Exercises week 1

Overview

Week 1: Introduction to circuit quantization and the transmon

Keywords: Qubit basics, circuit quantization, transmon, anharmonicity.

[1, Chapters: I, IIa-b, Fig. 14(a-c)] Philip Krantz, Morten Kjaergaard, Fei Yan, Terry P. Orlando, Simon Gustavsson, and William D. Oliver. A Quantum Engineer's Guide to Superconducting Qubits. *Applied Physics Reviews*, 6(2):021318, June 2019. arXiv: 1904.06560

[2, Chapters: 1-2.1, 3.1.1-3.1.2] Uri Vool and Michel Devoret. Introduction to quantum electromagnetic circuits. *International Journal of Circuit Theory and Applications*, 45(7):897–934, jun 2017

[3, Chapters: I-II, VI] Jens Koch, Terri M. Yu, Jay Gambetta, A. A. Houck, D. I. Schuster, J. Majer, Alexandre Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf. Charge-insensitive qubit design derived from the Cooper pair box. *Physical Review A*, 76(4):042319, October 2007

E1

Exercises concerning [1]:

(a) Derive equations (5) - (9) and (13) as detailed in the text.

(b) Derive ω_r as found in Eq. (14) and the text above. A trick is to compare Eq. (13) to the standard harmonic oscillator,

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega_r^2 x^2,$$

and isolate ω_r.

E2

Exercises concerning [1]:

(a) Derive Eq. (24).

(b) Expand the cosine at half flux bias ($\phi_e = \pi$) to fourth order. What should γ/N be for the quadratic term to vanish?

(c) Plot the potential for $\gamma/N < 1$, $\gamma/N = 1$ and $\gamma/N > 1$. How will the qubit states qualitatively be different in the three cases?

(d) What is the relative anharmonicity for the value found in (b)? The relative anharmonicity is the anharmonicity divided by the qubit frequency $\alpha_r = \alpha/\omega_r$.

E3

Exercises concerning Fig. 1.

(a) How many loops do you find in Fig. 1?

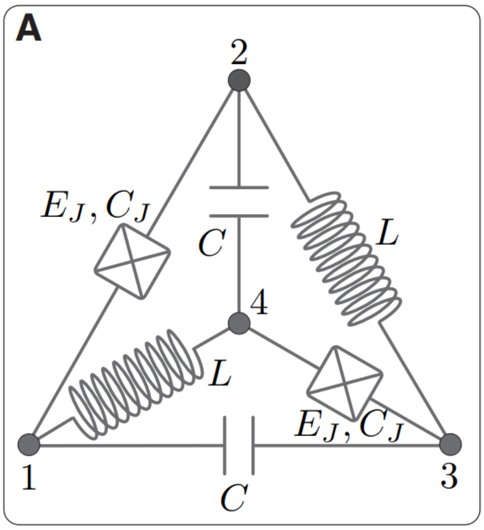

Figure 1: An advanced circuit adapted from Gyenis et al.

(b) Using the coordinates

$$\theta = \phi_2 + \phi_3 - (\phi_1 + \phi_4),$$
$$\phi = \phi_1 + \phi_3 - (\phi_2 + \phi_4),$$
$$\zeta = \phi_1 + \phi_2 - (\phi_3 + \phi_4),$$

show that the Hamiltonian becomes

$$H = 4E_C^\theta(n_\theta - n_g^\theta)^2 + 4E_C^\phi(n_\phi - n_g^\phi)^2 - E_J\cos(\theta)\cos(\phi + \phi_e) + \frac{1}{2}E_L\phi^2 + H_\zeta.$$

What are $E_C^{\theta/\phi}$, E_L and H_ζ?

(c) Given that C_J is small and C is large, which modes are “heavy” and which are “light”?

(d) Plot the potential for smallish E_L for zero and half flux bias. Do you think that the qubit states will be qualitatively different in the two cases?

References

[1] Philip Krantz, Morten Kjaergaard, Fei Yan, Terry P. Orlando, Simon Gustavsson, and William D. Oliver. A Quantum Engineer’s Guide to Superconducting Qubits. *Applied Physics Reviews*, 6(2):021318, June 2019. arXiv: 1904.06560.

[2] Uri Vool and Michel Devoret. Introduction to quantum electromagnetic circuits. *International Journal of Circuit Theory and Applications*, 45(7):897–934, jun 2017.

[3] Jens Koch, Terri M. Yu, Jay Gambetta, A. A. Houck, D. I. Schuster, J. Majer, Alexandre Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf. Charge-insensitive qubit design derived from the Cooper pair box. *Physical Review A*, 76(4):042319, October 2007.