UNIVERSITY OF COPENHAGEN

DEPARTMENT OF ECONOMICS

Assignment I | HANC with ex-ante Heterogeneity

Advanced Macroeconomics: Heterogeneous Agent Models

Johan Ølgaard, jlh601

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a) Setup

In equilibrium along the transition path, capital, labour, and prices are derived from the utility maximisation of households and profit-maximising firms clearing the markets. The model has a market for capital, goods, and two types of labour, high- and low-skilled, with 3 different levels of patience drawn from predetermined probabilities. The transition path is defined as

$$\mathbf{0} = \begin{bmatrix} r_t^K - F'_K(\Gamma_t, K_{t-1}, L_{0,t}, L_{1,t}) \\ w_t^j - F'_{Lj}(\Gamma_t, K_{t-1}, L_t^0, L_t^1) \text{ for } j \in \{0,1\} \\ r_t - (r_t^K - \delta) \\ A_t - K_t \\ \boldsymbol{D}_t - \Pi'_z \underline{\boldsymbol{D}}_t \\ \underline{\boldsymbol{D}}_{t+1} - \Lambda'_t \boldsymbol{D}_t \\ A_t - A_t^{hh} \\ L_t^j - L_t^{j,hh} \text{ for } j \in \{0,1\} \\ \forall\ t \in 0,1,\ldots \text{ given } \underline{\boldsymbol{D}}_0 \end{bmatrix} \tag{1}$$

The stationary equilibrium can subsequently be defined as when the variables are constant over time, but households still move around in the distribution as a result of stochastic shocks to income.

The model is set up with a household block, a production block, a market-clearing block, and households, taking the inputs ϕ_0, ϕ_1 and Γ as illustrated in the DAG below

Figure 1: DAG of the model

b) Stationary equilibrium

I solve for the stationary equilibrium and get

	ss
Y_{ss}	1.2831
I_{ss}	0.4110
K_{ss}	4.1097
r^K_{ss}	0.1124
L^0_{ss}	0.6667
L^1_{ss}	0.6667
w^0_{ss}	0.6159
w^1_{ss}	0.6159

Table 1: Values in the stationary equilibrium for $\phi_0 = 1$ and $\phi_1 = 2$. NB. labour supply is measured in effective terms. As households of type 1 are twice as effective, they supply the same effective amount of labour though they only are half the number

I find that as types $\chi = 0$ and $\chi = 1$ provide effectively the same labour, their wage effective wage is equivalent, $w_0 = w_1 = 0.6159$. However, types of χ_1 are twice as productive, and I find that their salary is twice as high. In this model, the different levels of productivity are, therefore, a major reason for wealth inequality, as reflected by the graphs below.

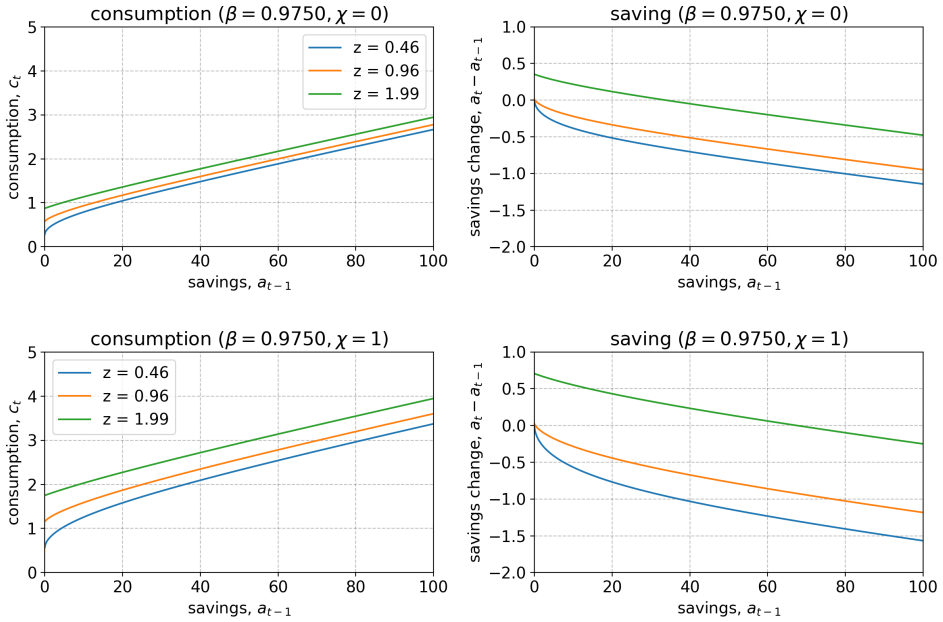

Figure 2: Household behaviour for different χ

In figure 2, I find that the more productive households, ($\chi = 1$), will always consume more than the less productive households, given equal savings. Further, it is seen how the less productive households are less inclined to change their savings, as they know they are less productive, resulting in flatter curves in the savings change diagrams, making them more likely to hold on to their wealth.

