

Summary of symmetry calculations

November 4, 2021

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Chapter 1

Lotka_Volterra

Run 11_54PM_03_November-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\begin{aligned}\frac{dN}{dt} &= N(-Pb + a), \\ \frac{dP}{dt} &= P(Nc - d).\end{aligned}$$

The calculated generators are:

$$X_1 = (1) \partial t,$$

$$X_2 = \left(-1 + \frac{1}{a}\right) \partial t$$

$$X_3 = \left(\frac{1}{c} + f_1(t)\right) \partial t + \left(\frac{Na f_1(t)}{c} - \frac{NPb f_1(t)}{c}\right) \partial N + \left(NP f_1(t) - \frac{P d f_1(t)}{c}\right) \partial P$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

WARNING:

Some of the calculated generators did not satisfy the linearised symmetry conditions. Thus, the presented list here is not complete and consists exclusively of the calculated generators that satisfy the linearised symmetry conditions.

Equation: $-C_3 + \frac{C_6 b^2 e^{-2at}}{c^2} - \frac{C_6 b^2 d}{ac^2} + \frac{C_6 b^2 d e^{-2at}}{ac^2} = 0$ Basis functions:

$$[1.0, e^{-2at}]$$

Solutions *before* processing:

$$C_6 = 0$$

$$C_3 = -\frac{C_6 b^2 d}{ac^2}$$

Solutions *after* processing:

$$C_3 = 0$$

$$C_6 = 0$$

Equation: $-\frac{C_{15}ace^{2dt}}{2ad^2+4d^3} - \frac{C_{15}cde^{2dt}}{2ad^2+4d^3} + \frac{C_{15}c}{a^2e^{at}+2ade^{at}} + \frac{C_{15}ce^{dt}}{d^2} - \frac{C_{15}c}{2d^2} - \frac{C_{15}c}{2ad} + \frac{C_2ce^{dt}}{d} - \frac{C_2c}{d} - \frac{C_4b}{a} + \frac{C_4be^{-at}}{a} - C_5 + \frac{C_7abc^2e^{2dt}}{2a^3d^2-2a^2d^3-8ad^4+8d^5} - \frac{C_7abc^2e^{at}}{2a^4d-6a^3d^2+4a^2d^3} + \frac{C_7bc^2de^{2dt}}{2a^3d^2-2a^2d^3-8ad^4+8d^5} - \frac{C_7bc^2de^{at}}{2a^4d-6a^3d^2+4a^2d^3} - \frac{C_7bc^2}{4a^4e^{at}+10a^3de^{at}+4a^2d^2e^{at}} + \frac{C_7bc^2e^{at}e^{dt}}{2a^3d-a^2d^2-ad^3} - \frac{C_7bc^2e^{dt}}{a^2d^2-ad^3} + \frac{C_7bc^2}{2a^2d^2} + \frac{C_8ac^2e^{2dt}}{2a^2d^2+2ad^3-4d^4} + \frac{C_8c^2de^{2dt}}{2a^2d^2+2ad^3-4d^4} - \frac{C_8c^2}{2a^3e^{at}+5a^2de^{at}+2ad^2e^{at}} - \frac{C_8c^2e^{at}e^{dt}}{2a^3-a^2d-ad^2} - \frac{C_8c^2e^{dt}}{ad^2} + \frac{C_8c^2}{2ad^2} = 0$ Basis functions:

$$[1.0, e^{-at}, e^{2dt}, e^{dt}, e^{at}e^{dt}, e^{at}]$$

Solutions *before* processing:

$$C_4 = 0$$

$$C_7 = \frac{C_{15}a^2}{bc} - \frac{3C_{15}ad}{bc} + \frac{2C_{15}d^2}{bc} - \frac{C_8a}{b} + \frac{2C_8d}{b}$$

$$C_2 = -\frac{2C_{15}a^3}{2a^3d-a^2d^2-ad^3} + \frac{C_{15}a^2d}{2a^3d-a^2d^2-ad^3} + \frac{C_{15}ad^2}{2a^3d-a^2d^2-ad^3} + \frac{2C_7abc}{2a^3d-a^2d^2-ad^3} - \frac{C_7bcde^{at}}{2a^3d-a^2d^2-ad^3} + \frac{C_7b}{2a^3d-a^2d^2-ad^3}$$

