Universidad Nacional Autónoma de México

Facultad de Ciencias

Tarea 3

Matemáticas Aplicadas para las Ciencias III

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Link al código fuente: https://github.com/JohannGordillo/Mates-lll-Tarea-2

Ejercicios

1. Evaluar las siguientes integrales a lo largo de las trayectorias $\int_c f(x,y,z)ds$ donde

   a) $f(x,y,z = e^{\sqrt{z}}$,y $c :\rightarrow (1,2,t^2)$,y $t \in [0,1]$

   $$\int_c f\,ds = \int_0^1 f(c(t))\|\vec{c}'(t)\|\,dt = \int_0^1 f(c(t)))(\sqrt{0^2+0^2+(2t)^2})\,dt$$
   $$= \int_0^1 f(c(t))(2t)\,dt = \int_0^1 f(c(t))(2t)\,dt = \int_0^1 e^{\sqrt{t^2}}(2t)\,dt$$
   $$= \int_0^1 e^t(2t)\,dt = 2\int_0^1 e^t(t)\,dt$$

   haciendolo por partes sera $u = t$ y $v = e^t$

   asi por $\int udv = uv - \int vdu$

   $$= 2(\left[(e^t)(t) - \int e^t\,dt\right]_0^1) = 2(\left[(e^t)(t) - e^t\right]_0^1) = 2([(t)]_0^1) = 2(1) = 2$$

   b) $f(x,y,z = yz$,y $c :\rightarrow (t,3t,2t)$, $t \in [0,3]$

   $$\int_c f\,ds = \int_1^3 f(c(t))\|\vec{c}'(t)\|\,dt = \int_1^3 f(c(t)))(\sqrt{1^2+3^2+2^2})\,dt$$
   $$= \int_1^3 f(c(t))(\sqrt{14})\,dt = \int_1^3 f(c(t))(\sqrt{14})\,dt = \int_1^3 ((2t)(3t))(\sqrt{14})\,dt$$
   $$= \int_1^3 6t^2(\sqrt{14})\,dt = 6\sqrt{14}\int_1^3 t^2\,dt = 6\sqrt{14}\left[\frac{t^3}{3}\right]_1^3$$
   $$= 6\sqrt{14}[\frac{3^3}{3} - \frac{1^3}{3}] = \frac{6\sqrt{14}}{3}(9-1) = 52\sqrt{14}$$

2. Sea $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Evaluar la integral de $\mathbf{F}$ a lo largo de cada una de las siguientes trayectorias:

   a) $c(t) = (t,t,t)$, $0 \leq t \leq 1$

   Queremos encontrar $\int_c F \cdot ds$, y sabemos que $\int_c F \cdot ds = \int_a^b F(c(t)) \cdot c'(t)dt$.

   $$c'(t) = \left(\frac{d}{dt}t, \frac{d}{dt}t, \frac{d}{dt}t\right)$$

   $\therefore c'(t) = (1,1,1)$

   Y como $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $F(c(t)) = (t,t,t)$.

   De esta manera:

$$
\begin{aligned}
\int_c F \cdot ds &= \int_a^b F(c(t)) \cdot c'(t)dt \\
&= \int_0^1 (t,t,t) \cdot (1,1,1)dt \\
&= \int_0^1 (t+t+t)dt \\
&= \int_0^1 3tdt \\
&= 3\int_0^1 tdt \\
&= 3\left[\frac{t^2}{2}\right]_0^1 \\
&= 3\left(\frac{1^2}{2} - \frac{0^2}{2}\right) \\
&= 3\left(\frac{1}{2}\right) \\
&= \frac{3}{2}
\end{aligned}
$$

b) $c(t) = (\cos t, \sin t, 0)$, $0 \le t \le 2\pi$

Queremos encontrar $\int_c F \cdot ds$, y sabemos que $\int_c F \cdot ds = \int_a^b F(c(t)) \cdot c'(t)dt$.

$c'(t) = \left(\frac{d}{dt}\cos t, \frac{d}{dt}\sin t, \frac{d}{dt}0\right)$

$\therefore c'(t) = (-\sin t, \cos t, 0)$

Y como $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $F(c(t)) = (\cos t, \sin t, 0)$.

De esta manera:

$$
\begin{aligned}
\int_c F \cdot ds &= \int_a^b F(c(t)) \cdot c'(t)dt \\
&= \int_0^{2\pi} (\cos t, \sin t, 0) \cdot (-\sin t, \cos t, 0)dt \\
&= \int_0^{2\pi} (-\sin t\cos t + \sin t\cos t + 0)dt \\
&= \int_0^{2\pi} 0 + 0dt \\
&= \int_0^{2\pi} 0dt \\
&= 0
\end{aligned}
$$

c) $c(t) = (\sin t, 0, \cos t)$, $0 \le t \le 2\pi$

Queremos encontrar $\int_c F \cdot ds$, y sabemos que $\int_c F \cdot ds = \int_a^b F(c(t)) \cdot c'(t)dt$.

