Mathematical Modelling of UAS

- Guideline for Computer Lab -

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I. Introduction

In this computer lab, the 6 degree-of-freedom (DoF) dynamics of UAS is simulated with MATLAB. The 6 DoF dynamics simulator is completed and simple simulations are conducted to understand and acclimatize to the simulator.

Please open the m-file named 'UAS_dynamics_Exercise' and mdl-file named 'UAS_Model_ Exercise' to start.

II. Initial Conditions

States	Values
Velocity (V_{T_0})	400 ft / s
Angle-of-Attack (α_0)	0°
Sideslip Angle (β_0)	0°
Euler Angles ($[\phi_0 \ \theta_0 \ \psi_0]$)	$[0 \ 5 \ 0]$ °
Body Angular Rates ($[p_0 \ q_0 \ r_0]$)	$[0 \ 0 \ 0]$ °/ s
Position ($[x_{E_0} \ x_{N_0} \ h_0]$)	$[0 \ 0 \ 1500]$ ft

III. Activity

The activity instructions are addressed below.

1. Completing 6 Degree-of-Freedom Simulator of UAS

(1) Review on 6 DoF Model of UAS

Recall the nonlinear 6 DoF dynamics model of UAS.

i. Velocity in Body Axis

$$\dot{u} = rv - qw - g\sin\theta + \frac{X_b + X_T}{m}$$

$$\dot{v} = pw - ru + g\sin\phi\cos\theta + \frac{Y_b}{m}$$

$$\dot{w} = qu - pv + g\cos\phi\cos\theta + \frac{Z_b + Z_T}{m}$$

ii. Euler Angles

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta$$

$$\dot{\theta} = q\cos\phi - r\sin\phi$$

$$\dot{\psi} = q\sin\phi\sec\theta + r\cos\phi\sec\theta$$

iii. Body Angular Rates

$$I_{xx}\dot{p} - I_{xz}(\dot{r} + pq) + qr(I_{zz} - I_{yy}) = L_b$$
$$I_{yy}\dot{q} + I_{xz}(p^2 - r^2) + pr(I_{xx} - I_{zz}) = M_b$$
$$I_{zz}\dot{r} - I_{xz}\dot{p} + pq(I_{yy} - I_{xx}) + I_{xz}qr = N_b$$

iv. Position in Inertial Frame

$$\dot{x}_{E_0} = u\cos\psi\cos\theta + v(-\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi) + w(\sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi)$$
$$\dot{x}_{N_0} = u\sin\psi\cos\theta + v(\cos\psi\cos\phi + \sin\psi\sin\theta\sin\phi) + w(-\cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi)$$
$$\dot{h}_0 = u\sin\theta - v\cos\theta\sin\phi - w\cos\theta\cos\phi$$

$[u\ v\ w]$: Velocity in Body Frame

$[\phi\ \theta\ \psi]$: Euler Angles

$[p\ q\ r]$: Body Angular Rates

$[x_E\ x_N\ h]$: Position in Inertial Frame

$[X_b\ Y_b\ Z_b]$: Aerodynamic Forces in Body Axis

$[L_b\ M_b\ N_b]$: Aerodynamic Moments in Body Axis

$[X_T\ 0\ Z_T]$: Thrust in Body Axis

m : UAS Mass

$I_{xx}\ I_{yy}\ I_{zz}\ I_{xz}$: UAS Moment of Inertia

g : Gravitational Acceleration

Hint : Body angular rate dynamics is needed to be rewritten for each body angular rate element.

(2) Instruction for Coding

i. Open "UAS_dynamics_Exercise.m".

ii. Read the code carefully and understand the meaning of each part.

iii. Find missing parts of the code.

iv. Fill in the missing parts with the dynamic equations.

v. Open [UAS_Model_Exercise / Aircraft Model / A/C Frame] block and change 'S-function name' to 'UAS_dynamics_Exercise'.

vi. Run the completed simulator and check the results.

2. Simulation with Completed Simulator

(1) Changes in Initial Conditions

(a) Longitudinal Motion

i. Change the initial condition of α to $5°$ and observe the simulation results.

   What are the differences from the original initial condition case? Could you explain why?

ii. Explain the effects of initial α and θ by changing their values and observing the results.

(b) Lateral Motion

i. Change the initial condition of β to $5°$ and observe the difference in the simulation results.

   What are the differences from the original initial condition case? Could you explain why?

ii. Explain the effects of initial β, ϕ and ψ by changing their values and observing the results.

(2) Effects of Control Surfaces

(a) Longitudinal Motion

i. Change the elevator deflection angle to $10°$ and observe the simulation results.

   What are the differences from the original initial condition case? Could you explain why?

   What if the elevator deflection angle is $-10°$?

ii. Explain how the magnitude change in elevator deflection angle affects the motion of UAS using the simulator.

(b) Lateral Motion

i. Change the aileron deflection angle to $10°$ and observe the simulation results.

   What are the differences from the original initial condition case? Could you explain why?

   What if the aileron deflection angle is $-10°$?

ii. Explain how the magnitude change in aileron deflection angle affects the motion of UAS using the simulator.

iii. Perform i. and ii. for rudder deflection angle.