UNIVERSITY
OF OSLO

Master's thesis

-sometitle-

Classifying N-body simulations with and without relativistic corrections using machine learning techniques

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Abstract

Here come 3–6 sentences describing your thesis.

Sammendrag

Here comes the abstract in a different language.

Contents

List of Figures

List of Tables

Preface

Here comes your preface, including acknowledgments and thanks.

Chapter 1

Introduction

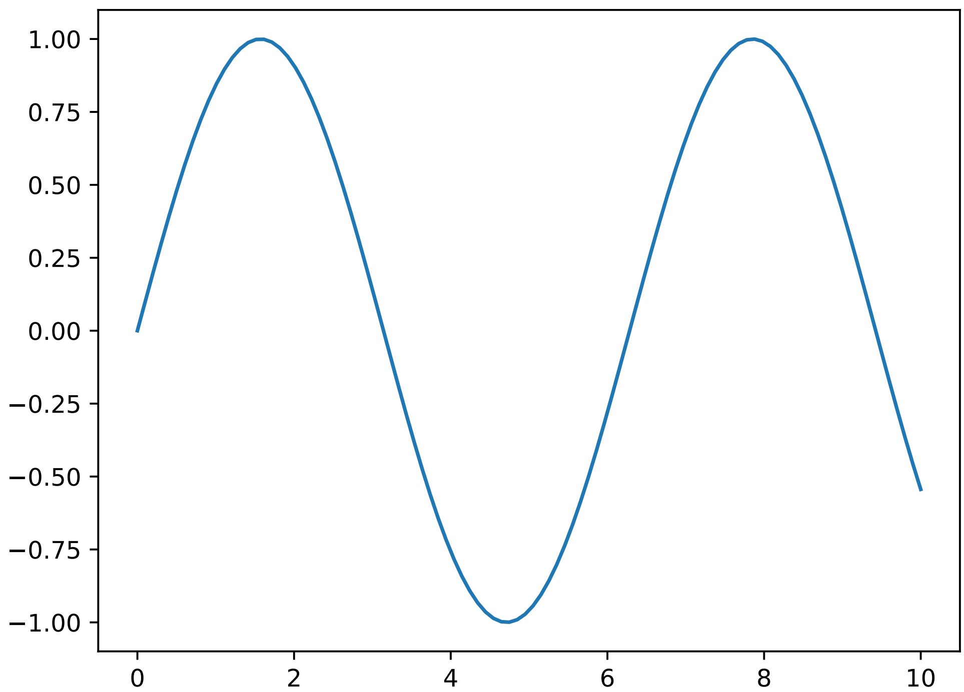
This is the introduction that will shortly be written. How fast does things change.

1.1 Motivation

1.2 Outline

1.3 Aim

1.4 Nomenclature


1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
2
4
6
8
10

Part I

Cosmological Structure Formation

Chapter 2

Preliminaries

2.1 General Relativity

2.1.1 Einstein's Field Equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{2.1}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \tag{2.2}$$

$$R = g^{\mu\nu} R_{\mu\nu} \tag{2.3}$$

$$R_{\mu\nu} = \partial_\rho \Gamma^\rho_{\mu\nu} - \partial_\nu \Gamma^\rho_{\mu\rho} + \Gamma^\rho_{\mu\nu}\Gamma^\sigma_{\rho\sigma} - \Gamma^\rho_{\mu\sigma}\Gamma^\sigma_{\nu\rho} \tag{2.4}$$

2.1.2 Riemann Connection and Covariant Derivatives

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}\right) \tag{2.5}$$

$$\nabla_\mu T^\mu_\nu = \partial_\mu T^\mu_\nu + \Gamma^\mu_{\mu\alpha} T^\alpha_\nu - \Gamma^\alpha_{\mu\nu} T^\mu_\alpha \tag{2.6}$$

2.1.3 Geodesic Equation

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0 \tag{2.7}$$

2.1.4 The Stress-Energy Tensor

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu} \tag{2.8}$$

2.2 Useful Relations

Chapter 3

Background Cosmology

3.1 The homogeneous Universe

In this chapter I will focus on explaining the background cosmology in light of a homogeneous universe. A natural place to start is the cosmological principle, followed by a description of the geometry of space itself. If not otherwise stated, the development of this chapter is based on Dodelson and Schmidt 2020, Weinberg 2008 and TODO: cite Baumann

