Math Methods Assignment #4

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1. Using the following conversions between cartesian coordinates and the coordinates of our system:

$$x_1 = l_1 \sin\varphi_1 \quad y_1 = -l_1 \cos\varphi_1$$
$$x_2 = l_1 \sin\varphi_1 + l_2 \sin\varphi_2 \quad y_2 = -l_1 \cos\varphi_1 - l_2 \cos\varphi_2$$

Taking the derivatives:

$$\dot{x}_1 = l_1 \dot{\varphi}_1 \cos\varphi_1 \quad \dot{y}_1 = l_1 \dot{\varphi}_1 \sin\varphi_1$$
$$\dot{x}_2 = l_1 \dot{\varphi}_1 \cos\varphi_1 + l_2 \dot{\varphi}_2 \cos\varphi_2 \quad \dot{y}_2 = l_1 \dot{\varphi}_1 \sin\varphi_1 + l_2 \dot{\varphi}_2 \sin\varphi_2$$

This gives us the following values for the kinetic and potential energy:

$$V = m_1 g y_1 + m_2 g y_2 = -m_1 g l_1 \cos\varphi_1 + m_2 g \left(-l_1 \cos\varphi_1 - l_2 \cos\varphi_2\right)$$
$$T = \frac{1}{2}\left[m_1\left(\dot{x}_1^2 + \dot{y}_1^2\right) + m_2\left(\dot{x}_2^2 + \dot{y}_2^2\right)\right]$$
$$T = \frac{1}{2}\left[m_1\left[(l_1\dot{\varphi}_1 \cos\varphi_1)^2 + (l_1\dot{\varphi}_1 \sin\varphi_1)^2\right] +\right.$$
$$\left. m_2\left[(l_1\dot{\varphi}_1\cos\varphi_1 + l_2\dot{\varphi}_2\cos\varphi_2)^2 + (l_1\dot{\varphi}_1\sin\varphi_1 + l_2\dot{\varphi}_2\sin\varphi_2)^2\right]\right]$$
$$T = \frac{1}{2}\left[m_1 l_1^2 \dot{\varphi}_1^2 + m_2\left(l_1^2\dot{\varphi}_1^2 + l_2^2\dot{\varphi}_2^2 + 2 l_1 l_2 \dot{\varphi}_1 \dot{\varphi}_2 \cos(\varphi_1 - \varphi_2)\right)\right]$$

2. In the case where $l_1 = l_2 = l$ and $m_1 = m_2 = m$:

$$V = mgl(2\cos\varphi_1 - \cos\varphi_2)$$
$$T = \frac{1}{2} l^2 m\left[2\dot{\varphi}_1^2 + \dot{\varphi}_2^2 + 2\dot{\varphi}_1\dot{\varphi}_2\cos(\varphi_1 - \varphi_2)\right]$$
$$L = \frac{1}{2} l^2 m\left[2\dot{\varphi}_1^2 + \dot{\varphi}_2^2 + 2\dot{\varphi}_1\dot{\varphi}_2\cos(\varphi_1 - \varphi_2)\right] - mgl(2\cos\varphi_1 - \cos\varphi_2)$$