

# Calculate the CMB power spectrum: Cosmology II

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February 23, 2023 GitHub repo link: <https://github.com/Johanmkr/AST5220/tree/main/project>

## ABSTRACT

SOME ABSTRACT

## Contents

### 1 Introduction

### 2 Milestone I - Background Cosmology

- 2.1 Theory . . . . .
- 2.2 Methods . . . . .
  - 2.2.1 Initial equation . . . . .
  - 2.2.2 ODEs . . . . .
- 2.3 Results . . . . .
  - 2.3.1 Tests . . . . .
  - 2.3.2 Analysis . . . . .

### 3 Milestone II

- 3.1 Theory . . . . .
- 3.2 Methods . . . . .
- 3.3 Results . . . . .

### 4 Milestone III

- 4.1 Theory . . . . .
- 4.2 Methods . . . . .
- 4.3 Results . . . . .

### 5 Milestone IV

- 5.1 Theory . . . . .
- 5.2 Methods . . . . .
- 5.3 Results . . . . .

### 6 Conclusion

### A Some appendix

### B Some appendix

## Nomenclature

### Constants of nature

- $G$  - Gravitational constant.  
 $G = 6.6743 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ .
- $k_B$  - Boltzmann constant.  
 $k_B = 1.3806 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$ .
- $\hbar$  - Reduced Planck constant.  
 $\hbar = 1.0546 \times 10^{-34} \text{ J s}^{-1}$ .
- $c$  - Speed of light in vacuum.  
 $c = 2.9979 \times 10^8 \text{ m s}^{-1}$ .

### Cosmological parameters

- 1  $H$  - Hubble parameter.
- 1  $H_0$  - Hubble constant **fill in stuff**.
- 1  $e^x \mathcal{H}$  - Scaled Hubble parameter.
- 1  $T_{\text{CMB}0}$  - Temperature of CMB today.
- 2  $T_{\text{CMB}0} = 2.7255 \text{ K}$ .
- 2  $\eta$  - Conformal time.
- 2  $\chi$  - Co-moving distance.

### Density parameters

- 2 Density parameter  $\Omega_X = \rho_X / \rho_c$  where  $\rho_X$  is the density
- 2 and  $\rho_c = 8\pi G / 3H^2$  the critical density.  $X$  can take the
- 2 following values:
- 3  $b$  - Baryons.
- 3  $\text{CDM}$  - Cold dark matter.
- 3  $\gamma$  - Electromagnetic radiation.
- 3  $\nu$  - Neutrinos.
- 3  $k$  - Spatial curvature.
- 4  $\Lambda$  - Cosmological constant.

A 0 in the subscript indicates the present day value.

## 1. Introduction

Some citation: [Pedregosa et al. \(2011\)](#) and another citation [Goodfellow et al. \(2016\)](#). Figure 1 makes sure that you do all the work.

## 2. Milestone I - Background Cosmology

Some introduction to milestone 1

### 2.1. Theory

The main time variable will be  $x = \ln a$ .

The Hubble equation, where we allow for curvature is **citation?**:

$$H(x) = H_0 \sqrt{\Omega_{\text{M}0} e^{-3x} + \Omega_{\text{R}0} e^{-4x} + \Omega_{k0} e^{-2x} + \Omega_{\Lambda0}}, \quad (1)$$

where  $\Omega_{\text{M}0} = \Omega_{b0} + \Omega_{\text{CDM}0}$  and  $\Omega_{\text{R}0} = \Omega_{\gamma0} + \Omega_{\nu0}$  are the present day values of the total matter and radiation densities.



**Fig. 1.** Penguin making sure that you do all the work necessary!

Something aboutu the curvature, and the evolution of density parameters here.  
derive?

$$\frac{d\eta}{dx} = \frac{c}{\mathcal{H}(x)}.$$

$$\frac{dt}{dx} = \frac{1}{H(x)}.$$

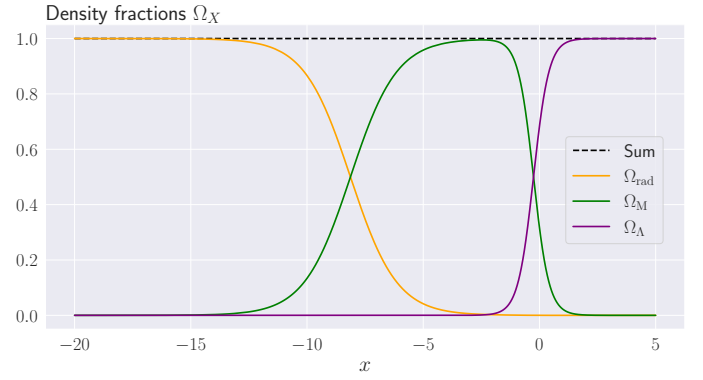
$$\chi(x) = \eta_0 - \eta(x).$$

$$r(\chi) = \begin{cases} \chi \cdot \frac{\sin(\sqrt{|\Omega_{k0}|}H_0\chi/c)}{\sqrt{|\Omega_{k0}|}H_0\chi/c} & \Omega_{k0} < 0 \\ \chi & \Omega_{k0} = 0 \\ \chi \cdot \frac{\sinh(\sqrt{|\Omega_{k0}|}H_0\chi/c)}{\sqrt{|\Omega_{k0}|}H_0\chi/c} & \Omega_{k0} > 0 \end{cases}$$

$$d_A(x) = e^x r(\chi(x)).$$

$$d_L = e^{-x} r(\chi(x)).$$

$$\chi^2(h, \Omega_{m0}, \Omega_{k0}) = \sum_{i=1}^N \frac{(d_L(z, \Omega_{m0}, \Omega_{k0}) - d_L^{\text{obs}}(z_i))^2}{\sigma_i^2} \quad (8)$$



