HW Feb 18, Johannes Byle

5.1 (a)

$$mg = -kl_1$$
$$F_{pull} + mg = -kx - kl_1$$

Since $mg = -kl_1$:

$$F_{pull} = -kx$$

(b) If F_{pull} is removed, then $F = -kx$:

$$-\int kx dx = \frac{1}{2}kx^2 + C$$

5.4 (a) If we set mgh to be the distance from the center we can set $U = mgh$. The radial location where the rope is contacting the cylinder is also ϕ. Thus the length of the rope is $l_0 + R\phi$. Since the vertical is $y = Rsin\phi$:

$$U = mg((l_0 + R\phi)cos\phi - Rsin\phi)$$

If we assume that $sin\phi = \phi$ and $cos\phi = 1 - \frac{\phi^2}{2}$

$$U = mg((l_0 + R\phi)(1 - \frac{\phi^2}{2}) - R\phi)$$
$$U = mg(l_0 - l_0\frac{\phi^2}{2} + R\phi - R\phi\frac{\phi^2}{2} - R\phi)$$
$$U = mg(l_0 - l_0\frac{\phi^2}{2} - R\frac{\phi^3}{2})$$

If we assume that since ϕ is small ϕ^3 can be ignored:

$$U = mgl_0\frac{\phi^2}{2} + C$$

5.8 (a) Since $F = m\ddot{x} = -kx$. A solution to this differential equation is $x = Asin(\omega t)$. Thus:

$$-mA\omega^2 sin(\omega t) = -kAsin(\omega t)$$
$$-m\omega^2 = -k$$
$$\omega = \sqrt{\frac{k}{m}}$$
$$\omega = \frac{2\pi}{T} = 2\pi f$$
$$T = 2\pi\sqrt{\frac{m}{k}}, f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

(b)

$$0 = Acos(\delta)$$
$$40 = -A\omega sin(\delta)$$
$$\delta = \pi, A = -\frac{40}{\omega}$$