

Mainly consist of precalculus, and basic Calculus.

Algebra

Laws of Exponents.

$$\begin{aligned}x^{\frac{m}{n}} &= \sqrt[n]{m} \\(x^m)^n &= x^{mn} \\x^m x^n &= x^{m+n} \\x^a y^a &= (xy)^a\end{aligned}$$

Special Factorization.

$$\begin{aligned}x^2 - y^2 &= (x + y)(x - y) \\x^3 - y^3 &= (x - y)(x^2 + xy + y^2) \\x^3 + y^3 &= (x + y)(x^2 - xy + y^2)\end{aligned}$$

Quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \begin{cases} D > 0 & \text{re}(2) \\ D = 0 & \text{re}(1) \\ D < 0 & \text{im}(2) \end{cases}$$

Binomial theorem.

$$(a + b)^n = \sum_{k=0}^{\infty} \binom{n}{k} a^{n-k} b^k$$

with

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{\Gamma(n+1)}{\Gamma(k+1)\Gamma(n-k+1)}$$

Trigonometry

Trigonometry Definition.

$$\begin{aligned}\sin \theta &= \frac{1}{\csc \theta} \\\cos \theta &= \frac{1}{\sec \theta} \\\tan \theta &= \frac{1}{\cot \theta}\end{aligned}$$

Pythagorean Identity.

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\\sec^2 \theta - \tan^2 \theta &= 1 \\\csc^2 \theta - \cot^2 \theta &= 1\end{aligned}$$

Law of Sines.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Trigonometry Double Angle Identity.

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= \cos^2 \theta - \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Trigonometry Addition and Difference Identity.

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Trigonometry Product Rule.

$$\cos x \cos y = \frac{1}{2} [\cos(x - y) + \cos(x + y)]$$

$$\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$$

$$\sin x \cos y = \frac{1}{2} [\sin(x + y) + \sin(x - y)]$$

$$\cos x \sin y = \frac{1}{2} [\sin(x + y) - \sin(x - y)]$$

Neat Mnemonics.

$$\begin{vmatrix} S^+ \\ S^- \\ C^+ \\ C^- \end{vmatrix} = \begin{vmatrix} SC + CS \\ SC - CS \\ CC - SS \\ CC + SS \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} CC \\ SS \\ SC \\ CS \end{vmatrix} = \begin{vmatrix} C^- + C^+ \\ C^- - C^+ \\ S^+ + S^- \\ S^+ - S^- \end{vmatrix}$$

Logarithm

Definition (informal). $\log_a b$ means a to the power of what equal b .

Few important log rule.

$$\log_c(ab) = \log_c(a) + \log_c(b)$$

$$\log_c\left(\frac{a}{b}\right) = \log_c(a) - \log_c(b)$$

$$\log_a b = \frac{\log_c(b)}{\log_c(a)}$$

$$a^{\log_a b} = b$$

Limit

Few Important Limits.

$$\lim_{x \rightarrow a} c = c$$

$$\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty \rightarrow \frac{1}{0^+} = \infty$$

$$\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty \rightarrow \frac{1}{0^-} = -\infty$$

Limit as Definition of Derivative.

$$\frac{d}{dx}y = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Derivative

How to determine the order of derivation: last computation is the first thing to do.

General Formula.

$$D x^n = nx^{n-1}$$

$$D(uv) = D u \cdot v + u \cdot D v$$

$$D\left(\frac{u}{v}\right) = \frac{D u \cdot v - u \cdot D v}{v^2}$$

Trigonometry Formula.

$$D \sin x = \cos x$$

$$D \cos x = -\sin x$$

$$D \tan x = \sec^2 x$$

$$D \cot x = -\csc^2 x$$

$$D \sec x = \sec x \tan x$$

$$D \csc x = -\cot x \csc x$$

Neat Mnemonics.

sec	sec	tan	↓ cofunction
csc	- csc	cot	
↔ multiply			

Exponential and Logarithmic Functions.

$$D \ln x = \frac{1}{x}$$

$$D a^n = a^n \ln a$$

$$D \log_a b = \frac{1}{b \ln a}$$

Minima and Maxima test. First derivative test:

- Determine critical points ($Dy = 0$), then divide into region;
- Pick value from each region and plug into *derivative*;
- Do the sign-graph.
- Determine local minima and maxima, then plug into *original function*

Second derivative test:

- determine critical points;
- plug critical into second derivative; and
- positive D^2y means concave up (\smile), negative means concave down (\frown), and 0 means inconclusive.

