HW March 4, Johannes Byle

8.4

$$\mathcal{L} = \frac{1}{2}\mu\dot{x}^2 - U(x)$$
$$\mathcal{L} = \frac{1}{2}\mu\dot{y}^2 - U(y)$$
$$\mathcal{L} = \frac{1}{2}\mu\dot{z}^2 - U(z)$$

The motion of a single particle in three dimension would be:

$$\mathcal{L} = \frac{1}{2}\mu(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - U(x, y, z)$$

Since $r = \sqrt{x^2 + y^2 + z^2}$

$$\mathcal{L} = \frac{1}{2}\mu(\dot{x}^2+\dot{y}^2+\dot{z}^2)-U(x, y, z) = \frac{1}{2}\mu\dot{r}-U(r)$$

8.12 (a)

$$U_{eff}(r) = -\frac{Gm_1m_2}{r} + \frac{l^2}{2\mu r^2}$$
$$\frac{dU_{eff}}{dr} = \frac{Gm_1m_2}{r^2} - \frac{l^2}{\mu r^3}$$
$$0 = r^2\Big(\frac{1}{Gm_1m_2} - \frac{r}{l^2}\Big)$$

It will orbit at:

$$r = \frac{l^2}{Gm_1m_2}$$

(b)

$$\frac{d^2U_{eff}}{dr^2} = -\frac{2Gm_1m_2}{r^3} + \frac{3l^2}{\mu r^4}$$
$$\frac{d^2U_{eff}}{dr^2} = -\frac{2Gm_1m_2}{(\frac{l^2}{Gm_1m_2})^3} + \frac{3l^2}{\mu(\frac{l^2}{Gm_1m_2})^4}$$
$$\frac{d^2U_{eff}}{dr^2} = -\frac{2(Gm_1m_2)^4}{l^6}+\frac{3(Gm_1m_2)^4l^2}{\mu l^8}$$
$$\frac{d^2U_{eff}}{dr^2} = -\frac{2(Gm_1m_2)^4}{l^6}+\frac{3(Gm_1m_2)^4l^2}{\mu l^8}$$
$$\frac{d^2U_{eff}}{dr^2} = \frac{(Gm_1m_2)^4}{l^6}\Big(\frac{3}{\mu} - 2\Big)$$

Whether or it is in a stable orbit is dependent on the value of μ