Summary of symmetry calculations

April 27, 2022

Contents

Chapter 1

DBH_model

Run 01_27PM_01_February-2022

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1 w_2 - w_1 w_3 + w_2 w_3,$$
$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1 w_2 + w_1 w_3 - w_2 w_3,$$
$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1 w_2 - w_1 w_3 - w_2 w_3.$$

The calculated generators are:

$$X_1 = (-t)\,\partial t + (w_1)\,\partial w_1 + (w_2)\,\partial w_2 + (w_3)\,\partial w_3,$$

$$X_2 = (1)\,\partial t,$$

$$X_3 = (1)\,\partial t,$$

$$X_4 = \left(t^2\right)\partial t + (1 - 2tw_1)\,\partial w_1 + (1 - 2tw_2)\,\partial w_2 + (1 + -2tw_3)\,\partial w_3$$

$$X_5 = (\mathrm{f}_1(t))\,\partial t + (w_2 w_3\,\mathrm{f}_1(t) - w_1 w_2\,\mathrm{f}_1(t) - w_1 w_3\,\mathrm{f}_1(t))\,\partial w_1 + (w_1 w_3\,\mathrm{f}_1(t) - w_1 w_2\,\mathrm{f}_1(t) + -w_2 w_3\,\mathrm{f}_1(t))\,\partial w_2 + (w_1 w_2\,\mathrm{f}_1(t) - w_1 w_3\,\mathrm{f}_1(t) - w_2 w_3\,\mathrm{f}_1(t))\,\partial w_3.$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 5 minutes 15 seconds.

Run 01_51PM_27_April-2022

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1 w_2 - w_1 w_3 + w_2 w_3,$$
$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1 w_2 + w_1 w_3 - w_2 w_3,$$
$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1 w_2 - w_1 w_3 - w_2 w_3.$$

The calculated generators are:

$$X_1 = (1)\,\partial t,$$

$$X_2 = \left(t^2\right)\partial t + (1 - 2tw_1)\,\partial w_1 + (1 - 2tw_2)\,\partial w_2 + (1 + -2tw_3)\,\partial w_3$$

$$X_3 = (-t)\,\partial t + (w_1)\,\partial w_1 + (w_2)\,\partial w_2 + (w_3)\,\partial w_3,$$

$$X_4 = (1)\,\partial t,$$

$$X_5 = (\mathrm{f}_1(t))\,\partial t + (w_2 w_3 \mathrm{f}_1(t) - w_1 w_2 \mathrm{f}_1(t) - w_1 w_3 \mathrm{f}_1(t))\,\partial w_1 + (w_1 w_3 \mathrm{f}_1(t) - w_1 w_2 \mathrm{f}_1(t) + -w_2 w_3 \mathrm{f}_1(t))\,\partial w_2 + (w_1 w_2 \mathrm{f}_1(t) - w_1 w_3 \mathrm{f}_1(t) - w_2 w_3 \mathrm{f}_1(t))\,\partial w_3.$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 3 minutes 34 seconds.

Chapter 2

LV

Run 01_28PM_01_February-2022

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u\left(1 - v\right),$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = av\left(u - 1\right).$$

The calculated generators are:

$$X_1 = \left(1\right)\partial t,$$

$$X_2 = \left(-1\right)\partial t,$$

$$X_3 = \left(\frac{\mathrm{f}_1\left(t\right)}{a}\right)\partial t + \left(\frac{u\,\mathrm{f}_1\left(t\right)}{a} - \frac{uv\,\mathrm{f}_1\left(t\right)}{a}\right)\partial u + \left(uv\,\mathrm{f}_1\left(t\right) - v\,\mathrm{f}_1\left(t\right)\right)\partial v.$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 0 minutes 41 seconds.

