Summary of symmetry calculations

January 24, 2022

Contents

Chapter 1

DBH_model

Run 11_09AM_24_January-2022

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\begin{aligned}\frac{\mathrm{d}w_1}{\mathrm{d}t} &= -w_1w_2 - w_1w_3 + w_2w_3,\\ \frac{\mathrm{d}w_2}{\mathrm{d}t} &= -w_1w_2 + w_1w_3 - w_2w_3,\\ \frac{\mathrm{d}w_3}{\mathrm{d}t} &= w_1w_2 - w_1w_3 - w_2w_3.\end{aligned}$$

The calculated generators are:

$$X_1 = (1)\,\partial t,$$

$$X_2 = (1)\,\partial t,$$

$$X_3 = (-t)\,\partial t + (w_1)\,\partial w_1 + (w_2)\,\partial w_2 + (w_3)\,\partial w_3,$$

$$\begin{aligned}X_4 = \left(t^2\right)\partial t + (1 - 2tw_1)\,\partial w_1 + (1 - 2tw_2)\,\partial w_2 + (1\\ + \ -2tw_3)\,\partial w_3\end{aligned}$$

$$\begin{aligned}X_5 = (\mathrm{f}_1(t))\,\partial t + (w_2w_3\,\mathrm{f}_1(t) - w_1w_2\,\mathrm{f}_1(t) - w_1w_3\,\mathrm{f}_1(t))\,\partial w_1 + (w_1w_3\,\mathrm{f}_1(t) - w_1w_2\,\mathrm{f}_1(t)\\ + \ -w_2w_3\,\mathrm{f}_1(t))\,\partial w_2 + (w_1w_2\,\mathrm{f}_1(t) - w_1w_3\,\mathrm{f}_1(t) - w_2w_3\,\mathrm{f}_1(t))\,\partial w_3.\end{aligned}$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

The execution time of the script was:

0 hours 5 minutes 7 seconds.

Chapter 2

SIR

Run 11_10AM_24_January-2022

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\begin{aligned}\frac{\mathrm{d}S}{\mathrm{d}t} &= -ISr,\\ \frac{\mathrm{d}I}{\mathrm{d}t} &= ISr - Ia,\\ \frac{\mathrm{d}R}{\mathrm{d}t} &= Ia.\end{aligned}$$

The calculated generators are:

$$X_1 = (1)\,\partial R,$$

$$X_2 = (I + R + S)\,\partial t$$

$$X_3 = (I + R + S)\,\partial R$$

$$X_4 = (1)\,\partial t.$$

The execution time of the script was:

0 hours 0 minutes 23 seconds.

Run 11_13AM_24_January-2022

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -ISr,$$
$$\frac{\mathrm{d}I}{\mathrm{d}t} = ISr - Ia,$$
$$\frac{\mathrm{d}R}{\mathrm{d}t} = Ia.$$

The calculated generators are:

$$X_1 = \left(-\frac{1}{r}\right)\partial t,$$

$$X_2 = (1)\,\partial R,$$

$$X_3 = (1)\,\partial t,$$

$$X_4 = (I + R + S)\,\partial t$$

$$X_5 = \left(I^2 + R^2 + S^2 + 2IR + 2IS + 2RS\right)\partial t$$

$$X_6 = \left(\frac{\mathrm{f}_1(t)}{a}\right)\partial t + \left(-\frac{ISr\,\mathrm{f}_1(t)}{a}\right)\partial S + \left(\frac{ISr\,\mathrm{f}_1(t)}{a} - I\,\mathrm{f}_1(t)\right)\partial I + (I\,\mathrm{f}_1(t))\,\partial R.$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

WARNING:

Some of the calculated generators did not satisfy the linearised symmetry conditions. Thus, the presented list here is not complete and consists exclusively of the calculated generators that satisfy the linearised symmetry conditions.

The execution time of the script was:

0 hours 2 minutes 52 seconds.

Chapter 3

Brusselator

Run 11_13AM_24_January-2022

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = au^2v - u\,(b+1) + 1,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = -au^2v + bu.$$

The calculated generators are:

$$X_1 = (1)\,\partial t.$$

The execution time of the script was:

0 hours 0 minutes 6 seconds.

Chapter 4

linear_model

Run 11_03AM_24_January-2022

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

The calculated generators are:

$$X_1 = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_2 = \left(\frac{e^{2t}}{4} - \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right)\partial u + \left(\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$X_3 = \left(\frac{e^{2t}}{2} - \frac{1}{2}\right)\partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_4 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(\frac{e^{2t}}{2} - \frac{1}{2}\right)\partial v,$$

$$X_5 = \left(-\frac{e^{2t}}{4} - \frac{1}{2} - \frac{e^{-2t}}{4}\right)\partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{2t}}{2} - \frac{ve^{-2t}}{2}\right)\partial u + \left(-\frac{ve^{2t}}{2} - \frac{v}{2}\right)\partial v,$$

$$X_6 = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right)\partial u + \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v$$

$$X_7 = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} + \frac{ve^{2t}}{2}\right)\partial u + \left(\frac{ve^{2t}}{2} - \frac{v}{2}\right)\partial v,$$

$$X_8 = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_9 = (1)\,\partial t,$$

$$X_{10} = (\mathrm{f}_1(t))\,\partial t + (u\,\mathrm{f}_1(t) + v\,\mathrm{f}_1(t))\,\partial u + (u\,\mathrm{f}_1(t) + v\,\mathrm{f}_1(t))\,\partial v$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 0 minutes 12 seconds.

