Learning-Based Optimal Control for Safe Quadrotor Navigation in Windy Environments

Midterm Colloquium Johanna Probst

Delft University of Technology, The Netherlands

October 27, 2022

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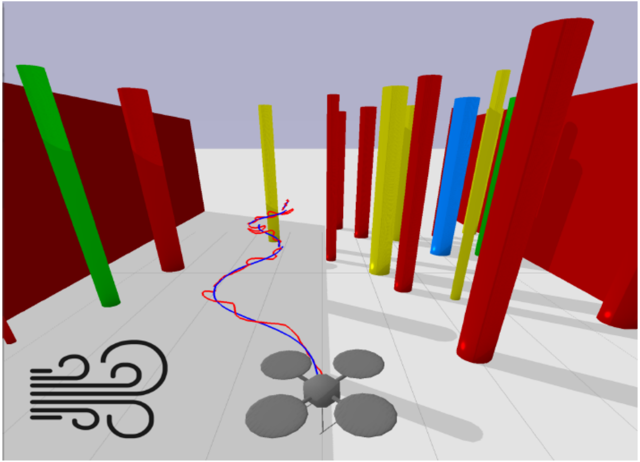
Motivation

[1]

[1] A. Mirzaeinia and M. Hassanalian (2019). "Minimum-cost drone–nest matching through the kuhn–munkres algorithm in smart cities: energy management and efficiency enhancement".

Motivation

 2

A. E. Gurgen, A. Majumdar, and S. Veer (2021). "Learning Provably Robust Motion Planners Using Funnel Libraries".

[2] A. E. Gurgen, A. Majumdar, and S. Veer (2021). "Learning Provably Robust Motion Planners Using Funnel Libraries".

Key Challenges

Modelling/Description of Wind Disturbances

Safe Quadrotor Navigation

Related Literature - Wind Disturbances

Disturbance Estimation and Rejection

- Classic Observer Design [3]
- Extended/Unscented Kalman Filter [4]
- Nonlinear Dynamic Inversion [5]

Disturbance Modelling

- Aerodynamic Models [6]
- Robust Models [7]

[3] T. Tomić and S. Haddadin (2014). "A unified framework for external wrench estimation, interaction control and collision reflexes for flying robots".

[4] D. Hentzen, T. Stastny, R. Siegwart, and R. Brockers (2019). "Disturbance estimation and rejection for high-precision multirotor position control".

[5] E. J. Smeur, G. C. de Croon, and Q. Chu (2018). "Cascaded incremental nonlinear dynamic inversion for MAV disturbance rejection".

[6] S. Waslander and C. Wang (2009). "Wind disturbance estimation and rejection for quadrotor position control".

[7] C. T. Ton and W. Mackunis (2012). "Robust attitude tracking control of a quadrotor helicopter in the presence of uncertainty".

Related Literature - Wind Disturbances

Learning-based Disturbance Model

- Set-Membership Identification [8]
- Neural Networks [9]
- Gaussian Processes

[8] M. Lorenzen, M. Cannon, and F. Allgöwer (2019). "Robust MPC with recursive model update".

[9] M. O'Connell, G. Shi, X. Shi, and S.-J. Chung (n.d.). "Meta-learning-based robust adaptive flight control under uncertain wind conditions". ()

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Related Literature - Wind Disturbances

Learning-based Disturbance Model

- Set-Membership Identification
- Neural Networks
- **Gaussian Processes**


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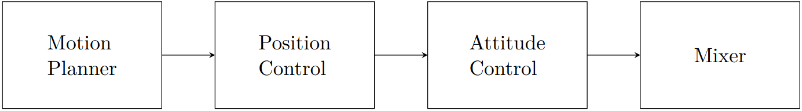
[10] C. J. Ostafew, A. P. Schoellig, and T. D. Barfoot (2016). "Robust constrained learning-based NMPC enabling reliable mobile robot path tracking".

[11] G. Torrente, E. Kaufmann, P. Föhn, and D. Scaramuzza (2021). "Data-driven MPC for quadrotors".

