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6.5 10. 6 types of croissants

a) A dozen croissants: $C(n+r-1, r)$, $n=6$, $r=12$

$= C(17, 12)$

$= \frac{17!}{12!(17-12)!}$

$= 6188$

b) 3 dozen croissants: $C(n+r-1, r)$, $n=6$, $r=36$

$= C(41, 36)$

$= \frac{41!}{36!(41-36)!}$

$= 749,398$

c) 2 dozen croissants with at least 2 of each kind.

$= C(n+r-1, r)$, $n=6$, $r=12$

$= C(17,12)$

$= 6188$

d) 2 dozen with no more than 2 broccoli

No broccoli	1 brocoli	2 broccoli
$n=5$, $r=24$	$n=5$, $r=23$	$n=5$, $r=22$
$C(28,24)=20475$	$C(27,23)=17,550$	$C(26,22)=14,950$

Total $= 52975$.

e) 2 dozen, at least 5 choc and at least 3 almond.

$n=6$ $r=24-(5+3)$

$=16$

$C(21,16) = 20,349$

$\boxed{C(n+r-1, r)}$

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f) 2 dozen, at least: 1 plain, 2 cherry, 3 choco, 1 almond, 2 apple } 9.

no more: 3 broccoli

$r = 24 - 9$

$r = 15$

No broccoli: $n=5$.	1 broccoli: $n=5$, $r=14$	2 broccoli: $n=5$, $r=13$	3 broccoli, $n=5$, $r=12$
$C(19, 15) = 3875$.	$C(18, 14) = 3060$	$C(17, 13) = 2380$	$C(16, 12) = 1820$

Total = 11,135.

16. $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29$.

a) $x_i \geq 1$ for $i = 1,2,3,4,5,6$

let $x'_i = x_i - 2$, $i = 1,2,3,4,5,6$

$$x'_1 + x'_2 + x'_3 + x'_4 + x'_5 + x'_6 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 - 2 \cdot 6$$
$$= 29 - 12$$
$$r = 17.$$

$C(n+r-1, r)$, $n=6$, $r=17$

$C(22, 17) = 26,334.$

b) $x_1 \geq 1$, $x_2 \geq 2$, $x_3 \geq 3$, $x_4 \geq 4$, $x_5 \geq 5$, $x_6 \geq 6$

let $x'_1 = x_1 - 1$

$x'_2 = x_2 - 2$

$x'_3 = x_3 - 3$

$x'_4 = x_4 - 4$

$x'_5 = x_5 - 6$

$x'_6 = x_6 - 6$

$$x'_1 + x'_2 + x'_3 + x'_4 + x'_5 + x'_6 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 - 1 - 2 - 3 - 4 - 6 - 6$$
$$= 29 - 22$$
$$= 7.$$

$C(n+r-1, r)$, $n=6$, $r=7$

$C(22, 7) = 792$

c) $x_1 \le 5 \rightarrow \text{Total} - (x_1 > 5)$

Grand total $= n = 6$
$r = 29$

$C(34, 29) = 278,256.$

$x_1 > 5$, define $x'_1 = x_1 - 6$

$$x'_1 + x_2 + x_3 + x_4 + x_5 + x_6 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 - 6$$
$$= 29 - 6$$
$$= 23$$

$C(n+r-1, r)$, $n=6$, $r=23$

$C(28, 23) = 98,280$

$x_1 \le 5 = 278,256 - 98,280$
$= 179,976$

d) $x_1 < 8 \quad x_2 > 8.$

$x_2 > 8.$

$x'_2 = x_2 - 9$

$$x_1 + x'_2 + x_3 + x_4 + x_5 + x_6 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 - 9$$
$$r = 29 - 9 = 20$$

$C(n+r-1, r)$, $n=6$, $r=20$

$C(25, 20) = 53,130$

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$x_1 > 8$ and $x_2 > 8$

let $x_1' = x_1 - 8$

$x_2' = x_2 - 9$

$x_1' + x_2' + x_3 + x_4 + x_5 + x_6 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 - 8 - 9$

$r = 29 - 17$

$= 12.$

$C(n+r-1, r)$, $n=6$, $r=12$

$C(17, 12) = 6188$

N° of solutions $x_1 < 8$ and $x_2 > 8$

$= 58130 - 6188$

$= 46,942$

26. Integer $< 1000,000$, 1 digit $= 9$, $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 13$

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 13$

$x_6 = 9 \quad \therefore \; x_1 + x_2 + x_3 + x_4 + x_5 = 4$

$C(n+r-1, r)$, $n = 5$, $r = 4$

$C(8, 4) = 70$

70×6 possibilities $= 420$.

32. AARDVARK, All As are consecutive.

$= \frac{6!}{2!1!1!1!1!} = 360.$

46. 12 books, 5 books such that no 2 adjacent books are chosen.

$\binom{6+3-1}{5} = \binom{8}{5}$

$= 56$

_, O, _, O, _, O, _, O, _, O, _, _

1 2 3 4 5 6 7

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50. 5 distinguishable objects into 3 indistinguishable boxes.

$S(5,1) + S(5,2) + S(5,3)$

$= 1 + 15 + 25$

$= 41$.

$$S(n,j) = \frac{1}{j!}\sum_{i=0}^{j-1} (-1)^i \binom{j}{i} (j-i)^n$$

54. 5 indistinguishable objects into 3 indistinguishable boxes.

i) 0, 0, 5

ii) 0, 1, 4

iii) 0, 2, 3

iv) 1, 1, 3

v) 1, 2, 2

∴ 5 ways.

61. 5 sites twice.

1 site per day.

Cannot go to site X on two consecutive day.

$$\frac{10!}{2!2!2!2!2!} - \frac{9!}{2!2!2!2!} = 90{,}720$$

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6.6. 6f) 23587416 → 23587461

7. 1234, 1243, 1324, 1342, 1423, 1432,
2123, 2143, 2314, 2341, 2413, 2431,
3124, 3142, 3214, 3241, 3412, 3421,
4123, 4132, 4213, 4231, 4312, 4321.

9. $\{1,2,3,4,5\}$. 3 combinations.

$\{1,2,3\}$, $\{1,2,4\}$, $\{1,2,5\}$
$\{1,3,4\}$, $\{1,3,5\}$, $\{1,4,5\}$
$\{2,3,4\}$, $\{2,3,5\}$, $\{2,4,5\}$
$\{3,4,5\}$