Escuela Tecnológica Instituto Técnico Central (ETITC)

Facultad de sistemas

Taller 2: Forma Polar, Potencias y Raíces de los Complejos
Matemáticas Especiales

Autores

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Presentado a:

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Multiplicación

En cada uno de los ejercicios **(1)** al **(3)** realizar lo siguiente:

a) Calcular el producto de los números complejos $\mathbf{z_1} \cdot \mathbf{z_2}$, usando la fórmula de la multiplicación en forma polar.

b) Escriba **el resultado del producto** en la forma cartesiana $\mathbf{z} = \mathbf{x} + \mathbf{i}\,\mathbf{y}$ y en la forma exponencial $\mathbf{z} = \mathbf{r}\,\mathbf{e}^{\mathbf{i}\,\theta}$ ó $\mathbf{z} = \mathbf{r}\,\mathbf{exp}\,(\mathbf{i}\,\theta)$.

c) Realizar en un plano complejo la gráfica de $\mathbf{z_1}$, $\mathbf{z_2}$ y $\mathbf{z_1} \cdot \mathbf{z_2}$.

(1) $\mathbf{z_1 = 4 - 4\,i \quad ; \quad z_2 = 6 - i\,6\,\sqrt{3}}$

a) Calcule $\mathbf{z_1} \cdot \mathbf{z_2}$ en forma polar:

$$z_1 \cdot z_2 = r_1 \cdot r_2 \, [\cos(\theta_1 + \theta_2) + i \, \sin(\theta_1 + \theta_2)]$$

✏ Forma polar de $\mathbf{z_1}$ y $\mathbf{z_2}$.

✎ Módulo y ángulo de $\mathbf{z_1}$

$$r_1 = |z_1| = \sqrt{(4)^2 + (-4)^2} = \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$$

$$\theta_1 = \tan^{-1}\left(\frac{-4}{4}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$$

$$\boxed{r_1 = 4\sqrt{2} \quad ; \quad \theta_1 = -\frac{\pi}{4}}$$

✎ Módulo y ángulo de $\mathbf{z_2}$

$$r_2 = |z_2| = \sqrt{(6)^2 + \left(-6\sqrt{3}\right)^2} = \sqrt{36 + 36(3)} = \sqrt{36 + 108} = \sqrt{144} = 12$$

$$\theta_2 = \tan^{-1}\left(\frac{-6\sqrt{3}}{6}\right) = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$$

$$\boxed{r_2 = 12 \quad ; \quad \theta_2 = -\frac{\pi}{3}}$$

✏ Calcular $\mathbf{z_1} \cdot \mathbf{z_2}$

$$z_3 = z_1 \cdot z_2$$

$$z_3 = \left(4\sqrt{2}\right)(12)\left[\cos\left(-\frac{\pi}{4} + \left(-\frac{\pi}{3}\right)\right) + i\,\sin\left(-\frac{\pi}{4} + \left(-\frac{\pi}{3}\right)\right)\right]$$

$$= 48\sqrt{2}\left[\cos\left(-\frac{\pi}{4} - \frac{\pi}{3}\right) + i\,\sin\left(-\frac{\pi}{4} - \frac{\pi}{3}\right)\right]$$

$$= 48\sqrt{2}\left[\cos\left(\frac{-3\pi - 4\pi}{12}\right) + i\,\sin\left(\frac{-3\pi - 4\pi}{12}\right)\right]$$

$$z_3 \;=\; 48\sqrt{2}\left[\cos\left(-\,\frac{7\,\pi}{12}\right) \,+\, i\,\sin\left(-\,\frac{7\,\pi}{12}\right)\right]$$

b) Formas cartesiana y exponenccial $\mathbf{z_3}$

✏ Cartesiana

$$x \;=\; 48\sqrt{2}\,\cos\left(-\,\frac{7\,\pi}{12}\right) \,\approx\, -\,17{,}57$$

$$y \;=\; 48\sqrt{2}\,\sin\left(-\,\frac{7\pi}{12}\right) \,\approx\, -\,65{,}57$$

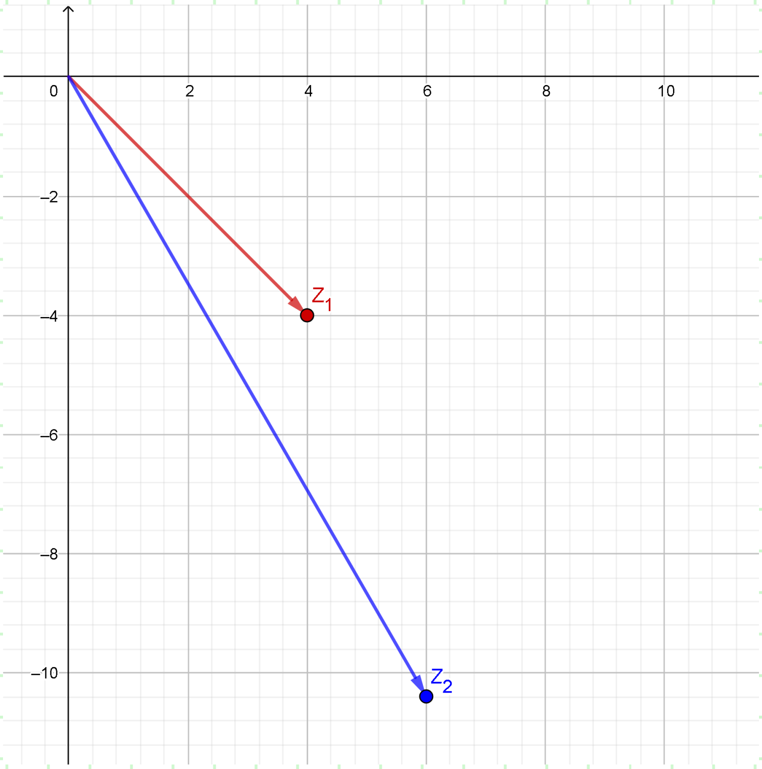
$$z_3 \;=\; -\,17{,}57 \,-\, 65{,}57\,i$$

✏ Exponencial

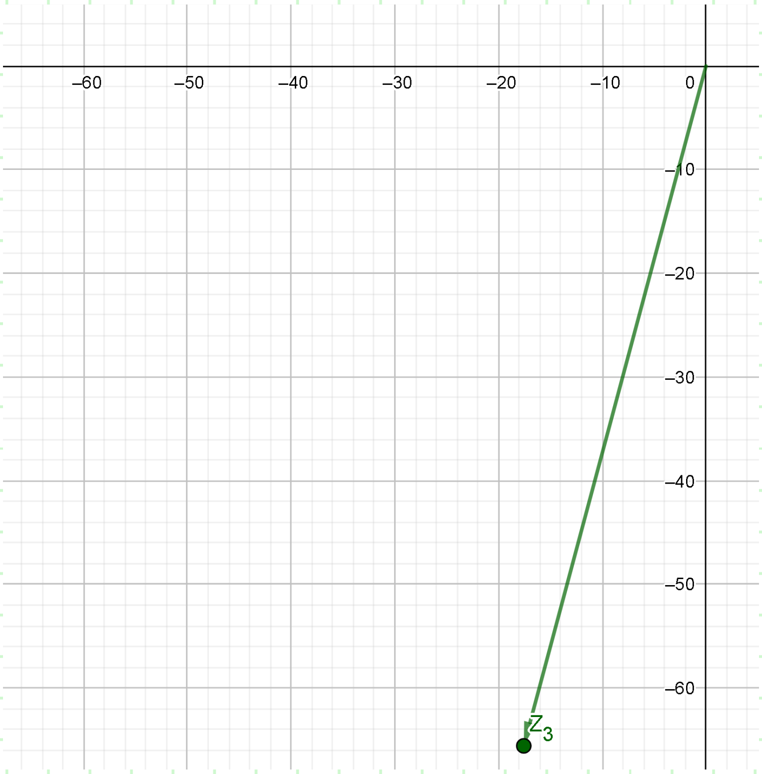
$$z_3 \;=\; 48\sqrt{2}\,\cdot\, exp\left(-\,\frac{7\,\pi}{12}\,i\right)$$

c) Graficar plano complejo

✏ Gráfica $\mathbf{z_1}$ y $\mathbf{z_2}$



✏ Gráfica $\mathbf{z_3} = \mathbf{z_1} \cdot \mathbf{z_2}$



(2) $\mathbf{z_1} = \mathbf{2} - \mathbf{i}$; $\mathbf{z_2} = -\mathbf{2} + \mathbf{2\,i}$

(a) Calcule $\mathbf{z_1} \cdot \mathbf{z_2}$ en forma polar:

$$z_1 \cdot z_2 = r_1 \cdot r_2 \left[\cos\left(\theta_1 + \theta_2\right) + i \sin\left(\theta_1 + \theta_2\right)\right]$$

✏ Forma polar de $\mathbf{z_1}$ y $\mathbf{z_2}$.