[12] A. Carron, E. Arcari, M. Wermelinger, L. Hewing, M. Hutter, and M. N. Zeilinger (2019). "Data-driven model predictive control for trajectory tracking with a robotic arm".

Related Literature - Control Structure




[13]

[13] HoverGames (n.d.). *https://www.hovergames.com/*.

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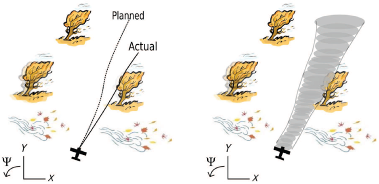
Related Literature - Motion Planning and Control

1. Reachability-based methods [14] [15]
2. Model predictive control (MPC)



(a) A plane deviating from its nominal planned trajectory due to a heavy cross-wind. (b) The "funnel" of possible trajectories.

[16]

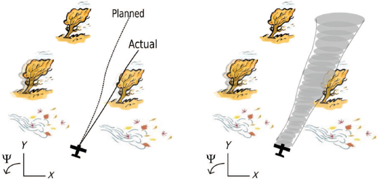
[14] S. L. Herbert, M. Chen, S. Han, S. Bansal, J. F. Fisac, and C. J. Tomlin (2017). "FaSTrack: A modular framework for fast and guaranteed safe motion planning". *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*. IEEE, pp. 1517–1522

[15] S. Kousik, S. Vaskov, F. Bu, M. Johnson-Roberson, and R. Vasudevan (2020). "Bridging the gap between safety and real-time performance in receding-horizon trajectory design for mobile robots".

[16] A. Majumdar and R. Tedrake (2017). "Funnel libraries for real-time robust feedback motion planning".

Related Literature - Motion Planning and Control

1. Reachability-based methods
2. **Model predictive control (MPC)** [17] [18]



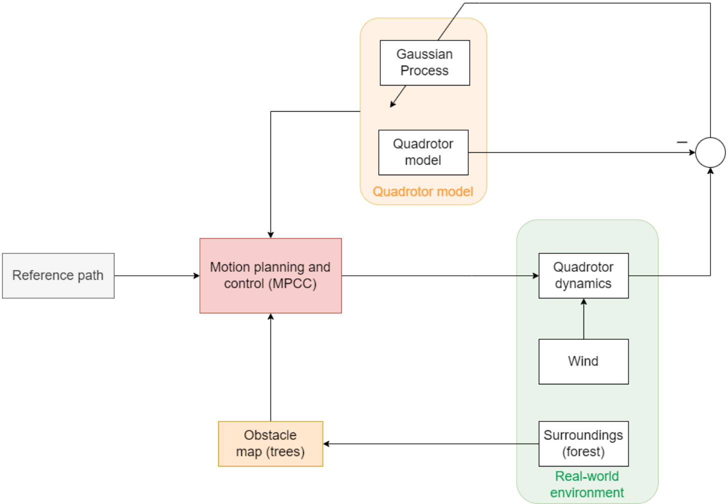
(a) A plane deviating from its nominal planned trajectory due to a heavy cross-wind.

(b) The "funnel" of possible trajectories.

[17] S. Singh, B. Landry, A. Majumdar, J.-J. Slotine, and M. Pavone (2019). "Robust feedback motion planning via contraction theory".

[18] H. Zhu and J. Alonso-Mora (2019). "Chance-constrained collision avoidance for MAVs in dynamic environments".

Problem Formulation



Research Questions

1. How well are Gaussian processes performing to model external wind disturbances using the quadrotor state information?
2. By how much can we increase the robustness of the MPCC controller by including the learned Gaussian process model into the MPCC formulation?

