Summary of symmetry calculations

July 21, 2021

Contents

Chapter 1

DBH_model

Run 08_11AM_21_July-2021

Degree in tangential ansätze: 1
The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1w_2 - w_1w_3 + w_2w_3,$$
$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1w_2 + w_1w_3 - w_2w_3,$$
$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1w_2 - w_1w_3 - w_2w_3.$$

The calculated generators are:

$$X_1 = (1)\,\partial t,$$
$$X_2 = \left(\frac{t^2}{2}\right)\partial t + (-tw_1 + 1)\,\partial w_1 + (-tw_2)\,\partial w_2 + (-tw_3)\,\partial w_3,$$
$$X_3 = \left(\frac{t^2}{2}\right)\partial t + (-tw_1)\,\partial w_1 + (-tw_2 + 1)\,\partial w_2 + (-tw_3)\,\partial w_3,$$
$$X_4 = (1)\,\partial w_3,$$
$$X_5 = (-t)\,\partial t + (w_1)\,\partial w_1 + (w_2)\,\partial w_2 + (w_3)\,\partial w_3$$

Run 08_15AM_21_July-2021

Degree in tangential ansätze: 2
The system of ODEs is given by:

$$\frac{\mathrm{d}w_1}{\mathrm{d}t} = -w_1w_2 - w_1w_3 + w_2w_3,$$
$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = -w_1w_2 + w_1w_3 - w_2w_3,$$
$$\frac{\mathrm{d}w_3}{\mathrm{d}t} = w_1w_2 - w_1w_3 - w_2w_3.$$

The calculated generators are:

$$
\begin{aligned}
X_1 &= (1)\,\partial t,\\
X_2 &= (w_3)\,\partial t,\\
X_3 &= (w_2)\,\partial t,\\
X_4 &= (w_1)\,\partial t,\\
X_5 &= (-tw_2 - tw_3 + 1)\,\partial w_1 + (-tw_2 + tw_3)\,\partial w_2 + (tw_2 - tw_3)\,\partial w_3,\\
X_6 &= (w_3)\,\partial w_1,\\
X_7 &= (w_2)\,\partial w_1,\\
X_8 &= (-t)\,\partial t + (w_1)\,\partial w_1,\\
X_9 &= (-tw_1 + tw_3)\,\partial w_1 + (-tw_1 - tw_3 + 1)\,\partial w_2 + (tw_1 - tw_3)\,\partial w_3,\\
X_{10} &= (w_3)\,\partial w_2,\\
X_{11} &= (-t)\,\partial t + (w_2)\,\partial w_2,\\
X_{12} &= (w_1)\,\partial w_2,\\
X_{13} &= \left(t^2\right)\partial t + (-tw_1 + tw_2)\,\partial w_1 + (tw_1 - tw_2)\,\partial w_2 + (-tw_1 - tw_2 + 1)\,\partial w_3,\\
X_{14} &= (t)\,\partial t + (w_3)\,\partial w_3,\\
X_{15} &= (w_2)\,\partial w_3,\\
X_{16} &= (w_1)\,\partial w_3
\end{aligned}
$$

Run 09_10AM_21_July-2021

Degree in tangential ansätze: 3
The system of ODEs is given by:

$$
\begin{aligned}
\frac{\mathrm{d}w_1}{\mathrm{d}t} &= -w_1w_2 - w_1w_3 + w_2w_3,\\
\frac{\mathrm{d}w_2}{\mathrm{d}t} &= -w_1w_2 + w_1w_3 - w_2w_3,\\
\frac{\mathrm{d}w_3}{\mathrm{d}t} &= w_1w_2 - w_1w_3 - w_2w_3.
\end{aligned}
$$

The calculated generators are:

$$
\begin{aligned}
X_1 &= (1)\,\partial t,\\
X_2 &= (-tw_1w_2 + tw_1w_3 + tw_2w_3 + w_3)\,\partial t,\\
X_3 &= \left(w_3^2\right)\partial t,\\
X_4 &= (tw_1w_2 - tw_1w_3 + tw_2w_3 + w_2)\,\partial t,\\
X_5 &= (w_2w_3)\,\partial t,\\
X_6 &= \left(w_2^2\right)\partial t,\\
X_7 &= (tw_1w_2 + tw_1w_3 - tw_2w_3 + w_1)\,\partial t,\\
X_8 &= (w_1w_3)\,\partial t,\\
X_9 &= (w_1w_2)\,\partial t,\\
X_{10} &= \left(w_1^2\right)\partial t,\\
X_{11} &= \left(\frac{t^4w_1w_2}{3} - \frac{t^4w_1w_3}{3} + \frac{2t^3w_2}{3} - \frac{2t^3w_3}{3}\right)\partial t + \left(t^2w_2^2 - t^2w_2w_3 + t^2w_3^2 - tw_2 - tw_3 + 1\right)\partial w_1 + \left(t^2w_2w_3 - t^2w_3^2 - tw_2 + \right.\\
X_{12} &= \left(\frac{2t^3w_1w_2}{3} - \frac{2t^3w_1w_3}{3} + t^2w_2 - t^2w_3\right)\partial t + \left(-tw_1w_2 + tw_1w_3 - tw_3^2 + w_3\right)\partial w_1 + \left(-tw_2w_3 + tw_3^2\right)\partial w_2 + \left(tw_2w_3 - t\right.\\
X_{13} &= \left(w_3^2\right)\partial w_1,\\
X_{14} &= \left(-\frac{2t^3w_1w_2}{3} + \frac{2t^3w_1w_3}{3} - t^2w_2 + t^2w_3\right)\partial t + \left(tw_1w_2 - tw_1w_3 - tw_2^2 + w_2\right)\partial w_1 + \left(-tw_2^2 + tw_2w_3\right)\partial w_2 + \left(tw_2^2 - tw_2\right.\\
X_{15} &= (w_2w_3)\,\partial w_1,\\
X_{16} &= \left(w_2^2\right)\partial w_1,\\
X_{17} &= \left(\frac{t^3w_1w_2}{3} - \frac{t^3w_1w_3}{3} - \frac{t^3w_2w_3}{3} - t^2w_3 - t\right)\partial t + (tw_1w_2 + tw_1w_3 - 2tw_2w_3 + w_1)\,\partial w_1 + (tw_2w_3)\,\partial w_2 + (tw_2w_3)\,\partial w_3,\\
X_{18} &= \left(\frac{t^2w_1w_2}{2} - \frac{t^2w_1w_3}{2} - \frac{t^2w_2w_3}{2} - tw_3\right)\partial t + (w_1w_3)\,\partial w_1,\\
X_{19} &= \left(-\frac{t^2w_1w_2}{2} + \frac{t^2w_1w_3}{2} - \frac{t^2w_2w_3}{2} - tw_2\right)\partial t + (w_1w_2)\,\partial w_1,\\
X_{20} &= \left(-t^2w_1w_3 - tw_1 - tw_3\right)\partial t + \left(w_1^2\right)\partial w_1,\\
X_{21} &= \left(\frac{t^4w_1w_2}{3} - \frac{t^4w_2w_3}{3} + \frac{2t^3w_1}{3} - \frac{2t^3w_3}{3}\right)\partial t + \left(t^2w_1w_3 - t^2w_3^2 - tw_1 + tw_3\right)\partial w_1 + \left(t^2w_1^2 - t^2w_1w_3 + t^2w_3^2 - tw_1 - tw_3\right.\\
X_{22} &= \left(\frac{2t^3w_1w_2}{3} - \frac{2t^3w_2w_3}{3} + t^2w_1 - t^2w_3\right)\partial t + \left(-tw_1w_3 + tw_3^2\right)\partial w_1 + \left(-tw_1w_2 + tw_2w_3 - tw_3^2 + w_3\right)\partial w_2 + \left(tw_1w_3 - t\right.\\
X_{23} &= \left(w_3^2\right)\partial w_2,\\
X_{24} &= \left(\frac{t^3w_1w_2}{3} - \frac{t^3w_1w_3}{3} - \frac{t^3w_2w_3}{3} - t^2w_3 - t\right)\partial t + (tw_1w_3)\,\partial w_1 + (tw_1w_2 - 2tw_1w_3 + tw_2w_3 + w_2)\,\partial w_2 + (tw_1w_3)\,\partial w_3,\\
X_{25} &= \left(\frac{t^2w_1w_2}{2} - \frac{t^2w_1w_3}{2} - \frac{t^2w_2w_3}{2} - tw_3\right)\partial t + (w_2w_3)\,\partial w_2,\\
X_{26} &= \left(-t^2w_2w_3 - tw_2 - tw_3\right)\partial t + \left(w_2^2\right)\partial w_2,\\
X_{27} &= \left(-\frac{2t^3w_1w_2}{3} + \frac{2t^3w_2w_3}{3} - t^2w_1 + t^2w_3\right)\partial t + \left(-tw_1^2 + tw_1w_3\right)\partial w_1 + \left(-tw_1^2 + tw_1w_2 - tw_2w_3 + w_1\right)\partial w_2 + \left(tw_1^2 - tw\right.\\
X_{28} &= (w_1w_3)\,\partial w_2,\\
X_{29} &= \left(-\frac{t^2w_1w_2}{2} - \frac{t^2w_1w_3}{2} + \frac{t^2w_2w_3}{2} - tw_1\right)\partial t + (w_1w_2)\,\partial w_2,\\
X_{30} &= \left(w_1^2\right)\partial w_2,\\
X_{31} &= \left(-\frac{2t^4w_1w_2}{3} + \frac{t^4w_1w_3}{3} + \frac{t^4w_2w_3}{3} - \frac{2t^3w_1}{3} - \frac{2t^3w_2}{3} + \frac{4t^3w_3}{3} + t^2\right)\partial t + \left(-t^2w_1w_3 - t^2w_2^2 + t^2w_2w_3 - tw_1 + tw_2\right)\partial w_1\\
X_{32} &= \left(-\frac{2t^3w_1w_2}{3} + \frac{2t^3w_1w_3}{3} + \frac{2t^3w_2w_3}{3} + 2t^2w_3 + t\right)\partial t + (-tw_1w_2 - 2tw_1w_3 + 2tw_2w_3)\,\partial w_1 + (-tw_1w_2 + 2tw_1w_3 - 2tw\\
X_{33} &= \left(-t^2w_1w_2 + t^2w_1w_3 + t^2w_2w_3 + 2tw_3\right)\partial t + \left(w_3^2\right)\partial w_3,\\
&\left(\frac{t^3w_1w_2}{\ }\right.
\end{aligned}
$$

