

DBH_model_error_search_ODEsys_and_algSys

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1 Error search ODE system and algebra system DBH model ansatz degree 1

Date: 2021-09-16, *Written by:* Johannes Borgqvist.

This document contains an attempt of finding the error in the calculations of the symmetry generators for Hydon's model. The model at hand is the following two component ODE system:

$$\begin{aligned}\frac{dw_1}{dt} &= -w_1w_2 - w_1w_3 + w_2w_3 = \omega_1(t, w_1, w_2, w_3) \\ \frac{dw_2}{dt} &= -w_1w_2 + w_1w_3 - w_2w_3 = \omega_2(t, w_1, w_2, w_3) \\ \frac{dw_3}{dt} &= w_1w_2 - w_1w_3 - w_2w_3 = \omega_3(t, w_1, w_2, w_3)\end{aligned}$$

To this model, the aim is to find the most general form of the *infinitesimal generator of the Lie group* denoted by X which is defined as follows:

$$X = \xi(t, w_1, w_2, w_3)\partial_t + \eta_1(t, w_1, w_2, w_3)\partial_{w_1} + \eta_2(t, w_1, w_2, w_3)\partial_{w_2} + \eta_3(t, w_1, w_2, w_3)\partial_{w_3}.$$

To find this generator, a set of *linear ansätze* is used for the three tangents as follows:

$$\begin{aligned}\xi &= w_1 c_{01}(t) + w_2 c_{02}(t) + w_3 c_{03}(t) + c_{00}(t) \\ \eta_1 &= w_1 c_{11}(t) + w_2 c_{12}(t) + w_3 c_{13}(t) + c_{10}(t) \\ \eta_2 &= w_1 c_{21}(t) + w_2 c_{22}(t) + w_3 c_{23}(t) + c_{20}(t) \\ \eta_3 &= w_1 c_{31}(t) + w_2 c_{32}(t) + w_3 c_{33}(t) + c_{30}(t)\end{aligned}$$

The aim is to find the nine arbitrary functions $c_{ij}(t)$ for the two indices $i, j \in \{0, 1, 2\}$. The equations required in order to find these constants are given by the three *linearised symmetry conditions* given by

$$X^{(1)}(w'_k - \omega_k(t, y_1, y_2)) = 0, \text{ for } k \in \{1, 2, 3\}.$$

Here, $X^{(1)}$ corresponds to the prolonged generator given by

$$X^{(1)} = X + \eta_1^{(1)} \partial_{w'_1} + \eta_2^{(1)} \partial_{w'_2} + \eta_3^{(1)} \partial_{w'_3}$$

where the prolonged tangents are given by the *prolongation formula*:

$$\eta_k^{(1)} = D_t \eta_k - w'_k D_t \xi, \text{ for } k \in \{1, 2, 3\}$$

where the total derivative is defined as follows: $D_t = \partial_t + w'_1 \partial_{w_1} + w'_2 \partial_{w_2} + w'_3 \partial_{w_3}$.

What is nice about the DBH model is that it has at least three known generators, namely the *scaling generator* given by

$$X = w_1 \partial_{w_1} + w_2 \partial_{w_2} + w_3 \partial_{w_3}$$

and also the translation generator

$$X = \partial_t + \partial_{w_1} + \partial_{w_2} + \partial_{w_3}.$$

Thus, we now when the algorithm performs correctly in this case as the above generator should be returned as an output.

Moreover, plugging in these ansätze into the linearised symmetry conditions will result in a linear system of equations which can be formulated on matrix form as follows:

$$A \frac{d\mathbf{c}(t)}{dt} = B\mathbf{c}(t)$$

where the vector $\mathbf{c}(t) \in \mathcal{C}(\mathbb{R}^{16})$ contains the nine arbitrary coefficients in the tangential ansätze. Typically, the number of equations are much larger than the number of unknowns meaning that if $A, B \in \mathcal{C}(\mathbb{R}^{n \times m})$ then $n \gg m$ (in this case $m = 16$). After row reducing this system and simplifying it is (in the best of worlds) possible to write the system on the following form:

$$\frac{d\mathbf{c}(t)}{dt} = B\mathbf{c}(t), \tag{1}$$

$$B_{\text{algebraic}} \mathbf{c}(t) = \mathbf{0}. \tag{2}$$

The first ODE system is a quadratic ODE system which can be solved using the Jordan decomposition. That is if

$$B = P^{-1} J P$$

then the solution to the ODE system is given by

$$\mathbf{c}(t) = P^{-1} e^{Jt} P \mathbf{c}_0$$

for some initial condition \mathbf{c}_0 composed of arbitrary integration constants. Then the solution of the system of ODEs is plugged in to the algebraic equations given by the second matrix equation above. This will result in certain algebraic equations that can simplify the results even further.

Now, the problem is that certain generators are obtained that do not solve the linearised symmetry conditions. This implies that the implementation of the algorithm is wrong, as the methodology of ansätze can never yield non-solutions. Therefore, the Hydon example will be used to see if the error is introduced in the solution of the ODE system or if it is when certain simplifications are made when the algebraic equations are solved.

What will be done in the subsequent cells is that all matrices will be printed out and then we will try to track down the error.

