benchmarking_DBH

July 21, 2021

1 Benchmarking the DBH model

So, now I have implemented a simple benchmarking of the script. Here, I will present the output from the terminal when the calculations with tangential ansätze of degree n=1 and n=2 respectively are run.

1.1 Tangential ansätze n=1

==
INITIATING CALCULATIONS WITH INPUT: model = DBH_model, tangent degree = 1.

```
Step 1 out of 6: Loading the model...
        The model was successfully loaded!
        Done!

Step 2 out of 6: Creating tangent ansätze...
Time elapsed:   0.01186 seconds
        Done!
Step 3 out of 6: Calculating the linearised symmetry conditions...
Time elapsed:   0.99791 seconds
        Done!
Step 4 out of 6: Deriving the determining equations...
Time elapsed:   23.92862        seconds
        Done!
Step 5 out of 6: Solving the determining equations...
                Matrix formulation:     0.08027 seconds
                Column space:   0.27542 seconds
                Matrix remove columns:  0.00056 seconds
                Jordan form:    0.97991 seconds
                Exponential matrix:     1.17438 seconds
Time elapsed:   2.68064 seconds
        Done!
Step 6 out of 6: Saving the data...
        Done!

Total time elapsed:     27.67395        seconds

The calculations are finished.
```

==

1.2 Tangential ansätze n=2

==
INITIATING CALCULATIONS WITH INPUT: model = DBH_model, tangent degree = 2.

```
                    Step 1 out of 6: Loading the model...
                            The model was successfully loaded!
                            Done!

                    Step 2 out of 6: Creating tangent ansätze...
                    Time elapsed:   0.03009 seconds
                            Done!
                    Step 3 out of 6: Calculating the linearised symmetry conditions...
                    Time elapsed:   3.18804 seconds
                            Done!
                    Step 4 out of 6: Deriving the determining equations...
                    Time elapsed:   67.46304        seconds
                            Done!
                    Step 5 out of 6: Solving the determining equations...
                                    Matrix formulation:     0.38972 seconds
                                    Column space:   2.86611 seconds
                                    Matrix remove columns:  0.00888 seconds
                                    Jordan form:    63.32136        seconds
                                    Exponential matrix:     72.05849        seconds
                    Time elapsed:   139.92284       seconds
                            Done!
                    Step 6 out of 6: Saving the data...
                            Done!

                    Total time elapsed:     210.65830       seconds

                    The calculations are finished.
```

==

1.3 Tangential ansätze n=3

==
INITIATING CALCULATIONS WITH INPUT: model = DBH_model, tangent degree = 3.

```
                    Step 1 out of 6: Loading the model...
                            The model was successfully loaded!
                            Done!

                    Step 2 out of 6: Creating tangent ansätze...
                    Time elapsed:   0.06521 seconds
            Done!
                    Step 3 out of 6: Calculating the linearised symmetry conditions...
```

```
Time elapsed:   9.71043 seconds
        Done!
Step 4 out of 6: Deriving the determining equations...
Time elapsed:   407.76120       seconds
        Done!
Step 5 out of 6: Solving the determining equations...
                Matrix formulation:     1.44245 seconds
                Column space:   17.51598        seconds
                Matrix remove columns:  0.06122 seconds
                Jordan form:    1848.86839      seconds
                Exponential matrix:     1665.44864      seconds
Time elapsed:   3580.76763      seconds
        Done!
Step 6 out of 6: Saving the data...
        Done!

Total time elapsed:     3998.36207      seconds

The calculations are finished.
=========================================================================
```

1.4 Summary statistics

It seems like when the ansätze get big enough implying that the matrices get big enough, then the computations get costly. So, let's make a small table in this cell where we compare six steps of the solution algorithm: 1. The total time, 2. The time to formulate the linearised symmetry conditions as a percentage of the total time, 3. The time to derive the determining equations as a percentage of the total time, 4. The the time to reduce the system by using the column space as a percentage of the total time, 5. The time to calculate the Jordan form as a percentage of the total time, 6. The time to calculate the exponential matrix as a percentage of the total time.

