Learning-Based Optimal Control for Safe Quadrotor Navigation in Windy Environments

Midterm Colloquium Johanna Probst

Delft University of Technology, The Netherlands

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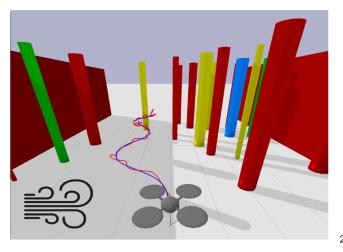
Motivation



¹A. Mirzaeinia and M. Hassanalian (2019). "Minimum-cost drone-nest matching through the kuhn-munkres algorithm in smart cities: energy management and efficiency enhancement".



Motivation



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²A. E. Gurgen, A. Majumdar, and S. Veer (2021). "Learning Provably Robust Motion Planners Using Funnel Libraries".



Key Challenges

Modelling/Description of Wind Disturbances

Safe Quadrotor Navigation

Related Literature - Wind Disturbances

Disturbance Estimation and Rejection

- Classic Observer Design ³
- Extended/Unscented Kalman Filter ⁴
- Nonlinear Dynamic Inversion ⁵

Disturbance Modelling

- Aerodynamic Models
 6
- Robust Models ⁷

⁷C. T. Ton and W. Mackunis (2012). "Robust attitude tracking control of a quadrotor helicopter in the presence of uncertainty".



³T. Tomić and S. Haddadin (2014). "A unified framework for external wrench estimation, interaction control and collision reflexes for flying robots".

⁴D. Hentzen, T. Stastny, R. Siegwart, and R. Brockers (2019). "Disturbance estimation and rejection for high-precision multirotor position control".

⁵E. J. Smeur, G. C. de Croon, and Q. Chu (2018). "Cascaded incremental nonlinear dynamic inversion for MAV disturbance rejection".

⁶S. Waslander and C. Wang (2009). "Wind disturbance estimation and rejection for quadrotor position control".

Related Literature - Wind Disturbances

Learning-based Disturbance Model

- Set-Membership Identification ⁸
- Neural Networks ⁹
- Gaussian Processes

⁹M. O'Connell, G. Shi, X. Shi, and S.-J. Chung (n.d.). "Meta-learning-based robust adaptive flight control under uncertain wind conditions". ()



⁸M. Lorenzen, M. Cannon, and F. Allgöwer (2019). "Robust MPC with recursive model update".

Related Literature - Wind Disturbances

Learning-based Disturbance Model

- Set-Membership Identification
- Neural Networks
- Gaussian Processes







2

 $^{^{12}}$ A. Carron, E. Arcari, M. Wermelinger, L. Hewing, M. Hutter, and M. N. Zeilinger (2019). "Data-driven model predictive control for trajectory tracking with a robotic arm".



¹⁰C. J. Ostafew, A. P. Schoellig, and T. D. Barfoot (2016). "Robust constrained learning-based NMPC enabling reliable mobile robot path tracking".

 $^{^{11}}$ G. Torrente, E. Kaufmann, P. Föhn, and D. Scaramuzza (2021). "Data-driven MPC for quadrotors".

Related Literature - Control Structure

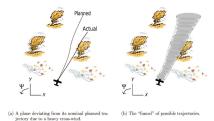




¹³HoverGames (n.d.). https://www.hovergames.com/.

Related Literature - Motion Planning and Control

- Reachabilitybased methods 14 15
- Model predictive control (MPC)



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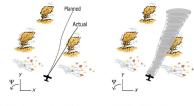
¹⁴S. L. Herbert, M. Chen, S. Han, S. Bansal, J. F. Fisac, and C. J. Tomlin (2017). "FaSTrack: A modular framework for fast and guaranteed safe motion planning". 2017 IEEE 56th Annual Conference on Decision and Control (CDC). IEEE, pp. 1517–1522

¹⁵ S. Kousik, S. Vaskov, F. Bu, M. Johnson-Roberson, and R. Vasudevan (2020). "Bridging the gap between safety and real-time performance in receding-horizon trajectory design for mobile robots".

¹⁶A. Majumdar and R. Tedrake (2017). "Funnel libraries for real-time robust feedback motion planning".

