Advanced Macroeconomics: Heterogeneous Agent Models
Assignment II

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January 8$^{\text{th}}$ 2024

HANC Welfare Model

a) Stationary Equilibrium without a Government

The model is set with 2 parameters that can be shocked, τ and χ, and with the following initial parameter values

Preferences	Income Process	Production	Government
$\beta = 0.96$	$\rho_z = 0.96$	$\Gamma^Y = 1.00$	$\tau = 0.00$
$\sigma = 2.00$	$\sigma_\psi = 0.15$	Γ^G 1.00	$\chi = 0.00$
$\omega = 2.00$		$\alpha_{ss} = 0.30$	
$\varphi = 1.00$		$\delta = 0.10$	
$\nu = 1.00$			
$\underline{S} = 10^{-8}$			

Table 1: Baseline parameter values

The model's interaction can be seen from the DAG, where we find 5 blocks: production, government, households, mutual funds, and market clearing.

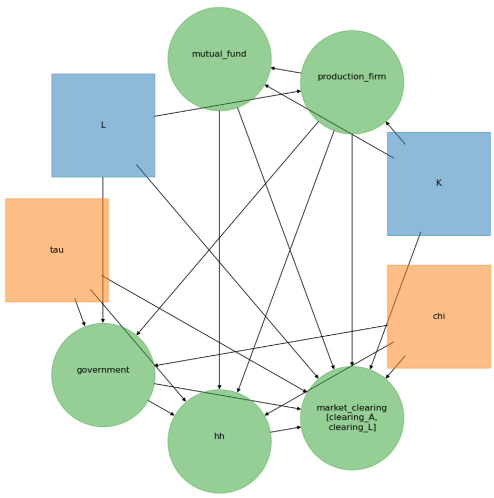

Figure 1: DAG of the model

Solving the model with $\tau = \chi = 0.00$, implying no government, gives the steady state values

K_{ss}	3.389
Y_{ss}	1.363
C_{ss}^{hh}	1.024
G_{ss}	0.000
I_{ss}	0.339
L_{ss}^{Y}	0.923
L_{ss}^{G}	0.000
r_{ss}	0.021
w_{ss}	1.034

Table 2: Steady state values for the model with no government

Summing the expected discounted utility for the first 500 periods

$$\bar{v} = \sum_{t=0}^{500} \beta^t u(c_{it}, S_t, \ell_{it}) d\mathbf{D}_{t+k} = \sum_{t=0}^{500} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \frac{(S_t + \underline{S})^{1-\omega}}{1-\omega} - \varphi \frac{\ell_{it}^{1+\nu}}{1+\nu} d\mathbf{D}_{t+k} \tag{1}$$

Yields an expected discounted utility of $\bar{v} = -2.5 \cdot 10^9$

b) Optimal Welfare Policies with Income Tax

Allowing for $\tau \in (0,1)$, the government can now start to produce a flow of services. Utilising a scalar minimizer, I find the tax rate on labour income that maximises expected as $\tau = 65.51\%$. In the new optimum, the steady state values are

K_{ss}	2.897
Y_{ss}	1.221
C_{ss}^{hh}	0.517
G_{ss}	0.415
I_{ss}	0.290
L_{ss}^{Y}	0.843
L_{ss}^{G}	0.415
r_{ss}	0.026
w_{ss}	1.014

Table 3: Steady state values for model with $\tau = 65.51\%$ and $\chi = 0$

We see that as households value consumption and government services equally ($\sigma = \omega$), the optimum is that the tax level is rather high. The capital government services output ratio is therefore $G_{ss}/Y_{ss} = 0.3398$.

c) Optimal Welfare Policies with Income Tax and Transfers

Now allowing for χ different from 0 I utilise a numerical minimizer to find the optimal tax rate and government transfer. I find that the optimal tax rate on labour income decreases to $\tau = 47.71\%$; however, whilst the $\chi = -0.2181$, i.e. it is optimal to have a lump sum tax. As households see this as exogenous, they choose to work more when the $\chi < 0$, and they, therefore, supply more labour in the steady state, whilst the lump sum tax ensures a minimum level of government services making consumption more attractive.

The new steady state values are

K_{ss}	3.479
Y_{ss}	1.399
C_{ss}^{hh}	0.606
G_{ss}	0.444
I_{ss}	0.348
L_{ss}^{Y}	0.946
L_{ss}^{G}	0.444
r_{ss}	0.021
w_{ss}	1.034

Table 4: Steady state values for model with $\tau = 47.71\%$ and $\chi = -0.2181$

In the new steady state, the capital government services output ratio is therefore $G_{ss}/Y_{ss} = 0.3177$.

d) Increase in Productivity

I now set $\Gamma^Y = 1.1$ and use the same minimizer to solve the social planner's problem to optimise welfare. I find that as productivity only increases in the private sector, it is optimal for the government to increase taxes as well so that the level of government services follows the increase in the private sector, again balancing private consumption and government services. The new optimal tax level, therefore, is $\tau = 48.15\%$ and $\chi = -0.2476$, the new steady state values are.

K_{ss}	3.731
Y_{ss}	1.495
C_{ss}^{hh}	0.657
G_{ss}	0.465
I_{ss}	0.373
L_{ss}^{Y}	0.882
L_{ss}^{G}	0.465
r_{ss}	0.020
w_{ss}	1.187

Table 5: Steady state values for model with $\tau = 48.15\%$, $\chi = -0.2476$ and $\Gamma^Y = 1.1$

As the government adjusts the taxes such that they again can balance private consumption and government services, the ratio of government services to output is just slightly lower than what was found in c), with the new level being $G_{ss}/Y_{ss} = 0.3112$.

e) Transition towards a new Equilibrium

In this section, I will evaluate 4 different ways of implementing the changes in taxes and transitioning towards the new equilibrium. Common to all the different implementations of the is the assumption that the government observes the change in Γ_Y in period 0 and, thus, cannot change the taxes until period 1.

