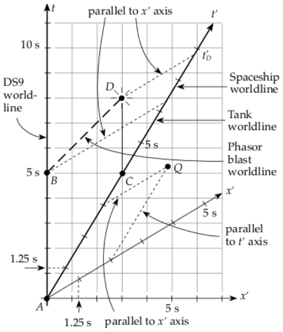
R5B.1 **(a)** According to the calibration procedure discussed in section R5.3, we first calculate the gamma factor for this situation

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(\frac{3}{5})^2}} = \frac{1}{\sqrt{1-\frac{9}{25}}}$$
$$= \frac{1}{\sqrt{\frac{16}{25}}} = \sqrt{\frac{25}{16}} = \frac{5}{4} \qquad (1)$$

Then if we want to draw marks that are $\Delta t'$ apart in the primed frame ($\Delta t' = 1$ s, in this case) we should space them along the t' axis so that these marks are $\gamma\Delta t'$ apart relative to the vertical axis. This means that marks along the t' axis for $t' = 1$ s, 2 s, 3 s, 4 s, … should be placed so their t-coordinates are $t = 1.25$ s, 2.5 s, 3.75 s, 5 s, etc. This has been correctly done in the drawing.



(b) To draw the x' axis, we draw a line with slope $\beta = 3/5$ from event A, then calibrate it in a similar way by drawing 1-s marks along the x' axis that are 1.25 s apart according to the horizontal axis, as shown in the diagram to the right.

(c) The ship drops the tank at event C. Since C occurs along the t' axis, we can read the t' coordinate of that event straight from the calibrated t' axis. According to the diagram, $t'_C = 4$ s.

(d) To determine the time when event D occurs in the spaceship frame, we draw from event D a line parallel to the x' axis until it hits the t' axis. I have done this on the diagram above, and I find that the line hits the t' axis at about 7.75 s. Therefore, $t'_D \approx 7.75$ s.

(e) The coordinates of event D in the DS9 frame are $t_D = 8$ s and $x_D = 3$ s. According to equation R5.11*a*,

$$t'_D = \gamma(t_D - \beta x_D) = \tfrac{5}{4}(8\text{ s} - \tfrac{3}{5}3\text{ s}) = \tfrac{5}{4}(\tfrac{40}{5} - \tfrac{9}{5})\text{ s} = \tfrac{31}{4}\text{ s} = 7\tfrac{3}{4}\text{ s} = 7.75\text{ s} \qquad (2)$$

[note that we calculated that $\gamma = \frac{5}{4}$ in part (a)].This is consistent with the answer to part (d).

(f) According to the diagram events B and C both occur at $t = 5$ s in the DS9 frame. Therefore, these events are simultaneous in that frame. To find the time when event B occurs in the spaceship's frame we draw from event B a line parallel to the x' axis until it hits the t' axis. I have done this on the diagram above, and I find that the line hits the t' axis at about $t'_B = 6.25$ s. On the other hand, we can read the time of event C directly from the t' axis since it lies on that axis: it looks like $t'_C = 4$ s. Therefore in the spaceship's frame, event C happens before event B. We can check the coordinate of event B using a Lorentz transformation equation. Event B has coordinates $t_B = 5$ s and $x_B = 0$ s in the DS9 frame. According to equation R5.11*a*, we have

$$t'_B = \gamma(t_B - \beta x_B) = \tfrac{5}{4}(5\text{ s} - \tfrac{3}{5}\cdot 0\text{ s}) = \tfrac{25}{4}\text{ s} = 6\tfrac{1}{4}\text{ s} = 6.25\text{ s} \qquad (3)$$

This is consistent with what the diagram appears to indicate.

(g) The coordinates of event D in the DS9 frame are $t_D = 8$ s and $x_D = 3$ s. According to equation R5.11*b*,

$$x'_D = \gamma(-\beta t_D + x_D) = \tfrac{5}{4}(-\tfrac{3}{5}8\text{ s} + 3\text{ s}) = \tfrac{5}{4}(\tfrac{24}{5} - \tfrac{15}{5})\text{ s} = -\tfrac{9}{4}\text{ s} = -2\tfrac{1}{4}\text{ s} = -2.25\text{ s} \qquad (4)$$

Note that in drawing, this event occurs to the *left* of the t' axis. This implies that the x' coordinate of that event should be negative, as we indeed find it to be.

