Quiz 1

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a)

I am expressing $|P\rangle$ as a matrix so that the elements can be more easily seen.

$$|P\rangle = \frac{1}{\sqrt{18}} \begin{bmatrix} -2\,|d_- u_+ u_+\rangle & |d_+ u_- u_+\rangle & |d_+ u_+ u_-\rangle \\ |u_- d_+ u_+\rangle & -2\,|u_+ d_- u_+\rangle & |u_+ d_+ u_-\rangle \\ |u_- u_+ d_+\rangle & |u_+ u_- d_+\rangle & -2\,|u_+ u_+ d_-\rangle \end{bmatrix}$$

$$\langle P|P\rangle = \left(\frac{1}{\sqrt{18}}\right)^2 \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix} = \frac{1}{18}\left(4+1+1+1+4+1+1+1+4\right) = 1$$

b)

$$\hat{\mathbf{S}} = \hat{S}_x \mathbf{i} + \hat{S}_y \mathbf{j} + \hat{S}_z \mathbf{k}$$

$$\hat{S}_x = \frac{\hat{S_+} + \hat{S}_-}{2}$$

$$\hat{S}_y = \frac{\hat{S_+} - \hat{S}_-}{2i}$$

$$\hat{S}_{1x}\hat{S}_{2x} = \frac{\left(\hat{S}_{1+} + \hat{S}_{1-}\right)\left(\hat{S}_{2+} + \hat{S}_{2-}\right)}{4} = \frac{\hat{S}_{1+}\hat{S}_{2+} + \hat{S}_{1-}\hat{S}_{2+} + \hat{S}_{2-}\hat{S}_{1+} + \hat{S}_{1-}\hat{S}_{2-}}{4}$$

$$\hat{S}_{1y}\hat{S}_{2y} = \frac{\left(\hat{S}_{1+} - \hat{S}_{1-}\right)\left(\hat{S}_{2+} - \hat{S}_{2-}\right)}{-4} = \frac{-\hat{S}_{1+}\hat{S}_{2+} + \hat{S}_{1-}\hat{S}_{2+} + \hat{S}_{2-}\hat{S}_{1+} - \hat{S}_{1-}\hat{S}_{2-}}{4}$$

$$\hat{S}_{1x}\hat{S}_{2x} + \hat{S}_{1y}\hat{S}_{2y} = \frac{2\hat{S}_{1-}\hat{S}_{2+} + 2\hat{S}_{2-}\hat{S}_{1+}}{4} = \frac{1}{2}\hat{S}_{2-}\hat{S}_{1+} + \frac{1}{2}\hat{S}_{1-}\hat{S}_{2+}$$

c)

The m values of the proton states are all $\frac{1}{2}$ because the m value is the sum of the spins of each q, and every proton state has two quarks with $m_s = \frac{1}{2}$ and one quark with $m_s = -\frac{1}{2}$. Thus:

$$\hat{S}_+ |P\rangle = \frac{1}{\sqrt{18}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Since I have already shown that $\hat{S}_{1x}\hat{S}_{2x} + \hat{S}_{1y}\hat{S}_{2y} = \frac{1}{2}\hat{S}_{2-}\hat{S}_{1+} + \frac{1}{2}\hat{S}_{1-}\hat{S}_{2+}$ and since all $\hat{S}_+$ values are zero the $\hat{S}_x$ and $\hat{S}_y$ components are zero.

$$\lambda \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2 = \lambda \left(0 + 0 + \hat{S}_{1z}\hat{S}_{2z}\right)$$

$$\hat{S}_{1z}\hat{S}_{2z} |P\rangle = \frac{1}{\sqrt{18}} \frac{1}{2} \frac{1}{2} \begin{bmatrix} -2\,|d_- u_+ u_+\rangle & |d_+ u_- u_+\rangle & |d_+ u_+ u_-\rangle \\ |u_- d_+ u_+\rangle & -2\,|u_+ d_- u_+\rangle & |u_+ d_+ u_-\rangle \\ |u_- u_+ d_+\rangle & |u_+ u_- d_+\rangle & -2\,|u_+ u_+ d_-\rangle \end{bmatrix}$$

$$\lambda \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2 |P\rangle = \lambda \frac{1}{4\sqrt{18}} \begin{bmatrix} -2\,|d_- u_+ u_+\rangle & |d_+ u_- u_+\rangle & |d_+ u_+ u_-\rangle \\ |u_- d_+ u_+\rangle & -2\,|u_+ d_- u_+\rangle & |u_+ d_+ u_-\rangle \\ |u_- u_+ d_+\rangle & |u_+ u_- d_+\rangle & -2\,|u_+ u_+ d_-\rangle \end{bmatrix}$$

d)

$$\langle P| \lambda \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2 |P\rangle = \lambda \frac{1}{4\sqrt{18}} \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix} = \lambda \frac{1}{4\sqrt{18}} (4+1+1+1+4+1+1+1+4) = \lambda \frac{1}{4}$$

e)

$$\lambda \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2 |P\rangle = \lambda \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \lambda \frac{9}{4}$$