Baseline: In the baseline scenario, the government adjusts τ and χ in period 1 straight to the optimal level in the new steady state.

Scenario A: The government will adjust the tax rate linearly from the initial tax rate to the optimal tax rate over a number of periods.

Scenario B: The government will reduce the gap between the initial tax rate and the optimal tax rate by a fraction in each period. Therefore, the change in the tax rate will decrease over time.

Scenario C: The government will increase the taxes from the initial tax rate to the optimal tax rate by an unknown fraction in each period. Thereby, the change in the tax rate will be increased over time until it reaches the new optimum.

I find the socially optimal behaviour of the government as

	Expected Utility	Behaviour
Baseline	-129.19584	Set τ and χ to the optimal level in period 1
Scenario A	-129.08819	Increase τ and decrease χ with 1/20 of the initial gap each period
Scenario B	-129.09972	Decrease the gap in τ and χ by 10.76611% and 7.88065% respectively
Scenario C	-129.08742	Increase τ by 0.06207% and decrease χ by 0.59409% each period until new optimum is reached

Table 6: Optimal government behaviour for the transition towards new equilibrium

We see how the expected utility is highest and very similar in scenarios A and C, but all alternative scenarios outperform the baseline. Visually, we can in figure 2 see why scenarios A and C have similar effects. Though different ways of implementing the changes, the actual tax rates in every period are very similar.

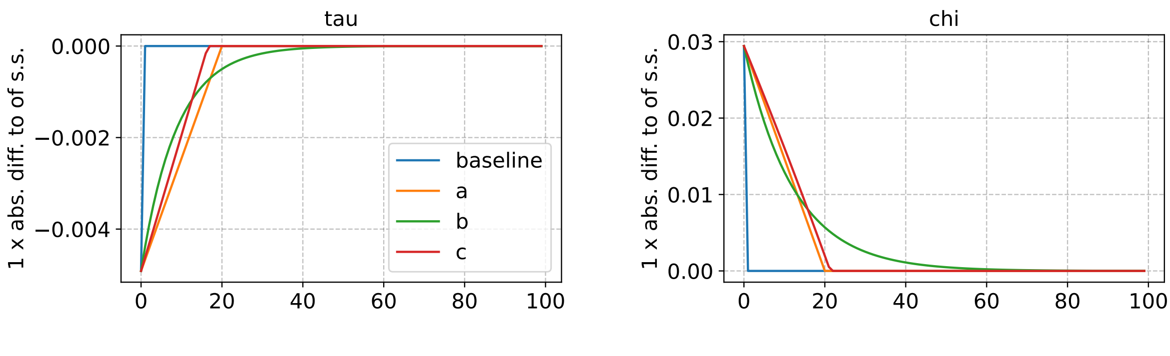

Figure 2: Different ways of implementing tax changes

As seen from table 6 of the tested implementation methods, the optimal way of implementing the new tax is with a constant increase in τ of 0.06207% and in χ of 0.59409% until the new optimal level is reached. This new level is reached in period 17 for τ and period 21 for χ.

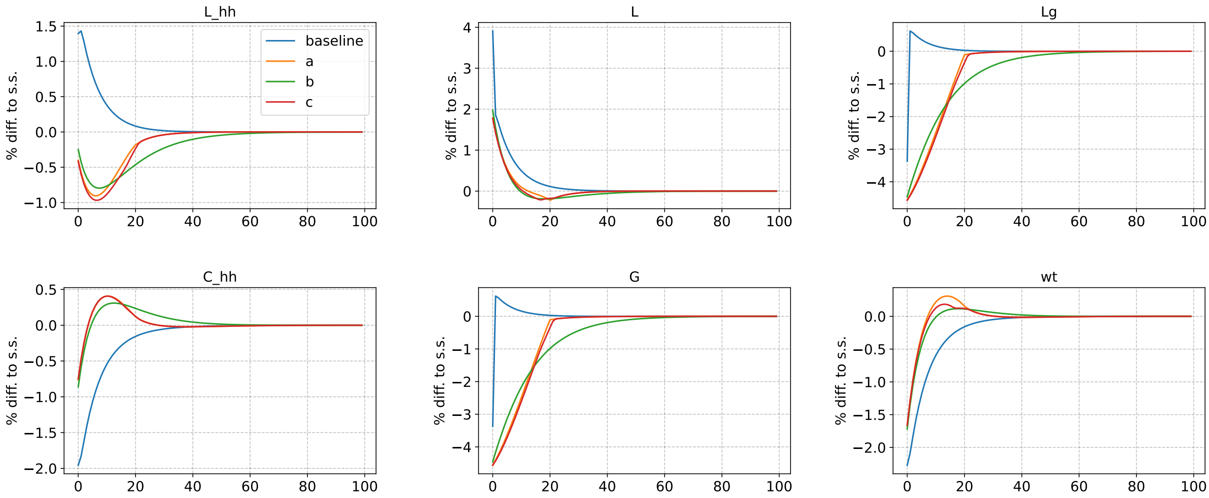

Figure 3: Household behaviour during transition

It is seen from figure 3 how with the implementation from scenario C. Compared to the naive baseline implementation it is seen how this implementation ensures that private consumption does not decrease too drastically as taxes are implemented gradually.