Financial Economics | Problem Set 3

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1 Present value of car payments

(a) With an annual interest of 5%

To calculate the PV of the offer from Montague Autos, we must discount the monthly payments. To do so, we can write up the following sum

$$
\begin{aligned}
\text{PV} &= \$750 + \sum_{t=1}^{42} \frac{\$250}{(1+5\%)^{\frac{t}{12}}} = \$750 + \frac{\$250}{(1+5\%)^{\frac{1}{12}} - 1}\left(1 - \left(\frac{1}{1+5\%}\right)^{\frac{42}{12}}\right) \\
&= \$750 + \$9{,}632.80 = \$10{,}382.80
\end{aligned} \tag{1}
$$

Here, we find that the discounted value of the payment stream made to Montague Autos is \$10,382.80, and thus, the better deal is to buy the car for \$10,250 today at Capulet Motors as you save $\sim$ \$130 in PV.

(b) With an annual interest of 6%

We use the formula as in (1) but this time using an annual interest rate of 6%.

$$
\begin{aligned}
\text{PV} &= \$750 + \sum_{t=1}^{42} \frac{\$250}{(1+6\%)^{\frac{t}{12}}} = \$750 + \frac{\$250}{(1+6\%)^{\frac{1}{12}} - 1}\left(1 - \left(\frac{1}{1+6\%}\right)^{\frac{42}{12}}\right) \\
&= \$750 + \$9475.49 = 10{,}225.49
\end{aligned} \tag{2}
$$

We now find that the offer from Montague Auto is the better deal, where you will save around $\sim$ \$25 in PV compared to the offer of Capulet Motors.

2 Excel Basics

To calculate the PV of this certain payment stream, we could use a similar approach as in section 1 with the formula

$$
\text{PV} = \sum_{t=1}^{10} \frac{\$100}{(1+10\%)^t} = \frac{\$100}{10\%}\left(1 - \left(\frac{1}{1+10\%}\right)^{10}\right) = \$614.457 \tag{3}
$$

In Excel, we can set up the following table to calculate the same present value

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Period	Payment	Discount Factor	PV of Payment	**Total PV**
1	$100	0.90909	$90.90909	**$614.457**
2	$100	0.82645	$82.64463	
3	$100	0.75131	$75.13148	
4	$100	0.68301	$68.30135	
5	$100	0.62092	$62.09213	
6	$100	0.56447	$56.44739	
7	$100	0.51316	$51.31581	
8	$100	0.46651	$46.65074	
9	$100	0.42410	$42.40976	
10	$100	0.38554	$38.55433	

Table 1: Calculated with 'Brute Force method' in Excel

We can also use the **PV** formula in Excel. Writing **=PV(10%;10;100)** yields

Total PV	$-614.457

Table 2: Calculated with 'Present Value' formula in Excel. NB the value is negative as this is the cash-outflow we are willing to have for this payment stream

3 More Excel

(a) Evaluating payment streams at a 5% interest

To calculate the present value, we will use the Excel formula for this (=-PV(rate;periods;payment;future value;0), NB. we include the '-' in front to make all values positive) for all but (iii). As (iii) is a perpetuity, the value of this will be calculated with the formula:

$$PV_{\text{Perpetuity}} = \frac{\text{payment}}{\text{rate}} \tag{4}$$

We first set up the different values for the different payment streams in a table

	(i)	(ii)	(iii)	(iv)
Payment	$0	$0	$80,000	$175,000
Periods	0	4	Infinite	10
Future Value	$1,200,000	$1,600,000	-	$0

Table 3: Values for the different payment streams to be used in the PV calculation

Using this, we can calculate the values of the payment streams as

(i)	(ii)	(iii)	(iv)
$1,200,000.00	$1,316,323.96	$1,600,000.00	$1,351,303.61

Table 4: Values of the different payment streams at an interest rate of 5%

From table 4 we can see that the most valuable payment stream is the perpetuity in (iii), next is (iv), third in (ii) and the least valuable is (i).

(b) Evaluating payment streams at a 9% interest

We will calculate this in the same way as we did for the interest rate of 5% but now just with a rate of 9%.

(i)	(ii)	(iii)	(iv)
$1,200,000.00	$1,133,480.34	$888,888.89	$1,123,090.10

Table 5: Values of the different payment streams at an interest rate of 9%

As seen in table 5, we now find the inverse of table 4. The most valuable payment stream is the upfront payment in (i), next is (ii), third is (iv), and the least valuable now is the perpetuity in (iii).

