# Escuela Tecnológica Instituto Técnico Central (ETITC)

Facultad de sistemas

## Taller 4 : Funciones, Límites y Derivadas en los Complejos
## Matemáticas Especiales

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**Presentado a:**

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**Funciones**

En los ejercicio **( 1 )** al **( 5 )** sea $f(z)$ la función que que actúa sobre el conjunto dado $S$. Hallar la imágen $S'$ correspondiente a cada conjunto y graficar su respectivo mapeo.

( 1 ) $S$ es $\boxed{\mathbf{y = 1 - x}}$ y $\boxed{\mathbf{f(z) = z + i\,(i - 2)}}$.

$$\underbrace{y = 1 - x}_{E_1}$$

✎ Sustituir $z = x + i\,y$ en $f(z)$.

$$f(x + i\,y) = x + i\,y + i\,(i - 2) = x + i\,y + \underbrace{i^2}_{-1} - 2\,i = x + i\,y - 1 - 2\,i$$

$$w = \underbrace{(x - 1)}_{u(x\,,\,y)} + i\,\underbrace{(y - 2)}_{v(x\,,\,y)}$$

$$\underbrace{u = x - 1}_{E_2} \quad ; \quad \underbrace{v = y - 2}_{E_3}$$

✎ Despejar $x$ en $E_1$.

$$y = 1 - x \iff \underbrace{x = 1 - y}_{E_4}$$

✎ Sustituir $E_4$ en $E_2$.

$$u = 1 - y - 1 \iff \underbrace{u = -y}_{E_5}$$

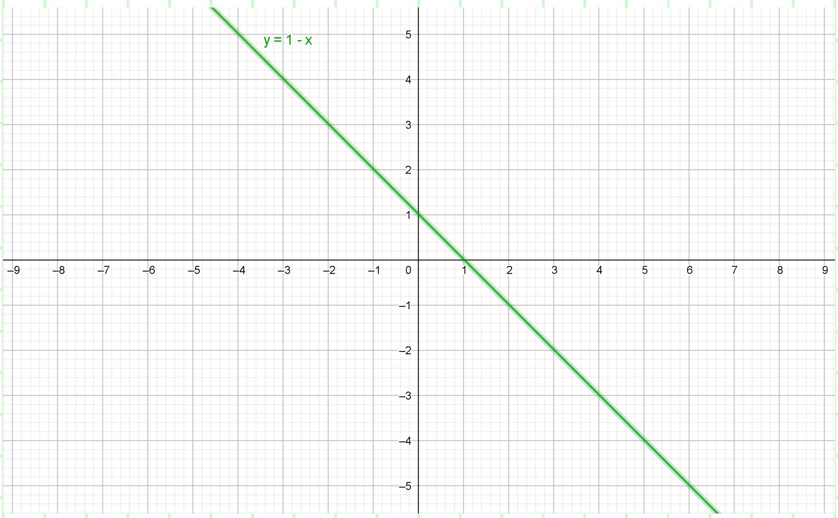
✎ Despejar $y$ en $E_5$.

$$u = -y \iff \underbrace{y = -u}_{E_6}$$

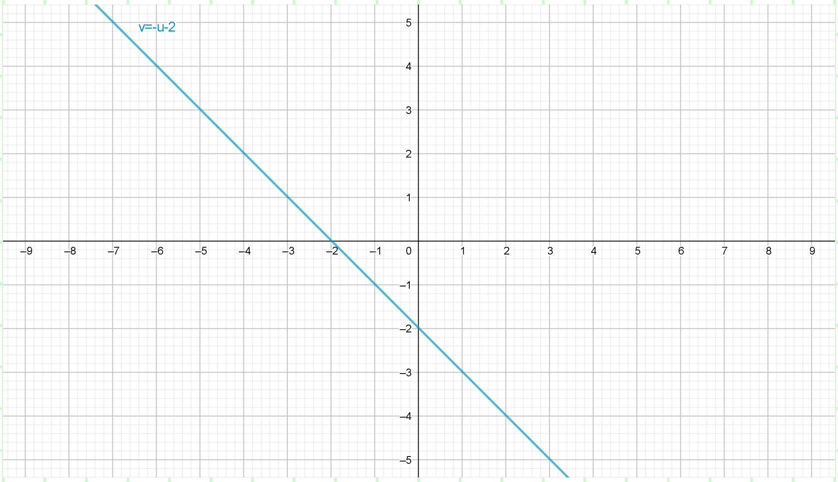
✎ Sustituir $E_6$ en $E_3$.

$$v = -u - 2$$

✎ Mapeo z-plano.



✎ Mapeo w-plano.




✎ Punto de prueba.

$$z_1 = 5 - 4i$$

$$f(z_1) = z_1 + i\,(i - 2) = 5 - 4i + i\,(i - 2) = 5 - 4i - 1 - 2\,i = 4 - 6\,i$$

$$z_2 = -3 + 4i$$

$$f(z_2) = z_2 + i\,(i - 2) = -3 + 4i + i\,(i - 2) = -3 + 4i - 1 - 2\,i = -4 + 2\,i$$

(2) $S$ es $\boxed{\mathbf{1} < Re(\mathbf{z}) < \mathbf{4}}$ y $\boxed{\mathbf{f(z) = 3\,z}}$.