$$C_7 = \frac{C_8d}{b}$$

$$C_7 = \frac{2C_8a^2de^{dt}}{2a^2be^{dt}-2a^2b-4abde^{dt}-3abd-bd^2} - \frac{4C_8ad^2e^{dt}}{2a^2be^{dt}-2a^2b-4abde^{dt}-3abd-bd^2}$$

$$C_2 = -\frac{C_{15}a^5ce^{at}}{2a^5cde^{at}+3a^4cd^2e^{at}-3a^3cd^3e^{at}-2a^2cd^4e^{at}} - \frac{5C_{15}a^4cde^{at}}{2(2a^5cde^{at}+3a^4cd^2e^{at}-3a^3cd^3e^{at}-2a^2cd^4e^{at})} + \frac{C_7b}{2a^5cde^{at}+3a^4cd^2e^{at}-3a^3cd^3e^{at}-2a^2cd^4e^{at}}$$

Solutions *after* processing:

$$C_4 = 0$$

$$C_2 = 0$$

$$C_5 = 0$$

$$C_8 = 0$$

$$C_7 = 0$$

$$C_{15} = 0$$

Equation: $-C_9 = 0$ Basis functions:

$$[1.0]$$

Solutions *before* processing:

$$C_9 = 0$$

Solutions *after* processing:

$$C_9 = 0$$

Equation: $\frac{C_{10}bdt}{a} - \frac{C_{10}b}{a} + \frac{C_{10}be^{-at}}{a} - \frac{C_{10}bd}{a^2} + \frac{C_{10}bde^{-at}}{a^2} - C_{11} + \frac{C_{13}a^2b^2}{a^3de^{at}-a^2d^2e^{at}} + \frac{2C_{13}a^2b^2}{a^3d+a^2d^2} - \frac{2C_{13}ab^2c^2}{a^2c^2de^{dt}-ac^2d^2e^{dt}} + \frac{2C_{13}ab^2d}{a^3de^{at}-a^2d^2e^{at}} + \frac{4C_{13}ab^2d}{a^3d+a^2d^2} - \frac{C_{13}ab^2}{a^2de^{at}e^{dt}+ad^2e^{at}e^{dt}} - \frac{2C_{13}b^2c^2d}{a^2c^2de^{dt}-ac^2d^2e^{dt}} + \frac{C_{13}b^2d^2}{a^3de^{at}-a^2d^2e^{at}} + \frac{C_{13}b^2d^2}{a^3d+a^2d^2} - \frac{2C_{13}b^2d}{a^2de^{at}e^{dt}+ad^2e^{at}e^{dt}} - \frac{C_{13}b^2t}{a} + C_{14}bt + \frac{C_{16}ab^2}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{2C_{16}b^2d}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{C_{16}b^2d}{a^2c+acd} - \frac{C_{16}b^2e^{-at}}{cd} - \frac{C_{16}b^2e^{-at}}{ac} - \frac{C_{18}be^{-at}}{c} + \frac{C_{18}bd}{ac} - \frac{C_{18}bde^{-at}}{ac} = 0$ Basis functions:

$$[1.0, e^{-at}, e^{-dt}, e^{-at}e^{-dt}, t]$$

Solutions *before* processing:

$$C_{10} = \frac{C_{16}ab}{cd} + \frac{C_{18}a}{c}$$

$$C_{10} = \frac{C_{13}b}{d} - \frac{C_{14}a}{d}$$

$$C_{10} = -\frac{C_{11}a^4cd}{a^3bcd + a^2bcd^2 - abcd^3 - bcd^4} + \frac{C_{11}a^2cd^3}{a^3bcd + a^2bcd^2 - abcd^3 - bcd^4} - \frac{C_{13}a^3b^2c}{a^3bcde^{at}e^{dt} + a^2bcd^2e^{at}e^{dt} - abcd^3e^{at}e^{dt} - bcd^4e^{at}e^{dt}}$$

