

Summary of symmetry calculations

October 25, 2021

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Chapter 1

DBH_model

Run 02_09PM_25_October-2021

Degree in tangential ansätze: 2.
The system of ODEs is given by:

$$\begin{aligned}\frac{dw_1}{dt} &= -w_1w_2 - w_1w_3 + w_2w_3, \\ \frac{dw_2}{dt} &= -w_1w_2 + w_1w_3 - w_2w_3, \\ \frac{dw_3}{dt} &= w_1w_2 - w_1w_3 - w_2w_3.\end{aligned}$$

The calculated generators are:

ODE solutions:

$$\mathbf{c} = \begin{bmatrix} C_1 - C_{17}t - C_{24}t + C_{31}t^2 + C_{32}t - C_{40} + c_{32}(t) \\ C_2 \\ C_3 \\ C_4 \\ C_2t + C_4t + C_5 - C_7t \\ C_6 \\ C_7 \\ C_2t - C_4t + C_7t + C_8 \\ -C_2t + C_4t + C_7t + C_9 \\ C_{10} \\ C_{11} \\ -C_{11}t + C_{12} + C_{21}t \\ C_{11}t^2 - C_{12}t + C_{13} - C_{21}t^2 + C_{22}t \\ -C_{11}t + C_{14} + C_{31}t \\ -C_{11}t^2 + C_{15} - 2C_{17}t + C_{31}t^2 + 2C_{32}t - C_{40} + c_{32}(t) \\ C_{11}t^2 - C_{14}t + C_{16} - C_{31}t^2 + C_{34}t \\ C_{17} - C_{21}t - C_{31}t \\ C_{12}t - C_{14}t + C_{17}t + C_{18} + C_{21}t^2 - C_{22}t + C_{24}t + C_{27}t - C_{31}t^2 - 2C_{32}t + C_{40} - c_{32}(t) \\ -C_{12}t + C_{14}t + C_{17}t + C_{19} - C_{32}t - C_{34}t + C_{37}t + C_{40} - c_{32}(t) \\ C_{20} - C_{27}t - C_{37}t \\ C_{21} \\ C_{11}t - C_{21}t + C_{22} \\ -C_{11}t^2 + C_{12}t + C_{21}t^2 - C_{22}t + C_{23} \\ -C_{11}t + C_{24} - C_{31}t \\ C_{11}t^2 - C_{12}t + C_{14}t + C_{17}t + C_{22}t + C_{24}t + C_{25} - C_{27}t - C_{31}t^2 - 2C_{32}t + C_{40} - c_{32}(t) \\ -C_{14}t + C_{26} - C_{34}t \\ -C_{21}t + C_{27} + C_{31}t \\ -C_{21}t^2 - 2C_{24}t + C_{28} + C_{31}t^2 + 2C_{32}t - C_{40} + c_{32}(t) \\ -C_{22}t + C_{24}t + C_{27}t + C_{29} - C_{32}t + C_{34}t - C_{37}t + C_{40} - c_{32}(t) \\ C_{21}t^2 - C_{27}t + C_{30} - C_{31}t^2 + C_{37}t \\ C_{31} \\ -C_{11}t - C_{21}t + C_{32} \\ -C_{12}t - C_{22}t + C_{33} \\ C_{11}t - C_{31}t + C_{34} \\ C_{11}t^2 + C_{12}t - C_{14}t + C_{17}t - C_{31}t^2 - C_{32}t + C_{34}t + C_{35} - C_{37}t + C_{40} - c_{32}(t) \\ -C_{11}t^2 + C_{14}t + C_{31}t^2 - C_{34}t + C_{36} \\ C_{21}t - C_{31}t + C_{37} \\ C_{21}t^2 + C_{22}t + C_{24}t - C_{27}t - C_{31}t^2 - C_{32}t - C_{34}t + C_{37}t + C_{38} + C_{40} - c_{32}(t) \\ -C_{21}t^2 + C_{27}t + C_{31}t^2 - C_{37}t + C_{39} \end{bmatrix}$$

