Discrete Mathematics, Spring & Summer 2020

Quiz 2 (100pts)

Name: ____________________ Student ID: _______________

1.(10pts) Describe the following sequences recursively. Include **initial conditions** and assume that the sequences begin with $\boldsymbol{a_1}$.

(a) a_n = the number of bit strings of length n that contain the string 01.

(b) a_n = the number of strings of letters of the ordinary alphabet that do not have adjacent vowels.

2.(5pts) What form does a particular solution of the linear nonhomogeneous recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + F(n)$ have when $F(n) = (n^2 + 1)2^n$?

3. (10pts) Find a closed form for the generating function for each of these sequences:
(a) 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, ….

(b) $a_n = C(10, n+1)$, for $n = 0, 1, 2, \ldots$

4.(5pts) If $G(x)$ is the generating function for $a_0, a_1, a_2, a_3, \ldots$, describe in terms of $G(x)$ the generating function for $a_0, 2a_1, 4a_2, 8a_3, 16a_4, \ldots$.

5. (5pts) What is the generating function for $\{\boldsymbol{a_k}\}$, where $\boldsymbol{a_k}$ is the number of solutions of $\boldsymbol{x}_1 + \boldsymbol{x}_2 + \boldsymbol{x}_3 + \boldsymbol{x}_4 = \boldsymbol{k}$ when $\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3$, and $\boldsymbol{x}_4$ are integers with $\boldsymbol{x}_1 \geqslant 3$, $1 \leqslant \boldsymbol{x}_2 \leqslant 5$, $0 \leqslant \boldsymbol{x}_3 \leqslant 4$, and $\boldsymbol{x}_4 \geqslant 1$?

6. (5pts) How many permutations of all 26 letters of the alphabet are there that contain at least one of the words: SWORD, PLANT, CARTS?

7. (5pts) How many ways can the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 be arranged so that no even digit is in its original position?

8. (8pts) In the questions below determine whether the binary relation is: (1) reflexive, (2) symmetric, (3) antisymmetric, (4) transitive.

a). The relation R on $\boldsymbol{Z}$ where aRb means $a \neq b$.

b). The relation R on the set $\{(a,b) \mid a,b \in \boldsymbol{Z}\}$ where $(a,b)R(c,d)$ means $a \leqslant c$ or $b = \leqslant d$.

9.(4pts) Suppose $|A| = 4$. Among all binary relations on A

a) How many anti-symmetric relations?

b) How many equivalence relations?

10. (3pts) In the questions below suppose R and S are relations on $\{a,b,c,d\}$, where $R = \{(a,b),(a,d),(b,c),(c,c),(d,a)\}$ and $S = \{(a,c),(b,d),(d,a)\}$.

Construct $R \circ S$.

11. (4pts) In the questions below find the matrix that represents the given relation. Use elements in the order given to determine rows and columns of the matrix.

a). R^{-1}, where R is the relation on $\{1,2,3,4\}$ such that aRb means $| a - b | \leq 1$.

b). $\overline{R}$, where R is the relation on $\{w,x,y,z\}$ such that $R =$

$\{(w,w),(w,x),(x,w),(x,x),(x,z),(y,y),(z,y),(z,z)\}$.

12 (4pts). Find the transitive closure of R if $\mathbf{M}_R$ is $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

13.(12pts, 2pts each) Suppose $A = \{2,3,6,9,10,12,14,18,20\}$ and R is the partial order relation defined on A where xRy means x is a divisor of y.

(a) Draw the Hasse diagram for R.

(b) Find all maximal elements.

(c) Find all minimal elements.

(d) Find lub($\{2,9\}$).

(f) Find glb($\{14,10\}$).

(g) Use a topological sort to order the elements of the poset (A, R).

14.（5pts） Let R be a relation that is reflexive and transitive. Prove that $R^n = R$ for all positive integers n.

15. (15 pts) Let a_n be the number of character strings of length n that contain odd number of As. Each character is one of the 26 upper-case English letters.

(1) Find a recurrence relation for a_n and give the necessary initial condition(s).

(2) Find an explicit formula for a_n by solving the recurrence relation in part (a) via its characteristic equation.

(3) Find an explicit formula for a_n by solving the recurrence relation in part (a) using generating functions.