1.4.1 Table with the benchmarking results for the DBH model

Degree Tangent	Total time	Sym Cond (%)	Det Eq (%)	Col (%)	Jordan (%)	Exp (%)
2	3m31s	1.513	32.025	1.3605	30.059	34.206
3	66m38s	0.146	6.119	0.263	27.7444	24.992

DBH_tangential_ansatz_degree_1

July 21, 2021

1 # DBH model all details for the case where the degree of the tangential ansätze is 1

Date: 2021-07-20

Now there is a problem with the implementation of the symmetry algorithm applied to the DBH model which we will try to find. The DBH model is formulated as follows:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1w_2 - w_1w_3 + w_2w_3$$
$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1w_2 + w_1w_3 - w_2w_3$$
$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1w_2 - w_1w_3 - w_2w_3$$

So let's see what goes wrong here…

1.1 The tangential ansätze and the coefficients

The tangential ansätze (first order polynomial):

$$\xi = w_1\,\mathrm{c}_{01}(t) + w_2\,\mathrm{c}_{02}(t) + w_3\,\mathrm{c}_{03}(t) + \mathrm{c}_{00}(t)$$
$$\eta_1 = w_1\,\mathrm{c}_{11}(t) + w_2\,\mathrm{c}_{12}(t) + w_3\,\mathrm{c}_{13}(t) + \mathrm{c}_{10}(t)$$
$$\eta_2 = w_1\,\mathrm{c}_{21}(t) + w_2\,\mathrm{c}_{22}(t) + w_3\,\mathrm{c}_{23}(t) + \mathrm{c}_{20}(t)$$
$$\eta_3 = w_1\,\mathrm{c}_{31}(t) + w_2\,\mathrm{c}_{32}(t) + w_3\,\mathrm{c}_{33}(t) + \mathrm{c}_{30}(t)$$

The coefficients:

$$\mathbf{c} = \begin{bmatrix} c_{00} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{33} \\ c_{32} \\ c_{31} \end{bmatrix}$$

So, we have three ODEs, implying sixteen unknown coefficients which we want to find.

1.2 Determining equations

The determining equations are listed below.

We have 69 determining equations!

So we have a 69×16 system, and this should be our dimensions when we go over to matrix form!

