Johannes Byle

4.5

$$|{+n}\rangle = \cos\frac{\theta}{2}\,|{+z}\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|{-z}\rangle$$

$$|{-n}\rangle = \sin\frac{\theta}{2}\,|{+z}\rangle - e^{i\phi}\cos\frac{\theta}{2}\,|{-z}\rangle$$

$$|{+y}\rangle = \frac{1}{\sqrt{2}}\,|{+z}\rangle + \frac{i}{\sqrt{2}}\,|{-z}\rangle$$

$$\langle{+n}|{+y}\rangle = \frac{1}{\sqrt{2}}\cos\frac{\theta}{2} + \frac{i}{\sqrt{2}}\sin\frac{\theta}{2}$$

$$\langle{-n}|{+y}\rangle = \frac{1}{\sqrt{2}}\sin\frac{\theta}{2} - \frac{i}{\sqrt{2}}\cos\frac{\theta}{2}$$

$$\langle{+y}|\psi(t)\rangle = e^{-i\omega_0 t/\hbar}\left(|{+n}\rangle + |{-n}\rangle\right)\frac{1}{\sqrt{2}}\begin{bmatrix}\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\\ \sin\frac{\theta}{2} - i\cos\frac{\theta}{2}\end{bmatrix}$$

$$\langle{+y}|\psi(t)\rangle = \frac{e^{-i\omega_0 t/\hbar}}{\sqrt{2}}\left[\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)|{+n}\rangle + \left(\sin\frac{\theta}{2} - i\cos\frac{\theta}{2}\right)|{-n}\rangle\right]$$

$$\phi = \frac{\theta}{2}$$

$$|\langle{+y}|\psi(t)\rangle|^2 = \frac{e^{-2i\omega_0 t/\hbar}}{2}\left[\cos^2\phi - \sin^2\phi + 2i\sin\phi\cos\phi + \sin^2\phi + \cos^2\phi - 2i\sin\phi\cos\phi\right]$$

$$|\langle{+y}|\psi(t)\rangle|^2 = e^{-2i\omega_0 t/\hbar}\cos^2\frac{\theta}{2}$$

$$|\langle{+y}|\psi(t)\rangle|^2 = e^{-2i\omega_0 T/\hbar}\cos^2\frac{0}{2} = e^{-2i\omega_0 T/\hbar}$$

$$|\langle{+y}|\psi(t)\rangle|^2 = e^{-2i\omega_0 T/\hbar}\cos^2\frac{\pi/2}{2} = e^{-2i\omega_0 T/\hbar}$$

4.6

$$\frac{d}{dt}\langle A\rangle = \frac{1}{\hbar}\langle\psi(t)|\,[\hat{H},\hat{A}]\,|\psi(t)\rangle + \langle\psi(t)|\,\frac{\delta\hat{A}}{\delta t}\,|\psi(t)\rangle$$

$$\frac{d}{dt}\langle S_z\rangle = \frac{1}{\hbar}\langle\psi(t)|\left(\omega_0\hat{S}_z S_z - S_z\omega_0\hat{S}_z\right)|\psi(t)\rangle + 0 = 0$$

$$\frac{d}{dt}\langle S_x\rangle = \frac{1}{\hbar}\langle\psi(t)|\left(\omega_0\hat{S}_z S_x - S_x\omega_0\hat{S}_z\right)|\psi(t)\rangle + \langle\psi(t)|\,\frac{\delta}{\delta t}\left(\frac{\hbar}{2}\cos\omega_0 t\right)|\psi(t)\rangle = -\omega_0\frac{\hbar}{2}\sin\omega_0 t$$

$$\frac{d}{dt}\langle S_y\rangle = \frac{1}{\hbar}\langle\psi(t)|\left(\omega_0\hat{S}_z S_y - S_y\omega_0\hat{S}_z\right)|\psi(t)\rangle + \langle\psi(t)|\,\frac{\delta}{\delta t}\left(\frac{\hbar}{2}\sin\omega_0 t\right)|\psi(t)\rangle = \omega_0\frac{\hbar}{2}\cos\omega_0 t$$

4.11

$$\omega_0 = \frac{gq}{2mc}$$

$$|\psi(t)\rangle = \sqrt{\frac{1+\sin\omega_0 t}{2}}\,|{+y}\rangle + \sqrt{\frac{1-\sin\omega_0 t}{2}}\,|{+y}\rangle$$

$$\frac{d}{dt}\langle S_z\rangle = 0$$

$$\frac{d}{dt}\langle S_x\rangle = -\omega_0\frac{\hbar}{2}\sin\omega_0 t$$

$$\frac{d}{dt}\langle S_y\rangle = \omega_0\frac{\hbar}{2}\cos\omega_0 t$$

4.13

a)

$$|\psi(t)\rangle = e^{-iE_1 t/\hbar}\,|2\rangle$$

b)

$$|\psi(t)\rangle = e^{-iE_0 t/\hbar}\,|3\rangle$$

4.15

$$\frac{\hbar}{2}\begin{bmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{bmatrix}$$

$$|\psi(0)\rangle = \left|\tfrac{3}{2},\tfrac{3}{2}\right\rangle\left\langle\tfrac{3}{2}\middle|\tfrac{3}{2}\right\rangle + \left|\tfrac{3}{2},\tfrac{1}{2}\right\rangle\left\langle\tfrac{3}{2}\middle|\tfrac{1}{2}\right\rangle + \left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle\left\langle\tfrac{3}{2}\middle|-\tfrac{1}{2}\right\rangle + \left|\tfrac{3}{2},-\tfrac{3}{2}\right\rangle\left\langle\tfrac{3}{2}\middle|-\tfrac{3}{2}\right\rangle$$

Using the Clebsch–Gordan coefficients:

$$|\psi(0)\rangle = \frac{1}{2\sqrt{2}}\left(\left|\tfrac{3}{2},\tfrac{3}{2}\right\rangle\left\langle\tfrac{3}{2}\middle|\tfrac{3}{2}\right\rangle + \sqrt{3}\left|\tfrac{3}{2},\tfrac{1}{2}\right\rangle\left\langle\tfrac{3}{2}\middle|\tfrac{1}{2}\right\rangle + \sqrt{3}\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle\left\langle\tfrac{3}{2}\middle|-\tfrac{1}{2}\right\rangle + \left|\tfrac{3}{2},-\tfrac{3}{2}\right\rangle\left\langle\tfrac{3}{2}\middle|-\tfrac{3}{2}\right\rangle\right)$$

$$|\psi(t)\rangle = \frac{e^{-i\omega_0 t/\hbar}}{2\sqrt{2}}\left(\left|\tfrac{3}{2},\tfrac{3}{2}\right\rangle\left\langle\tfrac{3}{2}\middle|\tfrac{3}{2}\right\rangle + \sqrt{3}\left|\tfrac{3}{2},\tfrac{1}{2}\right\rangle\left\langle\tfrac{3}{2}\middle|\tfrac{1}{2}\right\rangle + \sqrt{3}\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle\left\langle\tfrac{3}{2}\middle|-\tfrac{1}{2}\right\rangle + \left|\tfrac{3}{2},-\tfrac{3}{2}\right\rangle\left\langle\tfrac{3}{2}\middle|-\tfrac{3}{2}\right\rangle\right)$$

$$\left|\left\langle\tfrac{3}{2},-\tfrac{3}{2}\middle|\psi(\pi/\omega_0)\right\rangle\right|^2 = \left(\frac{e^{-i\pi/\hbar}}{2\sqrt{2}}\right)^2$$