4 Bond Pricing

(a) Calculate the price of a $1000 5% semi-annual pay coupon expiring in 5 years

To calculate the fair price of a $ 1,000 bond with semi-annual payments, we will use the spot rates of US Treasury strips to price the individual coupon payments as just as the US Treasury strips are seen as a risk-free investment. The price must therefore be:

$$P = \frac{\$1{,}000}{(1+\frac{S_{10}}{2})^{10}} + \sum_{t=1}^{2\cdot 5} \frac{\frac{\$1{,}000\cdot 5\%}{2}}{(1+\frac{S_t}{2})^t} \tag{5}$$

We can set this sum up in Excel to calculate the price of the bond

Period	Spot rate	Discount Factor	Payment	Present Value	**Price**
1	2%	0.99010	$25	$24.75248	**$968.073**
2	3%	0.97066	$25	$24.26654	
3	3.67%	0.94691	$25	$23.67276	
4	4.17%	0.92077	$25	$23.01931	
5	4.57%	0.89318	$25	$22.32956	
6	4.90%	0.86482	$25	$21.62062	
7	5.19%	0.83583	$25	$20.89569	
8	5.44%	0.80679	$25	$20.16972	
9	5.66%	0.77790	$25	$19.44740	
10	5.86%	0.74917	$1,025	$767.89906	

Table 6: Calculating the price of a $1,000 bond with a 5% coupon from the spot rate of US Treasury strips

We find that the price of the bond today must be $968,073.

(b) Calculate the yield-to-maturity of the bond

To calculate the yield to maturity, we will now find the R that solves

$$\$968{,}073 = \frac{\$1{,}000}{(1+\frac{R}{2})^{10}} + \sum_{t=1}^{2\cdot 5} \frac{\frac{\$1{,}000\cdot 5\%}{2}}{(1+\frac{R}{2})^t} \tag{6}$$

This can be done in a 'Brute-Force' fashion by using GoalSeek and finding the rate such that the price is equivalent.

Period	YTM	Discount Factor	Payment	Present Value	Price Difference
1	5.74%	0.97208	$25	$24.30208	$0.000
2	5.74%	0.94495	$25	$23.62365	
3	5.74%	0.91857	$25	$22.96416	
4	5.74%	0.89292	$25	$22.32308	
5	5.74%	0.86800	$25	$21.69989	
6	5.74%	0.84376	$25	$21.09410	
7	5.74%	0.82021	$25	$20.50523	
8	5.74%	0.79731	$25	$19.93279	
9	5.74%	0.77505	$25	$19.37633	
10	5.74%	0.75342	$1,025	$772.25193	

Table 7: Finding the yield to maturity in a 'Brute-Force' fashion

Another way to find the YTM in Excel would be to use the =RATE(periods;payment;-price;future value)·payments per year. The RATE function will calculate the YTM per period, and we thus just need to annualise the rate by multiplying it by the number of payments per year.

YTM	5.74%

Table 8: Calculating with =RATE(periods;payment;-price;future value)·payments per year

As seen, we, in both cases, calculate the YTM as 5.74%.

(c) Calculate the Macaulay and Modified Durations of the bond

We start by finding the Macaulay duration as

$$D = \frac{1}{m} \sum_{t=1}^{m \cdot T} t \cdot \frac{\frac{CF_t}{(1+R/m)^t}}{P} \tag{7}$$

Doing this in Excel yields

t	CF	Value of Sum	Multiplier	**D**
1	$25	0.025	0.5	**4.475**
2	$25	0.049		
3	$25	0.071		
4	$25	0.092		
5	$25	0.112		
6	$25	0.131		
7	$25	0.148		
8	$25	0.165		
9	$25	0.180		
10	$1,025	7.977		

Table 9: Calculating the Macaulay Duration in Excel

Thus, we find the Macaulay Duration to be $D = 4.475$. To subsequently find the modified duration, we can use the formula

$$D^* = \frac{D}{1 + R/m} \tag{8}$$

Or we could use the =MDURATION(settlement date;maturity date;coupon rate;yield to maturity;frequency) formula to find this directly.