$$-\frac{d}{dt}\,\mathrm{c}_{10}(t)=0$$
$$-\mathrm{c}_{10}(t)+\mathrm{c}_{20}(t)-\frac{d}{dt}\,\mathrm{c}_{13}(t)=0$$
$$-\mathrm{c}_{13}(t)+\mathrm{c}_{23}(t)=0$$
$$-\mathrm{c}_{10}(t)+\mathrm{c}_{30}(t)-\frac{d}{dt}\,\mathrm{c}_{12}(t)=0$$
$$-\mathrm{c}_{11}(t)+\mathrm{c}_{22}(t)+\mathrm{c}_{33}(t)+\frac{d}{dt}\,\mathrm{c}_{00}(t)=0$$
$$\frac{d}{dt}\,\mathrm{c}_{03}(t)=0$$
$$-\mathrm{c}_{12}(t)+\mathrm{c}_{32}(t)=0$$
$$\frac{d}{dt}\,\mathrm{c}_{02}(t)=0$$
$$\mathrm{c}_{01}(t)-\mathrm{c}_{02}(t)-\mathrm{c}_{03}(t)=0$$
$$-\mathrm{c}_{20}(t)-\mathrm{c}_{30}(t)-\frac{d}{dt}\,\mathrm{c}_{11}(t)=0$$
$$-\mathrm{c}_{12}(t)+\mathrm{c}_{13}(t)+\mathrm{c}_{21}(t)-\mathrm{c}_{23}(t)-\mathrm{c}_{33}(t)-\frac{d}{dt}\,\mathrm{c}_{00}(t)=0$$
$$-\frac{d}{dt}\,\mathrm{c}_{03}(t)=0$$
$$\mathrm{c}_{12}(t)-\mathrm{c}_{13}(t)-\mathrm{c}_{22}(t)+\mathrm{c}_{31}(t)-\mathrm{c}_{32}(t)-\frac{d}{dt}\,\mathrm{c}_{00}(t)=0$$
$$\frac{d}{dt}\,\mathrm{c}_{01}(t)-\frac{d}{dt}\,\mathrm{c}_{02}(t)-\frac{d}{dt}\,\mathrm{c}_{03}(t)=0$$
$$-2\,\mathrm{c}_{01}(t)+2\,\mathrm{c}_{02}(t)=0$$
$$-\frac{d}{dt}\,\mathrm{c}_{02}(t)=0$$
$$-2\,\mathrm{c}_{01}(t)+2\,\mathrm{c}_{03}(t)=0$$
$$-\mathrm{c}_{21}(t)-\mathrm{c}_{31}(t)=0$$
$$-\frac{d}{dt}\,\mathrm{c}_{01}(t)=0$$
$$\mathrm{c}_{01}(t)-\mathrm{c}_{02}(t)+\mathrm{c}_{03}(t)=0$$
$$-\frac{d}{dt}\,\mathrm{c}_{01}(t)=0$$
$$2\,\mathrm{c}_{01}(t)=0$$
$$\mathrm{c}_{01}(t)+\mathrm{c}_{02}(t)-\mathrm{c}_{03}(t)=0$$
$$-\frac{d}{dt}\,\mathrm{c}_{20}(t)=0$$
$$\mathrm{c}_{10}(t)-\mathrm{c}_{20}(t)-\frac{d}{dt}\,\mathrm{c}_{23}(t)=0$$
$$\mathrm{c}_{13}(t)-\mathrm{c}_{23}(t)=0$$
$$-\mathrm{c}_{10}(t)-\mathrm{c}_{30}(t)-\frac{d}{dt}\,\mathrm{c}_{22}(t)=0$$
$$\mathrm{c}_{12}(t)-\mathrm{c}_{13}(t)-\mathrm{c}_{21}(t)+\mathrm{c}_{23}(t)-\mathrm{c}_{33}(t)-\frac{d}{dt}\,\mathrm{c}_{00}(t)=0$$
$$-\frac{d}{dt}\,\mathrm{c}_{03}(t)=0$$
$$-\mathrm{c}_{12}(t)-\mathrm{c}_{32}(t)=0$$
$$\frac{d}{dt}\,\ldots(t)=0$$

1.3 Matrix formulation

Dimensions of A: (69, 16)
Dimensions of B: (69, 16)

$$
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
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0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0
\end{bmatrix}
$$

1.4 Reduced matrices

Dimensions of A: (27, 16)
Dimensions of B: (27, 16)

$$
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0
\end{bmatrix}
$$

1.5 Extracting algebraic equations

Dimensions of A: (16, 16) Dimensions of B: (16, 16) Dimensions of B_algebraic: (11, 16)

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$B_{\text{algebraic}} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

Look at that, we actually have a quadratic matrix directly! Yihaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa!

Let’s look at the Jordan form and the exponential matrix.