✎ Módulo y angilo de $\mathbf{z_1}$

$$r_1 = |z_1| = \sqrt{(2)^2 + (-1)^2} = \sqrt{5}$$

$$\theta_1 = \tan^{-1}\left(\frac{-1}{2}\right) = -\frac{3\pi}{20}$$

$$\boxed{r_1 = \sqrt{5} \quad ; \quad \theta_1 = -\frac{3\pi}{20}}$$

✎ Módulo y angilo de $\mathbf{z_2}$

$$r_2 = |z_2| = \sqrt{(-2)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$$

$$\theta_2 = \tan^{-1}\left(\frac{-2}{2}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$$

✎ Corrigiendo el angulo θ_2 en el cuadrante II

$$\theta_2 = \pi - |\theta_0| = \pi - \left|-\frac{\pi}{4}\right| = \pi - \frac{\pi}{4} = \frac{4\,\pi - \pi}{4} = \frac{3\,\pi}{4}$$

$$\boxed{r_2 = 2\sqrt{2} \quad ; \quad \theta_2 = \frac{3\pi}{4}}$$

✏ Calcular $\mathbf{z_1} \cdot \mathbf{z_2}$

$$z_3 = z_1 \cdot z_2$$

$$z_3 = \left(\sqrt{5}\right)\left(2\sqrt{2}\right)\left[\cos\left(-\frac{3\pi}{20} + \left(\frac{3\pi}{4}\right)\right) + i\,\sin\left(-\frac{3\pi}{20} + \left(\frac{3\pi}{4}\right)\right)\right]$$

$$= 2\sqrt{10}\left[\cos\left(-\frac{3\pi}{20} + \frac{3\pi}{4}\right) + i\,\sin\left(-\frac{3\pi}{20} + \frac{3\pi}{4}\right)\right]$$

$$= 2\sqrt{10}\left[\cos\left(\frac{-3\pi + 15\pi}{20}\right) + i\,\sin\left(\frac{-3\pi + 15\pi}{20}\right)\right]$$

$$z_3 = 2\sqrt{10}\left[\cos\left(\frac{3\pi}{5}\right) + i\,\sin\left(\frac{3\pi}{5}\right)\right]$$

(b) Formas cartesiana y exponenccial z_3

✏ Cartesiana

$$x = 2\sqrt{10}\,\cos\left(\frac{3\pi}{5}\right) \approx -1{,}95$$

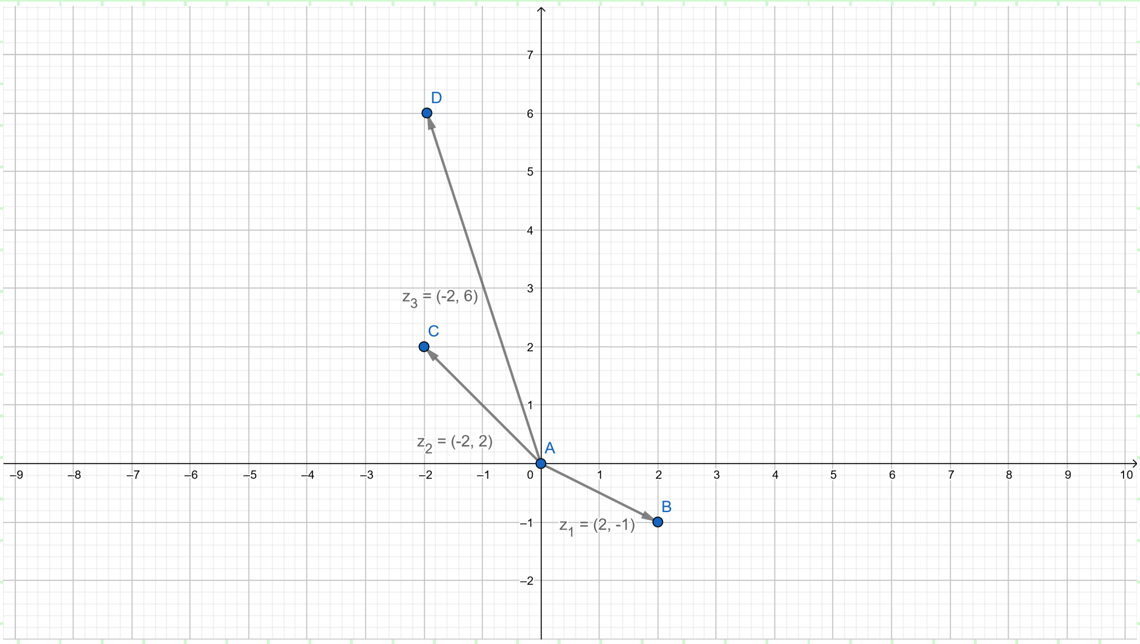
$$y = 2\sqrt{10}\,\sin\left(\frac{3\pi}{5}\right) \approx 6$$

$$z_3 = -1{,}95 + 6\,i$$

✏ Exponencial

$$z_3 = 2\sqrt{10} \cdot exp\left(\frac{3\pi}{5}\,i\right)$$

(c) Plano complejo la gráfica de $\mathbf{z_1}$, $\mathbf{z_2}$ y $\mathbf{z_1} \cdot \mathbf{z_2}$.



(3) $\mathbf{z_1 = 1 - i}$; $\mathbf{z_2 = 1 - i\sqrt{3}}$

a) Calcule $\mathbf{z_1 \cdot z_2}$ en forma polar:

$$z_1 \cdot z_2 = r_1 \cdot r_2 \left[\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)\right]$$

✏ Forma polar de $\mathbf{z_1}$ y $\mathbf{z_2}$.

✎ Módulo y ángulo de $\mathbf{z_1}$

$$r_1 = |z_1| = \sqrt{(1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$\theta_1 = \tan^{-1}\left(\frac{-1}{1}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$$

$$\boxed{r_1 = \sqrt{2} \quad ; \quad \theta_1 = -\frac{\pi}{4}}$$

✎ Módulo y ángulo de $\mathbf{z_2}$

$$r_2 = |z_2| = \sqrt{(1)^2 + \left(-\sqrt{3}\right)^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$\theta_2 = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$$

$$\boxed{r_2 = 2 \quad ; \quad \theta_2 = -\frac{\pi}{3}}$$

✏ Calcular $\mathbf{z_1 \cdot z_2}$

$$z_3 = z_1 \cdot z_2$$

$$z_3 = \left(2\sqrt{2}\right)\left[\cos\left(-\frac{\pi}{4} + \left(-\frac{\pi}{3}\right)\right) + i \sin\left(-\frac{\pi}{4} + \left(-\frac{\pi}{3}\right)\right)\right]$$

$$= 2\sqrt{2}\left[\cos\left(-\frac{\pi}{4} - \frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{4} - \frac{\pi}{3}\right)\right]$$

$$= 2\sqrt{2}\left[\cos\left(\frac{-3\pi - 4\pi}{12}\right) + i \sin\left(\frac{-3\pi - 4\pi}{12}\right)\right]$$

$$z_3 = 2\sqrt{2}\left[\cos\left(-\frac{7\pi}{12}\right) + i \sin\left(-\frac{7\pi}{12}\right)\right]$$

