in c) and subsequently use the same minimizer to solve the social planner's problem to optimise welfare. The new steady-state values are:

	Old Taxes	New Taxes
K_{ss}	3.667	3.731
Y_{ss}	1.474	1.495
C_{ss}^{hh}	0.669	0.657
G_{ss}	0.438	0.465
I_{ss}	0.367	0.373
L_{ss}^{hh}	1.308	1.347
L_{ss}^{Y}	0.870	0.882
L_{ss}^{G}	0.438	0.465
r_{ss}	0.021	0.020
w_{ss}	1.185	1.187

Table 5: Steady state values for models with $\Gamma^Y = 1.1$. Old taxes are $\tau = 47.71\%$ and $\chi = -0.2181$ and new taxes are $\tau = 48.15\%$, $\chi = -0.2476$.

I start by looking at what happens when only increasing productivity in the private sector. I find that the expected discounted utility increases to -129.09 as the consumption increases. However, as the private sector gets more productive, the wages increase, and as a result, the government service level decreases slightly as they cannot afford the same level with the higher wages. Additionally, as wages are increasing, households also start working less in production, decreasing the overall labour supply.

When optimising in this new scenario, I find that as productivity only increases in the private sector, it is optimal for the government to increase taxes as well so that the level of government services follows the increase in the private sector, again balancing private consumption and government services. The new optimal tax level, therefore, is $\tau = 48.15\%$ and $\chi = -0.2476$, resulting in an expected discounted utility of -128.84. As it is primarily an increase in χ, labour supply increases again, as discussed in c). This comes at the cost of slightly decreasing the consumption of private goods but a greater increase in government services, increasing overall production in the economy. As the government again can balance private consumption and government services, the ratio of government services to output is just slightly lower than what was found in c), with the new level being $G_{ss}/Y_{ss} = 0.3112$.

e) Transition Towards a New Equilibrium

In this section, I will evaluate 5 different ways of implementing the changes in taxes, 4 of which will transition towards the new equilibrium and 1 towards an out-of-equilibrium tax level. Common to all the different implementations of the is the assumption that the government observes the change in Γ_Y in period 0 and, thus, cannot change the taxes until period 1.

Baseline: In the baseline scenario, the government adjusts τ and χ in period 1 straight to the optimal level in the new steady state.

Scenario A: The government will adjust the tax rate linearly from the initial tax rate to the optimal tax rate over a number of periods.

Scenario B: The government will reduce the gap between the initial tax rate and the optimal tax rate by a fraction in each period. Therefore, the change in the tax rate will decrease over time.

Scenario C: The government will increase the taxes from the initial tax rate to the optimal tax rate by an unknown fraction in each period. Thereby, the change in the tax rate will be increased over time until it reaches the new optimum.

Scenario D: In the out-of-equilibrium scenario, the government does not optimise the length towards the equilibrium but rather optimises the tax and lump-sum transfer given the initial setting in the old steady state. I subsequently test for multiple linear phase-in lengths to find the optimal value of tax and phase-in.

I find the socially optimal behaviour of the government as

	Expected Utility	Behaviour
Baseline	-129.19584	Set τ and χ to the optimal level in period 1
Scenario A	-129.08819	Increase τ and decrease χ with 1/20 of the initial gap each period
Scenario B	-129.09972	Decrease the gap in τ and χ by 10.76611% and 7.88065% respectively
Scenario C	-129.08742	Increase τ by 0.06207% and decrease χ by 0.59409% each period until new optimum is reached
Scenario D	-128.71719	Increase τ to 14.480%-point above the steady state and χ to 0.15416 phased in linearly over 37 periods

Table 6: Optimal government behaviour for the transition towards the new equilibrium. NB; it should be noted that though "permanent" increase above the equilibrium in D is only 250 periods long as the _set_shocks function in GEModelClass.py sets shocks to 0 after half of the periods in T

When only looking at the scenarios transitioning towards the new equilibrium, I find the expected utility is highest and very similar in scenarios A and C, but all alternative scenarios outperform the baseline. Visually, we can in figure 4 see why scenarios A and C have similar effects. Though different ways of implementing the changes, the actual tax rates in every period are very similar.

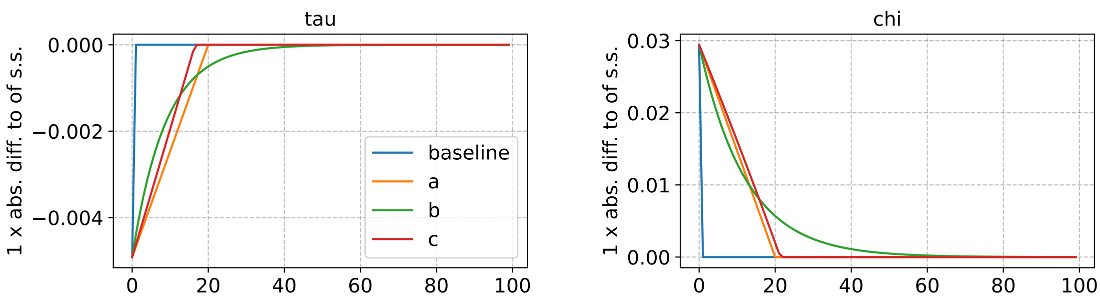

Figure 4: Different ways of implementing tax changes

As seen from table 6 of the tested implementation methods, the optimal way of implementing the new tax is with a constant increase in τ of 0.06207% and in χ of 0.59409% until the new optimal level is reached. This new level is reached in period 17 for τ and period 21 for χ.

