Summary of symmetry calculations

October 21, 2021

Contents

Chapter 1

DBH_model

Run 08_27PM_21_October-2021

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1 w_2 - w_1 w_3 + w_2 w_3,$$

$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1 w_2 + w_1 w_3 - w_2 w_3,$$

$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1 w_2 - w_1 w_3 - w_2 w_3.$$

The calculated generators are:

$$X_1 = (1)\,\partial t,$$

$$X_2 = (-1 + t)\,\partial t + (w_1)\,\partial w_1 + (w_2)\,\partial w_2 + (w_3)\,\partial w_3,$$

$$X_3 = (t + 2)\,\partial t + (1 - 2tw_1)\,\partial w_1 + (1 - 2tw_2)\,\partial w_2 + (1 - 2tw_3)\,\partial w_3$$

The execution time of the script was:

$$0 \text{ hours } 0 \text{ minutes } 25 \text{ seconds.}$$

Run 08_30PM_21_October-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1w_2 - w_1w_3 + w_2w_3,$$

$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1w_2 + w_1w_3 - w_2w_3,$$

$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1w_2 - w_1w_3 - w_2w_3.$$

The calculated generators are:

$$X_1 = (1)\,\partial t,$$

$$X_2 = (-1)\,\partial t,$$

$$X_3 = (-1+t)\,\partial t + (w_1)\,\partial w_1 + (w_2)\,\partial w_2 + (w_3)\,\partial w_3,$$

$$X_4 = (t)\,\partial t + (w_2w_3\,\mathrm{f}_1\,(t) - w_1w_2\,\mathrm{f}_1\,(t) - w_1w_3\,\mathrm{f}_1\,(t))\,\partial w_1 + (w_1w_3\,\mathrm{f}_1\,(t) - w_1w_2\,\mathrm{f}_1\,(t)$$
$$+ -w_2w_3\,\mathrm{f}_1\,(t))\,\partial w_2 + (w_1w_2\,\mathrm{f}_1\,(t) - w_1w_3\,\mathrm{f}_1\,(t) - w_2w_3\,\mathrm{f}_1\,(t))\,\partial w_3$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 3 minutes 29 seconds.

Chapter 2

hydons_model

Run 08_22PM_21_October-2021

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$
$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

The calculated generators are:

$$X_1 = (t)\,\partial t + (y_1)\,\partial y_1 + (y_2)\,\partial y_2.$$

The execution time of the script was:

0 hours 0 minutes 5 seconds.

Run 08_23PM_21_October-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$
$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

The calculated generators are:

$$X_1 = (t)\,\partial t + (y_1)\,\partial y_1 + (y_2)\,\partial y_2,$$

$$X_2 = \left(-t^2\, \mathrm{f}_1\,(t) + y_1 y_2\, \mathrm{f}_1\,(t)\right) \partial t + \left(y_2^2\, \mathrm{f}_1\,(t) + t y_1\, \mathrm{f}_1\,(t)\right) \partial y_1 + \left(y_1^2\, \mathrm{f}_1\,(t) + t y_2\, \mathrm{f}_1\,(t)\right) \partial y_2$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 0 minutes 23 seconds.

Chapter 3

linear_model

Run 08_23PM_21_October-2021

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

The calculated generators are:

$$X_1 = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right)\partial u$$
$$+ \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v$$

$$X_2 = \left(-\frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right)\partial u$$
$$+ \left(\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$X_3 = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_4 = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right.$$
$$\left. + \frac{ve^{2t}}{2}\right)\partial u + \left(-\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$X_5 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_6 = (1)\,\partial t,$$

$$X_7 = \left(-\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right.$$
$$\left. + -\frac{ve^{2t}}{2}\right)\partial u + \left(-\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v$$

$$X_8 = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_9 = \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_{10} = (t)\,\partial t + (u\,\mathrm{f}_1\,(t) + v\,\mathrm{f}_1\,(t))\,\partial u + (u\,\mathrm{f}_1\,(t) + v\,\mathrm{f}_1\,(t))\,\partial v$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 0 minutes 12 seconds.

Run 08_26PM_21_October-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

The calculated generators are:

$$X_1 = \left(-\frac{3v}{8} + \frac{u}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-4t}}{8}\right.$$
$$\left. + -\frac{ve^{-2t}}{8} + \frac{ue^{2t}}{8}\right)\partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} - \frac{uv}{2} - \frac{v^2e^{2t}}{2}\right.$$
$$\left. + -\frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2}\right)\partial u + \left(-\frac{v^2}{2} - \frac{v^2e^{2t}}{2}\right)\partial v$$

$$X_2 = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right.$$
$$\left. + \frac{ve^{2t}}{2}\right)\partial u + \left(-\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$X_3 = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_4 = \left(-\frac{u^2}{4} - \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2 e^{-4t}}{4} + \frac{v^2 e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial t$$

