Task 3.

Momentum eqs:

$$\boxed{x: \rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial U}{\partial y} = -\frac{dp}{dx} + \mu \frac{\partial^2 U}{\partial x^2} + \mu \frac{\partial^2 U}{\partial y^2}}$$

$$\boxed{y: \rho U \frac{\partial V}{\partial x} + \rho V \frac{\partial V}{\partial y} = -\frac{dp}{dy} + \mu \frac{\partial^2 V}{\partial x^2} + \mu \frac{\partial^2 V}{\partial y^2}}$$

$$\boxed{\text{Cont. eq:} \quad \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0}$$

Reorganized: only x:

$$\frac{\partial}{\partial x}\left(\rho U U - \mu \frac{\partial U}{\partial x} + p\right) + \frac{\partial}{\partial y}\left(\rho V U - \mu \frac{\partial U}{\partial y}\right) = 0$$

after gauss Thm:

$$\left((\rho U U)_e - (\rho U U)_w\right)\Delta y + \left((\rho V U)_n - (\rho V U)_s\right)\Delta x -$$

$$-\left(\left(\mu \frac{\partial U}{\partial x}\right)_e - \left(\mu \frac{\partial U}{\partial x}\right)_w\right)\Delta y - \left(\left(\mu \frac{\partial U}{\partial y}\right)_n - \left(\mu \frac{\partial U}{\partial y}\right)_s\right)\Delta x$$

$$+ \left(P_e - P_w\right)\Delta y = 0$$

Massflux: $F_x = \rho \Delta y U$
$F_y = \rho \Delta x V$: U, V interpolated:

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use Rhie chow-interpolation!
for face velocities!

$$U_e = 0.5\left(U_P + U_E\right) + \frac{dy}{4a_{p,e}}\left(P_{EE} - 3P_E + 3P_P - P_W\right)$$

$$U_w = 0.5\left(U_P + U_W\right) + \frac{dy}{4a_{p,w}}\left(P_E - 3P_P + 3P_W - P_{WW}\right)$$

Where: $a_{p,e} = \frac{a_{p,E} + a_{p,P}}{2}$ Simply.

Let: $\left(\frac{\partial U}{\partial x}\right)_e = \frac{U_E - U_P}{\Delta x}$, $\left(\frac{\partial U}{\partial x}\right)_w = \frac{U_P - U_W}{\Delta x}$ pss V.

The final expression:

$$F_{x,e}U_e - F_{x,w}U_w + F_{y,n}U_n - F_{y,s}U_s -$$

$$-\mu\left(\left(\frac{U_E-U_P}{\Delta x}\right)-\left(\frac{U_P-U_W}{\Delta x}\right)\right)\Delta y - \mu\left(\left(\frac{U_N-U_P}{\Delta y}\right)-\left(\frac{U_P-U_S}{\Delta y}\right)\right)\Delta x$$

$$+(P_e-P_w)\Delta y = 0 \quad \Rightarrow$$

$$\Rightarrow F_{x,e}\left(\frac{U_E+U_P}{2}\right) - F_{x,w}\left(\frac{U_P+U_W}{2}\right) + F_{y,n}\left(\frac{U_N+U_P}{2}\right)$$

$$- F_{y,s}\left(\frac{U_P+U_S}{2}\right) - \mu\left(\frac{\Delta y}{\Delta x}\left(U_E - 2U_P + U_W\right) - \left(\frac{\Delta x}{\Delta y}\left(U_N - 2U_P + U_S\right.\right.\right.$$

$$+(P_e-P_w)\Delta y = 0 \quad \Rightarrow$$

$$\Rightarrow U_P\left(\frac{F_{x,e}}{2}-\frac{F_{x,w}}{2}+\frac{F_{y,n}}{2}-\frac{F_{y,s}}{2}+\frac{2\mu\Delta y}{\Delta x}-\frac{2\mu\Delta x}{\Delta y}\right)+$$

$$+U_E\left(\frac{F_{x,e}}{2}-\mu\frac{\Delta y}{\Delta x}\right)+U_W\left(-\frac{F_{x,w}}{2}-\mu\frac{\Delta y}{\Delta x}\right)+$$

