DBH_model_error_search_ODEsys_and_algSys

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1 Error search ODE system and algebra system DBH model ansatz degree 1

Date: 2021-09-16, *Written by:* Johannes Borgqvist.

This document contains an attempt of finding the error in the calculations of the symmetry generators for Hydon's model. The model at hand is the following two component ODE system:

$$\begin{aligned}\frac{\mathrm{d}w_1}{\mathrm{d}t} &= -w_1w_2 - w_1w_3 + w_2w_3 = \omega_1(t, w_1, w_2, w_3)\\ \frac{\mathrm{d}w_2}{\mathrm{d}t} &= -w_1w_2 + w_1w_3 - w_2w_3 = \omega_2(t, w_1, w_2, w_3)\\ \frac{\mathrm{d}w_3}{\mathrm{d}t} &= w_1w_2 - w_1w_3 - w_2w_3 = \omega_3(t, w_1, w_2, w_3)\end{aligned}$$

To this model, the aim is to find the most general form of the *infinitesimal generator of the Lie group* denoted by X which is defined as follows:

$$X = \xi(t, w_1, w_2, w_3)\partial_t + \eta_1(t, w_1, w_2, w_3)\partial_{w_1} + \eta_2(t, w_1, w_2, w_3)\partial_{w_2} + \eta_3(t, w_1, w_2, w_3)\partial_{w_3}.$$

To find this generator, a set of *linear ansätze* is used for the three tangents as follows:

$$\begin{aligned}\xi &= w_1\,\mathrm{c}_{01}(t) + w_2\,\mathrm{c}_{02}(t) + w_3\,\mathrm{c}_{03}(t) + \mathrm{c}_{00}(t)\\ \eta_1 &= w_1\,\mathrm{c}_{11}(t) + w_2\,\mathrm{c}_{12}(t) + w_3\,\mathrm{c}_{13}(t) + \mathrm{c}_{10}(t)\\ \eta_2 &= w_1\,\mathrm{c}_{21}(t) + w_2\,\mathrm{c}_{22}(t) + w_3\,\mathrm{c}_{23}(t) + \mathrm{c}_{20}(t)\\ \eta_3 &= w_1\,\mathrm{c}_{31}(t) + w_2\,\mathrm{c}_{32}(t) + w_3\,\mathrm{c}_{33}(t) + \mathrm{c}_{30}(t)\end{aligned}$$

The aim is to find the nine arbitrary functions $c_{ij}(t)$ for the two indices $i, j \in \{0, 1, 2\}$. The equations required in order to find these constants are given by the three *linearised symmetry conditions* given by

$$X^{(1)}(w_k' - \omega_k(t, y_1, y_2)) = 0, \quad \text{for } k \in \{1, 2, 3\}.$$

Here, $X^{(1)}$ corresponds to the prolonged generator given by

$$X^{(1)} = X + \eta_1^{(1)}\partial_{w_1'} + \eta_2^{(1)}\partial_{w_2'} + \eta_3^{(1)}\partial_{w_3'}$$

where the prolonged tangents are given by the *prolongation formula*:

$$\eta_k^{(1)} = D_t\eta_k - w_k' D_t\xi, \quad \text{for } \ k \in \{1,2,3\}$$

where the total derivative is defined as follows: $D_t = \partial_t + w_1'\partial_{w_1} + w_2'\partial_{w_2} + w_3'\partial_{w_3}$.

What is nice about the DBH model is that it has at least three known generators, namely the *scaling generator* given by

$$X = w_1\partial_{w_1} + w_2\partial_{w_2} + w_3\partial_{w_3}$$

and also the translation generator

$$X = \partial_t + \partial_{w_1} + \partial_{w_2} + \partial_{w_3}.$$

Thus, we now when the algorithm performs correctly in this case as the above generator should be returned as an output.

Moreover, plugging in these ansätze into the linearised symmetry conditions will result in a linear system of equations which can be formulated on matrix form as follows:

$$A\frac{\mathrm{d}\mathbf{c}(t)}{\mathrm{d}t} = B\mathbf{c}(t)$$

where the vector $\mathbf{c}(t) \in \mathcal{C}(\mathbb{R}^{16})$ contains the nine arbitrary coefficients in the tangential ansätze. Typically, the number of equations are much larger than the number of unknowns meaning that if $A, B \in \mathcal{C}\left(\mathbb{R}^{n\times m}\right)$ then $n \gg m$ (in this case $m = 16$). After row reducing this system and simplifying it is (in the best of worlds) possible to write the system on the following form:

$$\frac{\mathrm{d}\mathbf{c}(t)}{\mathrm{d}t} = B\mathbf{c}(t), \tag{1}$$

$$B_{\text{algebraic}}\mathbf{c}(t) = \mathbf{0}. \tag{2}$$

The first ODE system is a quadratic ODE system which can be solved using the Jordan decomposition. That is if

$$B = P^{-1}JP$$

then the solution to the ODE system is given by

$$\mathbf{c}(t) = P^{-1}e^{Jt}P\mathbf{c}_0$$

for some initial condition $\mathbf{c}_0$ composed of arbitrary integration constants. Then the solution of the system of ODEs is plugged in to the algebraic equations given by the second matrix equation above. This will result in certain algebraic equations that can simplify the results even further.

Now, the problem is that certain generators are obtained that do not solve the linearised symmetry conditions. This implies that the implementation of the algorithm is wrong, as the methodology of ansätze can never yield non-solutions. Therefore, the Hydon example will be used to see if the error is introduced in the solution of the ODE system or if it is when certain simplifications are made when the algebraic equations are solved.

What will be done in the subsequent cells is that all matrices will be printed out and then we will try to track down the error.

2 Defining the tangents

The tangents are:

$$\begin{aligned}
\xi &= w_1\, \mathrm{c}_{01}(t) + w_2\, \mathrm{c}_{02}(t) + w_3\, \mathrm{c}_{03}(t) + \mathrm{c}_{00}(t) \\
\eta_1 &= w_1\, \mathrm{c}_{11}(t) + w_2\, \mathrm{c}_{12}(t) + w_3\, \mathrm{c}_{13}(t) + \mathrm{c}_{10}(t) \\
\eta_2 &= w_1\, \mathrm{c}_{21}(t) + w_2\, \mathrm{c}_{22}(t) + w_3\, \mathrm{c}_{23}(t) + \mathrm{c}_{20}(t) \\
\eta_3 &= w_1\, \mathrm{c}_{31}(t) + w_2\, \mathrm{c}_{32}(t) + w_3\, \mathrm{c}_{33}(t) + \mathrm{c}_{30}(t)
\end{aligned}$$