b) Formas cartesiana y exponenccial $\mathbf{z_3}$

✏ Cartesiana

$$x = 2\sqrt{2}\cos\left(-\frac{7\pi}{12}\right) \approx -0{,}73$$

$$y = 2\sqrt{2}\sin\left(-\frac{7\pi}{12}\right) \approx -2{,}73$$

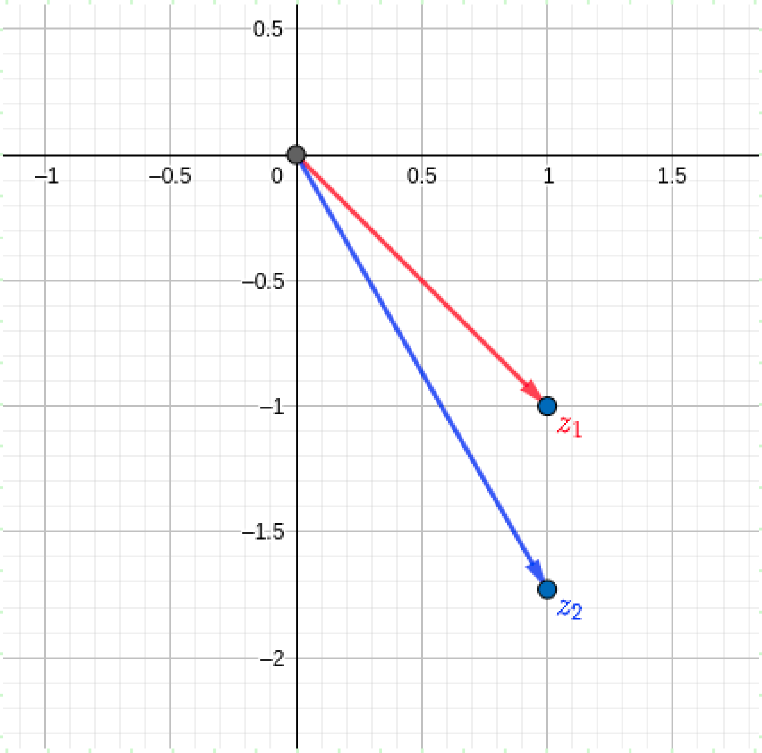
$$z_3 = -0{,}73 - 2{,}73\,i$$

✏ Exponencial

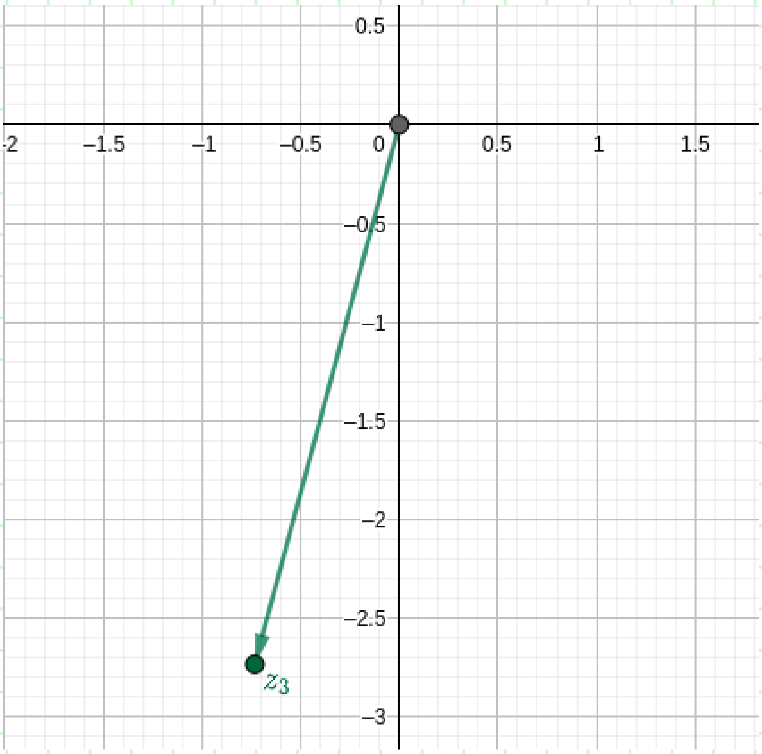
$$z_3 = 2\sqrt{2} \cdot exp\left(-\frac{7\,\pi}{12}\,i\right)$$

c) Graficar plano complejo

✏ Gráfica $\mathbf{z_1}$ y $\mathbf{z_2}$



✏ Gráfica $\mathbf{z_3} = \mathbf{z_1} \cdot \mathbf{z_2}$



División

En cada uno de los ejercicios **(4)** al **(6)** realizar lo siguiente:

a) Calcule $\mathbf{z_1 \div z_2}$, usando la fórmula de la división en forma polar.

b) Escriba **el resultado de la división** en la forma cartesiana $\mathbf{z = x + i\,y}$ y en la forma exponencial $\mathbf{z = r\,e^{i\,\theta}}$ ó $\mathbf{z = r\exp(i\,\theta)}$.

c) Realizar en un plano complejo la gráfica de $\mathbf{z_1}$, $\mathbf{z_2}$ y $\mathbf{z_1 \div z_2}$.

(4) $\mathbf{z_1 = 2 - 2\,i}$; $\mathbf{z_2 = 3 - i\,3\sqrt{3}}$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\,\sin(\theta_1 - \theta_2)\right]$$

✏ Forma polar de $\mathbf{z_1}$ y $\mathbf{z_2}$.

✎ Módulo y angilo de $\mathbf{z_1}$

$$r_1 = |z_1| = \sqrt{(2)^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

$$\theta_1 = \tan^{-1}\left(\frac{-2}{2}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$$

$$\boxed{r_1 = 2\sqrt{2} \quad ; \quad \theta_1 = -\frac{\pi}{4}}$$

✎ Módulo y angilo de $\mathbf{z_2}$

$$r_2 = |z_2| = \sqrt{(3)^2 + (-3\sqrt{3})^2} = \sqrt{9 + 27} = \sqrt{36} = 6$$

$$\theta_2 = \tan^{-1}\left(\frac{-3\sqrt{3}}{3}\right) = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$$

$$\boxed{r_2 = 6 \quad ; \quad \theta_2 = -\frac{\pi}{3}}$$

✏ Calcular $\mathbf{z_1 \div z_2}$

$$z_3 = \frac{z_1}{z_2}$$

$$z_3 = \frac{\sqrt{2}}{3}\left[\cos\left(-\frac{\pi}{4} + \frac{\pi}{3}\right) + i\,\sin\left(-\frac{\pi}{4} + \frac{\pi}{3}\right)\right]$$

$$= \frac{\sqrt{2}}{3}\left[\cos\left(\frac{-3\pi + 4\pi}{12}\right) + i\,\sin\left(\frac{-3\pi + 4\pi}{12}\right)\right]$$

$$z_3 = \frac{\sqrt{2}}{3}\left[\cos\left(\frac{\pi}{12}\right) + i\,\sin\left(\frac{\pi}{12}\right)\right]$$

