Normal Forms.

1-a) $((P \to Q) \leftrightarrow (\neg Q \to \neg P)) \wedge R$

$\equiv ((\neg P \to Q) \leftrightarrow (\neg\neg Q \vee \neg P)) \wedge R$

$\equiv ((\neg P \vee Q) \leftrightarrow (Q \vee \neg P)) \wedge R$

$\equiv ((\neg P \vee Q) \leftrightarrow (\neg P \vee Q)) \wedge R$

$\equiv T \wedge R$

$\equiv R$

DNF = CNF = R

b) $P \vee (\neg P \vee (Q \wedge \neg Q))$

$\equiv P \vee (\neg P \vee F)$

$\equiv P \vee \neg P$

$\equiv T$

DNF = CNF = T

c) $(P \wedge (Q \wedge S)) \vee (\neg P \wedge (Q \wedge S))$

$\equiv (P \vee \neg P) \wedge (Q \wedge S)$

$\equiv T \wedge (Q \wedge S)$

$\equiv Q \wedge S$

DNF = CNF = $Q \wedge S$

2 a) DNF $= (P \land \neg Q) \lor (\neg P \land Q) \lor (P \land Q)$

b) DNF $= (P \land Q \land R) \lor (\neg P \land Q \land R) \lor (P \land \neg Q \land R) \lor (P \land Q \land \neg R) \lor$
$(\neg P \land \neg Q \land R) \lor (\neg P \land \neg Q \land \neg R)$

c) DNF $= P \land Q \land R$

3 a) $(\forall x)(P(x) \to (\exists y) Q(x, y))$
$= \forall x (\neg P(x) \lor \exists_y Q(x, y))$
$= \forall x \exists y (\neg P(x) \lor Q(x, y))$

b) $(\forall x)(\forall y)(((\exists z) P(x, y, z) \land (\exists u) Q$
$\equiv (\forall x)(\forall y)(\neg((\exists z) P(x, y, z) \land (\exists u) Q(x, u)) (\exists v) Q(y, v))$
$\equiv (\forall x)(\forall y)((\neg(\exists z) P(x, y, z) \lor \neg(\exists u)(Q(x, u) \lor (\exists v) Q(y, v))$
$\equiv (\forall x)(\forall y)(((\forall z) \neg P(x, y, z) \lor (\forall u) \neg Q(x, u)) \lor (\exists v) Q(y, v))$
$\equiv (\forall x)(\forall y)(\forall z)(\forall u)(\exists v)((\neg p(x, y, z) \lor \neg Q(x, u)) \lor Q(y, v))$

4. a) $((\exists x) P(x) \lor (\exists x) Q(x)) \to (\exists x)(P(x) \lor Q(x))$
$\equiv (\neg((\exists x) P(x) \lor (\exists x) Q(x)) \lor (\exists x)(P(x) \lor Q(x))$
$\equiv (\neg(\exists x) p(x) \land \neg(\exists x) Q(x)) \lor (\exists x)(P(x) \lor Q(x))$
$\equiv ((\forall x) \neg p(x) \land (\forall x) \neg Q(x)) \lor (\exists x)(P(x) \lor Q(z))$
$\equiv \forall y \forall u \exists z ((\neg p(y) \land \neg Q(u)) \lor (P(z) \lor Q(z))$
$\equiv \forall y \forall u \exists z ((P(z) \lor Q(z) \lor \neg P(y) \land p(z) \lor Q(z) \lor \neg Q(u))$

Prenax DNF $= \forall y \forall u \exists z ((\neg p(y) \land \neg Q(u)) \lor p(z) \lor Q(z))$
Prenax CNF $= \forall y \forall u \exists z ((p(z) \lor Q(z) \lor \neg p(y)) \land (p(z) \lor (Q(z) \lor \neg Q(u))$

1) $(\forall x)(\forall y)(P(x) \rightarrow Q(x,y)) \rightarrow ((\exists y)(P(y) \wedge (\exists z) Q(y,z)))$

2) $(\forall x)(\forall y)(\neg P(x) \vee Q(x,y)) \rightarrow ((\exists y) P(y) \wedge (\exists z) Q(y,z))$

3) $\neg(\forall x)(\forall y) \neg(\neg P(x) \vee Q(x,y)) \vee ((\exists y)(P(y) \wedge (\exists z) Q(y,z))$

4) $\exists x \exists y \neg(\neg P(x) \vee Q(x,y)) \vee ((\exists y)(P(y) \wedge (\exists z) Q(y,z))$

5) $\exists x \exists y (P(x) \wedge \neg Q(x,y)) \vee ((\exists y)(P(y) \wedge (\exists z) Q(y,z))$

6) $\exists x \exists u (P(x) \wedge \neg Q(x,u)) \vee ((\exists m) P(m) \wedge (\exists z) Q(m,z))$

7) $\exists x \exists u \exists m \exists z ((P(x) \wedge \neg Q(x,u)) \vee (P(m) \wedge Q(m,z)))$

8) $\exists x \exists u \exists m \exists z ((P(x) \vee P(m)) \wedge (P(x) \vee Q(m,z)) \wedge Q(x,u) \vee P(m)) \wedge (Q(x,u) \vee Q(m,z)))$

Prenex DNF = $\exists x \exists u \exists m \exists z ((P(x) \wedge Q(x,u)) \vee (P(m) \wedge Q(m,z)))$

Prenex CNF = $\exists x \exists u \exists m \exists z ((P(x) \vee P(m)) \wedge (P(x) \vee Q(m,z)) \wedge (Q(x,u) \vee P(m)) \wedge (Q(x,u) \vee Q(m,z)))$