Summary of symmetry calculations

November 3, 2021

Contents

Chapter 1

Lotka_Volterra

Run 12_07PM_03_November-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = N\left(-Pb + a\right),$$
$$\frac{\mathrm{d}P}{\mathrm{d}t} = P\left(Nc - d\right).$$

The calculated generators are:

$$X_1 = (1)\,\partial t,$$

$$X_2 = \left(\frac{1}{c} + \mathrm{f}_1(t)\right)\partial t + \left(\frac{Na\,\mathrm{f}_1(t)}{c} - \frac{NPb\,\mathrm{f}_1(t)}{c}\right)\partial N + \left(NP\,\mathrm{f}_1(t) - \frac{Pd\,\mathrm{f}_1(t)}{c}\right)\partial P$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

WARNING:

Some of the calculated generators did not satisfy the linearised symmetry conditions. Thus, the presented list here is not complete and consists exclusively of the calculated generators that satisfy the linearised symmetry conditions.

Equation:$-C_3 + \frac{C_6 b^2 e^{-2at}}{c^2} - \frac{C_6 b^2 d}{ac^2} + \frac{C_6 b^2 d e^{-2at}}{ac^2} = 0$ Basis functions:

$$\left[1.0,\ e^{-2at}\right]$$

$$C_6 = 0$$

$$C_3 = -\frac{C_6 b^2 d}{ac^2}$$

Equation:$-\frac{C_{15}ace^{2dt}}{2ad^2+4d^3} - \frac{C_{15}cde^{2dt}}{2ad^2+4d^3} + \frac{C_{15}c}{a^2e^{at}+2ade^{at}} + \frac{C_{15}ce^{dt}}{d^2} - \frac{C_{15}c}{2d^2} - \frac{C_{15}c}{2ad} + \frac{C_2ce^{dt}}{d} - \frac{C_2c}{d} - \frac{C_4b}{a} + \frac{C_4be^{-at}}{a} - C_5 + \frac{C_7abc^2e^{2dt}}{2a^3d^2-2a^2d^3-8ad^4+8d^5} - \frac{C_7abc^2e^{at}}{2a^4d-6a^3d^2+4a^2d^3} + \frac{C_7bc^2de^{2dt}}{2a^3d^2-2a^2d^3-8ad^4+8d^5} - \frac{C_7bc^2de^{at}}{2a^4d-6a^3d^2+4a^2d^3} - \frac{C_7bc^2}{4a^4e^{at}+10a^3de^{at}+4a^2d^2e^{at}} + \frac{C_7bc^2e^{at}e^{dt}}{2a^3d-a^2d^2-ad^3} - \frac{C_7bc^2e^{dt}}{a^2d^2-ad^3} + \frac{C_7bc^2}{2a^2d^2} + \frac{C_8ac^2e^{2dt}}{2a^2d^2+2ad^3-4d^4} + \frac{C_8c^2de^{2dt}}{2a^2d^2+2ad^3-4d^4} - \frac{C_8c^2}{2a^3e^{at}+5a^2de^{at}+2ad^2e^{at}} - \frac{C_8c^2e^{at}e^{dt}}{2a^3-a^2d-ad^2} - \frac{C_8c^2e^{dt}}{ad^2} + \frac{C_8c^2}{2ad^2} = 0$ Basis functions:

$$\left[1.0,\ e^{dt},\ e^{at},\ e^{at}e^{dt},\ e^{-at},\ e^{2dt}\right]$$

$$C_2 = -\frac{2C_{15}a^3}{2a^3d-a^2d^2-ad^3} + \frac{C_{15}a^2d}{2a^3d-a^2d^2-ad^3} + \frac{C_{15}ad^2}{2a^3d-a^2d^2-ad^3} + \frac{2C_7abc}{2a^3d-a^2d^2-ad^3} - \frac{C_7bcde^{at}}{2a^3d-a^2d^2-ad^3} + \frac{C_7b}{2a^3d-a^2}$$

$$C_7 = \frac{2C_8a^2de^{dt}}{2a^2be^{dt}-2a^2b-4abde^{dt}-3abd-bd^2} - \frac{4C_8ad^2e^{dt}}{2a^2be^{dt}-2a^2b-4abde^{dt}-3abd-bd^2}$$

$$C_7 = \frac{C_8d}{b}$$

$$C_4 = 0$$

$$C_7 = \frac{C_{15}a^2}{bc} - \frac{3C_{15}ad}{bc} + \frac{2C_{15}d^2}{bc} - \frac{C_8a}{b} + \frac{2C_8d}{b}$$

$$C_2 = -\frac{C_{15}a^5ce^{at}}{2a^5cde^{at}+3a^4cd^2e^{at}-3a^3cd^3e^{at}-2a^2cd^4e^{at}} - \frac{5C_{15}a^4cde^{at}}{4a^5cde^{at}+6a^4cd^2e^{at}-6a^3cd^3e^{at}-4a^2cd^4e^{at}} + \frac{2C}{2a^5cde^{at}+3a^4cd^2e}$$