2 Jordan form and exponential matrix

Dimensions of J: (16, 16)

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Dimensions of J: (16, 16)

$$t \cdot J = \begin{bmatrix} 0 & t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Exponential matrix: 1.33032 seconds
Dimensions of J: (16, 16)

$$\exp(t \cdot J) = \begin{bmatrix} 1 & t & \frac{t^2}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Dimensions of sol: (16, 1)

$$c = \begin{bmatrix} C_1 - C_{14}t + \frac{C_5 t^2}{2} + \frac{C_9 t^2}{2} \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ -C_5 t + C_6 + C_9 t \\ C_{13}t - C_5 t + C_7 \\ -C_{13}t + C_8 - C_9 t \\ C_9 \\ C_{10} + C_5 t - C_9 t \\ C_{11} - C_{13}t - C_5 t \\ C_{12} + C_{13}t - C_9 t \\ C_{13} \\ C_{14} - C_5 t - C_9 t \\ -C_{13}t + C_{15} + C_5 t \\ -C_{13}t + C_{16} + C_9 t \end{bmatrix}$$

2.1 Algebraic equations

We need to define some of the arbitrary coefficients C_i in our solution c. To this end, we multiply the soution with the matrix B_{alg} in order to obtain some algebraic equations as follows:

$$B_{\text{algebraic}} c = 0.$$

Number of algebraic equations: (11, 1).

Algebraic equations:

$$
\begin{aligned}
C_2 &= 0 \\
C_3 &= 0 \\
C_4 &= 0 \\
-C_5t + C_6 + C_9t &= 0 \\
C_{13}t - C_5t + C_7 &= 0 \\
-C_{13}t - C_{14} + C_5t + C_8 &= 0 \\
C_{10} + C_5t - C_9t &= 0 \\
C_{11} - C_{13}t - C_{14} + C_9t &= 0 \\
C_{12} + C_{13}t - C_9t &= 0 \\
-C_{13}t + C_{15} + C_5t &= 0 \\
-C_{13}t + C_{16} + C_9t &= 0
\end{aligned}
$$

2.2 Solution after substitution

Solution after subsitution:

$$
c = \begin{bmatrix} C_1 - C_{14}t + \frac{C_5t^2}{2} + \frac{C_9t^2}{2} \\ 0 \\ 0 \\ 0 \\ C_5 \\ 0 \\ 0 \\ C_{14} - C_5t - C_9t \\ C_9 \\ 0 \\ C_{14} - C_5t - C_9t \\ 0 \\ C_{13} \\ C_{14} - C_5t - C_9t \\ 0 \\ 0 \end{bmatrix}.
$$

Arbitrary constants in final solution:

$$
\vec{C} = \begin{bmatrix} C_1 \\ C_5 \\ C_9 \\ C_{13} \\ C_{14} \end{bmatrix}
$$

Tangents after substitution Tangents after substitution:

$$\xi = C_1 - C_{14}t + \frac{C_5 t^2}{2} + \frac{C_9 t^2}{2}$$
$$\eta_1 = C_{14}w_1 - C_5 t w_1 + C_5 - C_9 t w_1$$
$$\eta_2 = C_{14}w_2 - C_5 t w_2 - C_9 t w_2 + C_9$$
$$\eta_3 = C_{13} + C_{14}w_3 - C_5 t w_3 - C_9 t w_3$$

[ ]:

```

```

DBH_tangential_ansatz_degree_2

July 21, 2021

1 DBH model all details for the case where the degree of the tangential ansätze is 2

Date: 2021-07-21

Now there is a problem with the implementation of the symmetry algorithm applied to the DBH model which we will try to find. The DBH model is formulated as follows:

$$\begin{aligned}
\frac{\mathrm{d}w_1}{\mathrm{d}t} &= -w_1w_2 - w_1w_3 + w_2w_3\\
\frac{\mathrm{d}w_2}{\mathrm{d}t} &= -w_1w_2 + w_1w_3 - w_2w_3\\
\frac{\mathrm{d}w_3}{\mathrm{d}t} &= w_1w_2 - w_1w_3 - w_2w_3
\end{aligned}$$

So let's see what goes wrong here...

