Quantum I Assignment #4

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1.13 Assuming that the observable is $|E\rangle$ with eigenkets $a\,|1\rangle + b\,|2\rangle$:

$$H\,|E\rangle = (H_{11}\,|1\rangle\,\langle 1| + H_{22}\,|2\rangle\,\langle 2| + H_{12}[|1\rangle\,\langle 2| + |2\rangle\,\langle 1|])\,(a\,|1\rangle + b\,|2\rangle) \tag{1}$$
$$H\,|E\rangle = H_{11}a\,|1\rangle + H_{22}b\,|2\rangle + H_{12}b\,|1\rangle + H_{12}a\,|2\rangle \tag{2}$$

Since $H\,|E\rangle = \lambda\,|E\rangle = \lambda a\,|1\rangle + \lambda b\,|2\rangle$:

$$\lambda a = H_{11}a + H_{12}b \tag{3}$$
$$\lambda b = H_{22}b + H_{12}a \tag{4}$$

Using substition to solve for λ:

$$\frac{\lambda(a - H_{11})}{H_{12}} = b \tag{5}$$

$$\frac{\lambda(a - H_{11})}{H_{12}}(\lambda - H_{22}) = H_{12}a \tag{6}$$