D* with formula (8)	4.350
D* with MDURATION	4.350

Table 10: Finding the modified duration in 2 different ways

As seen, no matter which method we use, we find the modified duration as $D^* = 4.350$

(d) Calculate the Convexity of the bond

To calculate the convexity of the bond, we will use the formula

$$C = \frac{1}{m^2(1 + \frac{R}{m})^2} \sum_{t=1}^{m \cdot T} (t^2 + t) \cdot \frac{\frac{CF_t}{(1+R/m)^t}}{P} \tag{9}$$

Doing so in Excel yields the following table

t	CF	Value of Sum	Multiplier	**C**
1	$25	0.050	0.236	**22.383**
2	$25	0.146		
3	$25	0.285		
4	$25	0.461		
5	$25	0.672		
6	$25	0.915		
7	$25	1.186		
8	$25	1.482		
9	$25	1.801		
10	$1.025	87.749		

Table 11: Calculating the convexity of the bond

Thus, we find the convexity of the bond to be $C = 22.383$.

(e) Calculate the Effective Duration of the bond

We will start calculating the new price for the bond when adding 100 basis points (1%) and then do the same when subtracting 100 basis points from all of the spot rates. This will be done in a similar fashion to what we did in (a).

For the scenario with a spot rate 100 basis points higher, we get

Period	Spot rate	Discount Factor	Payment	Present Value	**Price**
1	3%	0.98522	$25	$24.63054	**$927.140**
2	4%	0.96117	$25	$24.02922	
3	4.67%	0.93310	$25	$23.32746	
4	5.17%	0.90295	$25	$22.57379	
5	5.57%	0.87167	$25	$21.79171	
6	5.90%	0.83993	$25	$20.99819	
7	6.19%	0.80786	$25	$20.19653	
8	6.44%	0.77605	$25	$19.40123	
9	6.66%	0.74467	$25	$18.61668	
10	6.86%	0.71373	$1.025	$731.57460	

Table 12: Recalculating price for bond with all spot rates being 100 basis points higher

Doing the same for the bond now with all spot rates 100 basis points lower

Period	Spot rate	Discount Factor	Payment	Present Value	**Price**
1	1%	0.99502	$25	$24.87562	**$1,011.164**
2	2%	0.98030	$25	$24.50740	
3	2.67%	0.96100	$25	$24.02490	
4	3.17%	0.93903	$25	$23.47587	
5	3.57%	0.91534	$25	$22.88343	
6	3.90%	0.89059	$25	$22.26469	
7	4.19%	0.86491	$25	$21.62264	
8	4.44%	0.83891	$25	$20.97263	
9	4.66%	0.81278	$25	$20.31951	
10	4.86%	0.78655	$1.025	$806.21735	

Table 13: Recalculating price for bond with all spot rates being 100 basis points lower

With the new prices, we can calculate the effective duration as

$$\begin{aligned} D_{eff} &= \frac{P_d - P_u}{2 \cdot \Delta r \cdot P} \\ &= \frac{\$1{,}011.164 - \$927.140}{2 \cdot 1\% \cdot \$968.073} = \frac{\$84.024}{\$19.361} = 4.340 \end{aligned} \tag{10}$$

We thus find the effective duration to $D_eff = 4.340\%$, making it deviate with 0.01% from the modified duration and 0.135% from the Macaulay duration, i.e., it is closer to the modified duration.

(f) Compare the prices you derived in part (e) to the prices you get using the modified duration approximation

We know that the new price must be equal to the change plus the old price. We can, therefore, calculate this as

$$\begin{aligned} &\frac{dP}{P} = -D^* \cdot dR \Leftrightarrow \\ &dP = -D^* \cdot dR \cdot P \Leftrightarrow \\ &P^* = dP + P = -D^* \cdot dR \cdot P + P \end{aligned} \tag{11}$$

Where P^* is the updated price. Inputting our calculated variables give

$$\begin{aligned} P^{\Delta r=1\%} &= -4.350 \cdot 1\% \cdot \$968.073 + \$968.073 = \$925.959 \quad (\Delta\text{Price} = \$-1.181) \\ P^{\Delta r=-1\%} &= -4.350 \cdot (-1)\% \cdot \$968.073 + \$968.073 = \$1{,}010.187 \quad (\Delta\text{Price} = \$-0.977) \end{aligned} \tag{12}$$

We see, using this approximated calculation, we have a get price of the bond to be just around $\sim$ \$1 lower on both ends when compared to the more accurate method in (e), with a slightly larger deviation when yields increase.