Differentiation under integral sign. Differentiation under integral sign stated by Leibniz' rule

$$\frac{d}{dx} \int_{u(x)}^{v(x)} f(x, t) dt = \int_u^v \frac{\partial f}{\partial x} dt + f(x, v) \frac{dv}{dx} - f(x, u) \frac{du}{dx}$$

Proof. Suppose we want dI/dx where

$$I = \int_u^v f(t) dt$$

By the fundamental theorem of calculus

$$I = F(v) - F(u) = \mathcal{F}(v, u)$$

or I is a function of v and u . Finding dI/dx is then a partial differentiation problem. We can write

$$\frac{dI}{dx} = \frac{\partial I}{\partial v} \frac{dv}{dx} + \frac{\partial I}{\partial u} \frac{du}{dx}$$

By the fundamental theorem of calculus, we have

$$\begin{aligned} \frac{d}{dv} \int_a^v f(x) dt &= \frac{d}{dv} [F(v) - F(a)] = f(v) \\ \frac{d}{dv} \int_u^b f(x) dt &= \frac{d}{dv} [F(b) - F(u)] = -f(u) \end{aligned}$$

where u and v are a function of x , while a and b are a constant. This is the case when we consider $\partial I/\partial v$ or $\partial I/\partial u$; the other variable is constant. Then

$$\frac{d}{dx} \int_u^v f(t) dt = f(v) \frac{dv}{dx} - f(u) \frac{du}{dx}$$

Under not too restrictive conditions,

$$\frac{d}{dx} \int_a^b f(x, t) dt = \int_a^b \frac{\partial f(x, t)}{\partial x} dt$$

where, as before, a and b are constant. In other words, we can differentiate under the integral sign. It is convenient to collect these formulas into one formula known as Leibniz' rule:

$$\frac{d}{dx} \int_{u(x)}^{v(x)} f(x, t) dt = \int_u^v \frac{\partial f}{\partial x} dt + f(x, v) \frac{dv}{dx} - f(x, u) \frac{du}{dx} \quad \blacksquare$$

Leibniz' rule for differentiating a product.

$$\left(\frac{d}{dx}\right)^n fg = \sum_{k=0}^n \binom{n}{k} \left(\frac{d}{dx}\right)^{n-k} f \left(\frac{d}{dx}\right)^k g$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Integral

Basic Formula (integration constant omitted).

$$\begin{aligned} \int x^n dx &= \frac{1}{n+1} x^{n+1} \\ \int \frac{1}{x} dx &= \ln |x| \\ \int u dv &= uv - \int v du \\ \int a^x dx &= \frac{a^x}{\ln a} \end{aligned}$$

Trigonometry.

$$\begin{aligned} \int \sin x dx &= -\cos x \\ \int \cos x dx &= \sin x \\ \int \sec^2 x dx &= \tan x \\ \int \csc^2 x dx &= -\cot x \\ \int \sec x \tan x dx &= \sec x \\ \int \csc x \tan x dx &= -\csc x \end{aligned}$$

Root.

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a}$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln x + \sqrt{x^2 \pm a^2}$$

$$\int \frac{1}{\sqrt{a^2 + x^2}} dx = \frac{1}{a} \arctan \frac{x}{a}$$

Integration by part.

1. Splits the integrand. Choose u using LIATEN and let the rest be dv . (LIATEN: Log, Inverse trigonometry, Algebra, Trigonometry, Exponent)

2. Do the box:

u	v	\downarrow diff.
du	dv	\uparrow int.

3. $\int u dv = uv - \int v du$

Tabular Method. Refer to the table

	D	I
+	$a \searrow$	b
-	$a' \searrow$	b
+	$a'' \searrow$	b
\vdots	\vdots	\vdots

1. 0 in D column or use LIATEN
2. integrate a row
3. a row repeats

Trigonometry Integral. Pythagorean Identity.

