

# Summary of symmetry calculations

February 1, 2022



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# Chapter 1

## DBH\_model

Run 01\_27PM\_01\_February-2022

Degree in tangential ansätze: 2.  
The system of ODEs is given by:

$$\begin{aligned}\frac{dw_1}{dt} &= -w_1w_2 - w_1w_3 + w_2w_3, \\ \frac{dw_2}{dt} &= -w_1w_2 + w_1w_3 - w_2w_3, \\ \frac{dw_3}{dt} &= w_1w_2 - w_1w_3 - w_2w_3.\end{aligned}$$

The calculated generators are:

$$X_1 = (-t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$X_2 = (1) \partial t,$$

$$X_3 = (1) \partial t,$$

$$\begin{aligned}X_4 &= (t^2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 + (1 \\ &\quad + -2tw_3) \partial w_3\end{aligned}$$

$$\begin{aligned}X_5 &= (f_1(t)) \partial t + (w_2w_3 f_1(t) - w_1w_2 f_1(t) - w_1w_3 f_1(t)) \partial w_1 + (w_1w_3 f_1(t) - w_1w_2 f_1(t) \\ &\quad + -w_2w_3 f_1(t)) \partial w_2 + (w_1w_2 f_1(t) - w_1w_3 f_1(t) - w_2w_3 f_1(t)) \partial w_3.\end{aligned}$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

The execution time of the script was:

0 hours 5 minutes 15 seconds.

## Chapter 2

# SIR

**Run 01\_28PM\_01\_February-2022**

Degree in tangential ansätze: 1.  
The system of ODEs is given by:

$$\begin{aligned}\frac{dS}{dt} &= -ISr, \\ \frac{dI}{dt} &= ISr - Ia, \\ \frac{dR}{dt} &= Ia.\end{aligned}$$

The calculated generators are:

$$X_1 = (I + R + S) \partial t$$

$$X_2 = (1) \partial R,$$

$$X_3 = (I + R + S) \partial R$$

$$X_4 = (1) \partial t.$$

The execution time of the script was:

0 hours 0 minutes 24 seconds.

**Run 01\_31PM\_01\_February-2022**

Degree in tangential ansätze: 2.  
The system of ODEs is given by:

$$\begin{aligned}\frac{dS}{dt} &= -ISr, \\ \frac{dI}{dt} &= ISr - Ia, \\ \frac{dR}{dt} &= Ia.\end{aligned}$$

The calculated generators are:

$$X_1 = (1) \partial R,$$

$$X_2 = \left( \frac{I^2}{2} + \frac{R^2}{2} + \frac{S^2}{2} + IR + IS + RS \right) \partial t$$

$$X_3 = (I + R + S) \partial t$$

$$X_4 = (1) \partial t,$$

$$X_5 = (IS) \partial S + \left( -IS + \frac{Ia}{r} \right) \partial I + \left( \frac{Ra}{r} + \frac{Sa}{r} \right) \partial R$$

$$X_6 = \left( -\frac{1}{a} \right) \partial t + \left( \frac{ISr}{a} \right) \partial S + \left( I - \frac{ISr}{a} \right) \partial I + (R + S) \partial R$$

$$X_7 = \left( \frac{f_1(t)}{a} \right) \partial t + \left( -\frac{ISr f_1(t)}{a} \right) \partial S + \left( \frac{ISr f_1(t)}{a} - I f_1(t) \right) \partial I + (I f_1(t)) \partial R.$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

## WARNING:

*Some of the calculated generators did not satisfy the linearised symmetry conditions. Thus, the presented list here is not complete and consists exclusively of the calculated generators that satisfy the linearised symmetry conditions.*



The execution time of the script was:

0 hours 2 minutes 58 seconds.



## Chapter 3

# Brusselator

**Run 01\_31PM\_01\_February-2022**

Degree in tangential ansätze: 1.  
The system of ODEs is given by:

$$\begin{aligned}\frac{du}{dt} &= au^2v - u(b+1) + 1, \\ \frac{dv}{dt} &= -au^2v + bu.\end{aligned}$$

The calculated generators are:

$$X_1 = (1) \partial t.$$

The execution time of the script was:

0 hours 0 minutes 6 seconds.



