Summary of symmetry calculations

November 10, 2021

Contents

Chapter 1

Lotka_Volterra

Run 03_56PM_10_November-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = N\left(-Pb + a\right),$$
$$\frac{\mathrm{d}P}{\mathrm{d}t} = P\left(Nc - d\right).$$

The calculated generators are:

$$X_1 = \left(-\frac{1}{a}\right)\partial t,$$

$$X_2 = (1)\,\partial t,$$

$$X_3 = \left(\frac{\mathrm{f}_1(t)}{c}\right)\partial t + \left(\frac{Na\,\mathrm{f}_1(t)}{c} - \frac{NPb\,\mathrm{f}_1(t)}{c}\right)\partial N + \left(NP\,\mathrm{f}_1(t) - \frac{Pd\,\mathrm{f}_1(t)}{c}\right)\partial P.$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

Equation: $-C_3 + \frac{C_6 b^2 e^{-2at}}{c^2} - \frac{C_6 b^2 d}{ac^2} + \frac{C_6 b^2 d e^{-2at}}{ac^2} = 0$ Basis functions:

$$\left[1.0,\ e^{-2at}\right]$$

Solutions *after* processing:

$$C_3 = 0$$
$$C_6 = 0$$

Current ODE solutions:

$$\mathbf{c} = \Big[-\frac{C_{10}bdt}{a} + \frac{C_{10}b}{a} - \frac{C_{10}be^{-at}}{a} + \frac{C_{10}bd}{a^2} - \frac{C_{10}bde^{-at}}{a^2} + C_{11} - \frac{C_{13}a^2b^2}{a^3de^{at}-a^2d^2e^{at}} - \frac{2C_{13}a^2b^2}{a^3d+a^2d^2} + \frac{2C_{13}ab^2c^2}{a^2c^2de^{dt}-ac^2d^2e^{dt}} - \frac{2C_{13}ab^2d}{a^3de^{at}-a^2d^2e^{at}} - \cdots$$

Current algebraic equations solutions:

$$\Big[\frac{C_{10}bdt}{a} - \frac{C_{10}b}{a} + \frac{C_{10}be^{-at}}{a} - \frac{C_{10}bd}{a^2} + \frac{C_{10}bde^{-at}}{a^2} - C_{11} + \frac{C_{13}a^2b^2}{a^3de^{at}-a^2d^2e^{at}} + \frac{2C_{13}a^2b^2}{a^3d+a^2d^2} - \frac{2C_{13}ab^2c^2}{a^2c^2de^{dt}-ac^2d^2e^{dt}} + \frac{2C_{13}ab^2d}{a^3de^{at}-a^2d^2e^{at}} + \frac{4C_{13}ab^2\cdots}{a^3d+a^2\cdots}$$

Equation: $-\frac{C_{15}ace^{2dt}}{2ad^2+4d^3} - \frac{C_{15}cde^{2dt}}{2ad^2+4d^3} + \frac{C_{15}c}{a^2e^{at}+2ade^{at}} + \frac{C_{15}ce^{dt}}{d^2} - \frac{C_{15}c}{2d^2} - \frac{C_{15}c}{2ad} + \frac{C_2ce^{dt}}{d} - \frac{C_2c}{d} - \frac{C_4b}{a} + \frac{C_4be^{-at}}{a} - C_5 + \frac{C_7abc^2e^{2dt}}{2a^3d^2-2a^2d^3-8ad^4+8d^5} - \frac{C_7abc^2e^{at}}{2a^4d-6a^3d^2+4a^2d^3} + \frac{C_7bc^2de^{2dt}}{2a^3d^2-2a^2d^3-8ad^4+8d^5} - \frac{C_7bc^2de^{at}}{2a^4d-6a^3d^2+4a^2d^3} - \frac{C_7bc^2}{4a^4e^{at}+10a^3de^{at}+4a^2d^2e^{at}} + \frac{C_7bc^2e^{at}e^{dt}}{2a^3d-a^2d^2-ad^3} - \frac{C_7bc^2e^{dt}}{a^2d^2-ad^3} + \frac{C_7bc^2}{2a^2d^2} + \frac{C_8ac^2e^{2dt}}{2a^2d^2+2ad^3-4d^4} + \frac{C_8c^2de^{2dt}}{2a^2d^2+2ad^3-4d^4} - \frac{C_8c^2}{2a^3e^{at}+5a^2de^{at}+2ad^2e^{at}} - \frac{C_8c^2e^{at}e^{dt}}{2a^3-a^2d-ad^2} - \frac{C_8c^2e^{dt}}{ad^2} + \frac{C_8c^2}{2ad^2} = 0$ Basis functions:

$$\left[1.0,\ e^{dt},\ e^{at}e^{dt},\ e^{2dt},\ e^{-at},\ e^{at}\right]$$

Solutions *after* processing:

$$C_7 = 0$$
$$C_4 = 0$$
$$C_{15} = 0$$
$$C_5 = 0$$
$$C_8 = 0$$
$$C_2 = 0$$

Current ODE solutions:

$$\mathbf{c} = \left[\begin{array}{c} C_1 - \frac{C_{10}}{a} + \frac{C_{10}e^{-at}}{a} - \frac{C_{13}abc^2}{a^2c^2de^{dt} - ac^2d^2e^{dt}} - \frac{C \cdots}{a^2de^{at}e^{dt} \cdots} \\ C_{10}e^{-at} - \frac{C_{13}a^2b}{a^2de^{at}e^{dt} + ad^2e^{at}e^{dt}} + \cdots \\ -\frac{C_{10}bdt}{a} + \frac{C_{10}b}{a} - \frac{C_{10}be^{-at}}{a} + \frac{C_{10}bd}{a^2} - \frac{C_{10}bde^{-at}}{a^2} + C_{11} - \frac{C_{13}a^2b^2}{a^3de^{at} - a^2d^2e^{at}} - \frac{2C_{13}a^2b^2}{a^3d + a^2d^2} + \frac{2C_{13}ab^2c^2}{a^2c^2de^{dt} - ac^2d^2e^{dt}} - \frac{2C_{13}ab^2d}{a^3de^{at} - a^2d^2e^{at}} - \cdots \\ \frac{C_{10}d}{a} - \frac{C_{10}de^{-at}}{a} + \frac{C_{13}abc^2d}{a^2c^2de^{dt} - ac^2d^2e^{dt}} + \frac{C_{13} \cdots}{a^2e^{at}e^{dt} + \cdots} \end{array}\right.$$

Current algebraic equations solutions:

$$\left[\begin{array}{l} -\frac{C_{15}ace^{2dt}}{2ad^2 + 4d^3} - \frac{C_{15}cde^{2dt}}{2ad^2 + 4d^3} + \frac{C_{15}c}{a^2e^{at} + 2ade^{at}} + \frac{C_{15}ce^{dt}}{d^2} - \frac{C_{15}c}{2d^2} - \frac{C_{15}c}{2ad} + \frac{C_2ce^{dt}}{d} - \frac{C_2c}{d} - \frac{C_4b}{a} + \frac{C_4be^{-at}}{a} - C_5 + \frac{C_7abc^2e^{2dt}}{2a^3d^2 - 2a^2d^3 - 8ad^4 + 8d^5} - \cdots \\ \frac{C_{10}bdt}{a} - \frac{C_{10}b}{a} + \frac{C_{10}be^{-at}}{a} - \frac{C_{10}bd}{a^2} + \frac{C_{10}bde^{-at}}{a^2} - C_{11} + \frac{C_{13}a^2b^2}{a^3de^{at} - a^2d^2e^{at}} + \frac{2C_{13}a^2b^2}{a^3d + a^2d^2} - \frac{2C_{13}ab^2c^2}{a^2c^2de^{dt} - ac^2d^2e^{dt}} + \frac{2C_{13}ab^2d}{a^3de^{at} - a^2d^2e^{at}} + \frac{4C_{13}ab \cdots}{a^3d + a^2 \cdots} \end{array}\right.$$

Equation:$-C_9 = 0$ Basis functions:

$$[1.0]$$

Solutions *after* processing:

$$C_9 = 0$$

Current ODE solutions:

$$\mathbf{c} = \left[\begin{array}{c} C_1 - \frac{C_{10}}{a} + \frac{C_{10}e^{-at}}{a} - \frac{C_{13}abc^2}{a^2c^2de^{dt}-ac^2d^2e^{dt}} - \frac{C}{a^2de^{at}e^{dt}} \\ C_{10}e^{-at} - \frac{C_{13}a^2b}{a^2de^{at}e^{dt}+ad^2e^{at}e^{dt}} + \\ -\frac{C_{10}bdt}{a} + \frac{C_{10}b}{a} - \frac{C_{10}be^{-at}}{a} + \frac{C_{10}bd}{a^2} - \frac{C_{10}bde^{-at}}{a^2} + C_{11} - \frac{C_{13}a^2b^2}{a^3de^{at}-a^2d^2e^{at}} - \frac{2C_{13}a^2b^2}{a^3d+a^2d^2} + \frac{2C_{13}ab^2c^2}{a^2c^2de^{dt}-ac^2d^2e^{dt}} - \frac{2C_{13}ab^2d}{a^3de^{at}-a^2d^2e^{at}} - \\ \frac{C_{10}d}{a} - \frac{C_{10}de^{-at}}{a} + \frac{C_{13}abc^2d}{a^2c^2de^{dt}-ac^2d^2e^{dt}} + \frac{C_{13}}{a^2e^{at}e^{dt}+} \end{array}\right.$$

