Permutation and Combination

Permutation. Consider finite set A with n elements. An r-permutation is an ordered selection of r elements from A, with $1 \leq r \leq n$. In permutation, order does matter, unlike combination, and that all arrangements are distinct. r-permutation of an n elements set is defined as

$$P(n, r) = n(n-1)\dots(n-r+1)$$

or simply

$$P(n, r) = \frac{n!}{(n-r)!}$$

Combination. Combination counts the number of ways to chose r object form finite set A with n elements where order of selection does not matter. For all integer n and $1 \leq r \leq n$, the number of combination when r elements are chosen out of finite set with n elements $C(n, r)$ is

$$C(n, r) = \frac{P(n, r)}{r!} = \binom{n}{r}$$

or

$$C(n, r) = \frac{n!}{r!(n-r)!}$$

Difference. Suppose we are choosing 2 people out of 4 to be president and vice-president. Here order matter, thus we say that there are

$$P(4, 2) = \frac{4!}{(4-2)!} = 12$$

ways to choose 2 people out of 4 to be president and vice-president. Now, we change the situation into choosing 2 out of 4 people to be given a gift. Here, order does not matter, hence we say that there are

$$C(4, 2) = \frac{4!}{2!(4-2)!} = 6$$

ways to choose 2 people out of 4 to be given gift.