Chapter 2

Lotka_Volterra_realistic

Run 10_03AM_21_July-2021

Degree in tangential ansätze: 1
The system of ODEs is given by:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = N\left(-\frac{Pk}{D+N} + r\left(1 - \frac{N}{K}\right)\right),$$
$$\frac{\mathrm{d}P}{\mathrm{d}t} = Ps\left(1 - \frac{Ph}{N}\right).$$

The calculated generators are:

$$X_1 = (1)\,\partial t$$

Chapter 3

hydons_model

Run 08_10AM_21_July-2021

Degree in tangential ansätze: 1
The system of ODEs is given by:

$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{ty_1 + y_2^2}{-t^2 + y_1 y_2},$$
$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = \frac{ty_2 + y_1^2}{-t^2 + y_1 y_2}.$$

The calculated generators are:

$$X_1 = (t)\,\partial t + (y_1)\,\partial y_1 + (y_2)\,\partial y_2$$

Chapter 4

Lotka_Volterra

Run 10_02AM_21_July-2021

Degree in tangential ansätze: 1
The system of ODEs is given by:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = N\left(-Pb + a\right),$$
$$\frac{\mathrm{d}P}{\mathrm{d}t} = P\left(Nc - d\right).$$

The calculated generators are:

$$
\begin{aligned}
X_1 &= (1)\,\partial t,\\
X_2 &= \left(Ne^{-at} + \frac{Pbe^{-at}}{c}\right)\partial t,\\
X_3 &= \left(-\frac{e^{dt}}{-\frac{ad}{c} + \frac{d^2}{c}} + \frac{e^{at}}{-\frac{a^2}{c} + \frac{ad}{c}} - \frac{c}{ad}\right)\partial t\\
&\quad + \left(-\frac{Nace^{2dt}}{ade^{dt} - d^2e^{dt}} + \frac{Nace^{dt}}{ade^{dt} - d^2e^{dt}} - \frac{Nae^{dt}}{-\frac{ad}{c} + \frac{d^2}{c}} + \frac{2Ncde^{at}e^{dt}}{ade^{dt} - d^2e^{dt}} - \frac{Ncde^{2dt}}{ade^{dt} - d^2e^{dt}}\right.\\
&\quad \left. - \frac{Ncde^{dt}}{ade^{dt} - d^2e^{dt}} - \frac{Nc}{d} + \frac{Ne^{at}}{-\frac{a}{c} + \frac{d}{c}} + \frac{Pabe^{at}}{-a^2 + ad} - \frac{Pbde^{at}}{-a^2 + ad} + \frac{Pbe^{at}}{a} - \frac{ae^{at}}{-a + d} + \frac{de^{at}}{-a + d}\right)\partial N\\
&\quad + \left(-\frac{Pe^{at}}{-\frac{a}{c} + \frac{d}{c}} + \frac{Pe^{dt}}{-\frac{a}{c} + \frac{d}{c}}\right)\partial P,\\
X_4 &= \left(\frac{Nabe^{dt}}{ade^{dt} - d^2e^{dt}} - \frac{Nab}{ade^{dt} - d^2e^{dt}} - \frac{Nbde^{dt}}{ade^{dt} - d^2e^{dt}} + \frac{Nbd}{ade^{dt} - d^2e^{dt}} - \frac{Nb}{d} + \frac{Nbe^{-dt}}{d}\right)\partial N + \left(e^{-dt}\right)\partial P,\\
X_5 &= \left(-\frac{e^{dt}}{d} + \frac{1}{d}\right)\partial t + \left(\frac{Na^2e^{2dt}}{ade^{dt} - d^2e^{dt}} - \frac{Na^2e^{dt}}{ade^{dt} - d^2e^{dt}} + \frac{Nade^{dt}}{ade^{dt} - d^2e^{dt}} - \frac{Nae^{dt}}{d} + \frac{Na}{d} - \frac{Nd^2e^{2dt}}{ade^{dt} - d^2e^{dt}}\right)\partial N\\
&\quad + \left(Pe^{dt}\right)\partial P
\end{aligned}
$$

Run 10_09AM_21_July-2021

Degree in tangential ansätze: 2
The system of ODEs is given by:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = N\left(-Pb + a\right),$$
$$\frac{\mathrm{d}P}{\mathrm{d}t} = P\left(Nc - d\right).$$

The calculated generators are pasted in a file called temp.tex because they are way to big to compile.