Problem Set 2
Econ 136, Spring 2024

This problem set is due by 5PM Tuesday, February 20th, on Gradescope.

1. **Puts and calls**

   (a) The last two pages of the problem set contain option quotes for Microsoft from January 30, 2023. Verify the put-call parity condition for the options with the following strike prices: \$235, \$240, \$245, \$250. Use the bid prices for both the call and put options, and assume that $r_f = 0$. Note, the closing price of Microsoft on January 30 was \$242.71. Do you find deviations from put-call parity? What is your arbitrage portfolio for the options with strike price \$245?

   (b) What is wrong with using the bid prices for both the put and the call? If you actually want to engage in the arbitrage trade for the option with strike \$245 from (a), would you use the bid or the ask price for the put? For the call? Using the appropriate (bid or ask) prices, is your proposed arbitrage trade still making money?

   (c) Go to the Yahoo! Finance website at `http://finance.yahoo.com/` and type in the name or ticker symbol of a large publicly traded company other than Microsoft. What is the current stock price of the company? Now go to the "Options" page and click the drop down list (upper left just above the list of call options, next to where it says "In The Money") to get a listing of expiration dates. Choose "March 10" as the expiration date. Select the two strike prices surrounding the current stock price. Verify put-call parity for the options that trade at these two strike prices, using the bid prices for all options (use $r_f = 0$). How much money can you make per option if put-call parity is violated? What would be your arbitrage trade? Do you think that you can make money by engaging in this trade in practice? Include in your solution a printout of the option prices from Yahoo.

2. **Options and risk management**

   (a) You have \$10,000, which you want to invest in XYZ stock and options. The stock price of XYZ is $S_0 = 40$, and the price of a six-month call on XYZ with strike $X = 40$ is $C_0 = 1$. Consider the following three portfolios: (1) Invest all your wealth in XYZ stock; (2) invest all your wealth in call options on XYZ stock; (3) buy 100 calls and put the rest of your wealth in a bank-account that earns a six-month risk-free return of $r_f = 1\%$.

   In a table, compute the payoff of all three portfolios when S_T, the price of XYZ six months from now is \$30, \$40, \$50 and \$60. Use rows for these states of the world and columns for the payoffs of the three investments. Which portfolio appears to be the most risky? Which portfolio appears to be the safest? People sometimes say that call options are "safe", because the investor is protected when the price of the underlying falls. Does portfolio (2) confirm or contradict this reasoning? Are call options really safe? What happens when you use calls in combination with a bank account, as in portfolio (3), can call options then reduce risk relative to a pure stock investment?

   (b) Suppose you sell (write) a European put option on XYZ with strike \$50 and expiration T. Draw the payoff of your position as a function of S_T, the price of XYZ, in a payoff diagram. In the same diagram, draw the payoff of selling 2 put options. Now draw the payoff of selling ten put options. What happens to the slope of the payoff function in the range where $S_T < X$ when you sell more and more puts? If the stock price falls \$1 below the strike, how much does the seller of one put lose? How much does the seller of ten puts lose? Note: in your answer, ignore the price of the options at date zero; just focus on the payoffs on date T.

3. **Arbitrage and state prices**

The economy next year will be either in a boom or in a recession. We have the following data about the payoffs of the stocks and bonds of a company called Vitamin Co.:

	Vitamin Stock	Vitamin Bond
State 1: Boom	7	10
State 2: Recession	3	10
Price today	4	9

(a) Consider a portfolio of φ_s shares of Vitamin stock and φ_b shares of Vitamin bond. Express the payoff of this portfolio in state 1 and state 2 as a function of φ_s and φ_b.

(b) Suppose this portfolio is an Arrow-Debreu security for state 1. Equate your expression for the portfolio payoffs from part (a) with the corresponding payoffs of this Arrow-Debreu security. Solve this system of two equations for φ_s and φ_b. By the LOOP, what is the price of the Arrow-Debreu security for state 1?

(c) Can you construct an Arrow-Debreu security for state 2? What is its price? Is this market complete?

(d) What is the (net simple) rate of return R_f on an investment that gives you a sure payment one year from now (i.e., what is the risk-free rate of return)?

(e) What are the payoffs in the two states of the world of a European call option on Vitamin stock that has strike price $X = 5$ and expires in one year?

(f) Compute the price today of the call option in (e).

(g) What is the price of a European put option on Vitamin stock that has strike price $X = 5$ and expires one year from now?

4. **Dynamic trading**

In the coin toss economy from lecture, consider a call option with strike $X = 0.2$ and expiration in period 2, which is written on the number of heads. Formally, the payoff of this option is $\max(S - X, 0)$ where S is the total number of heads (0, 1 , or 2) and $X = 0.2$. Assume that the two securities traded in the economy are the Head and Tail assets discussed in class.

(a) According to the LOOP, what is the price of this call option at date zero?

(b) Use backward induction to construct a dynamic trading strategy that replicates the payoff of the call option.

(c) Suppose that the coins being tossed are replaced, and that the new coins are not fair, so that the probability of heads is now 3/5. However, the prices and payoffs of the Head and Tail security are unchanged. Is the price of the call option affected?

