Summary of symmetry calculations

November 10, 2021

Contents

Chapter 1

Lotka_Volterra

Run 01_12PM_10_November-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = N\left(-Pb + a\right),$$
$$\frac{\mathrm{d}P}{\mathrm{d}t} = P\left(Nc - d\right).$$

The calculated generators are:

$$X_1 = (1)\,\partial t,$$

$$X_2 = \left(\frac{1}{c} + \mathrm{f}_1(t)\right)\partial t + \left(\frac{Na\,\mathrm{f}_1(t)}{c} - \frac{NPb\,\mathrm{f}_1(t)}{c}\right)\partial N + \left(NP\,\mathrm{f}_1(t) - \frac{Pd\,\mathrm{f}_1(t)}{c}\right)\partial P$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

WARNING:

Some of the calculated generators did not satisfy the linearised symmetry conditions. Thus, the presented list here is not complete and consists exclusively of the calculated generators that satisfy the linearised symmetry conditions.

Equation:$-C_3 + \frac{C_6 b^2 e^{-2at}}{c^2} - \frac{C_6 b^2 d}{ac^2} + \frac{C_6 b^2 d e^{-2at}}{ac^2} = 0$ Basis functions:

$$\left[1.0,\ e^{-2at}\right]$$

Equation for the basis function e^{-2at}:

$$\frac{C_6 b^2}{c^2} + \frac{C_6 b^2 d}{ac^2}$$

This equation was solved for: C_6 which gave the solution:

$$0$$

Solutions *before* processing:

$$C_6 = 0$$
$$C_3 = -\frac{C_6 b^2 d}{ac^2}$$

Solutions *after* processing:

$$C_6 = 0$$
$$C_3 = -\frac{C_6 b^2 d}{ac^2}$$

Equation:$-\frac{C_{15} ace^{2dt}}{2ad^2+4d^3} - \frac{C_{15} cde^{2dt}}{2ad^2+4d^3} + \frac{C_{15} c}{a^2 e^{at}+2ade^{at}} + \frac{C_{15} ce^{dt}}{d^2} - \frac{C_{15} c}{2d^2} - \frac{C_{15} c}{2ad} + \frac{C_2 ce^{dt}}{d} - \frac{C_2 c}{d} - \frac{C_4 b}{a} + \frac{C_4 be^{-at}}{a} - C_5 + \frac{C_7 abc^2 e^{2dt}}{2a^3 d^2-2a^2 d^3-8ad^4+8d^5} - \frac{C_7 abc^2 e^{at}}{2a^4 d-6a^3 d^2+4a^2 d^3} + \frac{C_7 bc^2 de^{2dt}}{2a^3 d^2-2a^2 d^3-8ad^4+8d^5} - \frac{C_7 bc^2 de^{at}}{2a^4 d-6a^3 d^2+4a^2 d^3} - \frac{C_7 bc^2}{4a^4 e^{at}+10a^3 de^{at}+4a^2 d^2 e^{at}} + \frac{C_7 bc^2 e^{at} e^{dt}}{2a^3 d-a^2 d^2-ad^3} - \frac{C_7 bc^2 e^{dt}}{a^2 d^2-ad^3} + \frac{C_7 bc^2}{2a^2 d^2} + \frac{C_8 ac^2 e^{2dt}}{2a^2 d^2+2ad^3-4d^4} + \frac{C_8 c^2 de^{2dt}}{2a^2 d^2+2ad^3-4d^4} - \frac{C_8 c^2}{2a^3 e^{at}+5a^2 de^{at}+2ad^2 e^{at}} - \frac{C_8 c^2 e^{at} e^{dt}}{2a^3-a^2 d-ad^2} - \frac{C_8 c^2 e^{dt}}{ad^2} + \frac{C_8 c^2}{2ad^2} = 0$ Basis functions:

$$\left[e^{2dt},\ 1.0,\ e^{at} e^{dt},\ e^{dt},\ e^{at},\ e^{-at}\right]$$

Equation for the basis function e^{2dt}:

$$-\frac{C_{15} ac}{2ad^2 + 4d^3} - \frac{C_{15} cd}{2ad^2 + 4d^3} + \frac{C_7 abc^2}{2a^3 d^2 - 2a^2 d^3 - 8ad^4 + 8d^5} + \frac{C_7 bc^2 d}{2a^3 d^2 - 2a^2 d^3 - 8ad^4 + 8d^5} + \frac{C_8 ac^2}{2a^2 d^2 + 2ad^3 - 4d^4} + \frac{C_8 c^2 d}{2a^2 d^2 + 2ad^3 -}$$

