Mainly consist of precalculus, and basic Calculus.

Algebra

Laws of Exponents.

$$
\begin{aligned}
x^{\frac{m}{n}} &= \sqrt[n]{m} \\
(x^m)^n &= x^{mn} \\
x^m x^n &= x^{m+n} \\
x^a y^a &= (xy)^a
\end{aligned}
$$

Special Factorization.

$$
\begin{aligned}
x^2 - y^2 &= (x+y)(x-y) \\
x^3 - y^3 &= (x-y)(x^2+xy+y^2) \\
x^3 + y^3 &= (x+y)(x^2-xy+y^2)
\end{aligned}
$$

Quadratic formula.

$$
x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad \begin{cases} D > 0 & \mathrm{re}(2) \\ D = 0 & \mathrm{re}(1) \\ D < 0 & \mathrm{im}(2) \end{cases}
$$

Binomial theorem.

$$
(a+b)^n = \sum_{k=0}^{\infty} \binom{n}{k} a^{n-k} b^k
$$

with

$$
\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{\Gamma(n+1)}{\Gamma(k+1)\Gamma(n-k+1)}
$$

Trigonometry

Trigonometry Definition.

$$
\begin{aligned}
\sin\theta &= \frac{1}{\csc\theta} \\
\cos\theta &= \frac{1}{\sec\theta} \\
\tan\theta &= \frac{1}{\cot\theta}
\end{aligned}
$$

Pythagorean Identity.

$$
\begin{aligned}
\sin^2\theta + \cos^2\theta = 1 \\
\sec^2\theta - \tan^2\theta = 1 \\
\csc^2\theta - \cot^2\theta = 1
\end{aligned}
$$

Law of Sines.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Trigonometry Double Angle Identity.

$$\begin{aligned}
\sin 2\theta &= 2\sin\theta\cos\theta \\
\cos 2\theta &= 1 - 2\sin^2\theta \\
&= 2\cos^2\theta - 1 \\
&= \cos^2\theta - \sin^2\theta \\
\tan 2\theta &= \frac{2\tan\theta}{1 - \tan^2\theta}
\end{aligned}$$

Trigonometry Addition and Difference Identity.

$$\begin{aligned}
\sin(x+y) &= \sin x\cos y + \cos x\sin y \\
\sin(x-y) &= \sin x\cos y - \cos x\sin y \\
\cos(x+y) &= \cos x\cos y - \sin x\sin y \\
\cos(x-y) &= \cos x\cos y - \sin x\sin y \\
\tan(x+y) &= \frac{\tan x + \tan y}{1 - \tan x\tan y} \\
\tan(x-y) &= \frac{\tan x - \tan y}{1 + \tan x\tan y}
\end{aligned}$$

Trigonometry Product Rule.

$$\begin{aligned}
\cos x\cos y &= \frac{1}{2}[\cos(x-y) + \cos(x+y)] \\
\sin x\sin y &= \frac{1}{2}[\cos(x-y) - \cos(x+y)] \\
\sin x\cos y &= \frac{1}{2}[\sin(x+y) + \sin(x-y)] \\
\cos x\sin y &= \frac{1}{2}[\sin(x+y) - \sin(x-y)]
\end{aligned}$$

Neat Mnemonics.

$$\begin{vmatrix} \mathrm{S}^+ \\ \mathrm{S}^- \\ \mathrm{C}^+ \\ \mathrm{C}^- \end{vmatrix} = \begin{vmatrix} \mathrm{SC} + \mathrm{CS} \\ \mathrm{SC} - \mathrm{CS} \\ \mathrm{CC} - \mathrm{SS} \\ \mathrm{CC} + \mathrm{SS} \end{vmatrix} \qquad \text{and} \qquad \begin{vmatrix} \mathrm{CC} \\ \mathrm{SS} \\ \mathrm{SC} \\ \mathrm{CS} \end{vmatrix} = \begin{vmatrix} \mathrm{C}^- + \mathrm{C}^+ \\ \mathrm{C}^- - \mathrm{C}^+ \\ \mathrm{S}^+ + \mathrm{S}^- \\ \mathrm{S}^+ - \mathrm{S}^- \end{vmatrix}$$

Logarithm

Definition (informal). $\log_a b$ means a to the power of what equal b.

