Financial Economics | Problem Set 2

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1 Puts and Calls

(a) Finding Arbitrage on Microsoft

The put-call-parity states that the price of a call plus the discounted strike price is equal to the price of the put and the current price of the stock, i.e.

$$C_0 + \frac{X}{1 + r_f} = P_0 + S_0 \tag{1}$$

Under the give assumption of $r_f = 0$ this simplifies into

$$C_0 + X = P_0 + S_0 \tag{2}$$

As we know, the price is $S_0 = \$242.71$, we can fill out the table with information from the assignment

	C_0	X	P_0	S_0	Deviation
\$235	\$12.20	\$235.00	\$5.00	\$242.71	−\$0.51
\$240	\$9.60	\$240.00	\$6.75	\$242.71	\$0.14
\$245	\$7.00	\$245.00	\$7.95	\$242.71	\$1.34
\$250	\$4.95	\$250.00	\$10.05	\$242.71	\$2.19

Table 1: Put-Call parity and deviations. Deviations defined as Deviation $= C_0 + X - (P_0 + S_0)$

We find that for the strike prices \$235 and \$240, the put-call parity almost holds with a small deviation of \$0.51 and \$0.14, respectively – including trading costs will most likely make the parity hold. With the strikes of \$245 and \$250, however, we find a rather large deviation from the put-call parity of \$1.34 and \$2.19, respectively and thus, with low enough transaction costs, there could be arbitrage.

To exploit the potential arbitrage found for the strike of \$245, we would go long in the put and the MS stock while going short in the call and the bond. This would give us 0 exposure in the future but net us \$1.34 today, assuming no transaction costs.

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(b) Using Bid and Ask Prices

Using the bid price does not make sense as this is what people in the market currently are willing to buy the call and put contracts for. As we want to go long in the put, we would need to buy it at the asking price as no one is willing to sell it at the given time for a lower price; however, to short the call, we could still sell at the bid as people in the market are willing to buy at this price – hence underbidding the other market makers. The new put-call parity would, therefore, be

$$\$7.00 + \$245.00 = \$9.30 + \$242.71 \Leftrightarrow \text{Deviation} = -\$0.01 \tag{3}$$

Under these new, more realistic pricing assumptions, we no longer find arbitrage in the market.

(c) Looking if the Put-Call Parity holds for Lockheed Martin

Firstly, we will point out that no options expire on the *March 10, 2024* as it is a Sunday, not a Friday. We have chosen to look at options expiring on *March 8, 2024* instead.

We will look at the American aerospace, arms, and defense company Lockheed Martin Corporation (FMT) registered on the New York Stock Exchange (NYSE). As of market close on Monday, February 12, 2024, one share of FMT traded at \$428.07, and the nearest options with expiration on March 8$^{\text{th}}$ are with strike prices \$425.00 and \$430.00. Using the same calculations as in (a), we can calculate the deviation from the put-call parity.

	C_0	X	P_0	S_0	Deviation	Download Time
\$425	\$8.10	\$425.00	\$5.30	\$428.07	-\$0.27	2024-02-12 19:40:41
\$430	5.20	\$430.00	\$8.00	\$428.07	-\$0.87	2024-02-12 19:40:41

Table 2: Put-Call Parity for LMT with expiration date 2024-03-08

We find a small deviation from the put-call parity, in both cases buying the put and the stock is slightly more expensive than buying the call and a bond with value equal to the strike price. Thus an arbitrage trade would be to short the put and the stock while buying the call and the bond to pocket either \$0.27 – when using the strike of $X = \$425$ - or \$0.87 – when trading with the strike of $X = \$430$ today with zero exposure in the future.