Run 01_52PM_27_April-2022

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u\left(1 - v\right),$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = av\left(u - 1\right).$$

The calculated generators are:

$$X_1 = \left(1\right)\partial t,$$

$$X_2 = \left(1\right)\partial t,$$

$$X_3 = \left(\frac{\mathrm{f}_1\left(t\right)}{a}\right)\partial t + \left(\frac{u\,\mathrm{f}_1\left(t\right)}{a} - \frac{uv\,\mathrm{f}_1\left(t\right)}{a}\right)\partial u + \left(uv\,\mathrm{f}_1\left(t\right) - v\,\mathrm{f}_1\left(t\right)\right)\partial v.$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 0 minutes 27 seconds.

Chapter 3

hydons_model

Run 01_20PM_01_February-2022

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$
$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

The calculated generators are:

$$X_1 = (t)\,\partial t + (y_1)\,\partial y_1 + (y_2)\,\partial y_2,$$

$$X_2 = \left(-t^2\,\mathrm{f}_1\,(t) + y_1 y_2\,\mathrm{f}_1\,(t)\right)\partial t + \left(y_2^2\,\mathrm{f}_1\,(t) + ty_1\,\mathrm{f}_1\,(t)\right)\partial y_1 + \left(y_1^2\,\mathrm{f}_1\,(t) + ty_2\,\mathrm{f}_1\,(t)\right)\partial y_2$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 0 minutes 27 seconds.

Run 01_44PM_27_April-2022

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$
$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

The calculated generators are:

$$X_1 = (t)\,\partial t + (y_1)\,\partial y_1 + (y_2)\,\partial y_2,$$

$$X_2 = \left(-t^2\,\mathrm{f}_1\,(t) + y_1 y_2\,\mathrm{f}_1\,(t)\right)\partial t + \left(y_2^2\,\mathrm{f}_1\,(t) + ty_1\,\mathrm{f}_1\,(t)\right)\partial y_1 + \left(y_1^2\,\mathrm{f}_1\,(t) + ty_2\,\mathrm{f}_1\,(t)\right)\partial y_2$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 0 minutes 41 seconds.

Chapter 4

SIR

Run 01_28PM_01_February-2022

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -ISr,$$
$$\frac{\mathrm{d}I}{\mathrm{d}t} = ISr - Ia,$$
$$\frac{\mathrm{d}R}{\mathrm{d}t} = Ia.$$

The calculated generators are:

$$X_1 = (I + R + S)\,\partial t$$

$$X_2 = (1)\,\partial R,$$

$$X_3 = (I + R + S)\,\partial R$$

$$X_4 = (1)\,\partial t.$$

The execution time of the script was:

0 hours 0 minutes 24 seconds.

Run 01_31PM_01_February-2022

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -ISr,$$
$$\frac{\mathrm{d}I}{\mathrm{d}t} = ISr - Ia,$$
$$\frac{\mathrm{d}R}{\mathrm{d}t} = Ia.$$

The calculated generators are:

$$X_1 = (1)\,\partial R,$$

$$X_2 = \left(\frac{I^2}{2} + \frac{R^2}{2} + \frac{S^2}{2} + IR + IS + RS\right)\partial t$$

$$X_3 = (I + R + S)\,\partial t$$

$$X_4 = (1)\,\partial t,$$

$$X_5 = (IS)\,\partial S + \left(-IS + \frac{Ia}{r}\right)\partial I + \left(\frac{Ra}{r} + \frac{Sa}{r}\right)\partial R$$

$$X_6 = \left(-\frac{1}{a}\right)\partial t + \left(\frac{ISr}{a}\right)\partial S + \left(I - \frac{ISr}{a}\right)\partial I + (R + S)\,\partial R$$

$$X_7 = \left(\frac{\mathrm{f}_1(t)}{a}\right)\partial t + \left(-\frac{ISr\,\mathrm{f}_1(t)}{a}\right)\partial S + \left(\frac{ISr\,\mathrm{f}_1(t)}{a} - I\,\mathrm{f}_1(t)\right)\partial I + (I\,\mathrm{f}_1(t))\,\partial R.$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

WARNING:
Some of the calculated generators did not satisfy the linearised symmetry conditions. Thus, the presented list here is not complete and consists exclusively of the calculated generators that satisfy the linearised symmetry conditions.