Few important log rule.

$$
\begin{aligned}
\log_c(ab) &= \log_c(a) + \log_c(b) \\
\log_c(\frac{a}{b}) &= \log_c(a) - \log_c(b) \\
\log_a b &= \frac{\log_c(b)}{\log_c(a)} \\
a^{\log_a b} &= b
\end{aligned}
$$

Limit

Few Important Limits.

$$
\begin{aligned}
\lim_{x \to a} c &= c \\
\lim_{x \to 0^+} \frac{1}{x} &= \infty \to \frac{1}{0^+} = \infty \\
\lim_{x \to 0^-} \frac{1}{x} &= -\infty \to \frac{1}{0^-} = -\infty
\end{aligned}
$$

Limit as Definition of Derivative.

$$
\frac{d}{dx} y = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
$$

Derivative

How to determine the order of derivation: last computation is the first thing to do.

General Formula.

$$
\begin{aligned}
\mathrm{D}\ x^n &= nx^{n-1} \\
\mathrm{D}\ (uv) &= \mathrm{D}\ u \cdot v + u \cdot \mathrm{D}\ v \\
\mathrm{D}\left(\frac{u}{v}\right) &= \frac{\mathrm{D}\ u \cdot v - u \cdot \mathrm{D}\ v}{v^2}
\end{aligned}
$$

Trigonometry Formula.

$$
\begin{aligned}
\mathrm{D}\ \sin x &= \cos x \\
\mathrm{D}\ \cos x &= -\sin x \\
\mathrm{D}\ \tan x &= \sec^2 x \\
\mathrm{D}\ \cot x &= -\csc^2 x \\
\mathrm{D}\ \sec x &= \sec x \tan x \\
\mathrm{D}\ \csc x &= -\cot x \csc x
\end{aligned}
$$

Neat Mnemonics.

$$
\begin{array}{cccl}
\sec & \sec & \tan & \downarrow \text{cofunction} \\
\csc & -\csc & \cot & \\
 & \leftrightarrow \text{multiply} & &
\end{array}
$$

Exponential and Logarithmic Functions.

$$\begin{aligned}\mathrm{D}\ \ln x &= \frac{1}{x}\\ \mathrm{D}\ a^n &= a^n \ln x\\ \mathrm{D}\ \log_a b &= \frac{1}{b \ln a}\end{aligned}$$

Minima and Maxima test. First derivative test:

- Determine critical points ($\mathrm{D}y = 0$), then divine into region;
- Pick value from each region and plug into *derivative*;
- Do the sign-graph.
- Determine local minima and maxima, the plug into *original function*

Second derivative test:

- determine critical points;
- plug critical into second derivative; and
- positive $\mathrm{D}^2 y$ means concave up ($\smile$), negative means concave down ($\frown$), and 0 means inconclusive.

Differentiation under integral sign. Differentiation under integral sign stated by Leibniz' rule

$$\frac{d}{dx}\int_{u(x)}^{v(x)} f(x,t)\ dt = \int_u^v \frac{\partial f}{\partial x}\ dt + f(x,v)\frac{dv}{dx} - f(x,u)\frac{du}{dx}$$

Proof. Suppose we want dI/dx where

$$I = \int_u^v f(t)\ dt$$

By the fundamental theorem of calculus

$$I = F(v) - F(u) = \mathcal{F}(v,u)$$

or I is a function of v and u. Finding dI/dx is then a partial differentiation problem. We can write

$$\frac{dI}{dx} = \frac{\partial I}{\partial v}\frac{dv}{dx} + \frac{\partial I}{\partial u}\frac{du}{dx}$$