$$+U_N\left(\frac{F_{y,n}}{2}-\mu\frac{\Delta x}{\Delta y}\right)+U_S\left(-\frac{F_{y,s}}{2}-\mu\frac{\Delta x}{\Delta y}\right)+(P_e-P_w)\Delta y=0$$

$$a_E^u = \left(\mu \frac{\Delta y}{\Delta x} - \frac{F_{x,e}}{2}\right), \quad a_W^u = \left(\mu \frac{\Delta y}{\Delta x} + \frac{F_{x,w}}{2}\right)$$

$$a_N^u = \left(\mu \frac{\Delta x}{\Delta y} - \frac{F_{y,n}}{2}\right), \quad a_S^u = \left(\mu \frac{\Delta y}{\Delta x} - \frac{F_{y,s}}{2}\right)$$

Too avoid stability issues use upwind scheme:

$$a_E^u = \mu \frac{\Delta y}{\Delta x} + \max\left(-\frac{F_{x,e}}{2}, 0\right), \quad a_W^u = \mu \frac{\Delta y}{\Delta x} + \max\left(F_{x,w}, 0\right.$$

$$a_N^u = \mu \frac{\Delta x}{\Delta y} + \max\left(-\frac{F_{y,n}}{2}, 0\right), \quad a_S^u = \mu \frac{\Delta x}{\Delta y} + \max\left(F_{y,s}, 0\right.$$

$$a_P^u = a_E^u + a_W^u + a_N^u + a_S^u + S_u$$

$$S_u = -(P_e - P_w)\Delta y = (P_w - P_e)\Delta y = \left(\frac{P_P + P_W}{2} - \frac{(P_E + P_P)}{2}\right)$$

$$= (P_W - P_E)\frac{\Delta y}{2}$$

At boundary: east: $(P_w - P_e)\Delta y = (P_w - P_E)\Delta y =$

$$= \left(\frac{P_W + P_P}{2} - P_E\right)\Delta y$$

and vice versa.

Simple Method

Cont. eq: $(\rho U_e - \rho U_w)\Delta y + (\rho V_n - \rho V_s)\Delta x = 0$

or: $(F_{x,e} - F_{x,w}) + (F_{y,n} - F_{y,s}) = 0$

$u = u^* + u'$, $v = v^* + v'$, $p = p^* + p'$

$$a_p^u u_p^* = \sum a_{nb}^u u_{nb}^* + (P_W^* - P_E^*)\frac{\Delta y}{2}$$

$$a_p^u v_p^* = \sum a_{nb}^u v_{nb}^* + (P_S^* - P_N^*)\frac{\Delta x}{2}$$

(guessed P^* or Previous iteration)

Solve with gauss-seidel u^*, v^*

Then Scale so one $u^*(I,j) = 0$.

Guessed continuity error:

$$b_p' = -(F_{x,e}^* - F_{x,w}^*) - (F_{y,n}^* - F_{y,s}^*)$$

$U_e^*, U_w^*, V_n^*, V_s^*$ from <u>Rhie-Chow Interpolation!</u>

for $\underline{\text{Correction}}$: $\underline{\text{Pressure correction}}$

$\boxed{\text{eq. 6}}$ in <u>not</u> staggered grid:

$$\left(\rho U_e \Delta y - \rho U_w \Delta y\right) + \left(\rho V_n \Delta x - \rho V_s \Delta x\right) = b_p'$$

For the true expression:

$$a_p^u\left(U_p - U_p^*\right) = \sum a_{nb}^u\left(U_{nb} - U_{nb}^*\right) + \left(\left(P_w - P_w^*\right) - \left(P_e - P_e^*\right)\right)\Delta Y$$

$$\Downarrow$$

$$a_p^u U_p' = \sum a_{nb}^u U_{nb}' + \left(P_w' - P_e'\right)\Delta Y$$

approx: drop $\sum$!