2 Defining the tangents

The tangents are:

$$\begin{aligned}\xi &= w_1 c_{01}(t) + w_2 c_{02}(t) + w_3 c_{03}(t) + c_{00}(t) \\ \eta_1 &= w_1 c_{11}(t) + w_2 c_{12}(t) + w_3 c_{13}(t) + c_{10}(t) \\ \eta_2 &= w_1 c_{21}(t) + w_2 c_{22}(t) + w_3 c_{23}(t) + c_{20}(t) \\ \eta_3 &= w_1 c_{31}(t) + w_2 c_{32}(t) + w_3 c_{33}(t) + c_{30}(t)\end{aligned}$$

The unknown coefficients:

$$\mathbf{c} = \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} \quad (3)$$

3 The linearised symmetry conditions

Linearised symmetry condition 1:

$$\begin{aligned}
0 = & \left(-\frac{d}{dt} c_{10}(t) \right) + (w_2 c_{30}(t)) + (w_3 c_{20}(t)) + (w_2^2 c_{32}(t)) + (w_3^2 c_{23}(t)) + \left(-w_1 \frac{d}{dt} c_{11}(t) \right) \\
& + (-w_1 c_{20}(t)) + (-w_1 c_{30}(t)) + \left(-w_2 \frac{d}{dt} c_{12}(t) \right) + (-w_2 c_{10}(t)) + \left(-w_3 \frac{d}{dt} c_{13}(t) \right) \\
& + (-w_3 c_{10}(t)) + (-w_1^2 c_{21}(t)) + (-w_1^2 c_{31}(t)) + (-w_2^2 c_{12}(t)) + (-w_3^2 c_{13}(t)) \\
& + (w_1 w_2 c_{12}(t)) + (w_1 w_2 c_{31}(t)) + (w_1 w_3 c_{13}(t)) + (w_1 w_3 c_{21}(t)) + \left(w_2 w_3 \frac{d}{dt} c_{00}(t) \right) \\
& + (w_2 w_3 c_{22}(t)) + (w_2 w_3 c_{33}(t)) + \left(w_2 w_3^2 \frac{d}{dt} c_{03}(t) \right) + \left(w_2^2 w_3 \frac{d}{dt} c_{02}(t) \right) + (w_1^2 w_2^2 c_{01}(t)) \\
& + (w_1^2 w_2^2 c_{02}(t)) + (w_1^2 w_3^2 c_{01}(t)) + (w_1^2 w_3^2 c_{03}(t)) + (w_2^2 w_3^2 c_{01}(t)) + \left(-w_1 w_2 \frac{d}{dt} c_{00}(t) \right) \\
& + (-w_1 w_2 c_{13}(t)) + (-w_1 w_2 c_{22}(t)) + (-w_1 w_2 c_{32}(t)) + \left(-w_1 w_3 \frac{d}{dt} c_{00}(t) \right) + (-w_1 w_3 c_{12}(t)) \\
& + (-w_1 w_3 c_{23}(t)) + (-w_1 w_3 c_{33}(t)) + \left(-w_1 w_2^2 \frac{d}{dt} c_{02}(t) \right) + \left(-w_1 w_3^2 \frac{d}{dt} c_{03}(t) \right) + (-w_2 w_3 c_{11}(t)) \\
& + \left(-w_1^2 w_2 \frac{d}{dt} c_{01}(t) \right) + \left(-w_1^2 w_3 \frac{d}{dt} c_{01}(t) \right) + (-w_1^2 w_2^2 c_{03}(t)) + (-w_1^2 w_3^2 c_{02}(t)) + (-w_2^2 w_3^2 c_{02}(t)) \\
& + (-w_2^2 w_3^2 c_{03}(t)) + \left(w_1 w_2 w_3 \frac{d}{dt} c_{01}(t) \right) + \left(-w_1 w_2 w_3 \frac{d}{dt} c_{02}(t) \right) + \left(-w_1 w_2 w_3 \frac{d}{dt} c_{03}(t) \right) + (-2w_1 w_2 w_3^2 c_{01}(t))
\end{aligned}$$

Linearised symmetry condition 2:

$$\begin{aligned}
0 = & \left(-\frac{d}{dt} c_{20}(t) \right) + (w_1 c_{30}(t)) + (w_3 c_{10}(t)) + (w_1^2 c_{31}(t)) + (w_3^2 c_{13}(t)) + \left(-w_1 \frac{d}{dt} c_{21}(t) \right) \\
& + (-w_1 c_{20}(t)) + \left(-w_2 \frac{d}{dt} c_{22}(t) \right) + (-w_2 c_{10}(t)) + (-w_2 c_{30}(t)) + \left(-w_3 \frac{d}{dt} c_{23}(t) \right) \\
& + (-w_3 c_{20}(t)) + (-w_1^2 c_{21}(t)) + (-w_2^2 c_{12}(t)) + (-w_2^2 c_{32}(t)) + (-w_3^2 c_{23}(t)) \\
& + (w_1 w_2 c_{21}(t)) + (w_1 w_2 c_{32}(t)) + \left(w_1 w_3 \frac{d}{dt} c_{00}(t) \right) + (w_1 w_3 c_{11}(t)) + (w_1 w_3 c_{33}(t)) \\
& + \left(w_1 w_3^2 \frac{d}{dt} c_{03}(t) \right) + (w_2 w_3 c_{12}(t)) + (w_2 w_3 c_{23}(t)) + \left(w_1^2 w_3 \frac{d}{dt} c_{01}(t) \right) + (w_1^2 w_2^2 c_{01}(t)) \\
& + (w_1^2 w_2^2 c_{02}(t)) + (w_1^2 w_3^2 c_{02}(t)) + (w_2^2 w_3^2 c_{02}(t)) + (w_2^2 w_3^2 c_{03}(t)) + \left(-w_1 w_2 \frac{d}{dt} c_{00}(t) \right) \\
& + (-w_1 w_2 c_{11}(t)) + (-w_1 w_2 c_{23}(t)) + (-w_1 w_2 c_{31}(t)) + (-w_1 w_3 c_{22}(t)) + \left(-w_1 w_2^2 \frac{d}{dt} c_{02}(t) \right) \\
& + \left(-w_2 w_3 \frac{d}{dt} c_{00}(t) \right) + (-w_2 w_3 c_{13}(t)) + (-w_2 w_3 c_{21}(t)) + (-w_2 w_3 c_{33}(t)) + \left(-w_1^2 w_2 \frac{d}{dt} c_{01}(t) \right) \\
& + \left(-w_2 w_3^2 \frac{d}{dt} c_{03}(t) \right) + \left(-w_2^2 w_3 \frac{d}{dt} c_{02}(t) \right) + (-w_1^2 w_2^2 c_{03}(t)) + (-w_1^2 w_3^2 c_{01}(t)) + (-w_1^2 w_3^2 c_{03}(t)) \\
& + (-w_2^2 w_3^2 c_{01}(t)) + \left(w_1 w_2 w_3 \frac{d}{dt} c_{02}(t) \right) + \left(-w_1 w_2 w_3 \frac{d}{dt} c_{01}(t) \right) + \left(-w_1 w_2 w_3 \frac{d}{dt} c_{03}(t) \right) + (-2w_1 w_2 w_3^2 c_{02}(t))
\end{aligned}$$