By the fundamental theorem of calculus, we have

$$\begin{aligned}\frac{d}{dv}\int_a^v f(x)\ dt &= \frac{d}{dv}\big[F(v) - F(a)\big] = f(v)\\ \frac{d}{dv}\int_u^b f(x)\ dt &= \frac{d}{dv}\big[F(b) - F(u)\big] = -f(u)\end{aligned}$$

where u and v are a function of x, while a and b are a constant. This is the case when we consider $\partial I/\partial v$ or $\partial I/\partial v$; the other variable is constant. Then

$$\frac{d}{dx}\int_u^v f(t)\,dt = f(v)\frac{dv}{dx} - f(u)\frac{du}{dx}$$

Under not too restrictive conditions,

$$\frac{d}{dx}\int_a^b f(x,t)\,dt = \int_a^b \frac{\partial f(x,t)}{\partial x}\,dt$$

where, as before, a and b are constant. In other words, we can differentiate under the integral sign. It is convenient to collect these formulas into one formula known as Leibniz' rule:

$$\frac{d}{dx}\int_{u(x)}^{v(x)} f(x,t)\,dt = \int_u^v \frac{\partial f}{\partial x}\,dt + f(x,v)\frac{dv}{dx} - f(x,u)\frac{du}{dx} \quad \blacksquare$$

Leibniz' rule for differentiating a product.

$$\left(\frac{d}{dx}\right)^n fg = \sum_{k=0}^{n} \binom{n}{k} \left(\frac{d}{dx}\right)^{n-k} f \left(\frac{d}{dx}\right)^k g$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Integral

Basic Formula (integration constant omitted).

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}$$

$$\int \frac{1}{x}\,dx = \ln|x|$$

$$\int u\,dv = uv - \int v\,du$$

$$\int a^x\,dx = \frac{a^x}{\ln a}$$

Trigonometry.

$$\int \sin x\,dx = -\cos x$$

$$\int \cos x\,dx = \sin x$$

$$\int \sec^2 x\,dx = \tan x$$

$$\int \csc^2 x\,dx = -\cot x$$

$$\int \sec x \tan x\,dx = \sec x$$

$$\int \csc x \tan x\,dx = -\csc x$$

Root.

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \arcsin\frac{x}{a}$$
$$\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln x + \sqrt{x^2 \pm a^2}$$
$$\int \frac{1}{\sqrt{a^2 + x^2}}\,dx = \frac{1}{a}\arctan\frac{x}{a}$$

Integration by part.

1. Splits the integrand. Choose u using LIATEN and let the rest be dv. (LIATEN: Log, Inverse trigonometry, Algebra, Trigonometry, ExponeN)

2. Do the box:

u	v	$\downarrow$ diff.
du	dv	$\uparrow$ int.

3. $\int u\,dv = uv - \int v\,du$

Tabular Method. Refer to the table

	D	I
+	$a \searrow$	b
-	$a' \searrow$	b
+	$a'' \searrow$	b
$\vdots$	$\vdots$	$\vdots$

1. 0 in D collumn or use LIATEN

2. integrate a row

3. a row repeats

Trigonometry Integral. Phytagorian Identity.

$$\sin^2 x + \cos^2 x = 1$$
$$\sin^2 x = \frac{1 - \cos 2x}{2}$$
$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

note that argument inside quadratic trigonometry is half of trigonometry, which means $\cos^2 2x = (1 + \cos 4x)/2$. There are few cases of tricky trigonometry integral. First, if power of sin is odd and positive.

- lop one power off
- convert remaining (even power) using Phytagorian Identity in term of cosine
- integrate using subs method

If the power of sine is odd and positive.

- same as before

If the power of sine and cosine is even and nonegative, then:

- convert using Phytagorian Identity and solve

Trigonometry substitution. Trigonometry function and its radical pair

$$\tan\theta = \sqrt{u^2 + a^2}$$
$$\sin\theta = \sqrt{a^2 - u^2}$$
$$\sec\theta = \sqrt{u^2 - a^2}$$

where u is the variable we are differentiating with respect to. Mnemonics: + looks like tangent; - for sin and sec; and it is a $s$in. Trigonometry substitution step is then.

