Task 3.

Momentum eqs:

$$x: \quad \rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial U}{\partial y} = -\frac{dp}{dx} + \mu \frac{\partial^2 U}{\partial x^2} + \mu \frac{\partial^2 U}{\partial y^2}$$

$$y: \quad \rho U \frac{\partial V}{\partial x} + \rho V \frac{\partial V}{\partial y} = -\frac{dp}{dy} + \mu \frac{\partial^2 V}{\partial x^2} + \mu \frac{\partial^2 V}{\partial y^2}$$

Cont. eq:
$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

Reorganized: only x:

$$\frac{\partial}{\partial x}\left(\rho UU - \mu \frac{\partial U}{\partial x} + p \right) + \frac{\partial}{\partial y}\left(\rho VU - \mu \frac{\partial U}{\partial y} \right) = 0$$

after gauss Thm:

$$\left((\rho UU)_e - (\rho UU)_w \right) \Delta y + \left((\rho VU)_n - (\rho VU)_s \right) \Delta x -$$

$$- \left(\left(\mu \frac{\partial U}{\partial x} \right)_e - \left(\mu \frac{\partial U}{\partial x} \right)_w \right) \Delta y - \left(\left(\mu \frac{\partial U}{\partial y} \right)_n - \left(\mu \frac{\partial U}{\partial y} \right)_s \right) \Delta x$$

$$+ \left(P_e - P_w \right) \Delta y = 0$$

$$a_E^u = \left(\mu \frac{\Delta y}{\Delta x} - \frac{F_{x,e}}{2} \right), \quad a_W^u = \left(\mu \frac{\Delta y}{\Delta x} + \frac{F_{x,w}}{2} \right)$$

$$a_N^u = \left(\mu \frac{\Delta x}{\Delta y} - \frac{F_{y,n}}{2} \right), \quad a_S^u = \left(\mu \frac{\Delta y}{\Delta x} - \frac{F_{y,s}}{2} \right)$$

Too avoid stability issues use upwind scheme:

$$a_E^u = \mu \frac{\Delta y}{\Delta x} + \max\left(-F_{x,e}, 0 \right), \quad a_W^u = \mu \frac{\Delta y}{\Delta x} + \max\left(F_{x,w}, 0 \right)$$

$$a_N^u = \mu \frac{\Delta x}{\Delta y} + \max\left(-F_{y,n}, 0 \right), \quad a_S^u = \mu \frac{\Delta x}{\Delta y} + \max\left(F_{y,s}, 0 \right)$$

$$a_P^u = a_E^u + a_W^u + a_N^u + a_S^u + S_u - S_P$$

$$S_u = -(P_e - P_w)\Delta y = (P_w - P_e)\Delta y = \left(\frac{P_P + P_W}{2} - \frac{(P_E + P_P)}{2} \right)$$

$$= \boxed{(P_W - P_E) \frac{\Delta y}{2}}$$

$$S_P = -(F_e - F_w) - (F_n - F_s)$$

At boundary: east: $(P_w - P_e)\Delta y = (P_w - P_E)\Delta y =$

$$= \left(\frac{P_w + P_P}{2} - P_E \right)\Delta y \quad \text{and vice versa.}$$

Massflux: $\quad F_x = \rho \Delta y \cdot U$; $\quad U, V$ interpolated:
$\quad\quad\quad\quad\quad F_y = \rho \Delta x \cdot V$

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use Rhie chow-interpolation!
for face velocities!

$$U_e = 0.5 \left(U_P + U_E \right) + \frac{dy}{4 a_{P,e}} \left(P_{EE} - 3 P_E + 3 P_P - P_W \right)$$

$$U_w = 0.5 \left(U_P + U_W \right) + \frac{dy}{4 a_{P,w}} \left(P_E - 3 P_P + 3 P_W - P_{WW} \right)$$

Where: $\quad a_{P,e} = \dfrac{a_{P,E} + a_{P,P}}{2}$ Simply.

Let: $\left(\dfrac{\partial U}{\partial x} \right)_e = \dfrac{U_E - U_P}{\Delta x}$, $\left(\dfrac{\partial U}{\partial x} \right)_w = \dfrac{U_P - U_W}{\Delta x}$ $\quad$ PSS V.