This equation was solved for: C_7 which gave the solution:

$$\frac{C_{15} a^2}{bc} - \frac{3C_{15} ad}{bc} + \frac{2C_{15} d^2}{bc} - \frac{C_8 a}{b} + \frac{2C_8 d}{b}$$

Equation for the basis function $e^{at} e^{dt}$:

$$\frac{C_7 bc^2}{2a^3 d - a^2 d^2 - ad^3} - \frac{C_8 c^2}{2a^3 - a^2 d - ad^2}$$

This equation was solved for: C_7 which gave the solution:

$$\frac{C_8 d}{b}$$

Equation for the basis function e^{dt}:

$$\frac{C_{15} c}{d^2} + \frac{C_2 c}{d} - \frac{C_7 bc^2}{a^2 d^2 - ad^3} - \frac{C_8 c^2}{ad^2}$$

This equation was solved for: C_2 which gave the solution:

$$-\frac{C_{15}a^2}{a^2d-ad^2}+\frac{C_{15}ad}{a^2d-ad^2}+\frac{C_7bc}{a^2d-ad^2}+\frac{C_8ac}{a^2d-ad^2}-\frac{C_8cd}{a^2d-ad^2}$$

Equation for the basis function e^{at}:

$$-\frac{C_7abc^2}{2a^4d-6a^3d^2+4a^2d^3}-\frac{C_7bc^2d}{2a^4d-6a^3d^2+4a^2d^3}$$

This equation was solved for: C_7 which gave the solution:

$$0$$

Equation for the basis function e^{-at}:

$$\frac{C_4b}{a}$$

This equation was solved for: C_4 which gave the solution:

$$0$$

Solutions *before* processing:

$$C_7=\frac{C_{15}a^2}{bc}-\frac{3C_{15}ad}{bc}+\frac{2C_{15}d^2}{bc}-\frac{C_8a}{b}+\frac{2C_8d}{b}$$
$$C_7=\frac{C_8d}{b}$$
$$C_2=-\frac{C_{15}a^2}{a^2d-ad^2}+\frac{C_{15}ad}{a^2d-ad^2}+\frac{C_7bc}{a^2d-ad^2}+\frac{C_8ac}{a^2d-ad^2}-\frac{C_8cd}{a^2d-ad^2}$$
$$C_7=0$$
$$C_4=0$$
$$C_2=-\frac{C_{15}a^4ce^{at}}{2a^4cde^{at}+5a^3cd^2e^{at}+2a^2cd^3e^{at}}-\frac{7C_{15}a^3cde^{at}}{4a^4cde^{at}+10a^3cd^2e^{at}+4a^2cd^3e^{at}}-\frac{7C_{15}a^2cd^2e^{at}}{4a^4cde^{at}+10a^3cd^2e^{at}+4a^2cd^3e^{at}}+\frac{\cdots}{2a^4c\cdots}$$

Solutions *after* processing:

$$C_7=\frac{C_{15}a^2}{bc}-\frac{3C_{15}ad}{bc}+\frac{2C_{15}d^2}{bc}-\frac{C_8a}{b}+\frac{2C_8d}{b}$$
$$C_7=\frac{C_8d}{b}$$
$$C_2=-\frac{C_{15}a^2}{a^2d-ad^2}+\frac{C_{15}ad}{a^2d-ad^2}+\frac{C_7bc}{a^2d-ad^2}+\frac{C_8ac}{a^2d-ad^2}-\frac{C_8cd}{a^2d-ad^2}$$
$$C_7=0$$
$$C_4=0$$
$$C_2=-\frac{C_{15}a^4ce^{at}}{2a^4cde^{at}+5a^3cd^2e^{at}+2a^2cd^3e^{at}}-\frac{7C_{15}a^3cde^{at}}{4a^4cde^{at}+10a^3cd^2e^{at}+4a^2cd^3e^{at}}-\frac{7C_{15}a^2cd^2e^{at}}{4a^4cde^{at}+10a^3cd^2e^{at}+4a^2cd^3e^{at}}+\frac{\cdots}{2a^4c\cdots}$$

