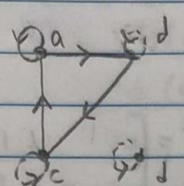
9.4 2. let R $\{(a,b) \mid a \neq b\}$

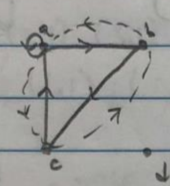
Reflexive closure of $R = \mathbb{Z} \times \mathbb{Z}$

6.



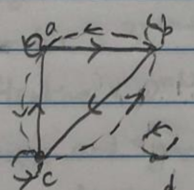
Reflexive closure

9. 6.



Symmetric closure

11. 6.



Reflexive & symmetric

20. let R (a,b)

a) $R^2 = R \circ R$

= There is an airline flight from a to b with exactly 1 stop in some intermediate city.

b) R^3 = There is an airline flight from a to b with exactly 2 stops in some intermediate cities

c) R^* = It is possible to fly from a to b.

Date ____________

28. a) Transitive closures

$\{(a,c),(b,d),(c,a),(d,b),(e,d)\}$

$$M = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$M = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

29. $\{(1,2),(1,4),(3,3),(4,1)\}$

a) Reflexive & transitive

$\{(1,1),(1,2),(1,4),(2,2),(3,3),(4,1),(4,2),(4,4)\}$

b) Symmetric & transitive

$\{(1,1),(1,2),(1,4),(2,1),(2,2),(2,4),(3,3),(4,1),(4,2),(4,4)\}$

c) Reflexive, Symmetric & transitive

$\{(1,1),(1,2),(1,4),(2,1),(2,2),(2,4),(3,3),(4,1),(4,2),(4,4)\}$

Date

9.5 3. a) Equivalence relation

b) Not transitive

c) Not reflexive, not symmetric, not transitive

d) Equivalence relation

e) Not reflexive, not transitive

10. Fix an arbitary representative for each of the equivalence classes of A under R, let us define a function f as $f: A \to A \ni f(x) = x_0$, where x_0 represents $[x]_R \ \forall x \in A$

16. $R = \{((a,b),(c,d)) \mid ad = cb\}$

A = Set of ordered pairs of positive integers

Reflexive

let $(a,b) \in A$

Since $ab = ba$ (commutative property of multiplication)

$((a,b),(a,b)) \in R$, R is reflexive.

Symmetric

let $((a,b),(c,d)) \in R$

$ad = bc$

$da = cb$ (commutative property of multiplication)

$cb = da$

therefore $((c,d),(a,b)) \in R$, R is symmetric

Transitive

let $((a,b),(c,d)) \in R$ and $((c,d),(e,f)) \in R$

$ad = bc$, $cf = de$

$a = \frac{bc}{d}$, $f = \frac{de}{c}$

$af = \frac{bc}{d} \cdot \frac{de}{c} = be$

Date

$af = be$ implies that

$((a,b),(e,f)) \in R$, R is transitive

R is reflexive, symmetric and transitive, therefore R is an equivalence relation

36. b) Congruence class $[4]_m$ when $m=3$

congruence class = 1

39. a) $[(1,2)] = \{(a,b) \mid a-b=-1\}$

$= \{(1,2),(3,4),(5,6),\cdots\}$

b) Each equivalence class can be interpreted as an integer

41. a) Not a partition

b) Partition

c) Partition

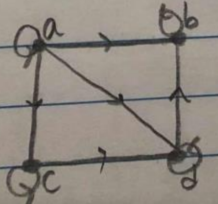
d) Not a partition

9.6 5. a) $(Z,=)$ Poset

b) $(Z,\neq)$ Not a poset

c) $(Z,\geq)$ Poset

d) $(Z,\not\mid)$ Not a poset

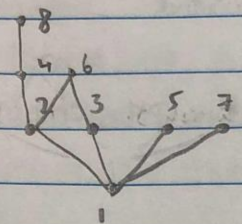
10. 

Reflexive, antisymmetric, not transitive

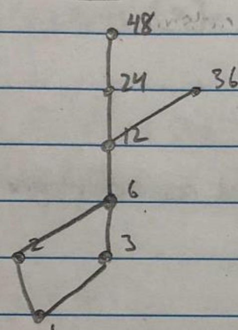
$\therefore$ Not a partial ordering.

