

Summary of symmetry calculations

October 21, 2021

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Chapter 1

DBH_model

Run 05_16PM_21_October-2021

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\begin{aligned}\frac{dw_1}{dt} &= -w_1w_2 - w_1w_3 + w_2w_3, \\ \frac{dw_2}{dt} &= -w_1w_2 + w_1w_3 - w_2w_3, \\ \frac{dw_3}{dt} &= w_1w_2 - w_1w_3 - w_2w_3.\end{aligned}$$

The calculated generators are:

$$X_1 = (1) \partial t,$$

$$X_2 = (-1 + t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$\begin{aligned}X_3 &= (t + 2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 \\ &\quad + (1 - 2tw_3) \partial w_3\end{aligned}$$

The execution time of the script was:

0 hours 0 minutes 25 seconds.

Run 05_20PM_21_October-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\begin{aligned}\frac{dw_1}{dt} &= -w_1w_2 - w_1w_3 + w_2w_3, \\ \frac{dw_2}{dt} &= -w_1w_2 + w_1w_3 - w_2w_3, \\ \frac{dw_3}{dt} &= w_1w_2 - w_1w_3 - w_2w_3.\end{aligned}$$

The calculated generators are:

$$X_1 = (1) \partial t,$$

$$\begin{aligned}X_2 &= (t + 2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 \\ &\quad + (1 - 2tw_3) \partial w_3\end{aligned}$$

$$X_3 = (-1) \partial t,$$

$$X_4 = (-1 + t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$\begin{aligned}X_5 &= (t) \partial t + (w_2w_3 f_1(t) - w_1w_2 f_1(t) - w_1w_3 f_1(t)) \partial w_1 + (w_1w_3 f_1(t) - w_1w_2 f_1(t) \\ &\quad + -w_2w_3 f_1(t)) \partial w_2 + (w_1w_2 f_1(t) - w_1w_3 f_1(t) - w_2w_3 f_1(t)) \partial w_3\end{aligned}$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

The execution time of the script was:

0 hours 3 minutes 37 seconds.

Chapter 2

hydons_model

Run 05_13PM_21_October-2021

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\begin{aligned}\frac{dy_1}{dt} &= \frac{ty_1 + y_2^2}{-t^2 + y_1y_2}, \\ \frac{dy_2}{dt} &= \frac{ty_2 + y_1^2}{-t^2 + y_1y_2}.\end{aligned}$$

The calculated generators are:

$$X_1 = (t) \partial t + (y_1) \partial y_1 + (y_2) \partial y_2.$$

The execution time of the script was:

0 hours 0 minutes 4 seconds.

Run 05_13PM_21_October-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\begin{aligned}\frac{dy_1}{dt} &= \frac{ty_1 + y_2^2}{-t^2 + y_1y_2}, \\ \frac{dy_2}{dt} &= \frac{ty_2 + y_1^2}{-t^2 + y_1y_2}.\end{aligned}$$

The calculated generators are:

$$X_1 = (t) \partial t + (y_1) \partial y_1 + (y_2) \partial y_2,$$

$$X_2 = \left(-t^2 f_1(t) + y_1 y_2 f_1(t) \right) \partial t + \left(y_2^2 f_1(t) + t y_1 f_1(t) \right) \partial y_1 + \left(y_1^2 f_1(t) + t y_2 f_1(t) \right) \partial y_2$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

The execution time of the script was:

0 hours 0 minutes 23 seconds.

Chapter 3

linear_model

Run 05_13PM_21_October-2021

Degree in tangential ansätze: 1.

The system of ODEs is given by:

$$\begin{aligned}\frac{du}{dt} &= u + v, \\ \frac{dv}{dt} &= u + v.\end{aligned}$$

The calculated generators are:

$$\begin{aligned}X_1 = & \left(-\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4} \right) \partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} \right. \\ & \left. + -\frac{ve^{2t}}{2} \right) \partial u + \left(-\frac{v}{2} - \frac{ve^{2t}}{2} \right) \partial v\end{aligned}$$

$$X_2 = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial t$$

$$\begin{aligned}X_3 = & \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4} \right) \partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right. \\ & \left. + \frac{ve^{2t}}{2} \right) \partial u + \left(-\frac{v}{2} + \frac{ve^{2t}}{2} \right) \partial v\end{aligned}$$

$$\begin{aligned}X_4 = & \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4} \right) \partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2} \right) \partial u \\ & + \left(\frac{v}{2} - \frac{ve^{2t}}{2} \right) \partial v\end{aligned}$$

$$X_5 = (1) \partial t,$$

$$X_6 = \left(-\frac{e^{-2t}}{4} + \frac{e^{2t}}{4} \right) \partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} \right) \partial u \\ + \left(\frac{v}{2} + \frac{ve^{2t}}{2} \right) \partial v$$

$$X_7 = \left(-\frac{1}{2} + \frac{e^{2t}}{2} \right) \partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2} \right) \partial v$$

$$X_8 = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial t$$

$$X_9 = \left(\frac{1}{2} + \frac{e^{2t}}{2} \right) \partial u + \left(-\frac{1}{2} + \frac{e^{2t}}{2} \right) \partial v$$

$$X_{10} = (t) \partial t + (u f_1(t) + v f_1(t)) \partial u + (u f_1(t) + v f_1(t)) \partial v$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

The execution time of the script was:

0 hours 0 minutes 12 seconds.