Calls for March 10, 2023

Contract Name	Last Trade Date	Strike	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
MSFT230310C00215000	2023-01-26 2:00PM EST	215.00	31.24	28.45	30.50	0.00	-	-	1	39.69%
MSFT230310C00220000	2023-01-30 10:53AM EST	220.00	25.00	23.55	25.70	-6.07	-19.54%	3	1	35.61%
MSFT230310C00225000	2023-01-27 3:44PM EST	225.00	26.40	19.70	21.65	0.00	-	1	1	34.47%
MSFT230310C00230000	2023-01-26 2:00PM EST	230.00	18.38	15.75	19.00	0.00	-	-	1	37.50%
MSFT230310C00235000	2023-01-30 1:06PM EST	235.00	13.78	12.20	13.85	-3.80	-21.62%	15	4	30.51%
MSFT230310C00240000	2023-01-30 1:04PM EST	240.00	9.90	9.60	12.00	-3.74	-27.42%	17	3	33.66%
MSFT230310C00245000	2023-01-30 3:59PM EST	245.00	7.30	7.00	7.50	-3.90	-34.82%	284	110	27.04%
MSFT230310C00250000	2023-01-30 3:28PM EST	250.00	5.00	4.95	5.20	-3.28	-39.61%	35	37	25.98%
MSFT230310C00255000	2023-01-30 3:43PM EST	255.00	3.42	3.25	3.55	-2.23	-39.47%	19	23	25.54%
MSFT230310C00260000	2023-01-30 3:46PM EST	260.00	2.26	2.05	2.24	-1.19	-34.49%	16	122	24.78%
MSFT230310C00265000	2023-01-30 3:47PM EST	265.00	1.41	1.18	1.41	-0.98	-41.00%	257	69	24.55%
MSFT230310C00270000	2023-01-30 3:58PM EST	270.00	0.80	0.79	0.89	-0.91	-53.22%	21	22	24.61%
MSFT230310C00275000	2023-01-30 3:32PM EST	275.00	0.51	0.44	0.58	-0.52	-50.49%	8	40	25.00%
MSFT230310C00280000	2023-01-30 11:57AM EST	280.00	0.35	0.00	0.37	-0.18	-33.96%	5	32	25.32%
MSFT230310C00285000	2023-01-30 10:52AM EST	285.00	0.22	0.00	0.26	-0.16	-42.11%	4	20	26.12%
MSFT230310C00290000	2023-01-27 11:40AM EST	290.00	0.20	0.07	0.17	0.00	-	1	4	26.56%
MSFT230310C00295000	2023-01-30 1:56PM EST	295.00	0.10	0.00	0.13	-0.04	-28.57%	2	5	27.54%
MSFT230310C00300000	2023-01-30 10:40AM EST	300.00	0.10	0.00	0.10	+0.03	+42.86%	11	25	28.52%

Puts for March 10, 2023

Contract Name	Last Trade Date	Strike	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
MSFT230310P00170000	2023-01-27 10:03AM EST	170.00	0.05	0.05	0.11	0.00	-	1	1	47.07%
MSFT230310P00175000	2023-01-27 12:23PM EST	175.00	0.05	0.08	0.12	0.00	-	5	6	44.14%
MSFT230310P00185000	2023-01-30 2:33PM EST	185.00	0.25	0.18	0.28	+0.10	+66.67%	4	4	42.48%
MSFT230310P00190000	2023-01-30 2:24PM EST	190.00	0.30	0.27	0.37	+0.11	+57.89%	3	5	40.75%
MSFT230310P00200000	2023-01-30 3:56PM EST	200.00	0.60	0.54	0.62	+0.21	+53.85%	10	35	37.04%
MSFT230310P00205000	2023-01-30 3:58PM EST	205.00	0.84	0.76	0.88	+0.35	+71.43%	14	1	35.91%
MSFT230310P00210000	2023-01-30 3:58PM EST	210.00	1.12	1.06	1.15	+0.43	+62.32%	35	14	34.12%
MSFT230310P00215000	2023-01-30 3:17PM EST	215.00	1.53	1.44	1.62	+0.55	+56.12%	168	19	33.06%
MSFT230310P00220000	2023-01-30 3:21PM EST	220.00	2.09	1.95	2.47	+0.72	+52.55%	269	80	33.09%
MSFT230310P00225000	2023-01-30 3:17PM EST	225.00	2.85	2.69	3.55	+1.05	+58.33%	57	99	32.86%
MSFT230310P00230000	2023-01-30 3:35PM EST	230.00	3.65	3.65	3.90	+1.20	+48.98%	39	431	28.70%
MSFT230310P00235000	2023-01-30 2:48PM EST	235.00	5.10	5.00	5.25	+1.52	+42.46%	72	37	27.49%
MSFT230310P00240000	2023-01-30 12:37PM EST	240.00	6.55	6.75	7.50	+1.90	+40.86%	31	19	27.94%
MSFT230310P00245000	2023-01-30 3:51PM EST	245.00	8.70	7.95	9.30	+2.10	+31.82%	53	141	25.50%
MSFT230310P00250000	2023-01-30 11:48AM EST	250.00	11.55	10.05	12.60	+2.75	+31.25%	7	7	26.35%
MSFT230310P00255000	2023-01-30 1:10PM EST	255.00	14.93	13.75	16.05	+1.27	+9.30%	1	0	26.32%
MSFT230310P00260000	2023-01-27 3:11PM EST	260.00	14.00	17.60	19.95	0.00	-	1	3	26.59%
MSFT230310P00265000	2023-01-27 2:58PM EST	265.00	17.88	21.95	24.00	0.00	-	3	3	26.16%
MSFT230310P00280000	2023-01-26 2:08PM EST	280.00	35.01	35.70	39.65	0.00	-	-	2	39.97%
MSFT230310P00285000	2023-01-26 9:35AM EST	285.00	40.67	40.60	44.50	0.00	-	-	5	42.36%