Another part that plays a role in wealth inequality is the discount factor of the households, β.

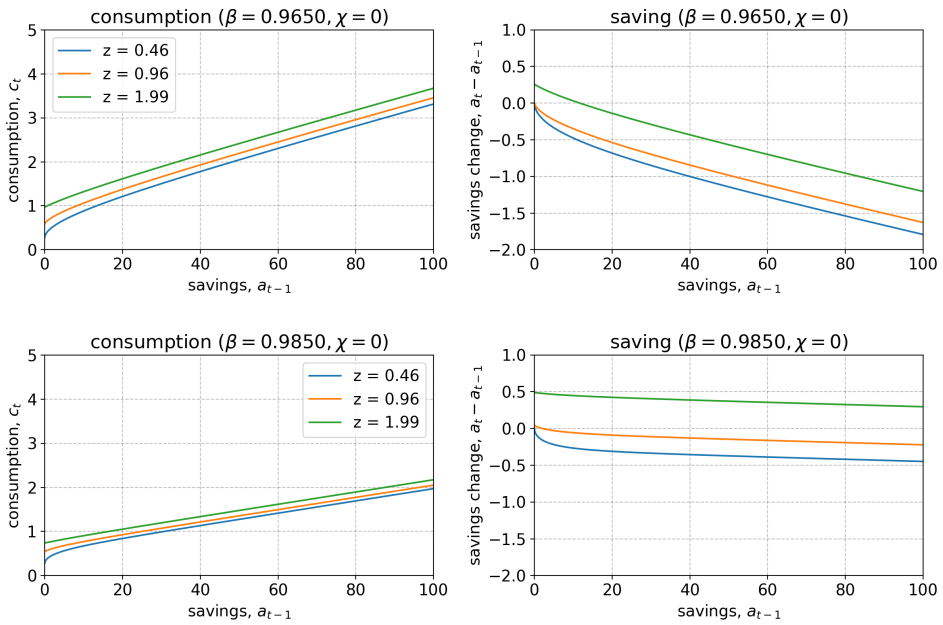

Figure 3: Household behaviour for different β. NB. similar behaviour is seen for $\chi = 1$

In figure 3, it is apparent that when β increases, households become more patient and thus less likely to spend and also more likely not to spend their savings. A large gap between preferences will, therefore, lead to more wealth inequality as the impatient will spend a larger share of their money and savings whilst the patient, to a larger degree, will hold on to their money, increasing wealth inequality.

Looking at the distribution of productivity and asset distribution for each type of household, I find a similar picture.

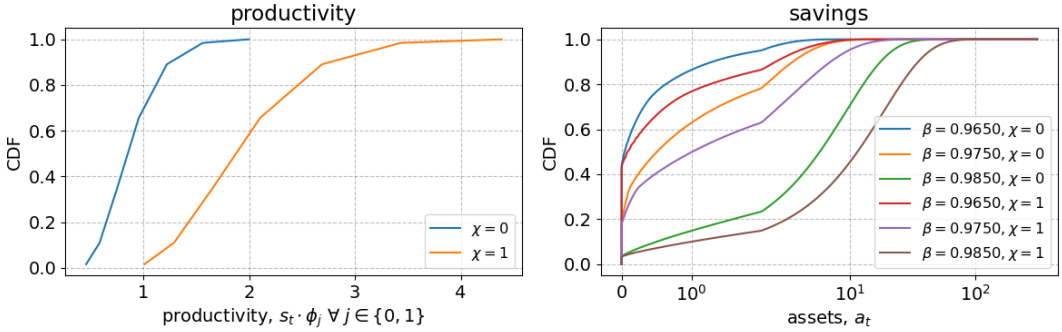

Figure 4: Distribution of productivity and assets

As the underlying stochastic process for productivity is identical, I multiply the stochastic productivity with the individual ability. When doing so, I find that the least productive households of high ability, $\chi = 1$, are still as productive as the 70^{th} percentile of households of low ability, $\chi = 0$. Looking at the asset distribution in figure 4, I in find, that households with higher abilities tend to save more than those with lower abilities. However, the most significant factor that affects wealth inequality is patience. Households that are more patient tend to have much higher savings than those with lower patience levels. Nevertheless, due to productivity shocks that vary from household to household, there is a

chance that a high-ability and high-patience household could be less wealthy than other households but, it is more likely that such a household is more wealthy.

c) Jacobians wrt. ϕ_1

The jacobians of the household block with respect to variable ϕ_1 represent the changes in consumption and savings when a shock in ϕ_1 occurs. This effect is calculated on an aggregate level, without differentiating between types of households. I have calculated the effect of four shocks occurring at periods 0, 50, 150, and 250, which can be seen below.

