Written Exam Economics winter 2023-24

Advanced Macroeconomics: Heterogeneous Agent Models

January 7 to January 9

This exam question consists of 8 pages in total

Answers only in English.

You should hand-in a single zip-file. The zip-file should have the following folder and file structure:

Assignment_I
Assignment_I.pdf – with text and all results
files for producing the results

Assignment_II
Assignment_II.pdf – with text and all results
files for producing the results

Assignment_III
Assignment_III.pdf

Exam
Exam.pdf
files for producing the results

Be careful not to cheat at exams!

Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules in the Faculty of Social Science's common part of the curriculum

You can read more about the rules on exam cheating on your Study Site and in the Faculty of Social Science's common part of the curriculum.

Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

The Fiscal Multiplier in HANK-SAM

In this exam set, we consider the HANK-SAM model from the course. The model description is reproduced on page 4 onwards. Code is provided which solves the model using a baseline calibration and an indirect approach for finding the steady state. You can continue to rely on the indirect method for finding the steady state unless stated otherwise.

Question 1) Solution method

a) Could the number of inputs to the household block be reduced?

b) Could the number of unknowns be reduced?
Hint: Consider whether $\boldsymbol{u}$ *needs to be an unknown.*

c) Is the impulse response to a fiscal spending shock to G_t with monthly persistence of 0.80 approximately linear in the size of the shock? Explain your result.

Note: You don't have to implement your answers to a) and b) in the code.

Question 2) Fiscal multiplier

Consider a fiscal spending shock to G_t of 1 percent with a monthly persistence of 0.80. The fiscal multiplier is defined as

$$\mathcal{M} = \frac{\sum_{t=0}^{\infty} \frac{Y_t - Y_{ss}}{1+r_{ss}}}{\sum_{t=0}^{\infty} \frac{\text{taxes}_t - \text{taxes}_{ss}}{1+r_{ss}}}$$

a) What is the fiscal multiplier? Is there crowding-in or crowding-out of private consumption? Is there differences across the different types of households? Explain the mechanisms behind your results.

b) Explain how your results change in a) when ω is increased.

c) Explain how your results change in a) when the share of hands-to-mouth households is increased.

Finally, we consider a version of the model without heterogeneous households, but with a representative household. The consumption decision of the representative household is summarized by the Euler-equation

$$\left(C_t^{RA}\right)^{-\sigma} = (1 + r_{t+1})\beta^{RA}\left(C_{t+1}^{RA}\right)^{-\sigma}$$

where $\beta^{RA} = \frac{1}{1+r_{ss}}$, $C_t^{\text{RA}} = Y_t - G_t$, and we assume $C_{ss}^{RA} = C_{ss}^{hh}$.

d) Explain how your results change in a) when there is a representative household instead of heterogeneous households. Does unemployment risk play a role in the transmission mechanism with a representative agent?

Question 3) Unemployment insurance duration

Consider a 1-month extension of the high unemployment insurance duration.

a) What is the fiscal spending multiplier when the high unemployment insurance duration is permanently 1-month longer? Explain your results.

b) What is the fiscal multiplier of a 1-month extension of the unemployment insurance duration lasting for 12 months? Explain your results.

c) What is the fiscal multiplier of a 1-month extension of the unemployment insurance duration announced in period 0 to last 12 months from period 12 to period 24? Explain your results.

d) What is the fiscal multiplier of a 1-month extension of the unemployment insurance duration announced in period 0 to last 12 months from period 12 to period 24, but which is announced canceled at the end of period 5? Explain your results.

Model

Households. The model has a continuum of infinitely lived households indexed by $i \in [0,1]$. Time is discrete and indexed by $t \in \{0,1,\dots\}$. Each period is one month.

The households are *ex ante* heterogeneous in terms of their discount factor β_i. There are three types of households:

1. Hands-to-mouth households with $\beta_i = \beta^{\text{HtM}}$.
2. Buffer-stock households with $\beta_i = \beta^{\text{BS}}$.
3. Permanent income hypothesis households with $\beta_i = \beta^{\text{PIH}}$.

Households are *ex post* heterogeneous in terms of their unemployment status, u_{it}, and lagged end-of-period savings, a_{it-1}. If $u_{it} = 0$ the household is employed. If $u_{it} > 0$ the household is in its u_{it}'th month of unemployment.

