Problem Set 3
Econ 136, Spring 2024

This problem set is due on March 4th, by 5PM (on Gradescope).

1. **Present value of car payments**

   Montague Autos is offering interest-free credit on a new car that costs \$11,250. The deal is that you pay down \$750 and then \$250 a month for the next 42 months. Capulet Motors does not offer free credit but will give you \$1000 off the list price, so you pay \$10,250 for the car today. Which dealership is offering a better deal

   (a) if the annual rate of interest is 5%,

   (b) if the annual rate of interest is 6%?

   Explain. Note, in computing the present values, use the *geometric* average monthly interest rate.

2. **Excel basics**

   You own an asset that entitles you to receive a payment of \$100 per year for 10 years. You receive the first payment one year from today. All ten payments will be made in full and on time. The interest rate is 10% per year with certainty over the life of the asset. How much would you be willing to accept today in exchange for this asset?

   First try the "brute force" approach to get used to Excel. Start a new workbook in Excel. Then number and enter the payments in the spreadsheet: Type the numbers 1 through 10 into column A, rows 1-10. In column B, enter the payments (\$100 for all ten cells). Column C should contain the discount factors for each payment $1/(1+r)^n$. To do this, in cell C1, type the text "=1/(1.10^A1)" (without the quotes) – the cell A1 contains the n (which is 1) that is relevant for the first payment. Now click on the cell C1 and grab the little box in lower right corner of the selection rectangle, and drag it down to cell C10. This will fill in the remaining cells of column C with the appropriate version of the formula: cell C2 references A2, etc. Next, using the same idea, fill in column D with a formula that multiplies each payment by its discount factor (e.g. "=B1*C1"). Lastly, in cell D11, sum up the first ten cells in column D ("=SUM(D1:D10)"). This should yield the present value of the asset.

   In cell E11, use the Excel function PV to accomplish the same. Look up the PV function in Excel help. The syntax of the PV function is PV (rate; nper; pmt; fv; type). Here *rate* is the discount rate r; *nper* is the number of periods T; *pmt* is the payment made each period; *fv* is the future value of the final payment (e.g., the face value for a coupon bond); and type = 0 or 1; if 0 or omitted, it means the first payment happens one period from now (i.e., it needs to be discounted). Once you plugged in all these numbers, do you get the same result?

   Please print out and turn in the spreadsheet together with your solution.

3. **More Excel**

   Now that you have learned the basics, calculate the present value of the following four alternative payment streams with interest rates $r = 5\%$ and $r = 9\%$, and rank the alternatives:

   (i) \$1,200,000 now

   (ii) \$1,600,000 at the end of four years

   (iii) \$80,000 per year in perpetuity, with payments made at the end of each year (so your first payment comes one year from today)

   (iv) \$175,000 per year for the next ten years, with payments made at the end of each year (so your first payment comes one year from today)

   Do this by entering the interest rate (e.g., 0.05) in cell A1. In B1, enter the payment of option (i), that is, 1,200,000. In B2, enter the present value of this payment ("= B1"). In C1, enter 1,600,000. In C2, enter the formula for the present value of this payment: "=C1/(1 + A1)^4"). Fill in columns D and E in the same spirit. Always refer to cell A1 when you use the interest rate.

   Once you are done, you can just replace the value in A1 by a different number (e.g., 0.09) and Excel will re-calculate all cells for you. So after you fill in the cells once, it is really easy to calculate the present values for different interest rates. This is why it's important to always refer to A1 when you use the interest rate: so that Excel knows how to re-calculate the formulas when you change the interest rate.

4. **Bond pricing**

Suppose the date is February 15, 2024, and the current spot rate curve (derived from US Treasury strips) is

TTM (yrs)	Spot Rate (%)
0.5	2.00
1.0	3.00
1.5	3.67
2.0	4.17
2.5	4.57
3.0	4.90
3.5	5.19
4.0	5.44
4.5	5.66
5.0	5.86

(a) Calculate the price of a \$1000 5% semi-annual pay coupon bond that matures on February 15, 2028. Assume the bond has no risk of default.

(b) Calculate the yield-to-maturity of the bond.

(c) Calculate the Macaulay and modified durations of the bond.

(d) Calculate the convexity of the bond.

(e) Calculate the *effective* duration of the bond by following these steps:

(i) Add 100 basis points to all of the spot rates (e.g., an increase of 1%). Using this new spot rate curve, recalculate the price and call it P_u.

(ii) Subtract 100 basis points from all of the spot rates (starting with the rates in the table). Using this new spot rate curve, recalculate the price and call it P_d.

(iii) The effective duration is then given by

$$D_{eff} = \frac{P_d - P_u}{2 \cdot \Delta r \cdot P}$$

where P is the price you calculated in part (a), and ΔR is 0.01 (100 basis points).

Is the effective duration closer to the Macaulay duration or the modified duration?

(f) Calculating P_u and P_d as in part (e) is the most accurate way of estimating what the price will be after a change in yield. Compare the prices you derived in part (e) to the prices you get using the modified duration based approximation.

(g) Compare the prices you derived in part (e) to the prices you get using the modified duration and convexity based approximation.

Look at the figure of the price-yield relationship for a 10% coupon bond with various maturities from lecture 9. Can you see at least one reason the convexity adjustment for the 5-year bond in this problem is so small? Explain.

(h) Repeat parts (e)-(g) using a change in rates of 25 basis points instead of 100 basis points. Are your approximations closer to P_u and P_d? Explain.