1.1 The tangential ansätze and the coefficients

The tangential ansätze (second order polynomial):

$$\begin{aligned}
\xi &= w_1^2\,\mathrm{c}_{01}(t) + w_1w_2\,\mathrm{c}_{02}(t) + w_1w_3\,\mathrm{c}_{03}(t) + w_1\,\mathrm{c}_{04}(t) + w_2^2\,\mathrm{c}_{05}(t) + w_2w_3\,\mathrm{c}_{06}(t) + w_2\,\mathrm{c}_{07}(t) + w_3^2\,\mathrm{c}_{08}(t) + w_3\,\mathrm{c}_{09}\\
\eta_1 &= w_1^2\,\mathrm{c}_{11}(t) + w_1w_2\,\mathrm{c}_{12}(t) + w_1w_3\,\mathrm{c}_{13}(t) + w_1\,\mathrm{c}_{14}(t) + w_2^2\,\mathrm{c}_{15}(t) + w_2w_3\,\mathrm{c}_{16}(t) + w_2\,\mathrm{c}_{17}(t) + w_3^2\,\mathrm{c}_{18}(t) + w_3\,\mathrm{c}_{19}\\
\eta_2 &= w_1^2\,\mathrm{c}_{21}(t) + w_1w_2\,\mathrm{c}_{22}(t) + w_1w_3\,\mathrm{c}_{23}(t) + w_1\,\mathrm{c}_{24}(t) + w_2^2\,\mathrm{c}_{25}(t) + w_2w_3\,\mathrm{c}_{26}(t) + w_2\,\mathrm{c}_{27}(t) + w_3^2\,\mathrm{c}_{28}(t) + w_3\,\mathrm{c}_{29}\\
\eta_3 &= w_1^2\,\mathrm{c}_{31}(t) + w_1w_2\,\mathrm{c}_{32}(t) + w_1w_3\,\mathrm{c}_{33}(t) + w_1\,\mathrm{c}_{34}(t) + w_2^2\,\mathrm{c}_{35}(t) + w_2w_3\,\mathrm{c}_{36}(t) + w_2\,\mathrm{c}_{37}(t) + w_3^2\,\mathrm{c}_{38}(t) + w_3\,\mathrm{c}_{39}
\end{aligned}$$

The coefficients:

$$\mathbf{c} = \begin{bmatrix} c_{00} \\ c_{09} \\ c_{08} \\ c_{07} \\ c_{06} \\ c_{05} \\ c_{04} \\ c_{03} \\ c_{02} \\ c_{01} \\ c_{10} \\ c_{19} \\ c_{18} \\ c_{17} \\ c_{16} \\ c_{15} \\ c_{14} \\ c_{13} \\ c_{12} \\ c_{11} \\ c_{20} \\ c_{29} \\ c_{28} \\ c_{27} \\ c_{26} \\ c_{25} \\ c_{24} \\ c_{23} \\ c_{22} \\ c_{21} \\ c_{30} \\ c_{39} \\ c_{38} \\ c_{37} \\ c_{36} \\ c_{35} \\ c_{34} \\ c_{33} \\ c_{31} \end{bmatrix}.$$

We have 40 unknown coefficients!

1.2 Determining equations

The determining equations are listed below.

We have 132 determining equations!

$$-\mathrm{c}_{10}(t)+\mathrm{c}_{20}($$

$$-\mathrm{c}_{19}(t)+\mathrm{c}_{29}($$

$$-\mathrm{c}$$

$$-\mathrm{c}_{10}(t)+\mathrm{c}_{30}($$

$$-\mathrm{c}_{14}(t)+\mathrm{c}_{27}(t)+\mathrm{c}_{39}(t)+\frac{d}{dt}\mathrm{c}_{00}($$

