Math Methods Assignment #4

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September 27, 2021

1. It would depend on whether the rocket rotated. Assuming that it's acceleration was constant in both the horizontal and vertical directions it would move in a parabola, unless v_0 is 0 in which case it would move in a straight line. If the rocket were allowed to rotate it would move in a parabolic path, as it would start asymptotically in horizontal direction, and then as it tilted down it would move asymptotically close to the vertical axis.

2. If we take a rhombus with corners A, B, C, D we can represent the sides as $\vec{AB}$, $\vec{BC}$, $\vec{CD}$, $\vec{DA}$. We know that $|\vec{AB}| = |\vec{BC}| = |\vec{CD}| = |\vec{DA}|$, and that $\vec{AC} = \vec{AB} + \vec{BC}$ and $\vec{BD} = \vec{BC} + \vec{CD}$. Since the sides of a rhombus are parallel we can for the sake of this problem consider $\vec{AC} = \vec{AB} + \vec{AD}$ and $\vec{BD} = \vec{AD} - \vec{AB}$. To show that the diagonals are orthogonal we need to show that the dot products are equal to zero: $\vec{AC} \cdot \vec{BD} = \left(\vec{AB} + \vec{AD}\right) \cdot \left(\vec{AD} - \vec{AB}\right) = \vec{AD}^2 - \vec{AB}^2 = 0$ since the sides all have equal length.

3. This is equivalent to $\sum_i \sum_j \sum_k \epsilon_{ijk}\epsilon_{ijk}$. The number of permutations of n numbers is $n!$, which means there are $3! = 6$ permutations in our case. Since whether the permutation is even or odd the term $\epsilon_{ijk}\epsilon_{ijk} = 1$ and otherwise 0 the sum is equal to 6.

4. Using the definition of the cross product $\boldsymbol{a} \times \boldsymbol{b} = \epsilon_{ijk}\hat{\boldsymbol{e}}_i a_j b_k$:

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{a} \times (\epsilon_{ijk}\hat{\boldsymbol{e}}_i b_j c_k) = \epsilon_{ijk}\hat{\boldsymbol{e}}_i a_j b_j c_k$$
$$(\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c} = (\epsilon_{ijk}\hat{\boldsymbol{e}}_i a_j b_k) \times \boldsymbol{c} = \epsilon_{ijk}\hat{\boldsymbol{e}}_i a_j b_j c_k$$