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

note that argument inside quadratic trigonometry is half of trigonometry, which means $\cos^2 2x = (1 + \cos 4x)/2$. There are few cases of tricky trigonometry integral. First, if power of sin is odd and positive.

- lop one power off
- convert remaining (even power) using Pythagorean Identity in term of cosine
- integrate using subs method

If the power of sine is odd and positive.

- same as before

If the power of sine and cosine is even and nonnegative, then:

- convert using Pythagorean Identity and solve

Trigonometry substitution. Trigonometry function and its radical pair

$$\tan \theta = \sqrt{u^2 + a^2}$$

$$\sin \theta = \frac{u}{\sqrt{a^2 - u^2}}$$

$$\sec \theta = \frac{a}{\sqrt{u^2 - a^2}}$$

where u is the variable we are differentiating with respect to. Mnemonics: + looks like tangent; - for sin and sec; and it is $a \sin$. Trigonometry substitution step is then.

1. Draw a right triangle where trigonometry pair equal $\frac{u}{a}$
2. using the trigonometry pair equation*, solve for x and dx
3. find trigonometry where $\frac{u}{a}$
4. subs again if equation* still contain θ and solve

Partial Fraction.

1. Factor out denominator
2. Breakup the function and put unknown (Capital Letter) into numerator. Put numerator normally if factor is linear, put $Px+Q$ Irreducible quadratic factor IQF. In general,

$$\frac{Ax^{n-1} + Bx^{n-2} + \dots}{x^n + x^{n-1} + \dots}$$

3. Multiply both side by left side's denominator
4. Take the roots of the linear factors and plug them into x , and solve for the unknowns
5. Put unknowns into step 2
6. Splits Integral, then solve
7. For equating coefficients like terms, after step 3, expand equation*. Then, collect like terms and equate coefficient of like terms from both side

Appendix

Integration Technique Example. 1. Trigonometry substitution. Find $\int \frac{dx}{\sqrt{9x^2 + 4}}$. Refer to the Mnemonics, the trigonometry pair is tangent.

$$I = \int \frac{dx}{\sqrt{(3x)^2 + 2^2}}$$
$$\tan \theta = \frac{3x}{2}$$

solving for x and dx

$$x = \frac{2}{3} \tan \theta$$
$$dx = \frac{2}{3} \sec^2 \theta d\theta$$

trigonometry where $\frac{y}{a}$ holds is secant, solving for radical

$$\sec \theta = \frac{\sqrt{9x^2 + 4}}{2}$$
$$\sqrt{9x^2 + 4} = 2 \sec \theta$$

the integral is then

$$I = \frac{1}{3} \int \sec \theta d\theta$$
$$= \frac{1}{3} \ln |\sec \theta + \tan \theta| + C$$

substituting the θ function

$$I = \frac{1}{3} \ln \left| \frac{\sqrt{9x^2 + 4}}{2} + \frac{3x}{2} \right| + C$$
$$= \frac{1}{3} \ln \left| \sqrt{9x^2 + 4} + 3x \right| + C$$

Basic.

$$\int \frac{1}{x} dx = \ln |x|$$
$$\int u dv = uv - \int v du$$
$$\int u dv = uv - \int v du$$
$$\int \frac{1}{ax + b} dx = \frac{1}{a} \ln |ax + b|$$

Integrals of Rational Functions.

$$\begin{aligned}\int \frac{1}{(x+a)^2} dx &= -\frac{1}{x+a} \\ \int (x+a)^n dx &= \frac{(x+a)^{n+1}}{n+1}, n \neq -1 \\ \int x(x+a)^n dx &= \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \\ \int \frac{1}{1+x^2} dx &= \tan^{-1} x \\ \int \frac{1}{a^2+x^2} dx &= \frac{1}{a} \tan^{-1} \frac{x}{a} \\ \int \frac{x}{a^2+x^2} dx &= \frac{1}{2} \ln |a^2+x^2| \\ \int \frac{x^2}{a^2+x^2} dx &= x - a \tan^{-1} \frac{x}{a} \\ \int \frac{x^3}{a^2+x^2} dx &= \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln |a^2+x^2| \\ \int \frac{1}{ax^2+bx+c} dx &= \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \\ \int \frac{1}{(x+a)(x+b)} dx &= \frac{1}{b-a} \ln \frac{a+x}{b+x}, \quad a \neq b \\ \int \frac{x}{(x+a)^2} dx &= \frac{a}{a+x} + \ln |a+x| \\ \int \frac{x}{ax^2+bx+c} dx &= \frac{1}{2a} \ln |ax^2+bx+c| - \\ &\quad \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}}\end{aligned}$$