The unknown coefficients:

$$\mathbf{c} = \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} \tag{3}$$

3 The linearised symmetry conditions

Linearised symmetry condition 1:

$$
\begin{aligned}
0 = {} & \left(-\frac{d}{dt}\,\mathrm{c}_{10}\left(t\right)\right) + \left(w_2\,\mathrm{c}_{30}\left(t\right)\right) + \left(w_3\,\mathrm{c}_{20}\left(t\right)\right) + \left(w_2^2\,\mathrm{c}_{32}\left(t\right)\right) + \left(w_3^2\,\mathrm{c}_{23}\left(t\right)\right) + \left(-w_1\frac{d}{dt}\,\mathrm{c}_{11}\left(t\right)\right) \\
& + \left(-w_1\,\mathrm{c}_{20}\left(t\right)\right) + \left(-w_1\,\mathrm{c}_{30}\left(t\right)\right) + \left(-w_2\frac{d}{dt}\,\mathrm{c}_{12}\left(t\right)\right) + \left(-w_2\,\mathrm{c}_{10}\left(t\right)\right) + \left(-w_3\frac{d}{dt}\,\mathrm{c}_{13}\left(t\right)\right) \\
& + \left(-w_3\,\mathrm{c}_{10}\left(t\right)\right) + \left(-w_1^2\,\mathrm{c}_{21}\left(t\right)\right) + \left(-w_1^2\,\mathrm{c}_{31}\left(t\right)\right) + \left(-w_2^2\,\mathrm{c}_{12}\left(t\right)\right) + \left(-w_3^2\,\mathrm{c}_{13}\left(t\right)\right) \\
& + \left(w_1w_2\,\mathrm{c}_{12}\left(t\right)\right) + \left(w_1w_2\,\mathrm{c}_{31}\left(t\right)\right) + \left(w_1w_3\,\mathrm{c}_{13}\left(t\right)\right) + \left(w_1w_3\,\mathrm{c}_{21}\left(t\right)\right) + \left(w_2w_3\frac{d}{dt}\,\mathrm{c}_{00}\left(t\right)\right) \\
& + \left(w_2w_3\,\mathrm{c}_{22}\left(t\right)\right) + \left(w_2w_3\,\mathrm{c}_{33}\left(t\right)\right) + \left(w_2w_3^2\frac{d}{dt}\,\mathrm{c}_{03}\left(t\right)\right) + \left(w_2^2w_3\frac{d}{dt}\,\mathrm{c}_{02}\left(t\right)\right) + \left(w_1^2w_2^2\,\mathrm{c}_{01}\left(t\right)\right) \\
& + \left(w_1^2w_2^2\,\mathrm{c}_{02}\left(t\right)\right) + \left(w_1^2w_3^2\,\mathrm{c}_{01}\left(t\right)\right) + \left(w_1^2w_3^2\,\mathrm{c}_{03}\left(t\right)\right) + \left(w_2^2w_3^2\,\mathrm{c}_{01}\left(t\right)\right) + \left(-w_1w_2\frac{d}{dt}\,\mathrm{c}_{00}\left(t\right)\right) \\
& + \left(-w_1w_2\,\mathrm{c}_{13}\left(t\right)\right) + \left(-w_1w_2\,\mathrm{c}_{22}\left(t\right)\right) + \left(-w_1w_2\,\mathrm{c}_{32}\left(t\right)\right) + \left(-w_1w_3\frac{d}{dt}\,\mathrm{c}_{00}\left(t\right)\right) + \left(-w_1w_3\,\mathrm{c}_{12}\left(t\right)\right) \\
& + \left(-w_1w_3\,\mathrm{c}_{23}\left(t\right)\right) + \left(-w_1w_3\,\mathrm{c}_{33}\left(t\right)\right) + \left(-w_1w_2^2\frac{d}{dt}\,\mathrm{c}_{02}\left(t\right)\right) + \left(-w_1w_3^2\frac{d}{dt}\,\mathrm{c}_{03}\left(t\right)\right) + \left(-w_2w_3\,\mathrm{c}_{11}\left(t\right)\right) \\
& + \left(-w_1^2w_2\frac{d}{dt}\,\mathrm{c}_{01}\left(t\right)\right) + \left(-w_1^2w_3\frac{d}{dt}\,\mathrm{c}_{01}\left(t\right)\right) + \left(-w_1^2w_2^2\,\mathrm{c}_{03}\left(t\right)\right) + \left(-w_1^2w_3^2\,\mathrm{c}_{02}\left(t\right)\right) + \left(-w_2^2w_3^2\,\mathrm{c}_{02}\left(t\right)\right) \\
& + \left(-w_2^2w_3^2\,\mathrm{c}_{03}\left(t\right)\right) + \left(w_1w_2w_3\frac{d}{dt}\,\mathrm{c}_{01}\left(t\right)\right) + \left(-w_1w_2w_3\frac{d}{dt}\,\mathrm{c}_{02}\left(t\right)\right) + \left(-w_1w_2w_3\frac{d}{dt}\,\mathrm{c}_{03}\left(t\right)\right) + \left(-2w_1w_2w_3^2\,\mathrm{c}_{01}\left(t\right)\right)
\end{aligned}
$$

Linearised symmetry condition 2:

$$
\begin{aligned}
0 = {} & \left(-\frac{d}{dt}\,\mathrm{c}_{20}\left(t\right)\right) + \left(w_1\,\mathrm{c}_{30}\left(t\right)\right) + \left(w_3\,\mathrm{c}_{10}\left(t\right)\right) + \left(w_1^2\,\mathrm{c}_{31}\left(t\right)\right) + \left(w_3^2\,\mathrm{c}_{13}\left(t\right)\right) + \left(-w_1\frac{d}{dt}\,\mathrm{c}_{21}\left(t\right)\right) \\
& + \left(-w_1\,\mathrm{c}_{20}\left(t\right)\right) + \left(-w_2\frac{d}{dt}\,\mathrm{c}_{22}\left(t\right)\right) + \left(-w_2\,\mathrm{c}_{10}\left(t\right)\right) + \left(-w_2\,\mathrm{c}_{30}\left(t\right)\right) + \left(-w_3\frac{d}{dt}\,\mathrm{c}_{23}\left(t\right)\right) \\
& + \left(-w_3\,\mathrm{c}_{20}\left(t\right)\right) + \left(-w_1^2\,\mathrm{c}_{21}\left(t\right)\right) + \left(-w_2^2\,\mathrm{c}_{12}\left(t\right)\right) + \left(-w_2^2\,\mathrm{c}_{32}\left(t\right)\right) + \left(-w_3^2\,\mathrm{c}_{23}\left(t\right)\right) \\
& + \left(w_1w_2\,\mathrm{c}_{21}\left(t\right)\right) + \left(w_1w_2\,\mathrm{c}_{32}\left(t\right)\right) + \left(w_1w_3\frac{d}{dt}\,\mathrm{c}_{00}\left(t\right)\right) + \left(w_1w_3\,\mathrm{c}_{11}\left(t\right)\right) + \left(w_1w_3\,\mathrm{c}_{33}\left(t\right)\right) \\
& + \left(w_1w_3^2\frac{d}{dt}\,\mathrm{c}_{03}\left(t\right)\right) + \left(w_2w_3\,\mathrm{c}_{12}\left(t\right)\right) + \left(w_2w_3\,\mathrm{c}_{23}\left(t\right)\right) + \left(w_1^2w_3\frac{d}{dt}\,\mathrm{c}_{01}\left(t\right)\right) + \left(w_1^2w_2^2\,\mathrm{c}_{01}\left(t\right)\right) \\
& + \left(w_1^2w_2^2\,\mathrm{c}_{02}\left(t\right)\right) + \left(w_1^2w_3^2\,\mathrm{c}_{02}\left(t\right)\right) + \left(w_2^2w_3^2\,\mathrm{c}_{02}\left(t\right)\right) + \left(w_2^2w_3^2\,\mathrm{c}_{03}\left(t\right)\right) + \left(-w_1w_2\frac{d}{dt}\,\mathrm{c}_{00}\left(t\right)\right) \\
& + \left(-w_1w_2\,\mathrm{c}_{11}\left(t\right)\right) + \left(-w_1w_2\,\mathrm{c}_{23}\left(t\right)\right) + \left(-w_1w_2\,\mathrm{c}_{31}\left(t\right)\right) + \left(-w_1w_3\,\mathrm{c}_{22}\left(t\right)\right) + \left(-w_1w_2^2\frac{d}{dt}\,\mathrm{c}_{02}\left(t\right)\right) \\
& + \left(-w_2w_3\frac{d}{dt}\,\mathrm{c}_{00}\left(t\right)\right) + \left(-w_2w_3\,\mathrm{c}_{13}\left(t\right)\right) + \left(-w_2w_3\,\mathrm{c}_{21}\left(t\right)\right) + \left(-w_2w_3\,\mathrm{c}_{33}\left(t\right)\right) + \left(-w_1^2w_2\frac{d}{dt}\,\mathrm{c}_{01}\left(t\right)\right) \\
& + \left(-w_2w_3^2\frac{d}{dt}\,\mathrm{c}_{03}\left(t\right)\right) + \left(-w_2^2w_3\frac{d}{dt}\,\mathrm{c}_{02}\left(t\right)\right) + \left(-w_1^2w_2^2\,\mathrm{c}_{03}\left(t\right)\right) + \left(-w_1^2w_3^2\,\mathrm{c}_{01}\left(t\right)\right) + \left(-w_1^2w_3^2\,\mathrm{c}_{03}\left(t\right)\right) \\
& + \left(-w_2^2w_3^2\,\mathrm{c}_{01}\left(t\right)\right) + \left(w_1w_2w_3\frac{d}{dt}\,\mathrm{c}_{02}\left(t\right)\right) + \left(-w_1w_2w_3\frac{d}{dt}\,\mathrm{c}_{01}\left(t\right)\right) + \left(-w_1w_2w_3\frac{d}{dt}\,\mathrm{c}_{03}\left(t\right)\right) + \left(-2w_1w_2w_3^2\,\mathrm{c}_{02}\left(t\right)\right)
\end{aligned}
$$

