Johannes Byle

9.4

$$\hat{\mathbf{R}} = \frac{m_1\hat{\mathbf{r}}_1 + m_2\hat{\mathbf{r}}_2}{m_1 + m_2}$$

$$\hat{\mathbf{p}} = \frac{m_1\hat{\mathbf{p}}_1 - m_2\hat{\mathbf{p}}_2}{m_1 + m_2}$$

$$\hat{\mathbf{r}} = \hat{\mathbf{r}}_1 - \hat{\mathbf{r}}_2$$

$$\hat{\mathbf{P}} = \hat{\mathbf{p}}_1 + \hat{\mathbf{p}}_2$$

$$[\hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2] = 0$$

$$\left[\hat{x}_i, \hat{P}_j\right] = [\hat{x}_{1i} - \hat{x}_{2i}, \hat{p}_{1j} + \hat{p}_{2j}] = (\hat{x}_{1i} - \hat{x}_{2i})(\hat{p}_{1j} + \hat{p}_{2j}) - (\hat{p}_{1j} + \hat{p}_{2j})(\hat{x}_{1i} - \hat{x}_{2i})$$

$$= [\hat{x}_{1i}, \hat{p}_{1j}] + [\hat{x}_{1i}, \hat{p}_{2j}] - [\hat{x}_{2i}, \hat{p}_{1j}] - [\hat{x}_{2i}, \hat{p}_{2j}] = i\hbar\delta_{ij} - i\hbar\delta_{ij} = 0$$

$$\mu_1 = \frac{m_1}{m_1 + m_2}$$

$$\mu_2 = \frac{m_2}{m_1 + m_2}$$

$$\left[\hat{X}_i, \hat{P}_j\right] = [\mu_1\hat{x}_{1i} + \mu_2\hat{x}_{2i}, \hat{p}_{1j} + \hat{p}_{2j}] = [\mu_1\hat{x}_{1i}, \hat{p}_{1j}] + [\mu_1\hat{x}_{1i}, \hat{p}_{2j}] + [\mu_2\hat{x}_{2i}, \hat{p}_{1j}] + [\mu_2\hat{x}_{2i}, \hat{p}_{2j}]$$

$$= \mu_1 i\hbar\delta_{ij} + \mu_2 i\hbar\delta_{ij} = i\hbar\delta_{ij}$$

$$[\hat{x}_i, \hat{p}_j] = [\hat{x}_{1i} - \hat{x}_{2i}, \mu_1\hat{p}_{1j} - \mu_2\hat{p}_{2j}] = [\hat{x}_{1i}, \mu_1\hat{p}_{1j}] - [\hat{x}_{1i}, \mu_2\hat{p}_{2j}] - [\hat{x}_{2i}, \mu_1\hat{p}_{1j}] - [\hat{x}_{2i}, \mu_2\hat{p}_{2j}]$$

$$= \mu_1 i\hbar\delta_{ij} + \mu_2 i\hbar\delta_{ij} = i\hbar\delta_{ij}$$

$$\left[\hat{X}_i, \hat{p}_j\right] = [\mu_1\hat{x}_{1i} + \mu_2\hat{x}_{2i}, \mu_1\hat{p}_{1j} - \mu_2\hat{p}_{2j}] = [\mu_1\hat{x}_{1i}, \mu_1\hat{p}_{1j}] + [\mu_1\hat{x}_{1i}, \mu_2\hat{p}_{2j}] - [\mu_2\hat{x}_{2i}, \mu_1\hat{p}_{1j}] - [\mu_2\hat{x}_{2i}, \mu_2\hat{p}_{2j}] = 0$$

9.9

$$\Delta E = hf = 6.63 \cdot 10^{-34} \cdot 1.15 \cdot 10^{11} = 7.62 \cdot 10^{-23} J$$

$$m_C = 2.00 \cdot 10^{-26} kg$$

$$m_O = 2.66 \cdot 10^{-26} kg$$

$$\mu = \frac{m_C m_O}{m_C + m_O} = 1.41 \cdot 10^{-26} kg$$

$$\Delta E = \frac{l(l+1)\hbar^2}{2\mu r_0^2} = \frac{\hbar^2}{\mu r_0^2}$$

$$r_0 = \sqrt{\frac{\hbar^2}{\mu \Delta E}} = \sqrt{\frac{(1.05 \cdot 10^{-34})^2}{1.41 \cdot 10^{-26} \cdot 7.62 \cdot 10^{-23}}} \approx 1.12 \cdot 10^{-10} m$$

9.13

$$\langle lm| L_x |lm\rangle = \langle lm| L_+ + L_- |lm\rangle$$

$$\langle lm| L_x^2 |lm\rangle = \langle lm| L_+^2 + L_-^2 + L_+L_- + L_-L_+ |lm\rangle$$

$$L_- |l,m\rangle = \sqrt{l(l+1) - m(m-1)}\hbar |l, m-1\rangle$$

$$L_+ |l,m\rangle = \sqrt{l(l+1) - m(m+1)}\hbar |l, m+1\rangle$$

$$\langle lm| L_x |lm\rangle = \langle lm| \sqrt{l(l+1) - m(m-1)}\hbar |l, m-1\rangle + \langle lm| \sqrt{l(l+1) - m(m+1)}\hbar |l, m+1\rangle = 0$$

$$L_+^2 |l,m\rangle = L_+\sqrt{l(l+1) - m(m+1)}\hbar |l, m+1\rangle = \sqrt{l(l+1) - m(m+1)}\hbar\sqrt{l(l+1) - (m+1)(m+2)}\hbar |l, m+2\rangle$$