$c'(t) = \left(\frac{d}{dt} \sin t, \frac{d}{dt} 0, \frac{d}{dt} \cos t \right)$

$\therefore c'(t) = (\cos t, 0, -\sin t)$

Y como $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $F(c(t)) = (\sin t, 0, \cos t)$.

De esta manera:

$$\begin{aligned}
\int_c F \cdot ds &= \int_a^b F(c(t)) \cdot c'(t)dt \\
&= \int_0^{2\pi} (\sin t, 0, \cos t) \cdot (\cos t, 0, -\sin t)dt \\
&= \int_0^{2\pi} (\sin t \cos t + 0 - \sin t \cos t)dt \\
&= \int_0^{2\pi} 0 + 0dt \\
&= \int_0^{2\pi} 0dt \\
&= 0
\end{aligned}$$

d) $c(t) = (t^2, 3t, 2t^3)$, $-1 \leq t \leq 2$

Queremos encontrar $\int_c F \cdot ds$, y sabemos que $\int_c F \cdot ds = \int_a^b F(c(t)) \cdot c'(t)dt$.

$c'(t) = \left(\frac{d}{dt} t^2, \frac{d}{dt} 3t, \frac{d}{dt} 2t^3 \right)$

$\therefore c'(t) = \left(2t, 3, 6t^2\right)$

Y como $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $F(c(t)) = (t^2, 3t, 2t^3)$.

De esta manera:

$$
\begin{aligned}
\int_c F \cdot ds &= \int_a^b F(c(t)) \cdot c'(t)dt \\
&= \int_{-1}^{2} (t^2, 3t, 2t^3) \cdot (2t, 3, 6t^2)dt \\
&= \int_{-1}^{2} (2t^3 + 9t + 12t^5)dt \\
&= \int_{-1}^{2} (t^3)dt + 9\int_{-1}^{2} (t)dt + 12\int_{-1}^{2} (t^5)dt \\
&= 2\left[\frac{t^4}{4}\right]_{-1}^{2} + 9\left[\frac{t^2}{2}\right]_{-1}^{2} + 12\left[\frac{t^6}{6}\right]_{-1}^{2} \\
&= 2\left(\frac{16}{4} - \frac{1}{4}\right) + 9\left(\frac{4}{2} - \frac{1}{2}\right) + 12\left(\frac{64}{6} - \frac{1}{6}\right) \\
&= 2\left(\frac{15}{4}\right) + 9\left(\frac{3}{2}\right) + 12\left(\frac{63}{6}\right) \\
&= \frac{30}{4} + \frac{27(2)}{4} + \frac{(63)(2)(4)}{4} \\
&= \frac{504 + 54 + 30}{4} \\
&= \frac{588}{4} \\
&= \frac{294}{2} \\
&= 147
\end{aligned}
$$

3. Hallar la ecuacion para el plano tangente a la superficie dada en el punto especificado

   a) $x = 2u$, $y = u^2 + v$, $z = v^2$ en $(0, 1, 1)$.
   Primero obtenemos los vectores tangentes T_u y T_v.

$$
\begin{aligned}
T_u &= \frac{\partial(2u)}{\partial u} + \frac{\partial(u^2 + v)}{\partial u} + \frac{\partial(v^2)}{\partial u} = 2\hat{\imath} + 2u\hat{\jmath} \\
T_v &= \frac{\partial(2u)}{\partial v} + \frac{\partial(u^2 + v)}{\partial v} + \frac{\partial(v^2)}{\partial v} = \hat{\jmath} + 2v\hat{k}
\end{aligned}
$$

   Luego calculamos $T_u \times T_v$.

$$
T_u \times T_v = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 2u & 0 \\ 0 & 1 & 2v \end{vmatrix} = 4uv\hat{\imath} - 4v\hat{\jmath} + 2\hat{k}
$$

   Ahora obtenemos los valores de u y v.

$$
\begin{aligned}
0 = x = 2u &\longrightarrow u = 0 \\
1 = z = v^2 &\longrightarrow v = 1
\end{aligned}
$$

   Calculamos $n = (T_u \times T_v)(0, 1)$.

$$
\begin{aligned}
n &= 4(0)(1)\hat{\imath} - 4(1)\hat{\jmath} + 2\hat{k} \\
&= -4(1)\hat{\jmath} + 2\hat{k} \\
&= (0, -4, 2)
\end{aligned}
$$

Y finalmente obténemos la ecuación del plano tangente.