Each period the household chooses consumption, c_{it}, and savings, a_{it}. Borrowing is not allowed and the utility function is CRRA.

The recursive household problem is

$$\begin{aligned} v_t(\beta_i, u_{it}, a_{it-1}) = & \max_{c_{it}, a_{it}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta_i \mathbb{E}_t \left[v_{t+1}\left(\beta_i, u_{it+1}, a_{it}\right)\right] \\ & \text{s.t. } a_{it} + c_{it} = (1+r_t)a_{it-1} + (1-\tau_t)y_t(u_{it}) + \text{div}_t + \text{transfer}_t \\ & a_{it} \geq 0 \end{aligned} \tag{1}$$

where r_t is the ex post return from period $t-1$ to t, $y_t(u_{it})$ is labor market income (including unemployment insurance), τ_t is the tax rate on labor market income, div_t is dividends, and transfer_t is a transfer from the government (or a lump-sum tax if negative).

The employment/unemployment transition probabilities are

$$\begin{aligned} \Pr[u_{it+1} = 0 \mid u_{it} = 0] &= 1 - \delta_t \\ \Pr[u_{it+1} = 1 \mid u_{it} = 0] &= \delta_t \\ \Pr[u_{it+1} > 1 \mid u_{it} = 0] &= 0 \\ \Pr[u_{it+1} = 0 \mid u_{it} > 0] &= \lambda_t^{u,s} s(u_{it-1}) \\ \Pr[u_{it+1} = u_{it} + 1 \mid u_{it} > 0] &= 1 - \lambda_t^{u,s} s(u_{it-1}) \\ \Pr[u_{it+1} \notin \{0, u_{it}+1\} \mid u_{it} > 0] &= 0 \end{aligned} \tag{2}$$

where δ_t is the separation rate, $\lambda_t^{u,s}$ is the job-finding rate per effective searcher, and $s(u_{it-1})$ determines the effectiveness of search conditional on unemployment status. When employed the households earn a fixed wage w_{ss}. When unemployed they get unemployment insurance. For the first $\overline{u}$ months this is $\overline{\phi}$. Afterwards it is $\underline{\phi}$. The income function thus is

$$y_{it}(u_{it}) = w_{ss} \cdot \begin{cases} 1 & \text{if } u_{it} = 0 \\ \overline{\phi}\mathrm{UI}_{it} + (1 - \mathrm{UI}_{it})\underline{\phi} & \text{else} \end{cases} \tag{3}$$

$$\mathrm{UI}_{it} = \begin{cases} 0 & \text{if } u_{it} = 0 \\ 1 & \text{else if } u_{it} < \overline{u}_t \\ 0 & \text{else if } u_{it} > \overline{u}_t + 1 \\ \overline{u} - (u_{it} - 1) & \text{else} \end{cases}$$

where $\mathrm{UI}_{it} \in [0, 1]$ is the share of high unemployment insurance in period t.

The aggregate quantities of central interest are

$$C_t^{hh} = \int c_{it} d\boldsymbol{D}_t \tag{4}$$

$$A_t^{hh} = \int a_{it} d\boldsymbol{D}_t \tag{5}$$

$$U_t^{hh} = \int 1\{u_{it} > 0\} d\boldsymbol{D}_t \tag{6}$$

$$\mathrm{UI}_t^{hh} = \int \mathrm{UI}_{it} d\boldsymbol{D}_t \tag{7}$$

$$S_t^{hh} = \int s(u_{it-1}) d\underline{\boldsymbol{D}}_t \tag{8}$$

Intermediate-good producers. Intermediate-good producers hire labor in a frictional labor market with search and matching frictions. Matches produce a homogeneous good sold in a perfectly competitive market. The Bellman equation for the value of a job is

$$V_t^j = p_t^x Z_t - w_{ss} + \beta^{\text{firm}} \mathbb{E}_t \left[(1 - \delta_{ss}) V_{t+1}^j \right] \tag{9}$$

where p_t^x is the intermediary goods price, Z_t is aggregate TFP, w_{ss} is the wage rate, β^{firm} is the firm discount factor, and δ_{ss} is the exogenous separation rate. The value of a vacancy is

