# Summary of symmetry calculations

October 29, 2021

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## Chapter 1

# $DBH_{model}$

#### $Run~08\_30PM\_29\_October-2021$

Degree in tangential ansätze: 1. The system of ODEs is given by:

$$\frac{dw_1}{dt} = -w_1w_2 - w_1w_3 + w_2w_3,$$

$$\frac{dw_2}{dt} = -w_1w_2 + w_1w_3 - w_2w_3,$$

$$\frac{dw_3}{dt} = w_1w_2 - w_1w_3 - w_2w_3.$$

The calculated generators are:

$$X_1 = (1) \partial t,$$

$$X_2 = (t+2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 + (1 - 2tw_3) \partial w_3$$

$$X_3 = (-1+t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3.$$

The execution time of the script was:

0 hours 0 minutes 25 seconds.

#### $Run~08\_31PM\_29\_October-2021$

Degree in tangential ansätze: 1. The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1w_2 - w_1w_3 + w_2w_3,$$

$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1w_2 + w_1w_3 - w_2w_3,$$

$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1w_2 - w_1w_3 - w_2w_3.$$

The calculated generators are:

$$X_1 = (t+2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 + (1 - 2tw_3) \partial w_3$$

$$X_2 = (1) \partial t$$
,

$$X_3 = (-1+t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3.$$

The execution time of the script was:

0 hours 0 minutes 25 seconds.

#### $Run~08\_31PM\_29\_October-2021$

Degree in tangential ansätze: 1. The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1w_2 - w_1w_3 + w_2w_3,$$

$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1w_2 + w_1w_3 - w_2w_3,$$

$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1w_2 - w_1w_3 - w_2w_3.$$

The calculated generators are:

$$X_1 = (t+2) \partial t + (1-2tw_1) \partial w_1 + (1-2tw_2) \partial w_2 + (1-2tw_3) \partial w_3$$

$$X_2 = (-1+t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$X_3 = (1) \partial t$$
.

The execution time of the script was:

0 hours 0 minutes 25 seconds.

#### $Run~08\_31PM\_29\_October-2021$

Degree in tangential ansätze: 1. The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1w_2 - w_1w_3 + w_2w_3,$$

$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1w_2 + w_1w_3 - w_2w_3,$$

$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1w_2 - w_1w_3 - w_2w_3.$$

The calculated generators are:

$$X_1 = (-1+t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$X_2 = (t+2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 + (1 - 2tw_3) \partial w_3$$

$$X_3 = (1) \partial t$$
.

The execution time of the script was:

0 hours 0 minutes 25 seconds.

#### Run 08\_32PM\_29\_October-2021

Degree in tangential ansätze: 1. The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1w_2 - w_1w_3 + w_2w_3,$$

$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1w_2 + w_1w_3 - w_2w_3,$$

$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1w_2 - w_1w_3 - w_2w_3.$$

$$X_1 = (t+2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 + (1 - 2tw_3) \partial w_3$$

$$X_2 = (-1+t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$X_3 = (1) \partial t$$
.

The execution time of the script was:

0 hours 0 minutes 25 seconds.

#### Run 08\_35PM\_29\_October-2021

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1w_2 - w_1w_3 + w_2w_3,$$

$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1w_2 + w_1w_3 - w_2w_3,$$

$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1w_2 - w_1w_3 - w_2w_3.$$

The calculated generators are:

$$X_1 = (1) \partial t$$
,

$$X_2 = (-1) \partial t$$
,

$$X_3 = (-1+t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$X_4 = (t+2) \partial t + (1-2tw_1) \partial w_1 + (1-2tw_2) \partial w_2 + (1-2tw_3) \partial w_3$$

$$X_5 = (t) \partial t + (w_2 w_3 f_1(t) - w_1 w_2 f_1(t) - w_1 w_3 f_1(t)) \partial w_1 + (w_1 w_3 f_1(t) - w_1 w_2 f_1(t) + -w_2 w_3 f_1(t)) \partial w_2 + (w_1 w_2 f_1(t) - w_1 w_3 f_1(t) - w_2 w_3 f_1(t)) \partial w_3$$

Some of the generators might contain the following arbitrary functions:

 $f_1$ 

The execution time of the script was:

0 hours 3 minutes 29 seconds.

#### Run $08_39PM_29_October-2021$

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\frac{dw_1}{dt} = -w_1w_2 - w_1w_3 + w_2w_3,$$

$$\frac{dw_2}{dt} = -w_1w_2 + w_1w_3 - w_2w_3,$$

$$\frac{dw_3}{dt} = w_1w_2 - w_1w_3 - w_2w_3.$$

The calculated generators are:

$$X_1 = (-1+t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$X_2 = (-1) \partial t$$
,

$$X_3 = (1) \partial t$$
,

$$X_4 = (t+2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 + (1 - 2tw_3) \partial w_3$$

$$X_5 = (t) \, \partial t + (w_2 w_3 \, \mathbf{f}_1 \, (t) - w_1 w_2 \, \mathbf{f}_1 \, (t) - w_1 w_3 \, \mathbf{f}_1 \, (t)) \, \partial w_1 + (w_1 w_3 \, \mathbf{f}_1 \, (t) - w_1 w_2 \, \mathbf{f}_1 \, (t) \\ + \, - w_2 w_3 \, \mathbf{f}_1 \, (t)) \, \partial w_2 + (w_1 w_2 \, \mathbf{f}_1 \, (t) - w_1 w_3 \, \mathbf{f}_1 \, (t) - w_2 w_3 \, \mathbf{f}_1 \, (t)) \, \partial w_3$$

Some of the generators might contain the following arbitrary functions:

 $f_1$ 

The execution time of the script was:

0 hours 3 minutes 25 seconds.