The execution time of the script was:

0 hours 2 minutes 58 seconds.

Run 01_52PM_27_April-2022

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -ISr,$$
$$\frac{\mathrm{d}I}{\mathrm{d}t} = ISr - Ia,$$
$$\frac{\mathrm{d}R}{\mathrm{d}t} = Ia.$$

The calculated generators are:

$$X_1 = (1)\,\partial R,$$

$$X_2 = (1)\,\partial t,$$

$$X_3 = (I + R + S)\,\partial t$$

$$X_4 = (I + R + S)\,\partial R$$

The execution time of the script was:

0 hours 0 minutes 14 seconds.

Run 01_54PM_27_April-2022

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -ISr,$$
$$\frac{\mathrm{d}I}{\mathrm{d}t} = ISr - Ia,$$
$$\frac{\mathrm{d}R}{\mathrm{d}t} = Ia.$$

The calculated generators are:

$$X_1 = (1)\, \partial R,$$

$$X_2 = \left(\frac{I^2}{2} + \frac{R^2}{2} + \frac{S^2}{2} + IR + IS + RS\right) \partial t$$

$$X_3 = \left(\frac{I^2}{2} + \frac{R^2}{2} + \frac{S^2}{2} + IR + IS + RS\right) \partial R$$

$$X_4 = (1)\, \partial t,$$

$$X_5 = \left(-\frac{1}{r}\right) \partial t,$$

$$X_6 = (I + R + S)\, \partial t$$

$$X_7 = \left(\frac{\mathrm{f}_1(t)}{a}\right) \partial t + \left(-\frac{ISr\, \mathrm{f}_1(t)}{a}\right) \partial S + \left(\frac{ISr\, \mathrm{f}_1(t)}{a} - I\, \mathrm{f}_1(t)\right) \partial I + \left(I\, \mathrm{f}_1(t)\right) \partial R.$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

WARNING:

Some of the calculated generators did not satisfy the linearised symmetry conditions. Thus, the presented list here is not complete and consists exclusively of the calculated generators that satisfy the linearised symmetry conditions.

The execution time of the script was:

0 hours 2 minutes 5 seconds.

Chapter 5

linear_model

Run 01_21PM_01_February-2022

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

The calculated generators are:

$$X_1 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(\frac{e^{2t}}{2} - \frac{1}{2}\right)\partial v,$$

$$X_2 = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right)\partial u + \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v$$

$$X_3 = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_4 = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_5 = (1)\,\partial t,$$

$$X_6 = \left(\frac{e^{2t}}{2} - \frac{1}{2}\right)\partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_7 = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} + \frac{ve^{2t}}{2}\right)\partial u + \left(\frac{ve^{2t}}{2} - \frac{v}{2}\right)\partial v,$$

$$X_8 = \left(-\frac{e^{2t}}{4} - \frac{1}{2} - \frac{e^{-2t}}{4}\right)\partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{2t}}{2} - \frac{ve^{-2t}}{2}\right)\partial u + \left(-\frac{ve^{2t}}{2} - \frac{v}{2}\right)\partial v,$$

$$X_9 = \left(\frac{e^{2t}}{4} - \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right)\partial u + \left(\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$X_{10} = (\mathrm{f}_1(t))\,\partial t + (u\,\mathrm{f}_1(t) + v\,\mathrm{f}_1(t))\,\partial u + (u\,\mathrm{f}_1(t) + v\,\mathrm{f}_1(t))\,\partial v$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 0 minutes 14 seconds.

