Escuela Tecnológica Instituto Técnico Central (ETITC)

Facultad de sistemas

Taller 1: Operaciones Básicas con Números Complejos
Matemáticas Especiales

Autores

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Presentado a:

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Gráfica y Módulo

Dado los números $\mathbf{z_1}$ y $\mathbf{z_2}$, en cada uno de los ejercicios ① al ③ grafique en un plano complejo los números $\mathbf{z_1}$, $\mathbf{z_2}$, $\mathbf{z_1} + \mathbf{z_2}$ y $\mathbf{z_1} - \mathbf{z_2}$. Finalmente calcule el módulo de los cuatro números.

① $\mathbf{z_1 = 6 - 2\,i} \quad ; \quad \mathbf{z_2 = 2 - 5\,i}$

$$z_1 + z_2 = 8 - 7\,i$$

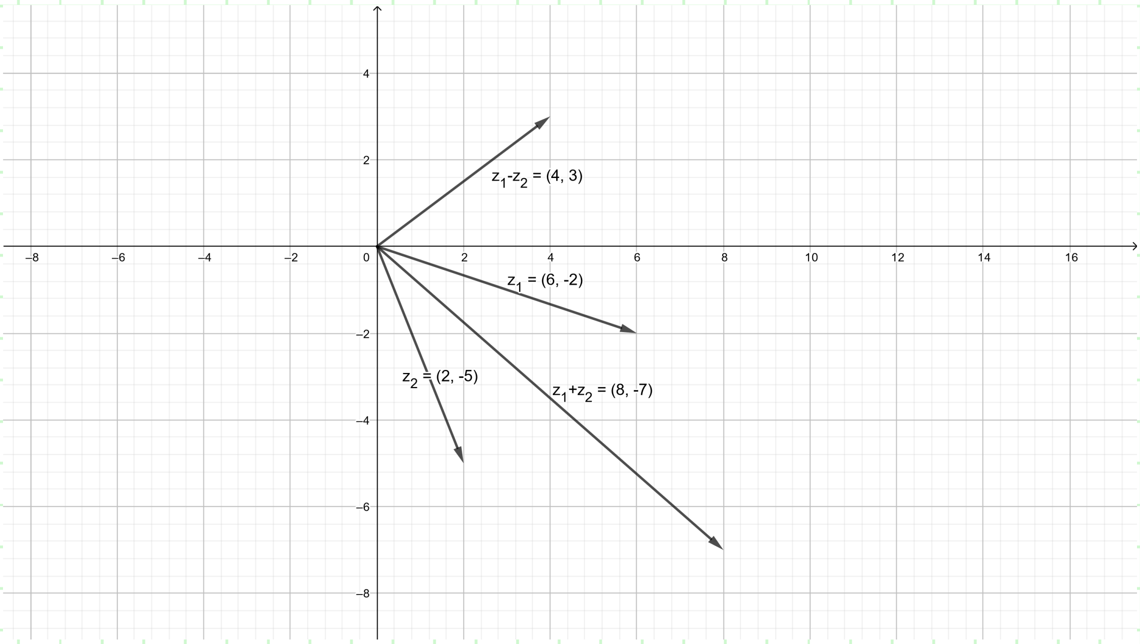
$$z_1 - z_2 = 4 + 3\,i$$

$$|z_1| = \sqrt{(6)^2 + (-2)^2} = \sqrt{36 + 4} = \boxed{2\sqrt{2}}$$

$$|z_2| = \sqrt{(2)^2 + (-5)^2} = \boxed{\sqrt{29}}$$

$$|z_1 + z_2| = \sqrt{(8)^2 + (-7)^2} = \sqrt{64 + 49} = \boxed{\sqrt{113}}$$

$$|z_1 - z_2| = \sqrt{(4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = \boxed{5}$$