Quadrotor Model

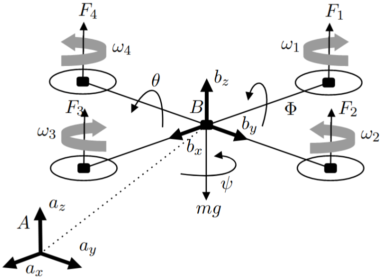
$$\dot{\mathbf{p}} = \mathbf{v} \tag{1a}$$

$$m\dot{\mathbf{v}} = -mg\mathbf{a}_3 + R\mathbf{T} \tag{1b}$$

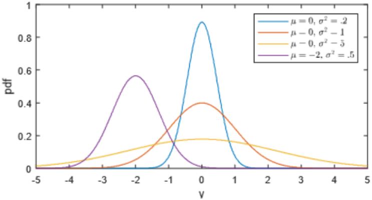
$$\dot{\phi} = \frac{1}{\tau_\phi}(k_\phi \phi_d - \phi) \tag{1c}$$

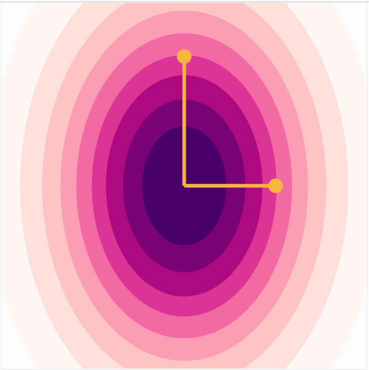
$$\dot{\theta} = \frac{1}{\tau_\theta}(k_\theta \theta_d - \theta) \tag{1d}$$

$$\dot{\psi} = \dot{\psi}_d \tag{1e}$$



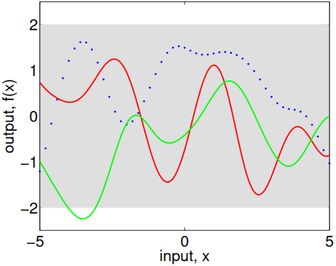
Gaussian Processes



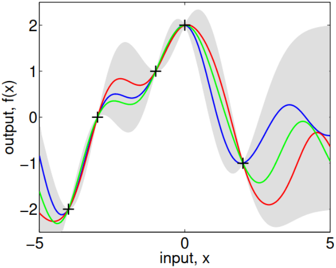


$$f(\mathbf{x}) \sim \mathcal{GP}\left(m(\mathbf{x}), k\left(\mathbf{x}, \mathbf{x}'\right)\right). \tag{2}$$

Gaussian Processes



(a), prior



(b), posterior

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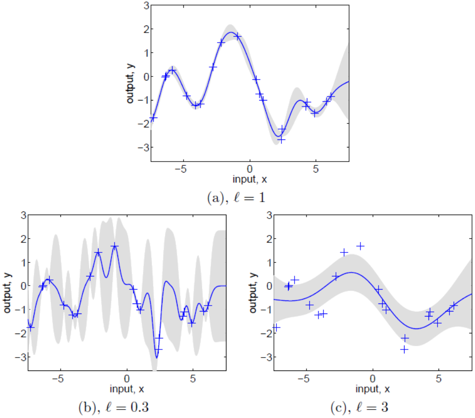
$$\begin{bmatrix} \mathbf{y} \\ \mathbf{f}_* \end{bmatrix} \sim \mathcal{N}\left(0, \begin{bmatrix} K(X,X) & K(X,X_*) \\ K(X_*,X) & K(X_*,X_*) \end{bmatrix}\right) \tag{3}$$

$$p(\mathbf{f}_* \mid X, \mathbf{y}, X_*) \sim \mathcal{N}(\boldsymbol{\mu}_*, \boldsymbol{\Sigma}_*) \tag{4}$$

[19] C. K. Williams and C. E. Rasmussen (2006). *Gaussian processes for machine learning*.

Gaussian Processes



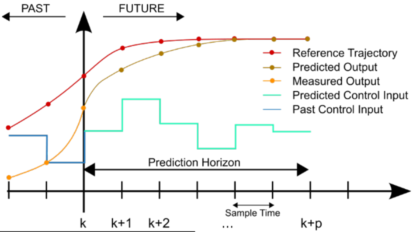
[20]

$$k_{\mathrm{SE}}\left(\mathbf{x}, \mathbf{x}'\right) = \sigma_f^2 \exp\left(-\frac{1}{2}\sum_{d=1}^{D}\left(x_d - x_d'\right)^2 l_d^{-2}\right). \tag{5}$$

[20] C. K. Williams and C. E. Rasmussen (2006). *Gaussian processes for machine learning*.