Integrals with Roots.

$$\int \sqrt{x-a} \, dx = \frac{2}{3}(x-a)^{3/2}$$

$$\int \frac{1}{\sqrt{x \pm a}} \, dx = 2\sqrt{x \pm a}$$

$$\int \frac{1}{\sqrt{a-x}} \, dx = -2\sqrt{a-x}$$

$$\int x\sqrt{x-a} \, dx = \begin{cases} \frac{2a}{3}(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2}, & \text{or} \\ \frac{2}{3}x(x-a)^{3/2} - \frac{4}{15}(x-a)^{5/2}, & \text{or} \\ \frac{2}{15}(2a+3x)(x-a)^{3/2} \end{cases}$$

$$\int \sqrt{ax+b} \, dx = \left(\frac{2b}{3a} + \frac{2x}{3} \right) \sqrt{ax+b}$$

$$\int (ax+b)^{3/2} \, dx = \frac{2}{5a}(ax+b)^{5/2}$$

$$\int \frac{x}{\sqrt{x \pm a}} \, dx = \frac{2}{3}(x \mp 2a)\sqrt{x \pm a}$$

$$\int \sqrt{\frac{x}{a-x}} \, dx = -\sqrt{x(a-x)} - a \tan^{-1} \frac{\sqrt{x(a-x)}}{x-a}$$

$$\int \sqrt{\frac{x}{a+x}} \, dx = \sqrt{x(a+x)} - a \ln[\sqrt{x} + \sqrt{x+a}]$$

$$\int x\sqrt{ax+b} \, dx = \frac{2}{15a^2}(-2b^2 + abx + 3a^2x^2)\sqrt{ax+b}$$

$$\int \sqrt{x^3(ax+b)} \, dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3} \right] \sqrt{x^3(ax+b)} \\ + \frac{b^3}{8a^{5/2}} \ln \left| a\sqrt{x} + \sqrt{a(ax+b)} \right|$$

$$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2 \tan^{-1} \frac{x}{\sqrt{a^2-x^2}}$$

$$\int x\sqrt{x^2 \pm a^2} \, dx = \frac{1}{3}(x^2 \pm a^2)^{3/2}$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} \, dx = \ln \left| x + \sqrt{x^2 \pm a^2} \right|$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1} \frac{x}{a}$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}} \, dx = \sqrt{x^2 \pm a^2}$$

$$\int \frac{x}{\sqrt{a^2-x^2}} \, dx = -\sqrt{a^2-x^2}$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}} \, dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2 \ln \left| x + \sqrt{x^2 \pm a^2} \right|$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}} \, dx = \frac{1}{\sqrt{a}} \ln \left| 2ax+b+2\sqrt{a(ax^2+bx+c)} \right|$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$$

Integrals with Logarithms.

$$\int \ln ax \, dx = x \ln(ax) - x$$

$$\int \frac{\ln ax}{x} \, dx = \frac{1}{2}(\ln ax)^2$$

$$\int \ln(ax + b)dx = (x + \frac{b}{a}) \ln(ax + b) - x, \quad a \neq 0$$

$$\int \ln(x^2 + a^2) \, dx = x \ln(x^2 + a^2) + 2a \tan^{-1} \frac{x}{a} - 2x$$

$$\int \ln(x^2 - a^2) \, dx = x \ln(x^2 - a^2) + a \ln \frac{x + a}{x - a} - 2x$$

$$\int \ln(x^2 - a^2) \, dx = x \ln(x^2 - a^2) + a \ln \frac{x + a}{x - a} - 2x$$

$$\int \ln(ax^2 + bx + c)dx = \frac{1}{a} \sqrt{4ac - b^2} \tan^{-1} \frac{2ax + b}{\sqrt{4ac - b^2}} - 2x$$

$$+ (\frac{b}{2a} + x) \ln (ax^2 + bx + c)$$

$$\int x \ln(ax + b)dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}(x^2 - \frac{b^2}{a^2}) \ln(ax + b)$$

$$\frac{1}{2}(x^2 - \frac{a^2}{b^2}) \ln(a^2 - b^2x^2)$$

$$\int x \ln(a^2 - b^2x^2)dx = -\frac{1}{2}x^2 +$$

Integrals with Exponential.

