1.3

22. c) $f(x) = \left(\frac{x+1}{x+2}\right)$

let x be 1. $f(1) = \left(\frac{(1)+1}{(1)+2}\right) \neq 1$ $\therefore$ R cannot be a bijection of R

34. Yes, g is one-to-one

Given $g: A \to B$

$f: B \to C$

Assume $g(a) = g(b)$

$f(g(a)) = f(g(b))$

$(f \circ g)(a) = (f \circ g)(b)$

$a = b$

40. a) $f(S \cup T) = f(S) \cup f(T)$

Given $f: A \to B$

$S \subseteq A$, $T \subseteq A$

Let $x \in f(S \cup T)$

There exists an element $y \in S \cup T$ such that $f(y) = x$

$y \in S \vee y \in T$

$f(y) \in f(S) \vee f(y) \in f(T)$

Since $f(y) = x$

$x \in f(S) \vee x \in f(T)$

Definition of Union: $x \in f(S) \cup f(T)$

Definition of Subset: $f(S \cup T) \subseteq f(S) \cup f(T)$

Let $x \in f(S) \cup f(T)$

Definition of union $x \in f(S) \vee x \in f(T)$

Then there exists an element y such that $f(y) = x$ $y \in S \vee y \in T$

Definition of union: $y \in S \cup T$

Since $f(y) = x$, $x \in f(S \cup T)$

Definition of subset: $f(S) \cup f(T) \subseteq f(S \cup T)$

Since $f(S) \cup f(T) \subseteq f(S \cup T)$ and $f(S \cup T) \subseteq f(S) \cup f(T)$, the two sets have to be equal.

72. Given: $f : A \to B$ and $|A| = |B|$

Because f is injective, $|A| = |f(A)|$

$f(A) \subseteq B$ so, $|B| = |A| = |f(A)| \leq |B|$

Thus, f is onto.

74. c) $\lceil \lceil x/2 \rceil / 2 \rceil = \lceil x/4 \rceil$

The ceiling function yields the smallest integer $\geq$ the value inserted. Since it is applied to $x/2$, we get the immediate integer greater than $x/2$, which is again divided by 2 and given as input to the ceiling function.

d) $\lfloor \sqrt{\lceil x \rceil} \rfloor \neq \lfloor \sqrt{x} \rfloor$

In the left hand side, since the ceiling function is applied to the value x, of which we take the square root, it will not equal to the right hand side. eg $\lfloor \sqrt{\lceil 37 \rceil} \rfloor = 6 \neq 5$...

2.5. 4. c) Countably infinite

$f : \mathbb{Z}^+ \to A$,

$1 + \frac{1^{a-1}}{10^a}$

could be $\frac{10}{9} \approx 1.\dot{1}\dot{1}$, $\frac{100}{9} \approx 11.\dot{1}\dot{1}$ etc.

d) The set is uncountable, because it is not possible to list all real numbers with decimal representations of all 1s and 9s.

35. $Z^+ \times Z^+$ is countable.

$f: Z^+ \times Z^+ \to Z^+$, $f(m,n) = 2^m \cdot 3^n$

Check if f is one-to-one: If $f(a,b) = f(m,n)$

$$2^a \cdot 3^b = 2^m \cdot 3^n$$

$$\therefore 2^a = 2^m$$

$$3^b = 3^n$$

Power rule:

$$a = m, \quad b = n$$

By the definition of one-to-one, we proved that f is a one-to-one function

$$|Z^+ \times Z^+| \leq |Z^+|$$

A set A is countable if and only if $|A| \leq |Z^+|$ Thus, $Z^+ \times Z^+$ is countable

36. There is a bijection between the set $(0,1)$ and $P(Z^+)$.

$|P(Z^+)| > |Z^+|$, there is a bijection between $(0,1)$ and R

Hence, $\aleph_0 = |Z^+| < |P(Z^+)| = |R|$

38. Given any function g from the positive integer to F.

F is the function from the positive integers to the given set. We can define a function 'h' in F by defining:

$h(n) = 4$, if $g(n)$ $n \neq 4$

$h(n) = 3$, if $g(n)$ $n = 4$

Then h cannot be in the range of 'g' since for all m, h of $g(m)$ differ on the element m. So, g is not subjective. There are no function from the positive integers onto F. Therefore F is uncountable.

3.1.

2. a) Characteristics: Input, Definiteness

Lacking: Output, correctness, Finiteness, Effectiveness, Generality.

b) Characteristics: Input, Definiteness, Finiteness

Lacking: Output, Correctness, Effectiveness, Generality.

c) Characteristics: Input, Definiteness, finiteness

Lacking: Output, correctness, Effectiveness, Generality.

d) Characteristics: Input, Finiteness, Generality

Lacking: Definiteness, Correctness, Effectiveness

4. Procedure maxdiff ($a_1, a_2 \ldots a_n$: integers with $n \geq 2$)

```
        maxdiff := a_2 - a_1
        for i := 3 to n
            diff := a_i - a_{i-1}
            if (maxdiff < diff)
                then maxdiff := diff
        return maxdiff
```