$$U_p' = \frac{\Delta Y}{a_p^u}\left(P_w' - P_e'\right) \quad \text{and same way:} \quad V_p' = \frac{\Delta X}{a_p^u}\left(P_S' - P_n'\right)$$

$$\Downarrow$$

$\boxed{\text{Put in eq. 6}}$

$$b_p' = \frac{\rho \Delta Y^2}{a_{p,e}^u}\left(P_p' - P_E'\right) + \frac{\rho \Delta Y^2}{a_{p,w}^u}\left(P_W' - P_p'\right) + \rho\frac{\Delta X^2}{a_{p,n}^u}\left(P_p' - P_N'\right) +$$

$$+ \rho\frac{\Delta X^2}{a_{p,S}^u}\left(P_S' - P_p'\right)$$

$$b_P' = P_P'\left(\frac{\rho\Delta y^2}{a_{P,e}^u} - \frac{\rho\Delta y^2}{a_{P,w}^u} + \frac{\rho\Delta x^2}{a_{P,n}^u} - \frac{\rho\Delta x^2}{a_{P,s}^u}\right) +$$

$$+ P_E'\left(-\frac{\rho\Delta y^2}{a_{P,e}^u}\right) + P_W'\left(\frac{\rho\Delta y^2}{a_{P,w}^u}\right) + P_N'\left(-\frac{\rho\Delta x^2}{a_{P,n}^u}\right) + P_S'\left(\frac{\rho\Delta x^2}{a_{P,s}^u}\right)$$

$$\Rightarrow \quad a_P P_P' = a_E P_E' + a_W P_W' + a_N P_N' + a_S P_S' + b_P'$$

$$a_E = \frac{\rho\Delta y^2}{a_{P,e}^u}, \quad a_W = -\frac{\rho\Delta y^2}{a_{P,w}^u}$$

$$a_N = \frac{\rho\Delta x^2}{a_{P,n}^u}, \quad a_S = -\frac{\rho\Delta x^2}{a_{P,s}^u}$$

$$a_P = a_E + a_W + a_N + a_S$$

$$b_P' = -\left(F_{x,e}^* - F_{x,w}^*\right) - \left(F_{y,n}^* - F_{y,s}^*\right)$$

$a_{P,b}$ from just pure interpolation!

Correction

$$p^{new} = p^* + \alpha_p p'$$

$$u^{new} = u^* + \frac{\Delta y\, d_u}{a_p^u}\left(p'_W - p'_E\right)$$

$$v^{new} = v^* + \frac{\Delta x\, d_u}{a_p^u}\left(p'_S - p'_N\right)$$

implicit

explicit

$$u^{new} = \alpha_u u + (1-\alpha_u) u^{(n-1)}$$

$$v^{new} = \alpha_u v + (1-\alpha_u) v^{(n-1)}$$

Interpolate Mass Fluxes: (Corrected)

$F_{x,e} =$

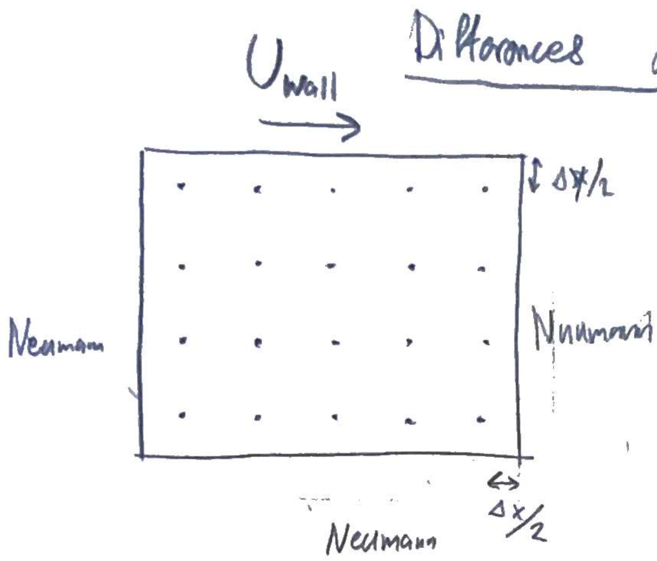
$F_{x,w} =$

$F_{x,n} =$

$F_{y,s} =$

Use these at the start of the next iteration!

Differences at the boundary.



no need to interpolate

For north boundary: $\text{Source} = \left(\frac{P_S + P_P}{2} - P_N\right)\Delta y + \left(2\mu \frac{\Delta x}{\Delta y} + \right.$

$+ \max(\underbrace{-F_{y,n}}, 0) \cdot U_{wall}$ Dirichlet cond.

already calculated mass flow

At all boundaries b: $a_b^u = 0$

For mass flow calculation:

no need to interpolate along boundaries here! eg. $U_e = U_E$

For pressure correction:

$a_b = 0$ at boundaries!