#### Run 08\_42PM\_29\_October-2021

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1w_2 - w_1w_3 + w_2w_3,$$

$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1w_2 + w_1w_3 - w_2w_3,$$

$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1w_2 - w_1w_3 - w_2w_3.$$

The calculated generators are:

$$X_1 = (-1) \partial t$$
,

$$X_2 = (t+2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 + (1 - 2tw_3) \partial w_3$$

$$X_3 = (1) \partial t$$
,

$$X_4 = (-1+t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$X_5 = (t) \partial t + (w_2 w_3 f_1(t) - w_1 w_2 f_1(t) - w_1 w_3 f_1(t)) \partial w_1 + (w_1 w_3 f_1(t) - w_1 w_2 f_1(t) + -w_2 w_3 f_1(t)) \partial w_2 + (w_1 w_2 f_1(t) - w_1 w_3 f_1(t) - w_2 w_3 f_1(t)) \partial w_3$$

Some of the generators might contain the following arbitrary functions:

 $f_1$ 

The execution time of the script was:

0 hours 3 minutes 30 seconds.

#### $Run~08\_46PM\_29\_October-2021$

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\begin{split} \frac{\mathrm{d}w_1}{\mathrm{d}t} &= -w_1w_2 - w_1w_3 + w_2w_3, \\ \frac{\mathrm{d}w_2}{\mathrm{d}t} &= -w_1w_2 + w_1w_3 - w_2w_3, \\ \frac{\mathrm{d}w_3}{\mathrm{d}t} &= w_1w_2 - w_1w_3 - w_2w_3. \end{split}$$

The calculated generators are:

$$X_1 = (-1) \, \partial t,$$

$$X_2 = (-1+t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$X_3 = (1) \partial t,$$

$$X_4 = (t+2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 + (1 - 2tw_3) \partial w_3$$

$$X_{5} = (t) \partial t + (w_{2}w_{3} f_{1}(t) - w_{1}w_{2} f_{1}(t) - w_{1}w_{3} f_{1}(t)) \partial w_{1} + (w_{1}w_{3} f_{1}(t) - w_{1}w_{2} f_{1}(t) + -w_{2}w_{3} f_{1}(t)) \partial w_{2} + (w_{1}w_{2} f_{1}(t) - w_{1}w_{3} f_{1}(t) - w_{2}w_{3} f_{1}(t)) \partial w_{3}$$

Some of the generators might contain the following arbitrary functions:

 $f_1$ 

The execution time of the script was:

0 hours 3 minutes 25 seconds.

## Chapter 2

# hydons\_model

#### $Run~08_16PM_29_October-2021$

Degree in tangential ansätze: 1. The system of ODEs is given by:

$$\begin{split} \frac{\mathrm{d}y_1}{\mathrm{d}t} &= \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2}, \\ \frac{\mathrm{d}y_2}{\mathrm{d}t} &= \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}. \end{split}$$

The calculated generators are:

$$X_1 = (t) \partial t + (y_1) \partial y_1 + (y_2) \partial y_2.$$

The execution time of the script was:

0 hours 0 minutes 5 seconds.

#### $Run~08\_16PM\_29\_October-2021$

Degree in tangential ansätze: 1. The system of ODEs is given by:

$$\begin{split} \frac{\mathrm{d}y_1}{\mathrm{d}t} &= \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2}, \\ \frac{\mathrm{d}y_2}{\mathrm{d}t} &= \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}. \end{split}$$

The calculated generators are:

$$X_1 = (t) \partial t + (y_1) \partial y_1 + (y_2) \partial y_2.$$

The execution time of the script was:

0 hours 0 minutes 4 seconds.

#### $Run~08\_16PM\_29\_October-2021$

Degree in tangential ansätze: 1. The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$
$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

The calculated generators are:

$$X_1 = (t) \partial t + (y_1) \partial y_1 + (y_2) \partial y_2.$$

The execution time of the script was:

0 hours 0 minutes 4 seconds.

#### $Run~08_16PM_29_October-2021$

Degree in tangential ansätze: 1. The system of ODEs is given by:

$$\begin{split} \frac{\mathrm{d}y_1}{\mathrm{d}t} &= \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2}, \\ \frac{\mathrm{d}y_2}{\mathrm{d}t} &= \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}. \end{split}$$

The calculated generators are:

$$X_1 = (t) \partial t + (y_1) \partial y_1 + (y_2) \partial y_2.$$

The execution time of the script was:

0 hours 0 minutes 4 seconds.