Linearised symmetry condition 3:

$$
\begin{aligned}
0 = &\left(-\frac{d}{dt}\,\mathrm{c}_{30}\left(t\right)\right) + \left(w_1\,\mathrm{c}_{20}\left(t\right)\right) + \left(w_2\,\mathrm{c}_{10}\left(t\right)\right) + \left(w_1^2\,\mathrm{c}_{21}\left(t\right)\right) + \left(w_2^2\,\mathrm{c}_{12}\left(t\right)\right) + \left(-w_1\frac{d}{dt}\,\mathrm{c}_{31}\left(t\right)\right) \\
&+ \left(-w_1\,\mathrm{c}_{30}\left(t\right)\right) + \left(-w_2\frac{d}{dt}\,\mathrm{c}_{32}\left(t\right)\right) + \left(-w_2\,\mathrm{c}_{30}\left(t\right)\right) + \left(-w_3\frac{d}{dt}\,\mathrm{c}_{33}\left(t\right)\right) + \left(-w_3\,\mathrm{c}_{10}\left(t\right)\right) \\
&+ \left(-w_3\,\mathrm{c}_{20}\left(t\right)\right) + \left(-w_1^2\,\mathrm{c}_{31}\left(t\right)\right) + \left(-w_2^2\,\mathrm{c}_{32}\left(t\right)\right) + \left(-w_3^2\,\mathrm{c}_{13}\left(t\right)\right) + \left(-w_3^2\,\mathrm{c}_{23}\left(t\right)\right) \\
&+ \left(w_1w_2\frac{d}{dt}\,\mathrm{c}_{00}\left(t\right)\right) + \left(w_1w_2\,\mathrm{c}_{11}\left(t\right)\right) + \left(w_1w_2\,\mathrm{c}_{22}\left(t\right)\right) + \left(w_1w_3\,\mathrm{c}_{23}\left(t\right)\right) + \left(w_1w_3\,\mathrm{c}_{31}\left(t\right)\right) \\
&+ \left(w_1w_2^2\frac{d}{dt}\,\mathrm{c}_{02}\left(t\right)\right) + \left(w_2w_3\,\mathrm{c}_{13}\left(t\right)\right) + \left(w_2w_3\,\mathrm{c}_{32}\left(t\right)\right) + \left(w_1^2w_2\frac{d}{dt}\,\mathrm{c}_{01}\left(t\right)\right) + \left(w_1^2w_2^2\,\mathrm{c}_{03}\left(t\right)\right) \\
&+ \left(w_1^2w_3^2\,\mathrm{c}_{01}\left(t\right)\right) + \left(w_1^2w_3^2\,\mathrm{c}_{03}\left(t\right)\right) + \left(w_2^2w_3^2\,\mathrm{c}_{02}\left(t\right)\right) + \left(w_2^2w_3^2\,\mathrm{c}_{03}\left(t\right)\right) + \left(-w_1w_2\,\mathrm{c}_{33}\left(t\right)\right) \\
&+ \left(-w_1w_3\frac{d}{dt}\,\mathrm{c}_{00}\left(t\right)\right) + \left(-w_1w_3\,\mathrm{c}_{11}\left(t\right)\right) + \left(-w_1w_3\,\mathrm{c}_{21}\left(t\right)\right) + \left(-w_1w_3\,\mathrm{c}_{32}\left(t\right)\right) + \left(-w_1w_3^2\frac{d}{dt}\,\mathrm{c}_{03}\left(t\right)\right) \\
&+ \left(-w_2w_3\frac{d}{dt}\,\mathrm{c}_{00}\left(t\right)\right) + \left(-w_2w_3\,\mathrm{c}_{12}\left(t\right)\right) + \left(-w_2w_3\,\mathrm{c}_{22}\left(t\right)\right) + \left(-w_2w_3\,\mathrm{c}_{31}\left(t\right)\right) + \left(-w_2w_3^2\frac{d}{dt}\,\mathrm{c}_{03}\left(t\right)\right) \\
&+ \left(-w_1^2w_3\frac{d}{dt}\,\mathrm{c}_{01}\left(t\right)\right) + \left(-w_2^2w_3\frac{d}{dt}\,\mathrm{c}_{02}\left(t\right)\right) + \left(-w_1^2w_2^2\,\mathrm{c}_{01}\left(t\right)\right) + \left(-w_1^2w_2^2\,\mathrm{c}_{02}\left(t\right)\right) + \left(-w_1^2w_3^2\,\mathrm{c}_{02}\left(t\right)\right) \\
&+ \left(-w_2^2w_3^2\,\mathrm{c}_{01}\left(t\right)\right) + \left(w_1w_2w_3\frac{d}{dt}\,\mathrm{c}_{03}\left(t\right)\right) + \left(-w_1w_2w_3\frac{d}{dt}\,\mathrm{c}_{01}\left(t\right)\right) + \left(-w_1w_2w_3\frac{d}{dt}\,\mathrm{c}_{02}\left(t\right)\right) + \left(-2w_1w_2^2w_3\,\mathrm{c}_{03}\left(t\right)\right)
\end{aligned}
$$

