Solution for Exersize 4

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Abstract

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1 Question 1

1.1 a)

Mean μ of givin points
$x : \frac{1+2+4+5}{4} = \frac{12}{4} = 3$
$y : \frac{-1+1+-1+1}{4} = \frac{0}{4} = 0$
$\mu = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

1.2 b)

The Covariancematrix C has the following form:

$$C = \begin{pmatrix} covariance(x,x) & covariance(x,y) \\ covariance(y,x) & covariance(y,y) \end{pmatrix} = \begin{pmatrix} variance(x) & covariance(x,y) \\ covariance(y,x) & variance(y) \end{pmatrix}$$

$$covariance(x,y) = covariance(y,x) = \frac{\sum_{i=0}^{n}(x_i - \bar{x}) * (y_i - \bar{y})}{n}$$

$$covariance(x,y) = \frac{(1-3)*(-1-0)+(2-3)*(1-0)+(4-3)*(-1-0)+(5-3)*(1-0)}{4}$$
$$= \frac{2}{4} = 2$$

$$variance(x) = \frac{\sum_{i=0}^{n}(x_i - \bar{x})^2}{n}$$

$$variance(x) = \frac{(1-3)^2+(2-3)^2+(4-3)^2+(5-3)^2}{4-1} = \frac{4+1+1+4}{4} = \frac{10}{4}$$

$$variance(y) = \frac{(-1-0)^2+(1-0)^2+(-1-0)^2+(1-0)^2}{4-1} = \frac{1++1+1+1}{4} = \frac{4}{4} = 1$$

$$C = \begin{pmatrix} \frac{10}{4} & 2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} \frac{5}{2} & 2 \\ 2 & 1 \end{pmatrix}$$

1.3 c)

$$det(C - I\lambda) = |C - I\lambda| = 0$$

$$\left| \begin{pmatrix} \frac{5}{2} & 2 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} * \lambda \right| = 0$$
$$\left| \begin{pmatrix} \frac{5}{2} & 2 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0$$
$$\begin{vmatrix} \frac{5}{2} - \lambda & 2 \\ 2 & 1 - \lambda \end{vmatrix} = 0$$

The formula for determinant is

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a * d - c * d$$

$$\begin{vmatrix} \frac{5}{2} - \lambda & 2 \\ 2 & 1 - \lambda \end{vmatrix} = (\frac{5}{2} - \lambda * 1 - \lambda) - (2 * 2) = \lambda^2 - \frac{5}{2}\lambda - \lambda + \frac{5}{2} - 4 = \lambda^2 - \frac{7}{2}\lambda - \frac{3}{2}$$

p-q formula

$$\lambda_{1,2} = -\frac{-\frac{7}{2}}{2} \pm \sqrt{\frac{-\frac{7}{2}}{2} - (-\frac{3}{2})} = \frac{7}{4} \pm \sqrt{\frac{49}{16} + \frac{24}{16}} = \frac{7}{4} \pm \frac{\sqrt{73}}{4}$$
$$\lambda_1 \approx \frac{7}{4} + 2 \approx \frac{15}{4}$$
$$\lambda_2 \approx \frac{7}{4} - 2 \approx \frac{-8}{4} \approx \frac{-8}{4} \approx= 2$$

1.4 d)

Insert λ_1 in $(C - I\lambda_1) * x = 0$

$$\left(\begin{pmatrix} \frac{5}{2} & 2 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} * \frac{15}{4} \right) * x = \left(\begin{pmatrix} \frac{5}{2} & 2 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} \frac{15}{4} & 0 \\ 0 & \frac{15}{4} \end{pmatrix} \right) * x$$
$$= \begin{pmatrix} \frac{5}{2} - \frac{15}{4} & 2 \\ 2 & 1 - \frac{15}{4} \end{pmatrix} * = 0$$

Now

$$\begin{pmatrix} \frac{5}{2} - \frac{15}{4} & 2 \\ 2 & 1 - \frac{15}{4} \end{pmatrix} * \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

Make a

$$(\frac{5}{2} - \frac{15}{4})x_1 + 2x_2 = 0 \tag{1}$$
$$2x_1 + (1 - \frac{15}{4})x_2 = 0 \tag{2}$$

$$-\frac{5}{4}x_1 + 2x_2 = 0 \tag{3}$$
$$2x_1 - \frac{11}{4}x_2 = 0 \tag{4}$$

$$x_1 = \frac{11}{8}$$
$$x_2 = 1$$
$$\text{the first eigenvector is} x = \begin{pmatrix} \frac{11}{8} \\ 1 \end{pmatrix}$$

And now the second eigenvector! Insert λ_2 in $(C - I\lambda_2) * x = 0$

$$\left(\begin{pmatrix} \frac{5}{2} & 2 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} * -2 \right) * x = \left(\begin{pmatrix} \frac{5}{2} & 2 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \right) * x$$
$$= \begin{pmatrix} \frac{5}{2} + 2 & 2 \\ 2 & 1 + 2 \end{pmatrix} * = 0$$

Now

$$\begin{pmatrix} \frac{5}{2} + 2 & 2 \\ 2 & 1 + 2 \end{pmatrix} * \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

Make a

$$(\frac{5}{2} + 2 + 2x_2 = 0 \tag{5}$$
$$2x_1 + 1 + 2 = 0 \tag{6}$$

$$\frac{9}{2}x_1 + 2x_2 = 0 \tag{7}$$

$$2x_1 + 3x_2 = 0 \tag{8}$$

$$x_1 = -\frac{5}{8}$$
$$x_2 = 1$$
$$\text{the first eigenvector is} x = \left(\tfrac{5}{8} \;\; 1\right)$$

2 Conclusions

We worked hard, and achieved very little.