HW 17

Johannes Byle

October 20, 2020

5.3.5

Proof. Suppose R is a relation from X to Y, S is a relation from Y to Z, and $A \subseteq X$.

Further suppose $x \in \mathcal{I}_{S\circ R}(A)$, by definition of image there exists $c \in A$ such that $(c, x) \in S \circ R$. By definition of composition there exists some $y \in Y$ such that $(c, y) \in R$ and $(y, x) \in S$. Then, by definition of image $y \in \mathcal{I}_R(A)$, so by definition of image $x \in \mathcal{I}_S(\mathcal{I}_R(A))$. Therefore $\mathcal{I}_{S\circ R}(A) \subseteq \mathcal{I}_S(\mathcal{I}_R(A))$.

Further suppose $x \in \mathcal{I}_S(\mathcal{I}_R(A))$, by definition of image there is some $y \in Y$ such that $y \in \mathcal{I}_R(A)$, and by definition of image $x \in \mathcal{I}_S(Y)$. By definition of composition there exists some $y \in Y$ such that $(c, y) \in R$ and $(y, x) \in S$. Therefore $\mathcal{I}_S(\mathcal{I}_R(A)) \subseteq \mathcal{I}_{S\circ R}(A)$.

Thus, by definition of equality $\mathcal{I}_{S\circ R}(A) = \mathcal{I}_S(\mathcal{I}_R(A))$.

5.3.11

False:

$A = \{a_1, a_2\}$, $B = \{b_1\}$, $R = \{(a_1, b_2)\}$

5.4.3

Proof. Suppose $x \in \mathbb{R}$, by definition of less than or equal to $x \leq x$.

Hence, by definition of reflexive, $\leq$ is reflexive.

5.4.4

$0 \leq 1$ but $1 \nleq 0$

5.4.5

Proof. Suppose $a, b, c \in \mathbb{R}$, and suppose that $a \leq b$ and $b \leq c$. By definition of less than or equal to $b \leq b$, therefore by definition of less than or equal $a \leq c$.

Hence $\leq$ is transitive.

5.4.22

Proof. Suppose $(x, y) \in R \cup R^{-1}$. By definition of union $(x, y) \in R$ or $(x, y) \in R^{-1}$.

Suppose $(x, y) \in R$, by definition of inverse $(y, x) \in R^{-1}$.

Suppose $(x, y) \in R^{-1}$, by definition of inverse $(y, x) \in (R^{-1})^{-1}$, by definition of inverse of inverse $(y, x) \in R$.

Therefore, by division into cases and by definition of symmetric $R \cup R^{-1}$ is symmetric.

5.4.24

Proof. Suppose $(a, b), (b, c) \in R \cap S$. By definition of intersect $(a, b), (b, c) \in R$ and $(a, b), (b, c) \in S$. By definition of transitive $(a, c) \in R$ and by definition of transitive $(a, c) \in S$. Therefore, by definition of intersect $(a, c) \in R \cap S$, thus $R \cap S$ is transitive.

5.4.25

Suppose R is reflexive. Further suppose $a \in A$. By definition of reflex ive $(a, a) \in R$, so by definition of image $a \in \mathcal{I}_R(A)$.

Therefore by definition of subset $A \subseteq \mathcal{I}_R(A)$.

5.5.7

Proof. Suppose R and S are equivalence relations, also suppose $(x, y) \in R \cap S$. By definition of intersect $(x, y) \in R$ and $(x, y) \in S$.

By definition of reflexivity and definition of equivalence $(x, x) \in R$ and $(x, x) \in S$. Therefore by definition of reflexivity and definition of intersect $R \cap S$ is reflexive.

By definition of symmetry and definition of equivalence $(y, x) \in R$ and $(y, x) \in S$. Therefore by definition of symmetry and definition of intersect $R \cap S$ is symmetric.

Further suppose $(y, z) \in R \cap S$. By definition of transitivity and definition of equivalence $(x, z) \in R$ and $(x, z) \in S$. Therefore by definition of transitivity and definition of intersect $R \cap S$ is transitive.

Hence, by definition of equivalence $R \cap S$ is an equivalence relation.

5.5.9

$A = \{1, 2, 3\}$ $R = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

5.5.10

Proof. Suppose R is symmetric and transitive, and suppose the domain of A is equal to the domain of $\mathcal{I}_R(A)$, further suppose $x, y \in A$ such that $(x, y) \in R$.

By definition of symmetric $(y, x) \in R$, so by definition of transitive $(x, x) \in R$.

Therefore, by definition of reflexive R is reflexive.