Equation:$-\frac{2C_8a^3b^2cde^{at}e^{2dt}}{2a^4bcde^{dt}-2a^4bcd-4a^3bcd^2e^{dt}-3a^3bcd^2-2a^2bcd^3e^{dt}+a^2bcd^3+4abcd^4e^{dt}+3abcd^4+bcd^5} + \frac{2C_8a^3b^2de^{at}e^{dt}}{2a^4bde^{dt}-2a^4bd-8a^3bd^2e^{dt}+a^3bd^2+8a^2bd^3e} \frac{6C_8a^2b^2cd^2e^{at}e^{2dt}}{2a^4bcde^{dt}-2a^4bcd-4a^3bcd^2e^{dt}-3a^3bcd^2-2a^2bcd^3e^{dt}+a^2bcd^3+4abcd^4e^{dt}+3abcd^4+bcd^5} - \frac{8C_8a^2b^2d^2e^{at}e^{dt}}{2a^4bde^{dt}-2a^4bd-8a^3bd^2e^{dt}+a^3bd^2+8a^2bd^3e^{dt}+5a^2bd^3+2abd^4} \frac{2C_8a^2b^2de^{dt}}{2a^4be^{dt}-2a^4b-2a^3bde^{dt}-5a^3bd-4a^2bd^2e^{dt}-4a^2bd^2-abd^3} - \frac{4C_8ab^2cd^3e^{at}e^{2dt}}{2a^4bcde^{dt}-2a^4bcd-4a^3bcd^2e^{dt}-3a^3bcd^2-2a^2bcd^3e^{dt}+a^2bcd^3+4abcd^4e^{dt}+3abcd^4+bcd^5} + \frac{8C_8ab^2d^3e^{at}e^{dt}}{2a^4bde^{dt}-2a^4bd-8a^3bd^2e^{dt}+a^3bd^2+8a^2bd^3e^{dt}+5a^2bd^3+2abd^4} + \frac{4C_8ab^2d^2e^{dt}}{2a^4be^{dt}-2a^4b-2a^3bde^{dt}-5a^3bd-4a^2bd^2e^{dt}-4a^2bd^2-abd^3} + \frac{C_8abce^{at}e^{dt}}{a^2c-cd^2} - \frac{C_8bcde^{at}e^{dt}}{a^2c-cd^2} - \frac{C_8b}{a+d} - C_9 = 0$ Basis functions:

$$\left[\frac{e^{dt}}{2a^2e^{dt}-2a^2-4ade^{dt}-3ad-d^2},\ 1.0,\ \frac{e^{at}e^{dt}}{2a^2e^{dt}-2a^2-4ade^{dt}-3ad-d^2},\ e^{at}e^{dt},\ \frac{e^{at}e^{2dt}}{2a^2e^{dt}-2a^2-4ade^{dt}-3ad-d^2}\right]$$

$$0 = 0$$

$$0 = 0$$

$$C_8 = 0$$

$$0 = 0$$

$$C_8 = -\frac{2C_9a^3e^{dt}}{2a^2be^{at}e^{2dt}+2a^2be^{dt}-2a^2b-4abde^{at}e^{2dt}-2abde^{dt}-3abd-4bd^2e^{dt}-bd^2} + \frac{2C}{2a^2be^{at}e^{2dt}+2a^2be^{dt}-2a^2b-4abde^{a}}$$

Equation:$\frac{C_{10}bdt}{a} - \frac{C_{10}b}{a} + \frac{C_{10}be^{-at}}{a} - \frac{C_{10}bd}{a^2} + \frac{C_{10}bde^{-at}}{a^2} - C_{11} + \frac{C_{13}a^2b^2}{a^3de^{at}-a^2d^2e^{at}} + \frac{2C_{13}a^2b^2}{a^3d+a^2d^2} - \frac{2C_{13}ab^2c^2}{a^2c^2de^{dt}-ac^2d^2e^{dt}} + \frac{2C_{13}ab^2d}{a^3de^{at}-a^2d^2e^{at}} + \frac{4C_{13}ab^2d}{a^3d+a^2d^2} - \frac{C_{13}ab^2}{a^2de^{at}e^{dt}+ad^2e^{at}e^{dt}} - \frac{2C_{13}b^2c^2d}{a^2c^2de^{dt}-ac^2d^2e^{dt}} + \frac{C_{13}b^2d^2}{a^3de^{at}-a^2d^2e^{at}} + \frac{C_{13}b^2d^2}{a^3d+a^2d^2} - \frac{2C_{13}b^2d}{a^2de^{at}e^{dt}+ad^2e^{at}e^{dt}} -$

