



ESCUELA TECNOLÓGICA INSTITUTO TÉCNICO CENTRAL (ETITC)

Facultad de sistemas

Taller 1: Operaciones Básicas con Números Complejos Matemáticas Especiales

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Presentado a:

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Gráfica y Módulo

Dado los números z_1 y z_2 , en cada uno de los ejercicios ① al ③ grafique en un plano complejo los números z_1 , z_2 , $z_1 + z_2$ y $z_1 - z_2$. Finalmente calcule el módulo de los cuatro números.

① $z_1 = 6 - 2i$; $z_2 = 2 - 5i$

$$z_1 + z_2 = 8 - 7i$$

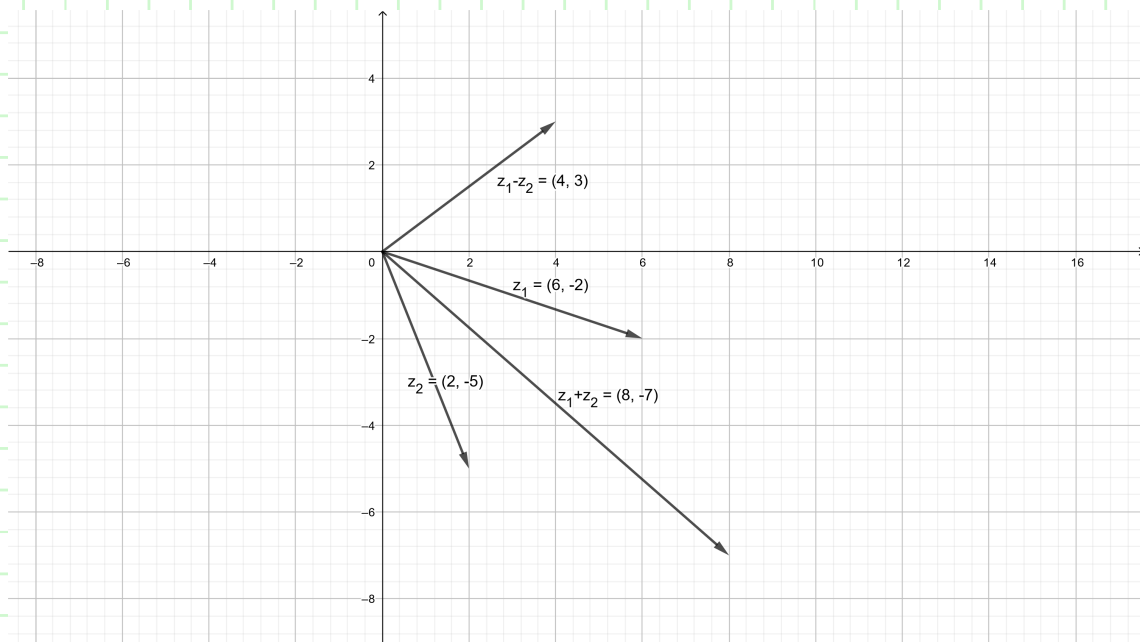
$$z_1 - z_2 = 4 + 3i$$

$$|z_1| = \sqrt{(6)^2 + (-2)^2} = \sqrt{36 + 4} = \boxed{2\sqrt{2}}$$

$$|z_2| = \sqrt{(2)^2 + (-5)^2} = \boxed{\sqrt{29}}$$

$$|z_1 + z_2| = \sqrt{(8)^2 + (-7)^2} = \sqrt{64 + 49} = \boxed{\sqrt{113}}$$

$$|z_1 - z_2| = \sqrt{(4)^2 + (3)^2} = \sqrt{16 + 9} = \sqrt{25} = \boxed{5}$$



② $z_1 = -5 + 5i$; $z_2 = -6 - 4i$

$$z_1 + z_2 = -11 + i$$

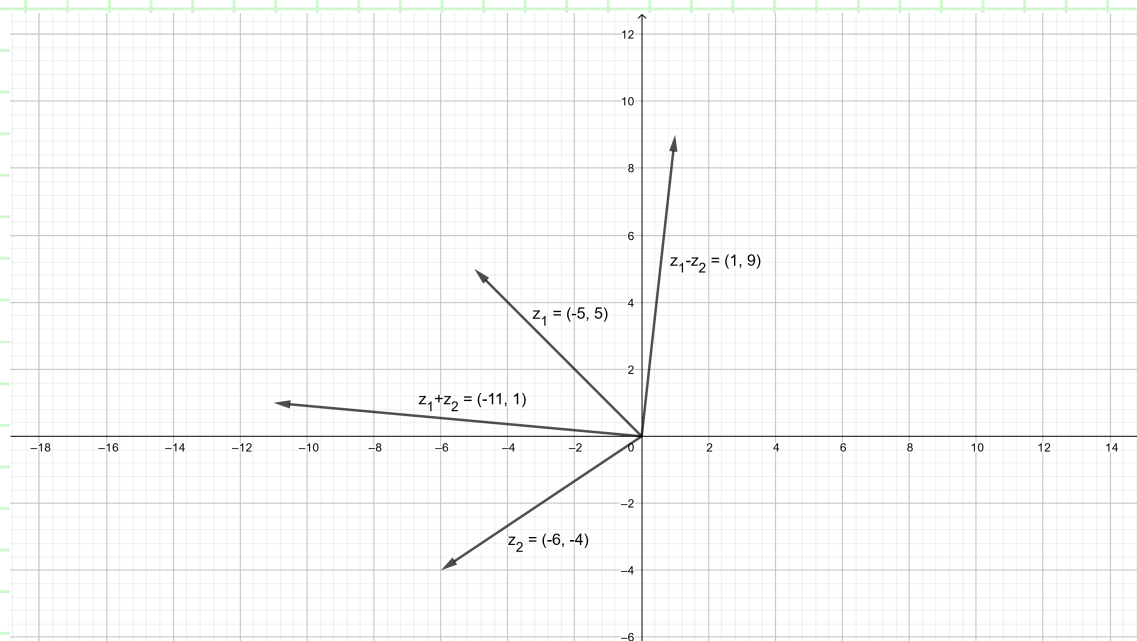
$$z_1 - z_2 = 1 + 9i$$

$$|z_1| = \sqrt{(5)^2 + (-5)^2} = \sqrt{25 + 25} = \boxed{5\sqrt{2}}$$

$$|z_2| = \sqrt{(-6)^2 + (-4)^2} = \sqrt{36 + 16} = \boxed{2\sqrt{13}}$$

$$|z_1 + z_2| = \sqrt{(-11)^2 + (1)^2} = \sqrt{121 + 1} = \boxed{\sqrt{122}}$$