Equation: $\frac{C_{15}a^3be^{at}}{a^2cd-2acd^2}-\frac{C_{15}a^3be^{at}e^{dt}}{a^2cd-cd^3}-\frac{5C_{15}a^2bde^{at}}{a^2cd-2acd^2}+\frac{4C_{15}a^2bde^{at}e^{dt}}{a^2cd-cd^3}-\frac{C_{15}a^2b}{a^2c+acd}+\frac{8C_{15}abd^2e^{at}}{a^2cd-2acd^2}-\frac{5C_{15}abd^2e^{at}e^{dt}}{a^2cd-cd^3}+\frac{3C_{15}abd}{a^2c+acd}-\frac{4C_{15}bd^3e^{at}}{a^2cd-2acd^2}+\frac{2C_{15}bd^3e^{at}e^{dt}}{a^2cd-cd^3}-\frac{2C_{15}bd^2}{a^2c+acd}+\frac{C_8a^2bce^{at}e^{dt}}{a^2cd-cd^3}-\frac{C_8a^2be^{at}}{a^2d-2ad^2}-\frac{3C_8abcde^{at}e^{dt}}{a^2cd-cd^3}+\frac{C_8abce^{at}e^{dt}}{a^2c-cd^2}+\frac{4C_8abde^{at}}{a^2d-2ad^2}+\frac{C_8ab}{a^2+ad}+\frac{2C_8bcd^2e^{at}e^{dt}}{a^2cd-cd^3}-\frac{C_8bcde^{at}e^{dt}}{a^2c-cd^2}-\frac{4C_8bd^2e^{at}}{a^2d-2ad^2}-\frac{2C_8bd}{a^2+ad}-\frac{C_8b}{a+d}-C_9=0$ Basis functions:

$$\left[e^{at},\ 1.0,\ e^{at}e^{dt}\right]$$

Equation for the basis function e^{at}:

$$\frac{C_{15}a^3b}{a^2cd-2acd^2}-\frac{5C_{15}a^2bd}{a^2cd-2acd^2}+\frac{8C_{15}abd^2}{a^2cd-2acd^2}-\frac{4C_{15}bd^3}{a^2cd-2acd^2}-\frac{C_8a^2b}{a^2d-2ad^2}+\frac{4C_8abd}{a^2d-2ad^2}-\frac{4C_8bd^2}{a^2d-2ad^2}$$

This equation was solved for: C_8 which gave the solution:

$$\frac{C_{15}a}{c}-\frac{C_{15}d}{c}$$

Equation for the basis function $e^{at}e^{dt}$:

$$-\frac{C_{15}a^3b}{a^2cd-cd^3}+\frac{4C_{15}a^2bd}{a^2cd-cd^3}-\frac{5C_{15}abd^2}{a^2cd-cd^3}+\frac{2C_{15}bd^3}{a^2cd-cd^3}+\frac{C_8a^2bc}{a^2cd-cd^3}-\frac{3C_8abcd}{a^2cd-cd^3}+\frac{C_8abc}{a^2c-cd^2}+\frac{2C_8bcd^2}{a^2cd-cd^3}-\frac{C_8bcd}{a^2c-cd^2}$$

This equation was solved for: C_8 which gave the solution:

$$\frac{C_{15}a}{c}-\frac{2C_{15}d}{c}$$

Solutions *before* processing:

$$C_8=\frac{C_{15}a}{c}-\frac{C_{15}d}{c}$$
$$C_8=\frac{C_{15}a}{c}-\frac{2C_{15}d}{c}$$
$$C_8=-\frac{C_{15}a^2}{2cd}+\frac{3C_{15}a}{2c}-\frac{C_{15}d}{c}-\frac{C_9a^2}{2bd}-\frac{C_9a}{2b}$$

Solutions *after* processing:

$$C_8=\frac{C_{15}a}{c}-\frac{C_{15}d}{c}$$
$$C_8=\frac{C_{15}a}{c}-\frac{2C_{15}d}{c}$$
$$C_8=-\frac{C_{15}a^2}{2cd}+\frac{3C_{15}a}{2c}-\frac{C_{15}d}{c}-\frac{C_9a^2}{2bd}-\frac{C_9a}{2b}$$

