Summary of symmetry calculations

October 21, 2021

Contents

Chapter 1

DBH_model

Run 03_24PM_21_October-2021

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\begin{aligned}\frac{\mathrm{d}w_1}{\mathrm{d}t} &= -w_1w_2 - w_1w_3 + w_2w_3,\\ \frac{\mathrm{d}w_2}{\mathrm{d}t} &= -w_1w_2 + w_1w_3 - w_2w_3,\\ \frac{\mathrm{d}w_3}{\mathrm{d}t} &= w_1w_2 - w_1w_3 - w_2w_3.\end{aligned}$$

The calculated generators are:

$$X_1 = (-1+t)\,\partial t + (w_1)\,\partial w_1 + (w_2)\,\partial w_2 + (w_3)\,\partial w_3,$$

$$X_2 = (1)\,\partial t,$$

$$\begin{aligned}X_3 = {} & (t+2)\,\partial t + (1-2tw_1)\,\partial w_1 + (1-2tw_2)\,\partial w_2\\ & + (1-2tw_3)\,\partial w_3\end{aligned}$$

The execution time of the script was:

0 hours 0 minutes 42 seconds.

Run 04_17PM_21_October-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1w_2 - w_1w_3 + w_2w_3,$$
$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1w_2 + w_1w_3 - w_2w_3,$$
$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1w_2 - w_1w_3 - w_2w_3.$$

The calculated generators are:

$$X_1 = (t+2)\,\partial t + (1-2tw_1)\,\partial w_1 + (1-2tw_2)\,\partial w_2 + (1-2tw_3)\,\partial w_3$$

$$X_2 = (1)\,\partial t,$$

$$X_3 = (t)\,\partial t + (w_2w_3\,\mathrm{f}_1(t) - w_1w_2\,\mathrm{f}_1(t) - w_1w_3\,\mathrm{f}_1(t))\,\partial w_1 + (w_1w_3\,\mathrm{f}_1(t) - w_1w_2\,\mathrm{f}_1(t) + -w_2w_3\,\mathrm{f}_1(t))\,\partial w_2 + (w_1w_2\,\mathrm{f}_1(t) - w_1w_3\,\mathrm{f}_1(t) - w_2w_3\,\mathrm{f}_1(t))\,\partial w_3$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 4 minutes 55 seconds.

Chapter 2

linear_model

Run 03_22PM_21_October-2021

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\begin{aligned}\frac{\mathrm{d}u}{\mathrm{d}t} &= u + v,\\ \frac{\mathrm{d}v}{\mathrm{d}t} &= u + v.\end{aligned}$$

The calculated generators are:

$$\begin{aligned}X_1 = &\left(-\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right.\\ &\left. + -\frac{ve^{2t}}{2}\right)\partial u + \left(-\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v\end{aligned}$$

$$X_2 = \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$\begin{aligned}X_3 = &\left(-\frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right)\partial u\\ &+ \left(\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v\end{aligned}$$

$$X_4 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$\begin{aligned}X_5 = &\left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right)\partial u\\ &+ \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v\end{aligned}$$

$$X_6 = (1)\,\partial t,$$

$$X_7 = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_8 = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_9 = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} + \frac{ve^{2t}}{2}\right)\partial u + \left(-\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$X_{10} = (t)\,\partial t + (u\,\mathrm{f}_1(t) + v\,\mathrm{f}_1(t))\,\partial u + (u\,\mathrm{f}_1(t) + v\,\mathrm{f}_1(t))\,\partial v$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 0 minutes 12 seconds.