(b) Formas cartesiana y exponenccial z_3

✏ Cartesiana

$$x = \frac{\sqrt{2}}{3} \cos\left(\frac{\pi}{12}\right) \approx 0{,}45$$

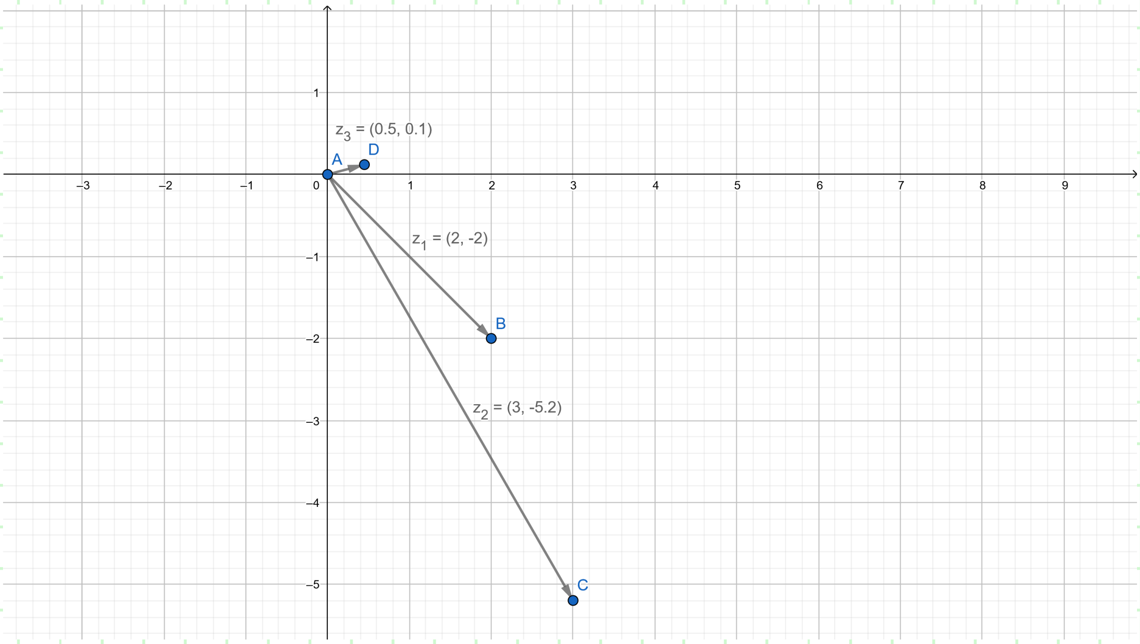
$$y = \frac{\sqrt{2}}{3} \sin\left(\frac{\pi}{12}\right) \approx 0{,}12$$

$$z_3 = 0{,}45 + 0{,}12\, i$$

✏ Exponencial

$$z_3 = \frac{\sqrt{2}}{3} \cdot exp\left(\frac{\pi}{12}\, i\right)$$

(c) Plano complejo la gráfica de $\mathbf{z_1}$, $\mathbf{z_2}$ y $\mathbf{z_1} \div \mathbf{z_2}$.



(5) $\mathbf{z_1 = -1 - i\sqrt{3}} \quad ; \quad \mathbf{z_2 = -4 + 4\,i}$

a) Calcule $\mathbf{z_1} \div \mathbf{z_2}$ en forma polar:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \left[\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)\right]$$

✏ Forma polar de $\mathbf{z_1}$ y $\mathbf{z_2}$.

✎ Módulo y ángulo de $\mathbf{z_1}$

$$r_1 = |z_1| = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$\theta_0 = \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

☞ Correción θ_0 Cuadrantre III

$$\theta_1 = |\theta_0| - \pi = \left|\frac{\pi}{3}\right| - \pi = \frac{\pi}{3} - \pi = \frac{\pi - 3\pi}{3} = -\frac{2\pi}{3}$$

$$\boxed{r_1 = 2 \quad ; \quad \theta_1 = -\frac{2\pi}{3}}$$

✎ Módulo y ángulo de $\mathbf{z_2}$

$$r_2 = |z_2| = \sqrt{(-4)^2 + (4)^2} = \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$$

$$\theta_0 = \tan^{-1}\left(\frac{4}{-4}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$$

☞ Correción θ_0 Cuadrantre II

$$\theta_2 = \pi - |\theta_0| = \pi - \left|-\frac{\pi}{4}\right| = \pi - \frac{\pi}{4} = \frac{4\pi - \pi}{4} = \frac{3\pi}{4}$$

$$\boxed{r_2 = 4\sqrt{2} \quad ; \quad \theta_2 = \frac{3\pi}{4}}$$

✏ Calcular $\mathbf{z_1} \div \mathbf{z_2}$

$$z_3 = \frac{z_1}{z_2}$$

$$z_3 = \frac{2}{4\sqrt{2}}\left[\cos\left(-\frac{2\pi}{3} - \frac{3\pi}{4}\right) + i\,\sin\left(-\frac{2\pi}{3} - \frac{3\pi}{4}\right)\right]$$

$$= \frac{1}{2\sqrt{2}}\left[\cos\left(\frac{-9\pi - 8\pi}{12}\right) + i\,\sin\left(\frac{-9\pi - 8\pi}{12}\right)\right]$$

$$z_3 = \frac{1}{2\sqrt{2}}\left[\cos\left(-\frac{17\pi}{12}\right) + i\,\sin\left(-\frac{17\pi}{12}\right)\right]$$

b) Formas cartesiana y exponenccial z_3

✏ Cartesiana

$$x = \frac{1}{2\sqrt{2}}\cos\left(-\frac{17\pi}{12}\right) \approx -0{,}1$$

$$y = \frac{1}{2\sqrt{2}}\sin\left(-\frac{17\pi}{12}\right) \approx 0{,}34$$

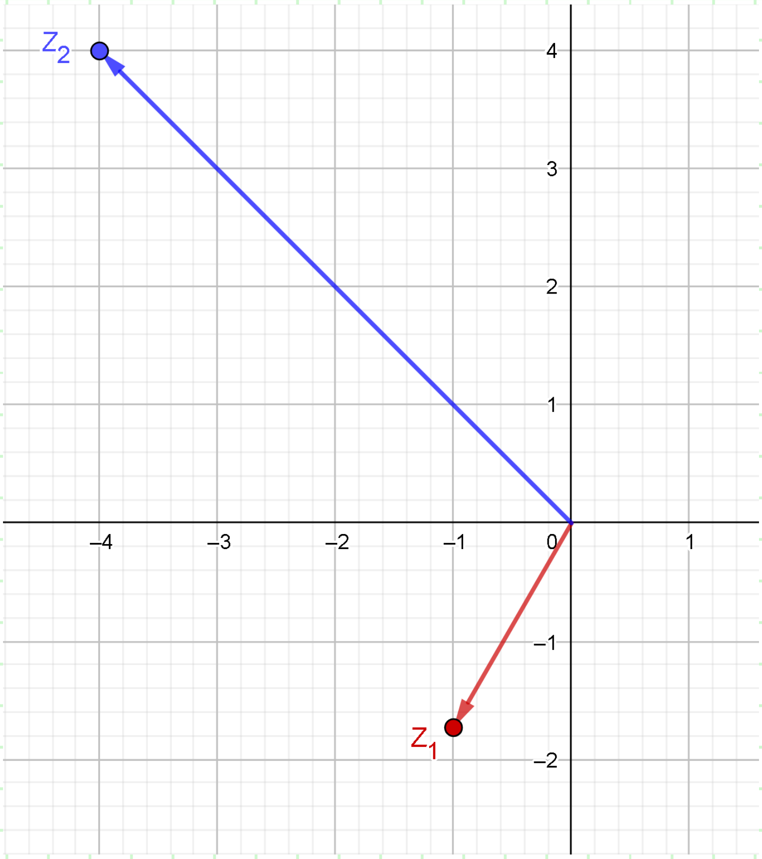
$$z_3 = -0{,}1 + 0{,}34\,i$$

✏ Exponencial

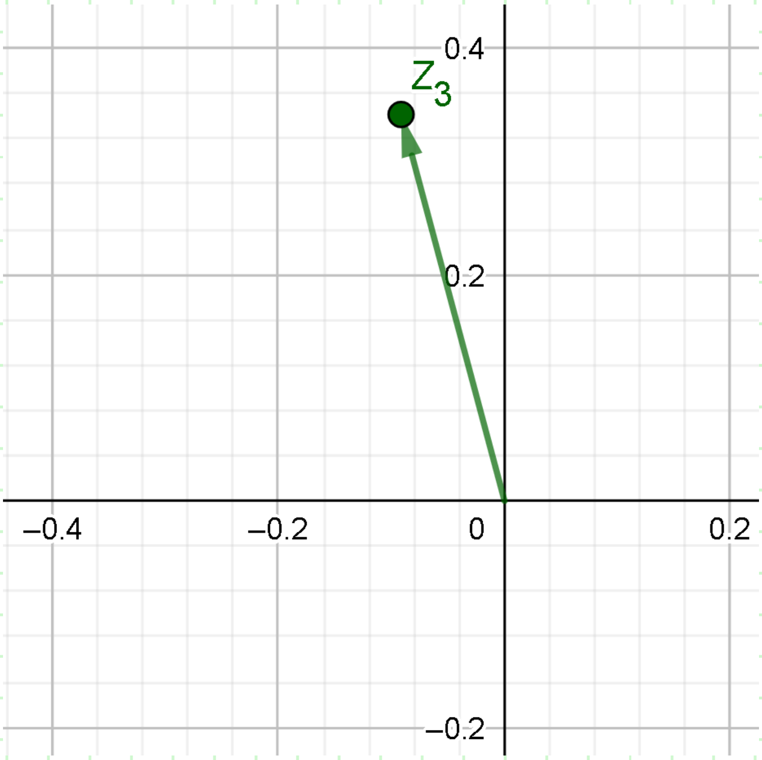
$$z_3 = \frac{1}{2\sqrt{2}} \cdot exp\left(-\frac{17\pi}{12}\,i\right)$$

