

Quantum I Assignment #1

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Q-1 (a) $\tilde{\mathbf{K}}$ is hermitian iff:

$$\tilde{\mathbf{K}} = \tilde{\mathbf{K}}^\dagger$$

(b)

$$\begin{aligned}\tilde{\mathbf{K}} &= |\phi\rangle \langle\psi| \\ \tilde{\mathbf{K}}^2 &= (|\phi\rangle \langle\psi|) (|\phi\rangle \langle\psi|)\end{aligned}$$

Because of the associative principle:

$$\tilde{\mathbf{K}}^2 = |\phi\rangle \langle\psi|\phi\rangle \langle\psi|$$

$\tilde{\mathbf{K}}^2$ is a projection operator if $\psi = \phi$

$$\begin{aligned}\tilde{\mathbf{K}}^2 &= |\psi\rangle \langle\psi|\psi\rangle \langle\psi| \\ \tilde{\mathbf{K}}^2 &= |\psi\rangle 1 \langle\psi| \\ \tilde{\mathbf{K}}^2 &= |\psi\rangle \langle\psi|\end{aligned}$$

(c) Assuming $\tilde{\mathbf{P}}_1 = |\phi\rangle \langle\phi|$ and $\tilde{\mathbf{P}}_2 = |\psi\rangle \langle\psi|$

$$\tilde{\mathbf{K}} = \lambda |\phi\rangle \langle\phi|\psi\rangle \langle\psi|$$

From equation 1.25 we know that the product of $\langle\phi|\psi\rangle$ is normally a complex number, therefore, assuming scalar multiplication with vectors is commutative:

$$\begin{aligned}\langle\phi|\psi\rangle &= c \\ \tilde{\mathbf{K}} &= \lambda |\phi\rangle c \langle\psi| \\ \tilde{\mathbf{K}} &= \lambda |\phi\rangle \langle\psi| \quad \text{We can just combine } c \text{ into } \lambda\end{aligned}$$

1.1 Assumptions:

$$\begin{aligned}v_{th} &= \sqrt{\frac{2k_b T}{m}} \\ t &= \frac{\Delta x}{v_{th}} \\ F_z &= ma_z = \mu_z \frac{\delta B_z}{\delta z} \\ \Delta z &= \frac{1}{2} a_z t^2\end{aligned}$$

Substituting variables:

$$\begin{aligned}\Delta z &= \frac{1}{2} \mu_z \frac{\delta B_z}{\delta z} \frac{1}{m} \frac{\sqrt{\Delta x}}{2k_b T} m \\ \Delta z &= \mu_z \frac{\delta B_z}{\delta z} \frac{\sqrt{\Delta x}}{4k_b T} \\ \Delta z &= 9.27 \cdot 10^{-24} \cdot 10 \cdot \frac{1}{4 \cdot 1.38 \cdot 10^{-23} \cdot 1273.15} \approx 0.0013 \text{ m}\end{aligned}$$

1.3

$$\langle (\Delta A)^2 \rangle \langle (\Delta B)^2 \rangle \geq \frac{1}{4} |\langle [A, B] \rangle|^2$$

For $A = S_x$ and $B = S_y$:

$$\begin{aligned}\langle (\Delta S_x)^2 \rangle \langle (\Delta S_y)^2 \rangle &\geq \frac{1}{4} |\langle [S_x, S_y] \rangle|^2 \\ (\langle S_x^2 \rangle - \langle S_x \rangle^2) (\langle S_y^2 \rangle - \langle S_y \rangle^2) &\geq \frac{1}{4} |i\hbar S_z|^2 \\ (0) \left(\frac{\hbar}{2} \right) &\geq \frac{1}{4} |i\hbar S_z|^2 \\ 0 &\geq \frac{1}{4} |i\hbar \langle S_y; \pm | S_x; + \rangle|^2 \\ 0 &\geq \frac{1}{4} \left| i\hbar \frac{1}{\sqrt{2}} \right|^2 \\ 0 &\geq -\frac{\hbar^2}{16}\end{aligned}$$

For $A = S_z$ and $B = S_y$:

$$\begin{aligned}\langle (\Delta S_z)^2 \rangle \langle (\Delta S_y)^2 \rangle &\geq \frac{1}{4} |\langle [S_z, S_y] \rangle|^2 \\ (\langle S_z^2 \rangle - \langle S_z \rangle^2) (\langle S_y^2 \rangle - \langle S_y \rangle^2) &\geq \frac{1}{4} |i\hbar S_x|^2 \\ \left(\frac{\hbar}{2} \right) \left(\frac{\hbar}{2} \right) &\geq \frac{1}{4} |i\hbar S_x|^2 \\ \frac{\hbar^2}{4} &\geq \frac{1}{4} |i\hbar \langle S_x; \pm | S_x; + \rangle|^2 \\ \frac{\hbar^2}{4} &\geq \frac{1}{4} |i\hbar 1|^2 \\ \frac{\hbar^2}{4} &\geq -\frac{\hbar^2}{4}\end{aligned}$$

1.7 (a)

$$\begin{bmatrix} \langle a^{(1)} | \alpha \rangle \langle \beta | a^{(1)} \rangle & \langle a^{(1)} | \alpha \rangle \langle \beta | a^{(2)} \rangle & \dots \\ \langle a^{(2)} | \alpha \rangle \langle \beta | a^{(1)} \rangle & \langle a^{(2)} | \alpha \rangle \langle \beta | a^{(2)} \rangle & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

(b)

$$\begin{bmatrix} \langle a^{(1)} | S_z; + \rangle \langle S_x; + | a^{(1)} \rangle & \langle a^{(1)} | S_z; + \rangle \langle S_x; + | a^{(2)} \rangle \\ \langle a^{(2)} | S_z; + \rangle \langle S_x; + | a^{(1)} \rangle & \langle a^{(2)} | S_z; + \rangle \langle S_x; + | a^{(2)} \rangle \end{bmatrix}$$

$$\begin{bmatrix} \frac{\hbar}{2} \cdot 0 & 0 \cdot \frac{\hbar}{2} \\ 0 \cdot \frac{\hbar}{2} & -\frac{\hbar}{2} \cdot 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

1.8