Current algebraic equations solutions:

$$\left[\begin{array}{l} -\frac{C_{15}ace^{2dt}}{2ad^2+4d^3} - \frac{C_{15}cde^{2dt}}{2ad^2+4d^3} + \frac{C_{15}c}{a^2e^{at}+2ade^{at}} + \frac{C_{15}ce^{dt}}{d^2} - \frac{C_{15}c}{2d^2} - \frac{C_{15}c}{2ad} + \frac{C_2ce^{dt}}{d} - \frac{C_2c}{d} - \frac{C_4b}{a} + \frac{C_4be^{-at}}{a} - C_5 + \frac{C_7abc^2e^{2dt}}{2a^3d^2-2a^2d^3-8ad^4+8d^5} - \\ \frac{C_{10}bdt}{a} - \frac{C_{10}b}{a} + \frac{C_{10}be^{-at}}{a} - \frac{C_{10}bd}{a^2} + \frac{C_{10}bde^{-at}}{a^2} - C_{11} + \frac{C_{13}a^2b^2}{a^3de^{at}-a^2d^2e^{at}} + \frac{2C_{13}a^2b^2}{a^3d+a^2d^2} - \frac{2C_{13}ab^2c^2}{a^2c^2de^{dt}-ac^2d^2e^{dt}} + \frac{2C_{13}ab^2d}{a^3de^{at}-a^2d^2e^{at}} + \frac{4C_{13}ab}{a^3d+a^2} \end{array}\right.$$

Equation: $\frac{C_{10}bdt}{a} - \frac{C_{10}b}{a} + \frac{C_{10}be^{-at}}{a} - \frac{C_{10}bd}{a^2} + \frac{C_{10}bde^{-at}}{a^2} - C_{11} + \frac{C_{13}a^2b^2}{a^3de^{at}-a^2d^2e^{at}} + \frac{2C_{13}a^2b^2}{a^3d+a^2d^2} - \frac{2C_{13}ab^2c^2}{a^2c^2de^{dt}-ac^2d^2e^{dt}} + \frac{2C_{13}ab^2d}{a^3de^{at}-a^2d^2e^{at}} + \frac{4C_{13}ab^2d}{a^3d+a^2d^2} - \frac{C_{13}ab^2}{a^2de^{at}e^{dt}+ad^2e^{at}e^{dt}} - \frac{2C_{13}b^2c^2d}{a^2c^2de^{dt}-ac^2d^2e^{dt}} + \frac{C_{13}b^2d^2}{a^3de^{at}-a^2d^2e^{at}} + \frac{C_{13}b^2d^2}{a^3d+a^2d^2} - \frac{2C_{13}b^2d}{a^2de^{at}e^{dt}+ad^2e^{at}e^{dt}} - \frac{C_{13}b^2t}{a} + C_{14}bt + \frac{C_{16}ab^2}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{2C_{16}b^2d}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{C_{16}b^2d}{a^2c+acd} - \frac{C_{16}b^2e^{-at}}{cd} - \frac{C_{16}b^2e^{-at}}{ac} - \frac{C_{18}be^{-at}}{c} + \frac{C_{18}bd}{ac} - \frac{C_{18}bde^{-at}}{ac} = 0$ Basis functions:

$$\left[1.0,\ e^{-dt},\ t,\ e^{-at},\ e^{-at}e^{-dt}\right]$$

Solutions *after* processing:

$$C_{14} = -\frac{C_{10}d}{a}$$
$$C_{16} = 0$$
$$C_{18} = \frac{C_{10}c}{a}$$
$$C_{11} = -\frac{C_{10}b}{a}$$
$$C_{13} = 0$$

Current ODE solutions:

$$\mathbf{c} = \begin{bmatrix} C_1 - \frac{C_{10}}{a} + \frac{\mathrm{c}_{22}(t)}{c} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{a\,\mathrm{c}_{22}(t)}{c} \\ -\frac{b\,\mathrm{c}_{22}(t)}{c} \\ C_{12} \\ 0 \\ -\frac{d\,\mathrm{c}_{22}(t)}{c} \\ 0 \\ 0 \\ C_{17} \end{bmatrix}$$