#### Run 08\_17PM\_29\_October-2021

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$

$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

The calculated generators are:

$$X_1 = (t) \partial t + (y_1) \partial y_1 + (y_2) \partial y_2,$$

$$X_{2} = \left(-t^{2} f_{1}\left(t\right) + y_{1} y_{2} f_{1}\left(t\right)\right) \partial t + \left(y_{2}^{2} f_{1}\left(t\right) + t y_{1} f_{1}\left(t\right)\right) \partial y_{1} + \left(y_{1}^{2} f_{1}\left(t\right) + t y_{2} f_{1}\left(t\right)\right) \partial y_{2}$$

Some of the generators might contain the following arbitrary functions:

 $f_1$ 

The execution time of the script was:

0 hours 0 minutes 23 seconds.

#### Run 08\_17PM\_29\_October-2021

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$

$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

The calculated generators are:

$$X_1 = (t) \partial t + (y_1) \partial y_1 + (y_2) \partial y_2,$$

$$X_{2} = \left(-t^{2} f_{1}\left(t\right) + y_{1} y_{2} f_{1}\left(t\right)\right) \partial t + \left(y_{2}^{2} f_{1}\left(t\right) + t y_{1} f_{1}\left(t\right)\right) \partial y_{1} + \left(y_{1}^{2} f_{1}\left(t\right) + t y_{2} f_{1}\left(t\right)\right) \partial y_{2}$$

Some of the generators might contain the following arbitrary functions:

 $f_1$ 

The execution time of the script was:

0 hours 0 minutes 23 seconds.

#### Run 08\_17PM\_29\_October-2021

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$

$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

The calculated generators are:

$$X_1 = (t) \partial t + (y_1) \partial y_1 + (y_2) \partial y_2,$$

$$X_{2} = \left(-t^{2} f_{1}\left(t\right) + y_{1} y_{2} f_{1}\left(t\right)\right) \partial t + \left(y_{2}^{2} f_{1}\left(t\right) + t y_{1} f_{1}\left(t\right)\right) \partial y_{1} + \left(y_{1}^{2} f_{1}\left(t\right) + t y_{2} f_{1}\left(t\right)\right) \partial y_{2}$$

Some of the generators might contain the following arbitrary functions:

 $f_1$ 

The execution time of the script was:

0 hours 0 minutes 23 seconds.

#### $Run~08_18PM_29_October-2021$

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$

$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

$$X_1 = (t) \partial t + (y_1) \partial y_1 + (y_2) \partial y_2,$$

$$X_{2} = \left(-t^{2} f_{1}(t) + y_{1} y_{2} f_{1}(t)\right) \partial t + \left(y_{2}^{2} f_{1}(t) + t y_{1} f_{1}(t)\right) \partial y_{1} + \left(y_{1}^{2} f_{1}(t) + t y_{2} f_{1}(t)\right) \partial y_{2}$$

 $f_1$ 

The execution time of the script was:

0 hours 0 minutes 23 seconds.

#### Run 08\_18PM\_29\_October-2021

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$
$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

The calculated generators are:

$$X_1 = (t) \partial t + (y_1) \partial y_1 + (y_2) \partial y_2,$$

$$X_{2} = \left(-t^{2} f_{1}\left(t\right) + y_{1} y_{2} f_{1}\left(t\right)\right) \partial t + \left(y_{2}^{2} f_{1}\left(t\right) + t y_{1} f_{1}\left(t\right)\right) \partial y_{1} + \left(y_{1}^{2} f_{1}\left(t\right) + t y_{2} f_{1}\left(t\right)\right) \partial y_{2}$$

Some of the generators might contain the following arbitrary functions:

 $f_1$ 

The execution time of the script was:

0 hours 0 minutes 23 seconds.

## Chapter 3

## linear\_model

#### Run 08\_18PM\_29\_October-2021

Degree in tangential ansätze: 1. The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

$$X_{1} = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right) \partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right) + \frac{ve^{2t}}{2} \partial u + \left(-\frac{v}{2} + \frac{ve^{2t}}{2}\right) \partial v$$

$$X_2 = (1) \partial t$$
,

$$X_{3} = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right) \partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right) \partial u + \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right) \partial v$$

$$X_4 = \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_5 = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_6 = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_7 = \left(-\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4}\right) \partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right) + \left(-\frac{ve^{2t}}{2}\right) \partial u + \left(-\frac{v}{2} - \frac{ve^{2t}}{2}\right) \partial v$$

$$X_8 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_{9} = \left(-\frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right) \partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right) \partial u + \left(\frac{v}{2} + \frac{ve^{2t}}{2}\right) \partial v$$

$$X_{10} = (t) \partial t + (u f_1 (t) + v f_1 (t)) \partial u + (u f_1 (t) + v f_1 (t)) \partial v$$

 $f_1$ 

The execution time of the script was:

0 hours 0 minutes 12 seconds.