$$f(z) = 3\,z \iff 3\,x + 3\,i\,y$$

$$U = 3\,x \quad V = 3\,y$$

Despejando en terminos de x Y y

$$x = \frac{U}{3}$$

$$y = \frac{V}{3}$$

Remplazo x en S

$$1 < x < 4 \iff 1 < \frac{U}{3} < 4$$

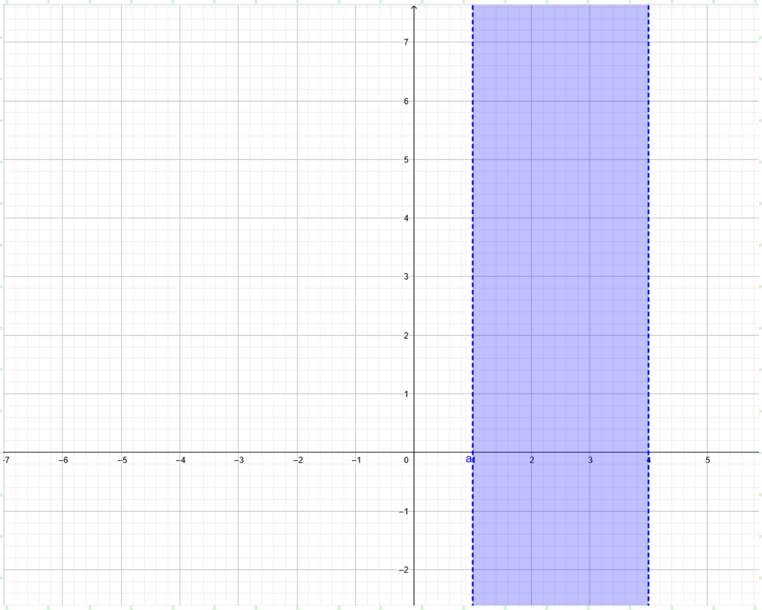
$$3 < U < 12$$

Remplazo y segun S

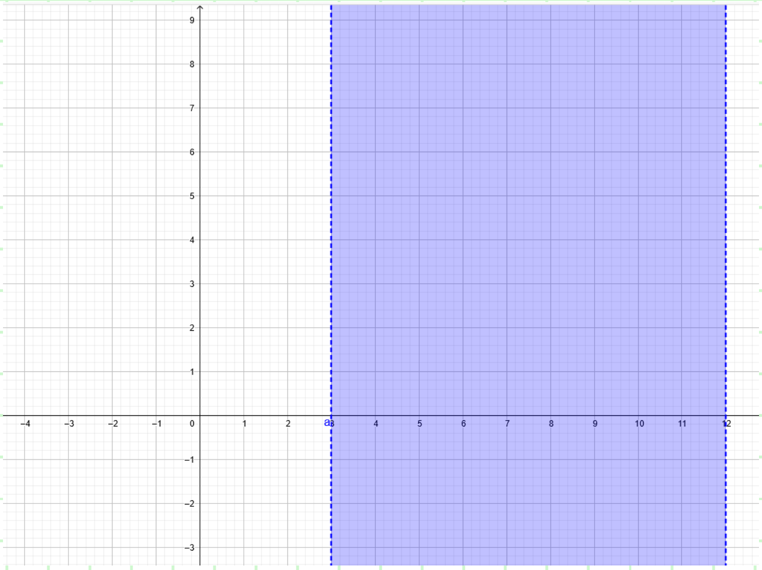
$$-\infty < y < \infty$$

$$-\infty < \frac{V}{3} < \infty \iff -\infty < V < \infty$$

✎ Mapeo z-plano.



✎ Mapeo w-plano.



(3) $S$ es $\boxed{\mathbf{y} = -\mathbf{2}}$ y $\boxed{\mathbf{f}(\mathbf{z}) = \mathbf{2}\,\mathbf{i} + \mathbf{z}\,(\mathbf{1} + \mathbf{i})}$.

$$\underbrace{y = -2}_{E_1}$$

✎ Sustituir $z = x + i\,y$ en $f(z)$.

$$f(x + i\,y) = 2\,i + (x + i\,y)\,(1 + i) = 2\,i + x + x\,i + y\,i - y$$

$$w = \underbrace{(x - y)}_{u(x\,,\,y)} + i\,\underbrace{(2 + x + y)}_{v(x\,,\,y)}$$

$$\underbrace{u = x - y}_{E_2} \quad ; \quad \underbrace{v = 2 + x + y}_{E_3}$$

✎ Sustituir $E_1$ en $E_2$ y $E_3$.

