UNIVERSITY OF COPENHAGEN

DEPARTMENT OF ECONOMICS

Assignment III | Do HANC-hh Bunch at Kink Points?

Advanced Macroeconomics: Heterogeneous Agent Models

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January 8th, 2024

Vision: I want to examine household behaviour around kink points in the Danish tax system. Unlike a lump sum or flat income tax, the Danish tax system is progressive with kinks. Recently, the government proposed a new "upper-upper" tax bracket that starts at DKK 2.500.000. However, the Danish Economic Council has raised concerns about the Ministry of Finance's calculations regarding the expected tax revenue[1], as many people are expected to adjust their income to avoid paying this new tax.

To shed light on this issue, I propose a Heterogeneous Agent Neo-Classical (HANC) model with a government that supplies a flow of services and collects taxes to pay for those services. The main deviation from assignment II will be in the system of taxation, where the government will have lump sum transfers, income tax and an "upper" tax bracket with a different income tax at their disposal to examine how households modify behaviour around kink points in the tax system.

Model

Setup: The model is based on and similar to the model in assignment 2.

Households: Just as in assignment 2, the model consists of a continuum of infinitely lived households indexed by $i \in [0,1]$. They are *ex-ante* homogeneous, but they face idiosyncratic uninsurable income shocks, z_{it} and can choose end-of-period savings, a_{it-1}, as they please, and thus, are *ex-post* heterogeneous. Savings are rented out to firms at the interest rate r_t whilst households are restricted from borrowing.

The distribution of households over idiosyncratic states is denoted $\underline{\boldsymbol{D}}_t$ before the realisation of shocks and $\boldsymbol{D}_t$ after. The real wage is w_t, and firms earn profits Π_t. The government imposes lump-sum transfers χ_t on all households and a flat tax-rate τ_{1t} on all income. Additionally, the government impose an additional upper tax, τ_{2t}, on incomes above $\underline{\Theta}$. For this, the government delivers a flow of services, S_t, that households benefit from. Households choose their labour supply $\ell_{it} \in \mathbb{R}_+$ and consumption, $c_{it} \in \mathbb{R}_+$. The household utility function is

$$u(c_{it},S_t,\ell_{it}) = \frac{c_{it}^{1-\sigma}}{1-\sigma} + \frac{S_t^{1-\omega}}{1-\omega} - \varphi\frac{\ell_{it}^{1+\nu}}{1+\nu} \tag{1}$$

The households solve the problem

$$\begin{aligned} v_t(z_{it},a_{it-1}) &= \max_{c_{it},\ell_{it}} u(c_{it},S_t,\ell_{it}) + \beta\mathbb{E}(v_{t+1}(z_{it+1},a_{it})) \\ &\text{s.t.} \\ a_{it} + c_{it} &= (1+r_t)a_{it-1} + (1-\tau_{1t})w_t z_{it}\ell_{it} + \chi_t + \Pi_t - \mathbf{1}_{w_t z_{it}\ell_{it}>\underline{\Theta}}\tau_{2t}(w_t z_{it}\ell_{it} - \underline{\Theta}) \\ \log z_{it} &= \rho_z \log z_{it-1} + \psi_{it+1},\ \psi_{it+1} \sim \mathcal{N}(\mu_\psi,\sigma_\psi),\ \mathbb{E}[z_{it}] = 1 \\ a_{it} &\geq 0 \end{aligned} \tag{2}$$

[1] https://dors.dk/vismandsrapporter/dansk-oekonomi-foraar-2023/kapitel-aktuel-oekonomisk-politik

The aggregate quantities relevant for further analysis are

$$\begin{aligned} A_t^{hh} &= \int a_{it} d\boldsymbol{D}_t \\ C_t^{hh} &= \int c_{it} d\boldsymbol{D}_t \\ L_t^{hh} &= \int \ell_{it} z_{it} d\boldsymbol{D}_t \end{aligned} \tag{3}$$

Firms: The representative firm follows a Cobb-Douglas production function and rents capital, K_{t-1} and hires labour L_t^Y

$$Y_t = \Gamma^Y K_{t-1}^{\alpha} \left(L_t^Y\right)^{1-\alpha} \tag{4}$$

Where Γ^Y is TFP and α is the Cobb-Douglas weight parameter on capital. As capital depreciates with $\delta \in (0,1)$, the real rental price of capital, r_t^K, is equal to $r_t^K \equiv r_t + \delta$ and law of motion of capital $K_t = (1-\delta)K_{t-1} + I_t$. With wages, w_t, I can write up the firm's profits as $\Pi_t = Y_t - w_t L_t^Y - r_t^K K_{t-1}$.