Solving equation:

$$-2C_7t - C_9 = 0$$

$$C_7 = 0$$

$$C_9 = 0$$

Solving equation:

$$-C_{10} = 0$$

$$C_{10} = 0$$

Solving equation:

$$-C_{11}t^2 + C_{12}t - C_{13} + C_{21}t^2 - C_{22}t = 0$$

$$C_{12} = C_{22}$$

$$C_{11} = C_{21}$$

$$C_{13} = 0$$

Solving equation:

$$-C_{15} + 2C_{17}t + C_{21}t^2 - C_{31}t^2 - 2C_{32}t + C_{40} = 0$$

$$C_{21} = C_{31}$$

$$C_{17} = C_{32}$$

$$C_{15} = C_{40}$$

Solving equation:

$$C_{14}t - C_{16} - C_{34}t = 0$$

$$C_{14} = C_{34}$$

$$C_{16} = 0$$

Solving equation:

$$-C_{18} - C_{24}t - C_{27}t + C_{32}t + C_{34}t - C_{40} = 0$$

$$C_{24} = -C_{27} + C_{32} + C_{34}$$

$$C_{18} = -C_{40}$$

Solving equation:

$$-C_{19} + C_{22}t - C_{37}t - C_{40} = 0$$

$$C_{22} = C_{37}$$

$$C_{19} = -C_{40}$$

Solving equation:

$$-C_{20} + C_{27}t + C_{37}t = 0$$

$$C_{27} = -C_{37}$$

$$C_{20} = 0$$

Solving equation:

$$-C_{23} = 0$$

$$C_{23} = 0$$

Solving equation:

$$-C_{25} - 2C_{34}t - 2C_{37}t - C_{40} = 0$$

$$C_{34} = -C_{37}$$

$$C_{25} = -C_{40}$$

Solving equation:

$$-C_{26} - 2C_{37}t = 0$$

$$C_{37} = 0$$

$$C_{26} = 0$$

Solving equation:

$$-C_{28} + C_{40} = 0$$

$$C_{28} = C_{40}$$

Solving equation:

$$-C_{29} - C_{40} = 0$$

$$C_{29} = -C_{40}$$

Solving equation:

$$-C_{30} = 0$$

$$C_{30} = 0$$

Solving equation:

$$-C_{33} = 0$$

$$C_{33} = 0$$

Solving equation:

$$-C_{35} - C_{40} = 0$$

$$C_{35} = -C_{40}$$

Solving equation:

$$-C_{36} = 0$$

$$C_{36} = 0$$

Solving equation:

$$-C_{38} - C_{40} = 0$$

$$C_{38} = -C_{40}$$

Solving equation:

$$-C_{39} = 0$$

$$C_{39} = 0$$

$$X_1 = (-1) \partial t,$$

$$X_2 = (-1 + t) \partial t + (w_1) \partial w_1 + (w_2) \partial w_2 + (w_3) \partial w_3,$$

$$X_3 = (t + 2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 \\ + (1 - 2tw_3) \partial w_3$$

$$X_4 = (1) \partial t,$$

$$X_5 = (t) \partial t + (w_2 w_3 f_1(t) - w_1 w_2 f_1(t) - w_1 w_3 f_1(t)) \partial w_1 + (w_1 w_3 f_1(t) - w_1 w_2 f_1(t) \\ + -w_2 w_3 f_1(t)) \partial w_2 + (w_1 w_2 f_1(t) - w_1 w_3 f_1(t) - w_2 w_3 f_1(t)) \partial w_3$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

The execution time of the script was:

0 hours 5 minutes 0 seconds.

Run 02_14PM_25_October-2021

Degree in tangential ansätze: 2.