4 The determining equations

Determining equations from linearised symmetry condition 1:

$$
\begin{aligned}
w_1^0 w_2^0 w_3^0 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{10}(t)\\
w_1^0 w_2^0 w_3^1 &: 0 = -\mathrm{c}_{10}(t) + \mathrm{c}_{20}(t) - \frac{d}{dt}\,\mathrm{c}_{13}(t)\\
w_1^0 w_2^0 w_3^2 &: 0 = -\mathrm{c}_{13}(t) + \mathrm{c}_{23}(t)\\
w_1^0 w_2^1 w_3^0 &: 0 = -\mathrm{c}_{10}(t) + \mathrm{c}_{30}(t) - \frac{d}{dt}\,\mathrm{c}_{12}(t)\\
w_1^0 w_2^1 w_3^1 &: 0 = -\mathrm{c}_{11}(t) + \mathrm{c}_{22}(t) + \mathrm{c}_{33}(t) + \frac{d}{dt}\,\mathrm{c}_{00}(t)\\
w_1^0 w_2^1 w_3^2 &: 0 = \frac{d}{dt}\,\mathrm{c}_{03}(t)\\
w_1^0 w_2^2 w_3^0 &: 0 = -\mathrm{c}_{12}(t) + \mathrm{c}_{32}(t)\\
w_1^0 w_2^2 w_3^1 &: 0 = \frac{d}{dt}\,\mathrm{c}_{02}(t)\\
w_1^0 w_2^2 w_3^2 &: 0 = \mathrm{c}_{01}(t) - \mathrm{c}_{02}(t) - \mathrm{c}_{03}(t)\\
w_1^1 w_2^0 w_3^0 &: 0 = -\mathrm{c}_{20}(t) - \mathrm{c}_{30}(t) - \frac{d}{dt}\,\mathrm{c}_{11}(t)\\
w_1^1 w_2^0 w_3^1 &: 0 = -\mathrm{c}_{12}(t) + \mathrm{c}_{13}(t) + \mathrm{c}_{21}(t) - \mathrm{c}_{23}(t) - \mathrm{c}_{33}(t) - \frac{d}{dt}\,\mathrm{c}_{00}(t)\\
w_1^1 w_2^0 w_3^2 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{03}(t)\\
w_1^1 w_2^1 w_3^0 &: 0 = \mathrm{c}_{12}(t) - \mathrm{c}_{13}(t) - \mathrm{c}_{22}(t) + \mathrm{c}_{31}(t) - \mathrm{c}_{32}(t) - \frac{d}{dt}\,\mathrm{c}_{00}(t)\\
w_1^1 w_2^1 w_3^1 &: 0 = \frac{d}{dt}\,\mathrm{c}_{01}(t) - \frac{d}{dt}\,\mathrm{c}_{02}(t) - \frac{d}{dt}\,\mathrm{c}_{03}(t)\\
w_1^1 w_2^1 w_3^2 &: 0 = -2\,\mathrm{c}_{01}(t) + 2\,\mathrm{c}_{02}(t)\\
w_1^1 w_2^2 w_3^0 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{02}(t)\\
w_1^1 w_2^2 w_3^1 &: 0 = -2\,\mathrm{c}_{01}(t) + 2\,\mathrm{c}_{03}(t)\\
w_1^2 w_2^0 w_3^0 &: 0 = -\mathrm{c}_{21}(t) - \mathrm{c}_{31}(t)\\
w_1^2 w_2^0 w_3^1 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{01}(t)\\
w_1^2 w_2^0 w_3^2 &: 0 = \mathrm{c}_{01}(t) - \mathrm{c}_{02}(t) + \mathrm{c}_{03}(t)\\
w_1^2 w_2^1 w_3^0 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{01}(t)\\
w_1^2 w_2^1 w_3^1 &: 0 = 2\,\mathrm{c}_{01}(t)\\
w_1^2 w_2^2 w_3^0 &: 0 = \mathrm{c}_{01}(t) + \mathrm{c}_{02}(t) - \mathrm{c}_{03}(t)
\end{aligned}
$$

Determining equations from linearised symmetry condition 2:

$$
\begin{aligned}
w_1^0 w_2^0 w_3^0 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{20}\,(t) \\
w_1^0 w_2^0 w_3^1 &: 0 = \mathrm{c}_{10}\,(t) - \mathrm{c}_{20}\,(t) - \frac{d}{dt}\,\mathrm{c}_{23}\,(t) \\
w_1^0 w_2^0 w_3^2 &: 0 = \mathrm{c}_{13}\,(t) - \mathrm{c}_{23}\,(t) \\
w_1^0 w_2^1 w_3^0 &: 0 = -\,\mathrm{c}_{10}\,(t) - \mathrm{c}_{30}\,(t) - \frac{d}{dt}\,\mathrm{c}_{22}\,(t) \\
w_1^0 w_2^1 w_3^1 &: 0 = \mathrm{c}_{12}\,(t) - \mathrm{c}_{13}\,(t) - \mathrm{c}_{21}\,(t) + \mathrm{c}_{23}\,(t) - \mathrm{c}_{33}\,(t) - \frac{d}{dt}\,\mathrm{c}_{00}\,(t) \\
w_1^0 w_2^1 w_3^2 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{03}\,(t) \\
w_1^0 w_2^2 w_3^0 &: 0 = -\,\mathrm{c}_{12}\,(t) - \mathrm{c}_{32}\,(t) \\
w_1^0 w_2^2 w_3^1 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{02}\,(t) \\
w_1^0 w_2^2 w_3^2 &: 0 = -\,\mathrm{c}_{01}\,(t) + \mathrm{c}_{02}\,(t) + \mathrm{c}_{03}\,(t) \\
w_1^1 w_2^0 w_3^0 &: 0 = -\,\mathrm{c}_{20}\,(t) + \mathrm{c}_{30}\,(t) - \frac{d}{dt}\,\mathrm{c}_{21}\,(t) \\
w_1^1 w_2^0 w_3^1 &: 0 = \mathrm{c}_{11}\,(t) - \mathrm{c}_{22}\,(t) + \mathrm{c}_{33}\,(t) + \frac{d}{dt}\,\mathrm{c}_{00}\,(t) \\
w_1^1 w_2^0 w_3^2 &: 0 = \frac{d}{dt}\,\mathrm{c}_{03}\,(t) \\
w_1^1 w_2^1 w_3^0 &: 0 = -\,\mathrm{c}_{11}\,(t) + \mathrm{c}_{21}\,(t) - \mathrm{c}_{23}\,(t) - \mathrm{c}_{31}\,(t) + \mathrm{c}_{32}\,(t) - \frac{d}{dt}\,\mathrm{c}_{00}\,(t) \\
w_1^1 w_2^1 w_3^1 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{01}\,(t) + \frac{d}{dt}\,\mathrm{c}_{02}\,(t) - \frac{d}{dt}\,\mathrm{c}_{03}\,(t) \\
w_1^1 w_2^1 w_3^2 &: 0 = 2\,\mathrm{c}_{01}\,(t) - 2\,\mathrm{c}_{02}\,(t) \\
w_1^1 w_2^2 w_3^0 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{02}\,(t) \\
w_1^1 w_2^2 w_3^1 &: 0 = 2\,\mathrm{c}_{02}\,(t) \\
w_1^2 w_2^0 w_3^0 &: 0 = -\,\mathrm{c}_{21}\,(t) + \mathrm{c}_{31}\,(t) \\
w_1^2 w_2^0 w_3^1 &: 0 = \frac{d}{dt}\,\mathrm{c}_{01}\,(t) \\
w_1^2 w_2^0 w_3^2 &: 0 = -\,\mathrm{c}_{01}\,(t) + \mathrm{c}_{02}\,(t) - \mathrm{c}_{03}\,(t) \\
w_1^2 w_2^1 w_3^0 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{01}\,(t) \\
w_1^2 w_2^1 w_3^1 &: 0 = -2\,\mathrm{c}_{02}\,(t) + 2\,\mathrm{c}_{03}\,(t) \\
w_1^2 w_2^2 w_3^0 &: 0 = \mathrm{c}_{01}\,(t) + \mathrm{c}_{02}\,(t) - \mathrm{c}_{03}\,(t)
\end{aligned}
$$