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int \sqrt{x} e^{ax} dx = \frac{1}{a} \sqrt{x} e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}} \operatorname{erf}(i\sqrt{ax})$$

$$\int x e^x dx = (x - 1) e^x$$

$$\int x e^{ax} dx = \left(\frac{x}{a} - \frac{1}{a^2} \right) e^{ax}$$

$$\int x^2 e^x dx = (x^2 - 2x + 2) e^x$$

$$\int x^2 e^{ax} dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3} \right) e^{ax}$$

$$\int x^3 e^x dx = (x^3 - 3x^2 + 6x - 6) e^x$$

$$\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx$$

$$\int x^n e^{ax} dx = \frac{(-1)^n}{a^{n+1}} \Gamma[1+n, -ax],$$

$$\text{where } \Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$$

$$\int e^{ax^2} dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}(ix\sqrt{a})$$

$$\int e^{-ax^2} dx = \frac{\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}(x\sqrt{a})$$

$$\int x e^{-ax^2} dx = -\frac{1}{2a} e^{-ax^2}$$

$$\int x^2 e^{-ax^2} dx = \frac{1}{4} \sqrt{\frac{\pi}{a^3}} \operatorname{erf}(x\sqrt{a}) - \frac{x}{2a} e^{-ax^2}$$

Integrals with Trigonometry Functions.

$$\int \sin ax dx = -\frac{1}{a} \cos ax$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int \sin^n ax dx =$$

$$-\frac{1}{a} \cos ax {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right]$$

$$\int \sin^3 ax dx = -\frac{3 \cos ax}{4a} + \frac{\cos 3ax}{12a}$$

$$\int \cos ax dx = \frac{1}{a} \sin ax$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int \cos^p ax dx = -\frac{1}{a(1+p)} \cos^{1+p} ax \times$$

$${}_2F_1\left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right]$$

$$\int \cos^3 ax dx = \frac{3 \sin ax}{4a} + \frac{\sin 3ax}{12a}$$

$$\int \cos ax \sin bxdx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b$$

$$\begin{aligned}\int \sin^2 ax \cos bxdx &= -\frac{\sin[(2a-b)x]}{4(2a-b)} \\ &\quad + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)}\end{aligned}$$

$$\int \sin^2 x \cos x dx = \frac{1}{3} \sin^3 x$$

$$\begin{aligned}\int \cos^2 ax \sin bxdx &= \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} \\ &\quad - \frac{\cos[(2a+b)x]}{4(2a+b)}\end{aligned}$$

$$\int \cos^2 ax \sin ax dx = -\frac{1}{3a} \cos^3 ax$$

$$\begin{aligned}\int \sin^2 ax \cos^2 bxdx &= \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} \\ &\quad + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)}\end{aligned}$$

$$\int \sin^2 ax \cos^2 ax dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$$

$$\int \tan ax dx = -\frac{1}{a} \ln \cos ax$$

$$\int \tan^2 ax dx = -x + \frac{1}{a} \tan ax$$

$$\begin{aligned}\int \tan^n ax dx &= \frac{\tan^{n+1} ax}{a(1+n)} \times \\ &\quad {}_2F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right)\end{aligned}$$

$$\int \tan^3 ax dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax$$

$$\int \sec x dx = \ln |\sec x + \tan x| = 2 \tanh^{-1} \left(\tan \frac{x}{2} \right)$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax$$

$$\int \sec^3 x dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x|$$

$$\int \sec x \tan x dx = \sec x$$

$$\int \sec^2 x \tan x dx = \frac{1}{2} \sec^2 x$$

$$\int \sec^n x \tan x dx = \frac{1}{n} \sec^n x, n \neq 0$$

$$\int \csc x dx = \ln \left| \tan \frac{x}{2} \right| = \ln |\csc x - \cot x| + C$$

$$\int \csc^2 ax dx = -\frac{1}{a} \cot ax$$

$$\int \csc^3 x dx = -\frac{1}{2} \cot x \csc x + \frac{1}{2} \ln |\csc x - \cot x|$$

$$\int \csc^n x \cot x dx = -\frac{1}{n} \csc^n x, n \neq 0$$

$$\int \sec x \csc x dx = \ln |\tan x|$$

Products of Trigonometry Functions and Monomials.