The system of ODEs is given by:

$$\begin{aligned}\frac{dw_1}{dt} &= -w_1w_2 - w_1w_3 + w_2w_3, \\ \frac{dw_2}{dt} &= -w_1w_2 + w_1w_3 - w_2w_3, \\ \frac{dw_3}{dt} &= w_1w_2 - w_1w_3 - w_2w_3.\end{aligned}$$

The calculated generators are:

ODE solutions:

$$\mathbf{c} = \begin{bmatrix} C_1 - C_{17}t - C_{24}t + C_{31}t^2 + C_{32}t - C_{40} + c_{32}(t) \\ C_2 \\ C_3 \\ C_4 \\ C_2t + C_4t + C_5 - C_7t \\ C_6 \\ C_7 \\ C_2t - C_4t + C_7t + C_8 \\ -C_2t + C_4t + C_7t + C_9 \\ C_{10} \\ C_{11} \\ -C_{11}t + C_{12} + C_{21}t \\ C_{11}t^2 - C_{12}t + C_{13} - C_{21}t^2 + C_{22}t \\ -C_{11}t + C_{14} + C_{31}t \\ -C_{11}t^2 + C_{15} - 2C_{17}t + C_{31}t^2 + 2C_{32}t - C_{40} + c_{32}(t) \\ C_{11}t^2 - C_{14}t + C_{16} - C_{31}t^2 + C_{34}t \\ C_{17} - C_{21}t - C_{31}t \\ C_{12}t - C_{14}t + C_{17}t + C_{18} + C_{21}t^2 - C_{22}t + C_{24}t + C_{27}t - C_{31}t^2 - 2C_{32}t + C_{40} - c_{32}(t) \\ -C_{12}t + C_{14}t + C_{17}t + C_{19} - C_{32}t - C_{34}t + C_{37}t + C_{40} - c_{32}(t) \\ C_{20} - C_{27}t - C_{37}t \\ C_{21} \\ C_{11}t - C_{21}t + C_{22} \\ -C_{11}t^2 + C_{12}t + C_{21}t^2 - C_{22}t + C_{23} \\ -C_{11}t + C_{24} - C_{31}t \\ C_{11}t^2 - C_{12}t + C_{14}t + C_{17}t + C_{22}t + C_{24}t + C_{25} - C_{27}t - C_{31}t^2 - 2C_{32}t + C_{40} - c_{32}(t) \\ -C_{14}t + C_{26} - C_{34}t \\ -C_{21}t + C_{27} + C_{31}t \\ -C_{21}t^2 - 2C_{24}t + C_{28} + C_{31}t^2 + 2C_{32}t - C_{40} + c_{32}(t) \\ -C_{22}t + C_{24}t + C_{27}t + C_{29} - C_{32}t + C_{34}t - C_{37}t + C_{40} - c_{32}(t) \\ C_{21}t^2 - C_{27}t + C_{30} - C_{31}t^2 + C_{37}t \\ C_{31} \\ -C_{11}t - C_{21}t + C_{32} \\ -C_{12}t - C_{22}t + C_{33} \\ C_{11}t - C_{31}t + C_{34} \\ C_{11}t^2 + C_{12}t - C_{14}t + C_{17}t - C_{31}t^2 - C_{32}t + C_{34}t + C_{35} - C_{37}t + C_{40} - c_{32}(t) \\ -C_{11}t^2 + C_{14}t + C_{31}t^2 - C_{34}t + C_{36} \\ C_{21}t - C_{31}t + C_{37} \\ C_{21}t^2 + C_{22}t + C_{24}t - C_{27}t - C_{31}t^2 - C_{32}t - C_{34}t + C_{37}t + C_{38} + C_{40} - c_{32}(t) \\ -C_{21}t^2 + C_{27}t + C_{31}t^2 - C_{37}t + C_{39} \end{bmatrix}$$