Determining equations from linearised symmetry condition 3:

$$
\begin{aligned}
w_1^0 w_2^0 w_3^0 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{30}(t)\\
w_1^0 w_2^0 w_3^1 &: 0 = -\mathrm{c}_{10}(t) - \mathrm{c}_{20}(t) - \frac{d}{dt}\,\mathrm{c}_{33}(t)\\
w_1^0 w_2^0 w_3^2 &: 0 = -\mathrm{c}_{13}(t) - \mathrm{c}_{23}(t)\\
w_1^0 w_2^1 w_3^0 &: 0 = \mathrm{c}_{10}(t) - \mathrm{c}_{30}(t) - \frac{d}{dt}\,\mathrm{c}_{32}(t)\\
w_1^0 w_2^1 w_3^1 &: 0 = -\mathrm{c}_{12}(t) + \mathrm{c}_{13}(t) - \mathrm{c}_{22}(t) - \mathrm{c}_{31}(t) + \mathrm{c}_{32}(t) - \frac{d}{dt}\,\mathrm{c}_{00}(t)\\
w_1^0 w_2^1 w_3^2 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{03}(t)\\
w_1^0 w_2^2 w_3^0 &: 0 = \mathrm{c}_{12}(t) - \mathrm{c}_{32}(t)\\
w_1^0 w_2^2 w_3^1 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{02}(t)\\
w_1^0 w_2^2 w_3^2 &: 0 = -\mathrm{c}_{01}(t) + \mathrm{c}_{02}(t) + \mathrm{c}_{03}(t)\\
w_1^1 w_2^0 w_3^0 &: 0 = \mathrm{c}_{20}(t) - \mathrm{c}_{30}(t) - \frac{d}{dt}\,\mathrm{c}_{31}(t)\\
w_1^1 w_2^0 w_3^1 &: 0 = -\mathrm{c}_{11}(t) - \mathrm{c}_{21}(t) + \mathrm{c}_{23}(t) + \mathrm{c}_{31}(t) - \mathrm{c}_{32}(t) - \frac{d}{dt}\,\mathrm{c}_{00}(t)\\
w_1^1 w_2^0 w_3^2 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{03}(t)\\
w_1^1 w_2^1 w_3^0 &: 0 = \mathrm{c}_{11}(t) + \mathrm{c}_{22}(t) - \mathrm{c}_{33}(t) + \frac{d}{dt}\,\mathrm{c}_{00}(t)\\
w_1^1 w_2^1 w_3^1 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{01}(t) - \frac{d}{dt}\,\mathrm{c}_{02}(t) + \frac{d}{dt}\,\mathrm{c}_{03}(t)\\
w_1^1 w_2^1 w_3^2 &: 0 = 2\,\mathrm{c}_{03}(t)\\
w_1^1 w_2^2 w_3^0 &: 0 = \frac{d}{dt}\,\mathrm{c}_{02}(t)\\
w_1^1 w_2^2 w_3^1 &: 0 = 2\,\mathrm{c}_{01}(t) - 2\,\mathrm{c}_{03}(t)\\
w_1^2 w_2^0 w_3^0 &: 0 = \mathrm{c}_{21}(t) - \mathrm{c}_{31}(t)\\
w_1^2 w_2^0 w_3^1 &: 0 = -\frac{d}{dt}\,\mathrm{c}_{01}(t)\\
w_1^2 w_2^0 w_3^2 &: 0 = \mathrm{c}_{01}(t) - \mathrm{c}_{02}(t) + \mathrm{c}_{03}(t)\\
w_1^2 w_2^1 w_3^0 &: 0 = \frac{d}{dt}\,\mathrm{c}_{01}(t)\\
w_1^2 w_2^1 w_3^1 &: 0 = 2\,\mathrm{c}_{02}(t) - 2\,\mathrm{c}_{03}(t)\\
w_1^2 w_2^2 w_3^0 &: 0 = -\mathrm{c}_{01}(t) - \mathrm{c}_{02}(t) + \mathrm{c}_{03}(t)
\end{aligned}
$$

5 Solving the determining equations

5.1 Step 0 of 6: Defining the matrices from the determining equations

5.1.1 Determining equation 1 out of 69

$$-\frac{d}{dt}\,\mathrm{c}_{10}\left(t\right)=0 \tag{4}$$

$$\left[\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\mathrm{c}_{00}\\ \mathrm{c}_{03}\\ \mathrm{c}_{02}\\ \mathrm{c}_{01}\\ \mathrm{c}_{10}\\ \mathrm{c}_{13}\\ \mathrm{c}_{12}\\ \mathrm{c}_{11}\\ \mathrm{c}_{20}\\ \mathrm{c}_{23}\\ \mathrm{c}_{22}\\ \mathrm{c}_{21}\\ \mathrm{c}_{30}\\ \mathrm{c}_{33}\\ \mathrm{c}_{32}\\ \mathrm{c}_{31}\end{bmatrix}=\left[\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{5}$$

Determining equation 2 out of 69

$$-\mathrm{c}_{10}\left(t\right)+\mathrm{c}_{20}\left(t\right)-\frac{d}{dt}\,\mathrm{c}_{13}\left(t\right)=0 \tag{6}$$

$$\left[\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\mathrm{c}_{00}\\ \mathrm{c}_{03}\\ \mathrm{c}_{02}\\ \mathrm{c}_{01}\\ \mathrm{c}_{10}\\ \mathrm{c}_{13}\\ \mathrm{c}_{12}\\ \mathrm{c}_{11}\\ \mathrm{c}_{20}\\ \mathrm{c}_{23}\\ \mathrm{c}_{22}\\ \mathrm{c}_{21}\\ \mathrm{c}_{30}\\ \mathrm{c}_{33}\\ \mathrm{c}_{32}\\ \mathrm{c}_{31}\end{bmatrix}=\left[\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{7}$$

Determining equation 3 out of 69

$$-\mathrm{c}_{13}\left(t\right)+\mathrm{c}_{23}\left(t\right)=0 \tag{8}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(9)

Determining equation 4 out of 69

$$-\mathrm{c}_{10}(t) + \mathrm{c}_{30}(t) - \frac{d}{dt}\mathrm{c}_{12}(t) = 0 \tag{10}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(11)

Determining equation 5 out of 69

$$-\mathrm{c}_{11}(t) + \mathrm{c}_{22}(t) + \mathrm{c}_{33}(t) + \frac{d}{dt}\mathrm{c}_{00}(t) = 0 \tag{12}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(13)

Determining equation 6 out of 69

$$\frac{d}{dt} \mathrm{c}_{03}(t) = 0 \tag{14}$$

$$\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(15)

Determining equation 7 out of 69

$$-\mathrm{c}_{12}(t) + \mathrm{c}_{32}(t) = 0 \tag{16}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(17)

Determining equation 8 out of 69

$$\frac{d}{dt} \mathrm{c}_{02}(t) = 0 \tag{18}$$

$$\begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(19)

Determining equation 9 out of 69

$$\mathrm{c}_{01}(t) - \mathrm{c}_{02}(t) - \mathrm{c}_{03}(t) = 0 \tag{20}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(21)

Determining equation 10 out of 69

$$-\mathrm{c}_{20}(t) - \mathrm{c}_{30}(t) - \frac{d}{dt}\mathrm{c}_{11}(t) = 0 \tag{22}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

(23)