#### Run 08\_19PM\_29\_October-2021

Degree in tangential ansätze: 1. The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

$$X_{1} = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right) \partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right) + \frac{ve^{2t}}{2} \partial u + \left(-\frac{v}{2} + \frac{ve^{2t}}{2}\right) \partial v$$

$$X_{2} = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right) \partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right) \partial u + \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right) \partial v$$

$$X_3 = \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_4 = \left(-\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4}\right) \partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right) + \left(-\frac{ve^{2t}}{2}\right) \partial u + \left(-\frac{v}{2} - \frac{ve^{2t}}{2}\right) \partial v$$

$$X_{5} = \left(-\frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right) \partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right) \partial u + \left(\frac{v}{2} + \frac{ve^{2t}}{2}\right) \partial v$$

$$X_6 = (1) \partial t$$
,

$$X_7 = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_8 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_9 = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_{10} = (t) \, \partial t + \left( u \, \mathbf{f}_1 \, (t) + v \, \mathbf{f}_1 \, (t) \right) \partial u + \left( u \, \mathbf{f}_1 \, (t) + v \, \mathbf{f}_1 \, (t) \right) \partial v$$

 $f_1$ 

The execution time of the script was:

#### Run 08\_19PM\_29\_October-2021

Degree in tangential ansätze: 1. The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

$$X_1 = (1) \partial t$$
,

$$\begin{split} X_2 &= \left( -\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4} \right) \partial t + \left( -\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} \right) \\ &+ -\frac{ve^{2t}}{2} \right) \partial u + \left( -\frac{v}{2} - \frac{ve^{2t}}{2} \right) \partial v \\ X_3 &= \left( \frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial t \\ X_4 &= \left( -\frac{e^{-2t}}{4} + \frac{e^{2t}}{4} \right) \partial t + \left( \frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} \right) \partial u \\ &+ \left( \frac{v}{2} + \frac{ve^{2t}}{2} \right) \partial v \\ X_5 &= \left( \frac{1}{2} + \frac{e^{2t}}{2} \right) \partial u + \left( -\frac{1}{2} + \frac{e^{2t}}{2} \right) \partial v \\ X_7 &= \left( -\frac{e^{2t}}{4} + \frac{e^{-2t}}{4} \right) \partial t + \left( \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2} \right) \partial u \\ &+ \left( \frac{v}{2} - \frac{ve^{2t}}{2} \right) \partial v \\ X_8 &= \left( \frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial t \\ X_9 &= \left( -\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4} \right) \partial t + \left( -\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial v \\ &+ \frac{ve^{2t}}{2} \right) \partial u + \left( -\frac{v}{2} + \frac{ve^{2t}}{2} \right) \partial v \end{split}$$

$$X_{10} = (t) \partial t + (u f_1 (t) + v f_1 (t)) \partial u + (u f_1 (t) + v f_1 (t)) \partial v$$

 $f_1$ 

The execution time of the script was:

0 hours 0 minutes 12 seconds.

#### Run 08\_19PM\_29\_October-2021

Degree in tangential ansätze: 1. The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

$$X_{1} = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right) \partial t$$

$$X_{2} = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right) \partial t$$

$$X_{3} = \left(-\frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right) \partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right) \partial u$$

$$+ \left(\frac{v}{2} + \frac{ve^{2t}}{2}\right) \partial v$$

$$X_{4} = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial v$$

$$X_{5} = \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial v$$

$$X_{6} = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right) \partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right) \partial u$$

$$+ \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right) \partial v$$

$$X_7 = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right) \partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right) + \frac{ve^{2t}}{2} \partial u + \left(-\frac{v}{2} + \frac{ve^{2t}}{2}\right) \partial v$$

$$X_8 = \left(-\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4}\right) \partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right) + \left(-\frac{ve^{2t}}{2}\right) \partial u + \left(-\frac{v}{2} - \frac{ve^{2t}}{2}\right) \partial v$$

$$X_9 = (1) \partial t$$

$$X_{10} = (t) \, \partial t + \left( u \, \mathbf{f}_1 \, (t) + v \, \mathbf{f}_1 \, (t) \right) \partial u + \left( u \, \mathbf{f}_1 \, (t) + v \, \mathbf{f}_1 \, (t) \right) \partial v$$

 $f_1$ 

The execution time of the script was:

0 hours 0 minutes 11 seconds.

#### Run 08 22PM 29 October-2021

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

$$\begin{split} X_1 &= \left( -\frac{3v}{8} + \frac{u}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-4t}}{8} \right. \\ &\quad + \left. - \frac{ve^{-2t}}{8} + \frac{ue^{2t}}{8} \right) \partial t + \left( -\frac{v^2}{4} + \frac{u^2}{4} - \frac{uv}{2} - \frac{v^2e^{2t}}{2} \right. \\ &\quad + \left. - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left( -\frac{v^2}{2} - \frac{v^2e^{2t}}{2} \right) \partial v \\ X_2 &= \left( \frac{u^2}{4} + \frac{v^2}{4} + \frac{u^2e^{-2t}}{2} - \frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} \right. \\ &\quad + \left. \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial t \end{split}$$

$$X_3 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial u + \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial v$$

