Fundamental Inputs

- Principle of Superposition
- Coulombs Law ($F = \frac{1}{4\pi\epsilon_0}$)

$\oint_S E_n dA = \frac{1}{\varepsilon_0} Q_{inside}$
$\nabla \cdot E = \frac{1}{\epsilon_0}\rho | V(r) = -\int^r E \cdot dl$
$E = -\nabla V | C = \frac{Q}{V}$
$W = \frac{1}{2}CV^2 | W = \frac{\epsilon_0}{2}\int E^2 d\tau$
$V(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r)}{r_s} d\tau$
Potential is continuous at boundaries

Method of Images

- Replace the conducting plane with a mirror image charge
- Use Gausss law on each charge in isolation
- Sum up the electric field contribution from each charge

Poisson's equation: $\nabla^2 V = -\frac{1}{\epsilon_0}\rho$
Laplace's equation: $\nabla^2 V = 0$
Laplace's equation in Cartesian:
$\frac{\delta^2 V}{\delta x^2}\frac{\delta^2 V}{\delta y^2}\frac{\delta^2 V}{\delta z^2} = 0$
Converted to PDE
$V(x,y) =$
$(Ae^{kx} + B^{-kx})(Csin(ky) + Dcos(ky))$
Laplace's equation is true if ρ is zero
First Uniqueness Theorem: The solution to Laplace equation in some volume V is uniquely determined if V is specified on the boundary surface S
Corollary: The potential in a volume V is uniquely determined if (a) the charge density throughout the region, and (b) the value of V on all boundaries, are specified.
Second Uniqueness Theorem: In a volume V surrounded by a conductors and containing a specified charge density ρ, the electric field is uniquely determined if the *total charge* on each conductor is given. (The region as a whole can be bounded by another conductor or else unbounded)

Fourier s Trick

$V_0(y) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi y}{a}\right)$
$V_0(y)\sin\left(\frac{n'\pi y}{a}\right) =$
$\sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi y}{a}\right)\sin\left(\frac{n'\pi y}{a}\right)$
$\int_0^a V_0(y)\sin\left(\frac{n'\pi y}{a}\right)\, dy =$
$\sum_{n=1}^{\infty} C_n \int_0^a \sin\left(\frac{n\pi y}{a}\right)\sin\left(\frac{n'\pi y}{a}\right)\, dy =$
$\frac{a}{2}C_{n'}$
$C_{n'} = \frac{2}{a}\int_0^a V_0(y)\sin\left(\frac{n'\pi y}{a}\right)\, dy$

Legendre Polynomials

$P_0(x) = 1$
$P_1(x) = x$
$P_2(x) = \frac{1}{2}(3x^2 - 1)$
$P_3(x) = \frac{1}{2}(5x^3 - 3x)$
$V(r,\theta) = \sum_{l=0}^{\infty}(A_l r^l + \frac{B^l}{R^{l+1}})P_l(cos\theta)$

Monopole Expansions

Monopole ($V\ 1/r$)
Dipole ($V\ 1/r^2$)
Quadrupole ($V\ 1/r^3$)
Octopole ($V\ 1/r^4$)
$\rho = \sum_{i=1}^{n} q_i r_i$
$V_{mon}(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$
$V_{dip}(r) = \frac{1}{4\pi\epsilon_0}\frac{\rho\cdot\hat{r}}{r^2}$
$E_{dip}(r,\theta) = \frac{\rho}{4\pi\epsilon_0 r^3}(2cos\theta\hat{r} + sin\theta\hat{\theta})$
Charge is evenly distributed across capacitor plates
$F = QE$
Volume Charge $E(r) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(r)}{r^2}\hat{r}d\tau$

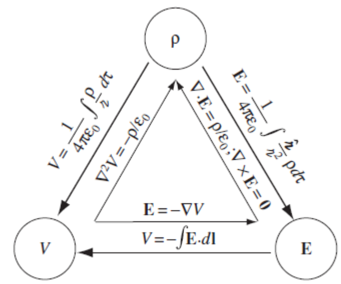

FIGURE 2.35