$$u = x - (-2) = x + 2$$

$$\underbrace{u = x + 2}_{E_4}$$

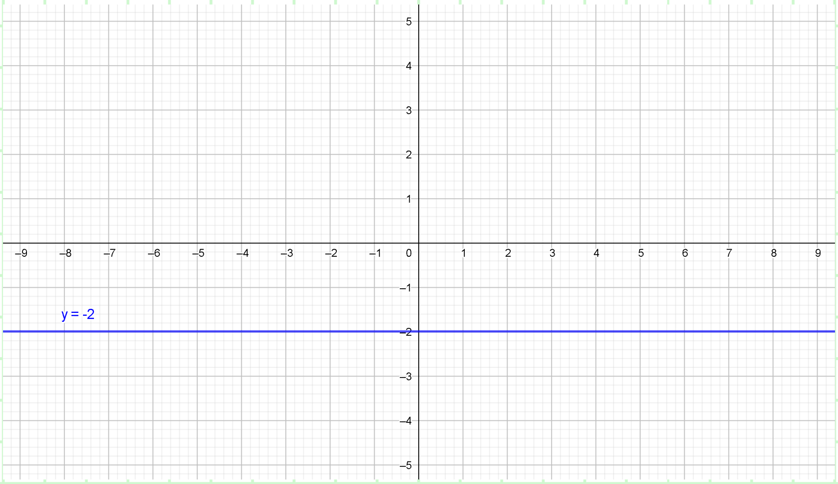
$$v = 2 + x + (-2) = 2 + x - 2 = x$$

$$v = x \iff \underbrace{x = v}_{E_5}$$

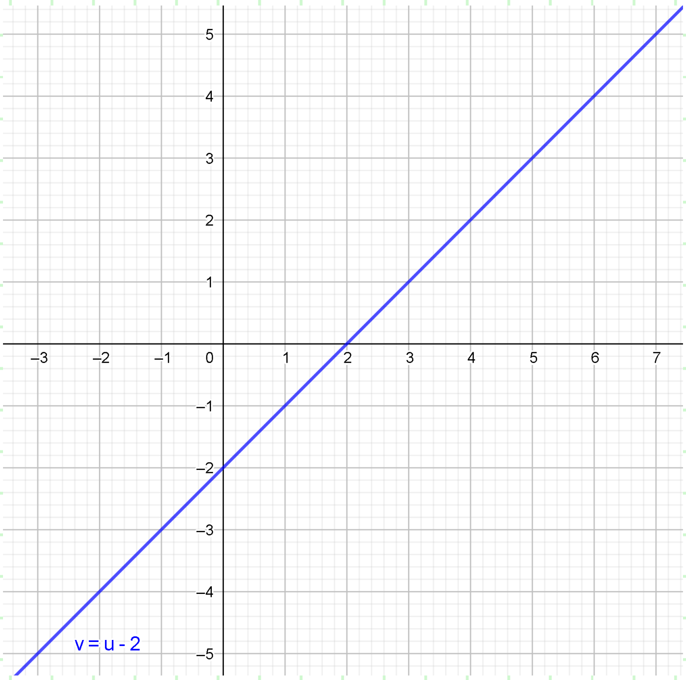
✎ Sustituir $E_5$ en $E_4$ y despejar $v$.

$$u = v + 2 \iff v = u - 2$$

✎ Mapeo z-plano.




✎ Mapeo w-plano.



(4) $S$ es $\boxed{\mathbf{-1 < Im(z) < 2}}$ y $\boxed{\mathbf{f(z) = i\,z + 4}}$.

✎ Sustituir $z = x + i\,y$ en $S$.

$$-1 < Im(z) < 2 \iff -1 < Im(x + i\,y) < 2 \iff \underbrace{-1 < y < 2}_{E_1}$$

✎ Sustituir $z = x + i\,y$ en $f(z)$.

$$f(x + i\,y) = x\,i + i^2 y + 4 = x\,i - y + 4$$

$$w = \underbrace{(4 - y)}_{u(x\,,y)} + i\,\underbrace{(x)}_{v(x\,,y)}$$

$$u = 4 - y \iff \underbrace{y = 4 - u}_{E_2} \quad ; \quad \underbrace{v = x}_{E_3}$$

✎ Sustituir $E_2$ en $E_1$.

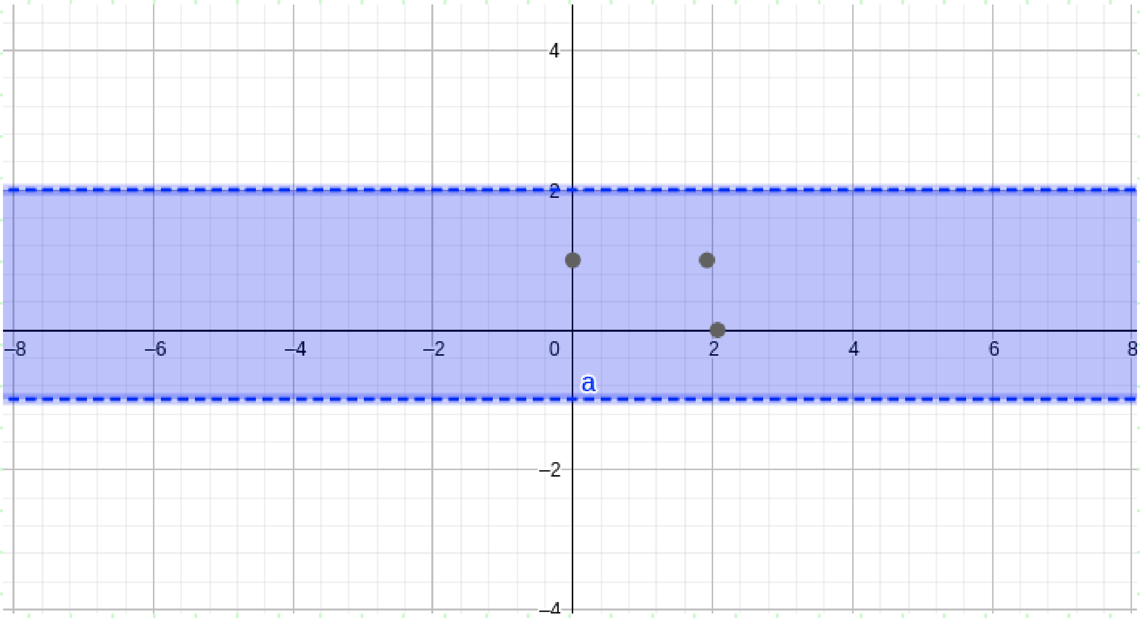
$$-1 < 4 - u < 2 \iff -5 < -u < -2 \iff 5 > u > 2$$

