Financial Economics | Problem Set 1

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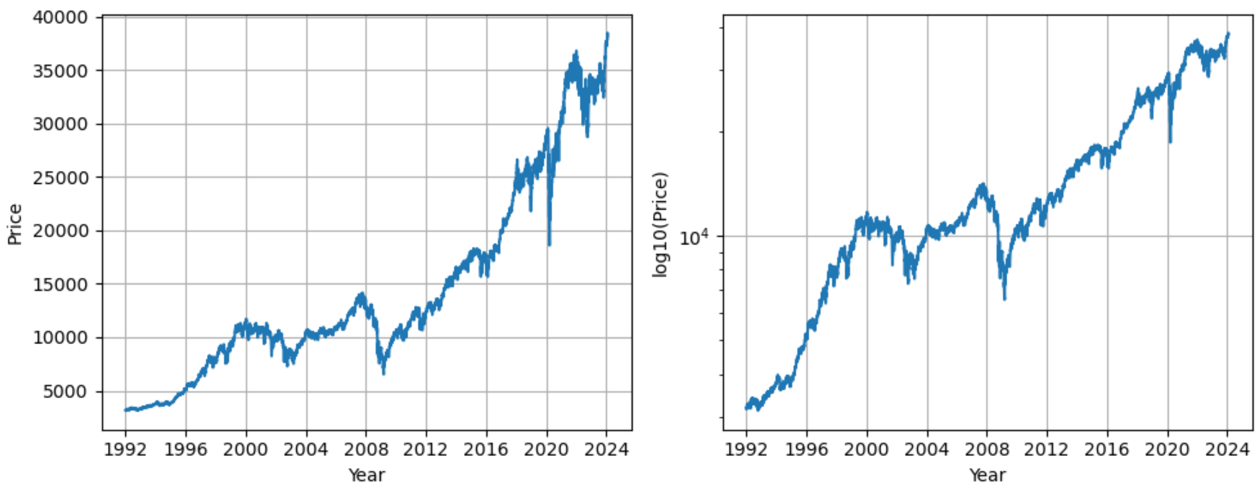
1 US and Foreign Stock Markets in the Past Five Years

(a) Advantages and Disadvantages of Linear or Logarithmic Scales

The major advantage of using a linear scale is the fact that every point on the y-axis is equally distanced, making it more intuitive to compare different points. However, as seen for the Dow Jones Industrial Average, when setting it to max, it is more difficult to observe the initial movement before the index took off around 1995. The logarithmic scale, on the other hand, makes it easier to visualise the initial movements but more difficult to compare two points directly. Personally, we prefer the linear as it is more intuitive and easier to see how the index has gone from around 3,000 in 1992 to over 35,000 in 2024.

Figure 1: Dow Jones Industrial Average



(b) Comparing Indices performances and Geometric and Arithmetic Average Returns

When just looking at the graphical illustration of the performance for the indices in figure 2, it is difficult to compare the performances of the indices when looking at their absolute value. After normalizing the prices, we find that the NASDAQ has been the index with the highest

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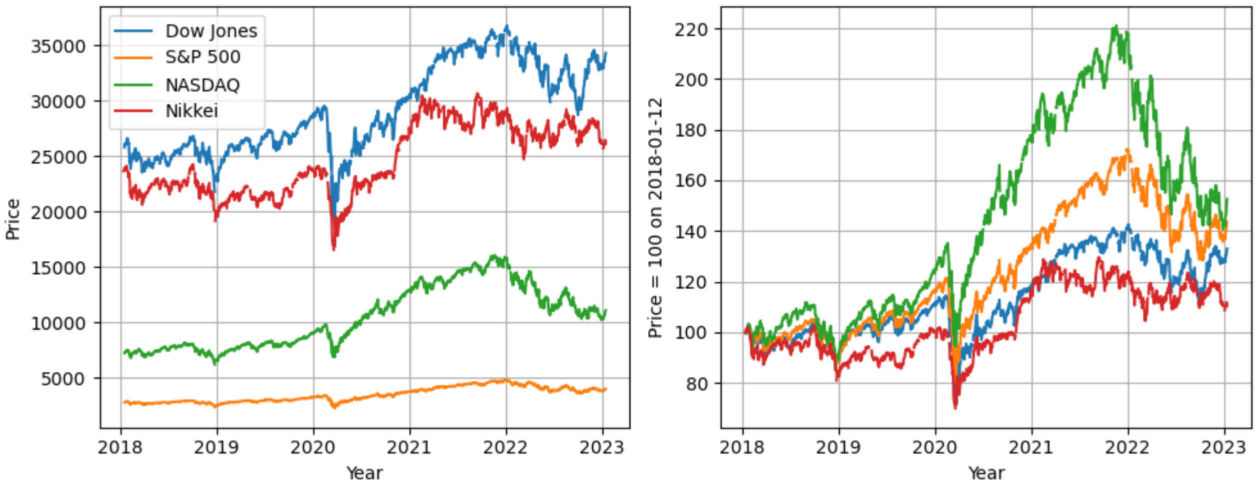
return since January 12th 2020 with a return of over 40%, followed by the S&P 500, Dow Jones and lastly, Nikkei.

