Financial Economics | Problem Set 4

Johan Oelgaard† and Rhys Blackmore‡

March 18th, 2024

1 Bank runs

(a) When Investors Keep their Money in Toxic Limited

If all investors keep their money until $t = 2$, the return on the CMOs will be

$$R = \frac{\$210}{\$200} - 1 = 5.00\% \tag{1}$$

Thus, the investors can indeed make 5% on their investment if all investors keep their money in the bank.

If 199 investors keep the money in the bank, Toxic Limited will not have any solvency issues. Thus, it will be rational for the 200th investor also to keep their money in the bank as we assume that 5% is high enough that it is optimal to keep the money in the bank as long as there is no fear of solvency issues.

(b) 180 other investors want to withdraw

If we know 180 other investors will withdraw their money from Toxic Limited at $t = 1$, Toxic Limited would have to liquidate their CMO position at the current market price of \$160. As the CMO position is liquidated and 180 investors choose to withdraw their money, there would no longer be any money in Toxic Limited to pay its remaining investors at $t = 2$, leading to a loss of all money for remaining investors, i.e. $R|180 \text{ investors withdraw} = -100\%$. If we instead choose to withdraw the money together with the 180 other investors, we would share the \$180 Toxic Limited would have. Depending on how many of the remaining 20 investors also choose to withdraw, our return will be in the range $[-0.55\%, -10\%]$.

$$\begin{aligned} R|181 \text{ investors withdraw} &= \frac{180}{181} - 1 = -0.55\% \\ R|200 \text{ investors withdraw} &= \frac{180}{200} - 1 = -10.00\% \end{aligned} \tag{2}$$

As it is rational for all investors to withdraw, this situation would indeed lead to a bank run, resulting in all investors losing -10%.

†Student ID: 3039806344
‡Student ID: 3039811141

(c) Intervention from a wealthy investor

If an investor injects additional capital of \$250, this could be used to pay off the \$180, leaving the CMO position untouched, and thus, Toxic Limited would still be able to honour its promised 5% return. It is, therefore, not optimal to withdraw the money in this case, and as an investor stepped in, the 180 other investors should have reestablished their faith in the bank and should have no incentive to withdraw their money. We, therefore, do not expect a bank run in this situation.

(d) Changing market conditions

Assuming investors are not able to coordinate, this scenario essentially boils down to a prisoner's dilemma with 200 players. If all players keep their money in the bank, the payoff would be

$$R = \frac{\$175 + \$20}{200} - 1 = -2.50\% \tag{3}$$

A loss, yes, but a smaller one than the -10% we expected to see in (b) due to the bank run. However, if they could withdraw their money at $t = 1$ while the others would not, their one would not lose any money. In an n-person prisoner's dilemma, we can describe the individual investor's payoff.

		Number of other investors withdrawing							
		0	1	19	20	100	179	180	199
Our choice	Keep	-2.5%	-2.5%	-2.5%	-2.5%	-12.5%	-94.8%	-100%	-100%
	Withdraw	0%	0%	0%	0%	0%	0%	-0.6%	-10%

Table 1: This shows only our return and assumes that the CMO position can be liquidated in smaller portions to accommodate an odd number of investors withdrawing. However, the same intuition holds if the position would be liquidated as a whole.

As seen in table 1, it is always optimal for the investor to withdraw in this situation no matter what the other investors do. Thus, the Nash Equilibrium in this hypothetical n-person prisoner's dilemma would be for all investors to withdraw their money. We would, therefore, expect a bank run to occur in this scenario. We do further not expect an investor to want to step in as Toxic Limited is insolvent in $t = 2$ even if everybody withdraws their money. A wealthy investor would essentially be handing money to the bank to cover its current obligations with no apparent benefit for himself.

2 Stock valuation I

(a) Price of ABC shares

Using Gordon's Growth formula, we can calculate the value of the growing perpetuity of dividends as

$$P_t = \frac{D_{t+1}}{R - g} = \frac{\$30}{10\% - 2\%} = \$375 \tag{4}$$

(b) Updated expected growth

Using the same equation as in (4) we can update the expected growth an recalculate the price as of a share to be

$$P_t = \frac{\$30}{10\% - 3\%} = \$428.57 \tag{5}$$

(c) Earnings and present value of growth opportunities per share

With the expected dividend and the retention ratio, we can back out the expected earnings per share next year as

$$EPS_{t+1} = \frac{D_{t+1}}{1-b} = \frac{\$30}{1-25\%} = \$40 \tag{6}$$

The present value of growth of this can be calculated as

$$PVGO = E_{t+1} \cdot \left[\frac{b \cdot (ROE - R)}{R \cdot (R - ROE \cdot b)}\right] = \$40 \cdot \left[\frac{25\% \cdot (8\% - 10\%)}{10\% \cdot (10\% - 8\% \cdot 25\%)}\right] = -\$25 \tag{7}$$

Thus we find a negative present value of growth per share indicating that the company could increase price by paying out higher dividends.

(d) Increasing the retention ratio

As the retention ration increases so would the growth – thus the new growth would be

$$g = 8\% \cdot 50\% = 4\% \tag{8}$$

Input this into the Gordon Growth model of (4) gives

$$P_t = \frac{\$40 \cdot (1 - 50\%)}{10\% - 4\%} = \$333.33 \tag{9}$$

If we knew this was the optimal price of the stock given a future payout policy we could profit from this either by buying puts with a strike above this price or we could go and directly short the stock.