Model Predictive Control

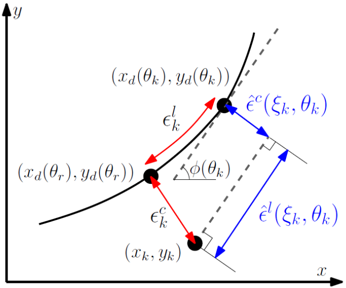
$$
\begin{aligned}
\min_{\mathbf{u}_{0:N-1}} \quad & \sum_{k=0}^{N-1} \ell(\mathbf{x}_k, \mathbf{u}_k) + V_f(\mathbf{x}_N) && \text{(6a)} \\
\text{s.t.} \quad & \mathbf{x}_0 = \mathbf{x}(0) && \text{(6b)} \\
& \mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k) && \text{(6c)} \\
& \mathbf{u} \in \mathbb{U} && \text{(6d)} \\
& \mathbf{x} \in \mathbb{X} && \text{(6e)} \\
& (\mathbf{x}, \mathbf{u}) \in \mathbb{Z} && \text{(6f)} \\
& \mathbf{x}_{k+N} \in X_f \subset \mathbb{X} && \text{(6g)}
\end{aligned}
$$



[21]

[21] M. Behrendt (n.d.). *Regelverhalten eines zeitdiskreten MPC-Modells.*

Model Predictive Contouring Control

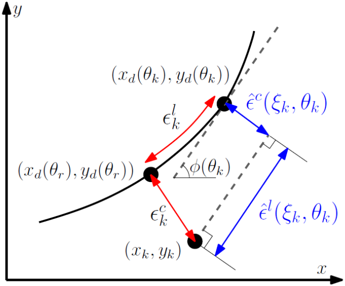


[22]

- Parametrize path $\mathbf{p}(\theta)$
- $V_{\text{progress}}(\mathbf{x}_k, \mathbf{u}_k, \theta)$ / $V_{\text{speed}}(\mathbf{x}_k, \mathbf{u}_k, \theta)$
- $V_{\text{contour}}(\mathbf{x}_k, \mathbf{u}_k, \theta)$
- $\mathcal{B}(\mathbf{x}_k) \cap \left(\mathcal{O}^{\text{static}} \cup \mathcal{O}_k^{\text{dyn}}\right) = \varnothing$

[22] D. Lam, C. Manzie, and M. Good (2010). "Model predictive contouring control".

Model Predictive Contouring Control

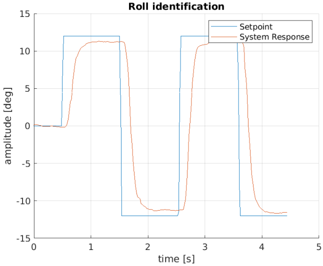


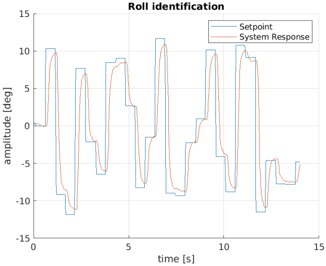
[23]

- Parametrize path $\mathbf{p}(\theta)$
- $V_{\text{progress}}(\mathbf{x}_k, \mathbf{u}_k, \theta)$ / $V_{\text{speed}}(\mathbf{x}_k, \mathbf{u}_k, \theta)$
- $V_{\text{contour}}(\mathbf{x}_k, \mathbf{u}_k, \theta)$
- $\Pr(\mathbf{x}_k \in \mathbb{X}) \geq 1 - \beta$

[23] D. Lam, C. Manzie, and M. Good (2010). "Model predictive contouring control".