Equation: $\frac{C_{10}bdt}{a}-\frac{C_{10}b}{a}+\frac{C_{10}be^{-at}}{a}-\frac{C_{10}bd}{a^2}+\frac{C_{10}bde^{-at}}{a^2}-C_{11}+\frac{C_{13}a^2b^2}{a^3de^{at}-a^2d^2e^{at}}+\frac{2C_{13}a^2b^2}{a^3d+a^2d^2}-\frac{2C_{13}ab^2c^2}{a^2c^2de^{dt}-ac^2d^2e^{dt}}+\frac{2C_{13}ab^2d}{a^3de^{at}-a^2d^2e^{at}}+\frac{4C_{13}ab^2d}{a^3d+a^2d^2}-\frac{C_{13}ab^2}{a^2de^{at}e^{dt}+ad^2e^{at}e^{dt}}-\frac{2C_{13}b^2c^2d}{a^2c^2de^{dt}-ac^2d^2e^{dt}}+\frac{C_{13}b^2d^2}{a^3de^{at}-a^2d^2e^{at}}+\frac{C_{13}b^2d^2}{a^3d+a^2d^2}-\frac{2C_{13}b^2d}{a^2de^{at}e^{dt}+ad^2e^{at}e^{dt}}-\frac{C_{13}b^2t}{a}+C_{14}bt+\frac{C_{15}a^3b}{2a^2bd+2abd^2}+\frac{4C_{15}a^3e^{at}e^{dt}}{2a^3+a^2d-2ad^2-d^3}-\frac{C_{15}a^3e^{2dt}}{2a^2d+2ad^2-4d^3}-\frac{C_{15}a^2bd}{2a^3be^{at}+5a^2bde^{at}+2abd^2e^{at}}-\frac{5C_{15}a^2bd}{2a^2bd+2abd^2}-\frac{9C_{15}a^2de^{at}e^{dt}}{2a^3+a^2d-2ad^2-d^3}+\frac{C_{15}a^2de^{2dt}}{2a^2d+2ad^2-4d^3}+\frac{C_{15}a^2e^{2dt}}{2ad+4d^2}+\frac{5C_{15}abd^2}{2a^2bd+2abd^2}+\frac{C_{15}abd}{a^2be^{at}+2abde^{at}}+\frac{2C_{15}ad^2e^{at}e^{dt}}{2a^3+a^2d-2ad^2-d^3}+\frac{5C_{15}ad^2e^{2dt}}{2a^2d+2ad^2-4d^3}+\frac{2C_{15}ae^{at}e^{dt}}{a-d}-\frac{2C_{15}ae^{2dt}}{a-d}-\frac{C_{15}a}{2d}+\frac{C_{15}bd^3}{2a^3be^{at}+5a^2bde^{at}+2abd^2e^{at}}-\frac{C_{15}bd^3}{2a^2bd+2abd^2}+\frac{C_{15}bd^2}{a^2be^{at}+2abde^{at}}+\frac{3C_{15}d^3e^{at}e^{dt}}{2a^3+a^2d-2ad^2-d^3}-\frac{5C_{15}d^3e^{2dt}}{2a^2d+2ad^2-4d^3}-\frac{5C_{15}d^2e^{2dt}}{2ad+4d^2}-\frac{2C_{15}de^{at}e^{dt}}{a-d}+\frac{2C_{15}de^{2dt}}{a-d}+2C_{15}e^{2dt}+C_{15}-\frac{C_{15}d}{2a}+\frac{C_{16}ab^2}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}}+\frac{2C_{16}b^2d}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}}+\frac{C_{16}b^2d}{a^2c+acd}-\frac{C_{16}b^2e^{-at}}{cd}-\frac{C_{16}b^2e^{-at}}{ac}-\frac{C_{18}be^{-at}}{c}+\frac{C_{18}bd}{ac}-\frac{C_{18}bde^{-at}}{ac}=0$ Basis functions:

$$\left[e^{2dt},\ 1.0,\ t,\ e^{at}e^{dt},\ e^{-at}e^{-dt},\ e^{-dt},\ e^{-at}\right]$$