c) Graficar plano complejo

✏ Gráfica $\mathbf{z_1}$ y $\mathbf{z_2}$



✏ Gráfica $\mathbf{z_3} = \mathbf{z_1} \div \mathbf{z_2}$



(6) $\mathbf{z_1 = 2 - 2\,i} \quad ; \quad \mathbf{z_2 = 2 - i\,2\sqrt{3}}$

a) Calcule $\mathbf{z_1 \div z_2}$ en forma polar

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,[\cos(\theta_1 - \theta_2) + i\,\sin(\theta_1 - \theta_2)]$$

✏ Forma polar de $\mathbf{z_1}$ y $\mathbf{z_2}$.

✎ Módulo y ángulo de $\mathbf{z_1}$

$$r_1 = |z_1| = \sqrt{(2)^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2} \approx 2{,}83$$

$$\theta_0 = \tan^{-1}\left(\frac{-2}{2}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$$

$$\boxed{r_1 = 2\sqrt{2} \quad ; \quad \theta_1 = -\frac{\pi}{4}}$$

✎ Módulo y ángulo de $\mathbf{z_2}$

$$r_2 = |z_2| = \sqrt{(2)^2 + (-2\sqrt{3})^2} = \sqrt{4 + 12} = \sqrt{16} = 4$$

$$\theta_0 = \tan^{-1}\left(\frac{-2\sqrt{3}}{2}\right) = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$$

$$\boxed{r_2 = 4 \quad ; \quad \theta_2 = -\frac{\pi}{3}}$$

✏ Calcular $\mathbf{z_1 \div z_2}$

$$z_3 = \frac{z_1}{z_2}$$

$$z_3 = \frac{2\sqrt{2}}{4}\left[\cos\left(-\frac{\pi}{4} - \left(-\frac{\pi}{3}\right)\right) + i\,\sin\left(-\frac{\pi}{4} - \left(-\frac{\pi}{3}\right)\right)\right]$$

$$z_3 = \frac{\sqrt{2}}{2}\left[\cos\left(-\frac{\pi}{4} + \frac{\pi}{3}\right) + i\,\sin\left(-\frac{\pi}{4} + \frac{\pi}{3}\right)\right]$$

$$z_3 = \frac{\sqrt{2}}{2}\left[\cos\left(\frac{\pi}{12}\right) + i\,\sin\left(\frac{\pi}{12}\right)\right]$$

b) Formas cartesiana y exponenccial z_3

✏ Cartesiana

$$x = \frac{\sqrt{2}}{2}\cos\left(\frac{\pi}{12}\right) \approx 0{,}68$$

$$y = \frac{\sqrt{2}}{2}\sin\left(\frac{\pi}{12}\right) \approx 0{,}18$$

$$z_3 = 0{,}68 + 0{,}18\,i$$

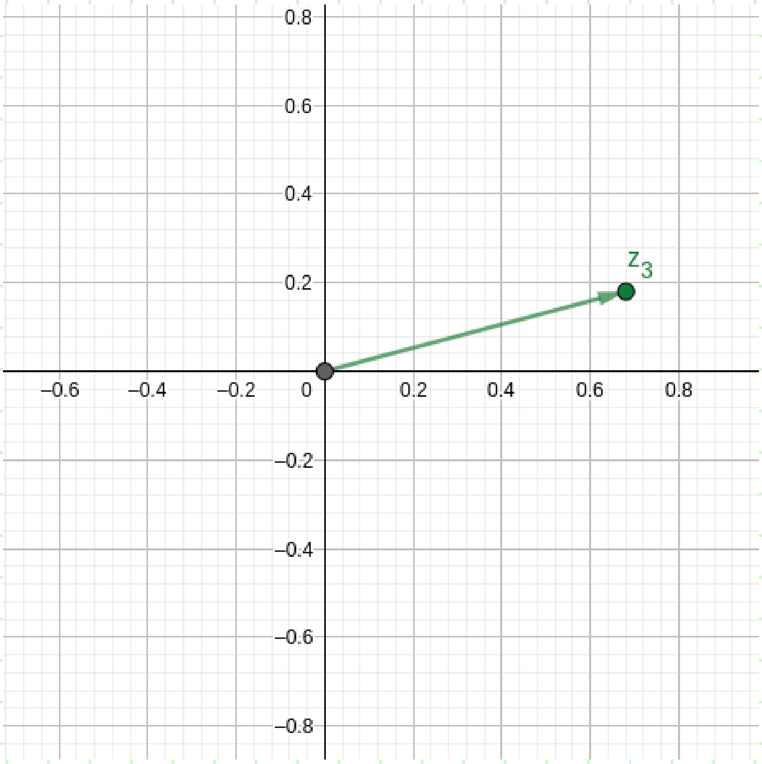
✏ Exponencial

$$z_3 = \frac{\sqrt{2}}{2} \cdot exp\left(\frac{\pi}{12}i\right)$$

c) Graficar plano complejo

✏ Gráfica $\mathbf{z_1}$ y $\mathbf{z_2}$

✏ Gráfica $\mathbf{z_3} = \mathbf{z_1} \div \mathbf{z_2}$



Potencias Enteras de Números Complejos

En cada uno de los ejercicios **(7)** al **(8)** realizar lo siguiente:

a) Calcule las potencias indicadas en forma exponencial.

b) Escriba el resultado final en la forma cartesiana $\mathbf{z^n = x + i\,y}$.

c) Dibujo en uno dos planos complejos los números, $\mathbf{z}$ y $\mathbf{z^n}$.

(7) $\left(\mathbf{1 + i\sqrt{3}}\right)^{\mathbf{9}}$

a) Potencia en forma exponnencial

$$z^n = r^n \cdot exp\,(i\,n\,\theta)$$

$$z = 1 + i\sqrt{3}$$

✏ Hallar el módulo de $\mathbf{z}$

$$r = |z| = \sqrt{(1)^2 + \left(\sqrt{3}\right)^2} = \sqrt{1 + 3} = \sqrt{4} = 2$$

✏ Hallar el angulo de $\mathbf{z}$

$$\theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

✏ Potencia donde $n = 9$

$$z^9 = 2^9 \cdot exp\left(i9\left(\frac{\pi}{3}\right)\right)$$

$$z^9 = 512 \cdot exp\,(3\,\pi\,i)$$

b) Forma cartesina $\mathbf{z^n = x + i\,y}$

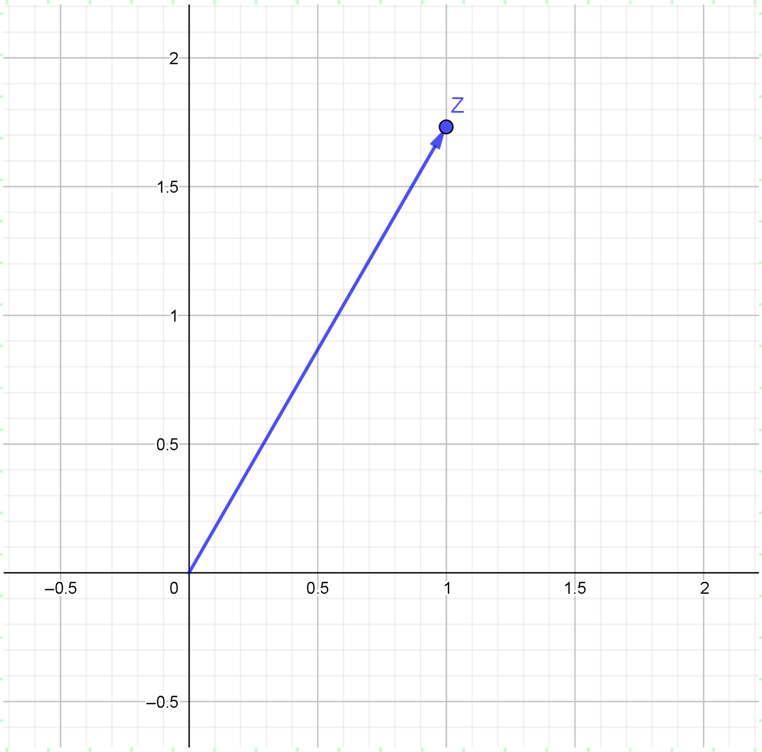
$$x = r\,\cos(\theta) = 512\,\cos(3\,\pi) = -512$$

$$y = r\,\sin(\theta) = 512\,\sin(3\,\pi) = 0$$

$$z^9 = -512$$

c) Graficar plano complejo

✏ Gráfica **z**



✏ Gráfica $\mathbf{z^9}$

(8) $\mathbf{(2 - 2\,i)^5}$

(a) Calcule las potencias indicadas en forma exponencial.