Current algebraic equations solutions:

$$\begin{bmatrix} -\frac{C_{15}ace^{2dt}}{2ad^2+4d^3} - \frac{C_{15}cde^{2dt}}{2ad^2+4d^3} + \frac{C_{15}c}{a^2e^{at}+2ade^{at}} + \frac{C_{15}ce^{dt}}{d^2} - \frac{C_{15}c}{2d^2} - \frac{C_{15}c}{2ad} + \frac{C_2ce^{dt}}{d} - \frac{C_2c}{d} - \frac{C_4b}{a} + \frac{C_4be^{-at}}{a} - C_5 + \frac{C_7abc^2e^{2dt}}{2a^3d^2-2a^2d^3-8ad^4+8d^5} - \\ \frac{C_{10}bdt}{a} - \frac{C_{10}b}{a} + \frac{C_{10}be^{-at}}{a} - \frac{C_{10}bd}{a^2} + \frac{C_{10}bde^{-at}}{a^2} - C_{11} + \frac{C_{13}a^2b^2}{a^3de^{at}-a^2d^2e^{at}} + \frac{2C_{13}a^2b^2}{a^3d+a^2d^2} - \frac{2C_{13}ab^2c^2}{a^2c^2de^{dt}-ac^2d^2e^{dt}} + \frac{2C_{13}ab^2d}{a^3de^{at}-a^2d^2e^{at}} + \frac{4C_{13}ab}{a^3d+a^2} \\ \\ \end{bmatrix}$$

Equation: $-C_{12} = 0$ Basis functions:

$$[1.0]$$

Solutions *after* processing:

$$C_{12} = 0$$

Current ODE solutions:

$$\mathbf{c} = \begin{bmatrix} C_1 - \frac{C_{10}}{a} + \frac{\mathrm{c}_{22}(t)}{c} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{a\,\mathrm{c}_{22}(t)}{c} \\ -\frac{b\,\mathrm{c}_{22}(t)}{c} \\ 0 \\ 0 \\ -\frac{d\,\mathrm{c}_{22}(t)}{c} \\ 0 \\ 0 \\ C_{17} \end{bmatrix}$$

Current algebraic equations solutions:

$$\left[\begin{matrix} -\frac{C_{15}ace^{2dt}}{2ad^2+4d^3} - \frac{C_{15}cde^{2dt}}{2ad^2+4d^3} + \frac{C_{15}c}{a^2e^{at}+2ade^{at}} + \frac{C_{15}ce^{dt}}{d^2} - \frac{C_{15}c}{2d^2} - \frac{C_{15}c}{2ad} + \frac{C_2ce^{dt}}{d} - \frac{C_2c}{d} - \frac{C_4b}{a} + \frac{C_4be^{-at}}{a} - C_5 + \frac{C_7abc^2e^{2dt}}{2a^3d^2-2a^2d^3-8ad^4+8d^5} - \\ \frac{C_{10}bdt}{a} - \frac{C_{10}b}{a} + \frac{C_{10}be^{-at}}{a} - \frac{C_{10}bd}{a^2} + \frac{C_{10}bde^{-at}}{a^2} - C_{11} + \frac{C_{13}a^2b^2}{a^3de^{at}-a^2d^2e^{at}} + \frac{2C_{13}a^2b^2}{a^3d+a^2d^2} - \frac{2C_{13}ab^2c^2}{a^2c^2de^{dt}-ac^2d^2e^{dt}} + \frac{2C_{13}ab^2d}{a^3de^{at}-a^2d^2e^{at}} + \frac{4C_{13}ab}{a^3d+a^2} \\ \\ \end{matrix}\right.$$

Equation:$-C_{17} = 0$ Basis functions:

$$[1.0]$$

Solutions *after* processing:

$$C_{17} = 0$$

Current ODE solutions:

$$\mathbf{c} = \begin{bmatrix} C_1 - \frac{C_{10}}{a} + \frac{\mathrm{c}_{22}(t)}{c} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{a\,\mathrm{c}_{22}(t)}{c} \\ -\frac{b\,\mathrm{c}_{22}(t)}{c} \\ 0 \\ 0 \\ -\frac{d\,\mathrm{c}_{22}(t)}{c} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Current algebraic equations solutions:

$$\left[\begin{matrix} -\frac{C_{15}ace^{2dt}}{2ad^2+4d^3} - \frac{C_{15}cde^{2dt}}{2ad^2+4d^3} + \frac{C_{15}c}{a^2e^{at}+2ade^{at}} + \frac{C_{15}ce^{dt}}{d^2} - \frac{C_{15}c}{2d^2} - \frac{C_{15}c}{2ad} + \frac{C_2ce^{dt}}{d} - \frac{C_2c}{d} - \frac{C_4b}{a} + \frac{C_4be^{-at}}{a} - C_5 + \frac{C_7abc^2e^{2dt}}{2a^3d^2-2a^2d^3-8ad^4+8d^5} - \\ \frac{C_{10}bdt}{a} - \frac{C_{10}b}{a} + \frac{C_{10}be^{-at}}{a} - \frac{C_{10}bd}{a^2} + \frac{C_{10}bde^{-at}}{a^2} - C_{11} + \frac{C_{13}a^2b^2}{a^3de^{at}-a^2d^2e^{at}} + \frac{2C_{13}a^2b^2}{a^3d+a^2d^2} - \frac{2C_{13}ab^2c^2}{a^2c^2de^{dt}-ac^2d^2e^{dt}} + \frac{2C_{13}ab^2d}{a^3de^{at}-a^2d^2e^{at}} + \frac{4C_{13}ab}{a^3d+a^2} \\ \\ \end{matrix}\right.$$

Solutions after all is done:

$$\mathbf{c} = \begin{bmatrix} C_1 - \frac{C_{10}}{a} + \frac{\mathrm{c}_{22}(t)}{c} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{a\,\mathrm{c}_{22}(t)}{c} \\ -\frac{b\,\mathrm{c}_{22}(t)}{c} \\ 0 \\ 0 \\ -\frac{d\,\mathrm{c}_{22}(t)}{c} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Tangents before any manipulation:

$$\eta_0 = C_1 - \frac{C_{10}}{a} + \frac{\mathrm{c}_{22}(t)}{c}$$
$$\eta_1 = \frac{aN\,\mathrm{c}_{22}(t)}{c} - \frac{bNP\,\mathrm{c}_{22}(t)}{c}$$
$$\eta_2 = NP\,\mathrm{c}_{22}(t) - \frac{dP\,\mathrm{c}_{22}(t)}{c}$$

Generators before some are removed:

$$\left[-\frac{1}{a},\ 0,\ 0\right]$$

$$[1,\ 0,\ 0]$$

$$\left[\frac{\mathrm{c}_{22}(t)}{c},\ \frac{aN\,\mathrm{c}_{22}(t)}{c} - \frac{bNP\,\mathrm{c}_{22}(t)}{c},\ NP\,\mathrm{c}_{22}(t) - \frac{dP\,\mathrm{c}_{22}(t)}{c}\right]$$

The tangents and their symmetry conditions:

$$\eta_0 = \left[-\frac{1}{a},\ 0,\ 0\right]$$

$$[0,\ 0]$$

$$\eta_1 = [1,\ 0,\ 0]$$

$$[0,\ 0]$$

$$\eta_2 = \left[\frac{\mathrm{c}_{22}(t)}{c},\ \frac{aN\,\mathrm{c}_{22}(t)}{c} - \frac{bNP\,\mathrm{c}_{22}(t)}{c},\ NP\,\mathrm{c}_{22}(t) - \frac{dP\,\mathrm{c}_{22}(t)}{c}\right]$$

$$[0,\ 0]$$

Generators after some were removed:

$$\left[-\frac{1}{a},\ 0,\ 0\right]$$

$$[1,\ 0,\ 0]$$

$$\left[\frac{c_{22}(t)}{c},\ \frac{aN\,c_{22}(t)}{c} - \frac{bNP\,c_{22}(t)}{c},\ NP\,c_{22}(t) - \frac{dP\,c_{22}(t)}{c}\right]$$

Generator:

$$-\frac{1}{a}$$

Component parts of generator:

$$['-frac1a']$$

Generator:

$$1$$

Component parts of generator:

$$['1']$$

Generator:

$$\frac{f_1(t)}{c}$$

Component parts of generator:

$$['fracoperatornamef_1left(tright)c']$$

Generator:

$$-\frac{NPb\,f_1(t)}{c} + \frac{Na\,f_1(t)}{c}$$

Component parts of generator:

$$['fracNaoperatornamef_1left(tright)c','-fracNPboperatornamef_1left(tright)c']$$

Generator:

$$NP\,f_1(t) - \frac{Pd\,f_1(t)}{c}$$

Component parts of generator:

$$['NPoperatornamef_1left(tright) - fracPdoperatornamef_1left(tright)c']$$

The execution time of the script was:

0 hours 1 minutes 9 seconds.