$$-\infty < x < \infty$$

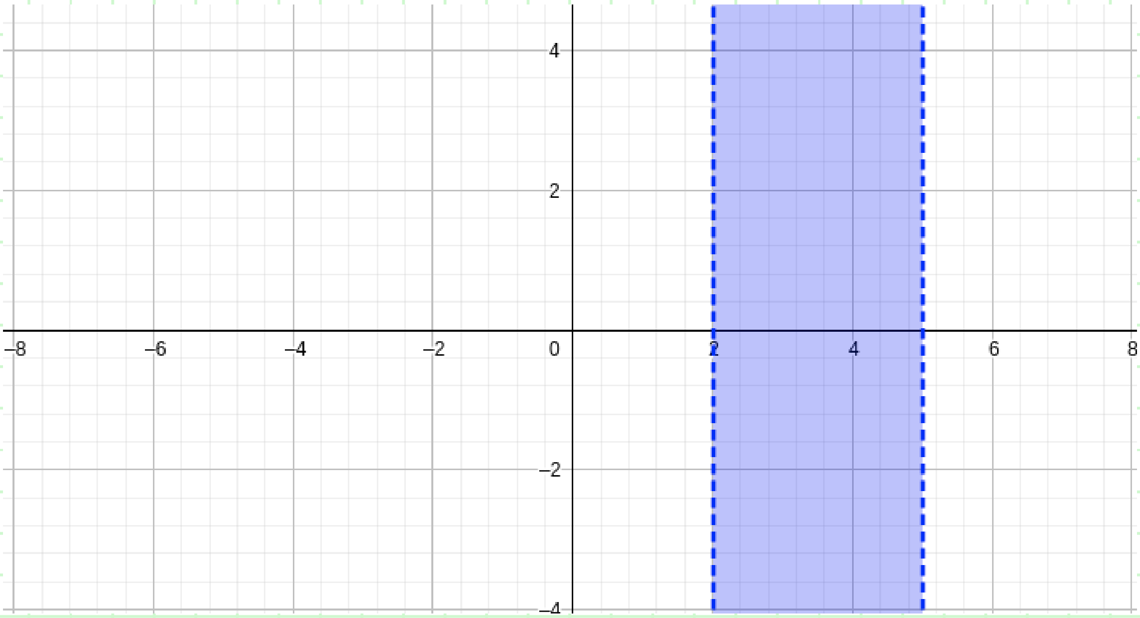
✎ Como en $E_2$ v = x , luego:

$$-\infty < v < \infty$$

✎ Mapeo z-plano.



✎ Mapeo w-plano.




(5) $S$ es $\boxed{Re(\mathbf{z}) > \mathbf{3}}$ y $\boxed{\mathbf{f}(\mathbf{z}) = \mathbf{z^2}}$.

✎ Sustituir $z = x + i\,y$ en $E_1$.

$$Re(z) > 3 \iff Re(x + i\,y) > 3 \iff \underbrace{x > 3}_{E_1}$$

✎ Sustituir $z = x + i\,y$ en $f(z)$.

$$f(x + i\,y) = (x + i\,y)^2 = x^2 + 2\,(x)\,(i\,y) + (i\,y)^2 = x^2 + i\,2\,x\,y - y^2$$

$$w = \underbrace{(x^2 - y^2)}_{u(x\,,\,y)} + i\,\underbrace{(2\,x\,y)}_{v(x\,,\,y)}$$

$$\underbrace{u = x^2 - y^2}_{E_2} \quad ; \quad \underbrace{v = 2\,x\,y}_{E_3}$$

✎ Sustituir $x = 3$ en $E_2$ y despejar $y$.

$$u = 3^2 - y^2 \iff u = 9 - y^2 \iff y^2 = 9 - u \iff \underbrace{y = \sqrt{9 - u}}_{E_4}$$

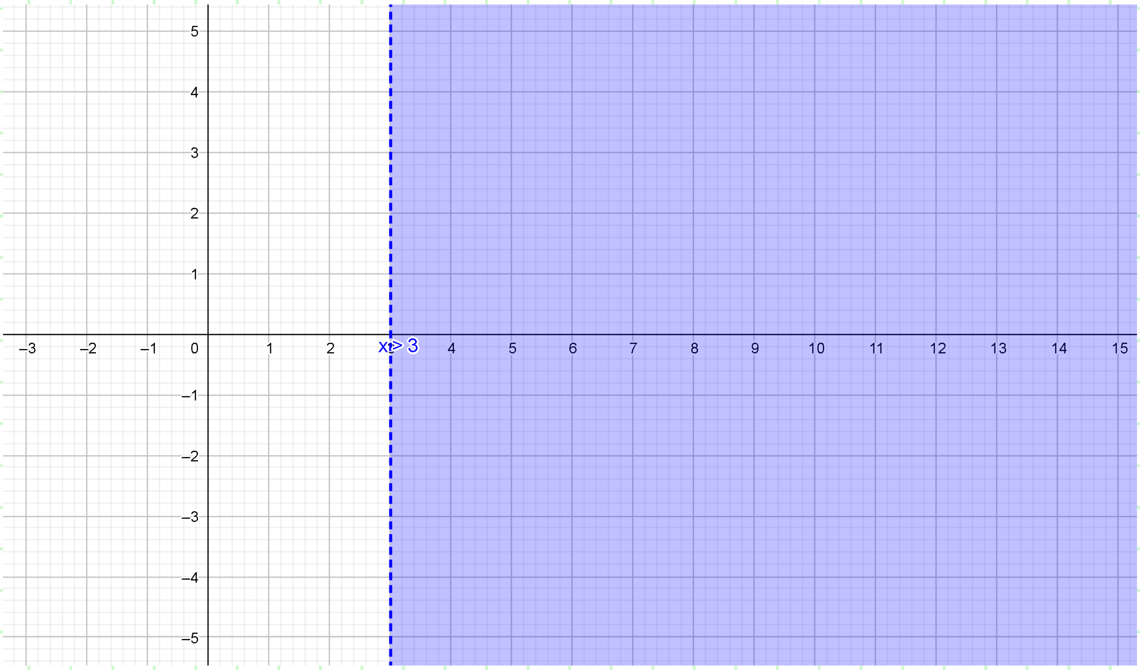
✎ Despejar x en $E_3$ y sustituir en $E_1$.

$$v = 2\,x\,y \iff x = \frac{v}{2\,y} \iff \frac{v}{2\,y} > 3 \iff \underbrace{v > 6\,y}_{E_5}$$