Determining equation 11 out of 69

$$-\mathrm{c}_{12}(t) + \mathrm{c}_{13}(t) + \mathrm{c}_{21}(t) - \mathrm{c}_{23}(t) - \mathrm{c}_{33}(t) - \frac{d}{dt}\mathrm{c}_{00}(t) = 0 \tag{24}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & 0 & 1 & 0 \end{bmatrix}$$

(25)

Determining equation 12 out of 69

$$-\frac{d}{dt}\,\mathrm{c}_{03}\,(t) = 0 \tag{26}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(27)

Determining equation 13 out of 69

$$\mathrm{c}_{12}\,(t) - \mathrm{c}_{13}\,(t) - \mathrm{c}_{22}\,(t) + \mathrm{c}_{31}\,(t) - \mathrm{c}_{32}\,(t) - \frac{d}{dt}\,\mathrm{c}_{00}\,(t) = 0 \tag{28}$$

$$\left[\begin{array}{cccccccccccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \left[\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & \end{array}\right.$$

(29)

Determining equation 14 out of 69

$$\frac{d}{dt}\,\mathrm{c}_{01}\,(t) - \frac{d}{dt}\,\mathrm{c}_{02}\,(t) - \frac{d}{dt}\,\mathrm{c}_{03}\,(t) = 0 \tag{30}$$

$$\left[\begin{array}{cccccccccccccccc} 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \left[\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \end{array}\right.$$

(31)

Determining equation 15 out of 69

$$-2\,\mathrm{c}_{01}\,(t) + 2\,\mathrm{c}_{02}\,(t) = 0 \tag{32}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(33)

Determining equation 16 out of 69

$$-\frac{d}{dt}\,\mathrm{c}_{02}\,(t) = 0 \tag{34}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(35)

Determining equation 17 out of 69

$$-2\,\mathrm{c}_{01}\,(t) + 2\,\mathrm{c}_{03}\,(t) = 0 \tag{36}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(37)

Determining equation 18 out of 69

$$-\mathrm{c}_{21}(t) - \mathrm{c}_{31}(t) = 0 \tag{38}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

(39)

Determining equation 19 out of 69

$$-\frac{d}{dt}\mathrm{c}_{01}(t) = 0 \tag{40}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(41)

Determining equation 20 out of 69

$$\mathrm{c}_{01}(t) - \mathrm{c}_{02}(t) + \mathrm{c}_{03}(t) = 0 \tag{42}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(43)

Determining equation 21 out of 69

$$-\frac{d}{dt}\mathrm{c}_{01}(t) = 0 \tag{44}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{45}$$

Determining equation 22 out of 69

$$2\,\mathrm{c}_{01}\left(t\right) = 0 \tag{46}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{47}$$

Determining equation 23 out of 69

$$\mathrm{c}_{01}\left(t\right) + \mathrm{c}_{02}\left(t\right) - \mathrm{c}_{03}\left(t\right) = 0 \tag{48}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(49)

Determining equation 24 out of 69

$$-\frac{d}{dt}\,\mathrm{c}_{20}\,(t) = 0 \tag{50}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(51)

Determining equation 25 out of 69

$$\mathrm{c}_{10}\,(t) - \mathrm{c}_{20}\,(t) - \frac{d}{dt}\,\mathrm{c}_{23}\,(t) = 0 \tag{52}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(53)

Determining equation 26 out of 69

$$\mathrm{c}_{13}(t) - \mathrm{c}_{23}(t) = 0 \tag{54}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(55)

Determining equation 27 out of 69

$$-\mathrm{c}_{10}(t) - \mathrm{c}_{30}(t) - \frac{d}{dt}\mathrm{c}_{22}(t) = 0 \tag{56}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

(57)

Determining equation 28 out of 69

$$\mathrm{c}_{12}(t) - \mathrm{c}_{13}(t) - \mathrm{c}_{21}(t) + \mathrm{c}_{23}(t) - \mathrm{c}_{33}(t) - \frac{d}{dt}\mathrm{c}_{00}(t) = 0 \tag{58}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

(59)

Determining equation 29 out of 69

$$-\frac{d}{dt}\mathrm{c}_{03}(t) = 0 \tag{60}$$

$$\left[\begin{array}{cccccccccccccccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \left[\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{61}$$

Determining equation 30 out of 69

$$-\mathrm{c}_{12}\left(t\right) - \mathrm{c}_{32}\left(t\right) = 0 \tag{62}$$

$$\left[\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \left[\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{63}$$

Determining equation 31 out of 69

$$-\frac{d}{dt}\,\mathrm{c}_{02}\left(t\right) = 0 \tag{64}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{65}$$

Determining equation 32 out of 69

$$-\mathrm{c}_{01}(t) + \mathrm{c}_{02}(t) + \mathrm{c}_{03}(t) = 0 \tag{66}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{67}$$

Determining equation 33 out of 69

$$-\mathrm{c}_{20}(t) + \mathrm{c}_{30}(t) - \frac{d}{dt}\mathrm{c}_{21}(t) = 0 \tag{68}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(69)

Determining equation 34 out of 69

$$\mathrm{c}_{11}(t) - \mathrm{c}_{22}(t) + \mathrm{c}_{33}(t) + \frac{d}{dt}\mathrm{c}_{00}(t) = 0 \tag{70}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

(71)

Determining equation 35 out of 69

$$\frac{d}{dt}\mathrm{c}_{03}(t) = 0 \tag{72}$$

$$\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(73)

Determining equation 36 out of 69

$$-\mathrm{c}_{11}(t) + \mathrm{c}_{21}(t) - \mathrm{c}_{23}(t) - \mathrm{c}_{31}(t) + \mathrm{c}_{32}(t) - \frac{d}{dt}\mathrm{c}_{00}(t) = 0 \tag{74}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(75)

Determining equation 37 out of 69

$$-\frac{d}{dt}\mathrm{c}_{01}(t) + \frac{d}{dt}\mathrm{c}_{02}(t) - \frac{d}{dt}\mathrm{c}_{03}(t) = 0 \tag{76}$$

$$\begin{bmatrix} 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(77)

Determining equation 38 out of 69

$$2\,\mathrm{c}_{01}\,(t) - 2\,\mathrm{c}_{02}\,(t) = 0 \tag{78}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(79)

Determining equation 39 out of 69

$$-\frac{d}{dt}\,\mathrm{c}_{02}\,(t) = 0 \tag{80}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{81}$$

Determining equation 40 out of 69

$$2\,\mathrm{c}_{02}\left(t\right) = 0 \tag{82}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{83}$$

Determining equation 41 out of 69

$$-\mathrm{c}_{21}\left(t\right) + \mathrm{c}_{31}\left(t\right) = 0 \tag{84}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

(85)

Determining equation 42 out of 69

$$\frac{d}{dt} \mathrm{c}_{01}(t) = 0 \tag{86}$$

$$\begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(87)

Determining equation 43 out of 69

$$-\mathrm{c}_{01}(t) + \mathrm{c}_{02}(t) - \mathrm{c}_{03}(t) = 0 \tag{88}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{89}$$

Determining equation 44 out of 69

$$-\frac{d}{dt}\,\mathrm{c}_{01}\,(t) = 0 \tag{90}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{91}$$

Determining equation 45 out of 69

$$-2\,\mathrm{c}_{02}\,(t) + 2\,\mathrm{c}_{03}\,(t) = 0 \tag{92}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(93)

Determining equation 46 out of 69

$$\mathrm{c}_{01}(t) + \mathrm{c}_{02}(t) - \mathrm{c}_{03}(t) = 0 \tag{94}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(95)