$\frac{C_{13}b^2t}{a} + C_{14}bt + \frac{C_{15}a^2e^{2dt}}{2ad+4d^2} + \frac{C_{15}abd}{a^2be^{at}+2abde^{at}} - \frac{C_{15}a}{2d} + \frac{C_{15}bd^2}{a^2be^{at}+2abde^{at}} - \frac{5C_{15}d^2e^{2dt}}{2ad+4d^2} + 2C_{15}e^{2dt} + C_{15} - \frac{C_{15}d}{2a} + \frac{C_{16}ab^2}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{2C_{16}b^2d}{acde^{at}e^{dt}+cd^2e^{at}e^{dt}} + \frac{C_{16}b^2d}{a^2c+acd} - \frac{C_{16}b^2e^{-at}}{cd} - \frac{C_{16}b^2e^{-at}}{ac} - \frac{C_{18}be^{-at}}{c} + \frac{C_{18}bd}{ac} - \frac{C_{18}bde^{-at}}{ac} = 0$
Basis functions:

$$\left[1.0,\ e^{-at}e^{-dt},\ e^{-at},\ t,\ e^{2dt},\ e^{-dt}\right]$$

$$0 = 0$$

$$C_{10} = \frac{C_{16}ab}{cd} + \frac{C_{18}a}{c}$$

$$C_{10} = \frac{C_{13}b}{d} - \frac{C_{14}a}{d}$$

$$C_{15} = 0$$

$$0 = 0$$

$$C_{10} = -\frac{C_{11}a^5cd}{a^4bcd + 3a^3bcd^2 + a^2bcd^3 - 3abcd^4 - 2bcd^5} - \frac{2C_{11}a^4cd^2}{a^4bcd + 3a^3bcd^2 + a^2bcd^3 - 3abcd^4 - 2bcd^5} + \frac{C_{11}a^3c}{a^4bcd + 3a^3bcd^2 + a^2bcd}$$

Equation: $-C_{12} + \frac{C_{13}bc}{a^2e^{at}e^{dt}+ade^{at}e^{dt}} + \frac{C_{13}bc}{ad+d^2} - \frac{C_{13}bce^{-dt}}{ad} - \frac{C_{16}b}{ae^{at}e^{dt}+de^{at}e^{dt}} + \frac{C_{16}b}{a+d} = 0$ Basis functions:

$$\left[1.0,\ e^{-dt},\ e^{-at}e^{-dt}\right]$$

$$C_{13} = 0$$

$$0 = 0$$

$$C_{12} = \frac{C_{13}abc}{a^2d + ad^2} + \frac{C_{13}bcd}{a^2de^{at}e^{dt} + ad^2e^{at}e^{dt}} - \frac{C_{16}abd}{a^2de^{at}e^{dt} + ad^2e^{at}e^{dt}} + \frac{C_{16}abd}{a^2d + ad^2}$$

Equation: $-\frac{C_{13}c^2}{a^2e^{at}e^{dt}+ade^{at}e^{dt}} - \frac{C_{13}c^2}{ad+d^2} + \frac{C_{13}c^2e^{-dt}}{ad} + \frac{C_{16}c}{ae^{at}e^{dt}+de^{at}e^{dt}} - \frac{C_{16}c}{a+d} - C_{17} = 0$ Basis functions:

$$\left[1.0,\ e^{-dt},\ e^{-at}e^{-dt}\right]$$

$$C_{13} = 0$$

$$0 = 0$$

$$C_{13} = -\frac{C_{16}acde^{at}e^{dt}}{ac^2e^{at}e^{dt} + c^2d} + \frac{C_{16}acd}{ac^2e^{at}e^{dt} + c^2d} - \frac{C_{17}a^2de^{at}e^{dt}}{ac^2e^{at}e^{dt} + c^2d} - \frac{C_{17}ad^2e^{at}e^{dt}}{ac^2e^{at}e^{dt} + c^2d}$$

The execution time of the script was:

0 hours 1 minutes 21 seconds.