In tables 3 and 4 the underlying data for our calculation can be found – all pulled via the python package yfinance on 2024-02-12 at 19:40:41.

contractSymbol	lastTradeDate	strike	lastPrice	bid	ask	volume
LMT240308C00425000	2024-02-12	\$425.00	\$9.30	\$8.10	\$8.60	4.00
LMT240308C00430000	2024-02-12	\$430.00	\$5.90	\$5.20	\$5.80	46.00

Table 3: Call Options for LMT with expiration date 2024-03-08

contractSymbol	lastTradeDate	strike	lastPrice	bid	ask	volume
LMT240308P00425000	2024-02-12	\$425.00	\$5.60	\$5.30	\$6.10	5.00
LMT240308P00430000	2024-02-09	\$430.00	\$9.90	\$8.00	\$8.50	2.00

Table 4: Put Options for LMT with expiration date 2024-03-08

2 Options and Risk Management

(a) Investing in Stock, Calls, or Bonds

In the first portfolio, as we are only buying the stock, we would buy 250 stocks.

$$\text{Stocks} = \frac{\$10{,}000}{\$40} = 250 \tag{4}$$

The payoff, therefore, is just 250 multiplied by the stock value six months from now, subtracted our initial investment.

$$\text{Payoff}_{(1)} = S_t \cdot 250 - \$10{,}000 \tag{5}$$

For the second portfolio, we can buy 10,000 call options at the price of \$1, and the payoff of this portfolio will, therefore, be

$$\text{Payoff}_{(2)} = \max\left(S_t - X{,}0\right) \cdot 10{,}000 - \$10{,}000 \tag{6}$$

For the third portfolio, we would buy 100 call options and put the remaining \$9,900 in the bank account, earning $r_f = 1\%$. The payoff, therefore, is

$$\text{Payoff}_{(3)} = \max\left(S_t - X{,}0\right) \cdot 100 + \$9{,}900 \cdot (1 + 0.01) - \$10{,}000 \tag{7}$$

	(1)	(2)	(3)
\$ 30	−\$2,500	−\$10,000	−\$1
\$ 40	\$0	−\$10,000	−\$1
\$ 50	\$2,500	\$90,000	\$999
\$ 60	\$5,000	\$190,000	\$1,999

Table 5: Payoffs of the three different portfolios

From 5 portfolio 2 seems to be the riskiest among the options, given the substantial gap between its lowest payoff of -\$10,000 (occurring at a price of \$30 and \$40) and its highest payoff of \$190,000 (at a price of \$60). This difference of \$200,000, depending on the direction of the price movement, is significantly greater than in any other portfolio, indicating higher inherent risk.

Portfolio 3 appears to be the safest because again the highest payoff is \$1,999 and the lowest is only -\$1. The difference between the two payoff extremes is a lot lower than the other portfolio meaning it is the most safe.

Though portfolio 2 seem to be the riskiest we still believe it reinforces the notion that call options are often perceived as "safe." When we use the term "safe," we are referring to the idea that call options come with limited losses, as evident in portfolio 2 where the losses are capped at -\$10,000. On the flip side, they offer unlimited gains, demonstrated by the increasing profits as the price rises. Thus, relative to the upside the potential loss is still small. While call options may not be considered as safe as other assets, such as risk-free government securities, they can be deemed relatively safe.

Call options can be employed to reduce the risk of a position, as they limit losses more effectively than simply purchasing the stock outright, while also offering unlimited gains. Hypothetically, by purchasing around $\sim$ 250 additional options, you could generate similar payoffs to owning the stock on the higher payoff side, while incurring lower losses on the downside.