(g) Compare the prices you derived in part (e) to the prices you get using the modified duration and convexity-based approximation

The formula to calculate the price, which also includes the convexity, is

$$\begin{aligned} \frac{dP}{P} &= -D^* \cdot dR + \frac{1}{2}C \cdot (dR)^2 \Leftrightarrow \\ dP &= \left(-D^* \cdot dR + \frac{1}{2}C \cdot (dR)^2\right) \cdot P \Leftrightarrow \\ P^* = dP + P &= \left(-D^* \cdot dR + \frac{1}{2}C \cdot (dR)^2\right) \cdot P + P \end{aligned} \tag{13}$$

Inputting our previous results yield

$$\begin{aligned} P^{\Delta r=1\%} &= \left(-4.350 \cdot 1\% + \frac{1}{2} \cdot 22.383 \cdot (1\%)^2\right) \cdot \$968.073 + \$968.073 = \$927.043 \\ & \qquad (\Delta\text{Price} = \$-0.097) \\ P^{\Delta r=-1\%} &= \left(-4.350 \cdot (-1\%) + \frac{1}{2} \cdot 22.383 \cdot (-1\%)^2\right) \cdot \$968.073 + \$968.073 = \$1{,}011.271 \\ & \qquad (\Delta\text{Price} = \$0.107) \end{aligned} \tag{14}$$

We now see that these approximations are much closer, only around $\sim$ \$0.1 difference. It does a little too well at correction for the convexity when the yield decreases and still does not correct enough when the yield increases.

Looking at the graph in question from lecture 9, we can infer from the relative positioning of the 10-, 20- and 30-year curves that the curve for a 5-year bond must be flatter and, thus, the correction smaller. However, we do not consider the inclusion of the convexity for a "small" correction as it reduces the deviation by around $\sim$ 90% in both directions.

(h) Repeat parts (e)-(g) using a change in rates of 25 basis points

We start by recalculating the prices as in (e).

Period	Spot rate	Discount Factor	Payment	Present Value	**Price**
1	2%	0.98888	$25	$24.72188	**$957.642**
2	3%	0.96828	$25	$24.20688	
3	3.92%	0.94343	$25	$23.58580	
4	4.42%	0.91628	$25	$22.90691	
5	4.82%	0.88774	$25	$22.19362	
6	5.15%	0.85852	$25	$21.46302	
7	5.44%	0.82873	$25	$20.71834	
8	5.69%	0.79898	$25	$19.97444	
9	5.91%	0.76944	$25	$19.23592	
10	6.11%	0.74013	$1.025	$758.63555	

Table 14: Recalculating price for bond with all spot rates being 25 basis points higher

Period	Spot rate	Discount Factor	Payment	Present Value	**Price**
1	2%	0.99133	$25	$24.78315	**$978.639**
2	3%	0.97306	$25	$24.32642	
3	3.42%	0.95041	$25	$23.76014	
4	3.92%	0.92530	$25	$23.13240	
5	4.32%	0.89866	$25	$22.46651	
6	4.65%	0.87118	$25	$21.77958	
7	4.94%	0.84299	$25	$21.07477	
8	5.19%	0.81469	$25	$20.36716	
9	5.41%	0.78646	$25	$19.66146	
10	5.61%	0.75833	$1.025	$777.28716	

Table 15: Recalculating price for bond with all spot rates being 25 basis points lower

Next, we can find the effective duration as in (10)

$$D_{eff} = \frac{\$978.639 - \$957.642}{2 \cdot 0.25\% \cdot \$968.073} = \frac{\$20.996}{\$4.840} = 4.338 \tag{15}$$

We see that compared to the effective duration found in (e), it is now slightly lower.

Now, we will calculate the prices using just the modified duration using the formula in (11).

$$\begin{aligned} P^{\Delta r=0.25\%} &= -4.350 \cdot 1\% \cdot \$968.073 + \$968.073 = \$957.545 \quad (\Delta\text{Price} = \$-0.098) \\ P^{\Delta r=-0.25\%} &= -4.350 \cdot (-1)\% \cdot \$968.073 + \$968.073 = \$978.602 \quad (\Delta\text{Price} = \$-0.037) \end{aligned} \tag{16}$$

We again see that both prices are slightly below; however, though we reduced the change by three quarters, we have made the average deviation 16 times smaller from around $\sim$ $1 to $\sim$ $0.06. Lastly, we will calculate the prices using both the modified duration and the convexity using (13).