Equation for the basis function e^{2dt}:

$$-\frac{C_{15}a^3}{2a^2d+2ad^2-4d^3}+\frac{C_{15}a^2d}{2a^2d+2ad^2-4d^3}+\frac{C_{15}a^2}{2ad+4d^2}+\frac{5C_{15}ad^2}{2a^2d+2ad^2-4d^3}-\frac{2C_{15}a}{a-d}-\frac{5C_{15}d^3}{2a^2d+2ad^2-4d^3}-\frac{5C_{15}d^2}{2ad+4d^2}+\frac{2C_{15}d}{a-d}+$$

This equation was solved for: C_{15} which gave the solution:

$$0$$

Equation for the basis function t:

$$\frac{C_{10}bd}{a} - \frac{C_{13}b^2}{a} + C_{14}b$$

This equation was solved for: C_{10} which gave the solution:

$$\frac{C_{13}b}{d} - \frac{C_{14}a}{d}$$

Equation for the basis function $e^{at}e^{dt}$:

$$\frac{4C_{15}a^3}{2a^3 + a^2d - 2ad^2 - d^3} - \frac{9C_{15}a^2d}{2a^3 + a^2d - 2ad^2 - d^3} + \frac{2C_{15}ad^2}{2a^3 + a^2d - 2ad^2 - d^3} + \frac{2C_{15}a}{a-d} + \frac{3C_{15}d^3}{2a^3 + a^2d - 2ad^2 - d^3} - \frac{2C_{15}d}{a-d}$$

This equation was solved for: C_{15} which gave the solution:

$$0$$

Equation for the basis function $e^{-at}e^{-dt}$:

$$0$$

This equation was solved for: 0 which gave the solution:

$$0$$

Equation for the basis function e^{-dt}:

$$0$$

This equation was solved for: 0 which gave the solution:

$$0$$

Equation for the basis function e^{-at}:

$$\frac{C_{10}b}{a} + \frac{C_{10}bd}{a^2} - \frac{C_{16}b^2}{cd} - \frac{C_{16}b^2}{ac} - \frac{C_{18}b}{c} - \frac{C_{18}bd}{ac}$$

This equation was solved for: C_{10} which gave the solution:

$$\frac{C_{16}ab}{cd} + \frac{C_{18}a}{c}$$

Solutions *before* processing:

$$C_{15} = 0$$
$$C_{10} = \frac{C_{13}b}{d} - \frac{C_{14}a}{d}$$
$$C_{15} = 0$$
$$0 = 0$$
$$0 = 0$$
$$C_{10} = \frac{C_{16}ab}{cd} + \frac{C_{18}a}{c}$$
$$C_{10} = -\frac{2C_{11}a^5cd}{2a^4bcd + 3a^3bcd^2 - a^2bcd^3 - 3abcd^4 - bcd^5} - \frac{C_{11}a^4cd^2}{2a^4bcd + 3a^3bcd^2 - a^2bcd^3 - 3abcd^4 - bcd^5} + \frac{2C_{11}a^3c}{2a^4bcd + 3a^3bcd^2 - a^2bc}$$

Solutions *after* processing:

$$C_{15} = 0$$
$$C_{10} = \frac{C_{13}b}{d} - \frac{C_{14}a}{d}$$
$$C_{15} = 0$$
$$0 = 0$$
$$0 = 0$$
$$C_{10} = \frac{C_{16}ab}{cd} + \frac{C_{18}a}{c}$$
$$C_{10} = -\frac{2C_{11}a^5cd}{2a^4bcd + 3a^3bcd^2 - a^2bcd^3 - 3abcd^4 - bcd^5} - \frac{C_{11}a^4cd^2}{2a^4bcd + 3a^3bcd^2 - a^2bcd^3 - 3abcd^4 - bcd^5} + \frac{2C_{11}a^3c}{2a^4bcd + 3a^3bcd^2 - a^2bc}$$