$$-\mathrm{c}_{13}(t)+\mathrm{c}_{18}(t)+\mathrm{c}_{26}(t)+\mathrm{c}_{38}($$

$$-\mathrm{c}_{17}(t)+\mathrm{c}_{37}($$

$$-\mathrm{c}_{12}(t)+\mathrm{c}_{15}(t)+\mathrm{c}_{25}(t)+\mathrm{c}_{36}($$

$$\mathrm{c}_{04}(t)-\mathrm{c}_{07}(t)-\mathrm{c}_{09}($$

$$\mathrm{c}_{03}(t)-\mathrm{c}_{06}$$

$$-\mathrm{c}$$

$$\mathrm{c}_{02}(t)-2\,\mathrm{c}$$

$$-\mathrm{c}_{20}(t)-\mathrm{c}_{30}($$

$$-\mathrm{c}_{17}(t)+\mathrm{c}_{19}(t)+\mathrm{c}_{24}(t)-\mathrm{c}_{29}(t)-\mathrm{c}_{39}(t)-\frac{d}{dt}\mathrm{c}_{00}($$

$$-\mathrm{c}_{16}(t)+2\,\mathrm{c}_{18}(t)+\mathrm{c}_{23}(t)-\mathrm{c}_{28}(t)-\mathrm{c}_{38}($$

$$\mathrm{c}_{17}(t)-\mathrm{c}_{19}(t)-\mathrm{c}_{27}(t)+\mathrm{c}_{34}(t)-\mathrm{c}_{37}(t)-\frac{d}{dt}\mathrm{c}_{00}($$

$$-2\,\mathrm{c}_{11}(t)+\mathrm{c}_{12}(t)+\mathrm{c}_{13}(t)-2\,\mathrm{c}_{15}(t)+2\,\mathrm{c}_{16}(t)-2\,\mathrm{c}_{18}(t)+\mathrm{c}_{22}(t)-\mathrm{c}_{26}(t)+\mathrm{c}_{33}(t)-\mathrm{c}_{36}(t)+\frac{d}{dt}\mathrm{c}_{04}(t)-\frac{d}{dt}\mathrm{c}_{07}($$

$$-2\,\mathrm{c}_{04}(t)+2\,\mathrm{c}_{07}(t)+\frac{d}{dt}\mathrm{c}_{03}(t)-\frac{d}{dt}\mathrm{c}_{06}($$

$$-2\,\mathrm{c}_{0}$$

$$2\,\mathrm{c}_{15}(t)-\mathrm{c}_{16}(t)-\mathrm{c}_{25}(t)+\mathrm{c}_{32}(t)-\mathrm{c}_{35}($$

$$-2\,\mathrm{c}_{04}(t)+2\,\mathrm{c}_{09}(t)+\frac{d}{dt}\mathrm{c}_{02}(t)-\frac{d}{dt}\mathrm{c}_{05}($$

$$2\,\mathrm{c}_{01}(t)-3\,\mathrm{c}_{02}(t)-3\,\mathrm{c}_{03}(t)+4\,\mathrm{c}_{05}$$

$$-2\,\mathrm{c}_{02}$$

$$-\mathrm{c}_{24}(t)-\mathrm{c}_{34}($$

So we have a 132×40 system, and this should be our dimensions when we go over to matrix form!

1.3 Matrix formulation

Dimensions of A: (132, 40)
Dimensions of B: (132, 40)

$$
\left[\begin{matrix}
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1.4 Reduced matrices

Dimensions of A: (62, 40)
Dimensions of B: (62, 40)

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1.5 Extracting algebraic equations

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\end{array}\right.
$$

2.1 Algebraic equations

Dimensions of sol: (39, 1)