Related Literature - Motion Planning and Control

- Reachabilitybased methods
- Model predictive control (MPC) 17 18



⁽a) A plane deviating from its nominal planned trajectory due to a heavy cross-wind.

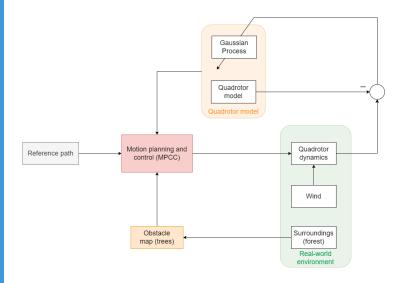
(b) The "funnel" of possible trajectories. $\,$

¹⁸H. Zhu and J. Alonso-Mora (2019). "Chance-constrained collision avoidance for MAVs in dynamic environments".



¹⁷S. Singh, B. Landry, A. Majumdar, J.-J. Slotine, and M. Pavone (2019). "Robust feedback motion planning via contraction theory".

Problem Formulation





Research Questions

- 1 How well are Gaussian processes performing to model external wind disturbances using the quadrotor state information?
- 2 By how much can we increase the robustness of the MPCC controller by including the learned Gaussian process model into the MPCC formulation?

Quadrotor Model

$$\dot{\mathbf{p}} = \mathbf{v}$$
 (1a)

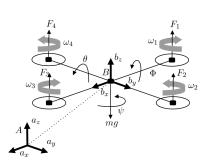
$$m\dot{\mathbf{v}} = -mg\mathbf{a}_3 + R\mathbf{T} \tag{1b}$$

$$\dot{\phi} = \frac{1}{\tau_{\phi}} (k_{\phi} \phi_{d} - \phi) \qquad \text{(1c)}$$

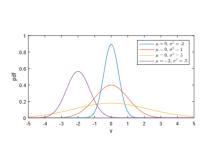
$$\dot{\theta} = \frac{1}{\tau_{\theta}} (k_{\theta} \theta_{d} - \theta) \qquad \text{(1d)}$$

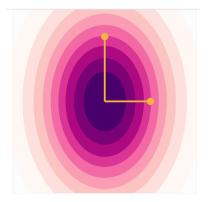
$$\dot{\theta} = \frac{1}{\tau_{\theta}} (k_{\theta} \theta_d - \theta)$$
 (1d)

$$\dot{\psi} = \dot{\psi}_d$$
 (1e)



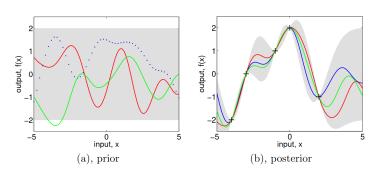
Gaussian Processes





$$f(\mathbf{x}) \sim \mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}')).$$
 (2)

Gaussian Processes



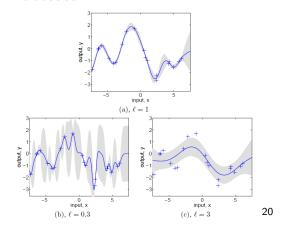
$$\begin{bmatrix} \mathbf{y} \\ \mathbf{f}_* \end{bmatrix} \sim \mathcal{N} \left(0, \begin{bmatrix} K(X, X) & K(X, X_*) \\ K(X_*, X) & K(X_*, X_*) \end{bmatrix} \right) \tag{3}$$

$$p\left(\mathbf{f}_{*} \mid X, \mathbf{y}, X_{*}\right) \sim \mathcal{N}\left(\boldsymbol{\mu}_{*}, \boldsymbol{\Sigma}_{*}\right) \tag{4}$$

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¹⁹C. K. Williams and C. E. Rasmussen (2006). Gaussian processes for machine learning.

Gaussian Processes



$$k_{\text{SE}}\left(\mathbf{x}, \mathbf{x}'\right) = \sigma_f^2 \exp\left(-\frac{1}{2} \sum_{d=1}^{D} \left(x_d - x_d'\right)^2 I_d^{-2}\right). \tag{5}$$



²⁰C. K. Williams and C. E. Rasmussen (2006). Gaussian processes for machine learning.