The final expression:

$$F_{x,e}U_e - F_{x,w}U_w + F_{y,n}U_n - F_{y,s}U_s -$$

$$- \mu\left(\left(\frac{U_E - U_p}{\Delta x}\right) - \left(\frac{U_p - U_w}{\Delta x}\right)\right)\Delta y - \mu\left(\left(\frac{U_N - U_p}{\Delta y}\right) - \left(\frac{U_p - U_s}{\Delta y}\right)\right)\Delta x$$

$$+ (P_e - P_w)\Delta y = 0 \qquad \Rightarrow$$

$$\Rightarrow F_{x,e}\left(\frac{U_E + U_p}{2}\right) - F_{x,w}\left(\frac{U_p + U_w}{2}\right) + F_{y,n}\left(\frac{U_N + U_p}{2}\right)$$

$$- F_{y,s}\left(\frac{U_p + U_s}{2}\right) - \mu\left(\frac{\Delta y}{\Delta x}\left(U_E - 2U_p + U_w\right) - \left(\frac{\Delta x}{\Delta y}\left(U_N - 2U_p + U_w\right)\right.\right.$$

$$+ (P_e - P_w)\Delta y = 0 \qquad \Rightarrow$$

$$\Rightarrow U_p\left(\frac{F_{x,e}}{2} - \frac{F_{x,w}}{2} + \frac{F_{y,n}}{2} - \frac{F_{y,s}}{2} + \frac{2\mu\Delta y}{\Delta x} - \frac{2\mu\Delta x}{\Delta y}\right) +$$

$$+ U_E\left(\frac{F_{x,e}}{2} - \mu\frac{\Delta y}{\Delta x}\right) + U_w\left(-\frac{F_{x,w}}{2} - \mu\frac{\Delta y}{\Delta x}\right) + $$

$$+ U_N\left(\frac{F_{y,n}}{2} - \mu\frac{\Delta x}{\Delta y}\right) + U_s\left(-\frac{F_{y,s}}{2} - \mu\frac{\Delta x}{\Delta y}\right) + (P_e - P_w)\Delta y = 0$$

Simple Method

Cont. eq:
$$\left(SU_e - SU_w\right)\Delta y + \left(SV_n - SV_s\right)\Delta x = 0$$

or:
$$\left(F_{x,e} - F_{x,w}\right) + \left(F_{y,n} - F_{y,s}\right) = 0$$

$$U = U^* + U', \qquad V = V^* + V', \qquad P = P^* + P'$$

$$a_p^u U_p^* = \sum a_{nb}^u U_{nb}^* + \left(P_W^* - P_E^*\right)\frac{\Delta y}{2}$$

$$a_p^u V_p^* = \sum a_{nb}^u V_{nb}^* + \left(P_S^* - P_N^*\right)\frac{\Delta x}{2} \qquad \left(\begin{array}{l}\text{guessed } P^* \text{ or}\\ \text{Previous iteration}\end{array}\right)$$

Solve with gauss-Saidel U^*, V^*

Then Scale So one $U^*(I,J) = 0$.

Guessed continuity error:

$$b_p' = -\left(F_{x,e}^* - F_{x,w}^*\right) - \left(F_{y,n}^* - F_{y,s}^*\right)$$

$U_e^*, U_w^*, V_n^*, V_s^*$ from <u>Rhie-Chow Interpolation</u>!

for Correction: Pressure correction

eq.6 in **not** staggered grid:

$$\left(\S U_e \Delta y - \S U_w \Delta y \right) + \left(\S V_n \Delta x - \S V_s \Delta x \right) = b_p'$$

For the true expression:

$$a_p^u \left(U_p - U_p^* \right) = \sum a_{nb}^u \left(U_{nb} - U_{nb}^* \right) + \left(\left(P_w - P_w^* \right) - \left(P_e - P_e^* \right) \right) \Delta y$$

$$\Downarrow$$

$$a_p^u U_p' = \sum a_{nb}^u U_{nb}' + \left(P_w' - P_e' \right) \Delta y$$

approx: drop $\sum$!