$$\text{n} = 5 \quad z = 2 - 2\,i$$

$$r = \sqrt{(2)^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{-2}{2}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$$

$$z^5 \;=\; 128\sqrt{2} \cdot exp\left(-\,\frac{5\,\pi}{4}\,i\right)$$

(b) Forma cartesiana de z^5

$$z^5 \;=\; 128\sqrt{2}\,\left[\cos\left(5 \cdot -\frac{\pi}{4}\right) \,+\, i\,\sin\left(5 \cdot -\frac{\pi}{4}\right)\right]$$

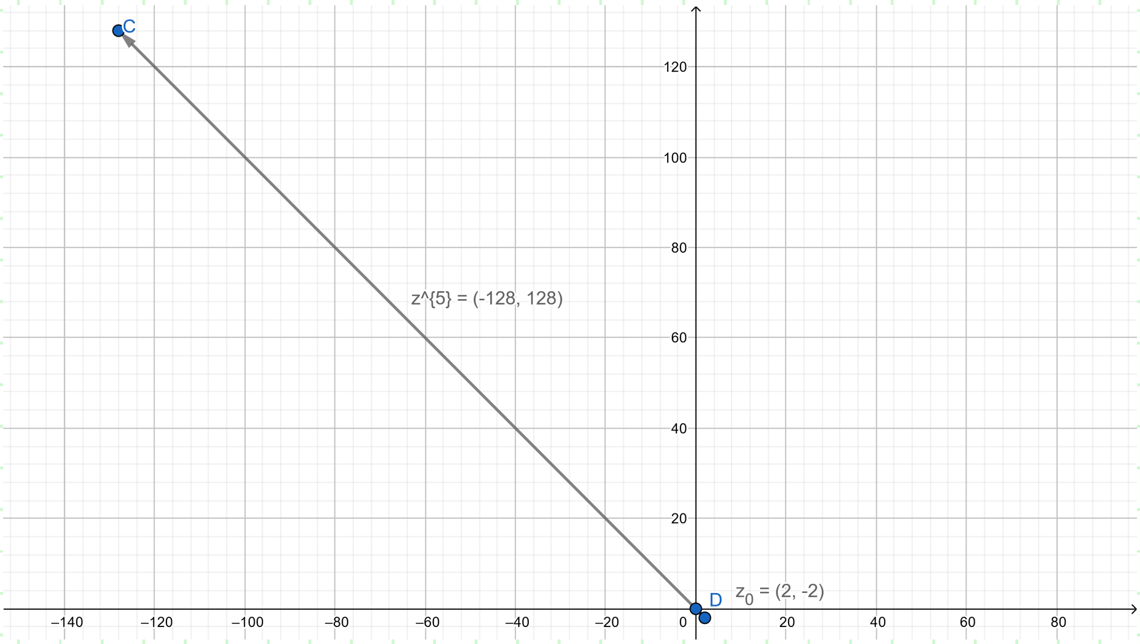
$$z^5 \;=\; 128\sqrt{2}\,\left[\cos\left(-\frac{5\pi}{4}\right) \,+\, i\,\sin\left(-\frac{5\,\pi}{4}\right)\right]$$

$$x \;=\; 128\sqrt{2}\,\cos\left(-\frac{5\,\pi}{4}\right) \;\approx\; -\,128$$

$$y \;=\; 128\sqrt{2}\,\sin\left(-\frac{5\pi}{4}\right) \;\approx\; 128$$

$$z^5 \;=\; -\,128 \,+\, 128\,i$$

(c) Dibujo en el plano complejo los números **z** y $\mathbf{z^n}$.



Raíces de Números Complejos

Hallar las raíces indicadas a continuación y realizar su correspondiente gráfica en el plano complejo.

(9) $\sqrt[3]{8}$

✏ Hallar el módulo y el angulo $\mathbf{z_0}$

$$z_0 \;=\; 8$$

✎ Módulo de $\mathbf{z_0}$

$$r_0 \;=\; |z_0| \;=\; \sqrt{(8)^2 \,+\, (0)^2} \;=\; \sqrt{64 \,+\, 0} \;=\; \sqrt{64} \;=\; 8$$

✎ Angulo de $\mathbf{z_0}$

$$\theta_0 = \tan^{-1}\left(\frac{0}{8}\right) = \tan^{-1}(0) = 0$$

✏ En este caso $\mathbf{n} = \mathbf{3}$ y $\mathbf{k} = \mathbf{0}\,,\,\mathbf{1}\,,\,\mathbf{2}$

✎ Si $\mathbf{k} = \mathbf{0}$

$$C_0 = 8 \cdot exp\left\{i\left(\frac{0 + 2\pi(0)}{3}\right)\right\} = 8 \cdot exp\{i\,(0)\}$$

$$C_0 = 8 \cdot exp\{i\,(0)\}$$

$$0 \iff 0^\circ$$

✎ Si $\mathbf{k} = \mathbf{1}$

$$C_1 = 8 \cdot exp\left\{i\left(\frac{0 + 2\pi(1)}{3}\right)\right\} = 8 \cdot exp\left\{i\left(\frac{2\pi}{3}\right)\right\}$$

$$C_1 = 8 \cdot exp\left\{i\left(\frac{2\pi}{3}\right)\right\}$$

$$\frac{2\pi}{3} \iff 120^\circ$$

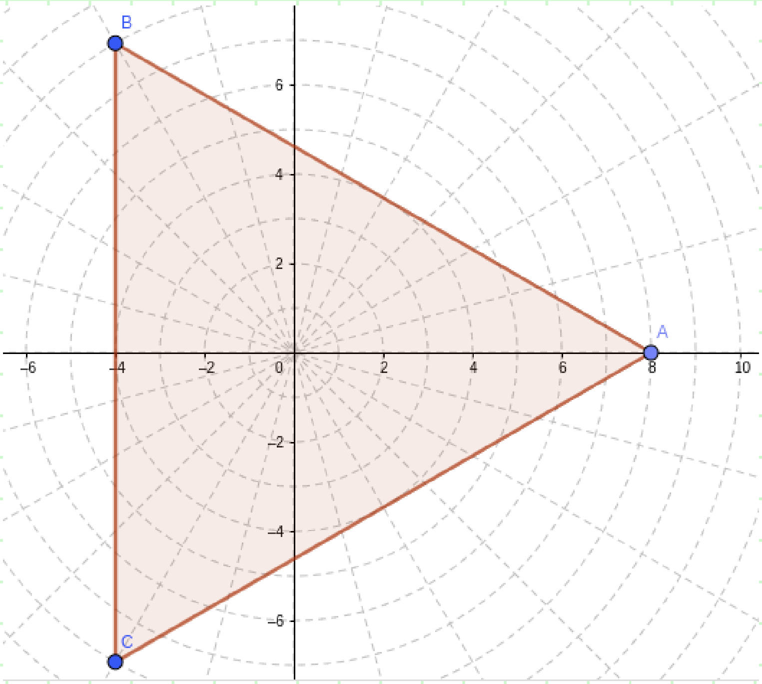
✎ Si $\mathbf{k} = \mathbf{2}$

$$C_2 = 8 \cdot exp\left\{i\left(\frac{0 + 2\pi(2)}{3}\right)\right\} = 8 \cdot exp\left\{i\left(\frac{4\pi}{3}\right)\right\}$$

$$C_2 = 8 \cdot exp\left\{i\left(\frac{4\pi}{3}\right)\right\}$$

$$\frac{4\pi}{3} \iff 240^\circ$$

✏ Graficar las raices



(10) $\sqrt[6]{-27\,i}$

$$\sqrt[6]{-27\,i}$$

$$z_0 = -27\,i$$

✏

$$r_0 = \sqrt{(-27)^{\,2}} = \sqrt{729} = \boxed{27}$$

✏

$$\theta_0 = \tan^{-1}\left(\frac{-27}{0}\right) = Error = \boxed{-\frac{\pi}{2}}$$

$$\text{n} = 6$$

$$\text{k=0,1,2,3,4,5}$$

Hallando raices:

✏

$$C_0 = \sqrt[6]{27} \cdot exp\left[i\left(\frac{-\frac{\pi}{2} + 2(0)\,\pi}{6}\right)\right]$$

$$C_0 = \sqrt[6]{27} \cdot exp\left[-\frac{\pi}{12}\,i\right]$$

$$-\frac{\pi}{12} \iff \boxed{-15^o}$$

✏

$$C_1 = \sqrt[6]{27} \cdot exp\left[i\left(\frac{-\frac{\pi}{2} + 2(1)\,\pi}{6}\right)\right]$$

$$C_1 = \sqrt[6]{27} \cdot exp\left[\frac{\pi}{4}\,i\right]$$

$$\frac{\pi}{4} \iff \boxed{45^o}$$

✏

$$C_2 = \sqrt[6]{27} \cdot exp\left[i\left(\frac{-\frac{\pi}{2} + 2(2)\,\pi}{6}\right)\right]$$

$$C_2 = \sqrt[6]{27} \cdot exp\left[\frac{7\,\pi}{12}\,i\right]$$

$$\frac{7\,\pi}{12} \iff \boxed{105^o}$$

✏

$$C_3 = \sqrt[6]{27} \cdot exp \left[i \left(\frac{-\dfrac{\pi}{2} + 2(3)\,\pi}{6} \right) \right]$$

$$C_3 = \sqrt[6]{27} \cdot exp \left[\frac{11\,\pi}{12}\, i \right]$$

$$\frac{11\,\pi}{12} \quad \Longleftrightarrow \quad \boxed{165^o}$$

✏

$$C_4 = \sqrt[6]{27} \cdot exp \left[i \left(\frac{-\dfrac{\pi}{2} + 2(4)\,\pi}{6} \right) \right]$$

$$C_4 = \sqrt[6]{27} \cdot exp \left[\frac{15\,\pi}{12}\, i \right]$$

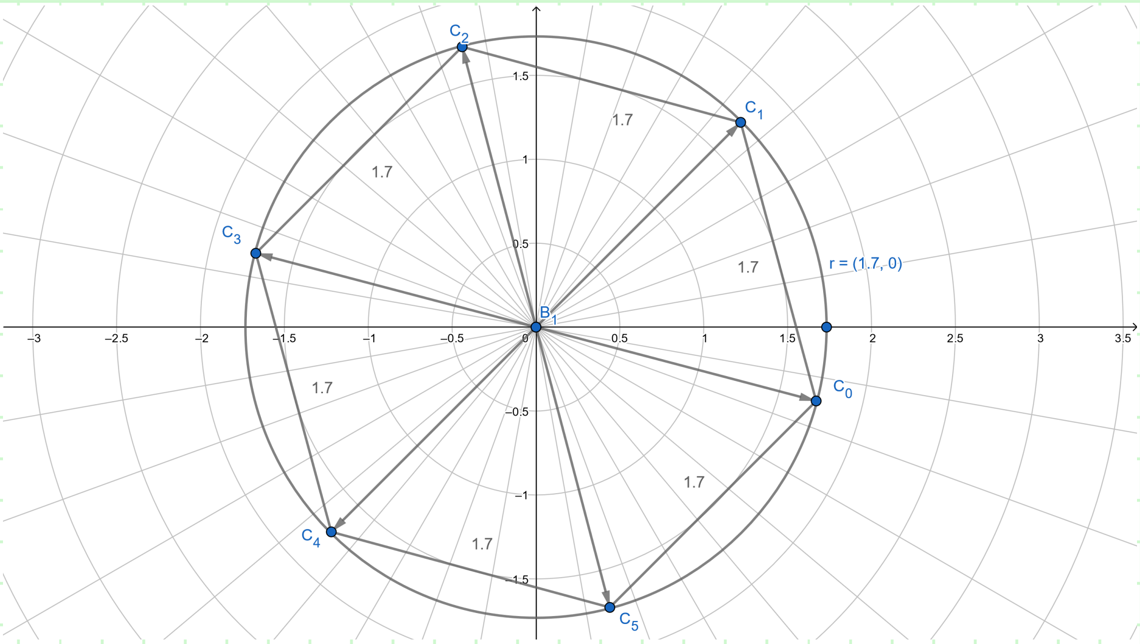
$$\frac{15\,\pi}{12} \quad \Longleftrightarrow \quad \boxed{225^o}$$

✏

$$C_5 = \sqrt[6]{27} \cdot exp \left[i \left(\frac{-\dfrac{\pi}{2} + 2(5)\,\pi}{6} \right) \right]$$

$$C_5 = \sqrt[6]{27} \cdot exp \left[\frac{19\,\pi}{12}\, i \right]$$

$$-\frac{19\,\pi}{12} \quad \Longleftrightarrow \quad \boxed{285^o}$$



(11) $\sqrt{\mathbf{4\sqrt{2} + i\,4\sqrt{2}}}$

✏ Hallar el módulo y el angulo $\mathbf{z_0}$.

$$z_0 = 4\sqrt{2} + i\,4\sqrt{2}$$

✎ Módulo de $\mathbf{z_0}$

$$r_0 = |z_0| = \sqrt{\left(4\sqrt{2}\right)^2 + \left(4\sqrt{2}\right)^2} = \sqrt{16\,(2) + 16\,(2)} = \sqrt{32 + 32} = \sqrt{64} = 8$$

✎ Angulo de $\mathbf{z_0}$

$$\theta_0 = \tan^{-1}\left(\frac{4\sqrt{2}}{4\sqrt{2}}\right) = \tan^{-1} 1 = \frac{\pi}{4}$$

✏ En este caso $\mathbf{n} = \mathbf{2}$ y $\mathbf{k} = \mathbf{0}\,,\,\mathbf{1}$

✎ Si $\mathbf{k} = \mathbf{0}$

$$C_0 = \sqrt{8}\cdot\ exp\left\{i\left(\frac{\frac{\pi}{4} + 2\,\pi\,(0)}{2}\right)\right\} = 2\sqrt{2}\cdot\ exp\left\{i\left(\frac{\frac{\pi}{4}}{2}\right)\right\}$$