Run 01_22PM_01_February-2022

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

The calculated generators are:

$$X_1 = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right)\partial u + \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v$$

$$X_2 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(\frac{e^{2t}}{2} - \frac{1}{2}\right)\partial v,$$

$$X_3 = (1)\,\partial t,$$

$$X_4 = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_5 = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right.$$
$$\left. + \frac{ve^{2t}}{2}\right)\partial u + \left(\frac{ve^{2t}}{2} - \frac{v}{2}\right)\partial v,$$

$$X_6 = \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{v^2e^{-2t}}{2} - \frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4}\right.$$
$$\left. + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial t$$

$$X_7 = \left(-\frac{3v}{8} + \frac{u}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-4t}}{8}\right.$$
$$\left. + -\frac{ve^{-2t}}{8} + \frac{ue^{2t}}{8}\right)\partial t + \left(\frac{u^2}{4} - \frac{u^2e^{-4t}}{4} - \frac{uv}{2} - \frac{uve^{-4t}}{2} - \frac{v^2e^{2t}}{2} - \frac{v^2}{4} - \frac{v^2e^{-4t}}{4}\right)\partial u + \left(-\frac{v^2e^{2t}}{2} - \frac{v^2}{2}\right)\partial v,$$

$$X_8 = \left(\frac{e^{2t}}{2} - \frac{1}{2}\right)\partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_9 = \left(\frac{u}{8} + \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8}\right.$$
$$\left. + \frac{ue^{-2t}}{8} + \frac{3ve^{2t}}{8}\right)\partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-2t}}{2}\right.$$
$$\left. + \frac{v^2e^{2t}}{2} - \frac{v^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2}\right)\partial u + \left(\frac{v^2}{2}\right.$$
$$\left. + \frac{v^2e^{2t}}{2}\right)\partial v$$

$$X_{10} = \left(-\frac{u}{8} + \frac{3v}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-2t}}{8} + \frac{ue^{-4t}}{8}\right.$$
$$\left. + \frac{ue^{2t}}{8} + \frac{ve^{-4t}}{8}\right)\partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} - \frac{v^2e^{2t}}{2}\right.$$
$$\left. + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial u + \left(\frac{v^2}{2} - \frac{v^2e^{2t}}{2}\right)\partial v$$

$$X_{11} = \Big(-\frac{3u}{8} + \frac{5v}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} + \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8}\Big)\partial t + \Big(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} - \frac{u^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2}\Big)\partial u + \Big(\frac{v^2}{2} + \frac{v^2e^{2t}}{2}\Big)\partial v$$

$$X_{12} = \Big(\frac{e^{2t}}{4} - \frac{e^{-2t}}{4}\Big)\partial t + \Big(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\Big)\partial u + \Big(\frac{v}{2} + \frac{ve^{2t}}{2}\Big)\partial v$$

$$X_{13} = \Big(-\frac{5v}{8} + \frac{3u}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ve^{-4t}}{8} + \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8}\Big)\partial t + \Big(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} + -\frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\Big)\partial u + \Big(\frac{v^2e^{2t}}{2} - \frac{v^2}{2}\Big)\partial v,$$

$$X_{14} = \Big(\frac{u^2}{4} + \frac{v^2}{4} + \frac{u^2e^{-2t}}{2} - \frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\Big)\partial t$$

$$X_{15} = \Big(-\frac{e^{2t}}{4} - \frac{1}{2} - \frac{e^{-2t}}{4}\Big)\partial t + \Big(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{2t}}{2} - \frac{ve^{-2t}}{2}\Big)\partial u + \Big(-\frac{ve^{2t}}{2} - \frac{v}{2}\Big)\partial v,$$

$$X_{16} = \Big(-\frac{u^2}{4} - \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\Big)\partial t$$

$$X_{17} = \Big(-\frac{u}{8} - \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{-2t}}{8} + \frac{ve^{-4t}}{8} + \frac{3ve^{2t}}{8}\Big)\partial t + \Big(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{u^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} + -\frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\Big)\partial u + \Big(\frac{v^2e^{2t}}{2} - \frac{v^2}{2}\Big)\partial v,$$

$$X_{18} = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right) \partial t$$