Government: The government purchases goods G_t and hires labour L_t^G to produce government services such that

$$S_t = \min\{G_t, \Gamma^G L_t^G\} \tag{5}$$

The government runs a balanced budget such that

$$\begin{aligned} G_t + w_t L_t^G + \chi_t &= \int \tau_{1t} w_t z_{it} \ell_{it} + \mathbf{1}_{w_t z_{it} \ell_{it} > \underline{\Theta}} \tau_{2t} (w_t z_{it} \ell_{it} - \underline{\Theta}) d\boldsymbol{D}_t \\ &= \tau_{1t} w_t L^{hh} + \tau_{2t} \int \mathbf{1}_{w_t z_{it} \ell_{it} > \underline{\Theta}} (w_t z_{it} \ell_{it} - \underline{\Theta}) d\boldsymbol{D}_t \end{aligned} \tag{6}$$

Market Clearing: The market clearing implies that Asset Market Clears: $K_t = A_t^{hh}$ Labour Market Clears: $L_t^Y + L_t^G = L_t^{hh}$ Goods Market Clears: $Y_t = C_t^{hh} + I_t + G_t$

Conducting Analysis

With my analysis, the question I want to answer is

> *How do taxpayers behave around kink point, and can we use this to predict how Danish taxpayers will bunch around the new tax brackets in the proposed tax system?*

This question is of interest with the recent talks of reforming income taxation in Denmark. Intuitively the expected behaviour would be that people just above the upper bracket limit choose to lower the number of worked hours, as it effectively reduces the benefit from working – as the level of services, S_t, is taken for given. However, at some point, people would earn too much for it to be beneficial to reduce the number of worked hours

such that they earn below the tax bracket, but instead, they would reduce the number of hours worked as the substitution effect of a higher marginal tax rate kicks in. Thus, there would be a spike in the number of people working just below the upper tax bracket limit, followed by a drop just after, and then the people earning more would be expected to reduce their hours compared to the old equilibrium.

The new proposed income tax system in Denmark would consist of multiple tax brackets; thus, this bunching would happen at multiple income levels, reducing the collected tax amount. To examine the extent of this, I would rely on the following procedure:

- First, I solve the model where $\tau_{2t} = 0$, essentially the baseline model from assignment 2 for some given parameter values.
- From the initial calibration of the model, I would set the appropriate level for $\underline{\Theta}$, to where 50% of the population is expected to pay the upper bracket tax – assuming no change in behaviour.
- With $\underline{\Theta}$ set exogenously in the model, I would then solve the model again for $\tau_{2t} \neq 0$, thereby having an upper-tax bracket in the model. This would be done at several different levels of τ_{2t}
- After having re-run the model, I would subsequently plot the income distribution for each different level of τ_{2t} to determine the amount of bunching for given levels of taxation.
- From here, I would quantitatively estimate the elasticity of taxable income in the upper bracket relative to the tax increase in the upper bracket. This could subsequently be used to predict taxpayer behaviour at different tax brackets.

Discussion and Potential Difficulties

This model is a simplification assuming that households can change their income only by adjusting the hours they work, which is not necessarily true, especially for those earning above DKK 2,500,000. The Danish debate has mainly circled around whether households in this bracket can adjust their salary structure to pay lower capital gains tax instead of income tax. If households can indeed modify their salary structure, then it is expected that everyone above the threshold would do so. However, if most households in the "upper-upper" bracket can do this, then this model would only apply to lower tax brackets, where diverting income towards dividends would not provide the same benefits or simply not be feasible.

Additionally, this analysis includes a model with kinks. This could create issues as the function is no longer differentiable and also requires numerical integration, making it more cumbersome to solve.