

Summary of symmetry calculations

October 26, 2021

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Chapter 1

DBH_model

Run 11_24AM_26_October-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\begin{aligned}\frac{dw_1}{dt} &= -w_1w_2 - w_1w_3 + w_2w_3, \\ \frac{dw_2}{dt} &= -w_1w_2 + w_1w_3 - w_2w_3, \\ \frac{dw_3}{dt} &= w_1w_2 - w_1w_3 - w_2w_3.\end{aligned}$$

The calculated generators are:

Solving the algebraic equations

Solving equation:

$$-C_3 = 0$$

Arguments: $[-1, C_3]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_3 = 0$$

Solving equation:

$$-C_2t - C_4t - C_5 + C_7t = 0$$

Arguments: $[-C_5, C_7t, -C_2t, -C_4t]$ Arbitrary functions: \square Basis functions:[$t, 1$] Solutions:

$$\begin{aligned}C_2 &= -C_4 + C_7 \\ C_5 &= 0\end{aligned}$$

Solving equation:

$$-C_6 = 0$$

Arguments: $[-1, C_6]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_6 = 0$$

Solving equation:

$$2C_4t - 2C_7t - C_8 = 0$$

Arguments: $[-C_8, -2C_7t, 2C_4t]$ Arbitrary functions: $[]$ Basis functions: $[2t, 1]$ Solutions:

$$\begin{aligned} C_4 &= C_7 \\ C_8 &= 0 \end{aligned}$$

Solving equation:

$$-2C_7t - C_9 = 0$$

Arguments: $[-C_9, -2C_7t]$ Arbitrary functions: $[]$ Basis functions: $[-2t, 1]$ Solutions:

$$\begin{aligned} C_7 &= 0 \\ C_9 &= 0 \end{aligned}$$

Solving equation:

$$-C_{10} = 0$$

Arguments: $[-1, C_{10}]$ Arbitrary functions: $[]$ Basis functions: $[1]$ Solutions:

$$C_{10} = 0$$

Solving equation:

$$-C_{11}t^2 + C_{12}t - C_{13} + C_{21}t^2 - C_{22}t = 0$$

Arguments: $[-C_{13}, C_{12}t, C_{21}t^2, -C_{11}t^2, -C_{22}t]$ Arbitrary functions: $[]$ Basis functions: $[t, t^2, 1]$ Solutions:

$$\begin{aligned} C_{12} &= C_{22} \\ C_{11} &= C_{21} \\ C_{13} &= 0 \end{aligned}$$

Solving equation:

$$-C_{15} + 2C_{17}t + C_{21}t^2 - C_{31}t^2 - 2C_{32}t + C_{40} = 0$$

Arguments: $[C_{40}, -C_{15}, C_{21}t^2, -C_{31}t^2, -2C_{32}t, 2C_{17}t]$ Arbitrary functions: $[]$ Basis functions: $[2t, 1, t^2]$ Solutions:

$$\begin{aligned} C_{17} &= C_{32} \\ C_{21} &= C_{31} \\ C_{15} &= C_{40} \end{aligned}$$

Solving equation:

$$C_{14}t - C_{16} - C_{34}t = 0$$

Arguments: $[-C_{16}, C_{14}t, -C_{34}t]$ Arbitrary functions: $[]$ Basis functions: $[t, 1]$ Solutions:

$$\begin{aligned} C_{14} &= C_{34} \\ C_{16} &= 0 \end{aligned}$$

Solving equation:

$$-C_{18} - C_{24}t - C_{27}t + C_{32}t + C_{34}t - C_{40} = 0$$

Arguments: $[-C_{18}, -C_{40}, C_{32}t, C_{34}t, -C_{24}t, -C_{27}t]$ Arbitrary functions: \square Basis functions: $[t, 1]$ Solutions:

$$\begin{aligned} C_{24} &= -C_{27} + C_{32} + C_{34} \\ C_{18} &= -C_{40} \end{aligned}$$