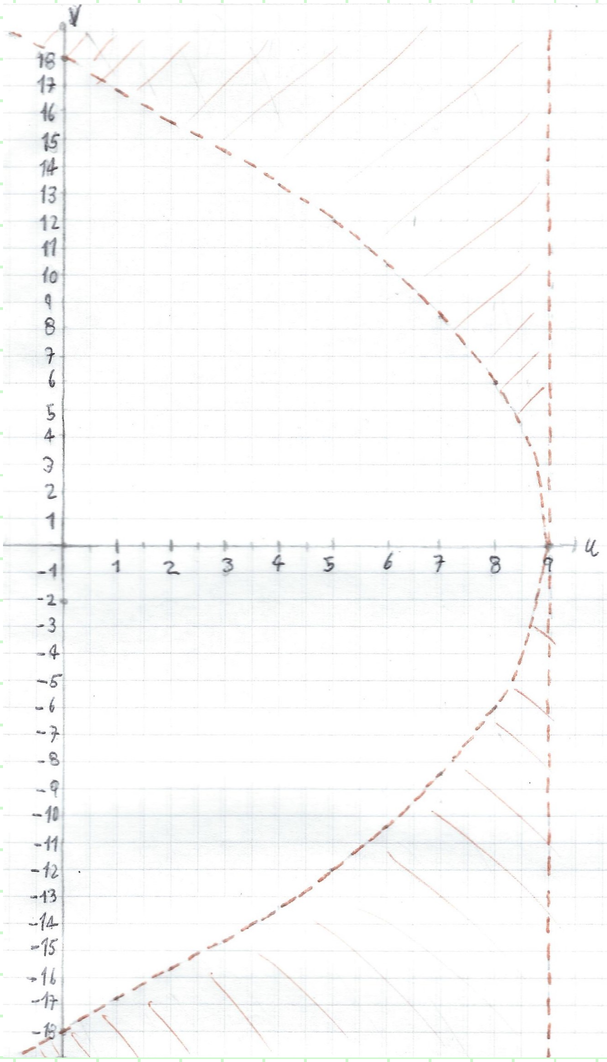
✎ Sustituir $E_4$ en $E_5$

$$v > 6\,\sqrt{9 - u}$$

✎ Mapeo z-plano.




✎ Mapeo w-plano.



✎ Punto de prueba.

$$z_1 = 5 - 4i$$

$$f(z_1) = (z_1)^2 = (5 - 4i)^2 = 25 - 40\,i - 16 = 9 - 40\,i$$

$$z_2 = 3 + 2i$$

$$f(z_2) = (z_2)^2 = (3 + 2i)^2 = 9 + 12\,i - 4 = 5 + 12\,i$$

**Límites**

En los ejercicio **( 6 )** al **( 11 )** use las **propiedades de los límites** para calcular los límites indicados.

( 6 ) $\lim\limits_{\mathbf{z \to 2\,i}} (\mathbf{z^2} - \overline{\mathbf{z}})$

$$(\,2\,i\,)^2 + 2\,i = \boxed{-4 + 2\,i}$$

( 7 ) $\lim\limits_{\mathbf{z \to 3\,i}} \dfrac{Im(\mathbf{z^2})}{\mathbf{z} + Re(\mathbf{z})}$

$$\lim_{z \to 3\,i} \frac{Im(z^2)}{z + Re(z)} = \frac{Im((3\,i)^2)}{3\,i + Re(3\,i)} = \frac{Im(9\,i^2)}{3\,i + 0} = \frac{Im(-9)}{3\,i} = \frac{0}{3\,i} = 0$$

( 8 ) $\lim\limits_{\mathbf{z \to 1 + i}} \dfrac{\mathbf{z^2 + 1}}{\mathbf{z^2 - 1}}$

$$\frac{(\,1 + i\,)^2 + 1}{(\,1 + i)^2 - 1}$$

$$(\,1 + i\,)^2 = 2\,i$$

$$\frac{2\,i + 1}{2\,i - 1} \iff \frac{1 + 2\,i}{-1 + 2\,i} \times \frac{-1 - 2\,i}{-1 - 2\,i} = \frac{-1 - 2\,i - 2\,i + 4}{(-1)^2 - (2\,i)^2} = \frac{3 - 4\,i}{1 + 4} = \frac{3 - 4\,i}{5} = \boxed{\frac{3}{5} - \frac{4\,i}{5}}$$

( 9 ) $\lim\limits_{\mathbf{z \to 3 + i\sqrt{2}}} \dfrac{\mathbf{z + 3 - i\sqrt{2}}}{\mathbf{z^2 + 6\,z + 11}}$

$$= \frac{3 + i\sqrt{2} + 3 - i\sqrt{2}}{(3 + i\sqrt{2})^2 + 6\,(3 + i\sqrt{2}) + 11} = \frac{6}{9 + 6\,i\sqrt{2} - 2 + 18 + 6\,i\sqrt{2} + 11}$$

$$= \frac{6}{36 + 12\,i\sqrt{2}} \times \frac{36 - 12\,i\sqrt{2}}{36 - 12\,i\sqrt{2}} = \frac{216 - 72\,i\sqrt{2}}{(36)^2 + (12\,i\sqrt{2})^2} = \frac{216 - 72\,i\sqrt{2}}{1296 + 288}$$

$$= \frac{216 - 72\,i\sqrt{2}}{1584} = \frac{216}{1584} - \frac{72\,i\sqrt{2}}{1584} = \frac{3}{22} - \frac{i\sqrt{2}}{22}$$

( 10 ) $\lim\limits_{\mathbf{z \to 2}} \dfrac{\mathbf{z\,2 - 5\,z + 6}}{\mathbf{z^2 - 4}}$

$$\frac{2\,(2) - 5\,(2) + 6}{(2)^2 - 4} \iff \frac{4 - 10 + 6}{4 - 4} \iff \frac{0}{0}$$

Aplicando Hopital:

$$f'(z_0) = \lim_{z\to 2} \frac{2 - 5}{2\,(2)} \iff \frac{-3}{4} \iff \boxed{-\frac{3}{4}}$$

(11) $\lim_{\mathbf{z}\to 3\,\mathbf{i}} \dfrac{\mathbf{z^4 + 10\,z^2 + 9}}{\mathbf{z^2 - 4\,i\,z - 3}}$

$$\lim_{z\to 3\,i} \frac{z^4 + 10\,z^2 + 9}{z^2 - 4\,i\,z - 3} = \frac{(3\,i)^4 + 10\,(3\,i)^2 + 9}{(3\,i)^2 - 4\,i\,(3\,i) - 3} = \frac{81\,i^4 + 90\,i^2 + 9}{9\,i^2 - 12\,i^2 - 3}$$

$$= \frac{81 - 90 + 9}{-9 + 12 - 3} = \frac{90 - 90}{12 - 12} = \frac{0}{0}$$

✎ Usamos la regla de L'Hopital.