Solving equation:

$$-C_8 - C_9 = 0$$

$$C_8 = -C_9$$

Solving equation:

$$-C_{10} = 0$$

$$C_{10} = 0$$

Solving equation:

$$-C_{11}t^2 + C_{12}t - C_{13} + C_{21}t^2 - C_{22}t = 0$$

$$C_{22} = 0$$

$$C_{11} = C_{21}$$

$$C_{12} = \frac{C_{13}}{t}$$

Solving equation:

$$-C_{15} + 2C_{17}t + C_{21}t^2 - C_{31}t^2 - 2C_{32}t + C_{40} = 0$$

$$C_{32} = 0$$

$$C_{21} = C_{31}$$

$$C_{15} = 2C_{17}t + C_{40}$$

Solving equation:

$$C_{14}t - C_{16} - C_{34}t = 0$$

$$C_{34} = 0$$

$$C_{14} = \frac{C_{16}}{t}$$

Solving equation:

$$-C_{13} + C_{16} - C_{17}t - C_{18} - C_{24}t - C_{27}t - C_{40} = 0$$

$$C_{17} = -C_{24} - C_{27}$$

$$C_{13} = C_{16} - C_{18} - C_{40}$$

Solving equation:

$$-C_{18} - C_{19} + C_{24}t + C_{27}t - C_{37}t - 2C_{40} = 0$$

$$C_{37} = 0$$

$$C_{18} = -C_{19} + C_{24}t + C_{27}t - 2C_{40}$$

Solving equation:

$$-C_{20} + C_{27}t = 0$$

$$C_{27} = 0$$

$$C_{20} = 0$$

Solving equation:

$$-C_{16} - C_{19} - C_{23} + C_{24}t - C_{40} = 0$$

$$C_{24} = 0$$

$$C_{16} = -C_{19} - C_{23} - C_{40}$$

Solving equation:

$$C_{19} - C_{25} = 0$$

$$C_{19} = C_{25}$$

Solving equation:

$$-C_{23} - C_{25} - C_{26} - C_{40} = 0$$

$$C_{23} = -C_{25} - C_{26} - C_{40}$$

Solving equation:

$$-C_{28} + C_{40} = 0$$

$$C_{28} = C_{40}$$

Solving equation:

$$-C_{29} - C_{40} = 0$$

$$C_{29} = -C_{40}$$

Solving equation:

$$-C_{30} = 0$$

$$C_{30} = 0$$

Solving equation:

$$C_{25} + C_{26} - C_{33} + C_{40} = 0$$

$$C_{25} = -C_{26} + C_{33} - C_{40}$$

Solving equation:

$$C_{26} - C_{33} - C_{35} - C_{40} = 0$$

$$C_{26} = C_{33} + C_{35} + C_{40}$$

Solving equation:

$$-C_{33} - C_{35} - C_{36} - C_{40} = 0$$

$$C_{33} = -C_{35} - C_{36} - C_{40}$$

Solving equation:

$$-C_{38} - C_{40} = 0$$

$$C_{38} = -C_{40}$$

Solving equation:

$$-C_{39} = 0$$

$$C_{39} = 0$$

$$X_1 = (t + 2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 \\ + (1 - 2tw_3) \partial w_3$$

$$X_2 = (1) \partial t,$$

$$X_3 = (t) \partial t + (w_2 w_3 f_1(t) - w_1 w_2 f_1(t) - w_1 w_3 f_1(t)) \partial w_1 + (w_1 w_3 f_1(t) - w_1 w_2 f_1(t) \\ + -w_2 w_3 f_1(t)) \partial w_2 + (w_1 w_2 f_1(t) - w_1 w_3 f_1(t) - w_2 w_3 f_1(t)) \partial w_3$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

WARNING:

Some of the calculated generators did not satisfy the linearised symmetry conditions. Thus, the presented list here is not complete and consists exclusively of the calculated generators that satisfy the linearised symmetry conditions.

The execution time of the script was:

0 hours 4 minutes 56 seconds.

Run 02_20PM_25_October-2021

Degree in tangential ansätze: 2.

