Test

Geometric Series (for r less than one):

$$S_n = \frac{a(1-r^n)}{1-r}$$
$$S = \lim_{n\to\infty} S_n = \frac{a}{1-r}$$

Preliminary Test:

$$\text{if } \lim_{n\to\infty} a_n \neq 0 \text{ , then series Diverges}$$

Comparation Test:

$$A \leq C \text{ , A Converges}$$
$$A \geq D \text{ , A Diverges}$$

Integral Test:

$$\int^{\infty} A \;\; dn \begin{cases} \text{A finite} \to \text{Converges} \\ \text{A infinite} \to \text{Diverges} \end{cases}$$

Ratio Test:

$$\rho_n = \left| \frac{a_{(n+1)}}{a_n} \right|$$
$$\rho = \lim_{n\to\infty} \rho_n \begin{cases} \rho > 1 \text{ Diverge} \\ \rho = 0 \text{ Inconclusive} \\ \rho < 1 \text{ Converges} \end{cases}$$

Special Comparation:

$$\lim_{n\to\infty} \frac{A}{C} \text{ Limit finite} \to \text{A Converges}$$
$$\lim_{n\to\infty} \frac{A}{D} \text{ Limit ¿ } 0 \to \text{A Diverges}$$

Raabe Test:

$$\rho \equiv \lim_{n\to\infty} \left[n\left(\frac{a_n}{a_{n+1}} - 1\right) \right] \begin{cases} \rho > 1, \text{ Converge} \\ \rho = 1, \text{ Inconclusive} \\ \rho < 1, \text{ Diverge} \end{cases}$$

Root Test:

$$\begin{aligned}\rho &\equiv \lim_{n\to\infty} \sqrt[n]{|a_n|} \\ &\equiv \lim_{n\to\infty} |a_n|^{\frac{1}{n}} \begin{cases} \rho > 1, & \text{Converge} \\ \rho = 1, & \text{Inconclusive} \\ \rho < 1, & \text{Diverge} \end{cases}\end{aligned}$$

Alternating series Test

$$\text{if} \quad |a_{n+1}| \leq |a_n| \quad \text{and} \quad \lim_{n\to\infty} a_n = 0$$

then series converges.

Function Expansion

Taylor Series about $x = a$:

$$\begin{aligned}\frac{1}{n!}(x-a)^n f^n(a) = f(a) + (x-a)f'(a) + \frac{1}{2!}(x-a)^2 f''(a) \\ + \frac{1}{3!}(x-a)^3 f'''(a) + \cdots\end{aligned}$$

Maclaurin Series:

$$\frac{1}{n!}(x)^n f^n(0) = f(0) + (x)f'(0) + \frac{1}{2!}(x)^2 f''(0) + \frac{1}{3!}(x)^3 f'''(0) + \cdots$$

Maclaurin Expansion for basic function:

$$\begin{aligned}\sin x &= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} && = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \\ \cos x &= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} && = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots \\ e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} && = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \quad -1 < x \leq 1 \\ \ln(1+x) &= \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{(n)} && = 1 + px + \frac{p(p-1)}{2!}x^2 \\ & && + \frac{p(p-1)(p-2)}{3!}x^3 + \cdots \quad |x| < 1\end{aligned}$$