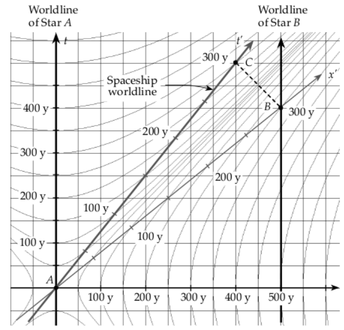
(h) To locate this event, we should draw a line parallel to the x' axis from the $t' = 3.0$ s mark on the t' axis, and then draw a line parallel to the t' axis from the $x' = 2.0$ s on the x' axis. The event is located where these two lines cross. I have done this on the drawing, and labeled the intersection event Q.

(i) The coordinates of event Q in the spaceship frame are $t'_Q = 3.0$ s and $x'_Q = 2$ s. According to equation R5.10*a* and R5.10*b*, we have $t_Q = \gamma(t'_Q + \beta x_Q)$ and $x_Q = \gamma(\beta t'_Q + x_Q)$, or in this case,

$$t_Q = \tfrac{5}{4}(3\text{ s} + \tfrac{3}{5}\cdot 2\text{ s}) = \tfrac{5}{4}(\tfrac{15}{5} + \tfrac{6}{5})\text{ s} = \tfrac{21}{4}\text{ s} = 5.25\text{ s}, \quad x_Q = \tfrac{5}{4}(\tfrac{3}{5}3\text{ s} + 2\text{ s}) = \tfrac{5}{4}(\tfrac{9}{5} + \tfrac{10}{5})\text{ s} = \tfrac{19}{4}\text{ s} = 4.75\text{ s} \qquad (5)$$

These results seem consistent with what the diagram shows.

R5M.1 **(a)** Let the explosion of star A be the origin event A that defines $t = t' = 0$ in both the Home Frame and the spaceship (Other) frame. In order for the stars to explode simultaneously in the spaceship's frame, the spaceship's x' axis must go through star B's explosion (event B) as well as event A. Since event B happens at $t_B = 400$ y and $x_B = 500$ y, this means that the slope of the x' axis in the spacetime diagram must be 4/5 relative to the Home Frame axes. Since this slope is supposed to be equal to β, the spaceship must be moving in the $+x$ direction with x-velocity $\beta = 4/5$. Therefore, the spaceship's worldline goes through event A and has a slope of $1/\beta = 5/4$. The two-observer spacetime diagram showing the situation appears below.



(b) If we use the hyperbola graph paper to calibrate the t' and x' axes, we see that event B occurs at $x'_B =$ 300 y in the spaceship's frame. We can check this with the Lorentz transformation equations as follows. Note that when $\beta = 4/5$, $\gamma = (1-\beta^2)^{-1/2} = (1-\frac{16}{25})^{-1/2} = (\frac{9}{25})^{-1/2} = \frac{5}{3}$. Therefore

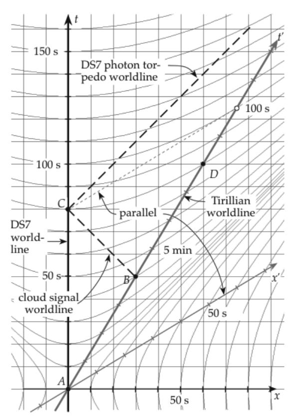
$$x'_B = \gamma(-\beta t_B + x_B) = \gamma(-\tfrac{4}{5}400\text{ y} + 500\text{ y}) = \gamma(-320\text{ y} + 500\text{ y}) = \tfrac{5}{3}(180\text{ y}) = 300\text{y}$$

This is consistent with the diagram.

(c) I have drawn the left-going light flash from event B on the diagram as a black dashed line. We see that the light from the explosion reaches the spaceship's worldline at event C, where $t'_C = 300$ y.

(d) In the spaceship's frame, event B happens at $t'_B = 0$ but 300 y from the spaceship. So it completely makes sense that the light arrives at the spaceship 300 y later.

R5M.6 **(a)** The spacetime diagram for this situation appears to the right.