Run 03_23PM_21_October-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

The calculated generators are:

$$X_1 = \left(-\frac{5v}{8} + \frac{3u}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ve^{-4t}}{8} + \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8}\right)\partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} + -\frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial u + \left(-\frac{v^2}{2} + \frac{v^2e^{2t}}{2}\right)\partial v$$

$$X_2 = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_3 = \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{u^2e^{-2t}}{2} - \frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial t$$

$$X_4 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_5 = \left(-\frac{3u}{8} + \frac{5v}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} + \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8}\right)\partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} - \frac{u^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2}\right)\partial u + \left(\frac{v^2}{2} + \frac{v^2e^{2t}}{2}\right)\partial v$$

$$X_6 = \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_7 = \left(\frac{u}{8} + \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} + \frac{ue^{-2t}}{8} + \frac{3ve^{2t}}{8}\right)\partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} - \frac{v^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2}\right)\partial u + \left(\frac{v^2}{2} + \frac{v^2e^{2t}}{2}\right)\partial v$$

$$X_8 = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right)\partial u + \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v$$

$$X_9 = \left(-\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} + -\frac{ve^{2t}}{2}\right)\partial u + \left(-\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v$$

$$X_{10} = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} + \frac{ve^{2t}}{2}\right)\partial u + \left(-\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$X_{11} = \left(-\frac{u}{8} + \frac{3v}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{2t}}{8} + \frac{ve^{-4t}}{8}\right)\partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} - \frac{v^2e^{2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial u + \left(\frac{v^2}{2} - \frac{v^2e^{2t}}{2}\right)\partial v$$

$$X_{12} = \left(-\frac{3v}{8} + \frac{u}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-4t}}{8} + -\frac{ve^{-2t}}{8} + \frac{ue^{2t}}{8}\right)\partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} - \frac{uv}{2} - \frac{v^2e^{2t}}{2} + -\frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2}\right)\partial u + \left(-\frac{v^2}{2} - \frac{v^2e^{2t}}{2}\right)\partial v$$

$$X_{13} = (1)\,\partial t,$$

$$X_{14} = \left(-\frac{u^2}{4} - \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial t$$

$$X_{15} = \left(-\frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right)\partial u + \left(\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$X_{16} = \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{v^2e^{-2t}}{2} - \frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial t$$

$$X_{17} = \left(-\frac{u}{8} - \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{-2t}}{8} + \frac{ve^{-4t}}{8} + \frac{3ve^{2t}}{8}\right)\partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{u^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} + -\frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial u + \left(-\frac{v^2}{2} + \frac{v^2e^{2t}}{2}\right)\partial v$$

$$X_{18} = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial t$$

$$\begin{aligned} X_{19} = & \left(u\,\mathrm{f}_2(t) + v\,\mathrm{f}_1(t) - v\,\mathrm{f}_2(t) + \mathrm{f}_3(t) \right) \partial t + \left(u\,\mathrm{f}_3(t) + v\,\mathrm{f}_3(t) \right. \\ & + u^2\,\mathrm{f}_2(t) + v^2\,\mathrm{f}_1(t) - v^2\,\mathrm{f}_2(t) + uv\,\mathrm{f}_1(t) \Big) \partial u + \left(u\,\mathrm{f}_3(t) + v\,\mathrm{f}_3(t) \right. \\ & + u^2\,\mathrm{f}_2(t) + v^2\,\mathrm{f}_1(t) - v^2\,\mathrm{f}_2(t) + uv\,\mathrm{f}_1(t) \Big) \partial v \end{aligned}$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$
$$\mathrm{f}_2$$
$$\mathrm{f}_3$$

The execution time of the script was:

0 hours 1 minutes 2 seconds.

Chapter 3

hydons_model

Run 03_21PM_21_October-2021

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$
$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

The calculated generators are:

$$X_1 = (t)\,\partial t + (y_1)\,\partial y_1 + (y_2)\,\partial y_2.$$

The execution time of the script was:

0 hours 0 minutes 6 seconds.

Run 03_22PM_21_October-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$
$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

The calculated generators are:

$$X_1 = (t)\,\partial t + (y_1)\,\partial y_1 + (y_2)\,\partial y_2,$$

$$X_2 = \left(-t^2 \,\mathrm{f}_1\left(t\right) + y_1 y_2 \,\mathrm{f}_1\left(t\right)\right) \partial t + \left(y_2^2 \,\mathrm{f}_1\left(t\right) + t y_1 \,\mathrm{f}_1\left(t\right)\right) \partial y_1 + \left(y_1^2 \,\mathrm{f}_1\left(t\right) + t y_2 \,\mathrm{f}_1\left(t\right)\right) \partial y_2$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 0 minutes 17 seconds.