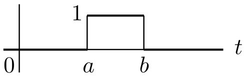
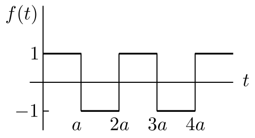
Appendix II: Laplace Table

$\mathcal{L}1$	1	$\dfrac{1}{p+a}$ Re $p>0$
$\mathcal{L}2$	e^{-at}	$\dfrac{1}{p}$ Re $p>0$
$\mathcal{L}3$	$\sin at$	$\dfrac{a}{p^2+a^2}$ Re $p>\lvert\text{Im } a\rvert$
$\mathcal{L}4$	$\cos at$	$\dfrac{p}{p^2+a^2}$ Re $p>\lvert\text{Im } a\rvert$
$\mathcal{L}5$	$t^k,\ k>-1$	$\dfrac{k!}{p^{k+1}}$ or $\dfrac{\Gamma(k+1)}{p^{k+1}}$ Re $p>0$
$\mathcal{L}6$	$t^k e^{-at}, k>-1$	$\dfrac{k!}{(p+a)^{k+1}}$ or $\dfrac{\Gamma(k+1)}{(p+a)^{k+1}}$ Re $(p+a)>0$
$\mathcal{L}7$	$\dfrac{e^{-at}-e^{-bt}}{b-a}$	$\dfrac{1}{(p+a)(p+b)}$ Re $(p+a)>0$ Re $(p+b)>0$
$\mathcal{L}8$	$\dfrac{ae^{-at}-be^{-bt}}{b-a}$	$\dfrac{p}{(p+a)(p+b)}$ Re $(p+a)>0$ Re $(p+b)>0$
$\mathcal{L}9$	$\sinh at$	$\dfrac{a}{p^2-a^2}$ Re $p>\lvert\text{Re } a\rvert$
$\mathcal{L}10$	$\cosh at$	$\dfrac{p}{p^2-a^2}$ Re $p>\lvert\text{Re } a\rvert$

$\mathcal{L}11$	$t\sin at$	$\dfrac{2ap}{(p^2+a^2)^2}$ $\text{Re } p > \lvert\text{Im } a\rvert$
$\mathcal{L}12$	$t\cos at$	$\dfrac{p^2-a^2}{(p^2+a^2)^2}$ $\text{Re } p > \lvert\text{Im } a\rvert$
$\mathcal{L}13$	$e^{-at}\sin bt$	$\dfrac{b}{(p+a)^2+b^2}$ $\text{Re } (p+a) > \lvert\text{Im } b\rvert$
$\mathcal{L}14$	$e^{-at}\cos bt$	$\dfrac{p+a}{(p+a)^2+b^2}$ $\text{Re } (p+a) > \lvert\text{Im } b\rvert$
$\mathcal{L}15$	$1-\cos at$	$\dfrac{a^2}{p(p^2+a^2)}$ $\text{Re } p > \lvert\text{Im } a\rvert$
$\mathcal{L}16$	$at-\sin at$	$\dfrac{a^3}{p^2(p^2+a^2)}$ $\text{Re } p > \lvert\text{Im } a\rvert$
$\mathcal{L}17$	$\sin at - at\cos at$	$\dfrac{2a^3}{(p^2+a^2)^2}$ $\text{Re } p > \lvert\text{Im } a\rvert$
$\mathcal{L}18$	$e^{-at}(1-at)$	$\dfrac{p}{(p+a)^2}$ $\text{Re } (p+a) > 0$
$\mathcal{L}19$	$\dfrac{\sin at}{t}$	$\arctan\dfrac{a}{p}$ $\text{Re } p > \lvert\text{Im } a\rvert$
$\mathcal{L}20$	$\dfrac{1}{t}\sin at\cos bt$	$\dfrac{1}{2}\left(\arctan\dfrac{a+b}{p}+\arctan\dfrac{a-b}{p}\right)$ $\text{Re } p > 0$

$\mathcal{L}21$	$\dfrac{e^{at} - e^{-bt}}{t}$	$\ln \dfrac{p+b}{p+a}$ Re $(p+a) > 0$ Re $(p+b) > 0$
$\mathcal{L}22$	$1 - \operatorname{erf}\left(\dfrac{a}{2\sqrt{t}}\right)$ $a > 0$	$\dfrac{1}{p} e^{-a\sqrt{p}}$ Re $p > 0$
$\mathcal{L}23$	$J_0(at)$	$(p^2 + a^2)^{-1/2}$ Re $p > \lvert \text{Im } a \rvert$ Re $p \geq 0$ for real $a \neq 0$
$\mathcal{L}24$	unit step, Heaviside function $u(t-a) = \begin{cases} 1, \\ t > a > 0 \\ 0, \ t < a \end{cases}$	$\dfrac{1}{p} e^{-pa}$ Re $p > 0$
$\mathcal{L}25$	$f(t) = u(t-a) - u(t-b)$ 	$\dfrac{e^{-ap} - e^{-}bp}{p}$ All p
$\mathcal{L}26$		$\dfrac{1}{p} \tanh \frac{ap}{2}$ All p
$\mathcal{L}27$	$\delta(t-a), \ a \geq 0$	e^{-pa}
$\mathcal{L}28$	$f(t) = \begin{cases} g(t-a), & t > a > 0 \\ 0, & t < a \end{cases}$ $f(t) = g(t-a)u(t-a)$	$e^{-pa} G(p)$ $G(p)$ means $\mathcal{L}(g)$ Therefore $e^{-pa} \mathcal{L}\left[g(t-a)\right]$
$\mathcal{L}29$	$e^{-at} g(t)$	$G(p+a)$
$\mathcal{L}30$	$g(at), a > 0$	$\dfrac{1}{a} G\left(\dfrac{p}{a}\right)$
$\mathcal{L}31$	$\dfrac{g(t)}{t}$ if integrable	$\displaystyle\int_p^\infty G(u)\, du$
$\mathcal{L}32$	$t^n g(t)$	$(-1)^n \left(\dfrac{d}{dp}\right)^n (G(p))$

$\mathcal{L}33$	$\int_0^t g(\tau)\, d\tau$	$\frac{1}{p}G(p)$
$\mathcal{L}34$	Convolution of g and h, often written as $g * h$ $= \int_0^t g(t-\tau)h(\tau)\, d\tau$ $= \int_0^t g(\tau)h(t-\tau)\, d\tau$	$G(p)H(p)$

Transforms of derivatives of y

$\mathcal{L}35$

$$\begin{aligned} \mathcal{L}(y') &= pY - y \\ \mathcal{L}(y'') &= p^2Y - py_0 - y_0' \\ \mathcal{L}(y''') &= p^3Y - p^2y_0 - py_0' - y_0 \\ \mathcal{L}(y^n) &= p^nY - p^{n-1}y_0 - p^{n-2}y_0' - \cdots - y_0^{n-1} \end{aligned}$$