Determining equation 47 out of 69

$$-\frac{d}{dt}\mathrm{c}_{30}(t) = 0 \tag{96}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{97}$$

Determining equation 48 out of 69

$$-\mathrm{c}_{10}(t) - \mathrm{c}_{20}(t) - \frac{d}{dt}\mathrm{c}_{33}(t) = 0 \tag{98}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{99}$$

Determining equation 49 out of 69

$$-\mathrm{c}_{13}(t) - \mathrm{c}_{23}(t) = 0 \tag{100}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

(101)

Determining equation 50 out of 69

$$\mathrm{c}_{10}(t) - \mathrm{c}_{30}(t) - \frac{d}{dt}\mathrm{c}_{32}(t) = 0 \tag{102}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

(103)

Determining equation 51 out of 69

$$-\mathrm{c}_{12}(t) + \mathrm{c}_{13}(t) - \mathrm{c}_{22}(t) - \mathrm{c}_{31}(t) + \mathrm{c}_{32}(t) - \frac{d}{dt}\mathrm{c}_{00}(t) = 0 \tag{104}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

(105)

Determining equation 52 out of 69

$$-\frac{d}{dt}\,\mathrm{c}_{03}\,(t) = 0 \tag{106}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(107)

Determining equation 53 out of 69

$$\mathrm{c}_{12}\,(t) - \mathrm{c}_{32}\,(t) = 0 \tag{108}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{109}$$

Determining equation 54 out of 69

$$-\frac{d}{dt}\,\mathrm{c}_{02}\,(t) = 0 \tag{110}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{111}$$

Determining equation 55 out of 69

$$-\mathrm{c}_{01}\,(t) + \mathrm{c}_{02}\,(t) + \mathrm{c}_{03}\,(t) = 0 \tag{112}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(113)

Determining equation 56 out of 69

$$\mathrm{c}_{20}(t) - \mathrm{c}_{30}(t) - \frac{d}{dt}\mathrm{c}_{31}(t) = 0 \tag{114}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

(115)

Determining equation 57 out of 69

$$-\mathrm{c}_{11}(t) - \mathrm{c}_{21}(t) + \mathrm{c}_{23}(t) + \mathrm{c}_{31}(t) - \mathrm{c}_{32}(t) - \frac{d}{dt}\mathrm{c}_{00}(t) = 0 \tag{116}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

(117)

Determining equation 58 out of 69

$$-\frac{d}{dt}\,\mathrm{c}_{03}\,(t) = 0 \tag{118}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(119)

Determining equation 59 out of 69

$$\mathrm{c}_{11}\,(t) + \mathrm{c}_{22}\,(t) - \mathrm{c}_{33}\,(t) + \frac{d}{dt}\,\mathrm{c}_{00}\,(t) = 0 \tag{120}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & - \end{bmatrix} \tag{121}$$

Determining equation 60 out of 69

$$-\frac{d}{dt}\,\mathrm{c}_{01}\,(t) - \frac{d}{dt}\,\mathrm{c}_{02}\,(t) + \frac{d}{dt}\,\mathrm{c}_{03}\,(t) = 0 \tag{122}$$

$$\begin{bmatrix} 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{123}$$

Determining equation 61 out of 69

$$2\,\mathrm{c}_{03}\,(t) = 0 \tag{124}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(125)

Determining equation 62 out of 69

$$\frac{d}{dt} \mathrm{c}_{02}(t) = 0 \tag{126}$$

$$\begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(127)

Determining equation 63 out of 69

$$2\,\mathrm{c}_{01}(t) - 2\,\mathrm{c}_{03}(t) = 0 \tag{128}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & -2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(129)

Determining equation 64 out of 69

$$\mathrm{c}_{21}(t) - \mathrm{c}_{31}(t) = 0 \tag{130}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(131)

Determining equation 65 out of 69

$$-\frac{d}{dt}\mathrm{c}_{01}(t) = 0 \tag{132}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(133)

Determining equation 66 out of 69

$$\mathrm{c}_{01}(t) - \mathrm{c}_{02}(t) + \mathrm{c}_{03}(t) = 0 \tag{134}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(135)

Determining equation 67 out of 69

$$\frac{d}{dt}\mathrm{c}_{01}(t) = 0 \tag{136}$$

$$\begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(137)

Determining equation 68 out of 69

$$2\,\mathrm{c}_{02}\,(t) - 2\,\mathrm{c}_{03}\,(t) = 0 \tag{138}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(139)

Determining equation 69 out of 69

$$-\,\mathrm{c}_{01}\,(t) - \mathrm{c}_{02}\,(t) + \mathrm{c}_{03}\,(t) = 0 \tag{140}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{141}$$

5.2 Step 1 of 6: the initial matrices

Dimension of matrices: 69X16
Matrix A
Matrix B

$$A = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{142}$$

$$B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & -1
\end{bmatrix} \tag{143}$$

Step 2 of 6: the reduced based on col(M^T) where M=[-A|B]
Dimension of matrices: 27X16
Matrix A

$$A = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{144}$$

Matrix B

$$B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{bmatrix} \tag{145}$$

Step 3 of 6: Splitting up to A, B and B_algebraic
Dimension of matrices A and B: 16X16
Dimension of matrices B_algebraic: 11X16
Matrix A

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{146}$$

Matrix B

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \tag{147}$$

Matrix B_algebraic

$$B_{\text{algebraic}} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{148}$$

Coefficient matrix c:

$$\mathbf{c} = \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} \tag{149}$$

Dimensions of A: 16X16

Dimensions of B_algebraic: 11X16

5.3 Step 4 of 6: Removing potential extra pivot columns

Dimension of matrices A and B: 16X16 Dimension of matrices B_algebraic: 11X16 Matrix A

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{150}$$

Matrix B

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \tag{151}$$

Dimensions of B: 16X16 Coefficients:

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \\ C_{11} \\ C_{12} \\ C_{13} \\ C_{14} \\ C_{15} \\ C_{16} \end{bmatrix} \tag{152}$$

Number of unknowns: 16 ## Step 5 of 6: Solving the ODE system Dimension of the matrix B: 16X16 Dimension of the matrix B_algebraic: 11X16 ODE system:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} \tag{153}$$

Solve the ODE system: Initial conditions for $\mathbf{c}$ denoted by $\mathbf{c}_0$ in terms of arbitrary integration constants:

$$\mathbf{c}_0 = \begin{bmatrix} \mathrm{c}_{00} \\ \mathrm{c}_{03} \\ \mathrm{c}_{02} \\ \mathrm{c}_{01} \\ \mathrm{c}_{10} \\ \mathrm{c}_{13} \\ \mathrm{c}_{12} \\ \mathrm{c}_{11} \\ \mathrm{c}_{20} \\ \mathrm{c}_{23} \\ \mathrm{c}_{22} \\ \mathrm{c}_{21} \\ \mathrm{c}_{30} \\ \mathrm{c}_{33} \\ \mathrm{c}_{32} \\ \mathrm{c}_{31} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \\ C_{11} \\ C_{12} \\ C_{13} \\ C_{14} \\ C_{15} \\ C_{16} \end{bmatrix} \tag{154}$$

Jordan form:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{155}$$

$$
J = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{156}
$$

Exponential form:

$$
\exp\left(J \cdot t\right) = \begin{bmatrix}
1 & t & \frac{t^2}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \tag{157}
$$