$$P^{\Delta r=0.25\%} = \left(-4.350 \cdot 1\% + \frac{1}{2} \cdot 22.383 \cdot (1\%)^2\right) \cdot \$968.073 + \$968.073 = \$957.612$$
$$(\Delta\text{Price} = \$-0.030)$$
$$P^{\Delta r=-0.25\%} = \left(-4.350 \cdot (-1\%) + \frac{1}{2} \cdot 22.383 \cdot (-1\%)^2\right) \cdot \$968.073 + \$968.073 = \$978.669$$
$$(\Delta\text{Price} = \$0.031) \tag{17}$$

We again see that it corrects a bit too much when yields decrease and not enough when yields increase; however, we are getting close to the correct prices with an average deviation now of $\sim$ \$0.03. Interestingly, where we in (g) saw a decrease in the deviation of $\sim$ 90% when including the convexity, the decrease now is only $\sim$ 50%. Thus, including the convexity term becomes less relevant as the change decreases.
As explained, we, in general, find that the deviations here are smaller when doing a 25 basis point shift compared to a 100 basis point shift. This is because both the modified duration and convexity are calculated at a specific point and thus are only accurate for an infinitesimally small change in yield to either side. The larger the deviation in yield, the less worse the slope and curvature of a specific point is to describe the entire curve.

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Excel Sheet

Problem 2

Interest Rate		10%
Periods		10
Payment	$	100

Calculated with 'Brute Force method' in Excel

Period	Payment	Discount Factor	PV of Payment	Total PV
1	$ 100	0,90909	$ 90,90909	$ 614,457
2	$ 100	0,82645	$ 82,64463	
3	$ 100	0,75131	$ 75,13148	
4	$ 100	0,68301	$ 68,30135	
5	$ 100	0,62092	$ 62,09213	
6	$ 100	0,56447	$ 56,44739	
7	$ 100	0,51316	$ 51,31581	
8	$ 100	0,46651	$ 46,65074	
9	$ 100	0,42410	$ 42,40976	
10	$ 100	0,38554	$ 38,55433	

Calculated with 'Present Value' formula in Excel

Total PV $ -614,457 ***<– negative as this is the cash-outflow we are willing to have for this payment stream***

Problem 3

	(i)	(ii)	(iii)	(iv)
Payment	$0	$0	$80.000	$175.000
Periods	0	4	Infinite	10
Future Value	$1.200.000	$1.600.000	-	$0

Value with 5% interest rate

Interest rate 5%

(i)	(ii)	(iii)	(iv)
$ 1.200.000,00	$ 1.316.323,96	$ 1.600.000,00	$ 1.351.303,61

Value with 9% interest rate

Interest rate 9%

(i)	(ii)	(iii)	(iv)
$ 1.200.000,00	$ 1.133.480,34	$ 888.888,89	$ 1.123.090,10

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Problem 4

Calculate the value of a $1,000 bond with 5% coupon rate

Face value	$1.000
Coupon rate	5%
Payment pr year	2
Periods	10

Period	Spot rate	Discount Factor	Payment	Present Value	**Price**
1	2%	0,99010	$25	$24,75248	**$968,073**
2	3%	0,97066	$25	$24,26654	
3	3,67%	0,94691	$25	$23,67276	
4	4,17%	0,92077	$25	$23,01931	
5	4,57%	0,89318	$25	$22,32956	
6	4,90%	0,86482	$25	$21,62062	
7	5,19%	0,83583	$25	$20,89569	
8	5,44%	0,80679	$25	$20,16972	
9	5,66%	0,77790	$25	$19,44740	
10	5,86%	0,74917	$1.025	$767,89906	

Calculate the YTM

Brute force with GoalSeek

YTM	5,74%

Period	Spot rate	Discount Factor	Payment	Present Value	**Difference**
1	5,74%	0,97208	$25	$24,30208	**$0,000**
2	5,74%	0,94495	$25	$23,62365	
3	5,74%	0,91857	$25	$22,96416	
4	5,74%	0,89292	$25	$22,32308	
5	5,74%	0,86800	$25	$21,69989	
6	5,74%	0,84376	$25	$21,09410	
7	5,74%	0,82021	$25	$20,50523	
8	5,74%	0,79731	$25	$19,93279	
9	5,74%	0,77505	$25	$19,37633	
10	5,74%	0,75342	$1.025	$772,25193	