Figure 2: Comparing indices' performance over past 5 years



When calculating the annual geometric average return and annual arithmetic average return, we use the dates 1/12/2018, 1/11/2019, 1/13/2020[1], 1/13/2021, 1/13/2022, and 1/13/2023. The geometric return is calculated as

$$R_{\text{Geo}} = ((1 + R_{2018,2019})(1 + R_{2019,\ 2020})(1 + R_{2020,\ 2021})(1 + R_{2021,\ 2022})(1 + R_{2022,\ 2023}))^{\frac{1}{5}} - 1 \tag{1}$$

While we calculate the arithmetic return as

$$R_{\text{Ari}} = \frac{R_{2018,2019} + R_{2019,\ 2020} + R_{2020,\ 2021} + R_{2021,\ 2022} + R_{2022,\ 2023}}{5} \tag{2}$$

	Geometric mean	Arithmetic mean
NASDAQ	0.088182	0.116423
S&P 500	0.074951	0.087643
Dow Jones	0.058597	0.064332
Nikkei	0.020030	0.028657

Table 1: Geometric average return and arithmetic average return; 1/12/2018-1/13/2023

We find that these numbers correlated with exactly what we also saw in figure 2 with NASDAQ as the best-performing index. As seen, the ranking in both geometric and arithmetic mean is equal, and thus, in this case, it does not matter what measure is used to select an index to invest in based on performance from 2018-2023.

However, the best measure of what return can be expected when investing (assuming the past equals our expectations) would be the geometric mean. The geometric mean takes into account the compounding effect and thus is more accurate of the actual return each year.

[1] 1/13/2020 was a public holiday in Japan thus, for the Nikkei index we have used the close value on Friday 1/10/2020

2 How much is your return?

(a) Return of Bang.com

We calculate the net simple return as

$$R_{2001,2003} = \frac{12.77 + 0.53}{80} - 1 = 0.16625 - 1 = -83.38\% \tag{3}$$

Thus, adding back 1 to get the gross simple return we can calculate the geometric average as

$$(0.16625)^{\frac{1}{2}} - 1 = -59.23\% \tag{4}$$

(b) Return of Rolling S Co.

We start by calculating the net simple nominal return

$$R^{\text{Nominal}}_{\text{Jan 1, Dec 31}} = \frac{107.20 + 0.80}{91} - 1 = 18.68\% \tag{5}$$

Deflating the return with the inflation rate to get real returns we get

$$R^{\text{Real}}_{\text{Jan 1, Dec 31}} = \frac{1.1868}{1.14} - 1 = 4.11\% \tag{6}$$

To compare the performance of Rolling S with other companies in 1980 both the nominal and real would be appropriate measures of performance, but one might argue that the nominal rate is better as it requires less calculations.

When comparing to the company's performance in a whole other year however, the nominal rate should *not* be used. Instead the appropriate measure is the real rate as inflation can vary drastically. And even so the greater macroeconomic context should also be taken into consideration to properly compare a company's performance across decades.

3 Create your portfolio

(a) Return in an Equal-Weighted, Price-Weighted, and Value-Weighted Portfolio

The equal weighted portfolio all the stocks will have the weight

$$w_i = \frac{1}{n} \tag{7}$$

Thus, the weight will be $w_i = 1/3 = 33.33\%\ \forall i$ for all the stocks. In the price-weighted portfolio the appropriate formula for the weight is

$$w_i = \frac{P_i}{\sum_{i \in I}^{n} P_i}, \quad I = \{\text{Historic Co., Present Ltd., Optimistic Inc.}\} \tag{8}$$

Where P_i is the current price of stock i. Thus, we can calculate the weights as

$$w_{\text{Historic Co.}} = \frac{60}{120} = 50.00\% \tag{9}$$

$$w_{\text{Present Ltd.}} = \frac{28}{120} = 23.33\% \tag{10}$$

$$w_{\text{Optimistic Inc.}} = \frac{32}{120} = 26.67\% \tag{11}$$

In the value-weighted portfolio, the appropriate weight is calculated as

$$w_i = \frac{P_i \cdot n_i}{\sum_{i\in I} P_i \cdot n_i}, \quad I = \{\text{Historic Co., Present Ltd., Optimistic Inc.}\} \tag{12}$$

Where n_i is the number of outstanding shares of stock i. Thus, the weights are calculated as

$$w_{\text{Historic Co.}} = \frac{3000}{7600} = 39.47\% \tag{13}$$

$$w_{\text{Present Ltd.}} = \frac{1400}{7600} = 18.42\% \tag{14}$$

$$w_{\text{Optimistic Inc.}} = \frac{3200}{7600} = 42.11\% \tag{15}$$

All the weights for the different portfolios can be seen in the table below.