1. Draw a right triangle where trigonometry pair equal $\frac{u}{a}$
2. using the trigonometry pair equation*, solve for x and dx
3. find trigonometry where $\frac{\surd}{a}$
4. subs again if equation* still contain θ and solve

Partial Fraction.

1. Factor out denominator
2. Breakup the function and put unknown (Capital Letter) into numerator. Put numerator normally if factor is linear, put $Px+Q$ Irreducible quadratic factor IQF. In general,

$$\frac{Ax^{n-1} + Bx^{n-2} + \cdots}{x^n + x^{n-1} + \cdots}$$

3. Multiply both side by left side's denominator
4. Take the roots of the linear factors and plug them into x, and solve for the unknowns
5. Put unknowns into step 2
6. Splits Integral, then solve
7. For equating coefficients like terms, after step 3, expand equation*. Then, collect like terms and equate coefficient of like terms from both side

Appendix

Integration Technique Example. 1. Trigonometry substitution. Find $\int \frac{dx}{\sqrt{9x^2+4}}$. Refer to the Mnemonics, the trigonometry pair is tangent.

$$I = \int \frac{dx}{\sqrt{(3x)^2+2^2}}$$
$$\tan\theta = \frac{3x}{2}$$

solving for x and dx

$$x = \frac{2}{3}\tan\theta$$
$$dx = \frac{2}{3}\sec^2\theta\ d\theta$$

trigonometry where $\frac{\sqrt{}}{a}$ holds is secant, solving for radical

$$\sec\theta = \frac{\sqrt{9x^2+4}}{2}$$
$$\sqrt{9x^2+4} = 2\sec\theta$$

the integral is then

$$I = \frac{1}{3}\int \sec\theta\ d\theta$$
$$= \frac{1}{3}\ln|\sec\theta + \tan\theta| + C$$

substituting the θ function

$$I = \frac{1}{3}\ln\left|\frac{\sqrt{9x^2+4}}{2} + \frac{3x}{2}\right| + C$$
$$= \frac{1}{3}\ln\left|\sqrt{9x^2+4} + 3\right| + C$$

Basic. Most common integrals.

$$\int \frac{1}{x}\ dx = \ln|x|$$
$$\int u\ dv = uv - \int v\ du$$
$$\int u\ dv = uv - \int v\ du$$
$$\int \frac{1}{ax+b}\ dx = \frac{1}{a}\ln|ax+b|$$

Rational Functions. Integrals of rational function

$$\int \frac{1}{(x+a)^2}\,dx = -\frac{1}{x+a}$$
$$\int (x+a)^n\,dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1$$
$$\int x(x+a)^n\,dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)}$$
$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x$$
$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$
$$\int \frac{x}{a^2+x^2}\,dx = \frac{1}{2}\ln|a^2+x^2|$$
$$\int \frac{x^2}{a^2+x^2}\,dx = x - a\tan^{-1}\frac{x}{a}$$
$$\int \frac{x^3}{a^2+x^2}\,dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln|a^2+x^2|$$
$$\int \frac{1}{ax^2+bx+c}\,dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}}$$
$$\int \frac{1}{(x+a)(x+b)}\,dx = \frac{1}{b-a}\ln\frac{a+x}{b+x}, \quad a \neq b$$
$$\int \frac{x}{(x+a)^2}\,dx = \frac{a}{a+x} + \ln|a+x|$$
$$\int \frac{x}{ax^2+bx+c}\,dx = \frac{1}{2a}\ln|ax^2+bx+c| -$$
$$\frac{b}{a\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}}$$

Roots. Integrals of roots.