Solving equation:

$$-C_{19} + C_{22}t - C_{37}t - C_{40} = 0$$

Arguments: $[-C_{19}, -C_{40}, C_{22}t, -C_{37}t]$ Arbitrary functions: \square Basis functions: $[t, 1]$ Solutions:

$$\begin{aligned} C_{22} &= C_{37} \\ C_{19} &= -C_{40} \end{aligned}$$

Solving equation:

$$-C_{20} + C_{27}t + C_{37}t = 0$$

Arguments: $[-C_{20}, C_{27}t, C_{37}t]$ Arbitrary functions: \square Basis functions: $[t, 1]$ Solutions:

$$\begin{aligned} C_{27} &= -C_{37} \\ C_{20} &= 0 \end{aligned}$$

Solving equation:

$$-C_{23} = 0$$

Arguments: $[-1, C_{23}]$ Arbitrary functions: \square Basis functions: $[1]$ Solutions:

$$C_{23} = 0$$

Solving equation:

$$-C_{25} - 2C_{34}t - 2C_{37}t - C_{40} = 0$$

Arguments: $[-C_{25}, -C_{40}, -2C_{34}t, -2C_{37}t]$ Arbitrary functions: \square Basis functions: $[-2t, 1]$ Solutions:

$$\begin{aligned} C_{34} &= -C_{37} \\ C_{25} &= -C_{40} \end{aligned}$$

Solving equation:

$$-C_{26} - 2C_{37}t = 0$$

Arguments: $[-C_{26}, -2C_{37}t]$ Arbitrary functions: \square Basis functions: $[-2t, 1]$ Solutions:

$$\begin{aligned} C_{37} &= 0 \\ C_{26} &= 0 \end{aligned}$$

Solving equation:

$$-C_{28} + C_{40} = 0$$

Arguments: $[C_{40}, -C_{28}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{28} = C_{40}$$

Solving equation:

$$-C_{29} - C_{40} = 0$$

Arguments: $[-C_{29}, -C_{40}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{29} = -C_{40}$$

Solving equation:

$$-C_{30} = 0$$

Arguments: $[-1, C_{30}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{30} = 0$$

Solving equation:

$$-C_{33} = 0$$

Arguments: $[-1, C_{33}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{33} = 0$$

Solving equation:

$$-C_{35} - C_{40} = 0$$

Arguments: $[-C_{35}, -C_{40}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{35} = -C_{40}$$

Solving equation:

$$-C_{36} = 0$$

Arguments: $[-1, C_{36}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{36} = 0$$

Solving equation:

$$-C_{38} - C_{40} = 0$$

Arguments: $[-C_{38}, -C_{40}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{38} = -C_{40}$$

Solving equation:

$$-C_{39} = 0$$

Arguments: $[-1, C_{39}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{39} = 0$$

$$X_1 = (-1) \partial t,$$

$$X_2 = (t + 2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 \\ + (1 - 2tw_3) \partial w_3$$

$$X_3 = (1) \partial t,$$

$$X_4 = (-1 + t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$X_5 = (t) \partial t + (w_2 w_3 f_1(t) - w_1 w_2 f_1(t) - w_1 w_3 f_1(t)) \partial w_1 + (w_1 w_3 f_1(t) - w_1 w_2 f_1(t) \\ + -w_2 w_3 f_1(t)) \partial w_2 + (w_1 w_2 f_1(t) - w_1 w_3 f_1(t) - w_2 w_3 f_1(t)) \partial w_3$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

The execution time of the script was:

0 hours 5 minutes 9 seconds.

Run 11_29AM_26_October-2021

Degree in tangential ansätze: 2.

The system of ODEs is given by:

$$\frac{dw_1}{dt} = -w_1 w_2 - w_1 w_3 + w_2 w_3, \\ \frac{dw_2}{dt} = -w_1 w_2 + w_1 w_3 - w_2 w_3, \\ \frac{dw_3}{dt} = w_1 w_2 - w_1 w_3 - w_2 w_3.$$

The calculated generators are:

Solving the algebraic equations

Solving equation:

$$-C_3 = 0$$

Arguments: $[-1, C_3]$ Arbitrary functions: $[]$ Basis functions:[1] Solutions:

$$C_3 = 0$$

Solving equation:

$$-C_2t - C_4t - C_5 + C_7t = 0$$

Arguments: $[-C_5, C_7t, -C_2t, -C_4t]$ Arbitrary functions: $[]$ Basis functions: $[t, 1]$ Solutions:

$$\begin{aligned} C_2 &= -C_4 + C_7 \\ C_5 &= 0 \end{aligned}$$

Solving equation:

$$-C_6 = 0$$

Arguments: $[-1, C_6]$ Arbitrary functions: $[]$ Basis functions: $[1]$ Solutions:

$$C_6 = 0$$

Solving equation:

$$2C_4t - 2C_7t - C_8 = 0$$

Arguments: $[-C_8, -2C_7t, 2C_4t]$ Arbitrary functions: $[]$ Basis functions: $[-2t, 1]$ Solutions:

$$\begin{aligned} C_7 &= 0 \\ C_4 &= \frac{C_8}{2t} \end{aligned}$$

Solving equation:

$$-C_8 - C_9 = 0$$

Arguments: $[-C_8, -C_9]$ Arbitrary functions: $[]$ Basis functions: $[1]$ Solutions:

$$C_8 = -C_9$$

Solving equation:

$$-C_{10} = 0$$

Arguments: $[-1, C_{10}]$ Arbitrary functions: $[]$ Basis functions: $[1]$ Solutions:

$$C_{10} = 0$$

Solving equation:

$$-C_{11}t^2 + C_{12}t - C_{13} + C_{21}t^2 - C_{22}t = 0$$

Arguments: $[-C_{13}, C_{12}t, C_{21}t^2, -C_{11}t^2, -C_{22}t]$ Arbitrary functions: $[]$ Basis functions: $[t^2, t, 1]$ Solutions:

$$\begin{aligned} C_{11} &= C_{21} \\ C_{12} &= C_{22} \\ C_{13} &= 0 \end{aligned}$$

Solving equation:

$$-C_{15} + 2C_{17}t + C_{21}t^2 - C_{31}t^2 - 2C_{32}t + C_{40} = 0$$

Arguments: $[C_{40}, -C_{15}, C_{21}t^2, -C_{31}t^2, -2C_{32}t, 2C_{17}t]$ Arbitrary functions: $[]$ Basis functions: $[1, t^2, t]$
 Solutions:

$$C_{21} = C_{31}$$

$$C_{17} = C_{32}$$

$$C_{15} = C_{40}$$

Solving equation:

$$C_{14}t - C_{16} - C_{34}t = 0$$

Arguments: $[-C_{16}, C_{14}t, -C_{34}t]$ Arbitrary functions: $[]$ Basis functions: $[t, 1]$ Solutions:

$$C_{14} = C_{34}$$

$$C_{16} = 0$$

Solving equation:

$$-C_{18} - C_{24}t - C_{27}t + C_{32}t + C_{34}t - C_{40} = 0$$

Arguments: $[-C_{18}, -C_{40}, C_{32}t, C_{34}t, -C_{24}t, -C_{27}t]$ Arbitrary functions: $[]$ Basis functions: $[t, 1]$ Solutions:

$$C_{24} = -C_{27} + C_{32} + C_{34}$$

$$C_{18} = -C_{40}$$

Solving equation:

$$-C_{19} + C_{22}t - C_{37}t - C_{40} = 0$$

Arguments: $[-C_{19}, -C_{40}, C_{22}t, -C_{37}t]$ Arbitrary functions: $[]$ Basis functions: $[t, 1]$ Solutions:

$$C_{22} = C_{37}$$

$$C_{19} = -C_{40}$$

Solving equation:

$$-C_{20} + C_{27}t + C_{37}t = 0$$

Arguments: $[-C_{20}, C_{27}t, C_{37}t]$ Arbitrary functions: $[]$ Basis functions: $[t, 1]$ Solutions:

$$C_{27} = -C_{37}$$

$$C_{20} = 0$$

Solving equation:

$$-C_{23} = 0$$

Arguments: $[-1, C_{23}]$ Arbitrary functions: $[]$ Basis functions: $[1]$ Solutions:

$$C_{23} = 0$$

Solving equation:

$$-C_{25} - 2C_{34}t - 2C_{37}t - C_{40} = 0$$

Arguments: $[-C_{25}, -C_{40}, -2C_{34}t, -2C_{37}t]$ Arbitrary functions: \square Basis functions: $[-2t, 1]$ Solutions:

$$\begin{aligned} C_{34} &= -C_{37} \\ C_{25} &= -C_{40} \end{aligned}$$

Solving equation:

$$-C_{26} - 2C_{37}t = 0$$

Arguments: $[-C_{26}, -2C_{37}t]$ Arbitrary functions: \square Basis functions: $[-2t, 1]$ Solutions:

$$\begin{aligned} C_{37} &= 0 \\ C_{26} &= 0 \end{aligned}$$

Solving equation:

$$-C_{28} + C_{40} = 0$$

Arguments: $[C_{40}, -C_{28}]$ Arbitrary functions: \square Basis functions: $[1]$ Solutions:

$$C_{28} = C_{40}$$

Solving equation:

$$-C_{29} - C_{40} = 0$$

Arguments: $[-C_{29}, -C_{40}]$ Arbitrary functions: \square Basis functions: $[1]$ Solutions:

$$C_{29} = -C_{40}$$

Solving equation:

$$-C_{30} = 0$$

Arguments: $[-1, C_{30}]$ Arbitrary functions: \square Basis functions: $[1]$ Solutions:

$$C_{30} = 0$$

Solving equation:

$$-C_{33} = 0$$

Arguments: $[-1, C_{33}]$ Arbitrary functions: \square Basis functions: $[1]$ Solutions:

$$C_{33} = 0$$

Solving equation:

$$-C_{35} - C_{40} = 0$$

Arguments: $[-C_{35}, -C_{40}]$ Arbitrary functions: \square Basis functions: $[1]$ Solutions:

$$C_{35} = -C_{40}$$

Solving equation:

$$-C_{36} = 0$$

Arguments: $[-1, C_{36}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{36} = 0$$