② $\mathbf{z_1 = -5 + 5\,i} \quad ; \quad \mathbf{z_2 = -6 - 4\,i}$

$$z_1 + z_2 = -11 + i$$

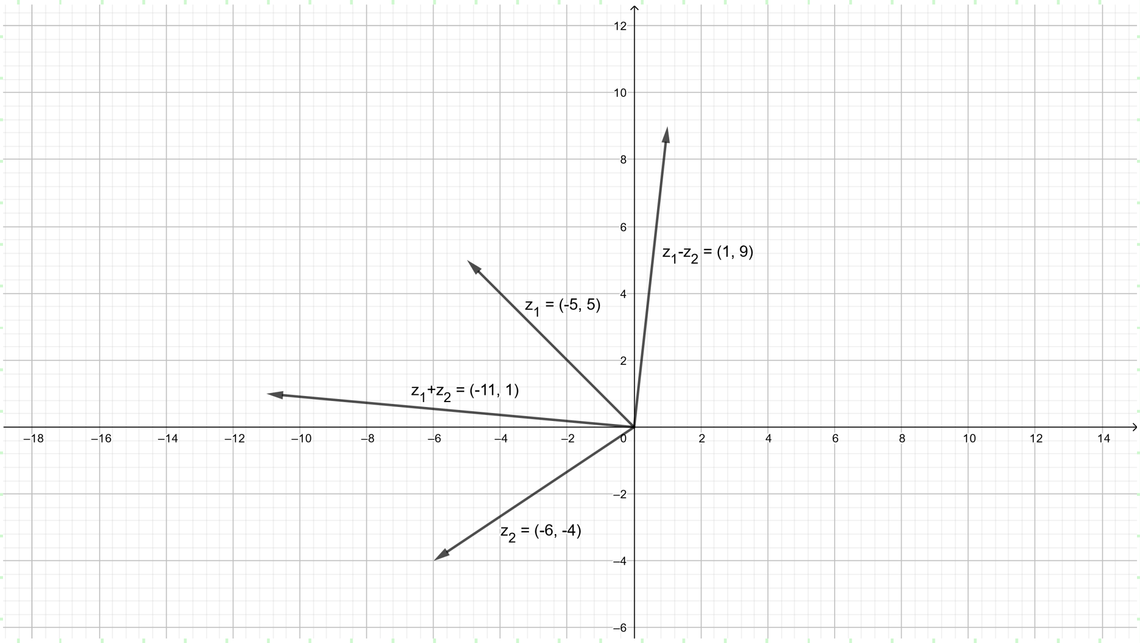
$$z_1 - z_2 = 1 + 9\,i$$

$$|z_1| = \sqrt{(5)^2 + (-5)^2} = \sqrt{25 + 25} = \boxed{5\sqrt{26}}$$

$$|z_2| = \sqrt{(-6)^2 + (-4)^2} = \sqrt{36 + 16} = \boxed{2\sqrt{13}}$$

$$|z_1 + z_2| = \sqrt{(-11)^2 + (1)^2} = \sqrt{121 + 1} = \boxed{\sqrt{122}}$$

$$|z_1 - z_2| = \sqrt{(1)^2 + (9)^2} = \sqrt{1 + 81} = \boxed{\sqrt{82}}$$



③ $\mathbf{z_1 = 8 - 6i}$; $\mathbf{z_2 = -4 - 8i}$

$$z_1 + z_2 = 4 - 14i$$

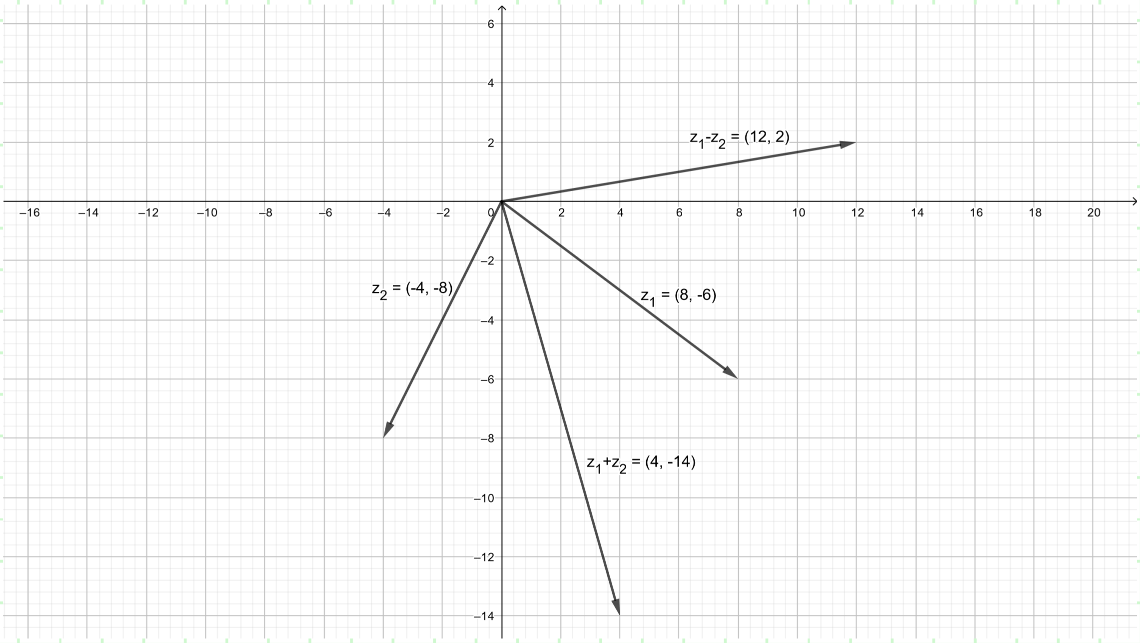
$$z_1 - z_2 = 12 + 2i$$

$$|z_1| = \sqrt{(8)^2 + (-6)^2} = \sqrt{64 + 36} = \boxed{10}$$

$$|z_2| = \sqrt{(4)^2 + (-8)^2} = \sqrt{16 + 64} = \boxed{4\sqrt{5}}$$

$$|z_1 + z_2| = \sqrt{(4)^2 + (-14)^2} = \sqrt{16 + 196} = \boxed{2\sqrt{53}}$$

$$|z_1 - z_2| = \sqrt{(12)^2 + (2)^2} = \sqrt{144 + 4} = \boxed{2\sqrt{37}}$$



Operaciones Aritméticas

En cada uno de los ejercicios ④ al ⑧ realice las operaciones indicadas simplificando tanto como sea posible (Recuerde que: $\mathbf{i^2 = -1}$)

