Summary of symmetry calculations

October 26, 2021

Contents

1 DBH_model 5

4 CONTENTS

Chapter 1

$DBH_{-}model$

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Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\begin{split} \frac{\mathrm{d}w_1}{\mathrm{d}t} &= -w_1w_2 - w_1w_3 + w_2w_3, \\ \frac{\mathrm{d}w_2}{\mathrm{d}t} &= -w_1w_2 + w_1w_3 - w_2w_3, \\ \frac{\mathrm{d}w_3}{\mathrm{d}t} &= w_1w_2 - w_1w_3 - w_2w_3. \end{split}$$

The calculated generators are:

Solving the algebraic equations

Solving equation:

$$-C_3 = 0$$

Arguments: $[-1, C_3]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_3 = 0$$

Solving equation:

$$-C_2t - C_4t - C_5 + C_7t = 0$$

Arguments: $[-C_5, C_7t, -C_2t, -C_4t]$ Arbitrary functions: [] Basis functions: [t, 1] Solutions:

$$C_2 = -C_4 + C_7$$
$$C_5 = 0$$

Solving equation:

$$-C_6 = 0$$

Arguments: $[-1, C_6]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_6 = 0$$

$$2C_4t - 2C_7t - C_8 = 0$$

Arguments: $[-C_8, -2C_7t, 2C_4t]$ Arbitrary functions: [] Basis functions: [2t, 1] Solutions:

$$C_4 = C_7$$
$$C_8 = 0$$

Solving equation:

$$-2C_7t - C_9 = 0$$

Arguments: $[-C_9, -2C_7t]$ Arbitrary functions: [] Basis functions: [1, t] Solutions:

$$C_7 = 0$$
$$C_9 = 0$$

Solving equation:

$$-C_{10} = 0$$

Arguments: $[-1, C_{10}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{10} = 0$$

Solving equation:

$$-C_{11}t^2 + C_{12}t - C_{13} + C_{21}t^2 - C_{22}t = 0$$

Arguments: $\left[-C_{13},\ C_{12}t,\ C_{21}t^2,\ -C_{11}t^2,\ -C_{22}t\right]$ Arbitrary functions: $\left[\right]$ Basis functions: $\left[t,\ t^2,\ 1\right]$ Solutions:

$$C_{12} = C_{22}$$

 $C_{11} = C_{21}$
 $C_{13} = 0$

Solving equation:

$$-C_{15} + 2C_{17}t + C_{21}t^2 - C_{31}t^2 - 2C_{32}t + C_{40} = 0$$

Arguments: $\left[C_{40},\ -C_{15},\ C_{21}t^2,\ -C_{31}t^2,\ -2C_{32}t,\ 2C_{17}t\right]$ Arbitrary functions: $\left[\right]$ Basis functions: $\left[1,\ t^2,\ t\right]$ Solutions:

$$C_{21} = C_{31}$$

 $C_{17} = C_{32}$
 $C_{15} = C_{40}$

Solving equation:

$$C_{14}t - C_{16} - C_{34}t = 0$$

Arguments: $[-C_{16}, C_{14}t, -C_{34}t]$ Arbitrary functions: [] Basis functions: [t, 1] Solutions:

$$C_{14} = C_{34}$$

 $C_{16} = 0$

$$-C_{18} - C_{24}t - C_{27}t + C_{32}t + C_{34}t - C_{40} = 0$$

Arguments: $[-C_{18}, -C_{40}, C_{32}t, C_{34}t, -C_{24}t, -C_{27}t]$ Arbitrary functions: [] Basis functions: [t, 1] Solutions:

$$C_{24} = -C_{27} + C_{32} + C_{34}$$
$$C_{18} = -C_{40}$$

Solving equation:

$$-C_{19} + C_{22}t - C_{37}t - C_{40} = 0$$

Arguments: $[-C_{19}, -C_{40}, C_{22}t, -C_{37}t]$ Arbitrary functions: [] Basis functions: [t, 1] Solutions:

$$C_{22} = C_{37}$$
$$C_{19} = -C_{40}$$

Solving equation:

$$-C_{20} + C_{27}t + C_{37}t = 0$$

Arguments: $[-C_{20}, C_{27}t, C_{37}t]$ Arbitrary functions: [] Basis functions: [t, 1] Solutions:

$$C_{27} = -C_{37}$$
$$C_{20} = 0$$

Solving equation:

$$-C_{23} = 0$$

Arguments: $[-1, C_{23}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{23} = 0$$

