State function, such as internal energy U and entropy S, can be thought as conservative field. The condition that must be satisfied by conservative field $\mathbf{V}$ is

$$\nabla \times \mathbf{V} = 0$$

Suppose we actually evaluate the curl of vector function $\mathbf{V}(x, y, z)$, we get

$$\nabla \times \mathbf{V} = \begin{pmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ V_x & V_y & V_z \end{pmatrix}$$

$$\nabla \times \mathbf{V} = \hat{\mathbf{i}}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) + \hat{\mathbf{j}}\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right) + \hat{\mathbf{k}}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

Since $\mathbf{V}$, as a conservative field, has curl of zero, those term inside parenthesis can be evaluated into

$$\frac{\partial V_z}{\partial y} = \frac{\partial V_y}{\partial z}, \quad \frac{\partial V_x}{\partial z} = \frac{\partial V_z}{\partial x}, \quad \frac{\partial V_y}{\partial x} = \frac{\partial V_x}{\partial y}$$

For state function $U(S, V, N)$, the equation reads

$$\frac{\partial U_N}{\partial V} = \frac{\partial U_V}{\partial N}, \quad \frac{\partial U_S}{\partial N} = \frac{\partial U_N}{\partial S}, \quad \frac{\partial U_V}{\partial S} = \frac{\partial U_S}{\partial V}$$

Of course you can't evaluate the curl of state function, but hear me out. What we consider is not the function U itself, but rather, the differential dU. Its total differential may be written as

$$dU(S, V, N) = \left.\frac{\partial U}{\partial S}\right|_{V,N} dS + \left.\frac{\partial U}{\partial V}\right|_{S,N} dV + \left.\frac{\partial U}{\partial N}\right|_{S,V} dN$$

Here, the differentials (dS, dT, dN) act like unit vector, thus we can pretend that dU is a vector field with components of

$$U_S = \left.\frac{\partial U}{\partial S}\right|_{V,N}, \quad U_V = \left.\frac{\partial U}{\partial V}\right|_{S,N}, \quad U_N = \left.\frac{\partial U}{\partial N}\right|_{S,V}$$

Therefore

$$\frac{\partial}{\partial V}\frac{\partial U}{\partial N} = \frac{\partial}{\partial N}\frac{\partial U}{\partial V}, \quad \frac{\partial}{\partial N}\frac{\partial U}{\partial S} = \frac{\partial}{\partial S}\frac{\partial U}{\partial N}, \quad \frac{\partial}{\partial S}\frac{\partial U}{\partial V} = \frac{\partial}{\partial V}\frac{\partial U}{\partial S}$$

This is what it means to be an exact differential.