Solution to the ODE system:

$$P \exp\left(J \cdot t\right) P^{-1} \mathbf{c}_0 = \begin{bmatrix} C_1 - C_{14} t + \frac{C_5 t^2}{2} + \frac{C_9 t^2}{2} \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ -C_5 t + C_6 + C_9 t \\ C_{13} t - C_5 t + C_7 \\ -C_{13} t + C_8 - C_9 t \\ C_9 \\ C_{10} + C_5 t - C_9 t \\ C_{11} - C_{13} t - C_5 t \\ C_{12} + C_{13} t - C_9 t \\ C_{13} \\ C_{14} - C_5 t - C_9 t \\ -C_{13} t + C_{15} + C_5 t \\ -C_{13} t + C_{16} + C_9 t \end{bmatrix} \tag{158}$$

5.4 Step 6 of 6: Solving the algebraic system

Number of algebraic equations: 11

Matrix B_algebraic

$$B_{\text{algebraic}} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{159}$$

Algebraic equations:

$$\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (160)$$

Algebraic equations after substitution of the solution to the ODE system:

$$\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} C_1 - C_{14}t + \frac{C_5t^2}{2} + \frac{C_9t^2}{2} \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ -C_5t + C_6 + C_9t \\ C_{13}t - C_5t + C_7 \\ -C_{13}t + C_8 - C_9t \\ C_9 \\ C_{10} + C_5t - C_9t \\ C_{11} - C_{13}t - C_5t \\ C_{12} + C_{13}t - C_9t \\ C_{13} \\ C_{14} - C_5t - C_9t \\ -C_{13}t + C_{15} + C_5t \\ -C_{13}t + C_{16} + C_9t \end{bmatrix} = \begin{bmatrix} C_5 \\ -C \\ C_{13}t - \\ -C \\ -C_{11} + \\ -C \\ C_1 \\ C_1 \end{bmatrix} \quad (161)$$

Equation: $C_2 = 0$, Solution: $C_2 = 0$ Equation: $C_3 = 0$, Solution: $C_3 = 0$ Equation: $C_4 = 0$, Solution: $C_4 = 0$ Equation: $-C_{13} + \frac{C_{15}}{t} + C_5 = 0$, Solution: $C_5 = C_{13} - \frac{C_{15}}{t}$ Equation: $C_{15} - C_{16} + C_6 = 0$, Solution: $C_6 = -C_{15} + C_{16}$ Equation: $C_{15} + C_7 = 0$, Solution: $C_7 = -C_{15}$ Equation: $-C_{14} - C_{15} + C_8 = 0$, Solution: $C_8 = C_{14} + C_{15}$ Equation: $-C_{13} + \frac{C_{16}}{t} + C_9 = 0$, Solution: $C_9 = C_{13} - \frac{C_{16}}{t}$ Equation: $C_{10} - C_{15} + C_{16} = 0$, Solution: $C_{10} = C_{15} - C_{16}$ Equation: $C_{11} - C_{14} - C_{16} = 0$, Solution: $C_{11} = C_{14} + C_{16}$ Equation: $C_{12} + C_{16} = 0$, Solution: $C_{12} = -C_{16}$

Solution *after* algebraic substitution:

$$\mathbf{c} = \begin{bmatrix} C_1 + C_{13}t^2 - C_{14}t - \frac{C_{15}t}{2} - \frac{C_{16}t}{2} \\ 0 \\ 0 \\ 0 \\ C_{13} - \frac{C_{15}}{t} \\ 0 \\ 0 \\ -2C_{13}t + C_{14} + C_{15} + C_{16} \\ C_{13} - \frac{C_{16}}{t} \\ 0 \\ -2C_{13}t + C_{14} + C_{15} + C_{16} \\ 0 \\ C_{13} \\ -2C_{13}t + C_{14} + C_{15} + C_{16} \\ 0 \\ 0 \end{bmatrix}$$

6 The very final step

The very final step: substituting the solution into the tangents and print the results: Arbitrary integration constants in the final solution:

$$\begin{bmatrix} C_{13} \\ C_{14} \\ C_{16} \\ C_1 \\ C_{15} \end{bmatrix}$$

Number of generators which are divided based on the number of constants: 5

Number of component tangents before removing: 5 Generator 1 out of 5:

$$\begin{aligned} \xi &= t^2 \\ \eta_1 &= -2tw_1 + 1 \\ \eta_2 &= -2tw_2 + 1 \\ \eta_3 &= -2tw_3 + 1 \end{aligned}$$

Checking the 3 linearised symmetry conditions of generator X_1: Lin syms

$$[0,\ 0,\ 0]$$

Generator 2 out of 5:

$$\xi = -t$$
$$\eta_1 = w_1$$
$$\eta_2 = w_2$$
$$\eta_3 = w_3$$

Checking the 3 linearised symmetry conditions of generator X_2: Lin syms

$$[0,\ 0,\ 0]$$

Generator 3 out of 5:

$$\xi = -\frac{t}{2}$$
$$\eta_1 = w_1$$
$$\eta_2 = w_2 - \frac{1}{t}$$
$$\eta_3 = w_3$$

Checking the 3 linearised symmetry conditions of generator X_3: Lin syms

$$\left[-\frac{x_1x_2}{2} - \frac{x_1x_3}{2} + \frac{x_2x_3}{2} + \frac{x_1}{x_0} - \frac{x_3}{x_0},\ -\frac{x_1x_2}{2} + \frac{x_1x_3}{2} - \frac{x_2x_3}{2} + \frac{x_1}{x_0} + \frac{x_3}{x_0} - \frac{1}{x_0^2},\ \frac{x_1x_2}{2} - \frac{x_1x_3}{2} - \frac{x_2x_3}{2} - \frac{x_1}{x_0} + \frac{x_3}{x_0}\right]$$

Generator 4 out of 5:

$$\xi = 1$$
$$\eta_1 = 0$$
$$\eta_2 = 0$$
$$\eta_3 = 0$$

Checking the 3 linearised symmetry conditions of generator X_4: Lin syms

$$[0,\ 0,\ 0]$$

Generator 5 out of 5:

$$\xi = -\frac{t}{2}$$
$$\eta_1 = w_1 - \frac{1}{t}$$
$$\eta_2 = w_2$$
$$\eta_3 = w_3$$

Checking the 3 linearised symmetry conditions of generator X_5: Lin syms

$$\left[-\frac{x_1x_2}{2}-\frac{x_1x_3}{2}+\frac{x_2x_3}{2}+\frac{x_2}{x_0}+\frac{x_3}{x_0}-\frac{1}{x_0^2},\ -\frac{x_1x_2}{2}+\frac{x_1x_3}{2}-\frac{x_2x_3}{2}+\frac{x_2}{x_0}-\frac{x_3}{x_0},\ \frac{x_1x_2}{2}-\frac{x_1x_3}{2}-\frac{x_2x_3}{2}-\frac{x_2}{x_0}+\frac{x_3}{x_0}\right]$$

Number of component tangents after removing: 3 The final generators are given by:

$$X_1 = \left(t^2\right)\partial t + \left(-2tw_1+1\right)\partial w_1 + \left(-2tw_2+1\right)\partial w_2 + \left(-2tw_3+1\right)\partial w_3,$$

,

$$X_2 = \left(-t\right)\partial t + \left(w_1\right)\partial w_1 + \left(w_2\right)\partial w_2 + \left(w_3\right)\partial w_3,$$

,

$$X_3 = \left(1\right)\partial t.$$

7 Conclusion

Something here is not completely right since we obtain generators that are not solutions to the linearised symmetry conditions.