$$X_4 = \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial v$$

$$X_5 = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right) \partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} + \frac{ve^{-2t}}{2}\right) \partial v$$

$$+ \frac{ve^{2t}}{2} \partial u + \left(-\frac{v}{2} + \frac{ve^{2t}}{2}\right) \partial v$$

$$X_6 = (1) \partial t,$$

$$X_7 = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$\begin{split} X_8 &= \left(-\frac{5v}{8} + \frac{3u}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ve^{-4t}}{8} \right. \\ &+ \left. \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} \right. \\ &+ \left. -\frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left(-\frac{v^2}{2} + \frac{v^2e^{2t}}{2} +$$

$$X_9 = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_{10} = \left(-\frac{u}{8} - \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{-2t}}{8} + \frac{ve^{-4t}}{8} + \frac{3ve^{2t}}{8}\right) \partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{u^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} + \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right) \partial u + \left(-\frac{v^2}{2} + \frac{v^2e^{2t}}{2}\right) \partial v$$

$$\begin{split} X_{11} &= \left( -\frac{u}{8} + \frac{3v}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-2t}}{8} + \frac{ue^{-4t}}{8} \right. \\ &+ \left. \frac{ue^{2t}}{8} + \frac{ve^{-4t}}{8} \right) \partial t + \left( -\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} - \frac{v^2e^{2t}}{2} \right. \\ &+ \left. \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left( \frac{v^2}{2} - \frac{v^2e^{2t}}{2} \right) \partial v \end{split}$$

$$\begin{split} X_{12} = & \left( -\frac{e^{2t}}{4} + \frac{e^{-2t}}{4} \right) \partial t + \left( \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2} \right) \partial u \\ & + \left( \frac{v}{2} - \frac{ve^{2t}}{2} \right) \partial v \end{split}$$

$$\begin{split} X_{13} &= \left(\frac{u}{8} + \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} \right. \\ &+ \left. \frac{ue^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left( -\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-2t}}{2} \right. \\ &+ \left. \frac{v^2e^{2t}}{2} - \frac{v^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left( \frac{v^2}{2} + \frac{v^2e^{2t}}{2} \right) \partial v \end{split}$$

$$X_{14} = \left(-\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4}\right) \partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right) + \left(-\frac{ve^{2t}}{2}\right) \partial u + \left(-\frac{v}{2} - \frac{ve^{2t}}{2}\right) \partial v$$

$$X_{15} = \left(-\frac{3u}{8} + \frac{5v}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8}\right)$$

$$+ \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8} \partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{v^2e^{-2t}}{2}\right)$$

$$+ \frac{v^2e^{2t}}{2} - \frac{u^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \partial u + \left(\frac{v^2}{2} + \frac{v^2e^{2t}}{2}\right) \partial v$$

$$X_{16} = \left(-\frac{u^2}{4} - \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial t$$

$$X_{17} = \left(-\frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right)\partial u + \left(\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$\begin{split} X_{18} = & \left( \frac{u^2}{4} + \frac{v^2}{4} + \frac{v^2 e^{-2t}}{2} - \frac{uv}{2} - \frac{u^2 e^{-2t}}{2} + \frac{u^2 e^{-4t}}{4} \right. \\ & + \left. \frac{v^2 e^{-4t}}{4} + \frac{uv e^{-4t}}{2} \right) \partial t \end{split}$$

$$X_{19} = (u f_2(t) + v f_1(t) - v f_2(t) + f_3(t)) \partial t + (u f_3(t) + v f_3(t) + u^2 f_2(t) + v^2 f_1(t) - v^2 f_2(t) + uv f_1(t)) \partial u + (u f_3(t) + v f_3(t) + u^2 f_2(t) + v^2 f_1(t) - v^2 f_2(t) + uv f_1(t)) \partial v$$

 $f_1$   $f_2$   $f_3$ 

The execution time of the script was:

0 hours 3 minutes 4 seconds.

#### Run 08\_26PM\_29\_October-2021

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

$$X_1 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial u + \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial v$$

$$X_2 = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right) \partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right) \partial u$$

$$+ \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right) \partial v$$

$$\begin{split} X_3 &= \left(\frac{u}{8} + \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} \right. \\ &+ \left. \frac{ue^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left( -\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-2t}}{2} \right. \\ &+ \left. \frac{v^2e^{2t}}{2} - \frac{v^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left( \frac{v^2}{2} + \frac{v^2e^{2t}}{2} \right) \partial v \end{split}$$

$$\begin{split} X_4 &= \left( -\frac{5v}{8} + \frac{3u}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ve^{-4t}}{8} \right. \\ &+ \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left( -\frac{v^2}{4} + \frac{u^2}{4} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} \right. \\ &+ -\frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left( -\frac{v^2}{2} + \frac{v^2e^{2t}}{2} \right) \partial v \\ &+ \left. \frac{v^2e^{2t}}{2} \right) \partial v \\ &X_5 = \left( -\frac{1}{2} + \frac{e^{2t}}{2} \right) \partial u + \left( \frac{1}{2} + \frac{e^{2t}}{2} \right) \partial v \\ &X_6 = \left( \frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial t \\ &X_7 = \left( -\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4} \right) \partial t + \left( -\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} + \frac{ve^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial v \end{split}$$