$$= \lim_{z\to 3\,i} \frac{4\,z^3 + 20\,z}{2\,z - 4\,i} = \frac{4\,(3\,i)^3 + 20\,(3\,i)}{2\,(3\,i) - 4\,i} = \frac{4\,(-27\,i) + 60\,i}{6\,i - 4\,i} = \frac{-108\,i + 60\,i}{2\,i} = \frac{-48\,i}{2\,i}$$

$$= \frac{-48\,i}{2\,i} \times \frac{-2\,i}{-2\,i} = \frac{96\,i^2}{-4\,i^2} = \frac{-96}{4} = -24$$

**Derivadas**

(12) Use la fórmula de la **Definición de Derivada** mostrada a continuación:

$$\mathbf{f'(z_0) = \lim_{\Delta z \to 0} \frac{f(z_0 + \Delta z) - f(z_0)}{\Delta z}}$$

para calcular la derivada de la función $\mathbf{f(z) = z}\,Im(\mathbf{z})$ en los puntos $\mathbf{z_0 = 0}$, $\mathbf{z_0 = 1 + i}$ y $\mathbf{z_0 = -2 - i}$. ¿Ésta función es Analítica?

**Recordemos que:**

☛ $z_0 = 0 \qquad \iff \qquad z_0 = 0 + 0\,i.$

☛ $\Delta\,z = \Delta\,x + i\,\Delta\,y$

✎ Calcular la derivada del punto $z_0 = 0$.

✏ Construir $f(z_0 + \Delta\,z)$ y $f(z_0)$.

$$f(z_0 + \Delta\,z) = (z_0 + \Delta\,z)\,Im(z_0 + \Delta\,z) = (0 + 0\,i + \Delta\,x + i\,\Delta\,y)\,Im(0 + 0\,i + \Delta\,x + i\,\Delta\,y)$$

$$= [(0 + \Delta\,x) + i(0 + \Delta\,y)]\,Im\,[(0 + \Delta\,x) + i(0 + \Delta\,y)] = (\Delta\,x + i\Delta\,y)\,Im(\Delta\,x + i\Delta\,y)$$

$$f(z_0 + \Delta\,z) = \Delta\,y\,(\Delta\,x + i\,\Delta\,y)$$

$$f(z_0) = z_0\,Im(z_0) = (0 + 0\,i)\,Im(0 + 0\,i) = (0 + 0\,i)\,0 = 0$$

✏ Sustituir $f(z_0 + \Delta z)$ y $f(z_0)$ en la defición de Derivadas.

$$f'(z_0) = \lim_{\Delta z \to 0} \frac{\Delta y(\Delta x + i\Delta y) - 0}{\Delta x + i\Delta y} = \lim_{\Delta z \to 0} \frac{\Delta y(\Delta x + i\Delta y)}{\Delta x + i\Delta y}$$

✏ Trayectoria $\parallel$ eje $x \quad \Rightarrow \quad \Delta y = 0$

$$f'(z_0) = \lim_{\Delta x \to 0} \frac{0(\Delta x + 0i)}{\Delta x + 0i} = \lim_{\Delta x \to 0} \frac{0}{\Delta x} = \lim_{\Delta x \to 0} 0 = 0$$

$$L_1 = 0$$

✏ Trayectoria $\parallel$ eje $y \quad \Rightarrow \quad \Delta x = 0$

$$f'(z_0) = \lim_{\Delta y \to 0} \frac{\Delta y(0 + i\Delta y)}{0 + i\Delta y} = \lim_{\Delta y \to 0} \frac{i(\Delta y)^{\cancel{2}}}{i\cancel{\Delta y}} = \lim_{\Delta y \to 0} \frac{i\Delta y}{i} = \frac{0i}{i} = 0$$

$$L_2 = 0$$

✎ Calcular la derivada del punto $z_0 = 1 + i$.

✏ Construir $f(z_0 + \Delta z)$ y $f(z_0)$.

$$f(z_0 + \Delta z) = (z_0 + \Delta z)\, Im(z_0 + \Delta z) = (1 + i + \Delta x + i\Delta y)\, Im(1 + i + \Delta x + i\Delta y)$$

$$= [(1 + \Delta x) + i(1 + \Delta y)]\, Im\,[(1 + \Delta x) + i(1 + \Delta y)] = [(1 + \Delta x) + i(1 + \Delta y)]\,(1 + \Delta y)$$

$$f(z_0 + \Delta z) = (1 + \Delta y)\,[(1 + \Delta x) + i(1 + \Delta y)]$$

$$f(z_0) = z_0\, Im(z_0) = (1 + i)\, Im(1 + i) = 1 + i$$

$$f(z_0) = 1 + i$$

✏ Sustituir $f(z_0 + \Delta z)$ y $f(z_0)$ en la defición de Derivadas.

$$f'(z_0) = \lim_{\Delta z \to 0} \frac{(1 + \Delta y)\,[(1 + \Delta x) + i(1 + \Delta y)] - (1 + i)}{\Delta x + i\Delta y}$$

$$f'(z_0) = \lim_{\Delta z \to 0} \frac{(1 + \Delta y)\,[(1 + \Delta x) + i(1 + \Delta y)] - 1 - i}{\Delta x + i\Delta y}$$