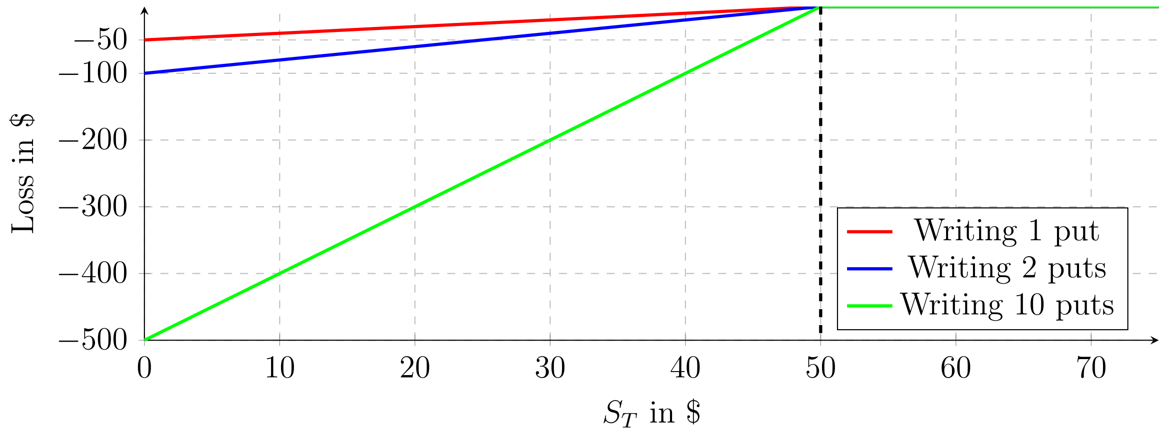
(b) Payoff when Writing a Put

When writing, we assume that the buyer of the set put will exercise only when they stand to gain something from it and thus will exercise when $S_T < X$. The payoff of writing a single put, therefore, is

$$\text{Pay off}_{\text{Writing put}} = -\max(X - S_T, 0) \tag{8}$$

Using this, we can plot the payoff profile from writing 1, 2 and 10 puts.

Figure 1: Payoff profile from writing puts at strike $X = \$50$



From figure 1, we find that the slope increases linearly in the number of puts written such that the slope can be described with the formula Slope = Puts written.

Using the formula from (8) we can calculate the loss if the price at T is \$1 below the strike as

$$\text{Loss} = -\max(\$40 - \$39, 0) = \$1 \tag{9}$$

Thus, the loss is \$1 per written option, and for 10 options written, the loss would therefore be \$10.

3 Arbitrage and State Prices

(a) Payoff of an Arbitrary Portfolio in Vitamin Co. Stocks and Bonds

We can express the payoff of a portfolio consisting of φ_s stocks and φ_b bonds as seen below in table 6.

	Vitamin Stock	Vitamin Bond	Portfolio Payoff
State 1	7	10	$7\varphi_s + 10\varphi_b$
State 2	3	10	$3\varphi_s + 10\varphi_b$
Price today	4	9	$4\varphi_s + 9\varphi_b$

Table 6: Payoff in different states for a portfolio of stocks and bonds

(b) Calculating the Price of an Arrow-Debreu Security for State 1

We start by setting up the equation system for the state 1 Arrow-Debreu security:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 3 & 10 \end{bmatrix} \cdot \begin{bmatrix} \varphi_s \\ \varphi_b \end{bmatrix} = \begin{bmatrix} 7\varphi_s & 10\varphi_b \\ 3\varphi_s & 10\varphi_b \end{bmatrix} \tag{10}$$

We can solve this system by subtracting the second row from the first, getting

$$
\begin{aligned}
1 - 0 &= 7\varphi_s - 3\varphi_s + 10\varphi_b - 10\varphi_b \\
&= 4\varphi_s \Leftrightarrow \\
\varphi_s &= \frac{1}{4} = 0.25 \\
\rightarrow 0 &= 3 \cdot \frac{1}{4} + 10\varphi_b \Leftrightarrow \\
\varphi_b &= -\frac{\frac{3}{4}}{10} = -\frac{3}{40} = -0.075
\end{aligned}
\tag{11}
$$

The price of the Arrow Debreau security for state 1, therefore, is

$$
\text{Price}_{\text{AD1}} = 4 \cdot 0.25 - 9 \cdot 0.075 = 0.325 \tag{12}
$$

(c) Calculating the Price of an Arrow-Debreu Security for State 2

Equal to in (b) we again can start by setting up the equation system:

$$
\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 3 & 10 \end{bmatrix} \cdot \begin{bmatrix} \varphi_s \\ \varphi_b \end{bmatrix} = \begin{bmatrix} 7\varphi_s & 10\varphi_b \\ 3\varphi_s & 10\varphi_b \end{bmatrix} \tag{13}
$$

Now, we subtract the second row from the first to solve the system

$$
\begin{aligned}
0 - 1 &= 7\varphi_s - 3\varphi_s + 10\varphi_b - 10\varphi_b \\
&= 4\varphi_s \Leftrightarrow \\
\varphi_s &= -\frac{1}{4} = -0.25 \\
\rightarrow 0 &= -7 \cdot \frac{1}{4} + 10\varphi_b \Leftrightarrow \\
\varphi_b &= \frac{\frac{7}{4}}{10} = \frac{7}{40} = 0.175
\end{aligned}
\tag{14}
$$

Now, we can find the price of the Arrow-Debreu in state 2.

$$
\text{Price}_{\text{AD2}} = -4 \cdot 0.25 + 9 \cdot 0.175 = 0.575 \tag{15}
$$

As both the price found in (b) for AD1 and the price found for AD2 are strictly positive, we know that the market is complete and there is no arbitrage.

