10.2 5. From theorem 2: An undirected graph has an even number of vertices of odd degree, the number of vertices of odd degree must be even. Therefore a graph with 15 vertices of odd degree 5 cannot exist.

24. The graph is bipartite

25. Not - bipartite

42 b) 6, 5, 4, 3, 2, 1 - Not graphic

f) 1, 1, 1, 1, 1, 1 - Graphic

h) 5, 5, 4, 3, 2, 1 - Not graphic

53. a) K_n For all $n \geq 1$

b) C_n For all $n \geq 3$

c) W_n For $n = 3$

d) Q_n For $n \geq 0$

60. $G = 15$

$\bar{G} = 13$

$$\frac{n(n-1)}{2} = G \cup \bar{G}$$

$$\frac{n(n-1)}{2} = 15 + 13$$

$$n(n-1) = 2(28)$$

$$n^2 - n - 56 = 0$$

$$(n-8)(n+7) = 0$$

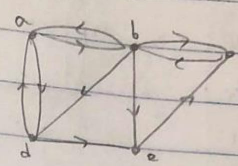
$$\underline{n=8} \text{ or } n = -7$$

$$n = 8$$

$\therefore$ G has 8 vertices.

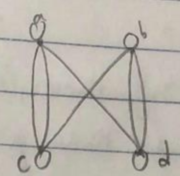
10.3 8.

$$\begin{array}{c|c} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

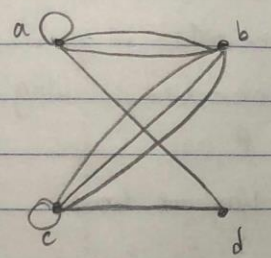


15.

$$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix} \end{array}$$



17.



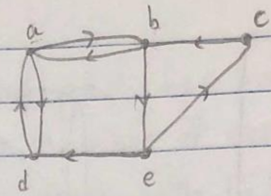
$$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

34. Isomorphic, One isomorphism is $f(u_1)=v_1, f(u_2)=v_2, f(u_3)=v_4, f(u_4)=v_5, f(u_5)=v_3$

35. Isomorphic, One isomorphism is $f(u_1)=v_1, f(u_2)=v_3, f(u_3)=v_5, f(u_4)=v_2, f(u_5)=v_4$

36. Not isomorphic, The second has a vertex of degree 4, whereas the first doesn't.

37. Isomorphic, Since 7-cycle $= f(u_1)=v_1, f(u_2)=v_3, f(u_3)=v_5, f(u_4)=v_7, f(u_5)=v_2, f(u_6)=v_4, f(u_7)=v_7$



10.4 22 e) length = 6.

Number of path = 5.

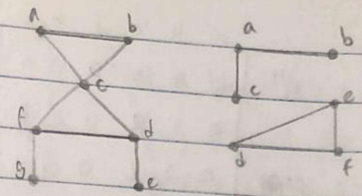
{a, b, a, b, a, b, e}, {a, b, a, d, a, b, e}, {a, d, a, b, a, b, e}
{a, d, a, d, a, b, e}, {a, b, e, d, a, b, e}

28. By induction

$n=1$, assume inductive hypothesis, let G be a connected graph with $n+1$ vertices and fewer than n edges, where $n \geq 1$.
Since the sum of the degrees of the vertices of G is equal to $2n$, sum of degree is less than $2n$, less than $2(n+1)$
Therefore some vertex has degree less than 2. Since G is connected, this vertex is not isolated, degree = 1.
Remove the vertex and edges, G is still connected, having n vertices and fewer than $n-1$ edges.
∴ Every connected graph with n vertices has at least $n-1$ edges.