✏ Trayectoria $\parallel$ eje $x \quad \Rightarrow \quad \Delta y = 0$

$$f'(z_0) = \lim_{\Delta x \to 0} \frac{(1 + 0)\,[(1 + \Delta x) + i(1 + 0)] - 1 - i}{\Delta x + 0i} = \lim_{\Delta x \to 0} \frac{1 + \Delta x + i - 1 - i}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x} = \lim_{\Delta x \to 0} 1 = 1$$

$$L_1 = 1$$

✏ Trayectoria $\|$ eje $y \quad \Rightarrow \quad \Delta x = 0$

$$f'(z_0) = \lim_{\Delta y \to 0} \frac{(1 + \Delta y)\,[(1+0) + i\,(1+\Delta y)] - 1 - i}{0 + i\,\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{(1 + \Delta y)\,[1 + i\,(1+\Delta y)] - 1 - i}{i\,\Delta y} = \lim_{\Delta y \to 0} \frac{1 + \Delta y + i\,(1+\Delta y)^2 - 1 - i}{i\,\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{\Delta y + i[1 + 2\,\Delta y + (\Delta y)^2] - i}{i\,\Delta y} = \lim_{\Delta y \to 0} \frac{\Delta y + i + 2\,i\,\Delta y + i\,(\Delta y)^2 - i}{i\,\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{\cancel{\Delta y} + 2\,i\,\cancel{\Delta y} + i\,(\Delta y)^{\cancel{2}}}{i\,\cancel{\Delta y}} = \lim_{\Delta y \to 0} \frac{1 + 2\,i + i\,\Delta y}{i} = \lim_{\Delta y \to 0} \frac{1 + i\,(2 + \Delta y)}{i}$$

$$= \frac{1 + i\,(2+0)}{i} = \frac{1 + 2\,i}{i}$$

$$L_2 = \frac{1 + 2\,i}{i} \times \frac{-\,i}{-\,i} = \frac{-\,i - 2\,i^2}{-\,i^2} = 2 - i$$

✎ Calcular la derivada del punto $z_0 = -2 - i$.

✏ Construir $f(z_0 + \Delta z)$ y $f(z_0)$.

$$f(z_0 + \Delta z) = (z_0 + \Delta z)\,Im(z_0 + \Delta z) = (-2 - i + \Delta x + i\,\Delta y)\,Im(-2 - i + \Delta x + i\,\Delta y)$$

$$= [(-2 + \Delta x) + i(-1 + \Delta y)]\; Im\,[(-2 + \Delta x) + i(-1 + \Delta y)]$$

$$= [(-2 + \Delta x) + i(-1 + \Delta y)]\;(-1 + \Delta y)$$

$$f(z_0 + \Delta z) = (-1 + \Delta y)\;[(-2 + \Delta x) + i(-1 + \Delta y)]$$

$$f(z_0) = z_0\,Im(z_0) = (-2 - i)\,Im(-2 - i) = (-2 - i)\,(-1)$$

$$f(z_0) = 2 + i$$

✏ Sustituir $f(z_0 + \Delta z)$ y $f(z_0)$ en la defición de Derivadas.

$$f'(z_0) = \lim_{\Delta z \to 0} \frac{(-1 + \Delta y)\;[(-2 + \Delta x) + i\,(-1 + \Delta y)] - (2 + i)}{\Delta x + i\,\Delta y}$$

$$f'(z_0) = \lim_{\Delta z \to 0} \frac{(-1 + \Delta y)\;[(-2 + \Delta x) + i\,(-1 + \Delta y)] - 2 - i}{\Delta x + i\,\Delta y}$$

✏ Trayectoria $\parallel$ eje $x \quad \Rightarrow \quad \Delta y = 0$

$$f'(z_0) = \lim_{\Delta x \to 0} \frac{(-1+0)\,[(-2+\Delta x) + i\,(-1+0)] - 2 - i}{\Delta x + 0\,i}$$

$$= \lim_{\Delta x \to 0} \frac{(-1)\,[(-2+\Delta x) - i] - 2 - i}{\Delta x + 0\,i} = \lim_{\Delta x \to 0} \frac{(-1)\,(-2+\Delta x) + i - 2 - i}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2 - \Delta x - 2}{\Delta x} = \lim_{\Delta x \to 0} \frac{-\Delta x}{\Delta x} = \lim_{\Delta x \to 0} -1 = -1$$

$$L_1 = -1$$

✏ Trayectoria $\parallel$ eje $y \quad \Rightarrow \quad \Delta x = 0$

$$f'(z_0) = \lim_{\Delta y \to 0} \frac{(-1+\Delta y)\,[(-2+0) + i\,(-1+\Delta y)] - 2 - i}{0 + i\,\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{2 - 2\,\Delta y + i\,(-1+\Delta y)^2 - 2 - i}{i\,\Delta y} = \lim_{\Delta y \to 0} \frac{-2\,\Delta y + i\,(-1+\Delta y)^2 - i}{i\,\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{-2\,\Delta y + i\,[1 - 2\,\Delta y + (\Delta y)^2] - i}{i\,\Delta y} = \lim_{\Delta y \to 0} \frac{-2\,\Delta y + i - 2\,i\,\Delta y + i\,(\Delta y)^2 - i}{i\,\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{-2\,\cancel{\Delta y} - 2\,i\,\cancel{\Delta y} + i\,(\Delta y)^{\cancel{2}}}{i\,\cancel{\Delta y}} = \lim_{\Delta y \to 0} \frac{-2 - 2\,i + i\,\Delta y}{i} = \lim_{\Delta y \to 0} \frac{-2 + i\,(-2+\Delta y)}{i}$$

$$= \frac{-2 + i\,(-2+0)}{i} = \frac{-2 - 2\,i}{i}$$

$$L_2 = \frac{-2 - 2\,i}{i} \times \frac{-i}{-i} = \frac{2\,i + 2\,i^2}{-i^2} = -2 + 2\,i$$

✎ Conclusión:

La función $f(z) = z\,Im(z)$ no es una función analítica, porque los puntos $z_0 = 1 + i$ y $z_0 = -2 - i$ sus límites $L_1 \neq L_2$ tomando 2 trayectorias diferentes; entonces no existe límite o derivada para los puntos anteriormente mecionados. Pero para el punto $z_0 = 0$ su límites $L_1 = L_2$ tomando 2 trayectorias diferentes; demuestra que exite límete unicamente en ese punto, por lo tanto tambien existe la derivada en dicho punto.

