Newton's Law

First law. in the absence of an external force, when viewed from an inertial frame, an object at rest remains at rest and an object in uniform motion in a straight line maintains that motion.

Second law. Simply put

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

Third law. States that if two objects interact, the force exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force exerted by object 2 on object 1.

Particle Under Constant Acceleration

Here's some kinematics equation for position

$$x(t) = x_i + \frac{1}{2}(v_i + v_f)t$$
$$x(t) = x_i + v_i t + \frac{1}{2}at^2$$

and for velocity

$$v(t) = v_i + at$$
$$v(t)^2 = v_i^2 + 2a(x_f - x_i)$$

Particle in Uniform Circular Motion

If a particle moves in a circular path of radius r with a constant speed v, the magnitude of its centripetal acceleration is given by

$$a_r = \frac{v^2}{r}$$

while its period and angular velocity is

$$T = \frac{2\pi r}{v}, \quad \omega = \frac{2\pi}{T}$$

Applying Newton's second law

$$\sum F = ma_r = m\frac{v^2}{r}$$

Rigid Object Under Constant Angular Acceleration

Analogous to those for translational motion of a particle under constant acceleration

$$\omega(t) = \omega_i + \alpha t$$

$$\omega(t)^2 = \omega_i^2 + 2\alpha(\theta_t - \theta_i)$$

$$\theta(t) = \theta_i + \omega t + \frac{1}{2}\alpha t^2$$

$$\theta(t) = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$$

Relation of Linear and Rotational Motion

The following equations show the relation of linear and rotational motion

$$s = r\theta, \quad v = r\omega, \quad a_t = r\alpha$$

Torque

The torque associated with a force $\mathbf{F}$ acting on an object

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = I\alpha = \frac{d\mathbf{L}}{dt}$$

Moment of Inertia

The moment of inertia of a rigid object is

$$I = \sum mr^2 = \int r^2 dm$$

Parallel Axis Theorem. To calculate the moment inertia from any axis, we use parallel axis theorem

$$I = I_{\mathrm{CM}} + Md^2$$

Terminal velocity

$\boldsymbol{r \propto v}$**.** The velocity as a function of time is

$$v = \frac{mg}{b}\left[1 - \exp\left(-\frac{bt}{m}\right)\right] = v_T\left[1 - \exp\left(-\frac{bt}{m}\right)\right]$$

where b is a resistive constant whose value depends on the properties of the medium.

$\boldsymbol{r \propto v^2}$**.** Given by

$$v_T = \sqrt{\frac{2mg}{D\rho A}}$$

where D is a dimensionless empirical quantity called the drag coefficient, ρ is the density of air, and A is the cross-sectional area of the moving object.

Escape velocity. the speed required bu an object to escape from any planet orbit is

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$$

Work Energy Theorem

It states that if work is done on a system by external forces and the only change in the system is in its speed,

$$W = \Delta T$$

Kinetic Energy

For an object in linear motion, the kinetic energy of said object is

$$T = \frac{1}{2}mv^2$$

whereas for rotational motion

$$T = \frac{1}{2}I\omega^2$$

Hence the total kinetic energy of a rigid object rolling on a rough surface without slipping

$$T = \frac{1}{2}mv_{\text{CM}}^2 + \frac{1}{2}I\omega_{\text{CM}}^2$$

Potential Energy Function

For conservative energy $\mathbf{F}$, applies

$$V_f - V_i = -\int_{\mathbf{r_i}}^{\mathbf{r_f}} \mathbf{F} \cdot d\mathbf{r}$$

For particle-Earth system, the gravitational potential energy is

$$V = mgy$$

and elastic potential stored in spring

$$V = \frac{1}{2}kx^2$$

Effective potential

Effective potential energy $U_{\text{eff}}(r)$ is the sum of the actual potential energy $U(r)$ and the centrifugal $U_{\text{cf}}(r)$:

$$U_{\text{eff}}(r) = U(r) + \frac{l^2}{2\mu r^2}$$

where l is the angular momentum and μ is the reduced mass

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Momentum Impulse

The linear momentum and impulse are defined as

$$\mathbf{p} = m\mathbf{v}, \quad \mathbf{I} = \int_{t_i}^{t_f} \sum \mathbf{F}\, dt$$

Angular Momentum The angular momentum about an axis through the origin of a particle having linear momentum

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

The z component of angular momentum of a rigid object rotating about a fixed z axis is

$$L_z = I\omega$$

Center of Mass and Velocity

The position vector of the center of mass of a system of particles is defined as

$$\mathbf{r}_{\text{CM}} = \frac{1}{M} \sum m\mathbf{r} = \frac{1}{M} \int \mathbf{r}\, dm$$

where M is the total mass. The velocity of the center of mass for a system of particles is

$$\mathbf{v}_{\text{CM}} = \frac{1}{M} \sum m\mathbf{v} =$$

Collision

Inelastic collision. One for which the total kinetic energy of the system of colliding particles is not conserved.

Elastic collision. One in which the kinetic energy of the system is conserved.

Perfectly inelastic. A collision which the colliding particles stick together after the collision.

Rocket Propulsion The expression for rocket propulsion is

$$v_f - v_i = v_e \ln \frac{M_i}{M_f}$$

Power

The rate at which work is done by an external force, called power, is

$$P = \frac{dE}{dt} = Fv = \tau\omega$$

Newton's Law on Gravity

$$\mathbf{F} = G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}$$

For an object at a distance h above the Earth's, the gravitational acceleration is

$$g = \frac{GM_E}{r^2} = \frac{GM_E}{(R_E + h)^2}$$

In general, the gravitational field experienced by mass m is

$$\mathbf{g} = \frac{\mathbf{F}}{m}$$

Kepler's Law

First Law. All planets move in elliptical orbits with the Sun at one focus.

Second Law The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals.

Third Law Simply put

$$T^2 = \frac{4\pi^2 a^3}{GM_S}$$

where a is semimajor axis and M_S is the mass of the sun.

Energy of Gravitational system

Potential energy. The gravitational potential energy associated with a system of two particles is

$$V = -\frac{Gm_1 m_2}{r}$$

Total energy. The total energy of the system is the sum of the kinetic and potential energies

$$E = \frac{1}{2}mv^2 - G\frac{Mm}{r} = -\frac{GMm}{2r}$$