(b) According to the problem statement, event B happens along the Tirillian worldline (at $x'_B = 0$) at Tirillian time $t'_B = 40$ s. Since the 40-s mark on the t' axis occurs at t coordinate of 50 s, event B must occur at $t_B = 50$ s. Since the spaceship moves at a speed of 3/5, the ship (and thus the event) must be 30 s from DS7 at this time. If we note that when $\beta = \frac{3}{5}$, then $\gamma = (1-\beta^2)^{-1/2} = (1-\frac{9}{25})^{-1/2} = (\frac{16}{25})^{-1/2} = \frac{5}{4}$, then we can use the Lorentz transformation equations to verify this:

$$t_B = \gamma[t'_B - \beta x'_B] = \tfrac{5}{4}[40\text{ s} + \tfrac{3}{5}\cdot 0\text{ s}] = 50\text{ s} \qquad (1a)$$
$$x_B = \gamma[\beta t'_B + x'_B] = \tfrac{5}{4}[\tfrac{3}{5}40\text{ s} + 0\text{ s}] = 30\text{ s} \qquad (1b)$$

(c) As the Tirillian spaceship is 30 s from DS7 when it goes through the cloud, it takes 30 s for the signal to get back to DS7. Since event B happened at $t_B = 50$ s in the Home (DS7) Frame, the signal is received at time $t_C = 50$ s + 30 s = 80 s. The event occurs on the DS9 worldline at this time.

(d) We can read t'_C from the diagram by drawing a line parallel to the x' axis from the event to the t' axis (thin gray line in the diagram). This suggests that $t'_C = 100$ s. We can use the Lorentz transformation equations to check this:

$$t_C = \gamma[t'_C - \beta x'_C] = \tfrac{5}{4}[80\text{ s} + \tfrac{3}{5}\cdot 0\text{ s}] = 100\text{ s} \qquad (2)$$

(e) As the dashed line shows, event D occurs at $t_D = 100$ s in the Home Frame, while event C occurs at $t_C = 80$ s. Therefore event C occurs first in the Home Frame. However, in the Tirillian frame, event D happens at $t'_D = 80$ s while event C happens at $t'_C = 100$ s, as we have just seen. Thus event D happens first in this frame.

(f) The Tirillians could *not* have made their decision to drop shields on the basis of knowing about event C, because in their frame it had not happened yet! (Note also that the photon torpedo travels as fast as anything can, and it has not even come close to reaching the Tirillians by the time of event D.)

R6B.1 We are given that the length L of the object in the given frame is half its rest length L_R, so $L = \frac{1}{2}L_R$. We can find the object's speed in the given frame by solving equation R6.2 for the speed $|\beta|$:

$$\tfrac{1}{2}L_R = L = \sqrt{1-\beta^2} \;\Rightarrow\; \tfrac{1}{2} = \sqrt{1-\beta^2} \;\Rightarrow\; \tfrac{1}{4} = 1-\beta^2 \;\Rightarrow\; |\beta| = \sqrt{1-\tfrac{1}{4}} = \sqrt{\tfrac{3}{4}} = 0.866.$$

R6B.8 ▾ Log Out

R6B.8 We are given that the rest length of the car is $L_R = 5.0$ m. The car's speed in SR units is

$$|\beta| = \left(30\,\frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)\left(\frac{1\,\cancel{\text{s}}}{3.0\times 10^{8}\,\cancel{\text{m}}}\right) = 1.0\times 10^{-7} \tag{1}$$

as measured by an observer on the ground. Note that this is much smaller than 1, so we are justified in using the binomial approximation. We can therefore use equation R6.2 and the binomial approximation to find the difference between the car's rest length and its length in the ground frame. The result is

$$L_R - L = L_R - L_R\sqrt{1-\beta^2} \approx L_R - L_R[1-\tfrac{1}{2}\beta^2] = \tfrac{1}{2}\beta^2 L_R = \tfrac{1}{2}(1.0\times 10^{-7})^2(5.0\,\cancel{\text{m}})\left(\frac{100\text{ fm}}{10^{-9}\,\cancel{\text{m}}}\right) = 25\text{ fm} \tag{2}$$

So the car is about 25 atomic nuclei shorter in the ground frame than it is in its own rest frame. This is incredibly tiny, but the car is really not moving very fast.