Linearised symmetry condition 3:

$$\begin{aligned}
0 = & \left(-\frac{d}{dt} c_{30}(t) \right) + (w_1 c_{20}(t)) + (w_2 c_{10}(t)) + (w_1^2 c_{21}(t)) + (w_2^2 c_{12}(t)) + \left(-w_1 \frac{d}{dt} c_{31}(t) \right) \\
& + (-w_1 c_{30}(t)) + \left(-w_2 \frac{d}{dt} c_{32}(t) \right) + (-w_2 c_{30}(t)) + \left(-w_3 \frac{d}{dt} c_{33}(t) \right) + (-w_3 c_{10}(t)) \\
& + (-w_3 c_{20}(t)) + (-w_1^2 c_{31}(t)) + (-w_2^2 c_{32}(t)) + (-w_3^2 c_{13}(t)) + (-w_3^2 c_{23}(t)) \\
& + \left(w_1 w_2 \frac{d}{dt} c_{00}(t) \right) + (w_1 w_2 c_{11}(t)) + (w_1 w_2 c_{22}(t)) + (w_1 w_3 c_{23}(t)) + (w_1 w_3 c_{31}(t)) \\
& + \left(w_1 w_2^2 \frac{d}{dt} c_{02}(t) \right) + (w_2 w_3 c_{13}(t)) + (w_2 w_3 c_{32}(t)) + \left(w_1^2 w_2 \frac{d}{dt} c_{01}(t) \right) + (w_1^2 w_2^2 c_{03}(t)) \\
& + (w_1^2 w_3^2 c_{01}(t)) + (w_1^2 w_3^2 c_{03}(t)) + (w_2^2 w_3^2 c_{02}(t)) + (w_2^2 w_3^2 c_{03}(t)) + (-w_1 w_2 c_{33}(t)) \\
& + \left(-w_1 w_3 \frac{d}{dt} c_{00}(t) \right) + (-w_1 w_3 c_{11}(t)) + (-w_1 w_3 c_{21}(t)) + (-w_1 w_3 c_{32}(t)) + \left(-w_1 w_3^2 \frac{d}{dt} c_{03}(t) \right) \\
& + \left(-w_2 w_3 \frac{d}{dt} c_{00}(t) \right) + (-w_2 w_3 c_{12}(t)) + (-w_2 w_3 c_{22}(t)) + (-w_2 w_3 c_{31}(t)) + \left(-w_2 w_3^2 \frac{d}{dt} c_{03}(t) \right) \\
& + \left(-w_1^2 w_3 \frac{d}{dt} c_{01}(t) \right) + \left(-w_2^2 w_3 \frac{d}{dt} c_{02}(t) \right) + (-w_1^2 w_2^2 c_{01}(t)) + (-w_1^2 w_2^2 c_{02}(t)) + (-w_1^2 w_3^2 c_{02}(t)) \\
& + (-w_2^2 w_3^2 c_{01}(t)) + \left(w_1 w_2 w_3 \frac{d}{dt} c_{03}(t) \right) + \left(-w_1 w_2 w_3 \frac{d}{dt} c_{01}(t) \right) + \left(-w_1 w_2 w_3 \frac{d}{dt} c_{02}(t) \right) + (-2w_1 w_2^2 w_3 c_{03}(t))
\end{aligned}$$

4 The determining equations

Determining equations from linearised symmetry condition 1:

$$\begin{aligned}
w_1^0 w_2^0 w_3^0 : 0 &= -\frac{d}{dt} c_{10}(t) \\
w_1^0 w_2^0 w_3^1 : 0 &= -c_{10}(t) + c_{20}(t) - \frac{d}{dt} c_{13}(t) \\
w_1^0 w_2^0 w_3^2 : 0 &= -c_{13}(t) + c_{23}(t) \\
w_1^0 w_2^1 w_3^0 : 0 &= -c_{10}(t) + c_{30}(t) - \frac{d}{dt} c_{12}(t) \\
w_1^0 w_2^1 w_3^1 : 0 &= -c_{11}(t) + c_{22}(t) + c_{33}(t) + \frac{d}{dt} c_{00}(t) \\
w_1^0 w_2^1 w_3^2 : 0 &= \frac{d}{dt} c_{03}(t) \\
w_1^0 w_2^2 w_3^0 : 0 &= -c_{12}(t) + c_{32}(t) \\
w_1^0 w_2^2 w_3^1 : 0 &= \frac{d}{dt} c_{02}(t) \\
w_1^0 w_2^2 w_3^2 : 0 &= c_{01}(t) - c_{02}(t) - c_{03}(t) \\
w_1^1 w_2^0 w_3^0 : 0 &= -c_{20}(t) - c_{30}(t) - \frac{d}{dt} c_{11}(t) \\
w_1^1 w_2^0 w_3^1 : 0 &= -c_{12}(t) + c_{13}(t) + c_{21}(t) - c_{23}(t) - c_{33}(t) - \frac{d}{dt} c_{00}(t) \\
w_1^1 w_2^0 w_3^2 : 0 &= -\frac{d}{dt} c_{03}(t) \\
w_1^1 w_2^1 w_3^0 : 0 &= c_{12}(t) - c_{13}(t) - c_{22}(t) + c_{31}(t) - c_{32}(t) - \frac{d}{dt} c_{00}(t) \\
w_1^1 w_2^1 w_3^1 : 0 &= \frac{d}{dt} c_{01}(t) - \frac{d}{dt} c_{02}(t) - \frac{d}{dt} c_{03}(t) \\
w_1^1 w_2^1 w_3^2 : 0 &= -2c_{01}(t) + 2c_{02}(t) \\
w_1^1 w_2^2 w_3^0 : 0 &= -\frac{d}{dt} c_{02}(t) \\
w_1^1 w_2^2 w_3^1 : 0 &= -2c_{01}(t) + 2c_{03}(t) \\
w_1^2 w_2^0 w_3^0 : 0 &= -c_{21}(t) - c_{31}(t) \\
w_1^2 w_2^0 w_3^1 : 0 &= -\frac{d}{dt} c_{01}(t) \\
w_1^2 w_2^0 w_3^2 : 0 &= c_{01}(t) - c_{02}(t) + c_{03}(t) \\
w_1^2 w_2^1 w_3^0 : 0 &= -\frac{d}{dt} c_{01}(t) \\
w_1^2 w_2^1 w_3^1 : 0 &= 2c_{01}(t) \\
w_1^2 w_2^2 w_3^0 : 0 &= c_{01}(t) + c_{02}(t) - c_{03}(t)
\end{aligned}$$