Model Predictive Control

$$\min_{\mathbf{u}_{0:N-1}} \sum_{k=0}^{N-1} \ell(\mathbf{x}_k, \mathbf{u}_k) + V_f(\mathbf{x}_N)$$
 (6a)

$$s.t. \quad \mathbf{x}_0 = \mathbf{x}(0) \tag{6b}$$

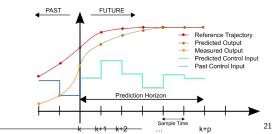
$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k) \tag{6c}$$

$$\mathbf{u} \in \mathbb{U}$$
 (6d)

$$\mathbf{x} \in \mathbb{X}$$

$$(\mathbf{x}, \mathbf{u}) \in \mathbb{Z}$$
 (6f)

$$\mathbf{x}_{k+N} \in X_f \subset \mathbb{X}$$
 (6g)

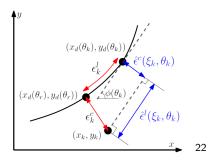


²¹M. Behrendt (n.d.). Regelverhalten eines zeitdiskreten MPC-Modells.



(6e)

Model Predictive Contouring Control

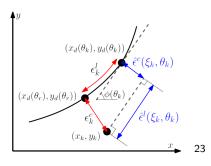


- Parametrize path $\mathbf{p}(\theta)$
- $V_{\text{progress}}(\mathbf{x}_k, \mathbf{u}_k, \theta) / V_{\text{speed}}(\mathbf{x}_k, \mathbf{u}_k, \theta)$
- $V_{\text{contour}}(\mathbf{x}_k, \mathbf{u}_k, \theta)$
- $\mathcal{B}(\mathbf{x}_k) \cap \left(\mathcal{O}^{\text{static}} \cup \mathcal{O}_k^{\text{dyn}}\right) = \emptyset$



²²D. Lam, C. Manzie, and M. Good (2010). "Model predictive contouring control".

Model Predictive Contouring Control

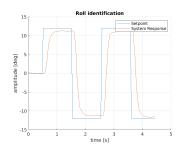


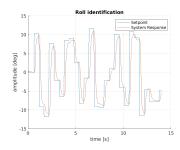
- Parametrize path $\mathbf{p}(\theta)$
- $V_{\text{progress}}(\mathbf{x}_k, \mathbf{u}_k, \theta) / V_{\text{speed}}(\mathbf{x}_k, \mathbf{u}_k, \theta)$
- $V_{\text{contour}}(\mathbf{x}_k, \mathbf{u}_k, \theta)$
- $\Pr\left(\mathbf{x}_k \in \mathbb{X}\right) \geq 1 \beta$



²³D. Lam, C. Manzie, and M. Good (2010). "Model predictive contouring control".

System Identification - Experiments



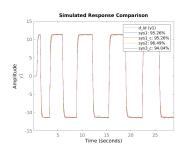


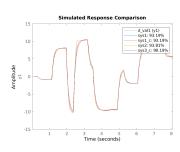
$$\dot{\phi} = \frac{1}{\tau_{\phi}} (\mathbf{k}_{\phi} \phi_{d} - \phi) \tag{7a}$$

$$\dot{\phi} = \frac{1}{\tau_{\phi}} (\mathbf{k}_{\phi} \phi_{d} - \phi)$$

$$\dot{\theta} = \frac{1}{\tau_{\theta}} (\mathbf{k}_{\theta} \theta_{d} - \theta)$$
(7a)

System Identification - Results



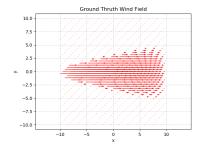


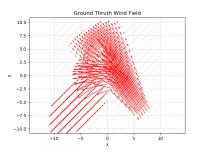
$$\dot{\phi} = \frac{1}{\tau_{\phi}} (\mathbf{k}_{\phi} \phi_{d} - \phi) \tag{8a}$$

$$\dot{\phi} = \frac{1}{\tau_{\phi}} (\mathbf{k}_{\phi} \phi_{d} - \phi)$$

$$\dot{\theta} = \frac{1}{\tau_{\theta}} (\mathbf{k}_{\theta} \theta_{d} - \theta)$$
(8a)