System Identification - Experiments





$$\dot{\phi} = \frac{1}{\tau_\phi}(k_\phi \phi_d - \phi) \tag{7a}$$

$$\dot{\theta} = \frac{1}{\tau_\theta}(k_\theta \theta_d - \theta) \tag{7b}$$

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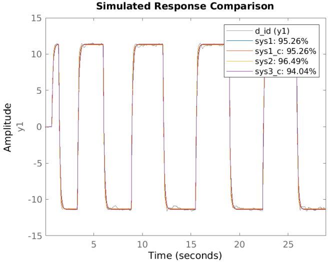
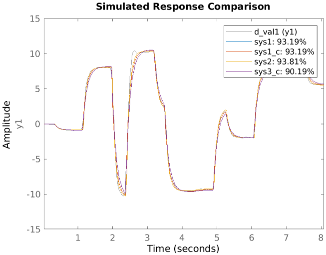
System Identification - Results





$$\dot{\phi} = \frac{1}{\tau_\phi}\left(k_\phi \phi_d - \phi\right) \tag{8a}$$

$$\dot{\theta} = \frac{1}{\tau_\theta}\left(k_\theta \theta_d - \theta\right) \tag{8b}$$

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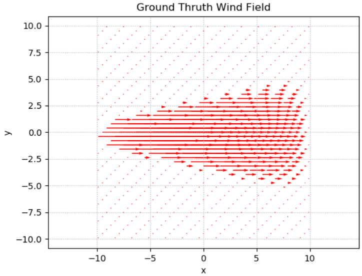
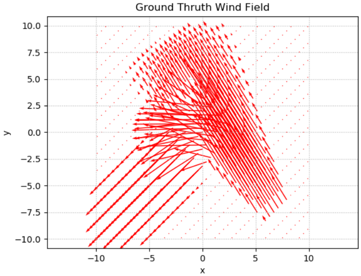
Gaussian Process Training - Wind Fields

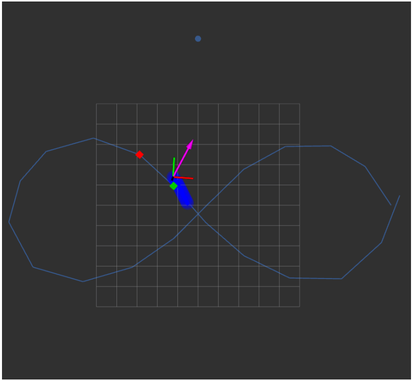




Gaussian Process Training - Full GP

$$\mathbf{d}(\mathbf{p}) = \begin{bmatrix} d_{v_x}(\mathbf{p}) \sim \mathcal{N}\left(\mu^{d_{vx}}(\mathbf{p}), \Sigma^{d_{vx}}(\mathbf{p})\right) \\ d_{v_y}(\mathbf{p}) \sim \mathcal{N}\left(\mu^{d_{vy}}(\mathbf{p}), \Sigma^{d_{vy}}(\mathbf{p})\right) \end{bmatrix} \tag{9}$$

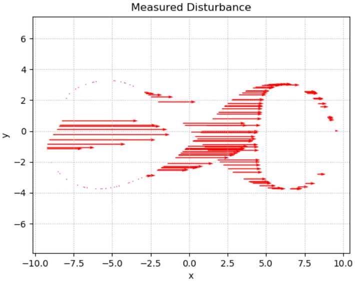
$$\begin{aligned} d_{v_x} &= \frac{\hat{v}_x - v_x}{\Delta T} \\ d_{v_y} &= \frac{\hat{v}_y - v_y}{\Delta T} \end{aligned} \tag{10}$$



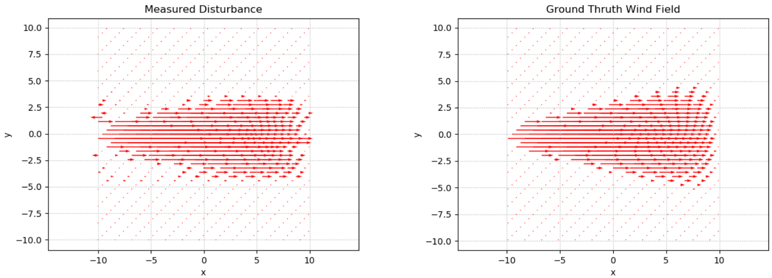
Gaussian Process Training - Full GP

$$\mathbf{d}(\mathbf{p}) = \begin{bmatrix} d_{v_x}(\mathbf{p}) \sim \mathcal{N}\left(\mu^{d_{v_x}}(\mathbf{p}), \Sigma^{d_{v_x}}(\mathbf{p})\right) \\ d_{v_y}(\mathbf{p}) \sim \mathcal{N}\left(\mu^{d_{v_y}}(\mathbf{p}), \Sigma^{d_{v_y}}(\mathbf{p})\right) \end{bmatrix} \tag{11}$$