$$c = \begin{bmatrix} C_1 - C_{17}t - C_{24}t + C_{31}t^2 + C_{32}t \\ C_2 \\ C_3 \\ C_4 \\ C_2t + C_4t + C_5 - C_7t \\ C_6 \\ C_7 \\ C_2t - C_4t + C_7t + C_8 \\ -C_2t + C_4t + C_7t + C_9 \\ C_{10} \\ C_{11} \\ -C_{11}t + C_{12} + C_{21}t \\ C_{11}t^2 - C_{12}t + C_{13} - C_{21}t^2 + C_{22}t \\ -C_{11}t + C_{14} + C_{31}t \\ -C_{11}t^2 + C_{15} - 2C_{17}t + C_{31}t^2 + 2C_{32}t \\ C_{11}t^2 - C_{14}t + C_{16} - C_{31}t^2 + C_{34}t \\ C_{17} - C_{21}t - C_{31}t \\ C_{12}t - C_{14}t + C_{17}t + C_{18} + C_{21}t^2 - C_{22}t + C_{24}t + C_{27}t - C_{31}t^2 - 2C_{32}t \\ -C_{12}t + C_{14}t + C_{17}t + C_{19} - C_{32}t - C_{34}t + C_{37}t \\ C_{20} - C_{27}t - C_{37}t \\ C_{21} \\ C_{11}t - C_{21}t + C_{22} \\ -C_{11}t^2 + C_{12}t + C_{21}t^2 - C_{22}t + C_{23} \\ -C_{11}t + C_{24} - C_{31}t \\ C_{11}t^2 - C_{12}t + C_{14}t + C_{17}t + C_{22}t + C_{24}t + C_{25} - C_{27}t - C_{31}t^2 - 2C_{32}t \\ -C_{14}t + C_{26} - C_{34}t \\ -C_{21}t + C_{27} + C_{31}t \\ -C_{21}t^2 - 2C_{24}t + C_{28} + C_{31}t^2 + 2C_{32}t \\ -C_{22}t + C_{24}t + C_{27}t + C_{29} - C_{32}t + C_{34}t - C_{37}t \\ C_{21}t^2 - C_{27}t + C_{30} - C_{31}t^2 + C_{37}t \\ C_{31} \\ -C_{11}t - C_{21}t + C_{32} \\ -C_{12}t - C_{22}t + C_{33} \\ C_{11}t - C_{31}t + C_{34} \\ C_{11}t^2 + C_{12}t - C_{14}t + C_{17}t - C_{31}t^2 - C_{32}t + C_{34}t + C_{35} - C_{37}t \\ -C_{11}t^2 + C_{14}t + C_{31}t^2 - C_{34}t + C_{36} \\ C_{21}t - C_{31}t + C_{37} \\ C_{21}t^2 + C_{22}t + C_{24}t - C_{27}t - C_{31}t^2 - C_{32}t - C_{34}t + C_{37}t + C_{38} \\ -C_{21}t^2 + C_{27}t + C_{31}t^2 - C_{37}t + C_{39} \end{bmatrix}$$

Number of algebraic equations: (23, 1)

$$C_3 = 0$$
$$C_2t + C_4t + C_5 - C_7t = 0$$
$$C_6 = 0$$
$$C_2t - C_4t + C_7t + C_8 = 0$$
$$-C_2t + C_4t + C_7t + C_9 = 0$$
$$C_{10} = 0$$
$$C_{11}t^2 - C_{12}t + C_{13} - C_{21}t^2 + C_{22}t = 0$$
$$-C_{11}t^2 + C_{15} - 2C_{17}t + C_{31}t^2 + 2C_{32}t = 0$$
$$C_{11}t^2 - C_{14}t + C_{16} - C_{31}t^2 + C_{34}t = 0$$
$$C_{12}t - C_{14}t + C_{17}t + C_{18} + C_{21}t^2 - C_{22}t + C_{24}t + C_{27}t - C_{31}t^2 - 2C_{32}t = 0$$
$$-C_{12}t + C_{14}t + C_{17}t + C_{19} - C_{32}t - C_{34}t + C_{37}t = 0$$
$$C_{20} - C_{27}t - C_{37}t = 0$$
$$-C_{11}t^2 + C_{12}t + C_{21}t^2 - C_{22}t + C_{23} = 0$$
$$C_{11}t^2 - C_{12}t + C_{14}t + C_{17}t + C_{22}t + C_{24}t + C_{25} - C_{27}t - C_{31}t^2 - 2C_{32}t = 0$$
$$-C_{14}t + C_{26} - C_{34}t = 0$$
$$-C_{21}t^2 - 2C_{24}t + C_{28} + C_{31}t^2 + 2C_{32}t = 0$$
$$-C_{22}t + C_{24}t + C_{27}t + C_{29} - C_{32}t + C_{34}t - C_{37}t = 0$$
$$C_{21}t^2 - C_{27}t + C_{30} - C_{31}t^2 + C_{37}t = 0$$
$$-C_{12}t - C_{22}t + C_{33} = 0$$
$$C_{11}t^2 + C_{12}t - C_{14}t + C_{17}t - C_{31}t^2 - C_{32}t + C_{34}t + C_{35} - C_{37}t = 0$$
$$-C_{11}t^2 + C_{14}t + C_{31}t^2 - C_{34}t + C_{36} = 0$$
$$C_{21}t^2 + C_{22}t + C_{24}t - C_{27}t - C_{31}t^2 - C_{32}t - C_{34}t + C_{37}t + C_{38} = 0$$
$$-C_{21}t^2 + C_{27}t + C_{31}t^2 - C_{37}t + C_{39} = 0$$