(e) Optimal payout policy

Because $ROE < R$ we should payout everything because our PVGO is negative. This means we should have a retention ratio of $b = 0$. The new maximised price with this new retention ratio is:

$$P_t = \frac{\$40}{10\%} = \$400 \tag{10}$$

3 Stock valuation II

(a) Price of a share in XYZ in year 7

Using the Gordon Growth model from (4) we can calculate the price of a share in period 7 as

$$P_7 = \frac{D_8}{R - g} = \frac{\$20}{6\% - 4\%} = \$1000 \tag{11}$$

(b) Price of a share in XYZ today

As they do not payout anything before period 7 we will just have to discount the price P_7 back to today in order to find the fair price of the share.

$$P_0 = \frac{\$1000}{(1 + 6\%)^7} = \$665.06 \tag{12}$$

(c) Growth in the share price until year 7

We can shuffle (12) around to see that the expected growth is exactly equal to the discount rate

$$P_7 = P_0 \cdot (1+6\%)^7 = P_0 \cdot (1+6\%) \cdot (1+6\%)^6 = P_1 \cdot (1+6\%)^6 \tag{13}$$

We see the price in $t = 1$ is exactly 6% higher than it is in period $t = 0$, which would also be the case for P_2 when comparing to P_1. Thus, the expected increase in price until $t = 7$ is exactly equal to the discount rate as this is the one we used to discount the price with in the first place.

(d) Growth in share price after year 7

We can push (11) one period ahead to get

$$P_8 = \frac{D_9}{R-g} = \frac{D_8 \cdot (1+g)}{R-g} = P_7 \cdot (1+g) = \$1000 \cdot (1+4\%) = \$1040 \tag{14}$$

Thus, the price is no longer expected to increase with the discount factor but will increase with the growth of the dividends. This happens as the firm is now paying out dividends and thus not reinvesting alf their earnings in the company as they did in the firsseven 7 years.

(e)

The investor should not earn a higher expected return after year seven than before year 7. This is because the only thing that has changed is the retention ratio and, implicitly, the dividend payout ratio. The return will be 6% either way; the only change is how that 6% return is distributed between dividends or the change in price. After year 7, 2%-points are from dividend growth, and 4%-points are from price growth. As opposed to before year 7, which is 6% strictly from price growth.

4 Two-stage growth

(a) Price today as a function of the price in the future

The price of a stock today, assuming it does not pay any dividends in the future, can be described as the future price discounted back to todayi.e..e:

$$P_0 = \frac{P_6}{(1+8\%)^6} \tag{15}$$

(b) Earnings growth during the first six years

We know that the earnings (as well as earnings per share when using capital per share) in the next period equals

$$E_{t+1} = ROE \cdot K_t \Leftrightarrow K_t = \frac{E_{t+1}}{ROE} \tag{16}$$

The capital in the following period will be given by

$$K_{t+1} = K_t + b \cdot E_{t+1} \tag{17}$$

We we can, derive the earnings per share in $t + 2$ from $t + 1$.

$$E_{t+2} = ROE \cdot K_{t+1} = ROE \cdot (K_t + b \cdot E_{t+1}) = (1 + b \cdot ROE)E_{t+1} \tag{18}$$

Rewriting this, we can get E_6 as a function of E_1 as

$$E_6 = (1 + b \cdot ROE)E_5 = (1 + b \cdot ROE)^2 E_4 = \cdots = (1 + b \cdot ROE)^6 E_0 \tag{19}$$

As we retain all earnings, we will have $b = 1$ and thus $E_6 = (1 + ROE)^6 E_0$. We can, with this, calculate E_6 as

$$E_6 = (1 + 15\%)^6 \cdot \$60 = \$138.78 \tag{20}$$

We note that we here have assumed that we, by E_6 and E_0, are talking about earning *per share* and not total earnings as we do not know the number of outstanding shares.

(c) Capital stock today and in the future

Using (16) we can derive the capital stock *per share* today as

$$K_0 = \frac{\$60}{15\%} = \$400 \tag{21}$$

We can now expand on (17) to calculate the capital stock in year 6 from the calculated capital stock in year 0

$$\begin{aligned} K_{t+1} &= K_t + b \cdot E_{t+1} = K_t + b \cdot ROE \cdot K_t = (1 + b \cdot ROE)K_t \\ K_{t+2} &= (1 + b \cdot ROE)K_{t+1} = (1 + b \cdot ROE)^2 K_t \\ &\vdots \\ K_6 &= (1 + b \cdot ROE)^6 K_0 = (1 + 15\%)^6 \cdot \$400 = \$925.22 \end{aligned} \tag{22}$$

Again, we note specifically that this is not the *total* capital stock of the company but rather the capital stock *per share* as we do not know the number of outstanding shares.

(d) Earnings in year 7

As the ROE now falls to only 5%, the new earnings per share as

$$E_7 = 5\% \cdot \$925.22 = \$46.26 \tag{23}$$

(e) Dividends in year 7

As their new dividend policy is to pay out everything, we simply have $D_7 = E_7$. This, in turn, also means that the capital stock bill will not increase as $b = 0$, and thus, there will be no growth in the dividends either, i.e., $g = 0$.

(f) Price in year 6

We will calculate the price P_6 from the dividends and growth for yearseven 7 and onwards found in (e).

$$P_6 = \frac{\$46.26}{8\% - 0\%} = \$578.27 \tag{24}$$

(g) Price today

To find the price today, we will discount the price from P_6 found in (f) back to today.

$$P_0 = \frac{\$578.27}{(1 + 8\%)^6} = \$364.41 \tag{25}$$