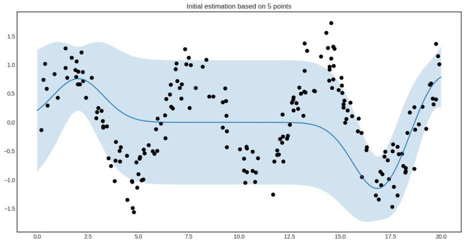
$$\begin{aligned} d_{v_x} &= \frac{\hat{v}_x - v_x}{\Delta T} \\ d_{v_y} &= \frac{\hat{v}_y - v_y}{\Delta T} \end{aligned} \tag{12}$$

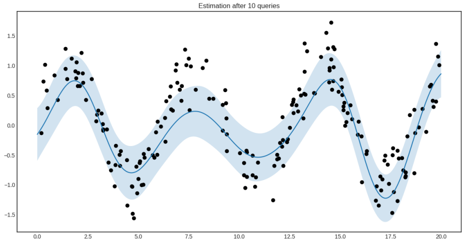


Gaussian Process Training - Full GP



Active Learning





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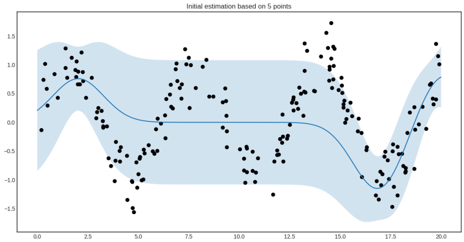
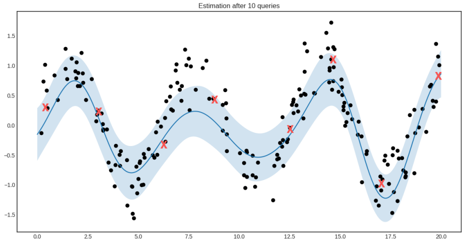
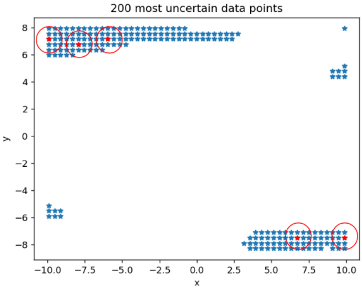
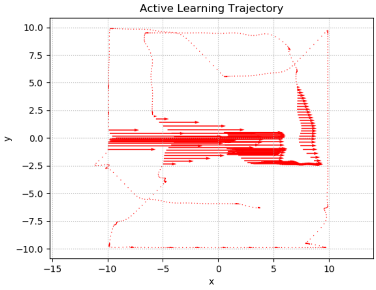
Sparse GP


Initial estimation based on 5 points
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0


Estimation after 10 queries
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

Active Learning plus TSP





Gaussian Process Training - Parameter Selection

Parameters

- Kernel
- Sampling Rate
- Epochs and batch size
- Points visited for TSP
- Number of Inducing Inputs

Gaussian Process Training - Parameter Selection

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Metrics

- MSE and MAE
- Training and prediction time
- Max und mean uncertainty of the GP
- Eploration time
- Data samples

Gaussian Process Training - Parameter Selection

Parameters

- Kernel
- Sampling Rate
- Epochs and batch size
- Points visited for TSP
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Metrics

- MSE and MAE
- Training and prediction time
- Max und mean uncertainty of the GP
- Eploration time
- Data samples

- Sparse GP: Prediction 100 times faster
- Active Learning: Uncertainty 2 times lower compared to structured path

Outlook

1. Extend nominal model with GP model
2. Make predictions with uncertain model
3. Include uncertain formulation into LMPCC
4. Compare LMPCC with uncertain model to state-of-the-art
5. Test GP training in Gazebo simulation and lab