Calculate the CMB power spectrum: Cosmology II

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February 28, 2023 GitHub repo link: https://github.com/Johanmkr/AST5220/tree/main/project

ABSTRACT

SOME ABSTRACT

Contents

Nomenclature

Constants of nature

G - Gravitational constant.
$G = 6.6743 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.
k_B - Boltzmann constant.
$k_B = 1.3806 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}.
$\hbar$ - Reduced Planck constant.
$\hbar = 1.0546 \times 10^{-34}$ J s^{-1}.
c - Speed of light in vacuum.
$c = 2.9979 \times 10^8$ m s^{-1}.

Cosmological parameters

H - Hubble parameter.
H_0 - Hubble constant fill in stuff.
$e^x\mathcal{H}$ - Scaled Hubble parameter.
T_{CMB0} - Temperature of CMB today.
$T_{\mathrm{CMB0}} = 2.7255$ K.
η - Conformal time.
χ - Co-moving distance.

Density parameters

Density parameter $\Omega_X = \rho_X/\rho_c$ where ρ_X is the density and $\rho_c = 8\pi G/3H^2$ the critical density. X can take the following values:

b - Baryons.
CDM - Cold dark matter.
γ - Electromagnetic radiation.
ν - Neutrinos.
k - Spatial curvature.
Λ - Cosmological constant.

A 0 in the subscript indicates the present day value.

1. Introduction

Some citation Dodelson & Schmidt (2020) and Weinberg (2008)

Also write about the following:

- Cosmological principle
- Einstein field equation
- Homogeneity and isotropy
- FLRW metric

In order to explain the connection between spacetime itself and the energy distribution within it we must solve the Einstein equation:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{1}$$

where $G_{\mu\nu}$ is the Einstein tensor describing the geometry of spacetime, G is the gravitational constant and $T_{\mu\nu}$ is the energy and momentum tensor.

Fig. 1. Penguin making sure that you do all the work necessary!

2. Milestone I - Background Cosmology

Some introduction to milestone 1

2.1. Theory

2.1.1. Fundamentals

If we assume the universe to be homogeneous and isotropic, the line elements ds is given by the FLWR-metric as follows (in polar coordinates)(Weinberg 2008, eq. 1.1.11):

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + e^{2x(t)}\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2\right], \quad (2)$$

where we have introduced $x(t) = \ln(a(t))$, the logarithm of the scale factor $a(t)$ include more as our first measure of time.

We further model all forms of energy in the universe as perfect fluids, only characterised by their rest frame density ρ and isotropic pressure p, and an equation of state relating the two:

$$\omega = \frac{\rho}{p}. \quad (3)$$

By conservation of energy and momentum we must satisfy $\nabla_\mu T^{\mu\nu} = 0$, which results in the following differential equations for the density include more here? of each fluid ρ_i:

$$\frac{\mathrm{d}\rho_i}{\mathrm{d}t} + 3H\rho_i(1+\omega) = 0, \quad (4)$$

where we have introduced the Hubble parameter $H \equiv \dot{a}/a = \mathrm{d}x/\mathrm{d}t$. The solution to eq. 4 is of the form:

$$\rho_i \propto e^{-3(1+\omega_i)x}, \quad (5)$$

where ω_i takes different values depending on the fluid it models (more on this later).

.... solution Friedmann eq... hubble equaiton.

2.1.2. Measure of time and space

2.1.3. ΛCDM-model

The Hubble equation, where we allow for curvature is citation?:

$$H(x) = H_0\sqrt{\Omega_{\mathrm{M}0}e^{-3x} + \Omega_{\mathrm{R}0}e^{-4x} + \Omega_{k0}e^{-2x} + \Omega_{\Lambda 0}}, \quad (6)$$

where $\Omega_{\mathrm{M}0} = \Omega_{b0} + \Omega_{\mathrm{CDM}0}$ and $\Omega_{\mathrm{R}0} = \Omega_{\gamma 0} + \Omega_{\nu 0}$ are the present day values of the total matter and radiation densities.

Something aboutu the curvature, and the evolution of density parameters here.

derive?

$$\frac{\mathrm{d}\eta}{\mathrm{d}x} = \frac{c}{\mathcal{H}(x)}. \quad (7)$$

$$\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{1}{H(x)}. \quad (8)$$

$$\chi(x) = \eta_0 - \eta(x). \quad (9)$$

$$r(\chi) = \begin{cases} \chi \cdot \frac{\sin\left(\sqrt{|\Omega_{k0}|}H_0\chi/c\right)}{\sqrt{|\Omega_{k0}|}H_0\chi/c} & \Omega_{k0} < 0 \\ \chi & \Omega_{k0} = 0 \\ \chi \cdot \frac{\sinh\left(\sqrt{|\Omega_{k0}|}H_0\chi/c\right)}{\sqrt{|\Omega_{k0}|}H_0\chi/c} & \Omega_{k0} > 0 \end{cases} \quad (10)$$

$$d_A(x) = e^x r(\chi(x)). \quad (11)$$

$$d_L = e^{-x} r(\chi(x)). \quad (12)$$

$$\chi^2(h, \Omega_{m0}, \Omega_{k0}) = \sum_{i=1}^{N} \frac{(d_L(z, \Omega_{m0}, \Omega_{k0}) - d_L^{\mathrm{obs}}(z_i))^2}{\sigma_i^2} \quad (13)$$