Determining equations from linearised symmetry condition 2:

$$\begin{aligned}
w_1^0 w_2^0 w_3^0 : 0 &= -\frac{d}{dt} c_{20}(t) \\
w_1^0 w_2^0 w_3^1 : 0 &= c_{10}(t) - c_{20}(t) - \frac{d}{dt} c_{23}(t) \\
w_1^0 w_2^0 w_3^2 : 0 &= c_{13}(t) - c_{23}(t) \\
w_1^0 w_2^1 w_3^0 : 0 &= -c_{10}(t) - c_{30}(t) - \frac{d}{dt} c_{22}(t) \\
w_1^0 w_2^1 w_3^1 : 0 &= c_{12}(t) - c_{13}(t) - c_{21}(t) + c_{23}(t) - c_{33}(t) - \frac{d}{dt} c_{00}(t) \\
w_1^0 w_2^1 w_3^2 : 0 &= -\frac{d}{dt} c_{03}(t) \\
w_1^0 w_2^2 w_3^0 : 0 &= -c_{12}(t) - c_{32}(t) \\
w_1^0 w_2^2 w_3^1 : 0 &= -\frac{d}{dt} c_{02}(t) \\
w_1^0 w_2^2 w_3^2 : 0 &= -c_{01}(t) + c_{02}(t) + c_{03}(t) \\
w_1^1 w_2^0 w_3^0 : 0 &= -c_{20}(t) + c_{30}(t) - \frac{d}{dt} c_{21}(t) \\
w_1^1 w_2^0 w_3^1 : 0 &= c_{11}(t) - c_{22}(t) + c_{33}(t) + \frac{d}{dt} c_{00}(t) \\
w_1^1 w_2^0 w_3^2 : 0 &= \frac{d}{dt} c_{03}(t) \\
w_1^1 w_2^1 w_3^0 : 0 &= -c_{11}(t) + c_{21}(t) - c_{23}(t) - c_{31}(t) + c_{32}(t) - \frac{d}{dt} c_{00}(t) \\
w_1^1 w_2^1 w_3^1 : 0 &= -\frac{d}{dt} c_{01}(t) + \frac{d}{dt} c_{02}(t) - \frac{d}{dt} c_{03}(t) \\
w_1^1 w_2^1 w_3^2 : 0 &= 2c_{01}(t) - 2c_{02}(t) \\
w_1^1 w_2^2 w_3^0 : 0 &= -\frac{d}{dt} c_{02}(t) \\
w_1^1 w_2^2 w_3^1 : 0 &= 2c_{02}(t) \\
w_1^2 w_2^0 w_3^0 : 0 &= -c_{21}(t) + c_{31}(t) \\
w_1^2 w_2^0 w_3^1 : 0 &= \frac{d}{dt} c_{01}(t) \\
w_1^2 w_2^0 w_3^2 : 0 &= -c_{01}(t) + c_{02}(t) - c_{03}(t) \\
w_1^2 w_2^1 w_3^0 : 0 &= -\frac{d}{dt} c_{01}(t) \\
w_1^2 w_2^1 w_3^1 : 0 &= -2c_{02}(t) + 2c_{03}(t) \\
w_1^2 w_2^2 w_3^0 : 0 &= c_{01}(t) + c_{02}(t) - c_{03}(t)
\end{aligned}$$

Determining equations from linearised symmetry condition 3:

$$\begin{aligned}
w_1^0 w_2^0 w_3^0 : 0 &= -\frac{d}{dt} c_{30}(t) \\
w_1^0 w_2^0 w_3^1 : 0 &= -c_{10}(t) - c_{20}(t) - \frac{d}{dt} c_{33}(t) \\
w_1^0 w_2^0 w_3^2 : 0 &= -c_{13}(t) - c_{23}(t) \\
w_1^0 w_2^1 w_3^0 : 0 &= c_{10}(t) - c_{30}(t) - \frac{d}{dt} c_{32}(t) \\
w_1^0 w_2^1 w_3^1 : 0 &= -c_{12}(t) + c_{13}(t) - c_{22}(t) - c_{31}(t) + c_{32}(t) - \frac{d}{dt} c_{00}(t) \\
w_1^0 w_2^1 w_3^2 : 0 &= -\frac{d}{dt} c_{03}(t) \\
w_1^0 w_2^2 w_3^0 : 0 &= c_{12}(t) - c_{32}(t) \\
w_1^0 w_2^2 w_3^1 : 0 &= -\frac{d}{dt} c_{02}(t) \\
w_1^0 w_2^2 w_3^2 : 0 &= -c_{01}(t) + c_{02}(t) + c_{03}(t) \\
w_1^1 w_2^0 w_3^0 : 0 &= c_{20}(t) - c_{30}(t) - \frac{d}{dt} c_{31}(t) \\
w_1^1 w_2^0 w_3^1 : 0 &= -c_{11}(t) - c_{21}(t) + c_{23}(t) + c_{31}(t) - c_{32}(t) - \frac{d}{dt} c_{00}(t) \\
w_1^1 w_2^0 w_3^2 : 0 &= -\frac{d}{dt} c_{03}(t) \\
w_1^1 w_2^1 w_3^0 : 0 &= c_{11}(t) + c_{22}(t) - c_{33}(t) + \frac{d}{dt} c_{00}(t) \\
w_1^1 w_2^1 w_3^1 : 0 &= -\frac{d}{dt} c_{01}(t) - \frac{d}{dt} c_{02}(t) + \frac{d}{dt} c_{03}(t) \\
w_1^1 w_2^1 w_3^2 : 0 &= 2 c_{03}(t) \\
w_1^1 w_2^2 w_3^0 : 0 &= \frac{d}{dt} c_{02}(t) \\
w_1^1 w_2^2 w_3^1 : 0 &= 2 c_{01}(t) - 2 c_{03}(t) \\
w_1^2 w_2^0 w_3^0 : 0 &= c_{21}(t) - c_{31}(t) \\
w_1^2 w_2^0 w_3^1 : 0 &= -\frac{d}{dt} c_{01}(t) \\
w_1^2 w_2^0 w_3^2 : 0 &= c_{01}(t) - c_{02}(t) + c_{03}(t) \\
w_1^2 w_2^1 w_3^0 : 0 &= \frac{d}{dt} c_{01}(t) \\
w_1^2 w_2^1 w_3^1 : 0 &= 2 c_{02}(t) - 2 c_{03}(t) \\
w_1^2 w_2^2 w_3^0 : 0 &= -c_{01}(t) - c_{02}(t) + c_{03}(t)
\end{aligned}$$

5 Solving the determining equations

5.1 Step 0 of 6: Defining the matrices from the determining equations

5.1.1 Determining equation 1 out of 69

$$-\frac{d}{dt}c_{10}(t) = 0 \quad (4)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

Determining equation 2 out of 69

$$-c_{10}(t) + c_{20}(t) - \frac{d}{dt}c_{13}(t) = 0 \quad (6)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (7)$$

Determining equation 3 out of 69

$$-c_{13}(t) + c_{23}(t) = 0 \quad (8)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

Determining equation 4 out of 69

$$-c_{10}(t) + c_{30}(t) - \frac{d}{dt} c_{12}(t) = 0 \quad (10)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (11)$$

Determining equation 5 out of 69

$$-c_{11}(t) + c_{22}(t) + c_{33}(t) + \frac{d}{dt} c_{00}(t) = 0 \quad (12)$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (13)$$

Determining equation 6 out of 69

$$\frac{d}{dt} c_{03}(t) = 0 \quad (14)$$

$$\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (15)$$

Determining equation 7 out of 69

$$-c_{12}(t) + c_{32}(t) = 0 \quad (16)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (17)$$

Determining equation 8 out of 69

$$\frac{d}{dt} c_{02}(t) = 0 \quad (18)$$

$$\begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (19)$$

Determining equation 9 out of 69

$$c_{01}(t) - c_{02}(t) - c_{03}(t) = 0 \quad (20)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(21)

Determining equation 10 out of 69

$$-c_{20}(t) - c_{30}(t) - \frac{d}{dt} c_{11}(t) = 0$$

(22)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

(23)

Determining equation 11 out of 69

$$-c_{12}(t) + c_{13}(t) + c_{21}(t) - c_{23}(t) - c_{33}(t) - \frac{d}{dt} c_{00}(t) = 0$$

(24)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & 0 & 1 & 0 \end{bmatrix} \quad (25)$$

Determining equation 12 out of 69

$$-\frac{d}{dt} c_{03}(t) = 0 \quad (26)$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (27)$$

Determining equation 13 out of 69

$$c_{12}(t) - c_{13}(t) - c_{22}(t) + c_{31}(t) - c_{32}(t) - \frac{d}{dt} c_{00}(t) = 0 \quad (28)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \quad (29)$$

Determining equation 14 out of 69

$$\frac{d}{dt} c_{01}(t) - \frac{d}{dt} c_{02}(t) - \frac{d}{dt} c_{03}(t) = 0 \quad (30)$$

$$\begin{bmatrix} 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (31)$$

Determining equation 15 out of 69

$$-2 c_{01}(t) + 2 c_{02}(t) = 0 \quad (32)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (33)$$

Determining equation 16 out of 69

$$-\frac{d}{dt} c_{02}(t) = 0 \quad (34)$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (35)$$

Determining equation 17 out of 69

$$-2 c_{01}(t) + 2 c_{03}(t) = 0 \quad (36)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(37)

Determining equation 18 out of 69

$$-c_{21}(t) - c_{31}(t) = 0$$

(38)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

(39)

Determining equation 19 out of 69

$$-\frac{d}{dt} c_{01}(t) = 0$$

(40)

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (41)$$

Determining equation 20 out of 69

$$c_{01}(t) - c_{02}(t) + c_{03}(t) = 0 \quad (42)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (43)$$

Determining equation 21 out of 69

$$-\frac{d}{dt} c_{01}(t) = 0 \quad (44)$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (45)$$

Determining equation 22 out of 69

$$2 \, c_{01} (t) = 0 \quad (46)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (47)$$

Determining equation 23 out of 69

$$c_{01} (t) + c_{02} (t) - c_{03} (t) = 0 \quad (48)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (49)$$

Determining equation 24 out of 69

$$-\frac{d}{dt} c_{20}(t) = 0 \quad (50)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (51)$$