Solutions *after* processing:

$$C_{14} = -\frac{C_{10}d}{a} + \frac{C_{13}b}{a}$$

$$C_{18} = \frac{C_{10}c}{a} - \frac{C_{16}b}{d}$$

$$C_{11} = -\frac{C_{10}b}{a} + \frac{C_{13}(2a^3b^2e^{at}e^{dt} - 2a^3b^2e^{at} + a^3b^2e^{dt} - a^3b^2 + 2a^2b^2de^{at}e^{dt} - 4a^2b^2de^{at} + 3a^2b^2de^{dt} - a^2b^2d - 3ab^2d^2e^{at}e^{dt})}{a^4de^{at}e^{dt} - a^2d^3e^{at}e^{dt}}$$

Equation: $-C_{12} + \frac{C_{13}bc}{a^2e^{at}e^{dt}+ade^{at}e^{dt}} + \frac{C_{13}bc}{ad+d^2} - \frac{C_{13}bce^{-dt}}{ad} - \frac{C_{16}b}{ae^{at}e^{dt}+de^{at}e^{dt}} + \frac{C_{16}b}{a+d} = 0$ Basis functions:

$$[1.0, e^{-dt}, e^{-at}e^{-dt}]$$

Solutions *before* processing:

$$C_{13} = 0$$

$$C_{12} = \frac{C_{13}abc}{a^2d + ad^2} + \frac{C_{13}bcd}{a^2de^{at}e^{dt} + ad^2e^{at}e^{dt}} - \frac{C_{16}abd}{a^2de^{at}e^{dt} + ad^2e^{at}e^{dt}} + \frac{C_{16}abd}{a^2d + ad^2}$$

Solutions *after* processing:

$$C_{12} = \frac{C_{16}(be^{at}e^{dt} - b)}{ae^{at}e^{dt} + de^{at}e^{dt}}$$

$$C_{13} = 0$$

Equation: $-\frac{C_{13}c^2}{a^2e^{at}e^{dt}+ade^{at}e^{dt}} - \frac{C_{13}c^2}{ad+d^2} + \frac{C_{13}c^2e^{-dt}}{ad} + \frac{C_{16}c}{ae^{at}e^{dt}+de^{at}e^{dt}} - \frac{C_{16}c}{a+d} - C_{17} = 0$ Basis functions:

$$[1.0, e^{-dt}, e^{-at}e^{-dt}]$$

Solutions *before* processing:

$$C_{13} = 0$$

$$C_{13} = -\frac{C_{16}acde^{at}e^{dt}}{ac^2e^{at}e^{dt}+c^2d} + \frac{C_{16}acd}{ac^2e^{at}e^{dt}+c^2d} - \frac{C_{17}a^2de^{at}e^{dt}}{ac^2e^{at}e^{dt}+c^2d} - \frac{C_{17}ad^2e^{at}e^{dt}}{ac^2e^{at}e^{dt}+c^2d}$$

Solutions *after* processing:

$$C_{17} = \frac{C_{16}(-ace^{2at}e^{2dt} + ace^{at}e^{dt} - cde^{at}e^{dt} + cd)}{a^2e^{2at}e^{2dt} + ade^{2at}e^{2dt} + ade^{at}e^{dt} + d^2e^{at}e^{dt}}$$

$$C_{13} = 0$$

Solutions after all is done:

$$\mathbf{c} = \begin{bmatrix} C_1 - \frac{C_{10}}{a} + \frac{C_{16}ab}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{C_{16}bd}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{c_{22}(t)}{c} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{C_{16}a^2b}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{C_{16}abd}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{ac_{22}(t)}{c} \\ -\frac{C_{16}b^2de^{at}e^{dt}}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} - \frac{C_{16}b^2d}{a^2c+acd} + \frac{C_{16}b^2}{ac} - \frac{bc_{22}(t)}{c} \\ \frac{C_{16}be^{at}e^{dt}}{ae^{at}e^{dt}+de^{at}e^{dt}} - \frac{C_{16}b}{a+d} \\ 0 \\ -\frac{C_{16}ab}{ace^{at}e^{dt}+cde^{at}e^{dt}} - \frac{C_{16}bd}{ace^{at}e^{dt}+cde^{at}e^{dt}} - \frac{dc_{22}(t)}{c} \\ 0 \\ \frac{C_{16}a}{ae^{at}e^{dt}+de^{at}e^{dt}} + \frac{C_{16}d}{ae^{at}e^{dt}+de^{at}e^{dt}} \\ -\frac{C_{16}ace^{2at}e^{2dt}}{a^2e^{2at}e^{2dt}+ade^{2at}e^{2dt}+ade^{at}e^{dt}+d^2e^{at}e^{dt}} + \frac{C_{16}ace^{at}e^{dt}}{a^2e^{2at}e^{2dt}+ade^{2at}e^{2dt}+ade^{at}e^{dt}+d^2e^{at}e^{dt}} - \frac{C_{16}cde^{at}e^{dt}}{a^2e^{2at}e^{2dt}+ade^{2at}e^{2dt}+ade^{at}e^{dt}+d^2e^{at}e^{dt}} + \frac{c_{22}(t)}{c} \end{bmatrix}$$

Tangents before any manipulation:

$$\eta_0 = C_1 - \frac{C_{10}}{a} + \frac{C_{16}ab}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{C_{16}bd}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{c_{22}(t)}{c}$$

$$\eta_1 = \frac{C_{16}a^2bN}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{C_{16}abdN}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} - \frac{C_{16}b^2dNP e^{at}e^{dt}}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} - \frac{C_{16}b^2dNP}{a^2c+acd} + \frac{C_{16}bN^2e^{at}e^{dt}}{ae^{at}e^{dt}+de^{at}e^{dt}} - \frac{C_{16}bN^2}{a+d} + \frac{c_{22}(t)}{c}$$

$$\eta_2 = -\frac{C_{16}abP}{ace^{at}e^{dt}+cde^{at}e^{dt}} - \frac{C_{16}acN^2e^{2at}e^{2dt}}{a^2e^{2at}e^{2dt}+ade^{2at}e^{2dt}+ade^{at}e^{dt}+d^2e^{at}e^{dt}} + \frac{C_{16}acN^2e^{at}e^{dt}}{a^2e^{2at}e^{2dt}+ade^{2at}e^{2dt}+ade^{at}e^{dt}+d^2e^{at}e^{dt}} + \frac{c_{22}(t)}{c}$$

Generators before some are removed:

$$[1, 0, 0]$$

$$\left[-\frac{1}{a}, 0, 0\right]$$

$$\left[\frac{ab}{acd e^{at} e^{dt} + cd^2 e^{at} e^{dt}} + \frac{bd}{acd e^{at} e^{dt} + cd^2 e^{at} e^{dt}}, \frac{a^2 b N}{acd e^{at} e^{dt} + cd^2 e^{at} e^{dt}} + \frac{abd N}{acd e^{at} e^{dt} + cd^2 e^{at} e^{dt}} - \frac{b^2 d N P e^{at} e^{dt}}{acd e^{at} e^{dt} + cd^2 e^{at} e^{dt}} - \frac{b^2}{a^2 c} \right. \\ \left. \left[\frac{c_{22}(t)}{c}, \frac{aN c_{22}(t)}{c} - \frac{bNP c_{22}(t)}{c}, NP c_{22}(t) - \frac{dP c_{22}(t)}{c} \right] \right]$$

Generators after some were removed:

$$\begin{aligned} & [1, 0, 0] \\ & \left[-\frac{1}{a}, 0, 0 \right] \\ & \left[\frac{c_{22}(t)}{c}, \frac{aN c_{22}(t)}{c} - \frac{bNP c_{22}(t)}{c}, NP c_{22}(t) - \frac{dP c_{22}(t)}{c} \right] \end{aligned}$$

The execution time of the script was:

0 hours 1 minutes 7 seconds.