**Fig. 2.** Omega tests

## 2.2. Methods

### 2.2.1. Initial equation

$$\Omega_k(a) = \Omega_{k0} e^{-2x} \left( \frac{H_0^2}{H(x)^2} \right)$$

$$\Omega_{\text{CDM}}(a) = \Omega_{\text{CDM}0} e^{-3x} \left( \frac{H_0^2}{H(x)^2} \right)$$

$$\Omega_{b0}(a) = \Omega_{b0} e^{-3x} \left( \frac{H_0^2}{H(x)^2} \right)$$

$$\Omega_{\gamma0}(a) = \Omega_{\gamma0} e^{-4x} \left( \frac{H_0^2}{H(x)^2} \right)$$

$$\Omega_{\nu0}(a) = \Omega_{\nu0} e^{-4x} \left( \frac{H_0^2}{H(x)^2} \right) \quad (3)$$

$$\Omega_{\Lambda0} = \Omega_{\Lambda0} \left( \frac{H_0^2}{H(x)^2} \right) \quad (9)$$

$$(4)$$

$$\Omega_{\gamma0} = \frac{16\pi^3 G}{90} \cdot \frac{(k_b T_{\text{CMB}0})^4}{\hbar^3 c^5 H_0^2}$$

$$\Omega_{\nu0} = N_{\text{eff}} \cdot \frac{7}{8} \cdot \left( \frac{4}{3} \right)^{4/3} \cdot \Omega_{\gamma0} \quad (10)$$

$$(5)$$

### 2.2.2. ODEs

We solve the differential equation for  $\eta(x)$ , eq. 2 using an ordinary differential equation solver, with Runge-Kutta 4 RK4? as the advancement method. The initial condition is given by  $\eta(x_{\text{start}}) = c/\mathcal{H}(x_{\text{start}})$