R6M.3 ▾ Log Out

R6M.3 Let Δt be the time between the muon's birth and decay in the earth frame and $\Delta\tau$ be the same in the muon's frame. The latter is a proper time because the muon is present at its own creation and decay events. If we assume that the muon moves at a constant velocity $\vec{\beta}$ between these events, then equation R4.7 tells us that these times are linked by the equation $\Delta\tau = [1-\beta^2]^{1/2}\,\Delta t$. In the earth frame, the muon moves a distance $L_R = 60$ km (this is the rest length of the part of the atmosphere that the muon covers), so we also have $|\beta| = L_R/\Delta t$. Putting these results together and eliminating the unknown time Δt yields

$$\Delta\tau = \Delta t\sqrt{1-\beta^2} = \frac{L}{|\beta|}\sqrt{1-\beta^2} = L\sqrt{\frac{1}{\beta^2}-1} \quad\Rightarrow\quad \left(\frac{\Delta\tau}{L_R}\right)^2 = \frac{1}{\beta^2}-1 \quad\Rightarrow\quad \beta^2 = \frac{1}{1+(\Delta\tau/L_R)^2} \tag{1}$$

So according to equation R6.2, the length to which the earth's atmosphere is contracted in the muon's frame is

$$L = L_R\sqrt{1-\beta^2} = L_R\sqrt{1-\frac{1}{1+(\Delta\tau/L_R)^2}} \tag{2}$$

Since $\Delta\tau/L_R = (1.52\times 10^{-6}\text{ s})(300{,}000\text{ km/s})/(60\text{ km}) = 7.6\times 10^{-3}$, this becomes

$$L = (60\text{ km})\sqrt{1-\frac{1}{1+(7.6\times 10^{-3})^2}} = 0.46\text{ km} \tag{3}$$

So in the muon's frame, the atmosphere is only about 460 m thick!

R6D.3 ▾ Log Out

R6D.3 **(a)** In the frame of the object, the object is at rest and my clock is observed to move along it from one end to the other. In the object's frame, the distance that the clock has to travel is simply L_R, the length of the object as measured in its own frame. The clock is moving at a speed $|\beta|$, so the time measured between events F and B in the object's frame must be $\Delta t' = L_R/|\beta|$.

(b) The distance between these events in the object's frame is $\Delta x' = L_R$, since the events occur at opposite ends of the object and the distance between the object's ends in its own frame is L_R by definition.

(c) Since my clock is inertial and present at both events, it measures the *spacetime interval* between these events: $\Delta t = \Delta s$. If the length L of the object in my frame is defined to be the time that it takes to pass my clock, then:

$$\begin{aligned} L &= |\beta|\Delta t = |\beta|\Delta s = |\beta|\sqrt{(\Delta t')^2-(\Delta x')^2} = |\beta|\sqrt{(L_R/|\beta|)^2 - L_R^2} \\ &= L_R|\beta|\sqrt{(1/\beta^2)-1} = L_R\sqrt{(\beta/\beta)^2-\beta^2} = L_R\sqrt{1-\beta^2} \end{aligned}$$

which is indeed equation R6.2.

R7B.1 ▾ Log Out

R7B.1 **(a)** In the earth frame, the time between these two events is $\Delta t = 0.0003\text{ s} \approx 300\ \mu\text{s}$. The distance between these events is:

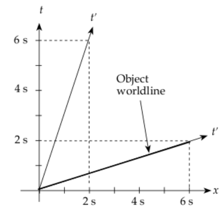
$$|\Delta\vec{d}| = 150\,\cancel{\text{km}}\left(\frac{1000\,\cancel{\text{m}}}{1\,\cancel{\text{km}}}\right)\left(\frac{1\,\cancel{\text{s}}}{3.0\times 10^{8}\,\cancel{\text{m}}}\right)\left(\frac{1\ \mu\text{s}}{10^{-6}\,\cancel{\text{s}}}\right) = 500\ \mu\text{s} \tag{1}$$

The events cannot be causally related, because a signal would have to travel faster than light in the earth frame to make it from event A to event B.

