Discrete Mathematics, Spring & Summer 2020

Quiz 1 (86 pts)

Student ID. ________________ Name: ________________ Score: ________________

1. (36 points) Determine whether the following statements are true or false. If it is true write a √ otherwise a × in the blank before the statement.

( ) (1) The proposition "1+1=3 if and only if 2+2=3" is True.

( ) (2) The proposition $((p \to q) \wedge \neg p) \to \neg q$ is a tautology.

( ) (3) The following system specifications are consistent.
Whenever the system software is being upgraded, users cannot access the file system.
If users can access the file system, then they can save new files.
If users cannot save new files, then the system software is not being upgraded.

( ) (4) $\forall x A(x) \vee \forall x B(x) \equiv \forall x(A(x) \vee B(x))$, where all variables are in the same domain .

( ) (5) If $P(A) \in P(B)$, then $A \in B$. ($P(S)$ is the power set of S.)

( ) (6) $A \oplus A = A$ ($\oplus$ is the symmetric difference of two sets.)

( ) (7) Let f be a function from the set A to the set B, let S and T be subsets of A, then

$$f(S \cap T) \subseteq f(S) \cap f(T)$$

( ) (8) Let N be the set of natural numbers. Then $|N|=|N\times N|$.

( ) (9) The set of real numbers that are solutions of quadratic equations $ax^2 + bx + c = 0$, where a, b, and c are integers, is countable.

( ) (10) Suppose g: R → R where $\mathrm{g(x)} = \left\lfloor \frac{x-1}{2} \right\rfloor$, g is a one-to-one function (injection).

( ) (11) Let $F=\{f \mid f: N \to \{0,1\}\}$,Where N is the set of natural numbers, then F is uncountable.

( ) (12) $\sum_{j=1}^{n} (j^3 + j)$ is $O(n^4)$.

2. (5 points)

Find a proposition using only p, q, $\neg$, and the connective $\vee$ that has this truth table.

p	q	?
T	T	F
T	F	T
F	T	T
F	F	F

3. (5 points)

Let ***F*(*A*)** be the predicate "***A* is a finite set**" and ***S*(*A*,*B*)** be the predicate "***A* is contained in *B***". Suppose the domain consists of all sets. Translate the statement "***Every subset of a finite set is finite.***" into symbols.

4. (5 points)

Consider the following argument:

She is a Math Major or a Computer Science Major.
If she does not know discrete math, she is not a Math Major.
If she knows discrete math, she is smart.
She is not a Computer Science Major.
Therefore, she is smart.

Let ***p***: "She is a Math Major;" ***q***: "She is a Computer Science Major;" ***r***: "She knows discrete math;" ***s***: "She is smart."

1) Write the formal form of the above argument.
2) Determine whether this argument is valid and justify your answer.

5.(5 points)

Give the full DNF of the following statement:

$$(p \to (q \wedge r)) \wedge (\neg p \to (\neg q \wedge \neg r))$$

6.(5 points)

Suppose g: $A \to B$ and $f: B \to C$ where $A = \{1,2,3,4\}$, $B = \{a,b,c\}$, $C = \{2,7,10\}$, and f and g are defined by $g = \{(1,b),(2,a),(3,a),(4,b)\}$ and $f = \{(a,10),(b,7),(c,2)\}$. Find $f \circ g$.

7.(16 points)

Determine whether the following set is countable or uncountable and **show your proof.**

a) The set $Z^+ \times Z^+$

b)The set of functions from the positive integers to the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

8.(6 points)

In the questions below find the best big-O function for the function. Choose your answer among the following:

$$1,\quad \log_2 n,\quad n,\quad n\log_2 n,\quad n^2,\quad n^3,\ldots,\quad 2^n,\quad n!.$$

(a) $\dfrac{3-2n^4-4n}{2n^3-3n}$

(b) $\lceil n+2 \rceil \cdot \lceil n/3 \rceil$

9.(3 points)

Arrange the functions $(1.5)^n, n^{100}, (logn)^3, \sqrt{n}logn, 10^n, (n!)^2, and\ n^{99}+n^{98}$ in a list so that each function is big-O of the next function.