$$X_8 = \left(-\frac{u^2}{4} - \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial t$$

$$X_{9} = \left(-\frac{3v}{8} + \frac{u}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-4t}}{8}\right)$$

$$+ -\frac{ve^{-2t}}{8} + \frac{ue^{2t}}{8}\right) \partial t + \left(-\frac{v^{2}}{4} + \frac{u^{2}}{4} - \frac{uv}{2} - \frac{v^{2}e^{2t}}{2}\right)$$

$$+ -\frac{u^{2}e^{-4t}}{4} - \frac{v^{2}e^{-4t}}{4} - \frac{uve^{-4t}}{2}\right) \partial u + \left(-\frac{v^{2}}{2} - \frac{v^{2}e^{2t}}{2}\right) \partial v$$

$$X_{10} = \left(\frac{u^{2}}{4} + \frac{v^{2}}{4} + \frac{u^{2}e^{-2t}}{2} - \frac{uv}{2} - \frac{v^{2}e^{-2t}}{2} + \frac{u^{2}e^{-4t}}{4}\right)$$

$$+ \frac{v^{2}e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right) \partial t$$

$$\begin{split} X_{11} &= \left( -\frac{3u}{8} + \frac{5v}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} \right. \\ &+ \left. \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left( -\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{v^2e^{-2t}}{2} \right. \\ &+ \left. \frac{v^2e^{2t}}{2} - \frac{u^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left( \frac{v^2}{2} + \frac{v^2e^{2t}}{2} \right) \partial v \end{split}$$

$$X_{12} = (1) \partial t$$
,

$$X_{13} = \left(-\frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right)\partial u + \left(\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$\begin{split} X_{14} &= \left( -\frac{u}{8} - \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{-2t}}{8} \right. \\ &\quad + \left. \frac{ve^{-4t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left( -\frac{v^2}{4} + \frac{u^2}{4} + \frac{u^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} \right. \\ &\quad + \left. -\frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left( -\frac{v^2}{2} + \frac{v^2e^{2t}}{2} \right) \partial v \end{split}$$

$$X_{15} = \left(-\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4}\right) \partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right) + \left(-\frac{ve^{2t}}{2}\right) \partial u + \left(-\frac{v}{2} - \frac{ve^{2t}}{2}\right) \partial v$$

$$X_{16} = \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{v^2 e^{-2t}}{2} - \frac{uv}{2} - \frac{u^2 e^{-2t}}{2} + \frac{u^2 e^{-4t}}{4} + \frac{v^2 e^{-4t}}{4} + \frac{uv e^{-4t}}{2}\right) \partial t$$

$$X_{17} = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$\begin{split} X_{18} &= \left(-\frac{u}{8} + \frac{3v}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-2t}}{8} + \frac{ue^{-4t}}{8} \right. \\ &+ \left. \frac{ue^{2t}}{8} + \frac{ve^{-4t}}{8} \right) \partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} - \frac{v^2e^{2t}}{2} \right. \\ &+ \left. \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left(\frac{v^2}{2} - \frac{v^2e^{2t}}{2} \right) \partial v \end{split}$$

$$X_{19} = (u f_2(t) + v f_1(t) - v f_2(t) + f_3(t)) \partial t + (u f_3(t) + v f_3(t) + u^2 f_2(t) + v^2 f_1(t) - v^2 f_2(t) + uv f_1(t)) \partial u + (u f_3(t) + v f_3(t) + u^2 f_2(t) + v^2 f_1(t) - v^2 f_2(t) + uv f_1(t)) \partial v$$

 $f_1$   $f_2$   $f_3$ 

The execution time of the script was:

0 hours 3 minutes 39 seconds.

#### Run 08\_27PM\_29\_October-2021

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

$$X_{1} = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right) \partial t$$

$$X_{2} = \left(-\frac{u}{8} + \frac{3v}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-2t}}{8} + \frac{ue^{-4t}}{8}\right)$$

$$+ \frac{ue^{2t}}{8} + \frac{ve^{-4t}}{8}\right) \partial t + \left(-\frac{u^{2}}{4} + \frac{v^{2}}{4} + \frac{uv}{2} - \frac{v^{2}e^{2t}}{2}\right)$$

$$+ \frac{u^{2}e^{-4t}}{4} + \frac{v^{2}e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right) \partial u + \left(\frac{v^{2}}{2} - \frac{v^{2}e^{2t}}{2}\right) \partial v$$

$$X_{3} = \left(-\frac{u^{2}}{4} - \frac{v^{2}}{4} + \frac{uv}{2} + \frac{u^{2}e^{-4t}}{4} + \frac{v^{2}e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right) \partial t$$

$$\begin{split} X_4 = \left( -\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4} \right) \partial t + \left( -\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} \right) \\ + -\frac{ve^{2t}}{2} \right) \partial u + \left( -\frac{v}{2} - \frac{ve^{2t}}{2} \right) \partial v \end{split}$$