(d) Calculating the Net Simple Return in the Economy

The risk-free rate can be found both by looking at the price of the two Arrow-Debreu securities as well as just looking at the price of the Vitamin Co. Bond. A tenth of this bond will be equal to buying the two Arrow-Debreu securities. We will find the net simple rate of return by looking at the Vitamin Co. Bond

$$
r_f = \frac{10}{9} - 1 = 11.11\% \tag{16}
$$

(e) Payoff of a European Call with Strike $X = 5$

The payoff of a European call option can be described by

$$\text{Payoff}_{\text{Call}} = \max\left(S_T - X{,}0\right) \tag{17}$$

With this we can calculate the payoffs in both states as seen in table 7

	Vitamin Stock	Vitamin Bond	AD1	AD2	Euro-call option with $X = 5$
State 1	7	10	1	0	$\max\left(7 - 5{,}0\right) = 2$
State 2	3	10	0	1	$\max\left(3 - 5{,}0\right) = 0$
Price today	4	9	0.325	0.575	0.650

Table 7: Payoff and prices in different states for stocks, bonds, AD securities and options

(f) Calculating the Price of a European Call with Strike $X = 5$

As seen from table 7, the payoff is equal to two AD1 securities, and therefore, for LOOP to hold, the price must be double that of an AD1 security:

$$\text{Price}_{\text{Euro-call option with } X\,=\,5} = 2 \cdot 0.325 = 0.650 \tag{18}$$

(g) Calculating the Price of a European Put with Strike $X = 5$

Looking at table 7, we can redo the table now with a put option instead, where the payoff is:

$$\text{Payoff}_{\text{Put}} = \max\left(X - S_T{,}0\right) \tag{19}$$

	Vitamin Stock	Vitamin Bond	AD1	AD2	Euro-put option with $X = 5$
State 1	7	10	1	0	$\max\left(5 - 7{,}0\right) = 0$
State 2	3	10	0	1	$\max\left(5 - 3{,}0\right) = 2$
Price today	4	9	0.325	0.575	1.150

Table 8: Payoff and prices in different states for stocks, bonds, AD securities and options

We see the payoff is equal to two AD2 securities, and the price must, therefore, also be equal to two AD2 securities for LOOP to hold

$$\text{Price}_{\text{Euro-put option with } X\,=\,5} = 2 \cdot 0.575 = 1.150 \tag{20}$$

