6. c) $\neg \exists x N(x)$	There doesn't exist a student in your school who has visited North Dakota
d) $\exists x \neg N(x)$	There exists a student in your school who has not visited North Dakota
e) $\neg \forall x N(x)$	Not all students in your school have visited North Dakota
f) $\forall x \neg N(x)$	All Students in your school have not visited North Dakota

9. b) $\exists x (P(x) \wedge \neg Q(x))$

d) $\neg \exists x (P(x) \vee Q(x))$

20. e) $\exists x (\neg P(x)) \wedge \left[\forall x ((x<0) \rightarrow P(x))\right]$ $P(x)\ \{-5, -3, -1, 1, 3, 5\}$

$\neg P(-5) \vee \neg P(-3) \vee \neg P(-1)$ $P(x)$ true for $x<0$

$\vee \neg P(1) \vee \neg P(3) \vee \neg P(5)$ $P(-5) \wedge P(-3) \wedge P(-1)$

$$\left(\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)\right) \wedge \left(P(-5) \wedge P(-3) \wedge P(-1)\right)$$

24. b) $\exists x\ M(x)$ — $M(x)$ is "has seen a foreign movie"

$\exists x (M(x) \wedge S(x))$ — $S(x)$ is "x is somebody in your class"

d) ~~$\exists x$~~ $\forall x\ Q(x)$ — $S(x)$ is "x is a student in your class"

$\forall x (S(x) \rightarrow Q(x))$ — $Q(x)$ is "x can solve quadratic eqn"

40. b) $\forall x R(z) \rightarrow [\forall x (\neg p(x)) \wedge \forall x (\neg q(y))]$

let. $p(x)$: Directory x in file system can be opened

$q(y)$: file y can be closed

$R(z)$: System error z have been detected.

44. $\forall x\,(P(x) \leftrightarrow Q(x))$ $\qquad$ $\forall x\, P(x) \leftrightarrow \forall x\, Q(x)$

Not logically equivalent

Let $P(x)$ be either T or F, let $Q(x)$ be F. Then $\forall x\,(P(x) \leftrightarrow Q(x))$ is false while $\cancel{\forall x\, P(x) \leftrightarrow \forall x\, Q(x)}$ $\forall x\, P(x) \leftrightarrow \forall x\, Q(x)$ is true.

49. a) $\forall x\,(P(x) \rightarrow A) \equiv \exists x\, P(x) \rightarrow A$

$$\equiv \neg(\forall x\,(P(x) \rightarrow A))$$

$$\equiv \exists x \neg P(x) \vee A$$

$$\equiv \exists x\, P(x) \rightarrow A$$

60. a) $\forall x\,(P(x) \rightarrow Q(x))$

b) $\exists x\,(R(x) \wedge \neg Q(x))$

c) $\exists x\,(R(x) \wedge \neg P(x))$

d) Yes.

1.5.

6. e) There exist a student x and a student y such that for every class z, student x and student y are not the same student and if student x is enrolled in class z then student is also enrolled in class z.

f) There exist a student x and a student y such that for every class z, student x and student y are not the same student and student x is enrolled in class z if and only if student y is enrolled in class z.

12. d) $\neg \exists x\, C(x, \text{Bob})$

$I(x)$ "x has an Internet conn

$C(x,y)$ "x and y have chatted over the internet"

h) $\exists! x\, I(x)$

k) $\exists x [I(x) \rightarrow \forall y \neg C(x,y)]$

n) $\exists x \exists z [x \neq z \rightarrow \forall y (\neg C(x,y) \wedge \neg C(z,y))]$

14. c) $\exists x [C(x, \text{Alaska}) \wedge \neg C(x, \text{Hawaii})]$

d) $\forall x \exists y\, C(x,y)$

e) $\exists x \exists y\, C(x,y)$

f) $\exists x \exists y (x \neq y \wedge C(x,y) \wedge \forall z (F(x,z) \rightarrow (x = z \vee y = z)))$

24. a) $\exists x \forall y (x + y = y)$

There exists a real number x that when added to any real number y, the result is the value of the real number y.

d) $\forall x \forall y ((x \neq 0) \wedge (y \neq 0) \Leftrightarrow (xy \neq 0))$

For every two real numbers ~~x, y~~, the two real numbers are both non-zero if ~~and~~ and only if their product is non-zero.

32. d) $\neg[\forall y \exists x \exists z (T(x,y,z) \vee Q(x,y))]$

$\equiv \exists y \forall x \forall z (\neg T(x,y,z) \wedge \neg Q(x,y))]$

34. $\forall x \forall y ((x \neq y) \rightarrow \forall z ((z = x) \vee (z = y)))$

True : $\{0,1\}$

False: $\mathbb{R}$

38. b) $\exists x \neg C(x)$ where $C(x)$ means "x have seen a computer"

$\forall x \, C(x)$

All students in this class have seen a computer

d) $\exists x \exists y \forall z \, C(x,y,z)$ where $C(x,y,z)$ means "student x has been in y room of z building"

$\forall x \forall y \exists z \, C(x,y,z)$

All students in this class have not been in any room of at least one building on campus.

42. $a(b+c) = ab + ac$

$\forall a \forall b \forall c [a \cdot (b+c) = (a \cdot b) + (a \cdot c)]$