Equation: $-C_{12} + \frac{C_{13}bc}{a^2e^{at}e^{dt}+ade^{at}e^{dt}} + \frac{C_{13}bc}{ad+d^2} - \frac{C_{13}bce^{-dt}}{ad} - \frac{C_{16}b}{ae^{at}e^{dt}+de^{at}e^{dt}} + \frac{C_{16}b}{a+d} = 0$ Basis functions:

$$\left[1.0,\ e^{-dt},\ e^{-at}e^{-dt}\right]$$

Equation for the basis function e^{-dt}:

$$-\frac{C_{13}bc}{ad}$$

This equation was solved for: C_{13} which gave the solution:

$$0$$

Equation for the basis function $e^{-at}e^{-dt}$:

$$0$$

This equation was solved for: 0 which gave the solution:

$$0$$

Solutions *before* processing:

$$C_{13} = 0$$
$$0 = 0$$
$$C_{12} = \frac{C_{13}abc}{a^2d + ad^2} + \frac{C_{13}bcd}{a^2de^{at}e^{dt} + ad^2e^{at}e^{dt}} - \frac{C_{16}abd}{a^2de^{at}e^{dt} + ad^2e^{at}e^{dt}} + \frac{C_{16}abd}{a^2d + ad^2}$$

Solutions *after* processing:

$$C_{13} = 0$$
$$0 = 0$$
$$C_{12} = \frac{C_{13}abc}{a^2d + ad^2} + \frac{C_{13}bcd}{a^2de^{at}e^{dt} + ad^2e^{at}e^{dt}} - \frac{C_{16}abd}{a^2de^{at}e^{dt} + ad^2e^{at}e^{dt}} + \frac{C_{16}abd}{a^2d + ad^2}$$

Equation: $-\frac{C_{13}c^2}{a^2e^{at}e^{dt}+ade^{at}e^{dt}} - \frac{C_{13}c^2}{ad+d^2} + \frac{C_{13}c^2e^{-dt}}{ad} + \frac{C_{16}c}{ae^{at}e^{dt}+de^{at}e^{dt}} - \frac{C_{16}c}{a+d} - C_{17} = 0$ Basis functions:

$$\left[1.0,\ e^{-dt},\ e^{-at}e^{-dt}\right]$$

Equation for the basis function e^{-dt}:

$$\frac{C_{13}c^2}{ad}$$

This equation was solved for: C_{13} which gave the solution:

$$0$$

Equation for the basis function $e^{-at}e^{-dt}$:

$$0$$

This equation was solved for: 0 which gave the solution:

$$0$$

Solutions *before* processing:

$$\begin{aligned} C_{13} &= 0 \\ 0 &= 0 \\ C_{13} &= -\frac{C_{16}acde^{at}e^{dt}}{ac^2e^{at}e^{dt}+c^2d} + \frac{C_{16}acd}{ac^2e^{at}e^{dt}+c^2d} - \frac{C_{17}a^2de^{at}e^{dt}}{ac^2e^{at}e^{dt}+c^2d} - \frac{C_{17}ad^2e^{at}e^{dt}}{ac^2e^{at}e^{dt}+c^2d} \end{aligned}$$

Solutions *after* processing:

$$\begin{aligned} C_{13} &= 0 \\ 0 &= 0 \\ C_{13} &= -\frac{C_{16}acde^{at}e^{dt}}{ac^2e^{at}e^{dt}+c^2d} + \frac{C_{16}acd}{ac^2e^{at}e^{dt}+c^2d} - \frac{C_{17}a^2de^{at}e^{dt}}{ac^2e^{at}e^{dt}+c^2d} - \frac{C_{17}ad^2e^{at}e^{dt}}{ac^2e^{at}e^{dt}+c^2d} \end{aligned}$$

Solutions after all is done:

$$\mathbf{c} = \begin{bmatrix} C_1 + \frac{C_{14}}{d} - \frac{C_{14}e^{-at}}{d} + \frac{C_{16}ab}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{C_{16}bd}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} - \frac{C_{16}be^{-at}}{cd} - \frac{C_{18}e^{-at}}{c} + \mathrm{c}_{22} \\ 0 \\ -\frac{C_6b^2d}{ac^2} \\ 0 \\ C_5 \\ 0 \\ 0 \\ 0 \\ C_9 \\ -\frac{C_{14}ae^{-at}}{d} + \frac{C_{16}a^2b}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{C_{16}abd}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} - \frac{C_{16}abe^{-at}}{cd} - \frac{C_{18}ae^{-at}}{c} + \frac{a\,\mathrm{c}_{22}(t)}{c} \\ C_{11} - \frac{C_{14}b}{d} + \frac{C_{14}be^{-at}}{d} - \frac{C_{14}b}{a} + \frac{C_{14}be^{-at}}{a} - \frac{C_{16}ab^2}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} - \frac{2C_{16}b^2d}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} - \frac{C_{16}b^2d}{a^2c+acd} + \frac{C_{16}b^2e^{-at}}{cd} + \frac{C_{16}b^2e^{-at}}{ac} + C \\ -\frac{C_{16}abd}{a^2de^{at}e^{dt}+ad^2e^{at}e^{dt}} + \frac{C_{16}abd}{a^2d+ad^2} + \frac{C_{16}b}{ae^{at}e^{dt}+de^{at}e^{dt}} - \frac{C_{16}b}{a+d} \\ 0 \\ C_{14}e^{-at} - \frac{C_{16}ab}{ace^{at}e^{dt}+cde^{at}e^{dt}} - \frac{C_{16}bd}{ace^{at}e^{dt}+cde^{at}e^{dt}} + \frac{C_{16}be^{-at}}{c} + \frac{C_{18}de^{-at}}{c} - \frac{d\,\mathrm{c}_{22}(t)}{c} \\ 0 \\ \frac{C_{16}a}{ae^{at}e^{dt}+de^{at}e^{dt}} + \frac{C_{16}d}{ae^{at}e^{dt}+de^{at}e^{dt}} \\ -\frac{C_{16}c}{ae^{at}e^{dt}+de^{at}e^{dt}} + \frac{C_{16}c}{a+d} + C_{17} \end{bmatrix}$$

Tangents before any manipulation:

$$\eta_0 = C_1 + \frac{C_{14}}{d} - \frac{C_{14}e^{-at}}{d} + \frac{C_{16}ab}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{C_{16}bd}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} - \frac{C_{16}be^{-at}}{cd} - \frac{C_{18}e^{-at}}{c} + C_5NP - \frac{C_6b^2dP^2}{ac^2} + \mathrm{c}_2$$

$$\eta_1 = C_{11}NP - \frac{C_{14}aNe^{-at}}{d} - \frac{C_{14}bNP}{d} + \frac{C_{14}bNPe^{-at}}{d} - \frac{C_{14}bNP}{a} + \frac{C_{14}bNPe^{-at}}{a} + \frac{C_{16}a^2bN}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} - \frac{C_{16}ab^2N}{acde^{at}e^{dt}+cd}$$

$$\eta_2 = C_{14}Pe^{-at} - \frac{C_{16}abP}{ace^{at}e^{dt}+cde^{at}e^{dt}} + \frac{C_{16}aN}{ae^{at}e^{dt}+de^{at}e^{dt}} - \frac{C_{16}bdP}{ace^{at}e^{dt}+cde^{at}e^{dt}} + \frac{C_{16}bPe^{-at}}{c} - \frac{C_{16}cN^2}{ae^{at}e^{dt}+de^{at}e^{dt}} + \frac{C_{16}cN^2}{a+d}$$

Generators before some are removed:

$$\left[-\frac{b^2dP^2}{ac^2},\ 0,\ 0\right]$$

$$\left[0,\ 0,\ N^2\right]$$

$$[NP,\ 0,\ 0]$$

$$\left[-\frac{e^{-at}}{c},\ -\frac{aNe^{-at}}{c}+\frac{bNPe^{-at}}{c}-\frac{bdNP}{ac}+\frac{bdNPe^{-at}}{ac},\ \frac{dPe^{-at}}{c}\right]$$

$$[0,\ NP,\ 0]$$

$$[1,\ 0,\ 0]$$

$$\left[0,\ P^2,\ 0\right]$$

$$\left[\frac{ab}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}}+\frac{bd}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}}-\frac{be^{-at}}{cd},\ \frac{a^2bN}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}}-\frac{ab^2NP}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}}-\frac{abdN^2}{a^2de^{at}e^{dt}+ad^2e^{at}e^{dt}}\right.$$

$$\left[\frac{1}{d}-\frac{e^{-at}}{d},\ -\frac{aNe^{-at}}{d}-\frac{bNP}{d}+\frac{bNPe^{-at}}{d}-\frac{bNP}{a}+\frac{bNPe^{-at}}{a},\ Pe^{-at}\right]$$

$$\left[\frac{c_{22}(t)}{c},\ \frac{aN\,c_{22}(t)}{c}-\frac{bNP\,c_{22}(t)}{c},\ NP\,c_{22}(t)-\frac{dP\,c_{22}(t)}{c}\right]$$