## Chapter 4

# linear\_model

Run 01\_21PM\_01\_February-2022

Degree in tangential ansätze: 1.  
The system of ODEs is given by:

$$\begin{aligned}\frac{du}{dt} &= u + v, \\ \frac{dv}{dt} &= u + v.\end{aligned}$$

The calculated generators are:

$$X_1 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial u + \left(\frac{e^{2t}}{2} - \frac{1}{2}\right) \partial v,$$

$$\begin{aligned}X_2 &= \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right) \partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right) \partial u \\ &\quad + \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right) \partial v\end{aligned}$$

$$X_3 = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right) \partial t$$

$$X_4 = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right) \partial t$$

$$X_5 = (1) \partial t,$$

$$X_6 = \left(\frac{e^{2t}}{2} - \frac{1}{2}\right) \partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial v$$

$$X_7 = \left( -\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4} \right) \partial t + \left( -\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} + \frac{ve^{2t}}{2} \right) \partial u + \left( \frac{ve^{2t}}{2} - \frac{v}{2} \right) \partial v,$$

$$X_8 = \left( -\frac{e^{2t}}{4} - \frac{1}{2} - \frac{e^{-2t}}{4} \right) \partial t + \left( -\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{2t}}{2} - \frac{ve^{-2t}}{2} \right) \partial u + \left( -\frac{ve^{2t}}{2} - \frac{v}{2} \right) \partial v,$$

$$X_9 = \left( \frac{e^{2t}}{4} - \frac{e^{-2t}}{4} \right) \partial t + \left( \frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} \right) \partial u + \left( \frac{v}{2} + \frac{ve^{2t}}{2} \right) \partial v$$

$$X_{10} = (f_1(t)) \partial t + (u f_1(t) + v f_1(t)) \partial u + (u f_1(t) + v f_1(t)) \partial v$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

The execution time of the script was:

0 hours 0 minutes 14 seconds.

## Run 01\_22PM\_01\_February-2022

Degree in tangential ansätze: 2.

The system of ODEs is given by:

$$\begin{aligned} \frac{du}{dt} &= u + v, \\ \frac{dv}{dt} &= u + v. \end{aligned}$$

The calculated generators are:

$$X_1 = \left( -\frac{e^{2t}}{4} + \frac{e^{-2t}}{4} \right) \partial t + \left( \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2} \right) \partial u + \left( \frac{v}{2} - \frac{ve^{2t}}{2} \right) \partial v$$

$$X_2 = \left( \frac{1}{2} + \frac{e^{2t}}{2} \right) \partial u + \left( \frac{e^{2t}}{2} - \frac{1}{2} \right) \partial v,$$

$$X_3 = (1) \partial t,$$

$$X_4 = \left( \frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial t$$

$$X_5 = \left( -\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4} \right) \partial t + \left( -\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} + \frac{ve^{2t}}{2} \right) \partial u + \left( \frac{ve^{2t}}{2} - \frac{v}{2} \right) \partial v,$$

$$X_6 = \left( \frac{u^2}{4} + \frac{v^2}{4} + \frac{v^2 e^{-2t}}{2} - \frac{uv}{2} - \frac{u^2 e^{-2t}}{2} + \frac{u^2 e^{-4t}}{4} + \frac{v^2 e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial t$$

$$X_7 = \left( -\frac{3v}{8} + \frac{u}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-4t}}{8} + -\frac{ve^{-2t}}{8} + \frac{ue^{2t}}{8} \right) \partial t + \left( \frac{u^2}{4} - \frac{u^2 e^{-4t}}{4} - \frac{uv}{2} - \frac{uve^{-4t}}{2} - \frac{v^2 e^{2t}}{2} - \frac{v^2}{4} - \frac{v^2 e^{-4t}}{4} \right) \partial u + \left( -\frac{v^2 e^{2t}}{2} - \frac{v^2}{2} \right) \partial v,$$

$$X_8 = \left( \frac{e^{2t}}{2} - \frac{1}{2} \right) \partial u + \left( \frac{1}{2} + \frac{e^{2t}}{2} \right) \partial v$$

$$X_9 = \left( \frac{u}{8} + \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} + \frac{ue^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left( -\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2 e^{-2t}}{2} + \frac{v^2 e^{2t}}{2} - \frac{v^2 e^{-2t}}{2} - \frac{u^2 e^{-4t}}{4} - \frac{v^2 e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left( \frac{v^2}{2} + \frac{v^2 e^{2t}}{2} \right) \partial v$$

$$X_{10} = \left( -\frac{u}{8} + \frac{3v}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{2t}}{8} + \frac{ve^{-4t}}{8} \right) \partial t + \left( -\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} - \frac{v^2 e^{2t}}{2} + \frac{u^2 e^{-4t}}{4} + \frac{v^2 e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left( \frac{v^2}{2} - \frac{v^2 e^{2t}}{2} \right) \partial v$$