29. R is reflexive by definition.
Assume that $(v,u) \in R$; then there is a path from u to v.
Then $(v,u) \in R$ because there is a path from v to u
Assume that $(u,v) \in R$ and $(v,w) \in R$; then there is a path from u to v and v to w.
Therefore there is a path from u to w. Hence $(u,w) \in R$
∴ R is transitive

Q2. Graph G1



Graph G2

Graph G1 =

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \end{array} \begin{array}{c} \begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A^2 = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \end{array} \begin{array}{c} \begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 4 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array} \qquad A^3 = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \end{array} \begin{array}{c} \begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\ \begin{bmatrix} 2 & 3 & 5 & 2 & 1 & 2 & 1 \\ 3 & 2 & 5 & 2 & 1 & 2 & 1 \\ 5 & 5 & 4 & 6 & 1 & 6 & 1 \\ 2 & 2 & 6 & 2 & 3 & 5 & 1 \\ 1 & 1 & 1 & 3 & 0 & 1 & 1 \\ 2 & 2 & 6 & 5 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 & 3 & 0 \end{bmatrix} \end{array}$$

There is a path of length 3 between every pair of distinct vertices.
Therefore graph G1 is connected.

$$G2: \quad A = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \qquad A^2 = \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 & 2 \end{bmatrix} \end{array}$$

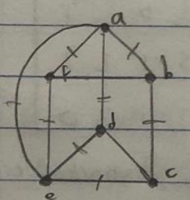
$$A^3 = \begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline a & & & & & & \\ \end{array}$$

$$A^3 = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 2 & 2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 3 & 3 \\ 0 & 0 & 0 & 3 & 2 & 3 \\ 0 & 0 & 0 & 3 & 3 & 2 \end{bmatrix} \end{array} \qquad A^4 = \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 5 & 5 \\ 0 & 0 & 0 & 5 & 6 & 5 \\ 0 & 0 & 0 & 5 & 5 & 6 \end{bmatrix} \end{array}$$

$$A^5 = \begin{bmatrix} 0 & 4 & 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 11 & 11 \\ 0 & 0 & 0 & 11 & 10 & 11 \\ 0 & 0 & 0 & 11 & 11 & 10 \end{bmatrix} \qquad A + A^2 + A^3 + A^4 + A^5 = \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 6 & 7 & 7 & 0 & 0 & 0 \\ 7 & 3 & 3 & 0 & 0 & 0 \\ 7 & 3 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 21 & 21 \\ 0 & 0 & 0 & 21 & 20 & 21 \\ 0 & 0 & 0 & 21 & 21 & 20 \end{bmatrix} \end{array}$$

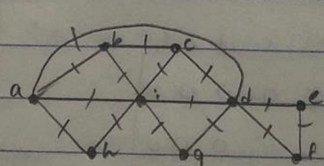
There is no path from a to d. The graph is not connected.

10.5 4.



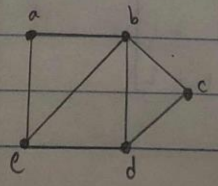
Not an Euler circuit because deg(c) = 3, odd.
Euler path = f, a, b, c, e, d, a, e, f, b.

6.

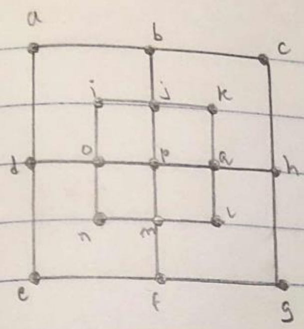


Not an Eular circuit because deg(b) = 3, odd.
Euler path = b, c, d, f, e, d, i, a, d, g, i, b, a, h, i, c

31.

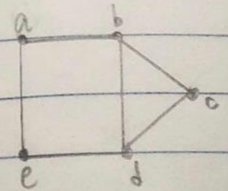


Hamilton circuit - a, b, c, d, e, a.

34. 

Not a hamilton circuit

Any circuit containing all vertices and containing b, c, h, g, f, d, a, b will pass through b more than once.

38. 

Hamilton path - a, b, c, d, e

41. Graph 34.

No hamilton path. There are eight vertices of degree 2, and only 2 of them can be end vertices of a path. For each of the other six, their two incident edges must be in a path. Exactly one of the inside corner vertices must be an end, and that is impossible.