2.2 Solution after substitution

Solution after subsitution:

$$c = \begin{bmatrix} C_1 - C_{17}t - C_{24}t + C_{31}t^2 + C_{32}t \\ C_2 \\ 0 \\ C_4 \\ 0 \\ 0 \\ C_7 \\ 0 \\ 0 \\ 0 \\ C_{11} \\ -C_{11}t + C_{12} + C_{21}t \\ 0 \\ -C_{11}t + C_{14} + C_{31}t \\ 0 \\ 0 \\ C_{17} - C_{21}t - C_{31}t \\ 0 \\ 0 \\ 0 \\ C_{21} \\ C_{11}t - C_{21}t + C_{22} \\ 0 \\ -C_{11}t + C_{24} - C_{31}t \\ 0 \\ 0 \\ -C_{21}t + C_{27} + C_{31}t \\ 0 \\ 0 \\ 0 \\ C_{31} \\ -C_{11}t - C_{21}t + C_{32} \\ 0 \\ C_{11}t - C_{31}t + C_{34} \\ 0 \\ 0 \\ C_{21}t - C_{31}t + C_{37} \\ 0 \\ 0 \end{bmatrix}$$

Arbitrary constants in final solution:

$$\vec{C} = \begin{bmatrix} C_1 \\ C_2 \\ C_4 \\ C_7 \\ C_{11} \\ C_{12} \\ C_{14} \\ C_{17} \\ C_{21} \\ C_{22} \\ C_{24} \\ C_{27} \\ C_{31} \\ C_{32} \\ C_{34} \\ C_{37} \end{bmatrix}$$

2.3 Tangents after substitution

Tangents after substitution:

$$\begin{aligned}
\xi &= C_1 - C_{17}t + C_2w_3 - C_{24}t + C_{31}t^2 + C_{32}t + C_4w_2 + C_7w_1 \\
\eta_1 &= -C_{11}tw_2 - C_{11}tw_3 + C_{11} + C_{12}w_3 + C_{14}w_2 + C_{17}w_1 - C_{21}tw_1 + C_{21}tw_3 - C_{31}tw_1 + C_{31}tw_2 \\
\eta_2 &= -C_{11}tw_2 + C_{11}tw_3 - C_{21}tw_1 - C_{21}tw_3 + C_{21} + C_{22}w_3 + C_{24}w_2 + C_{27}w_1 + C_{31}tw_1 - C_{31}tw_2 \\
\eta_3 &= C_{11}tw_2 - C_{11}tw_3 + C_{21}tw_1 - C_{21}tw_3 - C_{31}tw_1 - C_{31}tw_2 + C_{31} + C_{32}w_3 + C_{34}w_2 + C_{37}w_1
\end{aligned}$$