Summary of symmetry calculations

December 13, 2021

Contents

1	DBH_model	5
2	LV	7
3	hydons_model	9
4	SIR	11
5	Brusselator	15

4 CONTENTS

DBH_{model}

Run $08_09PM_13_December-2021$

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\begin{split} \frac{\mathrm{d}w_1}{\mathrm{d}t} &= -w_1w_2 - w_1w_3 + w_2w_3, \\ \frac{\mathrm{d}w_2}{\mathrm{d}t} &= -w_1w_2 + w_1w_3 - w_2w_3, \\ \frac{\mathrm{d}w_3}{\mathrm{d}t} &= w_1w_2 - w_1w_3 - w_2w_3. \end{split}$$

The calculated generators are:

$$X_1 = (1) \partial t$$
,

$$X_2 = (t^2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 + (1 + 2tw_3) \partial w_3$$

$$X_3 = (1) \partial t$$
,

$$X_4 = (-t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$X_5 = (\mathbf{f}_1\left(t\right))\,\partial t + (w_2w_3\,\mathbf{f}_1\left(t\right) - w_1w_2\,\mathbf{f}_1\left(t\right) - w_1w_3\,\mathbf{f}_1\left(t\right))\,\partial w_1 + (w_1w_3\,\mathbf{f}_1\left(t\right) - w_1w_2\,\mathbf{f}_1\left(t\right) \\ + -w_2w_3\,\mathbf{f}_1\left(t\right))\,\partial w_2 + (w_1w_2\,\mathbf{f}_1\left(t\right) - w_1w_3\,\mathbf{f}_1\left(t\right) - w_2w_3\,\mathbf{f}_1\left(t\right))\,\partial w_3.$$

Some of the generators might contain the following arbitrary functions:

 f_1

The execution time of the script was:

 $0~\mathrm{hours}~3~\mathrm{minutes}~54~\mathrm{seconds}.$

LV

Run $08_09PM_13_December-2021$

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u (1 - v),$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = av (u - 1).$$

The calculated generators are:

$$X_1 = (1) \partial t,$$

$$X_2 = \left(-\frac{1}{a}\right)\partial t,$$

$$X_{3} = \left(\frac{\mathbf{f}_{1}\left(t\right)}{a}\right)\partial t + \left(\frac{u\,\mathbf{f}_{1}\left(t\right)}{a} - \frac{uv\,\mathbf{f}_{1}\left(t\right)}{a}\right)\partial u + \left(uv\,\mathbf{f}_{1}\left(t\right) - v\,\mathbf{f}_{1}\left(t\right)\right)\partial v.$$

Some of the generators might contain the following arbitrary functions:

 f_1

The execution time of the script was:

0 hours 0 minutes 31 seconds.

8 CHAPTER 2. LV

$hydons_model$

Run 08_05 PM $_13_$ December-2021

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\begin{split} \frac{\mathrm{d}y_1}{\mathrm{d}t} &= \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2}, \\ \frac{\mathrm{d}y_2}{\mathrm{d}t} &= \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}. \end{split}$$

The calculated generators are:

$$X_1 = (t) \partial t + (y_1) \partial y_1 + (y_2) \partial y_2,$$

$$X_{2}=\left(-t^{2} \operatorname{f}_{1}\left(t\right)+y_{1} y_{2} \operatorname{f}_{1}\left(t\right)\right) \partial t+\left(y_{2}^{2} \operatorname{f}_{1}\left(t\right)+t y_{1} \operatorname{f}_{1}\left(t\right)\right) \partial y_{1}+\left(y_{1}^{2} \operatorname{f}_{1}\left(t\right)+t y_{2} \operatorname{f}_{1}\left(t\right)\right) \partial y_{2}$$

Some of the generators might contain the following arbitrary functions:

 f_1

The execution time of the script was:

0 hours 0 minutes 47 seconds.

SIR

Run $08_09PM_13_December-2021$

Degree in tangential ansätze: 1. The system of ODEs is given by:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -ISr,$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = ISr - Ia,$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = Ia.$$

The calculated generators are:

$$X_1 = (1) \partial t$$
,

$$X_2 = (I + R + S) \partial t$$

$$X_3 = (1) \partial R,$$

$$X_4 = (I + R + S) \partial R$$

The execution time of the script was:

0 hours 0 minutes 17 seconds.

Run $08_{12}PM_{13}$ December-2021

Degree in tangential ansätze: 2. The system of ODEs is given by:

12 CHAPTER 4. SIR

$$\begin{split} \frac{\mathrm{d}S}{\mathrm{d}t} &= -ISr, \\ \frac{\mathrm{d}I}{\mathrm{d}t} &= ISr - Ia, \\ \frac{\mathrm{d}R}{\mathrm{d}t} &= Ia. \end{split}$$

The calculated generators are:

$$X_1 = (1) \partial R$$
,

$$X_2 = (I + R + S) \,\partial t$$

$$X_3 = \left(-\frac{1}{r}\right) \partial t,$$

$$X_4 = \left(\frac{I^2}{2} + \frac{R^2}{2} + \frac{S^2}{2} + IR + IS + RS\right)\partial R$$

$$X_5 = (1) \partial t$$
,

$$X_6 = \left(\frac{I^2}{2} + \frac{R^2}{2} + \frac{S^2}{2} + IR + IS + RS\right) \partial t$$

$$X_{7} = \left(\frac{\mathbf{f}_{1}\left(t\right)}{a}\right)\partial t + \left(-\frac{ISr\,\mathbf{f}_{1}\left(t\right)}{a}\right)\partial S + \left(\frac{ISr\,\mathbf{f}_{1}\left(t\right)}{a} - I\,\mathbf{f}_{1}\left(t\right)\right)\partial I + \left(I\,\mathbf{f}_{1}\left(t\right)\right)\partial R.$$

Some of the generators might contain the following arbitrary functions:

 f_1

WARNING:
Some of the calculated generators did not satisfy the linearised symmetry conditions. Thus, the presented list here is not complete and consists exclusively of the calculated generators that satisfy the linearised symmetry conditions.

The execution time of the script was:

 $0~\mathrm{hours}~2~\mathrm{minutes}~21~\mathrm{seconds}.$

14 CHAPTER 4. SIR

Brusselator

Run $08_12PM_13_December-2021$

Degree in tangential ansätze: 1. The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = au^2v - u(b+1) + 1,$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -au^2v + bu.$$

The calculated generators are:

$$X_1 = (1) \partial t$$
.

The execution time of the script was:

0 hours 0 minutes 4 seconds.