The tangents and their symmetry conditions:

$$\eta_0=\left[-\frac{b^2dP^2}{ac^2},\ 0,\ 0\right]$$

$$\left[-2ab^2cdN^2P^2+2ab^2d^2NP^2+2b^3cdN^2P^3-2b^3d^2NP^3,\ -2b^2c^2dN^2P^3+4b^2cd^2NP^3-2b^2d^3P^3\right]$$

$$\eta_1=\left[0,\ 0,\ N^2\right]$$

$$\left[-bN^3,\ -2aN^2+2bN^2P+cN^3-dN^2\right]$$

$$\eta_2=[NP,\ 0,\ 0]$$

$$\left[a^2N^2P-2abN^2P^2+acN^3P-adN^2P+b^2N^2P^3-bcN^3P^2+bdN^2P^2,\ acN^2P^2-adNP^2-bcN^2P^3+bdNP^3+c^2N^3P^2\right.$$

$$\eta_3=\left[-\frac{e^{-at}}{c},\ -\frac{aNe^{-at}}{c}+\frac{bNPe^{-at}}{c}-\frac{bdNP}{ac}+\frac{bdNPe^{-at}}{ac},\ \frac{dPe^{-at}}{c}\right]$$

$$\left[-abcN^2P+abdNP+bcdN^2Pe^{at}-bcdN^2P-bd^2NPe^{at}+bd^2NP,\ abNP^2-bdNP^2e^{at}+bdNP^2\right]$$

$$\eta_4=[0,\ NP,\ 0]$$

$$\left[-cN^2P+dNP,\ cNP^2\right]$$

$$\eta_5=[1,\ 0,\ 0]$$

$$[0,\ 0]$$

$$\eta_6=\left[0,\ P^2,\ 0\right]$$

$$\left[aP^2-bP^3-2cNP^2+2dP^2,\ cP^3\right]$$

$$\eta_7=\left[\frac{ab}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}}+\frac{bd}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}}-\frac{be^{-at}}{cd},\ \frac{a^2bN}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}}-\frac{ab^2NP}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}}-\frac{abdN^2}{a^2de^{at}e^{dt}+a}\right.$$

$$\left[-a^2b^2cN^2Pe^{dt}+a^2b^2cN^2P+a^2b^2dNPe^{dt}-a^2b^2dNP-a^2bcdN^2-2ab^2cdN^2Pe^{dt}+2ab^2cdN^2P+2ab^2d^2NPe^{dt}-2ab^2d^2\right.$$

$$\eta_8=\left[\frac{1}{d}-\frac{e^{-at}}{d},\ -\frac{aNe^{-at}}{d}-\frac{bNP}{d}+\frac{bNPe^{-at}}{d}-\frac{bNP}{a}+\frac{bNPe^{-at}}{a},\ Pe^{-at}\right]$$

$$\left[abcN^2Pe^{at}-abcN^2P-abdNPe^{at}+abdNP+bcdN^2Pe^{at}-bcdN^2P-bd^2NPe^{at}+bd^2NP,\ -abcNP^2e^{at}+abcNP^2-bcd\right.$$

$$\eta_9 = \left[\frac{\mathrm{c}_{22}\left(t\right)}{c},\ \frac{aN\,\mathrm{c}_{22}\left(t\right)}{c} - \frac{bNP\,\mathrm{c}_{22}\left(t\right)}{c},\ NP\,\mathrm{c}_{22}\left(t\right) - \frac{dP\,\mathrm{c}_{22}\left(t\right)}{c}\right]$$

$$[0,\ 0]$$

Generators after some were removed:

$$[1,\ 0,\ 0]$$

$$\left[\frac{\mathrm{c}_{22}\left(t\right)}{c},\ \frac{aN\,\mathrm{c}_{22}\left(t\right)}{c} - \frac{bNP\,\mathrm{c}_{22}\left(t\right)}{c},\ NP\,\mathrm{c}_{22}\left(t\right) - \frac{dP\,\mathrm{c}_{22}\left(t\right)}{c}\right]$$

The execution time of the script was:

0 hours 1 minutes 18 seconds.