$$\int x \cos x dx = \cos x + x \sin x$$

$$\int x \cos ax dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax$$

$$\int x^2 \cos x dx = 2x \cos x + (x^2 - 2) \sin x$$

$$\int x^2 \cos ax dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax$$

$$\begin{aligned} \int x^n \cos x dx &= -\frac{1}{2} (i)^{n+1} [\Gamma(n+1, -ix) \\ &\quad + (-1)^n \Gamma(n+1, ix)] \end{aligned}$$

$$\begin{aligned} \int x^n \cos ax dx &= \frac{1}{2} (ia)^{1-n} [(-1)^n \Gamma(n+1, -iax) \\ &\quad - \Gamma(n+1, iax)] \end{aligned}$$

$$\int x \sin x dx = -x \cos x + \sin x$$

$$\int x \sin ax dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2}$$

$$\int x^2 \sin x dx = (2 - x^2) \cos x + 2x \sin x$$

$$\int x^2 \sin ax dx = \frac{2 - a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2}$$

$$\int x^n \sin x dx = -\frac{1}{2} (i)^n [\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, -ix)]$$

Products of Trigonometry Functions and Exponential.

$$\int e^x \sin x dx = \frac{1}{2} e^x (\sin x - \cos x)$$

$$\int e^{bx} \sin ax dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax)$$

$$\int e^x \cos x dx = \frac{1}{2} e^x (\sin x + \cos x)$$

$$\int e^{bx} \cos ax dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax)$$

$$\int x e^x \sin x dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x)$$

$$\int x e^x \cos x dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x)$$

Integrals of Hyperbolic Functions.

$$\int \cosh ax dx = \frac{1}{a} \sinh ax$$

$$\int e^{ax} \cosh bxdx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [a \cosh bx - b \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} + \frac{x}{2} & a = b \end{cases}$$

$$\int \sinh ax dx = \frac{1}{a} \cosh ax$$

$$\int e^{ax} \sinh bxdx = \begin{cases} \frac{e^{ax}}{a^2 - b^2} [-b \cosh bx + a \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} - \frac{x}{2} & a = b \end{cases}$$

$$\int e^{ax} \tanh bxdx = \begin{cases} \frac{e^{(a+2b)x}}{(a+2b)} {}_2F_1 \left[1 + \frac{a}{2b}, 1, 2 + \frac{a}{2b}, -e^{2bx} \right] \\ \quad - \frac{1}{a} e^{ax} {}_2F_1 \left[\frac{a}{2b}, 1, 1E, -e^{2bx} \right] & a \neq b \\ \frac{e^{ax} - 2 \tan^{-1}[e^{ax}]}{a} & a = b \end{cases}$$

$$\int \tanh ax dx = \frac{1}{a} \ln \cosh ax$$

$$\int \cos ax \cosh bxdx = \frac{1}{a^2 + b^2} [a \sin ax \cosh bx + b \cos ax \sinh bx]$$

$$\int \cos ax \sinh bxdx = \frac{1}{a^2 + b^2} [b \cos ax \cosh bx + a \sin ax \sinh bx]$$

$$\int \sin ax \cosh bxdx = \frac{1}{a^2 + b^2} [-a \cos ax \cosh bx + b \sin ax \sinh bx]$$

$$\int \sin ax \sinh bxdx = \frac{1}{a^2 + b^2} [b \cosh bx \sin ax - a \cos ax \sinh bx]$$

$$\int \sinh ax \cosh axdx = \frac{1}{4a} [-2ax + \sinh 2ax]$$

$$\int \sinh ax \cosh bxdx = \frac{1}{b^2 - a^2} [b \cosh bx \sinh ax - a \cosh ax \sinh bx]$$