Name	Equal-weighted	Price-weighted	Value-weighted
Historic Co.	33.33%	50.00%	39.47%
Present Ltd.	33.33%	23.33%	18.42%
Optimistic Inc.	33.33%	26.67%	42.11%

Table 2: Weights of individual stocks in different indexes

(b) Adjustments to Maintain the Weighting Scheme for Each of the Three Indices

For the price- and value-weighted indices, it is not necessary to rebalance the portfolio as their price is incorporated into the weights and thus adjusts accordingly. For the equal weighting, however, a rebalance is needed. We will assume a $100 initial investment and rebalance the portfolio from here.

We start by calculating the initial number of shares bought of each stock

$$\text{Shares}_{\text{Historic Co.}} = \frac{33.33}{60} = 0.5555 \tag{16}$$

$$\text{Shares}_{\text{Present Ltd.}} = \frac{33.33}{28} = 1.1906 \tag{17}$$

$$\text{Shares}_{\text{Optimistic Inc.}} = \frac{33.33}{32} = 1.0416 \tag{18}$$

Next we calculated the current value of the individual stocks as well as the portfolio to know the amount we have invested and what we want to invest in each

$$\text{Value}_{\text{Historic Co.}} = 0.5555 \cdot 63 = 33.9965 \tag{19}$$

$$\text{Value}_{\text{Present Ltd.}} = 1.1906 \cdot 21 = 25.0026 \tag{20}$$

$$\text{Value}_{\text{Optimistic Inc.}} = 1.0416 \cdot 36 = 37.4976 \tag{21}$$

$$\text{Value}_{\text{Portfolio}} = 33.9965 + 25.0026 + 37.4976 = 96.4967 \tag{22}$$

As it is an equal weighted portfolio we want $96.4967/3 = 32.1656$ invested into each stock and therefore we should rebalance the portfolio such that we own the following amount of each stock

$$\text{Shares}^{\text{Rebalanced}}_{\text{Historic Co.}} = \frac{32.1656}{63} = 0.5106 \tag{23}$$

$$\text{Shares}^{\text{Rebalanced}}_{\text{Present Ltd.}} = \frac{33.33}{28} = 1.5317 \tag{24}$$

$$\text{Shares}^{\text{Rebalanced}}_{\text{Optimistic Inc.}} = \frac{33.33}{32} = 0.8935 \tag{25}$$

4 Arbitrage and the Law of One Price

(a) First Arbitrage Opportunity

There is an arbitrage type I opportunity when shorting 1 of asset two while going long 5 of asset 1. This results in no future pay-offs but pockets 1.5 today.

State	Asset 1	Asset 2	Arbitrage Portfolio
State 1	1	5	0
State 2	-0.6	-3	0
Price	0.5	4	-1.5

Table 3: First Arbitrage Opportunity. Short 1 of asset 2 and long 5 of asset 1

(b) Second Arbitrage Opportunity

There is an arbitrage type II opportunity when shorting 1 of asset two while going long 3 of asset 1. This portfolio is free and while give 0 pay-off in state 1 and a pay-off of 0.5 in state 2.

State	Asset 1	Asset 2	Arbitrage Portfolio
State 1	1.5	4.5	0
State 2	2	5.5	0.5
Price	1	3	0

Table 4: Second Arbitrage Opportunity. Short 1 of asset 2 and long 3 asset 1

(c) Third Arbitrage Opportunity

There is an arbitrage type I opportunity when shorting the two arrow-seccurities and going long in asset 3. We suggest shorting 4 of asset 1 and 7 of asset asset two while buying 1 of asset 3. This results in no future pay-offs but pockets 0.3 today.

State	Asset 1	Asset 2	Asset 3	Arbitrage Portfolio
State 1	1	0	4	0
State 2	0	1	7	0
Price	0.6	0.3	4.2	-0.3

Table 5: Third Arbitrage Opportunity. Short 4 of asset 1, 7 of asset 2 and long 1 asset 3