1.6

12.

1.	$(p \wedge t) \to (r \vee s)$	Premise
~~2.~~	$q \to (u \wedge t)$	Premise
~~3.~~	$u \to p$	Premise
4.	$\neg s$	Premise
~~5.~~	q	Premise
~~6.~~	$u \wedge t$	Modus ponens 2,5
~~7.~~	u	Simplification 6.
8.	p	Modus ponens 3, 7
9.	t	Simplification 6.
10.	$p \wedge t$	Conjunction
11.	$r \vee s$	Modus ponens 1,10
12.	$r.$	Disjunctive syllogism 4, 11

11.

p_1	p_1
p_2	p_2
...	...
p_n	p_n
	q
$\therefore q \to r$	r

14. d) Modus ponens.

18. $\exists x S(x,x)$ cannot be true because x cannot be shorter than himself.

24. 3. $P(c)$
5. $Q(c)$ } Cannot be simplification because 2. $P(c) \vee Q(c)$ is a disjunction ($\vee$)

29.

~~1.~~	$\forall x(P(x) \vee Q(x))$	Premise
~~2.~~	$\forall x(\neg Q(x) \vee S(x))$	Premise
3.	$\forall x\,(R(x) \to \neg S(x))$	Premise
~~4.~~	$\exists x\, \neg P(x)$	Premise

5. $\neg P(c)$	Existential instantiation 4.
6. $P(c) \vee Q(c)$	Universal instantiation 1.
7. $Q(c)$	Disjunctive syllogism 5. 6.
8. $\neg Q(c) \vee S(c)$	Universal instantiation 2.
9. $S(c)$	Disjunctive syllogism 7. 8.
10. $R(c) \to \neg S(c)$	Universal instantiation 3.
11. $\neg R(c) \vee \neg S(c)$	Logical equivalence 10.
12. $\neg R(c)$	Disjunctive syllogism 11.
13. $\exists x \neg R(x)$	Existential generalization 12

34 a)

1. $p \vee \neg q$	Premise	p: Logic is difficult
2. $r \to \neg p$	Premise	q: Students like logic
3. $\neg r \vee \neg p$	Logical equivalence 2.	r: Mathematics is easy
4. $\neg(r \wedge p)$	De Morgan's law 3	
5. $\neg p \vee \neg r$	Commutative law 3.	a). $q \to \neg r$
6. $p \to \neg r$	Logical equivalence 5.	
7. $\neg q \vee \neg r$	Resolution 1, 3	
8. $q \to \neg r$	Logical equivalence 7.	

1.7

7. Every odd integer is the difference of two squares.

If x is an odd number $x = 2y+1$

If x is an even number $x = 2y$

$x = 2y+1$

$x = 2y+1+(y^2-y^2)$ from $(a+b)^2 = a^2+b^2+2ab$

$x = (y+1)^2 - y^2$

$\therefore$ x is the difference of the square of two integers

8. If n is a perfect square, then $n+2$ is not a perfect square.

let x be a perfect square so as $x = y^2$

$\therefore n = y^2$ — ①

let $n+2$ be a perfect square

$n+2 = z^2$ — ②

Replace ① in ②

$y^2+2 = z^2$

$2 = z^2 - y^2$ from $a^2-b^2 = (a-b)(a+b)$

$2 = (z-y)(z+y)$

$z - y = 1$ — ③

$z+y = 2$ — ④

Add ③ and ④ $2z = 3$

$z = 3/2$

z is supposed to be an integer if it was a perfect square. Here, z is not an integer therfore the assumption that $n+2$ is a perfect square is incorrect.

34. 1. $\sqrt{2x^2-1} = x$

2. $2x^2 - 1 = x^2$

3. $x^2 - 1 = 0$

4. $(x-1)(x+1) = 0$

5. $x = 1$ or $x = -1$

Correct for $x=1$ but not for $x=-1$ because

$$\sqrt{2(-1)^2 - 1} \neq -1$$

$$\sqrt{2-1} \neq -1$$

$$\sqrt{1} \neq -1$$

1.8. 22. $\boxed{x^2 + \frac{1}{x^2} \geq 2}$ $\left(x - \frac{1}{2}\right)^2 \geq 0$

$$\left(x - \frac{1}{x}\right)^2 \geq 0$$

$$x^2 - 2 + \frac{1}{x^2} \geq 0$$

$$x^2 + \frac{1}{x^2} \geq 2$$

24. $\sqrt{\frac{(x^2+y^2)}{2}} \geq \frac{x+y}{2}$

Quadratic Arithmatic	$\cancel{4}^{2}(x^2+y^2) \geq \cancel{2}^{1}(x^2+2xy+y^2)$
	$2x^2 + 2y^2 \geq x^2 + 2xy + y^2$
$\frac{x^2+y^2}{2} \geq \left(\frac{x+y}{2}\right)^2$	$x^2 + y^2 - 2xy \geq 0$
$\frac{x^2+y^2}{2} \geq \frac{x^2+2xy+y^2}{4}$	$(x-y)^2 \geq 0$

2.1

11. a) $x \in \{x\}$ | True
b) $\{x\} \subseteq \{x\}$ | True
c) $\{x\} \in \{x\}$ | False
d) $\{x\} \in \{\{x\}\}$ | True
e) $\emptyset \subseteq \{x\}$ | True
f) $\emptyset \in \{x\}$ | False.

18. $A \in B$ & $A \subseteq B$

$A = \{\}$
$B = \{\emptyset, \{1\}\}$

22. If A and B are two sets with same power set

$\therefore A = B$.

Yes.

24. a) $\emptyset$ | No. Set cannot be empty
b) $\{\emptyset, \{a\}\}$ | Yes $\emptyset \subseteq \{a\}$
c) $\{\emptyset, \{a\}, \{\emptyset, a\}\}$ | No Subset of $\{\emptyset, a\} = \emptyset, \{\emptyset\}, \{a\}$ $\emptyset$ missing.
d) $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ | Yes.

32. $A = \{a, b, c\}$ $B = \{x, y\}$ $C = \{0, 1\}$

a) $A \times B \times C$ =

$\{(a, x, 0)(a, x, 1)(a, y, 0)(a, y, 1), (b, x, 0), (b, x, 1), (b, y, 0), (b, y, 1), (c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)\}$

c) $C \times A \times B$

$\{(0, a, x), (0, a, y), (0, b, x), (0, b, y), (0, c, x), (0, c, y), (1, a, x), (1, a, y), (1, b, x), (1, b, y), (1, c, x)(1, c, y)\}$