④ $\mathbf{3\,(-1 + 4\,i) - 2\,(7 - i)}$

$$3\,(-1 + 4\,i) - 2\,(7 - i) = -3 + 12\,i - (14 - 2\,i)$$

$$= (-3 - 14) + i\,(12 - (-2)) = -17 + i\,(12 + 2) = -17 + 14\,i$$

⑤ $\mathbf{(i - 2)\,[2\,(1 + i) - 3\,(i - 1)]}$

$$(i - 2)\,[2\,(1 + i) - 3\,(i - 1)] = (i - 2)\,[2 + 2\,i - (3\,i - 3)] = (i - 2)\,[(2 - (-3)) + i\,(2 - 3)]$$

$$= (i - 2)\,[(2 + 3) - i] = (-2 + i)\,(5 - i) = -10 + 2\,i + 5\,i - \underbrace{i^2}_{\mathbf{-1}}$$

$$= (-10 + 1) + i\,(2 + 5) = -9 + 7\,i$$

⑥ $\mathbf{\dfrac{(2 + i)\,(3 - 2\,i)\,(1 + 2\,i)}{(1 - i)^2}}$

✏ Resolver el numerador:

$$[(2 + i)\,(3 - 2\,i)]\,(1 + 2\,i) = [6 - 4\,i + 3\,i + 4]\,(1 + 2\,i) = (10 - i)\,(1 + 2\,i)$$

$$= 10 + 20\,i - i + 2 = 12 - 19\,i$$

✏ Resolver el denominador:

$$(1 - i)^2 = 1 + 2\,(1)\,(-i) + \underbrace{i^2}_{-\mathbf{1}} = -2\,i$$

✏ Dividir:

$$\left(\frac{12 - 19\,i}{-2\,i}\right)\left(\frac{2\,i}{2\,i}\right) = \frac{24\,i + 38}{4} = \frac{38}{4} + \frac{24}{4}\,i = \frac{19}{2} + 6\,i$$

⑦ $\mathbf{(2\,i - 1)^2\left[\dfrac{4}{1 - i} + \dfrac{2 - i}{1 + i}\right]}$

$$\underbrace{(2\,i - 1)^2}_{\mathbf{z_1}}\left[\underbrace{\frac{4}{1 - i}}_{\mathbf{z_2}} + \underbrace{\frac{2 - i}{1 + i}}_{\mathbf{z_3}}\right]$$

✏ Resolver $\mathbf{z_1}$:

$$(2\,i - 1)^2 = -4 + 2\,(-1)\,(2\,i) + 1 = -3 - 4\,i$$

✏ Resolver $\mathbf{z_2}$:

$$\left(\frac{4}{1 - i}\right)\left(\frac{1 + i}{1 + i}\right) = \frac{4 + 4\,i}{2} = 2 + 2\,i$$

✏ Resolver $\mathbf{z_3}$:

$$\left(\frac{2 - i}{1 + i}\right)\left(\frac{1 - i}{1 - i}\right) = \frac{2 - 2\,i - i - 1}{1 - (-1)} = \frac{1 - 3\,i}{2} = \frac{1}{2} - \frac{3}{2}\,i$$

✏ Resolver $\mathbf{z_2} + \mathbf{z_3}$:

$$2 + 2\,i + \frac{1}{2} - \frac{3}{2}\,i = \frac{4 + 1}{2} + \frac{4 - 3}{2}\,i = \frac{5}{2} + \frac{1}{2}\,i$$

✏ Resolver $\mathbf{z_1}\,[\mathbf{z_2} + \mathbf{z_3}]$:

$$(-3 - 4\,i)\left(\frac{5}{2} + \frac{1}{2}\,i\right) = -\frac{15}{2} - \frac{3}{2}\,i - 10\,i + 2 = \frac{-15 + 4}{2} + i\left(\frac{-3 - 20}{2}\right) = -\frac{11}{2} - \frac{23}{2}\,i$$

⑧ $\mathbf{\dfrac{i^4 + i^9 + i^{16}}{2 - i^5 + i^{10} - i^{15}}}$

$$\frac{i^4 + i^9 + i^{16}}{2 - i^5 + i^{10} - i^{15}} = \frac{1 + i + 1}{2 - i - 1 + i} = \frac{2 + i}{1} = 2 + i$$