Using =RATE(period;payment;price;future value)*periods

YTM	**5,74%**

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Calculate the Macualay Duration

m	2
R	5,74%
P	$968,073

t	CF		Value of Sum	Multiplier	D
1	$	25	0,025	0,500	4,475
2	$	25	0,049		
3	$	25	0,071		
4	$	25	0,092		
5	$	25	0,112		
6	$	25	0,131		
7	$	25	0,148		
8	$	25	0,165		
9	$	25	0,180		
10	$	1.025	7,977		

Calculate the Duration

D* with formula	4,350
D* with MDURAT	4,350

Calculate the Convexity

t	CF		Value of Sum	Multiplier	C
1	$	25	0,050	0,236	22,383
2	$	25	0,146		
3	$	25	0,285		
4	$	25	0,461		
5	$	25	0,672		
6	$	25	0,915		
7	$	25	1,186		
8	$	25	1,482		
9	$	25	1,801		
10	$	1.025	87,749		

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Recalculate the Price 100 basis points higher

Delta-% in spot 1%

Period	Spot rate	Discount Factor	Payment	Present Value	Price
1	3%	0,98522	$25	$24,63054	**$927,140**
2	4%	0,96117	$25	$24,02922	
3	4,67%	0,93310	$25	$23,32746	
4	5,17%	0,90295	$25	$22,57379	
5	5,57%	0,87167	$25	$21,79171	
6	5,90%	0,83993	$25	$20,99819	
7	6,19%	0,80786	$25	$20,19653	
8	6,44%	0,77605	$25	$19,40123	
9	6,66%	0,74467	$25	$18,61668	
10	6,86%	0,71373	$1.025	$731,57460	

Recalculate the Price 100 basis points lower

Delta-% in spot -1%

Period	Spot rate	Discount Factor	Payment	Present Value	Price
1	1%	0,99502	$25	$24,87562	**$1.011,164**
2	2%	0,98030	$25	$24,50740	
3	2,67%	0,96100	$25	$24,02490	
4	3,17%	0,93903	$25	$23,47587	
5	3,57%	0,91534	$25	$22,88343	
6	3,90%	0,89059	$25	$22,26469	
7	4,19%	0,86491	$25	$21,62264	
8	4,44%	0,83891	$25	$20,97263	
9	4,66%	0,81278	$25	$20,31951	
10	4,86%	0,78655	$1.025	$806,21735	

Calculate the Effective Duration

D_eff 4,340

Calculate prices using the modified duration

			Delta price	
Delta r = 1%	$	925,959	$	-1,181
Delta r = -1%	$	1.010,187	$	-0,977

Calculate prices using the modified duration and convexity-based approximation

			Delta price	
Delta r = 1%	$	927,043	$	-0,097
Delta r = -1%	$	1.011,271	$	0,107

Johan Oelgaard
Rhys Blackmore

Recalculate the Price 25 basis points higher

Delta-% in spot 0,25%

Period	Spot rate	Discount Factor	Payment	Present Value	Price
1	2%	0,98888	$25	$24,72188	**$957,642**
2	3%	0,96828	$25	$24,20688	
3	3,92%	0,94343	$25	$23,58580	
4	4,42%	0,91628	$25	$22,90691	
5	4,82%	0,88774	$25	$22,19362	
6	5,15%	0,85852	$25	$21,46302	
7	5,44%	0,82873	$25	$20,71834	
8	5,69%	0,79898	$25	$19,97444	
9	5,91%	0,76944	$25	$19,23592	
10	6,11%	0,74013	$1.025	$758,63555	

Recalculate the Price 25 basis points lower

Delta-% in spot -0,25%

Period	Spot rate	Discount Factor	Payment	Present Value	Price
1	2%	0,99133	$25	$24,78315	**$978,639**
2	3%	0,97306	$25	$24,32642	
3	3,42%	0,95041	$25	$23,76014	
4	3,92%	0,92530	$25	$23,13240	
5	4,32%	0,89866	$25	$22,46651	
6	4,65%	0,87118	$25	$21,77958	
7	4,94%	0,84299	$25	$21,07477	
8	5,19%	0,81469	$25	$20,36716	
9	5,41%	0,78646	$25	$19,66146	
10	5,61%	0,75833	$1.025	$777,28716	

Calculate the Effective Duration

D_eff 4,338

Calculate prices using the modified duration

		Delta price
Delta r = 0,25%	$ 957,545	$ -0,098
Delta r = -0,25%	$ 978,602	$ -0,037

Calculate prices using the modified duration and convexity-based approximation

		Delta price
Delta r = 0,25%	$ 957,612	$ -0,030
Delta r = -0,25%	$ 978,669	$ 0,031