Solving equation:

$$-C_{38} - C_{40} = 0$$

Arguments: $[-C_{38}, -C_{40}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{38} = -C_{40}$$

Solving equation:

$$-C_{39} = 0$$

Arguments: $[-1, C_{39}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{39} = 0$$

$$X_1 = (1) \partial t,$$

$$X_2 = (-1) \partial t,$$

$$X_3 = (t + 2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 \\ + (1 - 2tw_3) \partial w_3$$

$$X_4 = (-1 + t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$X_5 = (t) \partial t + (w_2 w_3 f_1(t) - w_1 w_2 f_1(t) - w_1 w_3 f_1(t)) \partial w_1 + (w_1 w_3 f_1(t) - w_1 w_2 f_1(t) \\ + -w_2 w_3 f_1(t)) \partial w_2 + (w_1 w_2 f_1(t) - w_1 w_3 f_1(t) - w_2 w_3 f_1(t)) \partial w_3$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

WARNING:

Some of the calculated generators did not satisfy the linearised symmetry conditions. Thus, the presented list here is not complete and consists exclusively of the calculated generators that satisfy the linearised symmetry conditions.

The execution time of the script was:

0 hours 5 minutes 12 seconds.

Run 11_34AM_26_October-2021

Degree in tangential ansätze: 2.

The system of ODEs is given by:

$$\begin{aligned}\frac{dw_1}{dt} &= -w_1w_2 - w_1w_3 + w_2w_3, \\ \frac{dw_2}{dt} &= -w_1w_2 + w_1w_3 - w_2w_3, \\ \frac{dw_3}{dt} &= w_1w_2 - w_1w_3 - w_2w_3.\end{aligned}$$

The calculated generators are:

Solving the algebraic equations

Solving equation:

$$-C_3 = 0$$

Arguments: $[-1, C_3]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_3 = 0$$

Solving equation:

$$-C_2t - C_4t - C_5 + C_7t = 0$$

Arguments: $[-C_5, C_7t, -C_2t, -C_4t]$ Arbitrary functions: \square Basis functions:[$t, 1$] Solutions:

$$C_2 = -C_4 + C_7$$

$$C_5 = 0$$

Solving equation:

$$-C_6 = 0$$

Arguments: $[-1, C_6]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_6 = 0$$

Solving equation:

$$2C_4t - 2C_7t - C_8 = 0$$

Arguments: $[-C_8, -2C_7t, 2C_4t]$ Arbitrary functions: \square Basis functions:[$2t, 1$] Solutions:

$$C_4 = C_7$$

$$C_8 = 0$$

Solving equation:

$$-2C_7t - C_9 = 0$$

Arguments: $[-C_9, -2C_7t]$ Arbitrary functions: \square Basis functions:[$1, t$] Solutions:

$$C_7 = 0$$

$$C_9 = 0$$

Solving equation:

$$-C_{10} = 0$$

Arguments: $[-1, C_{10}]$ Arbitrary functions: \square Basis functions: $[1]$ Solutions:

$$C_{10} = 0$$

Solving equation:

$$-C_{11}t^2 + C_{12}t - C_{13} + C_{21}t^2 - C_{22}t = 0$$

Arguments: $[-C_{13}, C_{12}t, C_{21}t^2, -C_{11}t^2, -C_{22}t]$ Arbitrary functions: \square Basis functions: $[t^2, t, 1]$ Solutions:

$$C_{11} = C_{21}$$

$$C_{12} = C_{22}$$

$$C_{13} = 0$$

Solving equation:

$$-C_{15} + 2C_{17}t + C_{21}t^2 - C_{31}t^2 - 2C_{32}t + C_{40} = 0$$

Arguments: $[C_{40}, -C_{15}, C_{21}t^2, -C_{31}t^2, -2C_{32}t, 2C_{17}t]$ Arbitrary functions: \square Basis functions: $[t^2, 1, t]$ Solutions:

$$C_{21} = C_{31}$$

$$C_{17} = C_{32}$$

$$C_{15} = C_{40}$$

Solving equation:

$$C_{14}t - C_{16} - C_{34}t = 0$$

Arguments: $[-C_{16}, C_{14}t, -C_{34}t]$ Arbitrary functions: \square Basis functions: $[t, 1]$ Solutions:

$$C_{14} = C_{34}$$

$$C_{16} = 0$$

Solving equation:

$$-C_{18} - C_{24}t - C_{27}t + C_{32}t + C_{34}t - C_{40} = 0$$

Arguments: $[-C_{18}, -C_{40}, C_{32}t, C_{34}t, -C_{24}t, -C_{27}t]$ Arbitrary functions: \square Basis functions: $[t, 1]$ Solutions:

$$C_{24} = -C_{27} + C_{32} + C_{34}$$

$$C_{18} = -C_{40}$$

Solving equation:

$$-C_{19} + C_{22}t - C_{37}t - C_{40} = 0$$

Arguments: $[-C_{19}, -C_{40}, C_{22}t, -C_{37}t]$ Arbitrary functions: \square Basis functions: $[t, 1]$ Solutions:

$$C_{22} = C_{37}$$

$$C_{19} = -C_{40}$$

Solving equation:

$$-C_{20} + C_{27}t + C_{37}t = 0$$

Arguments: $[-C_{20}, C_{27}t, C_{37}t]$ Arbitrary functions: \square Basis functions: $[t, 1]$ Solutions:

$$\begin{aligned} C_{27} &= -C_{37} \\ C_{20} &= 0 \end{aligned}$$

Solving equation:

$$-C_{23} = 0$$

Arguments: $[-1, C_{23}]$ Arbitrary functions: \square Basis functions: $[1]$ Solutions:

$$C_{23} = 0$$

Solving equation:

$$-C_{25} - 2C_{34}t - 2C_{37}t - C_{40} = 0$$

Arguments: $[-C_{25}, -C_{40}, -2C_{34}t, -2C_{37}t]$ Arbitrary functions: \square Basis functions: $[1, t]$ Solutions:

$$\begin{aligned} C_{34} &= -C_{37} \\ C_{25} &= -C_{40} \end{aligned}$$

Solving equation:

$$-C_{26} - 2C_{37}t = 0$$

Arguments: $[-C_{26}, -2C_{37}t]$ Arbitrary functions: \square Basis functions: $[1, t]$ Solutions:

$$\begin{aligned} C_{37} &= 0 \\ C_{26} &= 0 \end{aligned}$$

Solving equation:

$$-C_{28} + C_{40} = 0$$

Arguments: $[C_{40}, -C_{28}]$ Arbitrary functions: \square Basis functions: $[1]$ Solutions:

$$C_{28} = C_{40}$$

Solving equation:

$$-C_{29} - C_{40} = 0$$

Arguments: $[-C_{29}, -C_{40}]$ Arbitrary functions: \square Basis functions: $[1]$ Solutions:

$$C_{29} = -C_{40}$$

Solving equation:

$$-C_{30} = 0$$

Arguments: $[-1, C_{30}]$ Arbitrary functions: \square Basis functions: $[1]$ Solutions:

$$C_{30} = 0$$

Solving equation:

$$-C_{33} = 0$$

Arguments: $[-1, C_{33}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{33} = 0$$

Solving equation:

$$-C_{35} - C_{40} = 0$$

Arguments: $[-C_{35}, -C_{40}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{35} = -C_{40}$$

Solving equation:

$$-C_{36} = 0$$

Arguments: $[-1, C_{36}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{36} = 0$$

Solving equation:

$$-C_{38} - C_{40} = 0$$

Arguments: $[-C_{38}, -C_{40}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{38} = -C_{40}$$

Solving equation:

$$-C_{39} = 0$$

Arguments: $[-1, C_{39}]$ Arbitrary functions: \square Basis functions:[1] Solutions:

$$C_{39} = 0$$

$$X_1 = (-1 + t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$X_2 = (t + 2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 \\ + (1 - 2tw_3) \partial w_3$$

$$X_3 = (-1) \partial t,$$

$$X_4 = (1) \partial t,$$

$$\begin{aligned}
X_5 = & (t) \partial t + (w_2 w_3 f_1(t) - w_1 w_2 f_1(t) - w_1 w_3 f_1(t)) \partial w_1 + (w_1 w_3 f_1(t) - w_1 w_2 f_1(t) \\
& + -w_2 w_3 f_1(t)) \partial w_2 + (w_1 w_2 f_1(t) - w_1 w_3 f_1(t) - w_2 w_3 f_1(t)) \partial w_3
\end{aligned}$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

The execution time of the script was:

0 hours 5 minutes 14 seconds.