$$U_p' = \frac{\Delta y}{a_p^u} \left(P_w' - P_e' \right) \quad \text{and same way:} \quad V_p' = \frac{\Delta x}{a_p^u} \left(P_s' - P_n' \right)$$

$$\Downarrow$$

Put in eq. 6

$$b_p' = \frac{\S \Delta y^2}{a_{p,e}^u} \left(P_p' - P_E' \right) + \frac{\S \Delta y^2}{a_{p,w}^u} \left(P_W' - P_p' \right) + \S \frac{\Delta x^2}{a_{p,n}^u} \left(P_p' - P_N' \right) +$$

$$+ \S \frac{\Delta x^2}{a_{p,s}^u} \left(P_S' - P_p' \right)$$

$$b_p' = P_P'\left(\frac{S\Delta y^2}{a_{P,e}^u} - \frac{S\Delta y^2}{a_{P,w}^u} + \frac{S\Delta x^2}{a_{P,n}^u} - \frac{S\Delta x^2}{a_{P,S}^u}\right) +$$

$$+ P_E'\left(-\frac{S\Delta y^2}{a_{P,e}^u}\right) + P_w'\left(\frac{S\Delta y^2}{a_{P,w}^u}\right) + P_N'\left(-\frac{S\Delta x^2}{a_{P,n}^u}\right) + P_S'\left(S\frac{\Delta x^2}{a_{P,S}^u}\right)$$

$$\Rightarrow \quad a_P P_P' = a_E P_E' + a_W P_w' + a_N P_N' + a_S P_S' + b_p'$$

$$a_E = \frac{S\Delta y^2}{a_{P,e}^u} \quad , \quad a_w = -\frac{S\Delta y^2}{a_{P,w}^u}$$

$$a_N = \frac{S\Delta x^2}{a_{P,n}^u} \quad , \quad a_S = -\frac{S\Delta x^2}{a_{P,S}^u}$$

$$a_P = a_E + a_w + a_N + a_S$$

$$b_p' = -\left(F_{x,e}^* - F_{x,w}^*\right) - \left(F_{y,n}^* - F_{y,S}^*\right)$$

$a_{P,b}$ from just pure interpolation!

Correction

$$p^{new} = p^* + \alpha_p p'$$

$$U^{new} = U^* + \frac{\Delta y \, d_u}{a_p^u}\left(P_W' - P_E'\right) \Bigg\} \text{implicit}$$

$$V^{new} = V^* + \frac{\Delta x \, d_u}{a_p^u}\left(P_S' - P_N'\right)$$

explicit

$$U^{new} = d_u U + (1-d_u)U^{(n-1)}$$

$$V^{new} = d_u V + (1-d_u)V^{(n-1)}$$

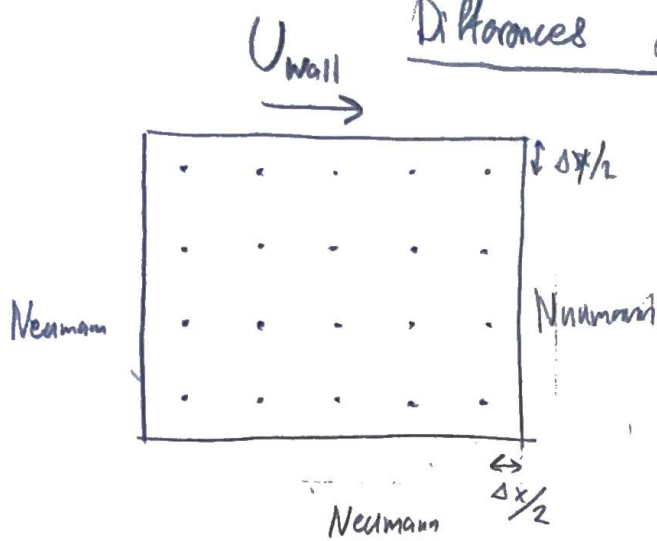## Interpolate Mass Fluxes: (Corrected)

$$F_{x,e} =$$

$$F_{x,w} =$$

$$F_{x,n} =$$

$$F_{y,s} =$$

Use these at the start of the next Iteration!

Differences at the boundary.

U_{wall} → (arrow pointing right)

Neumann (left), Neumann (right), Neumann (bottom)

$\downarrow \delta y / 2$

$\leftrightarrow \Delta x / 2$

For north boundary: $\quad Source = \left(\dfrac{P_S + P_P}{2} - P_N\right)\Delta y + \left(2\mu\dfrac{\Delta x}{\Delta y} + \right.$

no need to interpolate (with arrow)

$+ \max\left(-F_{y,n}, 0\right) \cdot U_{wall}$ Dirichlet cond.

$\underbrace{\phantom{-F_{y,n}}}$ already calculated mass flow

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At all boundaries b: $a_b^u = 0$

For mass flow calculation:

no need to interpolate along boundaries here! e.g. $U_e = U_E$

For pressure correction:

$a_b = 0$ at boundaries!