$$\begin{aligned}0(x-0)-4(y-1)+2(z-1)&=0\\-4y+4+2x-2&=0\\-4y+2z+2&=0\\2z&=4y-2\\z&=2y-1\end{aligned}$$

b) $x=u^2$, $y=u\sin e^v$, $z=\frac{1}{3}u\cos e^v$ en $(13,-2,1)$
Obténemos los valores de u y v.

$$\begin{aligned}13=x=u^2&\longrightarrow u=\sqrt{13}\\-2=y=u\sin e^v&\longrightarrow \sin e^v=-\frac{2}{\sqrt{13}}\\1=z=\frac{1}{3}u\cos e^v&\longrightarrow \cos e^v=\frac{3}{\sqrt{13}}\end{aligned}$$

Luego, obténemos los vectores tangentes T_u y T_v.

$$\begin{aligned}T_u&=\frac{\partial(u^2)}{\partial u}+\frac{\partial(u\sin e^v)}{\partial u}+\frac{\partial(\frac{1}{3}u\cos e^v)}{\partial u}=2u\hat{\imath}+\sin e^v\hat{\jmath}+\frac{1}{3}\cos e^v\hat{k}\\T_v&=\frac{\partial(u^2)}{\partial v}+\frac{\partial(u\sin e^v)}{\partial v}+\frac{\partial(\frac{1}{3}u\cos e^v)}{\partial v}=ue^v\cos e^v\hat{\jmath}-\frac{1}{3}ue^v\sin e^v\hat{k}\end{aligned}$$

Luego calculamos $T_u\times T_v$.

$$T_u\times T_v=\begin{vmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\2u&\sin e^v&\frac{1}{3}\cos e^v\\0&ue^v\cos e^v&-\frac{1}{3}ue^v\sin e^v\end{vmatrix}=-\frac{1}{3}ue^v\hat{\imath}+\frac{2}{3}u^2e^v\sin e^v\hat{\jmath}+2ue^v\cos e^v\hat{k}$$

Y obtenemos la ecuacion del plano tangente.

$$\begin{aligned}-\frac{1}{3}\sqrt{13}e^v(x-13)+\frac{2}{3}\sqrt{13}^2e^v(-\frac{2}{\sqrt{13}})(y+2)+2\sqrt{13}e^v(-3)(z-1)&=0\\\sqrt{13}e^v\big(-\frac{1}{3}(x-13)-\frac{4}{3}(y+2)+6(z-1)\big)&=0\\-\frac{1}{3}(x-13)-\frac{4}{3}(y+2)+6(z-1)&=0\\-\frac{1}{3}x+\frac{13}{3}-\frac{4}{3}y-\frac{8}{3}+6z+6&=0\\-\frac{1}{3}x-\frac{4}{3}y+6z+\frac{23}{3}&=0\\18z&=x+4y-23\end{aligned}$$

4. Sea D el rectángulo en el plano $\theta\phi$ definido por $0\le\theta\le 2\pi$, $0\le\phi\le\pi$ y sea S la superficie definida por la parametrización $\Phi:D\to\mathbb{R}^3$ dada por:

$$x=\cos\theta\sin\phi\quad y=\sin\theta\sin\phi\quad z=\cos\phi$$

Obtenemos $T_\theta=-i\sin\theta\sin\phi+j\cos\theta\sin\phi+k0$ $T_\phi=i\cos\theta\cos\phi+j\sin\theta\cos\phi-k\sin\phi$

$$|T_\theta \mathrm{x} T_\phi|=\begin{vmatrix}i&j&k\\-\sin\theta\sin\phi&\cos\theta\sin\phi&0\\\cos\theta\cos\phi&\sin\theta\cos\phi&-\sin\phi\end{vmatrix}=(\cos\theta\,\mathrm{sen}\,\phi(-\sin\phi))-(\sin\theta\cos\phi(0))-(\sin\theta\sin\phi(-\sin\phi))-0+(-\sin\theta\,\mathrm{s}$$

Obtenemos

$$F(x,y,z)\cdot(T_\theta \text{x} T_\phi) = -\cos^2\theta\sin^3\phi - \sin^2\theta\sin^3\phi - \sin\phi\cos^2\phi = -\sin\phi(\sin^2\phi+\cos^2\phi) = -\sin\phi$$

Obtenemos

$$\int_0^\pi\int_0^{2\pi} -\sin\phi \quad d\theta d\phi =$$
$$\int_0^\pi -\sin\phi\int_0^{2\pi} \quad d\theta d\phi =$$
$$\int_0^\pi -\sin\phi(\theta)|_0^{2\pi} d\phi = 2\pi\int_0^\pi -\sin\phi d\phi =$$
$$= 2\pi(-\cos\phi)|_0^\pi = 2\pi(-\cos\pi+\cos(0)) = 2\pi(1+1) = 4\pi$$