Solving equation:

$$-C_{25} - 2C_{34}t - 2C_{37}t - C_{40} = 0$$

 $\label{eq:arguments: optimization} \text{Arguments: } [-C_{25}, \ -C_{40}, \ -2C_{34}t, \ -2C_{37}t] \text{ Arbitrary functions: } [] \text{ Basis functions: } [1, \ t] \text{ Solutions: } [1, \ t] \text{ Solutions: } [1, \ t] \text{ Arbitrary functions: } [1, \ t] \text{ Solutions: } [1, \ t] \text{$

$$C_{34} = -C_{37}$$
$$C_{25} = -C_{40}$$

Solving equation:

$$-C_{26} - 2C_{37}t = 0$$

Arguments: $[-C_{26}, -2C_{37}t]$ Arbitrary functions: [] Basis functions: [1, t] Solutions:

$$C_{37} = 0$$
$$C_{26} = 0$$

$$-C_{28} + C_{40} = 0$$

Arguments: $[C_{40}, -C_{28}]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_{28} = C_{40}$$

Solving equation:

$$-C_{29} - C_{40} = 0$$

Arguments: $[-C_{29}, -C_{40}]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_{29} = -C_{40}$$

Solving equation:

$$-C_{30} = 0$$

Arguments: $[-1, C_{30}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{30} = 0$$

Solving equation:

$$-C_{33}=0$$

Arguments: $[-1, C_{33}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{33} = 0$$

Solving equation:

$$-C_{35} - C_{40} = 0$$

Arguments: $[-C_{35}, -C_{40}]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_{35} = -C_{40}$$

Solving equation:

$$-C_{36} = 0$$

Arguments: $[-1, C_{36}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{36} = 0$$

Solving equation:

$$-C_{38} - C_{40} = 0$$

Arguments: $[-C_{38}, -C_{40}]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_{38} = -C_{40}$$

Solving equation:

$$-C_{39} = 0$$

Arguments: $[-1, C_{39}]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_{39} = 0$$

$$X_1 = (-1+t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$X_2 = (1) \partial t$$
,

$$X_3 = (t+2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 + (1 - 2tw_3) \partial w_3$$

$$X_4 = (-1) \partial t$$
,

$$X_5 = (t) \partial t + (w_2 w_3 f_1(t) - w_1 w_2 f_1(t) - w_1 w_3 f_1(t)) \partial w_1 + (w_1 w_3 f_1(t) - w_1 w_2 f_1(t) + -w_2 w_3 f_1(t)) \partial w_2 + (w_1 w_2 f_1(t) - w_1 w_3 f_1(t) - w_2 w_3 f_1(t)) \partial w_3$$

Some of the generators might contain the following arbitrary functions:

 f_1

The execution time of the script was:

0 hours 5 minutes 24 seconds.

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Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1w_2 - w_1w_3 + w_2w_3,$$

$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1w_2 + w_1w_3 - w_2w_3,$$

$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1w_2 - w_1w_3 - w_2w_3.$$

The calculated generators are:

Solving the algebraic equations

Solving equation:

$$-C_3 = 0$$

Arguments: $[-1, C_3]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_3 = 0$$

$$-C_2t - C_4t - C_5 + C_7t = 0$$

Arguments: $[-C_5, C_7t, -C_2t, -C_4t]$ Arbitrary functions: [] Basis functions: [t, 1] Solutions:

$$C_2 = -C_4 + C_7$$
$$C_5 = 0$$

Solving equation:

$$-C_6 = 0$$

Arguments: $[-1, C_6]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_6 = 0$$

Solving equation:

$$2C_4t - 2C_7t - C_8 = 0$$

Arguments: $[-C_8, -2C_7t, 2C_4t]$ Arbitrary functions: [] Basis functions: [2t, 1] Solutions:

$$C_4 = C_7$$

$$C_8 = 0$$

Solving equation:

$$-2C_7t - C_9 = 0$$

Arguments: $[-C_9, -2C_7t]$ Arbitrary functions: [] Basis functions: [2t, 1] Solutions:

$$C_7 = 0$$

$$C_9 = 0$$

Solving equation:

$$-C_{10} = 0$$

Arguments: $[-1, C_{10}]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_{10} = 0$$