2.2. Methods

2.2.1. Initial equation

$$\begin{aligned} \Omega_k(a) &= \Omega_{k0}e^{-2x}\left(\frac{H_0^2}{H(x)^2}\right) \\ \Omega_{\mathrm{CDM}}(a) &= \Omega_{\mathrm{CDM}0}e^{-3x}\left(\frac{H_0^2}{H(x)^2}\right) \\ \Omega_{b0}(a) &= \Omega_{b0}e^{-3x}\left(\frac{H_0^2}{H(a)^2}\right) \\ \Omega_{\gamma 0}(a) &= \Omega_{\gamma 0}e^{-4x}\left(\frac{H_0^2}{H(x)^2}\right) \\ \Omega_{\nu 0}(a) &= \Omega_{\nu 0}e^{-4x}\left(\frac{H_0^2}{H(x)^2}\right) \\ \Omega_{\Lambda 0} &= \Omega_{\Lambda 0}\left(\frac{H_0^2}{H(x)^2}\right) \end{aligned} \quad (14)$$

$$\Omega_{\gamma 0} = \frac{16\pi^3 G}{90} \cdot \frac{(k_b T_{\mathrm{CMB}0})^4}{\hbar^3 c^5 H_0^2}$$

$$\Omega_{\nu 0} = N_{\mathrm{eff}} \cdot \frac{7}{8} \cdot \left(\frac{4}{3}\right)^{4/3} \cdot \Omega_{\gamma 0} \quad (15)$$

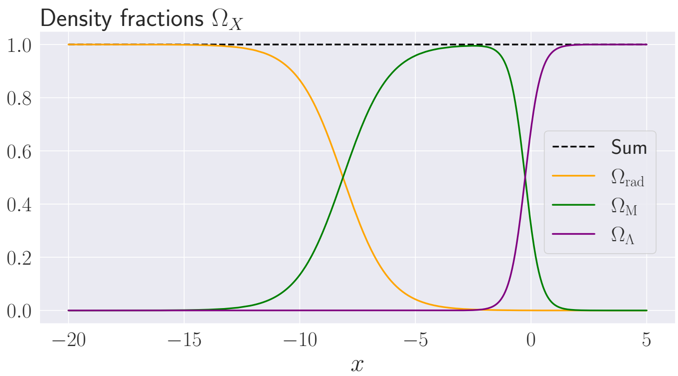

Fig. 2. Omega tests

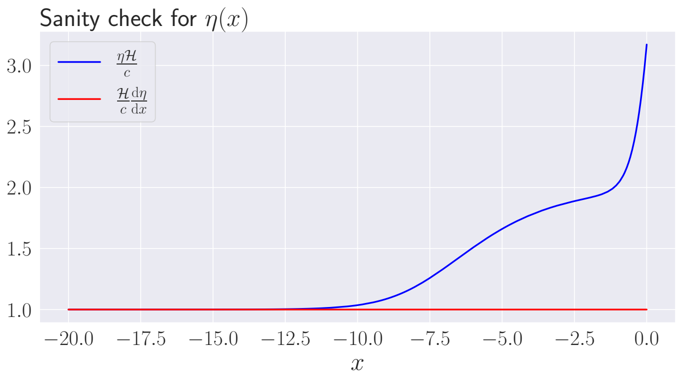

Fig. 3. Eta tests

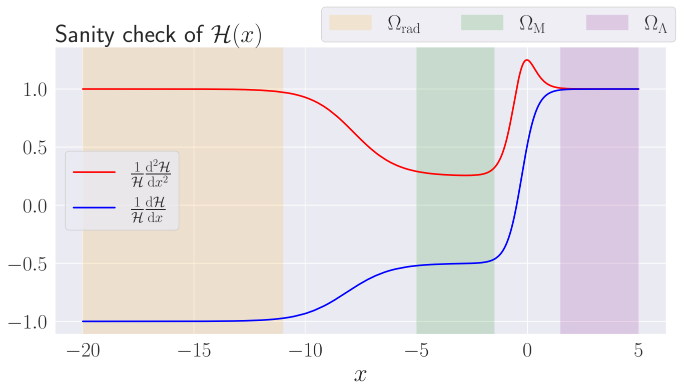

Fig. 4. HP tests

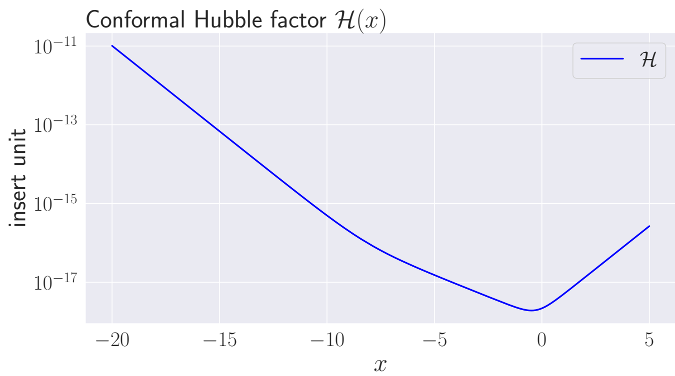

Fig. 5. Conformal Hubble factor.