Determining equation 25 out of 69

$$c_{10}(t) - c_{20}(t) - \frac{d}{dt} c_{23}(t) = 0 \quad (52)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (53)$$

Determining equation 26 out of 69

$$c_{13}(t) - c_{23}(t) = 0 \quad (54)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (55)$$

Determining equation 27 out of 69

$$-c_{10}(t) - c_{30}(t) - \frac{d}{dt} c_{22}(t) = 0 \quad (56)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad (57)$$

Determining equation 28 out of 69

$$c_{12}(t) - c_{13}(t) - c_{21}(t) + c_{23}(t) - c_{33}(t) - \frac{d}{dt} c_{00}(t) = 0 \quad (58)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix} \quad (59)$$

Determining equation 29 out of 69

$$- \frac{d}{dt} c_{03}(t) = 0 \quad (60)$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (61)$$

Determining equation 30 out of 69

$$-c_{12}(t) - c_{32}(t) = 0 \quad (62)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (63)$$

Determining equation 31 out of 69

$$-\frac{d}{dt} c_{02}(t) = 0 \quad (64)$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (65)$$

Determining equation 32 out of 69

$$-c_{01}(t) + c_{02}(t) + c_{03}(t) = 0 \quad (66)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (67)$$

Determining equation 33 out of 69

$$-c_{20}(t) + c_{30}(t) - \frac{d}{dt} c_{21}(t) = 0 \quad (68)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (69)$$

Determining equation 34 out of 69

$$c_{11}(t) - c_{22}(t) + c_{33}(t) + \frac{d}{dt} c_{00}(t) = 0 \quad (70)$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \quad (71)$$

Determining equation 35 out of 69

$$\frac{d}{dt} c_{03}(t) = 0 \quad (72)$$

$$\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(73)

Determining equation 36 out of 69

$$-c_{11}(t) + c_{21}(t) - c_{23}(t) - c_{31}(t) + c_{32}(t) - \frac{d}{dt} c_{00}(t) = 0$$

(74)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(75)

Determining equation 37 out of 69

$$-\frac{d}{dt} c_{01}(t) + \frac{d}{dt} c_{02}(t) - \frac{d}{dt} c_{03}(t) = 0$$

(76)

$$\begin{bmatrix} 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (77)$$

Determining equation 38 out of 69

$$2 \, c_{01} (t) - 2 \, c_{02} (t) = 0 \quad (78)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (79)$$

Determining equation 39 out of 69

$$- \frac{d}{dt} c_{02} (t) = 0 \quad (80)$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (81)$$

Determining equation 40 out of 69

$$2 \, c_{02} \left(t \right) = 0 \quad (82)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (83)$$

Determining equation 41 out of 69

$$- \, c_{21} \left(t \right) + c_{31} \left(t \right) = 0 \quad (84)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \quad (85)$$

Determining equation 42 out of 69

$$\frac{d}{dt} c_{01}(t) = 0 \quad (86)$$

$$\begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (87)$$

Determining equation 43 out of 69

$$-c_{01}(t) + c_{02}(t) - c_{03}(t) = 0 \quad (88)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (89)$$

Determining equation 44 out of 69

$$-\frac{d}{dt} c_{01}(t) = 0 \quad (90)$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (91)$$

Determining equation 45 out of 69

$$-2 c_{02}(t) + 2 c_{03}(t) = 0 \quad (92)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (93)$$

Determining equation 46 out of 69

$$c_{01}(t) + c_{02}(t) - c_{03}(t) = 0 \quad (94)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (95)$$

Determining equation 47 out of 69

$$-\frac{d}{dt} c_{30}(t) = 0 \quad (96)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (97)$$

Determining equation 48 out of 69

$$-c_{10}(t) - c_{20}(t) - \frac{d}{dt} c_{33}(t) = 0 \quad (98)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (99)$$

Determining equation 49 out of 69

$$-c_{13}(t) - c_{23}(t) = 0 \quad (100)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \quad (101)$$

Determining equation 50 out of 69

$$c_{10}(t) - c_{30}(t) - \frac{d}{dt} c_{32}(t) = 0 \quad (102)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \quad (103)$$

Determining equation 51 out of 69

$$-c_{12}(t) + c_{13}(t) - c_{22}(t) - c_{31}(t) + c_{32}(t) - \frac{d}{dt} c_{00}(t) = 0 \quad (104)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \quad (105)$$

Determining equation 52 out of 69

$$-\frac{d}{dt} c_{03}(t) = 0 \quad (106)$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (107)$$

Determining equation 53 out of 69

$$c_{12}(t) - c_{32}(t) = 0 \quad (108)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (109)$$

Determining equation 54 out of 69

$$-\frac{d}{dt} c_{02}(t) = 0 \quad (110)$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (111)$$

Determining equation 55 out of 69

$$-c_{01}(t) + c_{02}(t) + c_{03}(t) = 0 \quad (112)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (113)$$

Determining equation 56 out of 69

$$c_{20}(t) - c_{30}(t) - \frac{d}{dt} c_{31}(t) = 0 \quad (114)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \quad (115)$$

Determining equation 57 out of 69

$$-c_{11}(t) - c_{21}(t) + c_{23}(t) + c_{31}(t) - c_{32}(t) - \frac{d}{dt} c_{00}(t) = 0 \quad (116)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \end{bmatrix} \quad (117)$$

Determining equation 58 out of 69

$$-\frac{d}{dt} c_{03}(t) = 0 \quad (118)$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (119)$$

Determining equation 59 out of 69

$$c_{11}(t) + c_{22}(t) - c_{33}(t) + \frac{d}{dt} c_{00}(t) = 0 \quad (120)$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix}$$

(121)

Determining equation 60 out of 69

$$-\frac{d}{dt} c_{01}(t) - \frac{d}{dt} c_{02}(t) + \frac{d}{dt} c_{03}(t) = 0$$

