Summary of symmetry calculations

November 3, 2021

Contents

Chapter 1

Lotka_Volterra

Run 11_58AM_03_November-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = N\left(-Pb + a\right),$$
$$\frac{\mathrm{d}P}{\mathrm{d}t} = P\left(Nc - d\right).$$

The calculated generators are:

$$X_1 = (1)\,\partial t,$$

$$X_2 = \left(\frac{1}{c} + \mathrm{f}_1(t)\right)\partial t + \left(\frac{Na\,\mathrm{f}_1(t)}{c} - \frac{NPb\,\mathrm{f}_1(t)}{c}\right)\partial N + \left(NP\,\mathrm{f}_1(t) - \frac{Pd\,\mathrm{f}_1(t)}{c}\right)\partial P$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

WARNING:

Some of the calculated generators did not satisfy the linearised symmetry conditions. Thus, the presented list here is not complete and consists exclusively of the calculated generators that satisfy the linearised symmetry conditions.

Equation:$-C_3 + \frac{C_6 b^2 e^{-2at}}{c^2} - \frac{C_6 b^2 d}{ac^2} + \frac{C_6 b^2 d e^{-2at}}{ac^2} = 0$ Basis functions:

$$\left[1.0,\ e^{-2at}\right]$$

$$C_6 = 0$$
$$C_3 = -\frac{C_6 b^2 d}{ac^2}$$

Equation:$-\frac{C_{15} a c e^{2dt}}{2ad^2+4d^3} - \frac{C_{15} c d e^{2dt}}{2ad^2+4d^3} + \frac{C_{15} c}{a^2 e^{at} + 2ade^{at}} + \frac{C_{15} c e^{dt}}{d^2} - \frac{C_{15} c}{2d^2} - \frac{C_{15} c}{2ad} + \frac{C_2 c e^{dt}}{d} - \frac{C_2 c}{d} - \frac{C_4 b}{a} + \frac{C_4 b e^{-at}}{a} - C_5 + \frac{C_7 abc^2 e^{2dt}}{2a^3d^2 - 2a^2d^3 - 8ad^4 + 8d^5} - \frac{C_7 abc^2 e^{at}}{2a^4 d - 6a^3 d^2 + 4a^2 d^3} + \frac{C_7 bc^2 d e^{2dt}}{2a^3d^2 - 2a^2d^3 - 8ad^4 + 8d^5} - \frac{C_7 bc^2 d e^{at}}{2a^4 d - 6a^3 d^2 + 4a^2 d^3} - \frac{C_7 bc^2}{4a^4 e^{at} + 10a^3 d e^{at} + 4a^2 d^2 e^{at}} + \frac{C_7 bc^2 e^{at} e^{dt}}{2a^3 d - a^2 d^2 - ad^3} - \frac{C_7 bc^2 e^{dt}}{a^2 d^2 - ad^3} + \frac{C_7 bc^2}{2a^2 d^2} + \frac{C_8 ac^2 e^{2dt}}{2a^2 d^2 + 2ad^3 - 4d^4} + \frac{C_8 c^2 d e^{2dt}}{2a^2 d^2 + 2ad^3 - 4d^4} - \frac{C_8 c^2}{2a^3 e^{at} + 5a^2 d e^{at} + 2ad^2 e^{at}} - \frac{C_8 c^2 e^{at} e^{dt}}{2a^3 - a^2 d - ad^2} - \frac{C_8 c^2 e^{dt}}{ad^2} + \frac{C_8 c^2}{2ad^2} = 0$ Basis functions:

$$\left[e^{dt},\ 1.0,\ e^{at} e^{dt},\ e^{2dt},\ e^{-at},\ e^{at}\right]$$

$$C_2 = \frac{-2C_{15}a^3 + C_{15}a^2 d + C_{15}ad^2 + 2C_7 abc - C_7 bcde^{at} + C_7 bcd + 2C_8 a^2 c - C_8 acd + C_8 cd^2 e^{at} - C_8 cd^2}{ad\left(2a^2 - ad - d^2\right)}$$
$$C_7 = \frac{C_8 d}{b}$$
$$C_7 = \frac{C_{15}a^2 - 3C_{15}ad + 2C_{15}d^2 - C_8 ac + 2C_8 cd}{bc}$$
$$C_4 = 0$$
$$C_7 = \frac{2C_8 ad\left(a - 2d\right) e^{dt}}{b\left(2a^2 e^{dt} - 2a^2 - 4ade^{dt} - 3ad - d^2\right)}$$
$$C_2 = \frac{\Big(-C_{15}a^5 c e^{at} - \frac{5C_{15}a^4 cde^{at}}{2} + 2C_{15}a^3 cd^2 + \frac{5C_{15}a^2 cd^3 e^{at}}{2} - C_{15}a^2 cd^3 + C_{15}acd^4 e^{at} - C_{15}acd^4 - 2C_4 a^4 bd^2 e^{at} - 3C_4 a^3 bd^3 e}{}$$