The system of ODEs is given by:

$$\begin{aligned}\frac{dw_1}{dt} &= -w_1w_2 - w_1w_3 + w_2w_3, \\ \frac{dw_2}{dt} &= -w_1w_2 + w_1w_3 - w_2w_3, \\ \frac{dw_3}{dt} &= w_1w_2 - w_1w_3 - w_2w_3.\end{aligned}$$

The calculated generators are:

ODE solutions:

$$\mathbf{c} = \begin{bmatrix} C_1 - C_{17}t - C_{24}t + C_{31}t^2 + C_{32}t - C_{40} + c_{32}(t) \\ C_2 \\ C_3 \\ C_4 \\ C_2t + C_4t + C_5 - C_7t \\ C_6 \\ C_7 \\ C_2t - C_4t + C_7t + C_8 \\ -C_2t + C_4t + C_7t + C_9 \\ C_{10} \\ C_{11} \\ -C_{11}t + C_{12} + C_{21}t \\ C_{11}t^2 - C_{12}t + C_{13} - C_{21}t^2 + C_{22}t \\ -C_{11}t + C_{14} + C_{31}t \\ -C_{11}t^2 + C_{15} - 2C_{17}t + C_{31}t^2 + 2C_{32}t - C_{40} + c_{32}(t) \\ C_{11}t^2 - C_{14}t + C_{16} - C_{31}t^2 + C_{34}t \\ C_{17} - C_{21}t - C_{31}t \\ C_{12}t - C_{14}t + C_{17}t + C_{18} + C_{21}t^2 - C_{22}t + C_{24}t + C_{27}t - C_{31}t^2 - 2C_{32}t + C_{40} - c_{32}(t) \\ -C_{12}t + C_{14}t + C_{17}t + C_{19} - C_{32}t - C_{34}t + C_{37}t + C_{40} - c_{32}(t) \\ C_{20} - C_{27}t - C_{37}t \\ C_{21} \\ C_{11}t - C_{21}t + C_{22} \\ -C_{11}t^2 + C_{12}t + C_{21}t^2 - C_{22}t + C_{23} \\ -C_{11}t + C_{24} - C_{31}t \\ C_{11}t^2 - C_{12}t + C_{14}t + C_{17}t + C_{22}t + C_{24}t + C_{25} - C_{27}t - C_{31}t^2 - 2C_{32}t + C_{40} - c_{32}(t) \\ -C_{14}t + C_{26} - C_{34}t \\ -C_{21}t + C_{27} + C_{31}t \\ -C_{21}t^2 - 2C_{24}t + C_{28} + C_{31}t^2 + 2C_{32}t - C_{40} + c_{32}(t) \\ -C_{22}t + C_{24}t + C_{27}t + C_{29} - C_{32}t + C_{34}t - C_{37}t + C_{40} - c_{32}(t) \\ C_{21}t^2 - C_{27}t + C_{30} - C_{31}t^2 + C_{37}t \\ C_{31} \\ -C_{11}t - C_{21}t + C_{32} \\ -C_{12}t - C_{22}t + C_{33} \\ C_{11}t - C_{31}t + C_{34} \\ C_{11}t^2 + C_{12}t - C_{14}t + C_{17}t - C_{31}t^2 - C_{32}t + C_{34}t + C_{35} - C_{37}t + C_{40} - c_{32}(t) \\ -C_{11}t^2 + C_{14}t + C_{31}t^2 - C_{34}t + C_{36} \\ C_{21}t - C_{31}t + C_{37} \\ C_{21}t^2 + C_{22}t + C_{24}t - C_{27}t - C_{31}t^2 - C_{32}t - C_{34}t + C_{37}t + C_{38} + C_{40} - c_{32}(t) \\ -C_{21}t^2 + C_{27}t + C_{31}t^2 - C_{37}t + C_{39} \end{bmatrix}$$

$$C_4 = \frac{C_8}{2t}$$

Solving equation:

$$-C_8 - C_9 = 0$$

$$C_8 = -C_9$$

Solving equation:

$$-C_{10} = 0$$

$$C_{10} = 0$$

Solving equation:

$$-C_{11}t^2 + C_{12}t - C_{13} + C_{21}t^2 - C_{22}t = 0$$

$$C_{22} = 0$$

$$C_{11} = C_{21}$$

$$C_{12} = \frac{C_{13}}{t}$$

Solving equation:

$$-C_{15} + 2C_{17}t + C_{21}t^2 - C_{31}t^2 - 2C_{32}t + C_{40} = 0$$

$$C_{32} = 0$$

$$C_{21} = C_{31}$$

$$C_{15} = 2C_{17}t + C_{40}$$

Solving equation:

$$C_{14}t - C_{16} - C_{34}t = 0$$

$$C_{34} = 0$$

$$C_{14} = \frac{C_{16}}{t}$$

Solving equation:

$$-C_{13} + C_{16} - C_{17}t - C_{18} - C_{24}t - C_{27}t - C_{40} = 0$$

$$C_{17} = -C_{24} - C_{27}$$

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Solving equation:

$$-C_{18} - C_{19} + C_{24}t + C_{27}t - C_{37}t - 2C_{40} = 0$$

$$C_{37} = 0$$

$$C_{18} = -C_{19} + C_{24}t + C_{27}t - 2C_{40}$$

Solving equation:

$$-C_{20} + C_{27}t = 0$$

$$C_{27} = 0$$

$$C_{20} = 0$$

Solving equation:

$$-C_{16} - C_{19} - C_{23} + C_{24}t - C_{40} = 0$$

$$C_{24} = 0$$

$$C_{16} = -C_{19} - C_{23} - C_{40}$$

Solving equation:

$$C_{19} - C_{25} = 0$$

$$C_{19} = C_{25}$$

Solving equation:

$$-C_{23} - C_{25} - C_{26} - C_{40} = 0$$

$$C_{23} = -C_{25} - C_{26} - C_{40}$$

Solving equation:

$$-C_{28} + C_{40} = 0$$

$$C_{28} = C_{40}$$

Solving equation:

$$-C_{29} - C_{40} = 0$$

$$C_{29} = -C_{40}$$

Solving equation:

$$-C_{30} = 0$$

$$C_{30} = 0$$

Solving equation:

$$C_{25} + C_{26} - C_{33} + C_{40} = 0$$

$$C_{25} = -C_{26} + C_{33} - C_{40}$$

Solving equation:

$$C_{26} - C_{33} - C_{35} - C_{40} = 0$$

$$C_{26} = C_{33} + C_{35} + C_{40}$$

Solving equation:

$$-C_{33} - C_{35} - C_{36} - C_{40} = 0$$

$$C_{33} = -C_{35} - C_{36} - C_{40}$$

Solving equation:

$$-C_{38} - C_{40} = 0$$

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Solving equation:

$$-C_{39} = 0$$

$$C_{39} = 0$$

$$X_1 = (t + 2) \partial t + (1 - 2tw_1) \partial w_1 + (1 - 2tw_2) \partial w_2 \\ + (1 - 2tw_3) \partial w_3$$

$$X_2 = (1) \partial t,$$

$$X_3 = (t) \partial t + (w_2 w_3 f_1(t) - w_1 w_2 f_1(t) - w_1 w_3 f_1(t)) \partial w_1 + (w_1 w_3 f_1(t) - w_1 w_2 f_1(t) \\ + -w_2 w_3 f_1(t)) \partial w_2 + (w_1 w_2 f_1(t) - w_1 w_3 f_1(t) - w_2 w_3 f_1(t)) \partial w_3$$

Some of the generators might contain the following arbitrary functions:

$$f_1$$

WARNING:

Some of the calculated generators did not satisfy the linearised symmetry conditions. Thus, the presented list here is not complete and consists exclusively of the calculated generators that satisfy the linearised symmetry conditions.

The execution time of the script was:

0 hours 5 minutes 18 seconds.