Run 11_04AM_24_January-2022

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

The calculated generators are:

$$X_1 = \left(-\frac{5v}{8} + \frac{3u}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ve^{-4t}}{8} + \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8}\right)\partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} + -\frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial u + \left(\frac{v^2e^{2t}}{2} - \frac{v^2}{2}\right)\partial v,$$

$$X_2 = \left(-\frac{u}{8} + \frac{3v}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{2t}}{8} + \frac{ve^{-4t}}{8}\right)\partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} - \frac{v^2e^{2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial u + \left(\frac{v^2}{2} - \frac{v^2e^{2t}}{2}\right)\partial v$$

$$X_3 = \left(\frac{e^{2t}}{4} - \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right)\partial u + \left(\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$X_4 = \left(-\frac{u^2}{4} - \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial t$$

$$X_5 = \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{u^2e^{-2t}}{2} - \frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial t$$

$$X_6 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(\frac{e^{2t}}{2} - \frac{1}{2}\right)\partial v,$$

$$X_7 = \left(\frac{e^{2t}}{2} - \frac{1}{2}\right)\partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_8 = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right)\partial u + \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v$$

$$X_9 = \left(-\frac{3v}{8} + \frac{u}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-4t}}{8} + -\frac{ve^{-2t}}{8} + \frac{ue^{2t}}{8}\right)\partial t + \left(\frac{u^2}{4} - \frac{u^2e^{-4t}}{4} - \frac{uv}{2} - \frac{uve^{-4t}}{2} - \frac{v^2e^{2t}}{2} - \frac{v^2}{4} - \frac{v^2e^{-4t}}{4}\right)\partial u + \left(-\frac{v^2e^{2t}}{2} - \frac{v^2}{2}\right)\partial v,$$

$$X_{10} = \left(-\frac{u}{8} - \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{-2t}}{8} + \frac{ve^{-4t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{u^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} + -\frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left(\frac{v^2e^{2t}}{2} - \frac{v^2}{2} \right) \partial v,$$

$$X_{11} = \left(-\frac{3u}{8} + \frac{5v}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} + \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} - \frac{u^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left(\frac{v^2}{2} + \frac{v^2e^{2t}}{2} \right) \partial v$$

$$X_{12} = (1)\, \partial t,$$

$$X_{13} = \left(-\frac{e^{2t}}{4} - \frac{1}{2} - \frac{e^{-2t}}{4} \right) \partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{2t}}{2} - \frac{ve^{-2t}}{2} \right) \partial u + \left(-\frac{ve^{2t}}{2} - \frac{v}{2} \right) \partial v,$$

$$X_{14} = \left(\frac{u}{8} + \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} + \frac{ue^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} - \frac{v^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left(\frac{v^2}{2} + \frac{v^2e^{2t}}{2} \right) \partial v$$

$$X_{15} = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial t$$

$$X_{16} = \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{v^2e^{-2t}}{2} - \frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial t$$

$$X_{17} = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} + \frac{ve^{2t}}{2}\right)\partial u + \left(\frac{ve^{2t}}{2} - \frac{v}{2}\right)\partial v,$$

$$X_{18} = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$\begin{aligned} X_{19} = {} & \left(u\,\mathrm{f}_2(t) + v\,\mathrm{f}_1(t) - v\,\mathrm{f}_2(t) + \mathrm{f}_3(t)\right)\partial t + \left(u\,\mathrm{f}_3(t) + v\,\mathrm{f}_3(t)\right. \\ & \left. + u^2\,\mathrm{f}_2(t) + v^2\,\mathrm{f}_1(t) - v^2\,\mathrm{f}_2(t) + uv\,\mathrm{f}_1(t)\right)\partial u + \left(u\,\mathrm{f}_3(t) + v\,\mathrm{f}_3(t)\right. \\ & \left. + u^2\,\mathrm{f}_2(t) + v^2\,\mathrm{f}_1(t) - v^2\,\mathrm{f}_2(t) + uv\,\mathrm{f}_1(t)\right)\partial v \end{aligned}$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$
$$\mathrm{f}_2$$
$$\mathrm{f}_3$$

The execution time of the script was:

0 hours 1 minutes 7 seconds.

Chapter 5

hydons_model

Run 11_03AM_24_January-2022

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$
$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

The calculated generators are:

$$X_1 = (t)\,\partial t + (y_1)\,\partial y_1 + (y_2)\,\partial y_2,$$

$$X_2 = \left(-t^2\,\mathrm{f}_1(t) + y_1 y_2\,\mathrm{f}_1(t)\right)\partial t + \left(y_2^2\,\mathrm{f}_1(t) + ty_1\,\mathrm{f}_1(t)\right)\partial y_1 + \left(y_1^2\,\mathrm{f}_1(t) + ty_2\,\mathrm{f}_1(t)\right)\partial y_2$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 0 minutes 25 seconds.

Chapter 6

LV

Run 11_10AM_24_January-2022

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\begin{aligned}\frac{\mathrm{d}u}{\mathrm{d}t} &= u\left(1-v\right),\\ \frac{\mathrm{d}v}{\mathrm{d}t} &= av\left(u-1\right).\end{aligned}$$

The calculated generators are:

$$X_1 = \left(1\right)\partial t,$$

$$X_2 = \left(-\frac{1}{a}\right)\partial t,$$

$$X_3 = \left(\frac{\mathrm{f}_1\left(t\right)}{a}\right)\partial t + \left(\frac{u\,\mathrm{f}_1\left(t\right)}{a} - \frac{uv\,\mathrm{f}_1\left(t\right)}{a}\right)\partial u + \left(uv\,\mathrm{f}_1\left(t\right) - v\,\mathrm{f}_1\left(t\right)\right)\partial v.$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 0 minutes 36 seconds.