$$\begin{aligned}
X_{11} = & \left( -\frac{3u}{8} + \frac{5v}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} \right. \\
& + \left. \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left( -\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{v^2e^{-2t}}{2} \right. \\
& + \frac{v^2e^{2t}}{2} - \frac{u^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \left. \right) \partial u + \left( \frac{v^2}{2} \right. \\
& + \left. \frac{v^2e^{2t}}{2} \right) \partial v
\end{aligned}$$

$$\begin{aligned}
X_{12} = & \left( \frac{e^{2t}}{4} - \frac{e^{-2t}}{4} \right) \partial t + \left( \frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} \right) \partial u + \left( \frac{v}{2} \right. \\
& + \left. \frac{ve^{2t}}{2} \right) \partial v
\end{aligned}$$

$$\begin{aligned}
X_{13} = & \left( -\frac{5v}{8} + \frac{3u}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ve^{-4t}}{8} \right. \\
& + \left. \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left( -\frac{v^2}{4} + \frac{u^2}{4} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} \right. \\
& + \left. -\frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left( \frac{v^2e^{2t}}{2} - \frac{v^2}{2} \right) \partial v,
\end{aligned}$$

$$\begin{aligned}
X_{14} = & \left( \frac{u^2}{4} + \frac{v^2}{4} + \frac{u^2e^{-2t}}{2} - \frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} \right. \\
& + \left. \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial t
\end{aligned}$$

$$\begin{aligned}
X_{15} = & \left( -\frac{e^{2t}}{4} - \frac{1}{2} - \frac{e^{-2t}}{4} \right) \partial t + \left( -\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{2t}}{2} - \frac{ve^{-2t}}{2} \right) \partial u + \left( -\frac{ve^{2t}}{2} - \frac{v}{2} \right) \partial v,
\end{aligned}$$

$$\begin{aligned}
X_{16} = & \left( -\frac{u^2}{4} - \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial t
\end{aligned}$$

$$\begin{aligned}
X_{17} = & \left( -\frac{u}{8} - \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{-2t}}{8} \right. \\
& + \left. \frac{ve^{-4t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left( -\frac{v^2}{4} + \frac{u^2}{4} + \frac{u^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} \right. \\
& + \left. -\frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left( \frac{v^2e^{2t}}{2} - \frac{v^2}{2} \right) \partial v,
\end{aligned}$$



$$X_{18} = \left( \frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial t$$

$$\begin{aligned} X_{19} = & (u f_2(t) + v f_1(t) - v f_2(t) + f_3(t)) \partial t + (u f_3(t) + v f_3(t) \\ & + u^2 f_2(t) + v^2 f_1(t) - v^2 f_2(t) + uv f_1(t)) \partial u + (u f_3(t) + v f_3(t) \\ & + u^2 f_2(t) + v^2 f_1(t) - v^2 f_2(t) + uv f_1(t)) \partial v \end{aligned}$$

Some of the generators might contain the following arbitrary functions:

$f_1$

$f_2$

$f_3$

The execution time of the script was:

0 hours 1 minutes 10 seconds.



## Chapter 5

# hydons\_model

Run 01\_20PM\_01\_February-2022

Degree in tangential ansätze: 2.  
The system of ODEs is given by:

$$\begin{aligned}\frac{dy_1}{dt} &= \frac{ty_1 + y_2^2}{-t^2 + y_1y_2}, \\ \frac{dy_2}{dt} &= \frac{ty_2 + y_1^2}{-t^2 + y_1y_2}.\end{aligned}$$

The calculated generators are:

$$X_1 = (t) \partial t + (y_1) \partial y_1 + (y_2) \partial y_2,$$

$$X_2 = (-t^2 f_1(t) + y_1 y_2 f_1(t)) \partial t + (y_2^2 f_1(t) + t y_1 f_1(t)) \partial y_1 + (y_1^2 f_1(t) + t y_2 f_1(t)) \partial y_2$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

The execution time of the script was:

0 hours 0 minutes 27 seconds.



# Chapter 6

## LV

Run 01\_28PM\_01\_February-2022

Degree in tangential ansätze: 2.  
The system of ODEs is given by:

$$\begin{aligned}\frac{du}{dt} &= u(1-v), \\ \frac{dv}{dt} &= av(u-1).\end{aligned}$$

The calculated generators are:

$$X_1 = (1) \partial t,$$

$$X_2 = (-1) \partial t,$$

$$X_3 = \left( \frac{f_1(t)}{a} \right) \partial t + \left( \frac{u f_1(t)}{a} - \frac{uv f_1(t)}{a} \right) \partial u + (uv f_1(t) - v f_1(t)) \partial v.$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

The execution time of the script was:

0 hours 0 minutes 41 seconds.