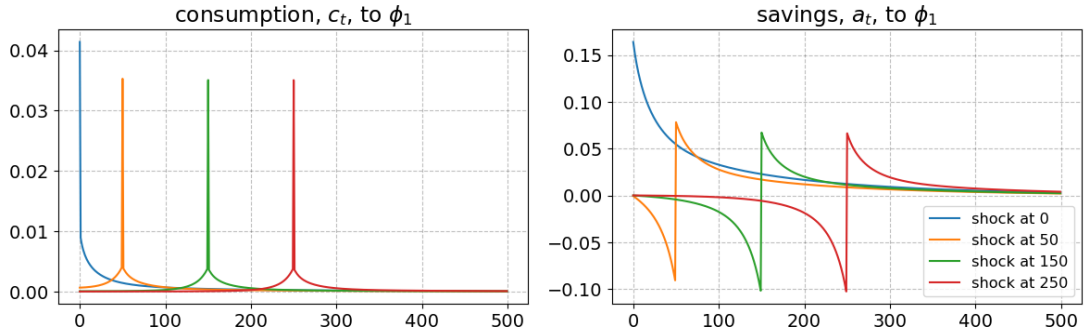

Figure 5: Aggregate consumption and savings as a result of shocks to ϕ_1

An increase in productivity, though just for 1 type of household, increases the overall supply of effective labour, in turn driving up production and therefore also demand for the other type of labour, increasing wages for all.

In anticipation of a future shock, I find that households reduce their savings and increase consumption, knowing that their income will rise with the approaching shock. Unsurprisingly, when the shock is not anticipated, as with the shock in period 0, households cannot increase consumption beforehand, and the effect of the shock thus drives up consumption even further in the period and also leads to higher levels of saving.

d) Transition path when ϕ_1 is 10 pct. higher for 10 periods

In the case of a temporary increase of 10% in ϕ_1, I find that the wage per effective labour for households of type $\chi = 1$ is lower than its steady state value, whilst the wage for households of type $\chi = 0$ is above its steady state, as seen in figure 6.

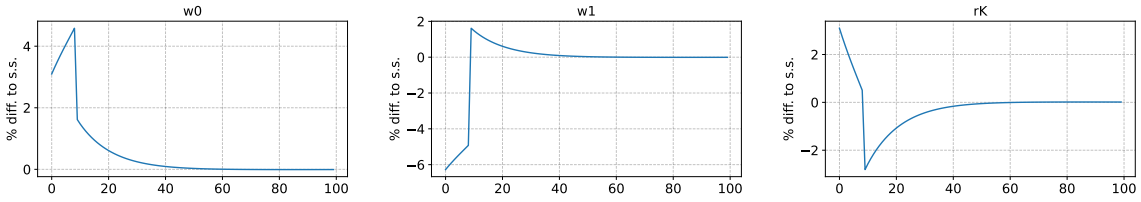

Figure 6: Effects of a temporary shock on wages and interest rates

As households of type $\chi = 1$ become more productive, the demand for their labour falls – due to diminishing marginal returns – while increasing the demand for labour from households of type $\chi = 0$, driving their wage up. Thus, counterintuitively, the increased

productivity actually benefits the less productive households. However, it should be noted that w_0 and w_1 denote the wage per effective labour – the wage actually paid out to households of type $\chi = 1$ is not lower than in its steady state; as the 10 % increase in productivity, more than make up for the wage being up to 6 % lower than the steady state wage. Households of type $\chi = 1$ just do not reap all the benefits of being more productive.

Furthermore, it is also apparent how the increased effective labour supply results in higher demand for capital, pushing up the interest rate in turn increasing demand for both types of labour.

Looking at savings, consumption, and utility for households of high and low ability, I find that during such a temporary shock, both types unsurprisingly increase their savings and their consumption, as seen in figure 7.