$$\int \sqrt{x-a}\,dx = \frac{2}{3}(x-a)^{3/2}$$
$$\int \frac{1}{\sqrt{x\pm a}}\,dx = 2\sqrt{x\pm a}$$
$$\int \frac{1}{\sqrt{a-x}}\,dx = -2\sqrt{a-x}$$
$$\int x\sqrt{x-a}\,dx = \begin{cases} \frac{2a}{3}(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2}, \text{ or} \\ \frac{2}{3}x(x-a)^{3/2} - \frac{4}{15}(x-a)^{5/2}, \text{ or} \\ \frac{2}{15}(2a+3x)(x-a)^{3/2} \end{cases}$$
$$\int \sqrt{ax+b}\,dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b}$$
$$\int (ax+b)^{3/2}\,dx = \frac{2}{5a}(ax+b)^{5/2}$$
$$\int \frac{x}{\sqrt{x\pm a}}\,dx = \frac{2}{3}(x\mp 2a)\sqrt{x\pm a}$$
$$\int \sqrt{\frac{x}{a-x}}\,dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a}$$

$$\int \sqrt{\frac{x}{a+x}}\, dx = \sqrt{x(a+x)} - a\ln[\sqrt{x}+\sqrt{x+a}]$$

$$\int x\sqrt{ax+b}\, dx = \frac{2}{15a^2}(-2b^2+abx+3a^2x^2)\sqrt{ax+b}$$

$$\int \sqrt{x^3(ax+b)}\, dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)}$$

$$+\frac{b^3}{8a^{5/2}}\ln\left|a\sqrt{x}+\sqrt{a(ax+b)}\right|$$

$$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$$

$$\int x\sqrt{x^2\pm a^2}\, dx = \frac{1}{3}\left(x^2\pm a^2\right)^{3/2}$$

$$\int \frac{1}{\sqrt{x^2\pm a^2}}\, dx = \ln\left|x+\sqrt{x^2\pm a^2}\right|$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}\frac{x}{a}$$

$$\int \frac{x}{\sqrt{x^2\pm a^2}}\, dx = \sqrt{x^2\pm a^2}$$

$$\int \frac{x}{\sqrt{a^2-x^2}}\, dx = -\sqrt{a^2-x^2}$$

$$\int \frac{x^2}{\sqrt{x^2\pm a^2}}\, dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \mp \frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right|$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}}\, dx = \frac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right|$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$$

Integrals with Logarithms.

$$\int \ln ax\; dx = x\ln(ax) - x$$

$$\int \frac{\ln ax}{x}\, dx = \frac{1}{2}(\ln ax)^2$$

$$\int \ln(ax+b)dx = (x+\frac{b}{a})\ln(ax+b) - x, \quad a\neq 0$$

$$\int \ln(x^2+a^2)\, dx = x\ln(x^2+a^2) + 2a\tan^{-1}\frac{x}{a} - 2x$$

$$\int \ln(x^2-a^2)\, dx = x\ln(x^2-a^2) + a\ln\frac{x+a}{x-a} - 2x$$

$$\int \ln(x^2-a^2)\, dx = x\ln(x^2-a^2) + a\ln\frac{x+a}{x-a} - 2x$$

$$\int \ln(ax^2+bx+c)dx = \frac{1}{a}\sqrt{4ac-b^2}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} - 2x$$

$$+(\frac{b}{2a}+x)\ln\,(ax^2+bx+c)$$

$$\int x\ln(ax+b)dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}(x^2-\frac{b^2}{a^2})\ln(ax+b)$$

$$\frac{1}{2}(x^2-\frac{a^2}{b^2})\ln(a^2-b^2x^2)$$

Integrals with Exponential.