$$|z_1 - z_2| = \sqrt{(1)^2 + (9)^2} = \sqrt{1 + 81} = \boxed{\sqrt{82}}$$



③ $z_1 = 8 - 6i$; $z_2 = -4 - 8i$

$$z_1 + z_2 = 4 - 14i$$

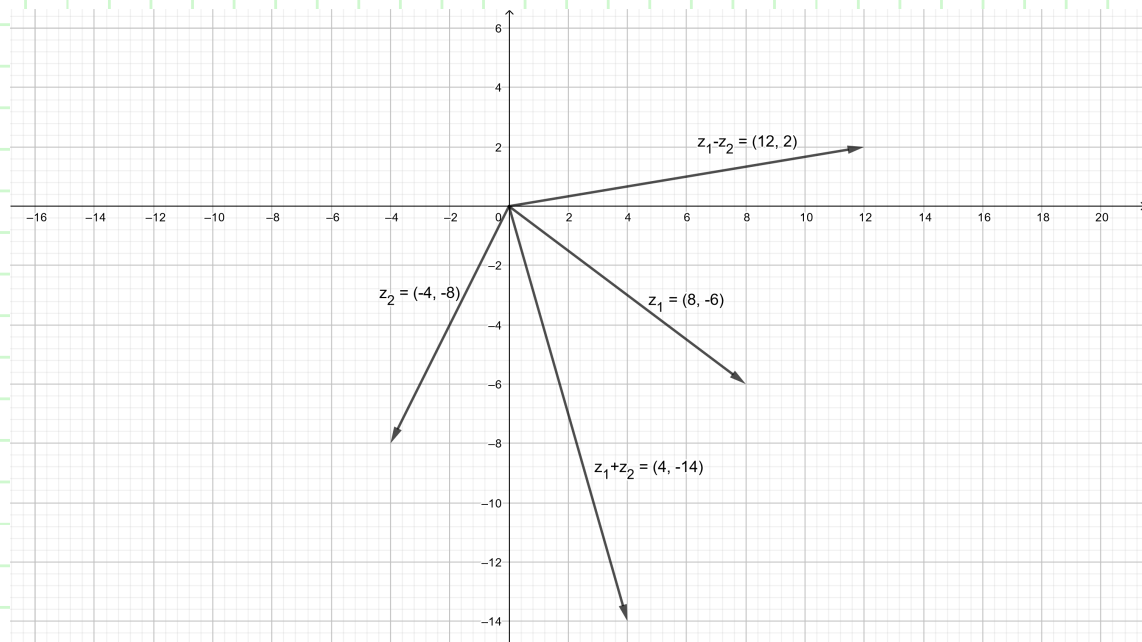
$$z_1 - z_2 = 12 + 2i$$

$$|z_1| = \sqrt{(8)^2 + (-6)^2} = \sqrt{64 + 36} = \boxed{10}$$

$$|z_2| = \sqrt{(4)^2 + (-8)^2} = \sqrt{16 + 64} = \boxed{4\sqrt{5}}$$

$$|z_1 + z_2| = \sqrt{(4)^2 + (-14)^2} = \sqrt{16 + 196} = \boxed{2\sqrt{53}}$$

$$|z_1 - z_2| = \sqrt{(12)^2 + (2)^2} = \sqrt{144 + 4} = \boxed{2\sqrt{37}}$$



Operaciones Aritméticas

En cada uno de los ejercicios ④ al ⑧ realice las operaciones indicadas simplificando tanto como sea posible (Recuerde que: $i^2 = -1$)

④ $3(-1 + 4i) - 2(7 - i)$

$$3(-1 + 4i) - 2(7 - i) = -3 + 12i - (14 - 2i)$$

$$= (-3 - 14) + i(12 - (-2)) = -17 + i(12 + 2) = -17 + 14i$$

⑤ $(i - 2)[2(1 + i) - 3(i - 1)]$

$$(i - 2)[2(1 + i) - 3(i - 1)] = (i - 2)[2 + 2i - (3i - 3)] = (i - 2)[(2 - (-3)) + i(2 - 3)]$$

$$= (i - 2)[(2 + 3) - i] = (-2 + i)(5 - i) = -10 + 2i + 5i - \underbrace{i^2}_{-1}$$

$$= (-10 + 1) + i(2 + 5) = -9 + 7i$$

⑥ $\frac{(2 + i)(3 - 2i)(1 + 2i)}{(1 - i)^2}$

⇒ Resolver el numerador:

$$[(2 + i)(3 - 2i)](1 + 2i) = [6 - 4i + 3i + 2](1 + 2i) = (8 - i)(1 + 2i)$$

$$= 8 + 16i - i + 2 = 10 + 15i$$

⇒ Resolver el denominador:

$$(1 - i)^2 = 1 + 2(1)(-i) + \underbrace{i^2}_{-1} = -2i$$