Run 05_16PM_21_October-2021

Degree in tangential ansätze: 2.

The system of ODEs is given by:

$$\frac{du}{dt} = u + v, \\ \frac{dv}{dt} = u + v.$$

The calculated generators are:

$$X_1 = \left(-\frac{5v}{8} + \frac{3u}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ve^{-4t}}{8} \right. \\ \left. + \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} \right. \\ \left. + \frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left(-\frac{v^2}{2} \right. \\ \left. + \frac{v^2e^{2t}}{2} \right) \partial v$$

$$X_2 = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4} \right) \partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2} \right) \partial u \\ + \left(\frac{v}{2} - \frac{ve^{2t}}{2} \right) \partial v$$

$$X_3 = \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{u^2e^{-2t}}{2} - \frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} \right. \\ \left. + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial t$$

$$X_4 = \left(\frac{1}{2} + \frac{e^{2t}}{2} \right) \partial u + \left(-\frac{1}{2} + \frac{e^{2t}}{2} \right) \partial v$$

$$X_5 = \left(-\frac{1}{2} + \frac{e^{2t}}{2} \right) \partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2} \right) \partial v$$

$$X_6 = \left(-\frac{e^{-2t}}{4} + \frac{e^{2t}}{4} \right) \partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} \right) \partial u \\ + \left(\frac{v}{2} + \frac{ve^{2t}}{2} \right) \partial v$$

$$X_7 = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4} \right) \partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right. \\ \left. + \frac{ve^{2t}}{2} \right) \partial u + \left(-\frac{v}{2} + \frac{ve^{2t}}{2} \right) \partial v$$

$$X_8 = \left(-\frac{u^2}{4} - \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial t$$

$$X_9 = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial t$$

$$X_{10} = \left(\frac{u}{8} + \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} \right. \\ \left. + \frac{ue^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-2t}}{2} \right. \\ \left. + \frac{v^2e^{2t}}{2} - \frac{v^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left(\frac{v^2}{2} \right. \\ \left. + \frac{v^2e^{2t}}{2} \right) \partial v$$

$$\begin{aligned}
X_{11} = & \left(-\frac{3u}{8} + \frac{5v}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} \right. \\
& + \left. \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{v^2e^{-2t}}{2} \right. \\
& + \frac{v^2e^{2t}}{2} - \frac{u^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \left. \right) \partial u + \left(\frac{v^2}{2} \right. \\
& + \left. \frac{v^2e^{2t}}{2} \right) \partial v
\end{aligned}$$

$$X_{12} = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial t$$

$$\begin{aligned}
X_{13} = & \left(-\frac{u}{8} - \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{-2t}}{8} \right. \\
& + \left. \frac{ve^{-4t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{u^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} \right. \\
& + \frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \left. \right) \partial u + \left(-\frac{v^2}{2} \right. \\
& + \left. \frac{v^2e^{2t}}{2} \right) \partial v
\end{aligned}$$

$$\begin{aligned}
X_{14} = & \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{v^2e^{-2t}}{2} - \frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} \right. \\
& + \left. \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial t
\end{aligned}$$

$$\begin{aligned}
X_{15} = & \left(-\frac{u}{8} + \frac{3v}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-2t}}{8} + \frac{ue^{-4t}}{8} \right. \\
& + \left. \frac{ue^{2t}}{8} + \frac{ve^{-4t}}{8} \right) \partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} - \frac{v^2e^{2t}}{2} \right. \\
& + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \left. \right) \partial u + \left(\frac{v^2}{2} - \frac{v^2e^{2t}}{2} \right) \partial v
\end{aligned}$$

$$X_{16} = (1) \partial t,$$

$$\begin{aligned}
X_{17} = & \left(-\frac{3v}{8} + \frac{u}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-4t}}{8} \right. \\
& + \left. \frac{ve^{-2t}}{8} + \frac{ue^{2t}}{8} \right) \partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} - \frac{uv}{2} - \frac{v^2e^{2t}}{2} \right. \\
& + \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \left. \right) \partial u + \left(-\frac{v^2}{2} - \frac{v^2e^{2t}}{2} \right) \partial v
\end{aligned}$$

$$X_{18} = \left(-\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4} \right) \partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} \right. \\ \left. + -\frac{ve^{2t}}{2} \right) \partial u + \left(-\frac{v}{2} - \frac{ve^{2t}}{2} \right) \partial v$$

$$X_{19} = (u f_2(t) + v f_1(t) - v f_2(t) + f_3(t)) \partial t + (u f_3(t) + v f_3(t) \\ + u^2 f_2(t) + v^2 f_1(t) - v^2 f_2(t) + uv f_1(t)) \partial u + (u f_3(t) + v f_3(t) \\ + u^2 f_2(t) + v^2 f_1(t) - v^2 f_2(t) + uv f_1(t)) \partial v$$

Some of the generators might contain the following arbitrary functions:

$$\begin{aligned} f_1 \\ f_2 \\ f_3 \end{aligned}$$

The execution time of the script was:

0 hours 2 minutes 41 seconds.