Summary of symmetry calculations

October 21, 2021

Contents

Chapter 1

DBH_model

Run 09_12PM_21_October-2021

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\begin{aligned}\frac{\mathrm{d}w_1}{\mathrm{d}t} &= -w_1w_2 - w_1w_3 + w_2w_3,\\ \frac{\mathrm{d}w_2}{\mathrm{d}t} &= -w_1w_2 + w_1w_3 - w_2w_3,\\ \frac{\mathrm{d}w_3}{\mathrm{d}t} &= w_1w_2 - w_1w_3 - w_2w_3.\end{aligned}$$

The calculated generators are:

$$\begin{aligned}X_1 = &(t+2)\,\partial t + (1-2tw_1)\,\partial w_1 + (1-2tw_2)\,\partial w_2\\ &+ (1-2tw_3)\,\partial w_3\end{aligned}$$

$$X_2 = (1)\,\partial t,$$

$$X_3 = (-1+t)\,\partial t + (w_1)\,\partial w_1 + (w_2)\,\partial w_2 + (w_3)\,\partial w_3.$$

The execution time of the script was:

0 hours 0 minutes 25 seconds.

Run 09_16PM_21_October-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1w_2 - w_1w_3 + w_2w_3,$$
$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1w_2 + w_1w_3 - w_2w_3,$$
$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1w_2 - w_1w_3 - w_2w_3.$$

The calculated generators are:

$$X_1 = (1)\,\partial t,$$

$$X_2 = (t+2)\,\partial t + (1-2tw_1)\,\partial w_1 + (1-2tw_2)\,\partial w_2 + (1-2tw_3)\,\partial w_3$$

$$X_3 = (t)\,\partial t + (w_2w_3\,\mathrm{f}_1(t) - w_1w_2\,\mathrm{f}_1(t) - w_1w_3\,\mathrm{f}_1(t))\,\partial w_1 + (w_1w_3\,\mathrm{f}_1(t) - w_1w_2\,\mathrm{f}_1(t) + -w_2w_3\,\mathrm{f}_1(t))\,\partial w_2 + (w_1w_2\,\mathrm{f}_1(t) - w_1w_3\,\mathrm{f}_1(t) - w_2w_3\,\mathrm{f}_1(t))\,\partial w_3$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

WARNING:

Some of the calculated generators did not satisfy the linearised symmetry conditions. Thus, the presented list here is not complete and consists exclusively of the calculated generators that satisfy the linearised symmetry conditions.

The execution time of the script was:

0 hours 3 minutes 32 seconds.

Chapter 2

hydons_model

Run 09_09PM_21_October-2021

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$
$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

The calculated generators are:

$$X_1 = (t)\,\partial t + (y_1)\,\partial y_1 + (y_2)\,\partial y_2.$$

The execution time of the script was:

0 hours 0 minutes 4 seconds.

Run 09_10PM_21_October-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$
$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

The calculated generators are:

$$X_1 = (t)\,\partial t + (y_1)\,\partial y_1 + (y_2)\,\partial y_2,$$

$$X_2 = \left(-t^2 \, \mathrm{f}_1(t) + y_1 y_2 \, \mathrm{f}_1(t)\right) \partial t + \left(y_2^2 \, \mathrm{f}_1(t) + t y_1 \, \mathrm{f}_1(t)\right) \partial y_1 + \left(y_1^2 \, \mathrm{f}_1(t) + t y_2 \, \mathrm{f}_1(t)\right) \partial y_2$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 0 minutes 23 seconds.

Chapter 3

linear_model

Run 09_10PM_21_October-2021

Degree in tangential ansätze: 1.
The system of ODEs is given by:

$$\begin{aligned}\frac{\mathrm{d}u}{\mathrm{d}t} &= u + v,\\ \frac{\mathrm{d}v}{\mathrm{d}t} &= u + v.\end{aligned}$$

The calculated generators are:

$$X_1 = (1)\,\partial t,$$

$$X_2 = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$\begin{aligned}X_3 = &\left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right.\\ &\left. + \frac{ve^{2t}}{2}\right)\partial u + \left(-\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v\end{aligned}$$

$$X_4 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_5 = \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$\begin{aligned}X_6 = &\left(-\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right.\\ &\left. + -\frac{ve^{2t}}{2}\right)\partial u + \left(-\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v\end{aligned}$$

$$X_7 = \left(-\frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2}\right)\partial u + \left(\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$X_8 = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right)\partial u + \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v$$

$$X_9 = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_{10} = (t)\,\partial t + (u\,\mathrm{f}_1(t) + v\,\mathrm{f}_1(t))\,\partial u + (u\,\mathrm{f}_1(t) + v\,\mathrm{f}_1(t))\,\partial v$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$

The execution time of the script was:

0 hours 0 minutes 11 seconds.

