Quantum Statistics

Based on their quantum statistics properties, there are three types of particle, namely

1. **Classics**. This particle obeys Maxwell-Boltzmann (MB) distribution. It assumes particles are distinguishable and can occupy any energy state without restriction.
2. **Boson.** This particle obeys the Bose-Einstein (BE) distribution. It is indistinguishable and can occupy the same quantum state without limit.
3. **Fermion.** This particle obeys Fermi-Dirac (FD) distribution. Fermions are indistinguishable and obey the Pauli exclusion principle, meaning no two fermions can occupy the same quantum state.

Discrete System

Microstate. The number of microstate based on energy level for classical particle is

$$\Omega_{\mathrm{MB}} = N! \prod_k \frac{g_k^{n_k}}{n_k!}$$

for boson

$$\Omega_{\mathrm{BE}} = \prod_k \frac{(n_k + g_k - 1)!}{n_k!(g_k - 1)!}$$

and for fermion

$$\Omega_{\mathrm{FD}} = \prod_k \frac{g_k!}{n_k!(g_k - n_k)!}$$

Example. Consider system with four particle and five unit energy, where each energy level have the following condition.

Table: Random system

n	(g_n, ϵ_n)
4	$(1, 3)$
3	$(3, 2)$
2	$(2, 1)$
1	$(3, 0)$

We have the following macrostate.

$$M_1 = (2, 0, 1, 1)$$
$$M_2 = (1, 2, 0, 1)$$
$$M_3 = (1, 1, 2, 0)$$
$$M_4 = (0, 3, 1, 0)$$

For classical particle, we have also the following number of microstate

$$\begin{aligned}
\Omega_1 &= 4!\frac{3^2}{2!}\frac{2^0}{0!}\frac{3^1}{1!}\frac{1^1}{1!} = 324\\
\Omega_2 &= 144\\
\Omega_3 &= 648\\
\Omega_4 &= 96
\end{aligned}$$

For boson

$$\begin{aligned}
\Omega_1 &= \frac{4!1!3!1!}{2!2!1!2!0!} = 18\\
\Omega_2 &= 9\\
\Omega_3 &= 36\\
\Omega_4 &= 12
\end{aligned}$$

and for fermion

$$\begin{aligned}
\Omega_1 &= \frac{3!}{2!}\frac{2!}{2!}\frac{3!}{2!}\frac{1!}{1!} = 9\\
\Omega_2 &= 3\\
\Omega_3 &= 18
\end{aligned}$$

No M_4 for fermion since those three particle cannot have the same energy.

Another example. This time I will do classical particle only. Consider system with $N = E = 7$ unit whose each level of energy obey the following condition.

Table: Random system

n	(g_n, ϵ_n)
7	$(3, 6)$
6	$(3, 5)$
5	$(3, 4)$
4	$(3, 3)$
3	$(3, 2)$
2	$(3, 1)$
1	$(3, 0)$

The corresponding macrostates are as follows.

$$\begin{aligned}
M_1 &= (5,0,0,0,0,0,7) & M_7 &= (2,3,0,1,0,0,0)\\
M_2 &= (4,1,0,0,0,1,0) & M_8 &= (3,0,3,0,0,0,0)\\
M_3 &= (4,0,1,0,1,0,0) & M_9 &= (2,2,2,0,0,0,0)\\
M_4 &= (3,2,0,0,1,0,0) & M_{10} &= (1,4,1,0,0,0,0)\\
M_5 &= (4,0,2,0,0,0,0) & M_{11} &= (0,6,0,0,0,0,0)\\
M_6 &= (3,1,1,1,0,0,0) & &
\end{aligned}$$

with the following microstate

$$
\begin{array}{ll}
\Omega_1 = 6 \cdot 3^6 & \Omega_7 = 60 \cdot 3^6 \\
\Omega_2 = 30 \cdot 3^6 & \Omega_8 = 20 \cdot 3^6 \\
\Omega_3 = 30 \cdot 3^6 & \Omega_9 = 90 \cdot 3^6 \\
\Omega_4 = 60 \cdot 3^6 & \Omega_{10} = 30 \cdot 3^6 \\
\Omega_5 = 15 \cdot 3^6 & \Omega_{11} = 3^6 \\
\Omega_6 = 120 \cdot 3^6 &
\end{array}
$$