⇒ Dividir:

$$\left(\frac{10 + 15i}{-2i}\right) \left(\frac{2i}{2i}\right) = \frac{20i - 30}{4} = -\frac{30}{4} + \frac{20}{4}i = -\frac{15}{2} + 5i$$

$$\textcircled{7} \quad (2i - 1)^2 \left[\frac{4}{1 - i} + \frac{2 - i}{1 + i} \right]$$

$$\underbrace{(2i - 1)^2}_{z_1} \left[\underbrace{\frac{4}{1 - i}}_{z_2} + \underbrace{\frac{2 - i}{1 + i}}_{z_3} \right]$$

⇒ Resolver z_1 :

$$(2i - 1)^2 = -4 + 2(-1)(2i) + 1 = -3 - 4i$$

⇒ Resolver z_2 :

$$\left(\frac{4}{1 - i}\right) \left(\frac{1 + i}{1 + i}\right) = \frac{4 + 4i}{2} = 2 + 2i$$

⇒ Resolver z_3 :

$$\left(\frac{2 - i}{1 + i}\right) \left(\frac{1 - i}{1 - i}\right) = \frac{2 - 2i - i - 1}{1 - (-1)} = \frac{1 - 3i}{2} = \frac{1}{2} - \frac{3}{2}i$$

⇒ Resolver $z_2 + z_3$:

$$2 + 2i + \frac{1}{2} - \frac{3}{2}i = \frac{4 + 1}{2} + \frac{4 - 3}{2}i = \frac{5}{2} + \frac{1}{2}i$$

⇒ Resolver $z_1 [z_2 + z_3]$:

$$(-3 - 4i) \left(\frac{5}{2} + \frac{1}{2}i\right) = -\frac{15}{2} - \frac{3}{2}i - 10i + 2 = \frac{-15 + 4}{2} + i \left(\frac{-3 - 20}{2}\right) = -\frac{11}{2} - \frac{23}{2}i$$

$$\textcircled{8} \quad \frac{i^4 + i^9 + i^{16}}{2 - i^5 + i^{10} - i^{15}}$$

$$\frac{i^4 + i^9 + i^{16}}{2 - i^5 + i^{10} - i^{15}} = \frac{1 + i + 1}{2 - i - 1 + i} = \frac{2 + i}{1} = 2 + i$$