3.1.1 The Cosmological Principle

3.1.2 The Robertson-Walker Metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{3.1}$$

3.1.3 The Friedmann Equations

3.2 My Universe is loaded with...

3.3 Thermal History of the Universe

Chapter 4

Perturbation Theory

4.1 Initial Conditions

4.2 Transfer Functions

4.3 Power Spectra

4.4 Non-linear Evolution

4.5 Bispectra

The bispectra are powerful tools for studying the non-linear evolution of the density field. The bispectrum is defined as the Fourier transform of the three-point correlation function, and is given by:

$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \tag{4.1}$$

Well this is rather awkward. Adamek et al. 2016 or (Falck et al. 2017)

Chapter 5

Simulation theory

Some theory and history as to how to conduct N-body simulations.

5.1 N-body simulations

5.1.1 Describing a box of particles

5.1.2 Forces and Fields

5.1.3 Mass Assignment Schemes

5.1.4 Validity of Box

5.2 Newtonian Approach

5.3 General Relativistic Approach

Part II

Machine Learning

Chapter 6

Fundamental Elements of Machine Learning

6.1 Introduction

In this chapter I will give a brief introduction into machine learning. This includes a mathematical description of some fundamental concepts common across numerous machine learning models. The more advanced models will be dealt with at a later stage. If not otherwise stated, the following chapter is based on Goodfellow, Bengio and Courville 2016 and Hastie, Tibshirani and Friedman 2009.

6.2 Linear Algebra

maybe

6.3 Probability and Information Theory

maybe

6.4 Basic Machine Learning

TODO: Fill more here

6.4.1 Estimators, Bias, Variance and Error

Estimators Based on the assumption that there exists some true parameter(s) $\boldsymbol{\theta}$ which remain unknown,[1] we are able to make predictions and estimations of such parameter(s). Let's say we have m independent and identically distributed (i.i.d.) random variables $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m\}$ drawn from the same probability distribution $p(\mathbf{x})$. An *estimator* of the true values $\boldsymbol{\theta}$ is any function of the data such that $\hat{\boldsymbol{\theta}}_m = g(\mathbf{x}_1, \ldots, \mathbf{x}_m)$, where $\hat{\boldsymbol{\theta}}$ is the estimate of $\boldsymbol{\theta}$. This is known as point estimation, as we are estimating a single value. This definition does not pose any restrictions on the function g. However, a good estimator would yield an estimate $\hat{\boldsymbol{\theta}}_m$ that is close to the true value $\boldsymbol{\theta}$.

[1] This is the frequentist perspective of statistics

Function estimators Say we want to predict a variable $\mathbf{y}$ given some vector $\mathbf{x}$. We assume the true variable $\mathbf{y}$ is given by some function approxiamation $f(\mathbf{x})$ plus some error $\boldsymbol{\epsilon}$: $\mathbf{y} = f(\mathbf{x}) + \boldsymbol{\epsilon}$. The aim is then to estimate the function f with the estimator $\hat{f}$. If we then realise that $\hat{f}$ is really just a point estimator in function space, the two above concepts are equivalent.

Bias The bias of the estimator $\hat{\boldsymbol{\theta}}_m$ is defined as the difference between the expected value of the estimator and the true value of the parameter: $\text{bias}(\hat{\boldsymbol{\theta}}_m) = \mathbb{E}[\hat{\boldsymbol{\theta}}_m] - \boldsymbol{\theta}$. An unbiased estimator has zero bias, i.e. $\mathbb{E}[\hat{\boldsymbol{\theta}}_m] = \boldsymbol{\theta}$. An estimator is asymptotically unbiased if its bias approaches zero as the number of data points m approaches infinity, i.e. $\lim_{m\to\infty} \mathbb{E}[\hat{\boldsymbol{\theta}}_m] = \boldsymbol{\theta}$.