Date

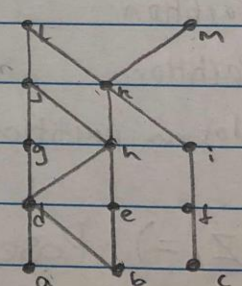
23. a) {1, 2, 3, 4, 5, 6, 7, 8}



c) {1, 2, 3, 6, 12, 24, 36, 48}



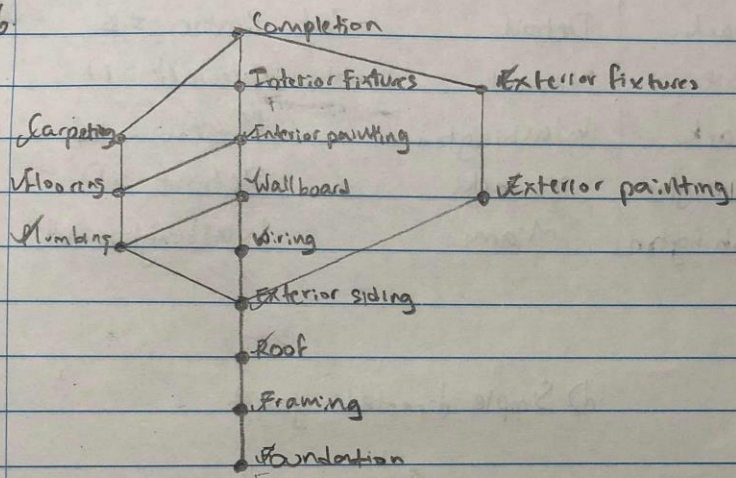
32. a) Max: l, m
b) Min: a, b, c
c) Greatest element: None
d) least element: None
e) Upper bounds {a, b, c} = l, M, k
f) lower bounds {a, b, c} = k
g) lower bounds {f, g, h} = None
h) Greatest lower bounds {f, g, h} = None



Date

44. a) $(\{1,3,6,9,12\}, |)$ Not a lattice
b) $(\{1,5,25,125\}, |)$ Lattice
c) $(\mathbb{Z}, \geq)$ Latice
d) $(P(S), \supseteq)$, where $P(S)$ is the power set of a set S. Latice

66.

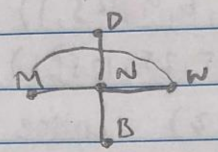


Foundation < Framing < Roof < Exterior siding < Exterior painting < Exterior fixtures < Wiring < Plumbing < Wallboard < Flooring < Interior Painting < Interior Fixtures < Carpeting < Completion

Date

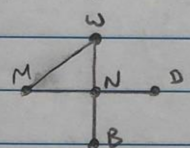
10.1 1.

Nº of flights	From	To
4	Boston	Newark
2	Newark	Boston
3	Newark	Miami
2	Miami	Newark
1	Newark	Detroit
2	Detroit	Newark
3	Newark	Washington
2	Washington	Newark
1	Washington	Miami

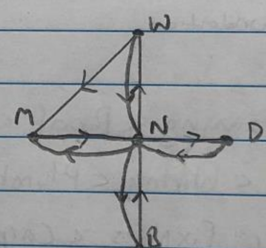


Let Boston = B
Newark = N
Miami = M
Detroit = D
Washington = W

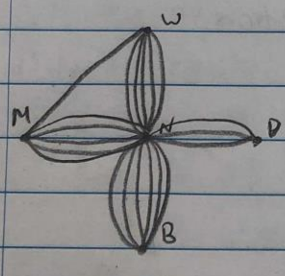
a) Simple graph



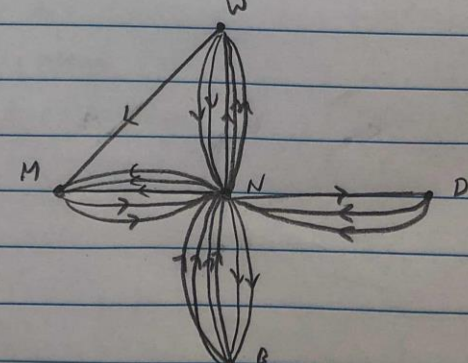
d) Simple directed graph



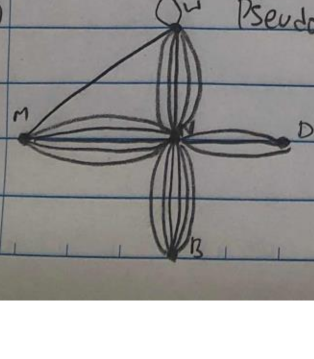
b) Multigraph



e) Directed multigraph



c) Pseudograph



Date

	Edges	Multiple Edges	loops.	Type of graph.
3.	Undirected	None	None	Simple
4.	Undirected	Yes	None	Multigraph
5.	Undirected	Yes	Yes	Pseudograph
6.	Undirected	Yes	None	Multigraph
7.	Directed	None	Yes	Directed graph
8.	Directed	Yes	Yes	Directed Multigraph
9.	Directed	Yes	Yes	Directed Multigraph