$$\begin{aligned} X_{19} = &(u\,\mathrm{f}_2(t) + v\,\mathrm{f}_1(t) - v\,\mathrm{f}_2(t) + \mathrm{f}_3(t))\,\partial t + (u\,\mathrm{f}_3(t) + v\,\mathrm{f}_3(t) \\ &+ u^2\,\mathrm{f}_2(t) + v^2\,\mathrm{f}_1(t) - v^2\,\mathrm{f}_2(t) + uv\,\mathrm{f}_1(t))\,\partial u + (u\,\mathrm{f}_3(t) + v\,\mathrm{f}_3(t) \\ &+ u^2\,\mathrm{f}_2(t) + v^2\,\mathrm{f}_1(t) - v^2\,\mathrm{f}_2(t) + uv\,\mathrm{f}_1(t))\,\partial v \end{aligned}$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$
$$\mathrm{f}_2$$
$$\mathrm{f}_3$$

The execution time of the script was:

0 hours 1 minutes 10 seconds.

Run 01_44PM_27_April-2022

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

The calculated generators are:

$$X_1 = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right) \partial t$$

$$\begin{aligned} X_2 = &\left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right) \partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right. \\ &\left. + \frac{ve^{2t}}{2}\right) \partial u + \left(\frac{ve^{2t}}{2} - \frac{v}{2}\right) \partial v, \end{aligned}$$

$$X_3 = \left(-\frac{e^{2t}}{4} - \frac{1}{2} - \frac{e^{-2t}}{4}\right) \partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{2t}}{2} - \frac{ve^{-2t}}{2}\right) \partial u + \left(-\frac{ve^{2t}}{2} - \frac{v}{2}\right) \partial v,$$

$$X_4 = \left(\frac{e^{2t}}{2} - \frac{1}{2}\right) \partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right) \partial v$$

$$X_5 = (1)\,\partial t,$$

$$X_6 = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_7 = \left(\frac{e^{2t}}{4} - \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right)\partial u + \left(\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$X_8 = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right)\partial u + \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v$$

$$X_9 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(\frac{e^{2t}}{2} - \frac{1}{2}\right)\partial v,$$

$$X_{10} = (\mathrm{f}_1(t))\,\partial t + (u\,\mathrm{f}_1(t) + v\,\mathrm{f}_1(t))\,\partial u + (u\,\mathrm{f}_1(t) + v\,\mathrm{f}_1(t))\,\partial v$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 0 minutes 11 seconds.

Run 01_48PM_27_April-2022

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

The calculated generators are:

$$X_1 = \left(-\frac{5v}{8} + \frac{3u}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ve^{-4t}}{8} \right.$$
$$+ \left. \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8}\right)\partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} \right.$$
$$+ \left. -\frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial u + \left(\frac{v^2e^{2t}}{2} - \frac{v^2}{2}\right)\partial v,$$

$$X_2 = \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{u^2e^{-2t}}{2} - \frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} \right.$$
$$+ \left. \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial t$$

$$X_3 = \left(-\frac{3u}{8} + \frac{5v}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} \right.$$
$$+ \left. \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8}\right)\partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{v^2e^{-2t}}{2} \right.$$
$$+ \left. \frac{v^2e^{2t}}{2} - \frac{u^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2}\right)\partial u + \left(\frac{v^2}{2} \right.$$
$$+ \left. \frac{v^2e^{2t}}{2}\right)\partial v$$

$$X_4 = \left(\frac{u}{8} + \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} \right.$$
$$+ \left. \frac{ue^{-2t}}{8} + \frac{3ve^{2t}}{8}\right)\partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-2t}}{2} \right.$$
$$+ \left. \frac{v^2e^{2t}}{2} - \frac{v^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2}\right)\partial u + \left(\frac{v^2}{2} \right.$$
$$+ \left. \frac{v^2e^{2t}}{2}\right)\partial v$$