$$V_t^v = -\kappa + \lambda_t^v V_t^j + (1 - \lambda_t^v)(1 - \delta_{ss})\beta^{\text{firm}} \mathbb{E}_t \left[V_{t+1}^v \right] \tag{10}$$

where κ is flow cost of posting vacancies, and λ_t^v is the job-filling rate. The assumption of free entry implies

$$V_t^v = 0 \tag{11}$$

Whole-sale and final-good producers. Wholesale firms buy intermediate goods and produce differentiated goods that they sell in a market with monopolistic competition. The wholesale firms set their prices subject to a Rotemberg adjustment cost. Final-good firms buy goods from wholesale firms and bundle them in a final good, which is sold in a perfectly competitive market. Together this implies a New Keynesian Phillips Curve,

$$1 - \epsilon + \epsilon p_t^x = \phi \pi_t (1 + \pi_t) - \phi \beta^{\text{firm}} \mathbb{E}_t \left[\pi_{t+1}(1 + \pi_{t+1}) \frac{Y_{t+1}}{Y_t} \right] \tag{12}$$

where ϵ is the elasticity of substitution between the differentiated goods, ϕ is the Rotemberg adjustment cost, π_t is the inflation rate from period $t-1$ to t, and Y_t is aggregate output given by

$$Y_t = Z_t(1 - u_t) \tag{13}$$

The adjustment costs are assumed to be virtual such that total dividends are

$$\text{div}_t = Z_t(1 - u_t) - w_t(1 - u_t) \tag{14}$$

Labor market dynamics. Labor market tightness is given by

$$\theta_t = \frac{v_t}{S_t} \tag{15}$$

A Cobb-Douglas matching function implies that the job-finding and job-finding rates are

$$\lambda_t^v = A\theta_t^{-\alpha} \tag{16}$$

$$\lambda_t^{u,s} = A\theta_t^{1-\alpha} \tag{17}$$

The law of motion for unemployment is

$$u_t = u_{t-1} + \delta_t(1 - u_{t-1}) - \lambda_t^{u,s} S_t \tag{18}$$

Central bank.

The central bank controls the nominal interest rate from period t to $t+1$, and follows a standard Taylor rule,

$$1 + i_t = (1 + i_{ss}) \left(\frac{1 + \pi_t}{1 + \pi_{ss}} \right)^{\delta_\pi} \tag{19}$$

Government.

The government can finance its expenses with long-term bonds, B_t, with a geometrically declining payment stream of $1, \delta, \delta^2, \ldots$ for $\delta \in [0, 1]$. The bond price is q_t. The expenses on unemployment insurance is

$$\Phi_t = w_{ss} \left(\overline{\phi} \mathrm{UI}_t^{hh} + \underline{\phi} \left(u_t - \mathrm{UI}_t^{hh} \right) \right) \tag{20}$$

Total expenses thus are

$$X_t = \Phi_t + G_t + \text{transfer}_t \tag{21}$$

Total taxes are

$$\text{taxes}_t = \tau_t \left(\Phi_t + w_{ss}(1 - u_t) \right) \tag{22}$$

The government budget is

$$q_t B_t = (1 + q_t \delta_q) B_{t-1} + X_t - \text{taxes}_t \tag{23}$$

The government adjust taxes to so that the value of government debt returns to its steady state value,

$$\tilde{\tau}_t = \frac{\left(1 + q_t \delta_q\right) B_{t-1} + X_t - q_{ss} B_{ss}}{\Phi_t + w_{ss}(1 - u_t)} \tag{24}$$

$$\tau_t = \omega \tilde{\tau}_t + (1 - \omega) \tau_{ss} \tag{25}$$

Financial markets. Arbitrage between government bonds and reserves implies that

$$\frac{1 + \delta_q q_{t+1}}{q_t} = \frac{1 + i_t}{1 + \pi_{t+1}} \tag{26}$$

The ex post realized return on savings is

$$1 + r_t = \begin{cases} \frac{(1+\delta_q q_0)B_{-1}}{A_{-1}^{hh}} & \text{if } t = 0 \\ \frac{1+i_{t-1}}{1+\pi_t} & \text{else} \end{cases} \tag{27}$$

Market clearing.

Asset and goods market clearing implies

$$A_t^{hh} = q_t B_t \tag{28}$$

$$Y_t = C_t^{hh} + G_t \tag{29}$$