Exam 1 Corrections

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Question 2

$$Z_{ABD} = \frac{1}{\sqrt{2}} e^{i\pi} \frac{1}{\sqrt{2}} e^{ikl_1} = -\frac{1}{2} e^{ikl_1}$$

In my answer I defined l_1 as the distance AB and l_2 as the distance AC. However, I see that it makes more sense to define l_1 as ABC and l_2 as ACD.

$$Z_{ACD} = \frac{1}{\sqrt{2}} e^{i\pi} e^{i\pi} \frac{1}{\sqrt{2}} e^{ikl_2} = -\frac{1}{2} e^{ikl_2}$$

In this problem I forgot about the second reflection, which would have added a second factor of $e^{i\pi}$. I also may have made a mistake with the amplitude, as $1 - r_1$ would not have been $\frac{1}{\sqrt{2}}$. For the second part, although I wrote down how to equation for Z I did not fully work it out. Thus all I need to do is simply plug in and solve from where I left of (using new definitions for l_1 and l_2).

$$P_{PM_2} = Z^* Z = \frac{1}{4}(-e^{-ikl_1} + e^{-ikl_2})(-e^{ikl_1} + e^{ikl_2})$$

$$P_{PM_2} = Z^* Z = \frac{1}{4}(2 - e^{-ik(l_1 - l_2)} - e^{ik(l_1 - l_2)})$$

I would not have known to use this trig identity, but using Euler's equation and $\sin^2 \theta + \cos^2 \theta = 1$ we get:

$$P_{PM_2} = \sin^2 [\frac{k(l_1 - l_2)}{2}]$$

In my answer I described what physically would need to occur to get 100% probability, but I did not solve any equation. If we set P_{PM_2} equal to one and solve the equation we get:

$$P_{PM_2} = \sin^2 [\frac{k(l_1 - l_2)}{2}] = 1$$

$$\frac{k(l_1 - l_2)}{2} = (n + \frac{1}{2})\pi$$

Question 3

I am not 100% sure if I actually made a mistake by having imaginary exponentials for $x > 0$ as I got my equation straight from the book (eq 4.116) and as long as $E < V_0$ k_0 is imaginary, making Ce^{ik_0x} real. I also checked my equations in Matlab and Wolfram alpha, and although T gives an imaginary number, R is indeed 1 as long as long as $E < V_0$ which means the equations should be at least half correct.

$$k = \frac{\sqrt{2mE}}{\hbar} = \sqrt{\frac{2m}{\hbar^2}E}$$

$$k_0 = \sqrt{k^2 - \frac{2mV_0}{\hbar^2}} = \sqrt{\frac{2m}{\hbar^2}(E - V_0)} = \sqrt{\frac{2m}{\hbar^2}\alpha}$$

$$R = \frac{(k-k_0)^2}{(k+k_0)^2} = \frac{(\sqrt{\frac{2m}{\hbar^2}E} - \sqrt{\frac{2m}{\hbar^2}\alpha})^2}{(\sqrt{\frac{2m}{\hbar^2}E} + \sqrt{\frac{2m}{\hbar^2}\alpha})^2} = \frac{(\sqrt{E} - \sqrt{\alpha})^2}{(\sqrt{E} + \sqrt{\alpha})^2}$$

If $E < V_0$ then $\alpha < 0$

$$R = \frac{(\sqrt{E} - \sqrt{-\alpha})^2}{(\sqrt{E} + \sqrt{-\alpha})^2} = \frac{(\sqrt{E} - i\sqrt{\alpha})^2}{(\sqrt{E} + i\sqrt{\alpha})^2}$$

$$R^*R = \left(\frac{(\sqrt{E} - i\sqrt{\alpha})^2}{(\sqrt{E} + i\sqrt{\alpha})^2}\right)\left(\frac{(\sqrt{E} + i\sqrt{\alpha})^2}{(\sqrt{E} - i\sqrt{\alpha})^2}\right) = 1$$

Question 4

This is the problem that I was the most confused by. I did not write down equation 3.96

$$c_n = \int_{-\infty}^{\infty} \psi_n^*(x)\Psi(x)dx$$

and the only equation I had down on my formula sheet for eigenvalues was equation 3.39

$$c_1(t) = \frac{1}{\sqrt{2}}e^{-iE_1t/\hbar}$$

Although I remembered the Dirac delta function from E&M I had no idea how to fit it in a integral, so I just pieced together the equations I had in the hopes of getting the right answer. If I had started correctly I would have probably gotten closer to solving it correctly.

$$c_n = \int_{-\infty}^{\infty} \psi_n^*(x)\Psi(x)dx$$

I knew that

$$\psi_n = \sin\frac{n\pi x}{L}$$

Thus

$$c_1 = \int_0^L \sin\frac{\pi x}{L}(x)\delta(x - \frac{L}{2})sdx$$

Since the delta function only exists where $x = \frac{L}{2}$ the integral becomes:

$$c_1 = \sin\frac{\pi}{2} = 1$$

$$c_2 = \sin\frac{2\pi}{2} = 0$$

$$P_n = |c_n|^2$$

Thus

$$\frac{P_2}{P_1} = \frac{|c_2|^2}{|c_1|^2} = 0$$

Editor - C:\Users\Johannes\Documents\Matlab\PHYS_331_Test.m

PHYS_331_Test.m

```matlab
m=1;
E=linspace(0,2,1000);
h=1.054571800*10^(-34);
V_0=1;
k=sqrt(2.*m.*E)./h;
k_0=sqrt(k.^2-(2.*m.*V_0)./h.^2);
R=((k-k_0).^2)./((k+k_0).^2);
T=(4.*k.*k_0)./((k+k_0).^2);
R_abs=abs(R);
T_abs=abs(T);
hold on
title('Energy vs Complex Conjugate of T and R')
xlabel('Energy')
ylabel('Complex Conjugates')
plot(E,T_abs,'LineWidth',2);
plot(E,R_abs,'LineWidth',2);
plot(E,T_abs+R_abs,'LineWidth',2);
legend({'T*T','R*R','T*T+R*R'},'Location','northeast')
```