Discrete Mathematics, Spring & Summer 2020
Midterm Exam

Name: ___________________ Student ID: ______________

I Short answer problems (45pts)
(*You do not need to give any reason or justification for your answer.*)

1(2pts). Find a proposition with three variables p, q, and r that is true when p and r are true and q is false, and false otherwise.

2(3pts). In the question below suppose the variables x and y represent real numbers, and
$L(x,y) : x < y$ $\quad G(x) : x > 0$ $\quad P(x)$: x is a prime number.
Write the following statement using these predicates and any needed quantifiers
"No matter what positive number is chosen, there is a larger prime."

3(2pts). Determine whether f is a function from the set of all bit strings to the set of integers if $f(S)$ is the position of a 1 bit in the bit string S.

4(3pts). Determine whether the set of all finite bit strings is countable

5(2pts). Give a big-O estimate for the following function. For the function g in your estimate $f(x)$ is $O(g(x))$, use a simple function of smallest order.
$f(x) = (n^3+n^2\log n)(\log n+1) + (17\log n+19)(n^3+2)$

6(2pts). Give a recursive definition with initial condition(s) of the set S
$\{0.1, 0.01, 0.001, 0.0001, \ldots\}$

7(3pts). How many different truth tables of compound propositions are there that involve the propositional variables p and q?

8(3pts). Suppose $|A| = 6$ and $|B| = 12$. Find the number of 1-1 functions $f: A \to B$.

9(3pts). Suppose $|A| = 12$ and $|B| = 6$. Find the number of 1-1 functions $f: A \to B$.

10(3pts). Find the next larger permutation in lexicographic order after 3254761

11(3pts). You have a pile of 20 identical blank cards. On each card you draw a circle, a plus, or a square. How many piles of 20 cards are possible?

12(3pts). Find the least number of cables required to connect eight computers to four printer to guarantee that for every choice of four of the eight computer, these four computers can directly access four different printers.

13(9pts). You have 20 cards and 12 envelopes (labeled 1, 2, …, 12). In how many ways can you put the 20 cards into the envelopes if
(a) the cards are distinct.
(b) the cards are identical.
(c) the cards are identical and no envelope can be left empty.

14(4pts). Find the next four largest 4-combinations of the set {1,2,3,4,5,6,7,8} after {1,2,3,5}

II Long answer problems (55pts)

(You need to give a complete explanation. WRITE CLEAR.)

1(8pts). Determine whether the following argument is valid and explain why.

> If you are not in the tennis tournament, you will not meet Ed.
> If you aren't in the tennis tournament or if you aren't in the play, you won't meet Kelly.
> You meet Kelly or you don't meet Ed.
> It is false that you are in the tennis tournament and in the play.
> Therefore, you are in the tennis tournament.

2(5pts). Give the prenex DNF and CNF of the following statements:

$$((\exists x)P(x) \vee (\exists x)Q(x)) \rightarrow (\exists x)(P(x) \vee Q(x))$$

3(8pts). Show that for every integer n, there is a multiple of n whose decimal expansion contains only 0s and 1s.

4(8pts). If f and $f \circ g$ are onto, does it follow that g is onto? Justify your answer.

5(8pts). Use the Principle of Mathematical Induction to prove that any integer amount of postage from 18 cents on up can be made from an infinite supply of 4-cent and 7-cent stamps

6(8pts). A game consisting of flipping a coin ends when the player gets two heads in a row, two tails in a row, or flips the coin four times. Draw a tree diagram to show the number of ways in which the game can end.

7(10pts).Show that in a group of ten people (where any two people are either friends or enemies) there are either three mutual friends or four mutual enemies.