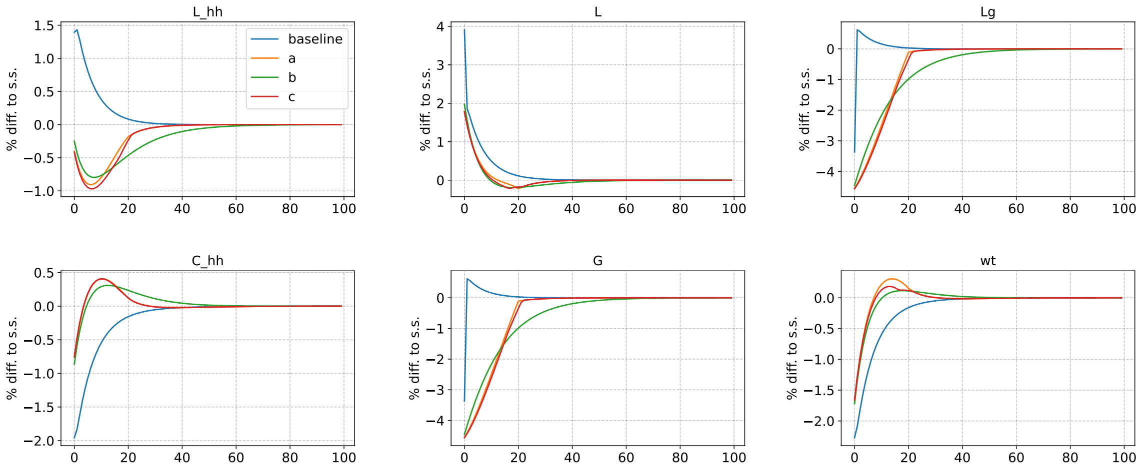

Figure 5: Household behaviour during transition

It is seen from figure 5 how with the implementation from scenario C. Compared to the naive baseline implementation, it is seen how this implementation ensures that private consumption does not decrease too drastically as taxes are implemented gradually.

Shifting our focus towards the best scenario of them all – the out-of-equilibrium scenario. I have in figure 6 plotted implementation together with the other scenarios.

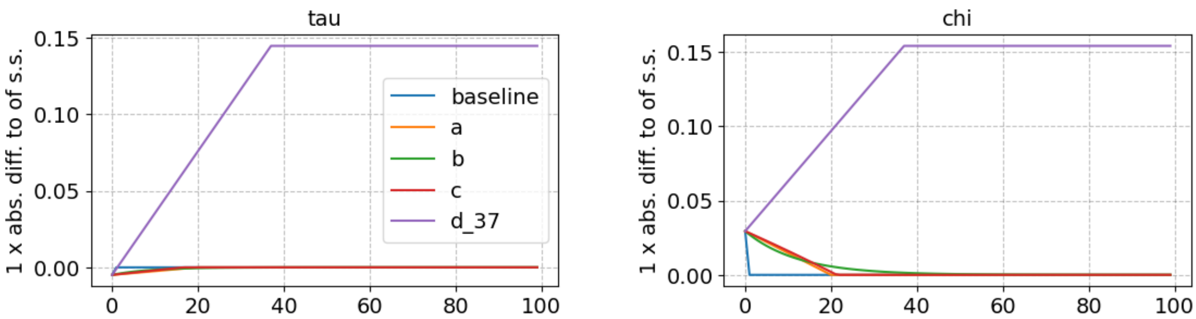

Figure 6: Different ways of implementing tax changes

It is immediately apparent that this implementation differs drastically as the income tax rises to 62.63%, (48.15% + 14.48%), and the lump-sum transfer drops to -0.0934, $(-0.2476 + 0.1542)$ over the first 37 periods.

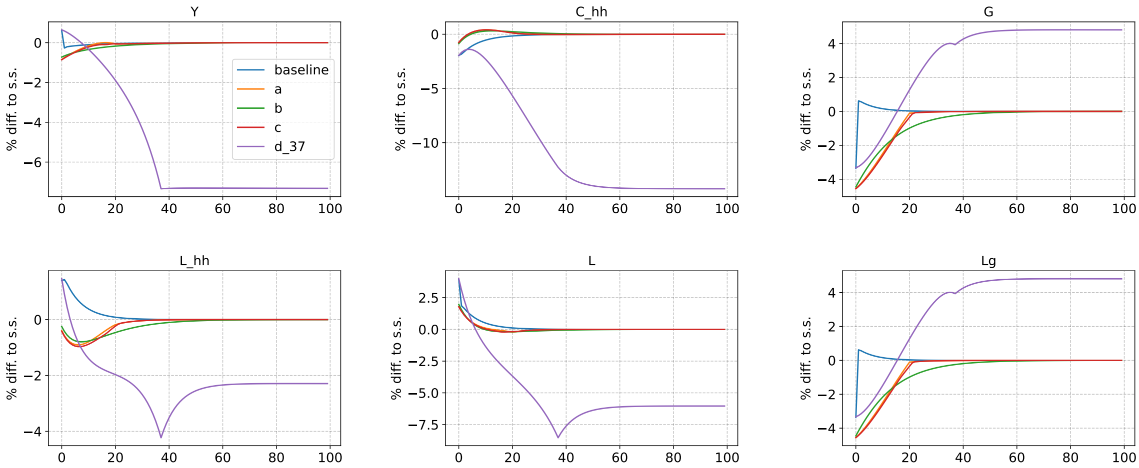

Figure 7: Household behaviour during transition

From figure 7, it is seen how the increase in the distortionary income tax combined with the decrease in the lump-sum tax leads to a decrease in labour supply of up to 4% as households start to substitute away from labour. This, in turn, leads to a drop in both consumption and production; however, as the tax revenue increases, the government flows of services, and government-employed labour does as well, which drives up the expected utility.