(122)

$$\begin{bmatrix} 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix}$$

(123)

Determining equation 61 out of 69

$$2 c_{03}(t) = 0$$

(124)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (125)$$

Determining equation 62 out of 69

$$\frac{d}{dt} c_{02}(t) = 0 \quad (126)$$

$$\begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (127)$$

Determining equation 63 out of 69

$$2 c_{01}(t) - 2 c_{03}(t) = 0 \quad (128)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & -2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (129)$$

Determining equation 64 out of 69

$$c_{21}(t) - c_{31}(t) = 0 \quad (130)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (131)$$

Determining equation 65 out of 69

$$-\frac{d}{dt} c_{01}(t) = 0 \quad (132)$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (133)$$

Determining equation 66 out of 69

$$c_{01}(t) - c_{02}(t) + c_{03}(t) = 0 \quad (134)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (135)$$

Determining equation 67 out of 69

$$\frac{d}{dt} c_{01}(t) = 0 \quad (136)$$

$$\begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (137)$$

Determining equation 68 out of 69

$$2 \, c_{02} \left(t \right) - 2 \, c_{03} \left(t \right) = 0 \quad (138)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (139)$$

Determining equation 69 out of 69

$$- \, c_{01} \left(t \right) - c_{02} \left(t \right) + c_{03} \left(t \right) = 0 \quad (140)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} \quad (141)$$

5.2 Step 1 of 6: the initial matrices

Dimension of matrices: 69X16 Matrix A

[illegible]

Matrix B

[illegible]

Step 2 of 6: the reduced based on $\text{col}(\mathbf{M}^T)$ where $\mathbf{M}=[-\mathbf{A}|\mathbf{B}]$ Dimension of matrices: 27X16
Matrix A

[illegible]

Matrix B

[illegible]

```
## Step 3 of 6: Splitting up to A, B and B_algebraic Dimension of matrices A and B: 16X16
Dimension of matrices B_algebraic: 11X16 Matrix A
```

[illegible]

[illegible][illegible]
$$\mathbf{c} = \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} \quad (149)$$

Dimensions of B_algebraic: 11X16

Dimension of matrices A and B: 16X16 Dimension of matrices B_algebraic: 11X16 Matrix A

[illegible]

[illegible]

Dimensions of B: 16X16 Coefficients:

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \\ C_{11} \\ C_{12} \\ C_{13} \\ C_{14} \\ C_{15} \\ C_{16} \end{bmatrix} \quad (152)$$

Number of unknowns: 16 ## Step 5 of 6: Solving the ODE system Dimension of the matrix B:
16X16 Dimension of the matrix B_algebraic: 11X16 ODE system:

$$\frac{d}{dt} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} \quad (153)$$

Solve the ODE system: Initial conditions for \mathbf{c} denoted by \mathbf{c}_0 in terms of arbitrary integration constants:

$$\mathbf{c}_0 = \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \\ C_{11} \\ C_{12} \\ C_{13} \\ C_{14} \\ C_{15} \\ C_{16} \end{bmatrix} \quad (154)$$

Jordan form:

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (155)$$

(156)

Exponential form:

(157)

Solution to the ODE system:

$$P \exp(J \cdot t) P^{-1} \mathbf{c}_0 = \begin{bmatrix} C_1 - C_{14}t + \frac{C_5 t^2}{2} + \frac{C_9 t^2}{2} \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ -C_5 t + C_6 + C_9 t \\ C_{13}t - C_5 t + C_7 \\ -C_{13}t + C_8 - C_9 t \\ C_9 \\ C_{10} + C_5 t - C_9 t \\ C_{11} - C_{13}t - C_5 t \\ C_{12} + C_{13}t - C_9 t \\ C_{13} \\ C_{14} - C_5 t - C_9 t \\ -C_{13}t + C_{15} + C_5 t \\ -C_{13}t + C_{16} + C_9 t \end{bmatrix} \quad (158)$$

Step 6 of 6: Solving the algebraic system Number of algebraic equations: 11

Matrix B_algebraic

$$B_{\text{algebraic}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (159)$$

Algebraic equations:

$$\begin{bmatrix}
 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
 \end{bmatrix}
 \begin{bmatrix}
 c_{00} \\
 c_{03} \\
 c_{02} \\
 c_{01} \\
 c_{10} \\
 c_{13} \\
 c_{12} \\
 c_{11} \\
 c_{20} \\
 c_{23} \\
 c_{22} \\
 c_{21} \\
 c_{30} \\
 c_{33} \\
 c_{32} \\
 c_{31}
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0
 \end{bmatrix}
 \quad (160)$$

Algebraic equations after substitution of the solution to the ODE system:

$$\begin{bmatrix}
 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
 \end{bmatrix}
 \begin{bmatrix}
 C_1 - C_{14}t + \frac{C_5t^2}{2} + \frac{C_9t^2}{2} \\
 C_2 \\
 C_3 \\
 C_4 \\
 C_5 \\
 -C_5t + C_6 + C_9t \\
 C_{13}t - C_5t + C_7 \\
 -C_{13}t + C_8 - C_9t \\
 C_9 \\
 C_{10} + C_5t - C_9t \\
 C_{11} - C_{13}t - C_5t \\
 C_{12} + C_{13}t - C_9t \\
 C_{13} \\
 C_{14} - C_5t - C_9t \\
 -C_{13}t + C_{15} + C_5t \\
 -C_{13}t + C_{16} + C_9t
 \end{bmatrix}
 =
 \begin{bmatrix}
 C_2 \\
 C_3 \\
 C_4 \\
 -C_5t + C_6 + C_9t \\
 C_{13}t - C_5t + C_7 \\
 -C_{13}t - C_{14} + C_5t + C_8 \\
 C_{10} + C_5t - C_9t \\
 C_{11} - C_{13}t - C_{14} + C_9t \\
 C_{12} + C_{13}t - C_9t \\
 -C_{13}t + C_{15} + C_5t \\
 -C_{13}t + C_{16} + C_9t
 \end{bmatrix}
 \quad (161)$$

