Quantum I Assignment #1

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Q-1 (a) $\tilde{\mathbf{K}}$ is hermitian iff:

$$\tilde{\mathbf{K}} = \tilde{\mathbf{K}}^\dagger$$

(b)

$$\tilde{\mathbf{K}} = |\phi\rangle\langle\psi|$$
$$\tilde{\mathbf{K}}^2 = (|\phi\rangle\langle\psi|)(|\phi\rangle\langle\psi|)$$

Because of the associative principle:

$$\tilde{\mathbf{K}}^2 = |\phi\rangle\langle\psi|\phi\rangle\langle\psi|$$

$\tilde{\mathbf{K}}^2$ is a projection operator if $\psi = \phi$

$$\tilde{\mathbf{K}}^2 = |\psi\rangle\langle\psi|\psi\rangle\langle\psi|$$
$$\tilde{\mathbf{K}}^2 = |\psi\rangle 1 \langle\psi|$$
$$\tilde{\mathbf{K}}^2 = |\psi\rangle\langle\psi|$$

(c) Assuming $\tilde{\mathbf{P}}_1 = |\phi\rangle\langle\phi|$ and $\tilde{\mathbf{P}}_2 = |\psi\rangle\langle\psi|$

$$\tilde{\mathbf{K}} = \lambda|\phi\rangle\langle\phi|\psi\rangle\langle\psi|$$

From equation 1.25 we know that the product of $\langle\phi|\psi\rangle$ is normally a complex number, therefore, assuming scalar multiplication with vectors is commutative:

$$\langle\phi|\psi\rangle = c$$
$$\tilde{\mathbf{K}} = \lambda|\phi\rangle c\langle\psi|$$

Since c is some arbitrary constant we can just combine c into λ

$$\tilde{\mathbf{K}} = \lambda|\phi\rangle\langle\psi|$$

1.1 Assumptions:

$$v_{th} = \sqrt{\frac{2k_bT}{m}}$$
$$t = \frac{\Delta x}{v_{th}}$$
$$F_z = ma_z = \mu_z\frac{\delta B_z}{\delta z}$$
$$\Delta z = \frac{1}{2}a_zt^2$$

Substituting variables:

$$\Delta z = \frac{1}{2}\mu_z \frac{\delta B_z}{\delta z}\frac{1}{m}\frac{\sqrt{\Delta x}}{2k_bT}m$$
$$\Delta z = \mu_z \frac{\delta B_z}{\delta z}\frac{\sqrt{\Delta x}}{4k_bT}$$
$$\Delta z = 9.27 \cdot 10^{-24} \cdot 10 \cdot \frac{1}{4 \cdot 1.38 \cdot 10^{-23} \cdot 1273.15} \approx 0.0013 \text{ m}$$

Since there are two spins that are sent into different directions:

$$\Delta z = 2 \cdot \left(9.27 \cdot 10^{-24} \cdot 10 \cdot \frac{1}{4 \cdot 1.38 \cdot 10^{-23} \cdot 1273.15}\right) \approx 2.6 \text{ mm}$$

1.3

$$\langle(\Delta A)^2\rangle \langle(\Delta B)^2\rangle \geq \frac{1}{4}\left|\langle[A,B]\rangle\right|^2$$

For $A = S_x$ and $B = S_y$:

$$\langle(\Delta S_x)^2\rangle \langle(\Delta S_y)^2\rangle \geq \frac{1}{4}\left|\langle[S_x,S_y]\rangle\right|^2$$
$$\left(\langle S_x^2\rangle - \langle S_x\rangle^2\right)\left(\langle S_y^2\rangle - \langle S_y\rangle^2\right) \geq \frac{1}{4}\left|i\hbar S_z\right|^2$$
$$(0)\left(\frac{\hbar}{2}\right) \geq \frac{1}{4}\left|i\hbar S_z\right|^2$$
$$0 \geq \frac{1}{4}\left|i\hbar \langle S_y;\pm|S_x;+\rangle\right|^2$$
$$0 \geq \frac{1}{4}\left|i\hbar\frac{1}{\sqrt{2}}\right|^2$$
$$0 \geq -\frac{\hbar^2}{16}$$

For $A = S_z$ and $B = S_y$:

$$\langle(\Delta S_z)^2\rangle \langle(\Delta S_y)^2\rangle \geq \frac{1}{4}\left|\langle[S_z,S_y]\rangle\right|^2$$
$$\left(\langle S_z^2\rangle - \langle S_z\rangle^2\right)\left(\langle S_y^2\rangle - \langle S_y\rangle^2\right) \geq \frac{1}{4}\left|i\hbar S_x\right|^2$$
$$\left(\frac{\hbar}{2}\right)\left(\frac{\hbar}{2}\right) \geq \frac{1}{4}\left|i\hbar S_z\right|^2$$
$$\frac{\hbar^2}{4} \geq \frac{1}{4}\left|i\hbar \langle S_x;\pm|S_x;+\rangle\right|^2$$
$$\frac{\hbar^2}{4} \geq \frac{1}{4}\left|i\hbar 1\right|^2$$
$$\frac{\hbar^2}{4} \geq -\frac{\hbar^2}{4}$$

1.7 (a)

$$\begin{bmatrix} \langle a^{(1)}|\alpha\rangle \langle\beta|a^{(1)}\rangle & \langle a^{(1)}|\alpha\rangle \langle\beta|a^{(2)}\rangle & \dots \\ \langle a^{(2)}|\alpha\rangle \langle\beta|a^{(1)}\rangle & \langle a^{(2)}|\alpha\rangle \langle\beta|a^{(2)}\rangle & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

(b)

$$\begin{bmatrix} \langle a^{(1)}|S_z;+\rangle \langle S_x;+|a^{(1)}\rangle & \langle a^{(1)}|S_z;+\rangle \langle S_x;+|a^{(2)}\rangle \\ \langle a^{(2)}|S_z;+\rangle \langle S_x;+|a^{(1)}\rangle & \langle a^{(2)}|S_z;+\rangle \langle S_x;+|a^{(2)}\rangle \end{bmatrix}$$
$$\begin{bmatrix} \frac{\hbar}{2}\cdot 0 & 0\cdot\frac{\hbar}{2} \\ 0\cdot\frac{\hbar}{2} & -\frac{\hbar}{2}\cdot 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

1.8 If $\tilde{\mathbf{A}}$ is degenerate. In that case both $|i\rangle$ and $|j\rangle$ have the same eigenvalue, so $\alpha\,|i\rangle + \alpha\,|j\rangle = \alpha\,(|i\rangle + |j\rangle)$ which is also an eigenket of $\tilde{\mathbf{A}}$.