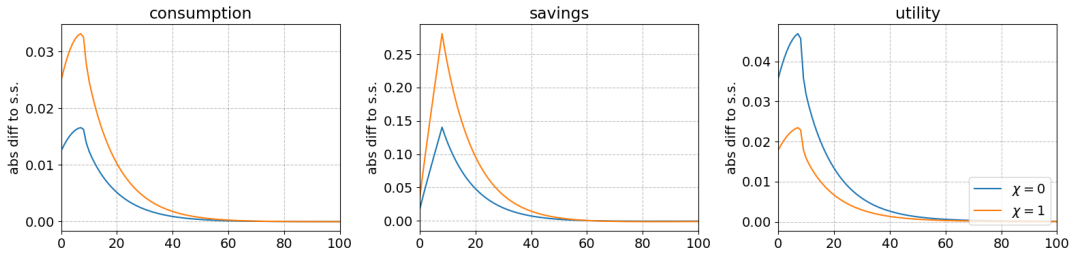

Figure 7: Household consumption and savings behaviour χ under a temporary shock

What is slightly surprising is that though the high-ability households increase their consumption the most absolutely, they experience the least increase in utility due to diminishing marginal utility. Thus, it is the low-ability households that experience the highest increase in utility during the period the shock is in effect, though not surpassing the utility of high-ability households. Additionally, high-ability households tend to save more, leading to an increase in wealth inequality in society. However, this effect is temporary, and savings slowly drop back to their normal levels once the shock disappears.

e) Transition path when ϕ_1 is permanently 10 pct. higher

In the case of a permanent increase of 10% in ϕ_1 the new steady state is in

	Old ss	New ss
Y_{ss}	1.2831	1.3458
I_{ss}	0.4110	0.4310
K_{ss}	4.1097	4.3103
r^K_{ss}	0.1124	0.1124
L^0_{ss}	0.6667	0.6667
L^1_{ss}	0.6667	0.7333
w^0_{ss}	0.6159	0.6460
w^1_{ss}	0.6159	0.5872

Table 2: Values in the new stationary equilibrium for $\phi_0 = 1$ and $\phi_1 = 2.2$

In the new steady state, I find that wages have changed to $w_0 = 0.6460$ and $w_1 = 0.5872$. Thus, I see an increase in the wage per effective labour for households of type 0, whilst

the wage per effective labour for households of type 2 decreases. This is again due to an increase in demand for labour of type 0 as explained in d).

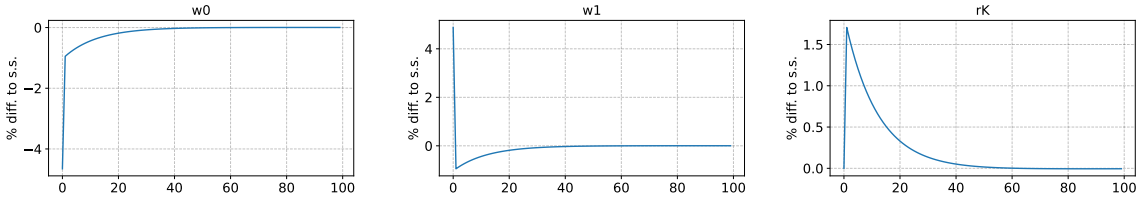

Figure 8: Household behaviour for different β. NB. similar behaviour is seen for $\chi = 1$

Looking at the transition towards the new equilibrium in the wage level, this transition is almost immediate, with wages in period 2 being of equilibrium with less than 0.5%. However, as seen from table 2, the low-ability households do not increase their amount of effective labour delivered, whilst the high-ability households do make up for the reduction in wages.

Thus, when I look at consumption and savings, I find that, as in figure 7, both households of high and low ability increase both consumption and savings.

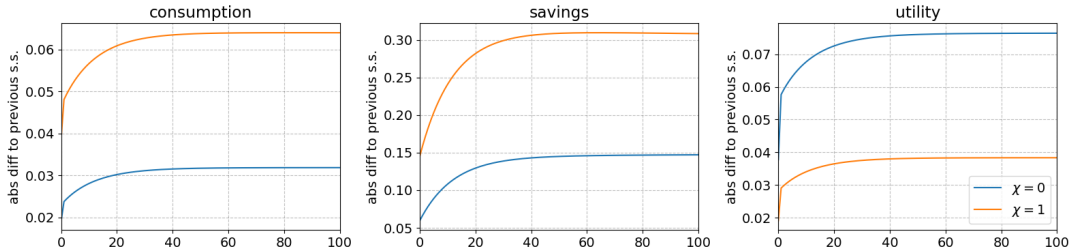

Figure 9: Household consumption and savings behaviour χ under a permanent shock, compared to previous steady-state values

Again as in figure 7, figure 9 shows that though the consumption increases the most for households of high ability, it is the ones of low ability experiencing the highest increase in utility due to the diminishing marginal utility. Further, the savings of high-ability households increase the most, thereby increasing the overall wealth inequality in society. Unlike the temporary rise in wealth inequality shown in Figure 7, this shock is permanent, which results in a long-term increase in wealth inequality.