Run 09_12PM_21_October-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u + v,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + v.$$

The calculated generators are:

$$X_1 = (1)\,\partial t,$$

$$X_2 = \left(\frac{v}{2} - \frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2}\right)\partial t$$

$$X_3 = \left(-\frac{e^{2t}}{4} + \frac{e^{-2t}}{4}\right)\partial t + \left(\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} - \frac{ve^{2t}}{2}\right)\partial u + \left(\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v$$

$$X_4 = \left(-\frac{u^2}{4} - \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial t$$

$$X_5 = \left(-\frac{1}{2} + \frac{e^{-2t}}{4} + \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} + \frac{ve^{2t}}{2}\right)\partial u + \left(-\frac{v}{2} + \frac{ve^{2t}}{2}\right)\partial v$$

$$X_6 = \left(-\frac{5v}{8} + \frac{3u}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ve^{-4t}}{8} + \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8}\right)\partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} + -\frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial u + \left(-\frac{v^2}{2} + \frac{v^2e^{2t}}{2}\right)\partial v$$

$$X_7 = \left(-\frac{3u}{8} + \frac{5v}{8} - \frac{3ue^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} + \frac{ve^{-2t}}{8} + \frac{3ve^{2t}}{8}\right)\partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{v^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} - \frac{u^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2}\right)\partial u + \left(\frac{v^2}{2} + \frac{v^2e^{2t}}{2}\right)\partial v$$

$$X_8 = \left(-\frac{1}{2} - \frac{e^{-2t}}{4} - \frac{e^{2t}}{4}\right)\partial t + \left(-\frac{u}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} + -\frac{ve^{2t}}{2}\right)\partial u + \left(-\frac{v}{2} - \frac{ve^{2t}}{2}\right)\partial v$$

$$X_9 = \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_{10} = \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{u^2e^{-2t}}{2} - \frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2}\right)\partial t$$

$$X_{11} = \left(-\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial u + \left(\frac{1}{2} + \frac{e^{2t}}{2}\right)\partial v$$

$$X_{12} = \left(-\frac{e^{-2t}}{4} + \frac{e^{2t}}{4} \right) \partial t + \left(\frac{u}{2} + \frac{ve^{2t}}{2} - \frac{ue^{-2t}}{2} - \frac{ve^{-2t}}{2} \right) \partial u + \left(\frac{v}{2} + \frac{ve^{2t}}{2} \right) \partial v$$

$$X_{13} = \left(\frac{u^2}{4} + \frac{v^2}{4} + \frac{v^2e^{-2t}}{2} - \frac{uv}{2} - \frac{u^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial t$$

$$X_{14} = \left(-\frac{u}{8} + \frac{3v}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{2t}}{8} + \frac{ve^{-4t}}{8} \right) \partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} - \frac{v^2e^{2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left(\frac{v^2}{2} - \frac{v^2e^{2t}}{2} \right) \partial v$$

$$X_{15} = \left(\frac{u}{8} + \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{2t}}{8} - \frac{ve^{-4t}}{8} + \frac{ue^{-2t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left(-\frac{u^2}{4} + \frac{v^2}{4} + \frac{uv}{2} + \frac{u^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} - \frac{v^2e^{-2t}}{2} - \frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left(\frac{v^2}{2} + \frac{v^2e^{2t}}{2} \right) \partial v$$

$$X_{16} = \left(-\frac{3v}{8} + \frac{u}{8} - \frac{3ve^{2t}}{8} - \frac{ue^{-4t}}{8} - \frac{ue^{-2t}}{8} - \frac{ve^{-4t}}{8} + -\frac{ve^{-2t}}{8} + \frac{ue^{2t}}{8} \right) \partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} - \frac{uv}{2} - \frac{v^2e^{2t}}{2} + -\frac{u^2e^{-4t}}{4} - \frac{v^2e^{-4t}}{4} - \frac{uve^{-4t}}{2} \right) \partial u + \left(-\frac{v^2}{2} - \frac{v^2e^{2t}}{2} \right) \partial v$$

$$X_{17} = \left(-\frac{u}{8} - \frac{v}{8} - \frac{3ve^{-2t}}{8} - \frac{ue^{2t}}{8} + \frac{ue^{-4t}}{8} + \frac{ue^{-2t}}{8} + \frac{ve^{-4t}}{8} + \frac{3ve^{2t}}{8} \right) \partial t + \left(-\frac{v^2}{4} + \frac{u^2}{4} + \frac{u^2e^{-2t}}{2} + \frac{v^2e^{2t}}{2} + -\frac{uv}{2} - \frac{v^2e^{-2t}}{2} + \frac{u^2e^{-4t}}{4} + \frac{v^2e^{-4t}}{4} + \frac{uve^{-4t}}{2} \right) \partial u + \left(-\frac{v^2}{2} + \frac{v^2e^{2t}}{2} \right) \partial v$$

$$X_{18} = \left(\frac{u}{2} - \frac{v}{2} + \frac{ue^{-2t}}{2} + \frac{ve^{-2t}}{2} \right) \partial t$$

$$\begin{aligned} X_{19} = &\left(u\,\mathrm{f}_2(t) + v\,\mathrm{f}_1(t) - v\,\mathrm{f}_2(t) + \mathrm{f}_3(t)\right)\partial t + \left(u\,\mathrm{f}_3(t) + v\,\mathrm{f}_3(t)\right. \\ &+ u^2\,\mathrm{f}_2(t) + v^2\,\mathrm{f}_1(t) - v^2\,\mathrm{f}_2(t) + uv\,\mathrm{f}_1(t)\Big)\partial u + \left(u\,\mathrm{f}_3(t) + v\,\mathrm{f}_3(t)\right. \\ &+ u^2\,\mathrm{f}_2(t) + v^2\,\mathrm{f}_1(t) - v^2\,\mathrm{f}_2(t) + uv\,\mathrm{f}_1(t)\Big)\partial v \end{aligned}$$

Some of the generators might contain the following arbitrary functions:

$$\mathrm{f}_1$$
$$\mathrm{f}_2$$
$$\mathrm{f}_3$$

The execution time of the script was:

0 hours 1 minutes 46 seconds.