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int \sqrt{x} e^{ax} dx = \frac{1}{a} \sqrt{x} e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}} \operatorname{erf}\left(i\sqrt{ax}\right)$$

$$\int x e^{x} dx = (x-1) e^{x}$$

$$\int x e^{ax} dx = \left(\frac{x}{a} - \frac{1}{a^2}\right) e^{ax}$$

$$\int x^2 e^{x} dx = \left(x^2 - 2x + 2\right) e^{x}$$

$$\int x^2 e^{ax} dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right) e^{ax}$$

$$\int x^3 e^{x} dx = \left(x^3 - 3x^2 + 6x - 6\right) e^{x}$$

$$\int x^n e^{ax}\ dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax}\ dx$$

$$\int x^n e^{ax}\ dx = \frac{(-1)^n}{a^{n+1}} \Gamma[1+n, -ax]$$

$$\int e^{ax^2}\ dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}\left(ix\sqrt{a}\right)$$

$$\int e^{-ax^2}\ dx = \frac{\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}\left(x\sqrt{a}\right)$$

$$\int x e^{-ax^2}\ dx = -\frac{1}{2a} e^{-ax^2}$$

$$\int x^2 e^{-ax^2}\ dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}} \operatorname{erf}(x\sqrt{a}) - \frac{x}{2a} e^{-ax^2}$$

Integrals with Trigonometry Functions.

$$\int \sin ax\ dx = -\frac{1}{a} \cos ax$$

$$\int \sin^2 ax\ dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int \sin^n ax\ dx = -\frac{1}{a} \cos ax \times {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right]$$

$$\int \sin^3 ax\ dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a}$$

$$\int \cos ax\ dx = \frac{1}{a} \sin ax$$

$$\int \cos^2 ax\ dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int \cos^p ax\ dx = -\frac{1}{a(1+p)} \cos^{1+p} ax \times {}_2F_1\left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right]$$

$$\int \cos^3 ax\ dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a}$$

$$\int \cos ax \sin bx \; dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, \quad a \neq b$$

$$\int \sin^2 ax \cos bx \; dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)}$$

$$\int \sin^2 x \cos x \; dx = \frac{1}{3} \sin^3 x$$

$$\int \cos^2 ax \sin bx \; dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)}$$

$$\int \cos^2 ax \sin ax \; dx = -\frac{1}{3a} \cos^3 ax$$

$$\int \sin^2 ax \cos^2 bx \; dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b}$$
$$- \frac{\sin[2(a+b)x]}{16(a+b)}$$

$$\int \sin^2 ax \cos^2 ax \; dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$$

$$\int \tan ax \; dx = -\frac{1}{a} \ln \cos ax$$

$$\int \tan^2 ax \; dx = -x + \frac{1}{a} \tan ax$$

$$\int \tan^n ax \; dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right)$$

$$\int \tan^3 ax \; dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax$$

$$\int \sec x \; dx = \ln |\sec x + \tan x| = 2 \tanh^{-1}\left(\tan \frac{x}{2}\right)$$

$$int \sec^2 ax \; dx = \frac{1}{a} \tan ax$$

$$\int \sec^3 x \; dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x|$$

$$\int \sec x \tan x \; dx = \sec x$$

$$\int \sec^2 x \tan x \; dx = \frac{1}{2} \sec^2 x$$

$$\int \sec^n x \tan x \; dx = \frac{1}{n} \sec^n x, \quad n \neq 0$$

$$\int \csc x \; dx = \ln \left|\tan \frac{x}{2}\right| = \ln |\csc x - \cot x| + C$$

$$\int \csc^2 ax \; dx = -\frac{1}{a} \cot ax$$

$$\int \csc^3 x \; dx = -\frac{1}{2} \cot x \csc x + \frac{1}{2} \ln |\csc x - \cot x|$$

$$\int \csc^n x \cot x \; dx = -\frac{1}{n} \csc^n x, n \neq 0$$

$$\int \sec x \csc x \; dx = \ln |\tan x|$$

Products of Trigonometry Functions and Monomials.