Solving equation:

$$-C_{11}t^2 + C_{12}t - C_{13} + C_{21}t^2 - C_{22}t = 0$$

Arguments: $\left[-C_{13},\ C_{12}t,\ C_{21}t^2,\ -C_{11}t^2,\ -C_{22}t\right]$ Arbitrary functions: $\left[\right]$ Basis functions: $\left[t^2,\ t,\ 1\right]$ Solutions:

$$C_{11} = C_{21}$$

$$C_{12} = C_{22}$$

$$C_{13} = 0$$

$$-C_{15} + 2C_{17}t + C_{21}t^2 - C_{31}t^2 - 2C_{32}t + C_{40} = 0$$

Arguments: $\left[C_{40},\ -C_{15},\ C_{21}t^2,\ -C_{31}t^2,\ -2C_{32}t,\ 2C_{17}t\right]$ Arbitrary functions: $\left[\right]$ Basis functions: $\left[1,\ t^2,\ t\right]$ Solutions:

$$C_{21} = C_{31}$$

 $C_{17} = C_{32}$

$$C_{15} = C_{40}$$

Solving equation:

$$C_{14}t - C_{16} - C_{34}t = 0$$

Arguments: $[-C_{16}, C_{14}t, -C_{34}t]$ Arbitrary functions: [] Basis functions: [t, 1] Solutions:

$$C_{14} = C_{34}$$

$$C_{16} = 0$$

Solving equation:

$$-C_{18} - C_{24}t - C_{27}t + C_{32}t + C_{34}t - C_{40} = 0$$

Arguments: $[-C_{18}, -C_{40}, C_{32}t, C_{34}t, -C_{24}t, -C_{27}t]$ Arbitrary functions: [] Basis functions: [t, 1] Solutions:

$$C_{24} = -C_{27} + C_{32} + C_{34}$$

$$C_{18} = -C_{40}$$

Solving equation:

$$-C_{19} + C_{22}t - C_{37}t - C_{40} = 0$$

Arguments: $[-C_{19}, -C_{40}, C_{22}t, -C_{37}t]$ Arbitrary functions: [] Basis functions: [t, 1] Solutions:

$$C_{22} = C_{37}$$

$$C_{19} = -C_{40}$$

Solving equation:

$$-C_{20} + C_{27}t + C_{37}t = 0$$

Arguments: $[-C_{20}, C_{27}t, C_{37}t]$ Arbitrary functions: [] Basis functions: [t, 1] Solutions:

$$C_{27} = -C_{37}$$

$$C_{20} = 0$$

Solving equation:

$$-C_{23} = 0$$

Arguments: $[-1, C_{23}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{23} = 0$$

$$-C_{25} - 2C_{34}t - 2C_{37}t - C_{40} = 0$$

 $\label{eq:arguments: of the constraint} \text{Arguments: } [-C_{25}, \ -C_{40}, \ -2C_{34}t, \ -2C_{37}t] \text{ Arbitrary functions: } [] \text{ Basis functions: } [2t, \ 1] \text{ Solutions: } [2t, \ 1] \text{ Soluti$

$$C_{34} = -C_{37}$$
$$C_{25} = -C_{40}$$

Solving equation:

$$-C_{26} - 2C_{37}t = 0$$

Arguments: $[-C_{26}, -2C_{37}t]$ Arbitrary functions: [] Basis functions: [2t, 1] Solutions:

$$C_{37} = 0$$
$$C_{26} = 0$$

Solving equation:

$$-C_{28} + C_{40} = 0$$

Arguments: $[C_{40}, -C_{28}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{28} = C_{40}$$

Solving equation:

$$-C_{29} - C_{40} = 0$$

Arguments: $[-C_{29}, -C_{40}]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_{29} = -C_{40}$$

Solving equation:

$$-C_{30} = 0$$

Arguments: $[-1, C_{30}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{30} = 0$$

Solving equation:

$$-C_{33}=0$$

Arguments: $[-1, C_{33}]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_{33} = 0$$