$$X_5 = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_6 = \left(-\frac{u}{8} - \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{-2t}}{8} \right.$$
$$+ \left. \frac{ve^{-4t}}{8} + \frac{3ve^{2t}}{8}\right)\partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{u^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} \right.$$
$$+ \left. -\frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial u + \left(\frac{v^2e^{2t}}{2} - \frac{v^2}{2}\right)\partial v,$$

$$X_7 = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_8 = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} + \frac{ve^{2t}}{2}\right)\partial u + \left(\frac{ve^{2t}}{2} - \frac{v}{2}\right)\partial v,$$

$$X_9 = \left(-\frac{3v}{8} + \frac{u}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-4t}}{8} + -\frac{ve^{-2t}}{8} + \frac{ue^{2t}}{8}\right)\partial t + \left(\frac{u^2}{4} - \frac{u^2e^{-4t}}{4} - \frac{uv}{2} - \frac{uve^{-4t}}{2} - \frac{v^2e^{2t}}{2} - \frac{v^2}{4} - \frac{v^2e^{-4t}}{4}\right)\partial u + \left(-\frac{v^2e^{2t}}{2} - \frac{v^2}{2}\right)\partial v,$$

$$X_{10} = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(\frac{e^{2t}}{2} - \frac{1}{2}\right)\partial v,$$

$$X_{11} = (1)\,\partial t,$$

$$X_{12} = \left(\frac{e^{2t}}{4} - \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right)\partial u + \left(\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$X_{13} = \left(\frac{e^{2t}}{2} - \frac{1}{2}\right)\partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_{14} = \left(-\frac{e^{2t}}{4} - \frac{1}{2} - \frac{e^{-2t}}{4}\right)\partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{2t}}{2} - \frac{ve^{-2t}}{2}\right)\partial u + \left(-\frac{ve^{2t}}{2} - \frac{v}{2}\right)\partial v,$$

$$X_{15} = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right)\partial u + \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v$$

$$X_{16} = \Big(-\frac{u}{8} + \frac{3v}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{2t}}{8} + \frac{ve^{-4t}}{8}\Big)\partial t + \Big(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} - \frac{v^2e^{2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\Big)\partial u + \Big(\frac{v^2}{2} - \frac{v^2e^{2t}}{2}\Big)\partial v$$

$$X_{17} = \Big(\frac{u^2}{4} + \frac{v^2}{4} + \frac{v^2e^{-2t}}{2} - \frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\Big)\partial t$$

$$X_{18} = \Big(-\frac{u^2}{4} - \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\Big)\partial t$$

$$X_{19} = \left(u\,\mathrm{f}_2(t) + v\,\mathrm{f}_1(t) - v\,\mathrm{f}_2(t) + \mathrm{f}_3(t)\right)\partial t + \big(u\,\mathrm{f}_3(t) + v\,\mathrm{f}_3(t) + u^2\,\mathrm{f}_2(t) + v^2\,\mathrm{f}_1(t) - v^2\,\mathrm{f}_2(t) + uv\,\mathrm{f}_1(t)\big)\partial u + \big(u\,\mathrm{f}_3(t) + v\,\mathrm{f}_3(t) + u^2\,\mathrm{f}_2(t) + v^2\,\mathrm{f}_1(t) - v^2\,\mathrm{f}_2(t) + uv\,\mathrm{f}_1(t)\big)\partial v$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$
$$\mathrm{f}_2$$
$$\mathrm{f}_3$$

The execution time of the script was:

0 hours 3 minutes 19 seconds.

Chapter 6

Brusselator

Run 01_31PM_01_February-2022

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = au^2v - u(b+1) + 1,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = -au^2v + bu.$$

The calculated generators are:

$$X_1 = (1)\,\partial t.$$

The execution time of the script was:

0 hours 0 minutes 6 seconds.

Run 01_54PM_27_April-2022

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = au^2v - u(b+1) + 1,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = -au^2v + bu.$$

The calculated generators are:

$$X_1 = (1)\,\partial t.$$

The execution time of the script was:

0 hours 0 minutes 4 seconds.