$$C_0 = 2\sqrt{2}\cdot\ exp\left\{i\left(\frac{\pi}{8}\right)\right\}$$

$$\frac{\pi}{8} \quad\Longleftrightarrow\quad 22{,}5^\circ$$

✎ Si $\mathbf{k} = \mathbf{1}$

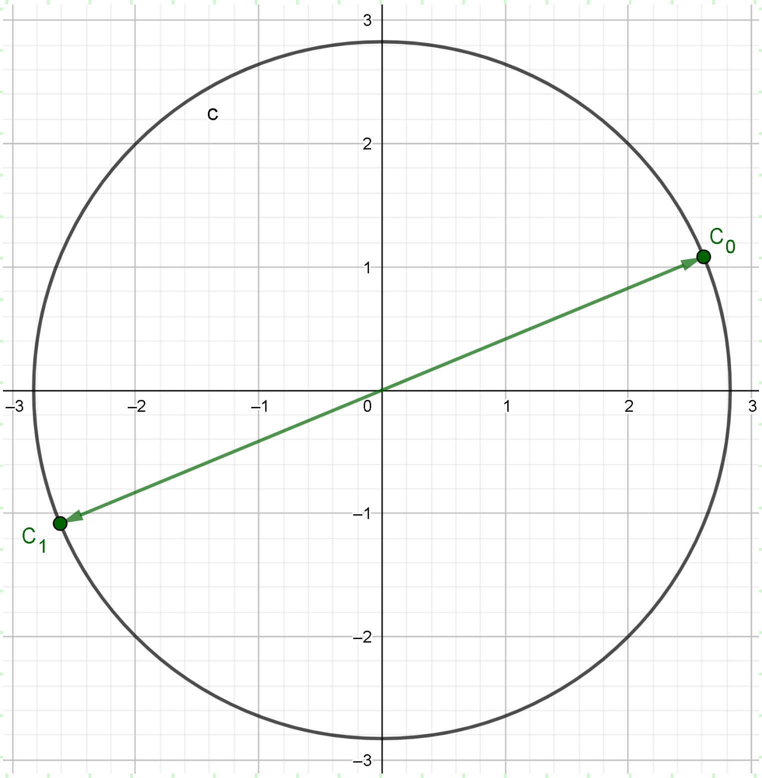
$$C_1 = \sqrt{8}\cdot\ exp\left\{i\left(\frac{\frac{\pi}{4} + 2\,\pi\,(1)}{2}\right)\right\} = 2\sqrt{2}\cdot\ exp\left\{i\left(\frac{\frac{\pi}{4} + 2\,\pi}{2}\right)\right\}$$

$$= 2\sqrt{2}\cdot\ exp\left\{i\left(\frac{\frac{\pi + 8\,\pi}{4}}{2}\right)\right\} = 2\sqrt{2}\cdot\ exp\left\{i\left(\frac{\frac{9\,\pi}{4}}{2}\right)\right\}$$

$$C_1 = 2\sqrt{2}\cdot\ exp\left\{i\left(\frac{9\,\pi}{8}\right)\right\}$$

$$\frac{9\,\pi}{8} \quad\Longleftrightarrow\quad 202{,}5^\circ$$

✏ Graficar las raices



(12) $\left(-\mathbf{16} + \mathbf{i}\,\mathbf{16}\,\sqrt{\mathbf{3}}\right)^{\mathbf{1/5}}$

✏ Hallar el módulo y el angulo $\mathbf{z_0}$

$$z_0 = -16 + i\,16\,\sqrt{3}$$

✎ Módulo de $\mathbf{z_0}$

$$r_0 = |z_0| = \sqrt{(-16)^2 + \left(16\,\sqrt{3}\right)^2} = \sqrt{256 + 256\,(3)} = \sqrt{1024} = 32$$

✎ Angulo de $\mathbf{z_0}$

$$\theta_0 = \tan^{-1}\left(\frac{16\,\sqrt{3}}{-16}\right) = \tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$$

☞ Correción θ_0 Cuadrantre II

$$\theta_2 = \pi - |\theta_0| = \pi - \left|-\frac{\pi}{3}\right| = \pi - \frac{\pi}{3} = \frac{3\,\pi - \pi}{3} = \frac{2\,\pi}{3}$$

✏ En este caso $\mathbf{n} = \mathbf{5}$ y $\mathbf{k} = \mathbf{0}\,,\,\mathbf{1}\,,\,\mathbf{2}\,,\mathbf{3}\,,\mathbf{4}$

✎ Si $\mathbf{k} = \mathbf{0}$

$$C_0 = 32\,\cdot\,exp\left\{i\left(\frac{\frac{2\,\pi}{3} + 2\,\pi\,(0)}{5}\right)\right\} = 32\,\cdot\,exp\left\{i\left(\frac{\frac{2\,\pi}{3} + 0}{5}\right)\right\}$$

$$= 32\,\cdot\,exp\left\{i\left(\frac{\frac{2\,\pi}{3}}{5}\right)\right\}$$

$$C_0 \;=\; 32 \,\cdot\, exp\left\{i\left(\frac{2\,\pi}{15}\right)\right\}$$

$$\frac{2\,\pi}{15} \quad\Longleftrightarrow\quad 24°$$

✎ Si $\mathbf{k} \;=\; \mathbf{1}$

$$C_1 \;=\; 32 \,\cdot\, exp\left\{i\left(\frac{\dfrac{2\,\pi}{3} \,+\, 2\,\pi\,(1)}{5}\right)\right\} = 32 \,\cdot\, exp\left\{i\left(\frac{\dfrac{2\,\pi}{3} \,+\, 2\,\pi}{5}\right)\right\}$$

$$= 32 \,\cdot\, exp\left\{i\left(\frac{\dfrac{2\,\pi \,+\, 6\,\pi}{3}}{5}\right)\right\} = 32 \,\cdot\, exp\left\{i\left(\frac{\dfrac{8\,\pi}{3}}{5}\right)\right\}$$

$$C_1 \;=\; 32 \,\cdot\, exp\left\{i\left(\frac{8\,\pi}{15}\right)\right\}$$

$$\frac{8\,\pi}{15} \quad\Longleftrightarrow\quad 96°$$

✎ Si $\mathbf{k} \;=\; \mathbf{2}$

$$C_2 \;=\; 32 \,\cdot\, exp\left\{i\left(\frac{\dfrac{2\,\pi}{3} \,+\, 2\,\pi\,(2)}{5}\right)\right\} = 32 \,\cdot\, exp\left\{i\left(\frac{\dfrac{2\,\pi}{3} \,+\, 4\,\pi}{5}\right)\right\}$$

$$= 32 \,\cdot\, exp\left\{i\left(\frac{\dfrac{2\,\pi \,+\, 12\,\pi}{3}}{5}\right)\right\} = 32 \,\cdot\, exp\left\{i\left(\frac{\dfrac{14\,\pi}{3}}{5}\right)\right\}$$

$$C_2 \;=\; 32 \,\cdot\, exp\left\{i\left(\frac{14\,\pi}{15}\right)\right\}$$

$$\frac{14\,\pi}{15} \quad\Longleftrightarrow\quad 168°$$

✎ Si $\mathbf{k} \;=\; \mathbf{3}$

$$C_3 \;=\; 32 \,\cdot\, exp\left\{i\left(\frac{\dfrac{2\,\pi}{3} \,+\, 2\,\pi\,(3)}{5}\right)\right\} = 32 \,\cdot\, exp\left\{i\left(\frac{\dfrac{2\,\pi}{3} \,+\, 6\,\pi}{5}\right)\right\}$$

$$= 32 \,\cdot\, exp\left\{i\left(\frac{\dfrac{2\,\pi \,+\, 18\,\pi}{3}}{5}\right)\right\} = 32 \,\cdot\, exp\left\{i\left(\frac{\dfrac{20\,\pi}{3}}{5}\right)\right\}$$

$$C_3 \;=\; 32 \,\cdot\, exp\left\{i\left(\frac{20\,\pi}{15}\right)\right\} \;=\; 32 \,\cdot\, exp\left\{i\left(\frac{4\,\pi}{3}\right)\right\}$$

$$\frac{4\,\pi}{3} \quad\Longleftrightarrow\quad 240°$$

✎ Si $\mathbf{k} = \mathbf{4}$

$$C_4 = 32 \cdot \ exp \left\{ i \left(\frac{\frac{2\pi}{3} + 2\pi\,(4)}{5} \right) \right\} = 32 \cdot \ exp \left\{ i \left(\frac{\frac{2\pi}{3} + 8\pi}{5} \right) \right\}$$

$$= 32 \cdot \ exp \left\{ i \left(\frac{\frac{2\pi + 24\pi}{3}}{5} \right) \right\} = 32 \cdot \ exp \left\{ i \left(\frac{\frac{26\pi}{3}}{5} \right) \right\}$$

$$C_4 = 32 \cdot \ exp \left\{ i \left(\frac{26\pi}{15} \right) \right\}$$

$$\frac{26\pi}{15} \quad \Longleftrightarrow \quad 312^\circ$$

✏ Graficar las raices