$$X_5 = \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_6 = \left(-\frac{u}{8} - \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{-2t}}{8}\right.$$
$$\left. + \frac{ve^{-4t}}{8} + \frac{3ve^{2t}}{8}\right)\partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{u^2 e^{-2t}}{2} + \frac{v^2 e^{2t}}{2}\right.$$
$$\left. + -\frac{uv}{2} - \frac{v^2 e^{-2t}}{2} + \frac{u^2 e^{-4t}}{4} + \frac{v^2 e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial u + \left(-\frac{v^2}{2}\right.$$
$$\left. + \frac{v^2 e^{2t}}{2}\right)\partial v$$

$$X_7 = \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{u^2 e^{-2t}}{2} - \frac{uv}{2} - \frac{v^2 e^{-2t}}{2} + \frac{u^2 e^{-4t}}{4}\right.$$
$$\left. + \frac{v^2 e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial t$$

$$X_8 = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_9 = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right)\partial u$$
$$+ \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v$$

$$X_{10} = \left(-\frac{u}{8} + \frac{3v}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-2t}}{8} + \frac{ue^{-4t}}{8}\right.$$
$$\left. + \frac{ue^{2t}}{8} + \frac{ve^{-4t}}{8}\right)\partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} - \frac{v^2 e^{2t}}{2}\right.$$
$$\left. + \frac{u^2 e^{-4t}}{4} + \frac{v^2 e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial u + \left(\frac{v^2}{2} - \frac{v^2 e^{2t}}{2}\right)\partial v$$

$$X_{11} = \left(\frac{u}{8} + \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} \right.$$
$$\left. + \frac{ue^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2 e^{-2t}}{2} \right.$$
$$\left. + \frac{v^2 e^{2t}}{2} - \frac{v^2 e^{-2t}}{2} - \frac{u^2 e^{-4t}}{4} - \frac{v^2 e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left(\frac{v^2}{2} \right.$$
$$\left. + \frac{v^2 e^{2t}}{2} \right) \partial v$$

$$X_{12} = \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{v^2 e^{-2t}}{2} - \frac{uv}{2} - \frac{u^2 e^{-2t}}{2} + \frac{u^2 e^{-4t}}{4} \right.$$
$$\left. + \frac{v^2 e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial t$$

$$X_{13} = \left(\frac{1}{2} + \frac{e^{2t}}{2} \right) \partial u + \left(-\frac{1}{2} + \frac{e^{2t}}{2} \right) \partial v$$

$$X_{14} = \left(-\frac{5v}{8} + \frac{3u}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ve^{-4t}}{8} \right.$$
$$\left. + \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{v^2 e^{-2t}}{2} + \frac{v^2 e^{2t}}{2} \right.$$
$$\left. + -\frac{uv}{2} - \frac{u^2 e^{-2t}}{2} + \frac{u^2 e^{-4t}}{4} + \frac{v^2 e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left(-\frac{v^2}{2} \right.$$
$$\left. + \frac{v^2 e^{2t}}{2} \right) \partial v$$

$$X_{15} = \left(-\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4} \right) \partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} \right.$$
$$\left. + -\frac{ve^{2t}}{2} \right) \partial u + \left(-\frac{v}{2} - \frac{ve^{2t}}{2} \right) \partial v$$

$$X_{16} = \left(-\frac{3u}{8} + \frac{5v}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} \right.$$
$$\left. + \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{v^2 e^{-2t}}{2} \right.$$
$$\left. + \frac{v^2 e^{2t}}{2} - \frac{u^2 e^{-2t}}{2} - \frac{u^2 e^{-4t}}{4} - \frac{v^2 e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left(\frac{v^2}{2} \right.$$
$$\left. + \frac{v^2 e^{2t}}{2} \right) \partial v$$

$$X_{17} = \left(-\frac{e^{-2t}}{4} + \frac{e^{2t}}{4} \right) \partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} \right) \partial u$$
$$+ \left(\frac{v}{2} + \frac{ve^{2t}}{2} \right) \partial v$$

$$X_{18} = (1)\,\partial t,$$

$$X_{19} = (u\,\mathrm{f}_2\,(t) + v\,\mathrm{f}_1\,(t) - v\,\mathrm{f}_2\,(t) + \mathrm{f}_3\,(t))\,\partial t + (u\,\mathrm{f}_3\,(t) + v\,\mathrm{f}_3\,(t)$$
$$+ \; u^2\,\mathrm{f}_2\,(t) + v^2\,\mathrm{f}_1\,(t) - v^2\,\mathrm{f}_2\,(t) + uv\,\mathrm{f}_1\,(t)\big)\,\partial u + (u\,\mathrm{f}_3\,(t) + v\,\mathrm{f}_3\,(t)$$
$$+ \; u^2\,\mathrm{f}_2\,(t) + v^2\,\mathrm{f}_1\,(t) - v^2\,\mathrm{f}_2\,(t) + uv\,\mathrm{f}_1\,(t)\big)\,\partial v$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$
$$\mathrm{f}_2$$
$$\mathrm{f}_3$$

The execution time of the script was:

0 hours 3 minutes 2 seconds.