4 Dynamic Trading

(a) Calculating the Price of the Call Option

We can draw the payoff of the option as

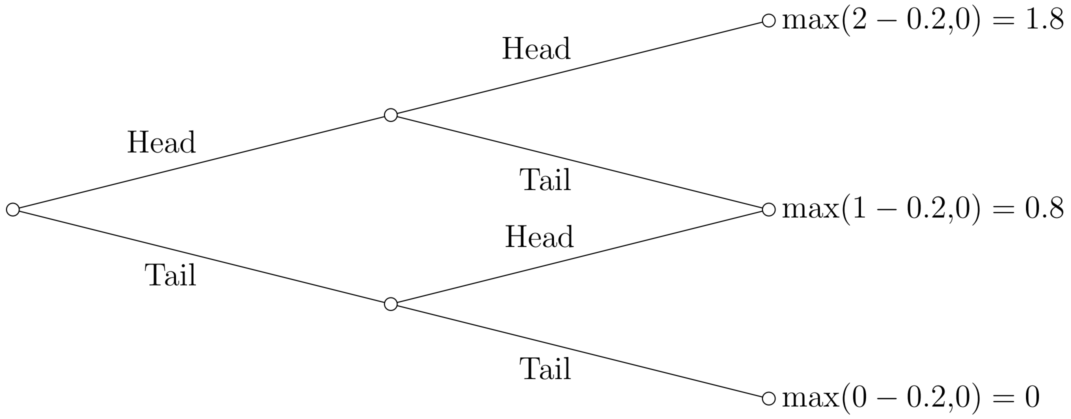


Figure 2: Payoff for the call option on heads with strike at $X = 0.2$

The other two securities in the economy are as described in the lecture

		Asset H	Asset T
Outcome	Heads	1	0
	Tail	0	1
	Price	0.5	0.5

Table 9: Assets other than the call options and their payoff in the economy

Additionally, as seen in the lecture, we can construct 3 AD securities

	AD1	AD2	AD3	Option
State 1 (HH)	1	0	0	1.8
State 2 (HT)	0	1	0	0.8
State 3 (TT)	0	0	1	0
Price	0.25	0.5	0.25	0.85

Table 10: Arrow-Debreu Securities in the economy as seen in the lecture and the option

We have additionally added the option that we can see replicates the payoff of 1.8 of the AD1 security and 0.8 of the AD2 security. The LOOP price of the option must, therefore, be $1.8 \cdot 0.25 + 0.8 \cdot 0.5 = 0.85$.

(b) Creating a Portfolio Replicating the Payoff of the Option

To find a self-financing strategy with similar pay-offs, we will use backwards induction.

We can update the binomial tree with the amount of each asset we need to buy of each asset at each node.

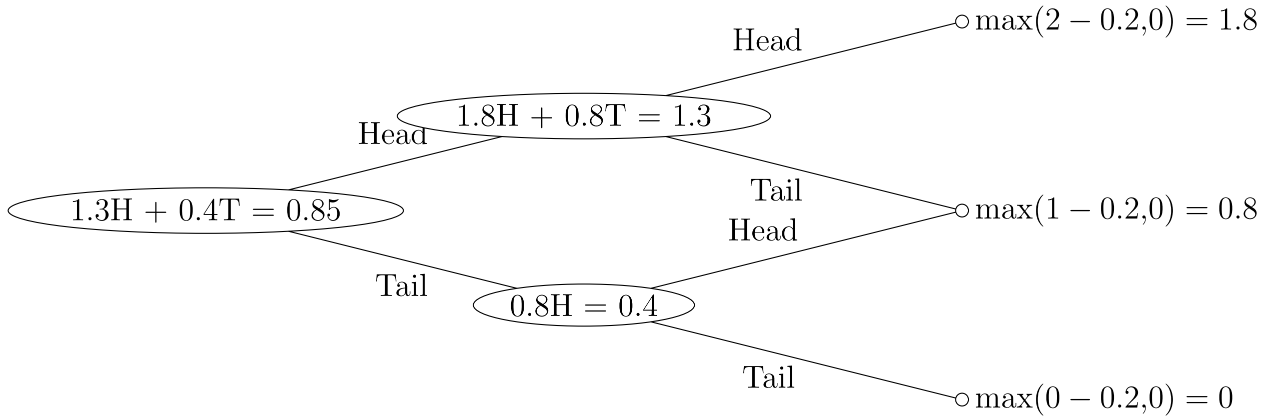


Figure 3: Payoff for the call option on heads with strike at $X = 0.2$

If the first flip is head, we would need to buy 1.8 of the head asset and 0.8 of the tail asset – costing us $1.8 \cdot 0.5 + 0.8 \cdot 0.5 = 1.3$. If the first flip is tails, we only need to buy 0.8 of the head asset – costing $0.8 \cdot 0.5 = 0.4$. Thus, to have a self-financing portfolio, we would need to get a payoff of 1.3 if the first round result is heads and 0.4 if the first round result is tails – i.e., a portfolio consisting of $1.3H + 0.4T$ at the price $1.3 \cdot 0.5 + 0.4 \cdot 0.5 = 0.85$. Thus, we find that the self-financing strategy costs 0.85, exactly as we found the option price to be in (a).

(c) The Effect of Outcome Probability on Option Prices

As the price of the underlying assets is not updated, the option price will not be affected. The probabilities in this problem are only implicitly assumed through the pricing of our original assets, but the LOOP holds no matter the probabilities and is only dependent on the price of the underlying asset. However, the asset prices with the updated coin are no longer risk-neutral, and thus, there is money to be made in the market if we are able to repeat the investment multiple times.