Variance

Standard Error

Mean Squared Error

6.4.2 Maximum Likelihood Estimation

6.4.3 Bayesian Statistics

6.4.4 Supervised Learning

6.4.5 Unsupervised Learning

Chapter 7

Neural Networks

7.1 Forward pass - Prediction

7.1.1 Activation functions

7.1.2 Loss functions

7.2 Backpropagation - Training

7.2.1 Gradient descent

7.2.2 Optimizers

7.2.3 Regularization

Chapter 8

Convolutional Neural Networks

8.1 Convolution

8.2 New Layers

8.2.1 Convolutional layers

8.2.2 Pooling layers

8.2.3 Dropout layers

Part III

Acquiring Data

Chapter 9

Simulations

Chapter 10

Data Verification

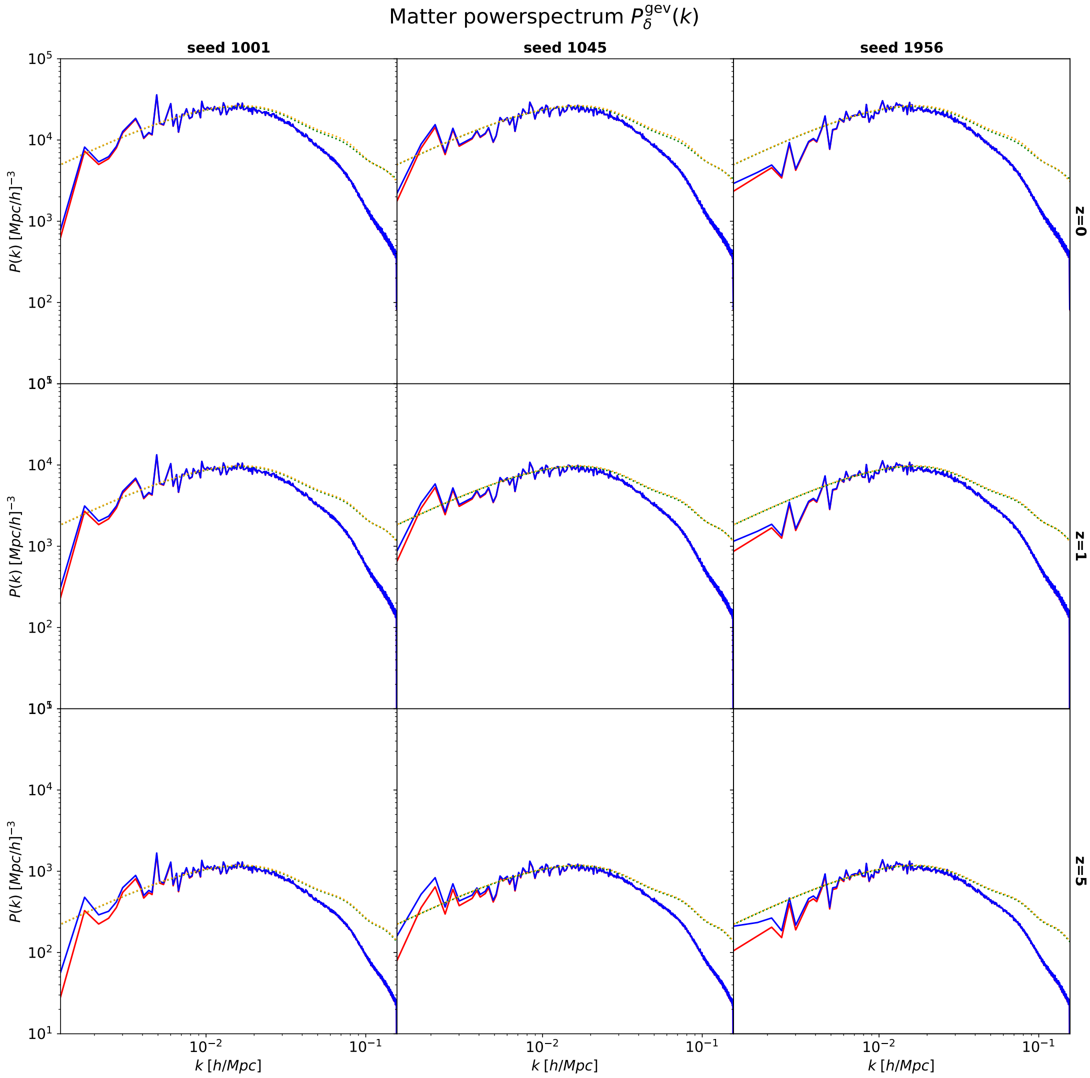
10.1 Slices of Datacubes

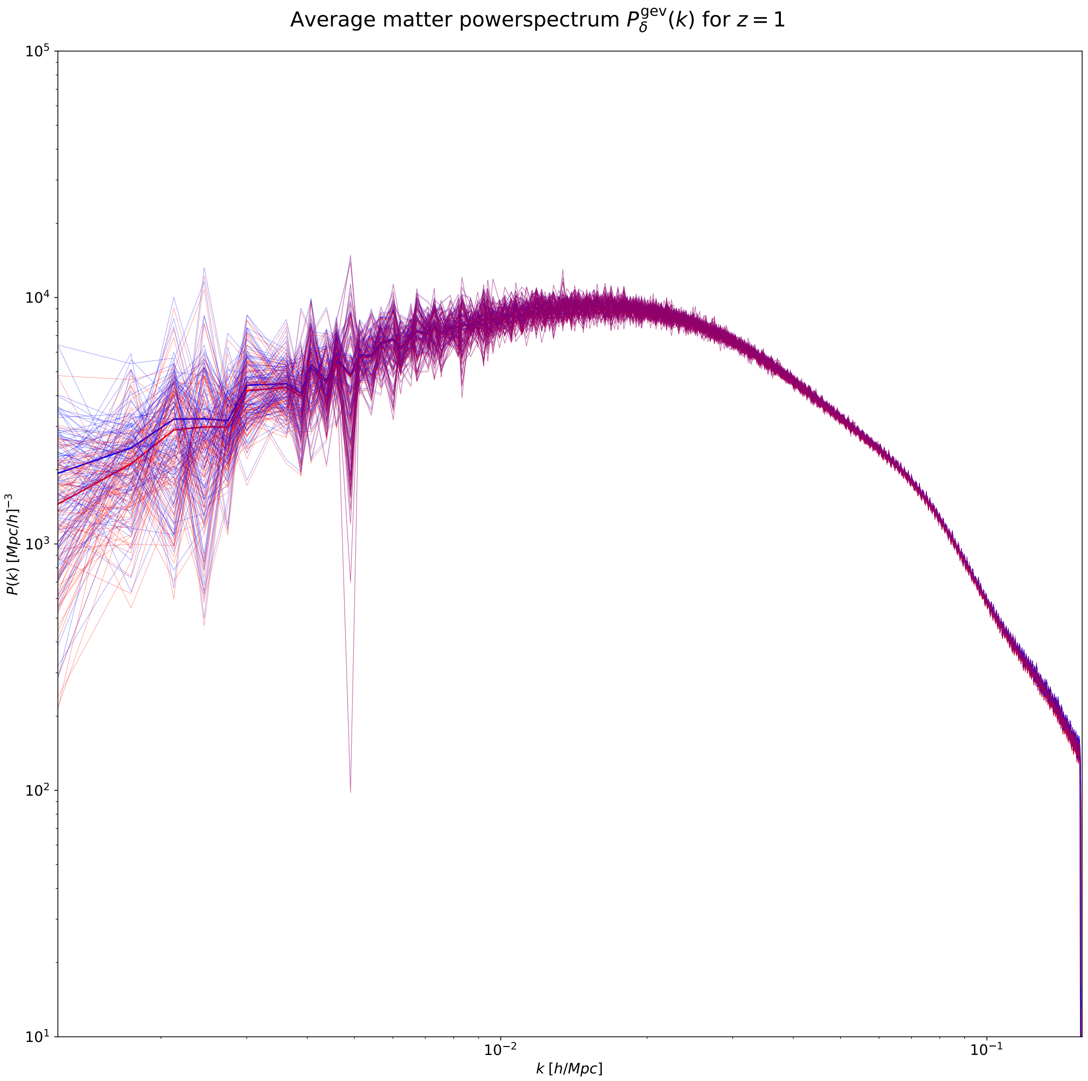
10.2 Power spectra from Theory

TODO: Provide some camb and class power spectra here.

10.3 Powerspectra from Simulations

10.4 Powerspectra from Datacubes


Matter powerspectrum $P_\delta^{\text{gev}}(k)$
seed 1001
seed 1045
seed 1956
10^5
10^4
10^3
10^2
10^1
$P(k)\ [Mpc/h]^{-3}$
z=0
z=1
z=5
10^{-2}
10^{-1}
$k\ [h/Mpc]$



Average matter powerspectrum $P_\delta^{\mathrm{gev}}(k)$ for $z=1$
10^5
10^4
10^3
10^2
10^1
$P(k)$ $[Mpc/h]^{-3}$
10^{-2}
10^{-1}
k $[h/Mpc]$

Chapter 11

Trainable Dataset

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Hastie, T., R. Tibshirani and J.H. Friedman (2009). *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*. Springer series in statistics. Springer. ISBN: 9780387848846. URL: https://books.google.no/books?id=eBSgoAEACAAJ.

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