$$X_5 = \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{v^2e^{-2t}}{2} - \frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial t$$

$$X_6 = \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial v$$
$$X_7 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial u + \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial v$$
$$X_8 = (1) \partial t,$$

$$\begin{split} X_9 &= \left(-\frac{5v}{8} + \frac{3u}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ve^{-4t}}{8} \right. \\ &+ \left. \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} \right. \\ &+ \left. -\frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left(-\frac{v^2}{2} + \frac{v^2e^{2t}}{2} +$$

$$X_{10} = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right) \partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right) \partial u + \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right) \partial v$$

$$\begin{split} X_{11} &= \left( -\frac{3v}{8} + \frac{u}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-4t}}{8} \right. \\ &+ \left. -\frac{ve^{-2t}}{8} + \frac{ue^{2t}}{8} \right) \partial t + \left( -\frac{v^2}{4} + \frac{u^2}{4} - \frac{uv}{2} - \frac{v^2e^{2t}}{2} \right. \\ &+ \left. -\frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left( -\frac{v^2}{2} - \frac{v^2e^{2t}}{2} \right) \partial v \end{split}$$

$$X_{12} = \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{u^2 e^{-2t}}{2} - \frac{uv}{2} - \frac{v^2 e^{-2t}}{2} + \frac{u^2 e^{-4t}}{4} + \frac{v^2 e^{-4t}}{4} + \frac{uv e^{-4t}}{2}\right) \partial t$$

$$\begin{split} X_{13} &= \left( -\frac{u}{8} - \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{-2t}}{8} \right. \\ &+ \left. \frac{ve^{-4t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left( -\frac{v^2}{4} + \frac{u^2}{4} + \frac{u^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} \right. \\ &+ \left. -\frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left( -\frac{v^2}{2} + \frac{v^2e^{2t}}{2} \right) \partial v \end{split}$$

$$\begin{split} X_{14} &= \left(\frac{u}{8} + \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} \right. \\ &+ \frac{ue^{-2t}}{8} + \frac{3ve^{2t}}{8} \left. \right) \partial t + \left( -\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-2t}}{2} \right. \\ &+ \frac{v^2e^{2t}}{2} - \frac{v^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left( \frac{v^2}{2} + \frac{v^2e^{2t}}{2} \right) \partial v \end{split}$$

$$X_{15} = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_{16} = \left(-\frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right)\partial u + \left(\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$X_{17} = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right) \partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right) + \frac{ve^{2t}}{2} \partial u + \left(-\frac{v}{2} + \frac{ve^{2t}}{2}\right) \partial v$$

$$\begin{split} X_{18} &= \left( -\frac{3u}{8} + \frac{5v}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} \right. \\ &+ \left. \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left( -\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{v^2e^{-2t}}{2} \right. \\ &+ \left. \frac{v^2e^{2t}}{2} - \frac{u^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left( \frac{v^2}{2} + \frac{v^2e^{2t}}{2} \right) \partial v \end{split}$$

$$\begin{split} X_{19} = & \left( u \, \mathrm{f}_2 \left( t \right) + v \, \mathrm{f}_1 \left( t \right) - v \, \mathrm{f}_2 \left( t \right) + \mathrm{f}_3 \left( t \right) \right) \partial t + \left( u \, \mathrm{f}_3 \left( t \right) + v \, \mathrm{f}_3 \left( t \right) \right. \\ & + \left. u^2 \, \mathrm{f}_2 \left( t \right) + v^2 \, \mathrm{f}_1 \left( t \right) - v^2 \, \mathrm{f}_2 \left( t \right) + u v \, \mathrm{f}_1 \left( t \right) \right) \partial u + \left( u \, \mathrm{f}_3 \left( t \right) + v \, \mathrm{f}_3 \left( t \right) \right. \\ & + \left. u^2 \, \mathrm{f}_2 \left( t \right) + v^2 \, \mathrm{f}_1 \left( t \right) - v^2 \, \mathrm{f}_2 \left( t \right) + u v \, \mathrm{f}_1 \left( t \right) \right) \partial v \end{split}$$

$$f_1$$
 $f_2$ 
 $f_3$ 

The execution time of the script was:

0 hours 1 minutes 30 seconds.

#### Run 08\_30PM\_29\_October-2021

Degree in tangential ansätze: 2. The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

$$\begin{split} X_1 &= \left( -\frac{e^{-2t}}{4} + \frac{e^{2t}}{4} \right) \partial t + \left( \frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} \right) \partial u \\ &+ \left( \frac{v}{2} + \frac{ve^{2t}}{2} \right) \partial v \\ \\ X_2 &= \left( \frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial t \\ \\ X_3 &= \left( \frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial t \\ \\ X_4 &= \left( -\frac{u}{8} - \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{-2t}}{8} \right) + \frac{ve^{-2t}}{8} \\ &+ \frac{ve^{-4t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left( -\frac{v^2}{4} + \frac{u^2}{4} + \frac{u^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} \right) + \frac{v^2e^{2t}}{2} \\ &+ -\frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left( -\frac{v^2}{2} + \frac{ve^{-2t}}{2} \right) \partial v \\ \\ X_5 &= \left( -\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4} \right) \partial t + \left( -\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial v \\ \\ X_6 &= \left( -\frac{1}{2} + \frac{e^{2t}}{2} \right) \partial u + \left( \frac{1}{2} + \frac{e^{2t}}{2} \right) \partial v \\ \\ X_7 &= \left( -\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4} \right) \partial t + \left( -\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} \right) \partial v \\ \\ X_7 &= \left( -\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4} \right) \partial t + \left( -\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} \right) \partial v \\ \\ \end{pmatrix}$$

