Date

9.1 7. a) $x \neq y$ | Symmetric

c) $x = y+1$ or $x = y-1$ | Symmetric

h) $x \geq y^2$ | Antisymmetric & transitive

26. $R = \{(a,b) \mid a \leq b\}$

a) $R^{-1} = \{(b,a) \mid (a,b) \in R\}$
$= \{(b,a) \mid a < b\}$
$= \{(a,b) \mid b < a\}$
$= \{(a,b) \mid a > b\}$

b) $\overline{R} = \{(a,b) \mid (a,b) \notin R\}$
$= \{(a,b) \mid a \nless b\}$
$= \{(a,b) \mid a \geq b\}$

32. Let R $\{(1,2),(1,3),(2,3),(2,4),(3,1)\}$
S $\{(2,1),(3,1),(3,2),(4,2)\}$
$S \circ R = \{(1,1),(1,2),(2,1),(2,2)\}$

47. a) Symmetric $= 2^{n(n+1)/2}$
b) Antisymmetric $= 2^n 3^{n(n-1)/2}$
c) Asymmetric $= 3^{n(n-1)/2}$
d) Irreflextive $= 2^{n(n-1)}$
e) Reflexive and symmetric $= 2^{n(n-1)/2}$
f) Neither reflexive nor irreflexive $= 2^{n^2} - 2 \cdot 2^{n(n-1)}$

51. Assume R is symmetric
Therefore $R^{-1} = \{(b,a) \mid (a,b) \in R\}$
$= \{(a,b) \mid (a,b) \in R\}$
$= R$

Thus $R = R^{-1}$

Date ______

Assume $R = R^{-1}$, let $(a,b) \in R$

Since $R^{-1} = \{(b,a) \mid (a,b) \in R\}$

$(b,a) \in R^{-1}$

Since $R = R^{-1}$

$(b,a) \in R$

$\therefore$ R is symmetric if and only if $R = R^{-1}$

9.3 13. $M_r = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

a) $R^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

b) $\bar{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

c) $R^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

14. $M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ $M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

a) $R_1 \cup R_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

b) $R_1 \cap R_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

c) $R_2 \circ R_1 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

d) $R_1 \circ R_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

e) $R_1 \oplus R_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Date

31. 23. Irreflexive

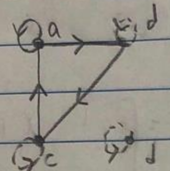
24. Reflexive, Antisymmetric, Transitive

25. Irreflexive, Antisymmetric

9.4 2. let R $\{(a,b) \mid a \neq b\}$

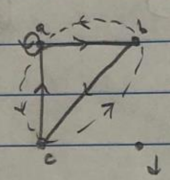
Reflexive closure of R = $\mathbb{Z} \times \mathbb{Z}$

6.



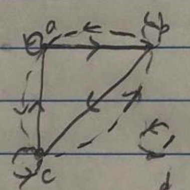
Reflexive closure

9. 6.



Symmetric closure

11. 6.



Reflexive & symmetric

20. let R (a,b)

a) $R^2 = R \circ R$

= There is an airline flight from a to b with exactly 1 stop in some intermediate city.

b) R^3 = There is an airline flight from a to b with exactly 2 stops in some intermediate cities

c) R^* = It is possible to fly from a to b.

Date

28. a) Transitive closures

$\{(a,c),(b,d),(c,a),(d,b),(e,d)\}$

$$M = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$M = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

29. $\{(1,2),(1,4),(3,3),(4,1)\}$

a) Reflexive & transitive

$\{(1,1),(1,2),(1,4),(2,2),(3,3),(4,1),(4,2),(4,4)\}$

b) Symmetric & transitive

$\{(1,1),(1,2),(1,4),(2,1),(2,2),(2,4),(3,3),(4,1),(4,2),(4,4)\}$

c) Reflexive, Symmetric & transitive

$\{(1,1),(1,2),(1,4),(2,1),(2,2),(2,4),(3,3),(4,1),(4,2),(4,4)\}$