Gaussian Process Training - Wind Fields





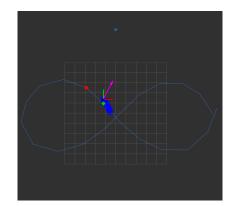


Gaussian Process Training - Full GP

$$\mathbf{d}(\mathbf{p}) = \begin{bmatrix} d_{v_x}(\mathbf{p}) \sim \mathcal{N}\left(\mu^{d_{v_x}}(\mathbf{p}), \Sigma^{d_{v_x}}(\mathbf{p})\right) \\ d_{v_y}(\mathbf{p}) \sim \mathcal{N}\left(\mu^{d_{v_y}}(\mathbf{p}), \Sigma^{d_{v_y}}(\mathbf{p})\right) \end{bmatrix}$$
(9)

$$d_{v_x} = \frac{\hat{v}_x - v_x}{\Delta T}$$

$$d_{v_y} = \frac{\hat{v}_y - v_y}{\Delta T}$$
(10)

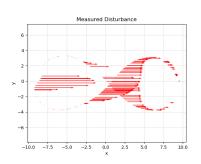


Gaussian Process Training - Full GP

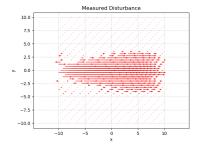
$$\mathbf{d}(\mathbf{p}) = \begin{bmatrix} d_{v_{x}}(\mathbf{p}) \sim \mathcal{N}\left(\mu^{d_{v_{x}}}(\mathbf{p}), \Sigma^{d_{v_{x}}}(\mathbf{p})\right) \\ d_{v_{y}}(\mathbf{p}) \sim \mathcal{N}\left(\mu^{d_{v_{y}}}(\mathbf{p}), \Sigma^{d_{v_{y}}}(\mathbf{p})\right) \end{bmatrix}$$
(11)

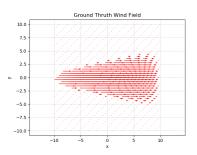
$$d_{v_x} = \frac{\hat{v}_x - v_x}{\Delta T}$$

$$d_{v_y} = \frac{\hat{v}_y - v_y}{\Delta T}$$
(12)

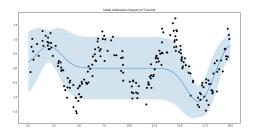


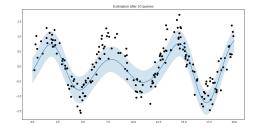
Gaussian Process Training - Full GP



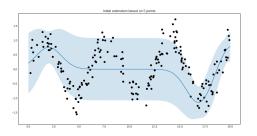


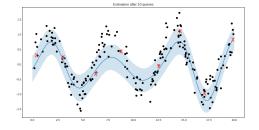
Active Learning



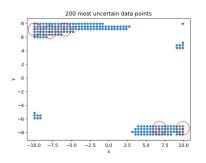


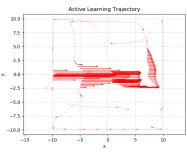
Sparse GP





Active Learning plus TSP







Gaussian Process Training - Parameter Selection

Parameters

- Kernel
- Sampling Rate
- Epochs and batch size
- Points visited for TSP
- Number of Inducing Inputs

Gaussian Process Training - Parameter Selection

Parameters

- Kernel
- Sampling Rate
- Epochs and batch size
- Points visited for TSP
- Number of Inducing Inputs

Metrics

- MSE and MAE
- Training and prediction time
- Max und mean uncertainty of the GP
- Eploration time
- Data samples



Gaussian Process Training - Parameter Selection

Parameters

- Kernel
- Sampling Rate
- Epochs and batch size
- Points visited for TSP
- Number of Inducing Inputs

Metrics

- MSE and MAE
- Training and prediction time
- Max und mean uncertainty of the GP
- Eploration time
- Data samples

- Sparse GP: Prediction 100 times faster
- Active Learning: Uncertainty 2 times lower compared to structured path



Outlook

- 1 Extend nominal model with GP model
- Make predictions with uncertain model
- Include uncertain formulation into LMPCC
- 4 Compare LMPCC with uncertain model to state-of-the-art
- 5 Test GP training in Gazebo simulation and lab

