

## Definition

If a function  $f(x_1, \dots, x_n)$  of  $n$  variables  $x_1, \dots, x_n$  is such that, for any constant  $\lambda$

$$f(\lambda x_1, \dots, \lambda x_n) = \lambda^m f(x_1, \dots, x_n)$$

then  $f(\lambda x_1, \dots, \lambda x_n)$  is called homogeneous of degree  $m$ , with  $m > 1$ .

**Euler's theorem on homogeneous functions.** The partial derivative of homogeneous function obey the relation

$$\sum_{i=1}^n x_i \frac{\partial f}{\partial x_i} = m f$$

## Thermodynamic

Mathematically, extensive properties are homogeneous functions of first order, while intensive properties are homogeneous functions of order zero.

**Extensive properties.** an extensive property scales linearly with the system's size. Properties such as  $U$ ,  $V$ ,  $m$ ,  $n$ , and  $N$  are all examples of extensive properties; they will double their values upon doubling the size of the system.

**Intensive properties.** An intensive property does not depend on the size (or extent) of the system; it is a scale invariant. The ratio between two extensive properties is an intensive property. The molar mass  $M$  is therefore an intensive property.