$$\langle l,m| L_+^2 |l,m\rangle = 0$$

$$L_-^2 |l,m\rangle = L_-\sqrt{l(l+1) - m(m-1)}\hbar |l, m-1\rangle = \sqrt{l(l+1) - m(m+1)}\hbar\sqrt{l(l+1) - (m-1)(m-2)}\hbar |l, m-2\rangle$$

$$\langle l,m| L_-^2 |l,m\rangle = 0$$

$$L_+L_- |lm\rangle = L_+\sqrt{l(l+1) - m(m-1)}\hbar |l, m-1\rangle = \sqrt{l(l+1) - (m-1)(m)}\sqrt{l(l+1) - m(m-1)}\hbar^2 |l,m\rangle$$

$$L_+L_- |lm\rangle = (l(l+1) - (m-1)m)\hbar^2$$

$$L_-L_+ |lm\rangle = L_-\sqrt{l(l+1)-m(m+1)}\hbar\, |l,m+1\rangle = \sqrt{l(l+1)-(m+1)(m)}\sqrt{l(l+1)-m(m1)}\hbar^2 \,|l,m\rangle$$

$$L_+L_- |lm\rangle = \left(l(l+1)-(m+1)m\right)\hbar^2$$

$$\Delta L_x = \langle lm| L_x^2 |lm\rangle - \left(\langle lm| L_x |lm\rangle\right)^2 = \left(2l(l+1)-(m+1)(m-1)m^2\right)\hbar^2$$

$$\Delta L_y = \langle lm| L_y^2 |lm\rangle - \left(\langle lm| L_y |lm\rangle\right)^2 = \left(2l(l+1)-(m+1)(m-1)m^2\right)\hbar^2$$

$$\left(2l(l+1)-(m+1)(m-1)m^2\right)^2\hbar^4 \geq \frac{\hbar}{2}\left|\langle L_z\rangle\right|$$

9.16
a)

$$\hat{L}_- \to \frac{\hbar}{i}e^{-i\phi}\left(-i\frac{\delta}{\delta\theta} - \cot\theta\frac{\delta}{\delta\phi}\right)$$

$$Y_{1,1} = \sqrt{\frac{3}{8\pi}}e^{i\phi}\sin\theta$$

$$\hat{L}_-Y_{1,1} = \sqrt{\frac{3}{8\pi}}\frac{\hbar}{i}e^{-i\phi}\left(-i\frac{\delta}{\delta\theta}e^{i\phi}\sin\theta - \cot\theta\frac{\delta}{\delta\phi}e^{i\phi}\sin\theta\right)$$

$$\hat{L}_-Y_{1,1} = \sqrt{\frac{3}{8\pi}}\frac{\hbar}{i}e^{-i\phi}\left(-ie^{i\phi}\cos\theta - ie^{i\phi}\cos\theta\right) = 0$$

b)

$$Y_{1,1} = \sqrt{\frac{3}{8\pi}}e^{i\phi}\sin\theta$$

$$\hat{\mathbf{L}}^2 \to -\hbar^2\left[\frac{1}{\sin\theta}\frac{\delta}{\delta\theta}\left(\sin\theta\frac{\delta}{\delta\theta}\right) + \frac{1}{\sin^2\theta}\frac{\delta^2}{\delta\phi^2}\right]$$

$$\hat{\mathbf{L}}^2Y_{1,1} = -\sqrt{\frac{3}{8\pi}}\hbar^2\left[\frac{1}{\sin\theta}\frac{\delta}{\delta\theta}\left(\sin\theta\frac{\delta}{\delta\theta}\right)e^{i\phi}\sin\theta + \frac{1}{\sin^2\theta}\frac{\delta^2}{\delta\phi^2}e^{i\phi}\sin\theta\right]$$

$$\hat{\mathbf{L}}^2Y_{1,1} = -\sqrt{\frac{3}{8\pi}}\hbar^2\left[\frac{1}{\sin\theta}\frac{\delta}{\delta\theta}\left(\sin\theta\frac{\delta}{\delta\theta}\right)e^{i\phi}\sin\theta - \frac{1}{\sin\theta}e^{i\phi}\right]$$

$$\hat{\mathbf{L}}^2Y_{1,1} = -\sqrt{\frac{3}{8\pi}}\hbar^2\left[\frac{1}{\sin\theta}\frac{\delta}{\delta\theta}\sin\theta\cos\theta e^{i\phi} - \frac{1}{\sin\theta}e^{i\phi}\right]$$

$$\hat{\mathbf{L}}^2Y_{1,1} = -\sqrt{\frac{3}{8\pi}}\hbar^2\left[\frac{1}{\sin\theta}\left(\cos^2\theta - \sin^2\theta\right)e^{i\phi} - \frac{1}{\sin\theta}e^{i\phi}\right]$$

$$\hat{\mathbf{L}}^2Y_{1,1} = -\sqrt{\frac{3}{8\pi}}\hbar^2\left[\frac{1}{\sin\theta}\left(1 - 2\sin^2\theta\right)e^{i\phi} - \frac{1}{\sin\theta}e^{i\phi}\right]$$

$$\hat{\mathbf{L}}^2Y_{1,1} = -\sqrt{\frac{3}{8\pi}}\hbar^2\left[-2\sin^2\theta e^{i\phi}\right]$$

9.21

$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}$$

$$|\psi(t)\rangle = e^{-i\left(\frac{\hat{\mathbf{L}}^2}{2I}+\omega_0\hat{L}_z\right)t/\hbar}$$

$$\sqrt{\frac{3}{4\pi}}\sin\theta\sin\phi = Y_{1,1} + Y_{1,-1}$$

$$\langle\theta,\phi|\psi(t)\rangle = e^{-i\left(\frac{\hat{\mathbf{L}}^2}{2I}+\omega_0\hat{L}_z\right)t/\hbar}\left(Y_{1,1} + Y_{1,-1}\right)$$