En los ejercicio **( 13 )** al **( 15 )** derive las siguientes funciones usando las **Reglas de la diferencición** (simplifique tanto como sea posible).

(13) $\mathbf{f(z) = -5\,z^2 + \dfrac{2+i}{z^2}}$

$$f(z) = -5\,z^2 + (2+i)\,z^{-2}$$

$$f'(z) = -5\,(2\,z) + (2+i)\,(-2\,z^{-3}) = -10\,z - 2\,(2+i)\,z^{-3}$$

$$f'(z) = -10\,z - \frac{4 + 2\,i}{z^3}$$

(14) $\mathbf{f(z) = (i\,z^2 + 3\,z)^5}$

$$\boxed{f'(z) = 5(i\,z^2 + 3\,z)^4 \cdot (2\,i\,z + 3)}$$

(15) $\mathbf{f(z) = (z^2 + 2\,z - 7\,i)^2\,(z^4 - 4\,i\,z)^3}$

$$f'(z) = 2\,(2\,z + 2)\,(z^2 + 2\,z - 7\,i)\,(z^4 - 4\,i\,z)^3 + 3\,(4\,z^3 - 4\,i)\,(z^2 + 2\,z - 7\,i)^2\,(z^4 - 4\,i\,z)^2$$

$$f'(z) = (4\,z + 4)\,(z^2 + 2\,z - 7\,i)\,(z^4 - 4\,i\,z)^3 + (12\,z^3 - 12\,i)\,(z^2 + 2\,z - 7\,i)^2\,(z^4 - 4\,i\,z)^2$$

**Ecuaciones de Cauchy-Riemann**

En los ejercicio **( 16 )** al **( 20 )**, use las Ecuaciones de Cauchy-Riemann para mostrar si las siguientes funciones complejas son o no Analíticas.

(16) $\mathbf{f(z) = z^2 + 5\,i\,z + 3 - i}$

$$f(z) = (x + i\,y)^2 + 5\,i\,(x + i\,y) + 3 - i$$

$$f(z) = x^2 + 2\,x\,i\,y - y^2 + 5\,i\,x - 5\,y + 3 - i$$

$$u = x^2 - y^2 - 5\,y + 3 \quad ; \quad v = 2\,x\,y + 5\,x - 1$$

✎ Usamos la ecuaciones de C-R.

$$\frac{\partial u}{\partial x} = 2\,x$$

$$\frac{\partial v}{\partial y} = 2\,x$$

$$\frac{\partial u}{\partial y} = -2y - 5$$

$$-\frac{\partial v}{\partial x} = 2y + 5$$

$$\frac{\partial v}{\partial x} = -2y - 5$$

La función es analítica.

(17) $\mathbf{f(z) = e^{2\,x}\,(\cos y + i \sin y)}$

$$f(z) = e^{2\,x}\cos y + i\,e^{2\,x}\sin y$$

$$u = e^{2\,x}\cos y \quad ; \quad v = e^{2\,x}\sin y$$

✎ Usamos la ecuaciones de C-R.

$$\frac{\partial u}{\partial x} = e^{2x} \cos y = 2e^{2x} \cos y$$

$$\frac{\partial v}{\partial y} = e^{2x} \sin y = e^{2x} \cos y$$

$$\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$$

$\therefore$ la función no es analítica.

(18) $\mathbf{f(z) = 3\,z^2 + 5\,z - 6\,i}$

$$f(z) = 3\,(x + i\,y)^2 + 5\,(x - i\,y) - 6\,i \iff 3\,(x^2 + 2\,i\,x\,y - y^2) + 5\,(x - i\,y) - 6\,i$$

$$f(z) = 3\,x^2 + 6\,i\,x\,y - 3\,y^2 + 5\,x + 5\,i\,y - 6\,i$$

$$u = 3\,x^2 + 5\,x - 3\,y^2 \quad ; \quad v = 6\,x\,y + 5\,y - 6$$

✎ Usamos la ecuaciones de C-R.

$$\frac{\partial u}{\partial x} = 6\,x + 5$$

$$\frac{\partial v}{\partial y} = 6\,x + 5$$

$$\frac{\partial u}{\partial y} = -6\,y$$

$$-\frac{\partial v}{\partial x} = 6\,y$$

$$\frac{\partial v}{\partial x} = -6\,y$$

La función es analítica.

(19) $\mathbf{f(z) = 4\,z - 6\,\overline{z} + 3}$

$$f(x + i\,y) = 4\,(x + i\,y) - 6\,(x - i\,y) + 3 = 4\,x + 4\,i\,y - 6\,x + 6\,i\,y + 3, = -2\,x + 10\,i\,y + 3$$

$$f(x + i\,y) = (3 - 2\,x) + 10\,i\,y$$

$$u = 3 - 2\,x \quad ; \quad v = 10\,y$$

✎ Usamos la ecuaciones de C-R.

$$\frac{\partial u}{\partial x} = 3 - 2x = -2$$

$$\frac{\partial v}{\partial y} = 10y = 10$$

$$\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$$

∴ la función no es analítica.

(20) $\mathbf{f(z) = e^{-x} (\cos y - i \sin y)}$

$$u = e^{-x} \cos y \quad ; \quad v = -e^{-x} i \sin y$$

✎ Usamos la ecuaciones de C-R.

$$\frac{\partial u}{\partial x} = -e^{-x} \cos y$$

$$\frac{\partial v}{\partial y} = -e^{-x} \cos y$$

$$\frac{\partial u}{\partial y} = -e^{-x} \sin y$$

$$-\frac{\partial v}{\partial x} = e^{-x} \sin y$$

$$\frac{\partial v}{\partial x} = -e^{-x} \sin y$$

La función es analítica.