Appendix: Using Gamma Function to Evaluate Integral

Ex. 1. Consider

$$\int_0^3 x^3 e^{-4x^2} = \frac{\gamma(4,32)}{2\cdot 4^2}$$

Ex. 2. Consider

$$\int_1^2 x^3 e^{-x}\,dx = \gamma(4,2) - \gamma(4,1)$$

Ex. 3. The distribution function for classical partial with respect to its energy is given by

$$n(\epsilon) = \frac{2\pi N}{(\pi kT)^{3/2}}\epsilon^{1/2}\exp\left(-\frac{\epsilon}{kT}\right)$$

Suppose we want to determine the fraction of particle with energy more than $\epsilon \geq 3kT$. Said fraction is determined first find the number of particle within the range of energy

$$n(\epsilon \geq 3kT) = \int_0^{3kT} \frac{2\pi N}{(\pi kT)^{3/2}}\epsilon^{1/2}\exp\left(-\frac{\epsilon}{kT}\right)\,d\epsilon$$

By substituting $\epsilon/kT = x$, we have $\epsilon = kTx$ and $d\epsilon = kT\,dx$

$$n(\epsilon \geq 3kT) = \frac{2N}{\sqrt{\pi}}\int_0^3 x^{1/2}\exp\left(-x\right)\,dx = \frac{2N}{\sqrt{\pi}} = \frac{2N}{\sqrt{\pi}}\gamma\left(\frac{3}{2},3\right)$$

Then the fraction

$$F = \frac{n(\epsilon \geq 3kT)}{N} = \frac{2}{\sqrt{\pi}}\gamma\left(\frac{3}{2},3\right)$$

Ex. 4. Now suppose we want to find the fraction of particle whose speed lies within $(\sqrt{2kT/m}, \sqrt{8kT/m})$ where its distribution function is given by

$$n(v) = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2\exp\left(-\frac{mv^2}{2kT}\right)$$

The number of particle within said range is

$$n\left(\sqrt{\frac{2kT}{m}} \leq v \leq \sqrt{\frac{8kT}{m}}\right) = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} \int_{\sqrt{2kT/m}}^{\sqrt{8kT/m}} v^2\exp\left(-\frac{mv^2}{2kT}\right)\,dv$$

By substituting $mv^2/2kT = x^2$, we have

$$v^2 = \frac{2kT}{m}x^2 \quad \text{and} \quad dv = \sqrt{\frac{2kT}{m}}\,dx$$

and limit of

$$v = \sqrt{\frac{2kT}{m}} \implies x = 1$$
$$v = \sqrt{\frac{8kT}{m}} \implies x = 2$$

Then

$$n = \frac{4N}{\sqrt{\pi}} \frac{m}{2kT} \int_1^2 \frac{2kT}{m} x^2 e^{-x^2} \sqrt{\frac{2kT}{m}}\, dx = \frac{4N}{\sqrt{\pi}} \left[\gamma(1.5, 1) - \gamma(1.5, 2)\right]$$