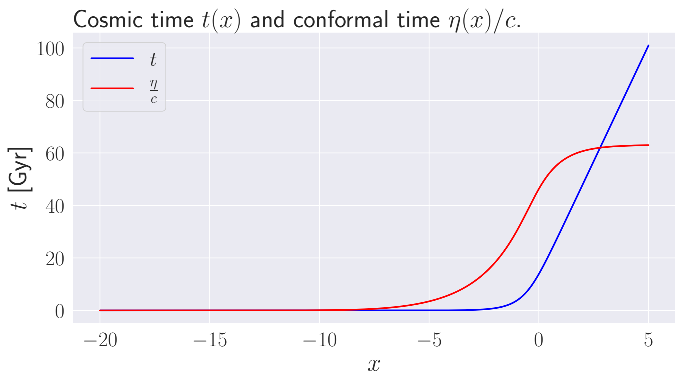

Fig. 6. cosmic time.

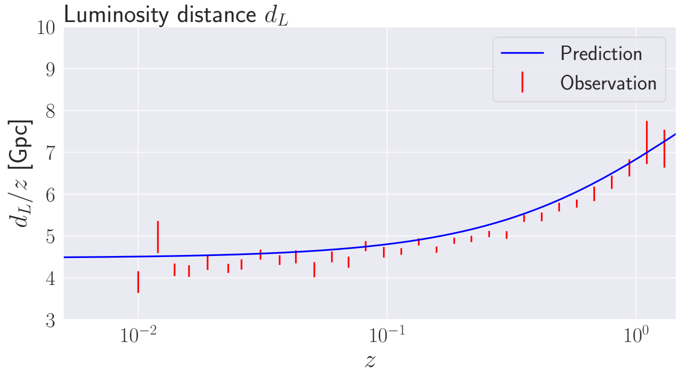

Fig. 7. Supernova data fitted

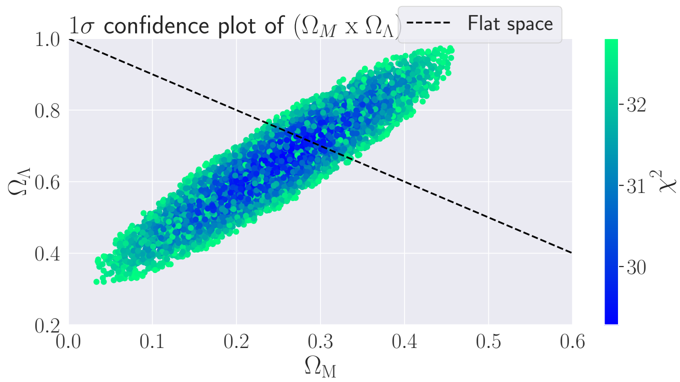

Fig. 8. one sigma confidence plot

2.2.2. ODEs

We solve the differential equation for $\eta(x)$, eq. 7 using an ordinary differential equation solver, with Runge-Kutta 4 RK4?as the advancement method. The initial condition is given by $\eta(x_{\rm start}) = c/\mathcal{H}(x_{\rm start})$

2.3. Results

2.3.1. Tests

2.3.2. Analysis

3. Milestone II

Some introduction to milestone 2

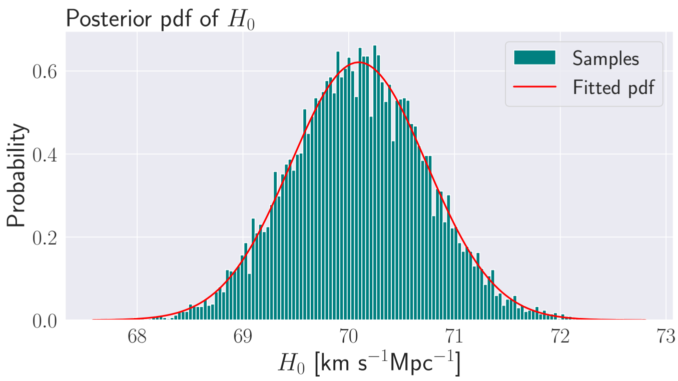

Fig. 9. posterior pdf.

3.1. Theory

Some theory

3.2. Methods

some methods

3.3. Results

4. Milestone III

Some introduction to milestone 3

4.1. Theory

Some theory

4.2. Methods

some methods

4.3. Results

5. Milestone IV

Some introduction to milestone 4

5.1. Theory

Some theory

5.2. Methods

some methods

5.3. Results

6. Conclusion

Some overall conclusion

References

Dodelson, S. & Schmidt, F. 2020, Modern Cosmology (Elsevier Science)
Weinberg, S. 2008, Cosmology, Cosmology (OUP Oxford)

Appendix A: Some appendix

Appendix B: Some appendix