$$\int x \cos x \; dx = \cos x + x \sin x$$

$$\int x \cos ax \; dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax$$

$$\int x^2 \cos x \; dx = 2x \cos x + \left(x^2 - 2\right) \sin x$$

$$\int x^2 \cos ax \; dx = \frac{2x \cos ax}{a^2} + \frac{a^2x^2 - 2}{a^3} \sin ax$$

$$\int x^n \cos x \; dx = -\frac{1}{2}(i)^{n+1} \left[\Gamma(n+1, -ix) + (-1)^n \Gamma(n+1, ix)\right]$$

$$\int x^n cosax \; dx = \frac{1}{2}(ia)^{1-n} \left[(-1)^n \Gamma(n+1, -iax) - \Gamma(n+1, ixa)\right]$$

$$\int x \sin x \; dx = -x \cos x + \sin x$$

$$\int x \sin ax \; dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2}$$

$$\int x^2 \sin x \; dx = \left(2 - x^2\right) \cos x + 2x \sin x$$

$$\int x^2 \sin ax \; dx = \frac{2 - a^2x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2}$$

$$\int x^n \sin x \; dx = -\frac{1}{2}(i)^n \left[\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, -ix)\right]$$

Products of Trigonometry Functions and Exponential.

$$\int e^x \sin x \; dx = \frac{1}{2} e^x (\sin x - \cos x)$$

$$\int e^{bx} \sin ax \; dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax)$$

$$\int e^x \cos x \; dx = \frac{1}{2} e^x (\sin x + \cos x)$$

$$\int e^{bx} \cos ax \; dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax)$$

$$\int xe^x \sin x \; dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x)$$

$$\int xe^x \cos x \; dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x)$$

Integrals of Hyperbolic Functions.

$$\int \cosh ax dx = \frac{1}{a} \sinh ax$$

$$\int e^{ax} \cosh bx dx = \begin{cases} \dfrac{e^{ax}}{a^2 - b^2} \left[a \cosh bx - b \sinh bx \right], & a \neq b \\ \\ \dfrac{e^{2ax}}{4a} + \dfrac{x}{2}, & a = b \end{cases}$$

$$\int \sinh ax dx = \frac{1}{a} \cosh ax$$

$$\int e^{ax} \sinh bx dx = \begin{cases} \dfrac{e^{ax}}{a^2 - b^2} \left[-b \cosh bx + a \sinh bx \right], & a \neq b \\ \\ \dfrac{e^{2ax}}{4a} - \dfrac{x}{2} a = b \end{cases}$$

$$\int e^{ax} \tanh bx dx = \begin{cases} \dfrac{e^{(a+2b)x}}{(a+2b)} {}_2F_1 \left[1 + \dfrac{a}{2b}, 1, 2 + \dfrac{a}{2b}, -e^{2bx} \right] \\ \\ -\dfrac{1}{a} e^{ax} {}_2F_1 \left[\dfrac{a}{2b}, 1, 1E, -e^{2bx} \right], & a \neq b \\ \\ \dfrac{e^{ax} - 2 \tan^{-1}[e^{ax}]}{a}, & a = b \end{cases}$$

$$\int \tanh ax \, dx = \frac{1}{a} \ln \cosh ax$$

$$\int \cos ax \cosh bx dx = \frac{1}{a^2 + b^2} \left[a \sin ax \cosh bx + b \cos ax \sinh bx \right]$$

$$\int \cos ax \sinh bx dx = \frac{1}{a^2 + b^2} \left[b \cos ax \cosh bx + a \sin ax \sinh bx \right]$$

$$\int \sin ax \cosh bx dx = \frac{1}{a^2 + b^2} \left[-a \cos ax \cosh bx + b \sin ax \sinh bx \right]$$

$$\int \sin ax \sinh bx dx = \frac{1}{a^2 + b^2} \left[b \cosh bx \sin ax - a \cos ax \sinh bx \right]$$

$$\int \sinh ax \cosh ax dx = \frac{1}{4a} \left[-2ax + \sinh 2ax \right]$$

$$\int \sinh ax \cosh bx dx = \frac{1}{b^2 - a^2} \left[b \cosh bx \sinh ax - a \cosh ax \sinh bx \right]$$