$$\begin{split} X_8 &= \left(-\frac{5v}{8} + \frac{3u}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ve^{-4t}}{8} \right. \\ &+ \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8}\right) \partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} \right. \\ &+ \left. -\frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right) \partial u + \left(-\frac{v^2}{2} + \frac{v^2e^{2t}}{2}\right) \partial v \end{split}$$

$$\begin{split} X_9 &= \left(-\frac{3u}{8} + \frac{5v}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} \right. \\ &\quad + \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8} \left.\right) \partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{v^2e^{-2t}}{2} \right. \\ &\quad + \frac{v^2e^{2t}}{2} - \frac{u^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \left.\right) \partial u + \left(\frac{v^2}{2} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{-2t}}{2} - \frac{v^2e^{-2t}}{2} - \frac{v^2e^{-2t}}{4} - \frac{v^2e^{-2t}}{4} - \frac{v^2e^{-2t}}{2} - \frac{v^2e^{-2t}}{4} - \frac{v^2e^{-2t}}{4} - \frac{v^2e^{-2t}}{2} - \frac{v^2e^{-2t}}{4} - \frac{v^2e^{-2t}}{4} - \frac{v^2e^{-2t}}{4} - \frac{v^2e^{-2t}}{2} - \frac{v^2e^{-2t}}{4} -$$

$$X_{10} = \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{v^2 e^{-2t}}{2} - \frac{uv}{2} - \frac{u^2 e^{-2t}}{2} + \frac{u^2 e^{-4t}}{4} + \frac{v^2 e^{-4t}}{4} + \frac{uv e^{-4t}}{2}\right) \partial t$$

$$\begin{split} X_{11} = & \left( \frac{u^2}{4} + \frac{v^2}{4} + \frac{u^2 e^{-2t}}{2} - \frac{uv}{2} - \frac{v^2 e^{-2t}}{2} + \frac{u^2 e^{-4t}}{4} \right. \\ & + \left. \frac{v^2 e^{-4t}}{4} + \frac{uv e^{-4t}}{2} \right) \partial t \end{split}$$

$$X_{12} = (1) \partial t$$
,

$$\begin{split} X_{13} = & \left( -\frac{e^{2t}}{4} + \frac{e^{-2t}}{4} \right) \partial t + \left( \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2} \right) \partial u \\ & + \left( \frac{v}{2} - \frac{ve^{2t}}{2} \right) \partial v \end{split}$$

$$X_{14} = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_{15} = \left(-\frac{u^2}{4} - \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial t$$

$$\begin{split} X_{16} &= \left(\frac{u}{8} + \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} \right. \\ &+ \left. \frac{ue^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left( -\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-2t}}{2} \right. \\ &+ \left. \frac{v^2e^{2t}}{2} - \frac{v^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left( \frac{v^2}{2} + \frac{v^2e^{2t}}{2} \right) \partial v \end{split}$$

$$\begin{split} X_{17} &= \left( -\frac{u}{8} + \frac{3v}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-2t}}{8} + \frac{ue^{-4t}}{8} \right. \\ &+ \left. \frac{ue^{2t}}{8} + \frac{ve^{-4t}}{8} \right) \partial t + \left( -\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} - \frac{v^2e^{2t}}{2} \right. \\ &+ \left. \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left( \frac{v^2}{2} - \frac{v^2e^{2t}}{2} \right) \partial v \end{split}$$

$$\begin{split} X_{18} &= \left(-\frac{3v}{8} + \frac{u}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-4t}}{8} \right. \\ &+ \left. - \frac{ve^{-2t}}{8} + \frac{ue^{2t}}{8} \right) \partial t + \left( -\frac{v^2}{4} + \frac{u^2}{4} - \frac{uv}{2} - \frac{v^2e^{2t}}{2} \right. \\ &+ \left. - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left( -\frac{v^2}{2} - \frac{v^2e^{2t}}{2} \right) \partial v \end{split}$$

$$X_{19} = (u f_2(t) + v f_1(t) - v f_2(t) + f_3(t)) \partial t + (u f_3(t) + v f_3(t) + u^2 f_2(t) + v^2 f_1(t) - v^2 f_2(t) + uv f_1(t)) \partial u + (u f_3(t) + v f_3(t) + u^2 f_2(t) + v^2 f_1(t) - v^2 f_2(t) + uv f_1(t)) \partial v$$

 $f_1$   $f_2$   $f_3$ 

The execution time of the script was:

0 hours 2 minutes 20 seconds.