$$(6)$$

## 2.3. Results

### 2.3.1. Tests

### 2.3.2. Analysis

$$(7)$$

## 3. Milestone II

Some introduction to milestone 2

### 3.1. Theory

Some theory

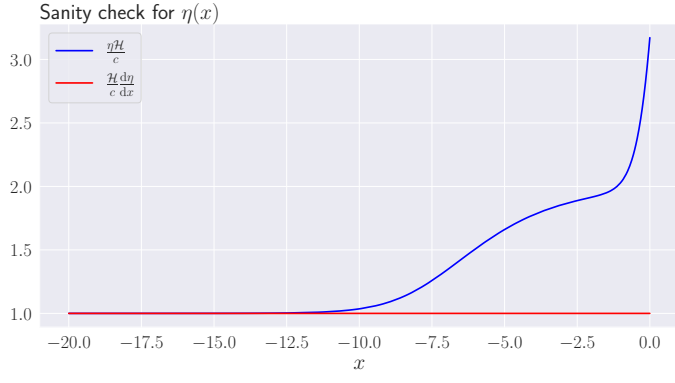


Fig. 3. Eta tests

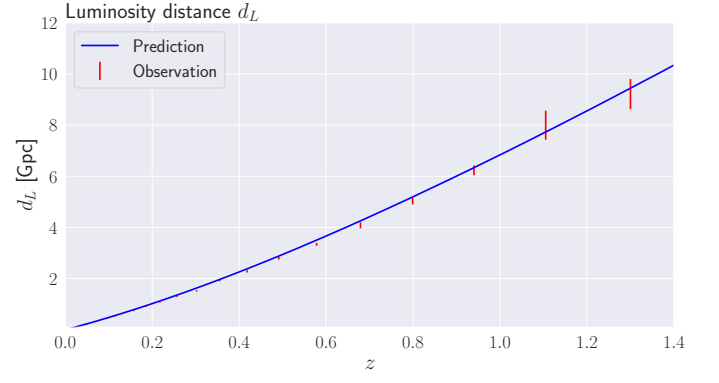


Fig. 7. Supernova data fitted

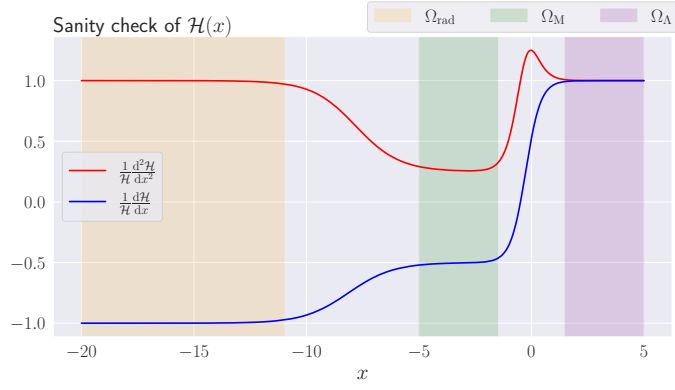


Fig. 4. HP tests

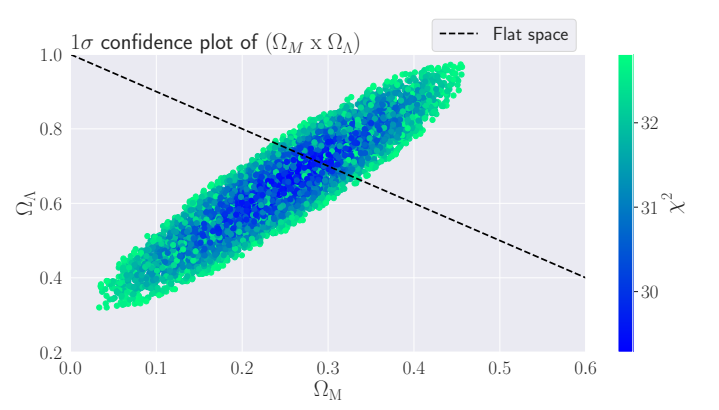


Fig. 8. one sigma confidence plot

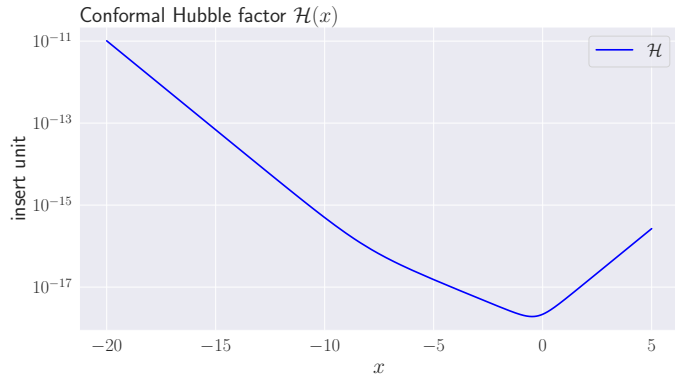


Fig. 5. Conformal Hubble factor.

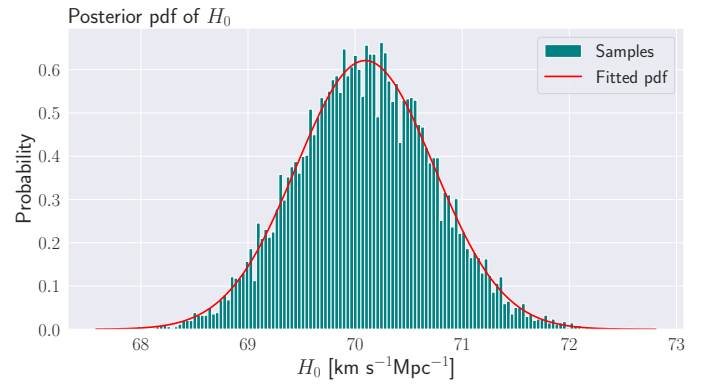


Fig. 9. posterior pdf.

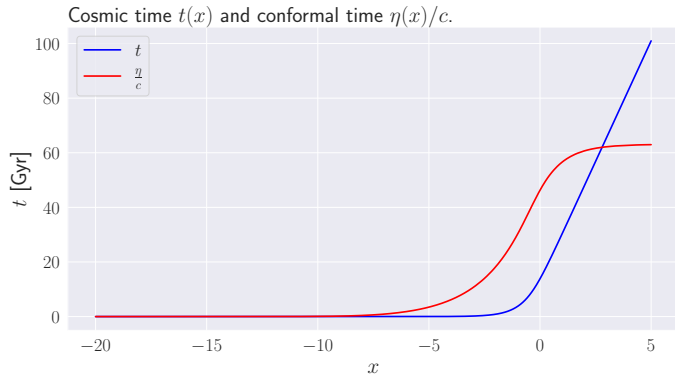


Fig. 6. cosmic time.

### 3.2. Methods

some methods

### 3.3. Results

## 4. Milestone III

Some introduction to milestone 3

### 4.1. Theory

Some theory

#### 4.2. *Methods*

some methods

#### 4.3. *Results*

### 5. Milestone IV

Some introduction to milestone 4

#### 5.1. *Theory*

Some theory

#### 5.2. *Methods*

some methods

#### 5.3. *Results*

### 6. Conclusion

Some overall conclusion

### References

- Goodfellow, I., Bengio, Y., & Courville, A. 2016, Deep Learning (MIT Press), accessed Nov. 5 2022 at <http://www.deeplearningbook.org>
- Pedregosa, F., Varoquaux, G., Gramfort, A., et al. 2011, Journal of Machine Learning Research, 12, 2825

**Appendix A: Some appendix**

**Appendix B: Some appendix**