Equation:$-\frac{C_8 abcde^{at}e^{dt}}{a^2cd - cd^3} + \frac{C_8 abce^{at}e^{dt}}{a^2c - cd^2} + \frac{C_8 abde^{at}}{a^2 d - 2ad^2} + \frac{C_8 bcd^2 e^{at} e^{dt}}{a^2cd - cd^3} - \frac{C_8 bcde^{at}e^{dt}}{a^2c - cd^2} - \frac{2C_8 bd^2 e^{at}}{a^2 d - 2ad^2} - \frac{C_8 bd}{a^2 + ad} - \frac{C_8 b}{a + d} - C_9 = 0$
Basis functions:

$$\left[1.0,\ e^{at} e^{dt},\ e^{at}\right]$$

$$C_8 = 0$$
$$C_8 = 0$$
$$C_8 = -\frac{C_9 a}{b}$$

Equation:$\frac{C_{10} bdt}{a} - \frac{C_{10} b}{a} + \frac{C_{10} be^{-at}}{a} - \frac{C_{10} bd}{a^2} + \frac{C_{10} bde^{-at}}{a^2} - C_{11} + \frac{C_{13} a^2 b^2}{a^3 de^{at} - a^2 d^2 e^{at}} + \frac{2C_{13} a^2 b^2}{a^3 d + a^2 d^2} - \frac{2C_{13} ab^2 c^2}{a^2 c^2 de^{dt} - ac^2 d^2 e^{dt}} + \frac{2C_{13} ab^2 d}{a^3 de^{at} - a^2 d^2 e^{at}} + \frac{4C_{13} ab^2 d}{a^3 d + a^2 d^2} - \frac{C_{13} ab^2}{a^2 de^{at} e^{dt} + ad^2 e^{at} e^{dt}} - \frac{2C_{13} b^2 c^2 d}{a^2 c^2 de^{dt} - ac^2 d^2 e^{dt}} + \frac{C_{13} b^2 d^2}{a^3 de^{at} - a^2 d^2 e^{at}} + \frac{C_{13} b^2 d^2}{a^3 d + a^2 d^2} - \frac{2C_{13} b^2 d}{a^2 de^{at} e^{dt} + ad^2 e^{at} e^{dt}} - \frac{C_{13} b^2 t}{a} + C_{14} bt + \frac{C_{15} a^2 e^{2dt}}{2ad + 4d^2} + \frac{C_{15} abd}{a^2 be^{at} + 2abde^{at}} - \frac{C_{15} a}{2d} + \frac{C_{15} bd^2}{a^2 be^{at} + 2abde^{at}} - \frac{5C_{15} d^2 e^{2dt}}{2ad + 4d^2} + 2C_{15} e^{2dt} + C_{15} - \frac{C_{15} d}{2a} + \frac{C_{16} ab^2}{acde^{at} e^{dt} + cd^2 e^{at} e^{dt}} + \frac{2C_{16} b^2 d}{acde^{at} e^{dt} + cd^2 e^{at} e^{dt}} + \frac{C_{16} b^2 d}{a^2 c + acd} - \frac{C_{16} b^2 e^{-at}}{cd} - \frac{C_{16} b^2 e^{-at}}{ac} - \frac{C_{18} be^{-at}}{c} + \frac{C_{18} bd}{ac} - \frac{C_{18} bde^{-at}}{ac} = 0$
Basis functions:

$$\left[1.0,\ t,\ e^{2dt},\ e^{-at},\ e^{-at} e^{-dt},\ e^{-dt}\right]$$

$$C_{10} = \frac{C_{13}b - C_{14}a}{d}$$
$$C_{15} = 0$$
$$C_{10} = \frac{a\left(C_{16}b + C_{18}d\right)}{cd}$$
$$0 = 0$$
$$0 = 0$$
$$C_{10} = \frac{\left(-C_{11}a^5cde^{t(a+d)} - 2C_{11}a^4cd^2e^{t(a+d)} + C_{11}a^3cd^3e^{t(a+d)} + 2C_{11}a^2cd^4e^{t(a+d)} - 2C_{13}a^4b^2ce^{at} + C_{13}a^4b^2ce^{dt} + 2C_{13}a^4b^2\right.}{}$$

Equation:$-C_{12} + \frac{C_{13}bc}{a^2e^{at}e^{dt}+ade^{at}e^{dt}} + \frac{C_{13}bc}{ad+d^2} - \frac{C_{13}bce^{-dt}}{ad} - \frac{C_{16}b}{ae^{at}e^{dt}+de^{at}e^{dt}} + \frac{C_{16}b}{a+d} = 0$ Basis functions:

$$\left[e^{-at}e^{-dt},\ 1.0,\ e^{-dt}\right]$$

$$0 = 0$$
$$C_{13} = 0$$
$$C_{12} = \frac{b\left(C_{13}ace^{t(a+d)} + C_{13}cd + C_{16}ade^{t(a+d)} - C_{16}ad\right)e^{-t(a+d)}}{ad\left(a+d\right)}$$

Equation:$-\frac{C_{13}c^2}{a^2e^{at}e^{dt}+ade^{at}e^{dt}} - \frac{C_{13}c^2}{ad+d^2} + \frac{C_{13}c^2e^{-dt}}{ad} + \frac{C_{16}c}{ae^{at}e^{dt}+de^{at}e^{dt}} - \frac{C_{16}c}{a+d} - C_{17} = 0$ Basis functions:

$$\left[e^{-at}e^{-dt},\ 1.0,\ e^{-dt}\right]$$

$$0 = 0$$
$$C_{13} = 0$$
$$C_{13} = \frac{ad\left(-C_{16}ce^{t(a+d)} + C_{16}c - C_{17}ae^{t(a+d)} - C_{17}de^{t(a+d)}\right)}{c^2\left(ae^{t(a+d)} + d\right)}$$

The execution time of the script was:

0 hours 1 minutes 12 seconds.