Solving equation:

$$-C_{35} - C_{40} = 0$$

Arguments: $[-C_{35}, -C_{40}]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_{35} = -C_{40}$$

$$-C_{36} = 0$$

Arguments: $[-1, C_{36}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{36} = 0$$

Solving equation:

$$-C_{38} - C_{40} = 0$$

Arguments: $[-C_{38}, -C_{40}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{38} = -C_{40}$$

Solving equation:

$$-C_{39} = 0$$

Arguments: $[-1, C_{39}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{39} = 0$$

$$X_1 = (-1+t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$X_2 = (t+2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 + (1 - 2tw_3) \partial w_3$$

$$X_3 = (1) \partial t$$
,

$$X_4 = (-1) \partial t$$
,

$$X_5 = (t) \partial t + (w_2 w_3 f_1(t) - w_1 w_2 f_1(t) - w_1 w_3 f_1(t)) \partial w_1 + (w_1 w_3 f_1(t) - w_1 w_2 f_1(t) + -w_2 w_3 f_1(t)) \partial w_2 + (w_1 w_2 f_1(t) - w_1 w_3 f_1(t) - w_2 w_3 f_1(t)) \partial w_3$$

Some of the generators might contain the following arbitrary functions:

 f_1

The execution time of the script was:

0 hours 5 minutes 14 seconds.

Run 11_54AM_26_October-2021

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1w_2 - w_1w_3 + w_2w_3,$$

$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1w_2 + w_1w_3 - w_2w_3,$$

$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1w_2 - w_1w_3 - w_2w_3.$$

The calculated generators are:

Solving the algebraic equations

Solving equation:

$$-C_3 = 0$$

Arguments: $[-1, C_3]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_3 = 0$$

Solving equation:

$$-C_2t - C_4t - C_5 + C_7t = 0$$

Arguments: $[-C_5, C_7t, -C_2t, -C_4t]$ Arbitrary functions: [] Basis functions: [t, 1] Solutions:

$$C_2 = -C_4 + C_7$$
$$C_5 = 0$$

Solving equation:

$$-C_6 = 0$$

Arguments: $[-1, C_6]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_6 = 0$$

Solving equation:

$$2C_4t - 2C_7t - C_8 = 0$$

Arguments: $[-C_8, -2C_7t, 2C_4t]$ Arbitrary functions: [] Basis functions: [2t, 1] Solutions:

$$C_4 = C_7$$
$$C_8 = 0$$

Solving equation:

$$-2C_7t - C_9 = 0$$

Arguments: $[-C_9, -2C_7t]$ Arbitrary functions: [] Basis functions: [2t, 1] Solutions:

$$C_7 = 0$$

$$C_9 = 0$$

$$-C_{10} = 0$$

Arguments: $[-1, C_{10}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{10} = 0$$

Solving equation:

$$-C_{11}t^2 + C_{12}t - C_{13} + C_{21}t^2 - C_{22}t = 0$$

Arguments: $\left[-C_{13},\ C_{12}t,\ C_{21}t^2,\ -C_{11}t^2,\ -C_{22}t\right]$ Arbitrary functions: $\left[\right]$ Basis functions: $\left[t,\ t^2,\ 1\right]$ Solutions:

$$C_{12} = C_{22}$$

 $C_{11} = C_{21}$
 $C_{13} = 0$

Solving equation:

$$-C_{15} + 2C_{17}t + C_{21}t^2 - C_{31}t^2 - 2C_{32}t + C_{40} = 0$$

Arguments: $\left[C_{40},\ -C_{15},\ C_{21}t^2,\ -C_{31}t^2,\ -2C_{32}t,\ 2C_{17}t\right]$ Arbitrary functions: $\left[\right]$ Basis functions: $\left[2t,\ 1,\ t^2\right]$ Solutions:

$$C_{17} = C_{32}$$

 $C_{21} = C_{31}$
 $C_{15} = C_{40}$

Solving equation:

$$C_{14}t - C_{16} - C_{34}t = 0$$

Arguments: $[-C_{16}, C_{14}t, -C_{34}t]$ Arbitrary functions: [] Basis functions: [t, 1] Solutions:

$$C_{14} = C_{34}$$

 $C_{16} = 0$

Solving equation:

$$-C_{18} - C_{24}t - C_{27}t + C_{32}t + C_{34}t - C_{40} = 0$$

Arguments: $[-C_{18}, -C_{40}, C_{32}t, C_{34}t, -C_{24}t, -C_{27}t]$ Arbitrary functions: [] Basis functions: [t, 1] Solutions:

$$C_{24} = -C_{27} + C_{32} + C_{34}$$
$$C_{18} = -C_{40}$$

Solving equation:

$$-C_{19} + C_{22}t - C_{37}t - C_{40} = 0$$

Arguments: $[-C_{19}, -C_{40}, C_{22}t, -C_{37}t]$ Arbitrary functions: [] Basis functions: [t, 1] Solutions:

$$C_{22} = C_{37}$$
$$C_{19} = -C_{40}$$

$$-C_{20} + C_{27}t + C_{37}t = 0$$

Arguments: $[-C_{20}, C_{27}t, C_{37}t]$ Arbitrary functions: [] Basis functions: [t, 1] Solutions:

$$C_{27} = -C_{37}$$
$$C_{20} = 0$$

Solving equation:

$$-C_{23} = 0$$

Arguments: $[-1, C_{23}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{23} = 0$$

Solving equation:

$$-C_{25} - 2C_{34}t - 2C_{37}t - C_{40} = 0$$

Arguments: $[-C_{25}, -C_{40}, -2C_{34}t, -2C_{37}t]$ Arbitrary functions: [] Basis functions: [2t, 1] Solutions:

$$C_{34} = -C_{37}$$
$$C_{25} = -C_{40}$$

Solving equation:

$$-C_{26} - 2C_{37}t = 0$$

Arguments: $[-C_{26}, -2C_{37}t]$ Arbitrary functions: [] Basis functions: [2t, 1] Solutions:

$$C_{37} = 0$$
$$C_{26} = 0$$

Solving equation:

$$-C_{28} + C_{40} = 0$$

Arguments: $[C_{40}, -C_{28}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{28} = C_{40}$$

Solving equation:

$$-C_{29} - C_{40} = 0$$

Arguments: $[-C_{29}, -C_{40}]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_{29} = -C_{40}$$

Solving equation:

$$-C_{30} = 0$$

Arguments: $[-1, C_{30}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{30} = 0$$

$$-C_{33} = 0$$

Arguments: $[-1, C_{33}]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_{33} = 0$$

Solving equation:

$$-C_{35} - C_{40} = 0$$

Arguments: $[-C_{35},\ -C_{40}]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_{35} = -C_{40}$$

Solving equation:

$$-C_{36} = 0$$

Arguments: $[-1, C_{36}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{36} = 0$$

Solving equation:

$$-C_{38} - C_{40} = 0$$

Arguments: $[-C_{38}, -C_{40}]$ Arbitrary functions: [] Basis functions:[1] Solutions:

$$C_{38} = -C_{40}$$

Solving equation:

$$-C_{39} = 0$$

Arguments: $[-1, C_{39}]$ Arbitrary functions: [] Basis functions: [1] Solutions:

$$C_{39} = 0$$

$$X_1 = (-1) \partial t$$
,

$$X_2 = (1) \partial t$$
,

$$X_3 = (-1+t)\,\partial t + (w_1)\,\partial w_1 + (w_2)\,\partial w_2 + (w_3)\,\partial w_3,$$

$$X_4 = (t+2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 + (1 - 2tw_3) \partial w_3$$

$$X_{5} = (t) \partial t + (w_{2}w_{3} f_{1}(t) - w_{1}w_{2} f_{1}(t) - w_{1}w_{3} f_{1}(t)) \partial w_{1} + (w_{1}w_{3} f_{1}(t) - w_{1}w_{2} f_{1}(t) + -w_{2}w_{3} f_{1}(t)) \partial w_{2} + (w_{1}w_{2} f_{1}(t) - w_{1}w_{3} f_{1}(t) - w_{2}w_{3} f_{1}(t)) \partial w_{3}$$

Some of the generators might contain the following arbitrary functions:

 f_1

The execution time of the script was:

0 hours 5 minutes 9 seconds.