Equation: $C_2 = 0$, Solution: $C_2 = 0$ Equation: $C_3 = 0$, Solution: $C_3 = 0$ Equation: $C_4 = 0$, Solution: $C_4 = 0$ Equation: $-C_{13} + \frac{C_{15}}{t} + C_5 = 0$, Solution: $C_5 = C_{13} - \frac{C_{15}}{t}$ Equation: $C_{15} - C_{16} + C_6 = 0$, Solution: $C_6 = -C_{15} + C_{16}$ Equation: $C_{15} + C_7 = 0$, Solution: $C_7 = -C_{15}$ Equation: $-C_{14} - C_{15} + C_8 = 0$, Solution: $C_8 = C_{14} + C_{15}$ Equation: $-C_{13} + \frac{C_{16}}{t} + C_9 = 0$, Solution: $C_9 = C_{13} - \frac{C_{16}}{t}$ Equation: $C_{10} - C_{15} + C_{16} = 0$, Solution: $C_{10} = C_{15} - C_{16}$ Equation: $C_{11} - C_{14} - C_{16} = 0$, Solution: $C_{11} = C_{14} + C_{16}$ Equation: $C_{12} + C_{16} = 0$, Solution: $C_{12} = -C_{16}$

Solution *after* algebraic substitution:

$$\mathbf{c} = \begin{bmatrix} C_1 + C_{13}t^2 - C_{14}t - \frac{C_{15}t}{2} - \frac{C_{16}t}{2} \\ 0 \\ 0 \\ 0 \\ C_{13} - \frac{C_{15}}{t} \\ 0 \\ 0 \\ -2C_{13}t + C_{14} + C_{15} + C_{16} \\ C_{13} - \frac{C_{16}}{t} \\ 0 \\ -2C_{13}t + C_{14} + C_{15} + C_{16} \\ 0 \\ 0 \\ C_{13} \\ -2C_{13}t + C_{14} + C_{15} + C_{16} \\ 0 \\ 0 \end{bmatrix}$$

6 The very final step

The very final step: substituting the solution into the tangents and print the results: Arbitrary integration constants in the final solution:

$$\begin{bmatrix} C_1 \\ C_{15} \\ C_{16} \\ C_{14} \\ C_{13} \end{bmatrix}$$

Number of generators which are divided based on the number of constants: 5

Number of component tangents before removing: 5 Generator 1 out of 5:

$$\xi = 1$$

$$\eta_1 = 0$$

$$\eta_2 = 0$$

$$\eta_3 = 0$$

Checking the 3 linearised symmetry conditions of generator X_1 : Lin syms

$$[0, 0, 0]$$

Generator 2 out of 5:

$$\begin{aligned}\xi &= -\frac{t}{2} \\ \eta_1 &= w_1 - \frac{1}{t} \\ \eta_2 &= w_2 \\ \eta_3 &= w_3\end{aligned}$$

Checking the 3 linearised symmetry conditions of generator X_2 : Lin syms

$$\left[-\frac{x_1x_2}{2} - \frac{x_1x_3}{2} + \frac{x_2x_3}{2} + \frac{x_2}{x_0} + \frac{x_3}{x_0} - \frac{1}{x_0^2}, -\frac{x_1x_2}{2} + \frac{x_1x_3}{2} - \frac{x_2x_3}{2} + \frac{x_2}{x_0} - \frac{x_3}{x_0}, \frac{x_1x_2}{2} - \frac{x_1x_3}{2} - \frac{x_2x_3}{2} - \frac{x_2}{x_0} + \frac{x_3}{x_0} \right]$$

Generator 3 out of 5:

$$\begin{aligned}\xi &= -\frac{t}{2} \\ \eta_1 &= w_1 \\ \eta_2 &= w_2 - \frac{1}{t} \\ \eta_3 &= w_3\end{aligned}$$

Checking the 3 linearised symmetry conditions of generator X_3 : Lin syms

$$\left[-\frac{x_1x_2}{2} - \frac{x_1x_3}{2} + \frac{x_2x_3}{2} + \frac{x_1}{x_0} - \frac{x_3}{x_0}, -\frac{x_1x_2}{2} + \frac{x_1x_3}{2} - \frac{x_2x_3}{2} + \frac{x_1}{x_0} + \frac{x_3}{x_0} - \frac{1}{x_0^2}, \frac{x_1x_2}{2} - \frac{x_1x_3}{2} - \frac{x_2x_3}{2} - \frac{x_1}{x_0} + \frac{x_3}{x_0} \right]$$

Generator 4 out of 5:

$$\begin{aligned}\xi &= -t \\ \eta_1 &= w_1 \\ \eta_2 &= w_2 \\ \eta_3 &= w_3\end{aligned}$$

Checking the 3 linearised symmetry conditions of generator X_4 : Lin syms

$$[0, 0, 0]$$

Generator 5 out of 5:

$$\begin{aligned}
\xi &= t^2 \\
\eta_1 &= -2tw_1 + 1 \\
\eta_2 &= -2tw_2 + 1 \\
\eta_3 &= -2tw_3 + 1
\end{aligned}$$

Checking the 3 linearised symmetry conditions of generator X_5 : Lin syms

$$[0, 0, 0]$$

Number of component tangents after removing: 3 The final generators are given by:

$$\begin{aligned}
X_1 &= (1) \partial t, \\
&, \\
X_2 &= (-t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3, \\
&, \\
X_3 &= (t^2) \partial t + (-2tw_1 + 1) \partial w_1 + (-2tw_2 + 1) \partial w_2 + (-2tw_3 + 1) \partial w_3.
\end{aligned}$$

7 Conclusion

Something here is not completely right since we obtain generators that are not solutions to the linearised symmetry conditions.