Classical Mechanics Assignment #1

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1. (a) Starting with the forces, simplifying, and substituting variables:

$$\begin{aligned} F_{\text{total}} &= F_{\text{spring}} + F_{\text{damping}} + F_{\text{piston}} \\ m\ddot{x} &= -kx - m\nu\dot{x} + kX(t) \\ \ddot{x} + \frac{k}{m}x + \nu\dot{x} &= \frac{k}{m}(t) \\ \ddot{x} + \nu\dot{x} + \omega_0^2 x &= F_0(t) \end{aligned}$$

   (b) Complementary solution:

$$x(t) = e^{-\beta t}\left[A_1 e^{-i\sqrt{\omega_0^2-\beta^2}t} + A_2 e^{i\sqrt{\omega_0^2-\beta^2}t}\right]$$

   Particular solution:

$$\left\{\left\{x(t) \to \frac{cw^3 \cos e^{at}\left(t - \frac{2}{a}\right)}{a^2} + c_2 t + c_1\right\}\right\}$$