1 System Model

State vector:

$$x^T = \left[x, y, z, v_x, v_y, v_z, \phi, \theta, \psi\right]^T \tag{1}$$

Input vector:

$$x^T = \left[\phi_d, \theta_d, \psi_d, T_d\right]^T \tag{2}$$

assuming

$$T = T_d \quad \text{with conversion:} \quad T_d = m(T_c - T_h)a_T \tag{3}$$

(T_c = commanded thrust to controller (0...1), T_d = desired thrust)

1.1 Nonlinear EOM

$$\begin{aligned}
\dot{p} &= v \\
m\dot{v}_x &= T\left(sin(\psi)sin(\phi) + cos(\phi)sin(\theta)cos(\psi)\right) - k_D v_x \\
m\dot{v}_y &= T\left(-sin(\phi)cos(\psi) + cos(\phi)sin(\theta)sin(\psi)\right) - k_D v_y \\
m\dot{v}_z &= T\left(cos(\phi)cos(\theta)\right) - mg \\
\dot{\phi} &= \frac{1}{\tau_\phi}(k_\phi \phi_d - \phi) \\
\dot{\theta} &= \frac{1}{\tau_\theta}(k_\theta \theta_d - \theta) \\
\dot{\psi} &= \dot{\psi}_d \quad / \quad (\dot{\psi} = \frac{1}{\tau_\psi}(k_\psi \psi_d - \psi))
\end{aligned} \tag{4}$$

1.2 Linear EOM

Linearizing around hovering conditions $\phi = \theta = \psi \approx 0$, $\dot{v_z} \approx 0$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{v_x} \\ \dot{v_y} \\ \dot{v_z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_D* & 0 & 0 & 0 & g & 0 \\ 0 & 0 & 0 & 0 & -k_D* & 0 & -g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_\phi} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_\theta} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau_\psi} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \\ \phi \\ \theta \\ \psi \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_\phi}{\tau_\phi} & 0 & 0 & 0 \\ 0 & \frac{k_\theta}{\tau_\theta} & 0 & 0 \\ 0 & 0 & \frac{k_\psi}{\tau_\psi} & 0 \end{bmatrix} \begin{bmatrix} \phi_d \\ \theta_d \\ \psi_d \\ T_d \end{bmatrix} \tag{5}$$

with $k_D* = \frac{k_D}{m}$

1.3 Identification

$\tau_\phi, \tau_\theta, (\tau_\psi), k_\phi, k_\theta, (k_\psi)$: Roll and pitch (and yaw) closed loop model
k_D: Drag coefficient
a_T, T_h: acceleration to thrust ratio and hovering thrust