(b) Choose the x direction to be the direction of motion of the spaceship (which is also the direction of Camden relative to New York). We are told that $\Delta t'_{AB} = 0$ in the spaceship frame. According to equation R5.11a, we have

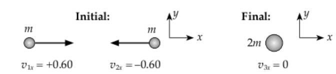
$$0 = \Delta t'_{AB} = \gamma[\Delta t_{AB} - \beta\Delta x_{AB}] \quad\Rightarrow\quad \Delta t_{AB} = \beta\Delta x_{AB} \quad\Rightarrow\quad \beta = \frac{\Delta t_{AB}}{\Delta x_{AB}} = \frac{300\,\cancel{\mu\text{s}}}{500\,\cancel{\mu\text{s}}} = \frac{3}{5} \tag{2}$$

R7D.3 If an object starting at $x = 0$ at $t = 0$ moves in the $+x$ direction at a speed of $|\vec{v}| > 1$ in the Home Frame, then the slope of its worldline drawn in a spacetime diagram based on the Home Frame will be $1/|\vec{v}| < 1$, as shown in the diagram below ($|\vec{v}| = 3$ in the case shown).



The slope of the x' axis for an Other Frame moving in the $+x$ direction with speed β is simply β. If we choose $\beta = 1/|\vec{v}|$ (= 1/3 in the case shown above), then the x' axis for the Other frame will have the same slope as the object's worldline, as shown. But since the x' axis by definition connects all events that occur at the same time $t' = 0$ in the Other Frame, we see that in the Other Frame, the object does not exist except at a single instant of time, and at that instant, the object is at many places at once. This is clearly absurd (another reason why objects cannot move faster than light).

R7M.7 The collision as viewed from the Home Frame looks as shown in the drawing below:

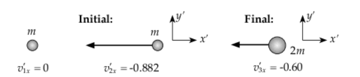


(a) We can use equation R7.14 to compute the velocities of these objects in an Other Frame that is moving in the $+x$ direction with speed $\beta = 0.60$. Equations R7.14*b* and R7.14*c* simply tell us that the velocities have no y or z components in the Other Frame if they do not in the Home Frame. Equation R7.14*a* tells us that

$$v'_{1x} = \frac{v_{1x} - \beta}{1 - \beta v_{1x}} = \frac{0.60 - 0.60}{1 - (0.60)^2} = 0, \qquad v'_{2x} = \frac{v_{2x} - \beta}{1 - \beta v_{2x}} = \frac{-0.60 - 0.60}{1 - (0.60)(-0.60)} = -\frac{1.20}{1.36} = -0.882 \qquad (1a,\ 1b)$$

$$v'_{3x} = \frac{v_{3x} - \beta}{1 - \beta v_{3x}} = \frac{0 - 0.60}{1 - (0.60)(0)} = -0.60 \qquad (1c)$$

It is no surprise that the right-going object has a speed of zero in this frame; in the Home Frame, both the object and the frame have the same x-velocity, so they should be at rest with respect to each other. Nor should it be a surprise that the final combined object, which was at rest in the Home Frame, is seen to move backward relative to the Other Frame at the same speed that the Home Frame does. The collision as viewed in the Other Frame thus looks like this:



(b) In the Home Frame, we have

$$\vec{p}_1 + \vec{p}_2 = m\begin{bmatrix} v_{1x} \\ v_{1y} \\ v_{1z} \end{bmatrix} + m\begin{bmatrix} v_{2x} \\ v_{2y} \\ v_{2z} \end{bmatrix} = m\begin{bmatrix} 0.60 - 0.60 \\ 0 \\ 0 \end{bmatrix} = 0, \quad \vec{p}_3 = 2m\begin{bmatrix} v_{3x} \\ v_{3y} \\ v_{3z} \end{bmatrix} = 2m\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 \qquad (2)$$

So $\vec{p}_1 + \vec{p}_2 = \vec{p}_3$ and momentum is indeed conserved in this frame.

(c) In the Other Frame, we see that

$$\vec{p}'_1 + \vec{p}'_2 = m\begin{bmatrix} v'_{1x} \\ v'_{1y} \\ v'_{1z} \end{bmatrix} + m\begin{bmatrix} v'_{2x} \\ v'_{2y} \\ v'_{2z} \end{bmatrix} = m\begin{bmatrix} 0 - 0.882 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.882m \\ 0 \\ 0 \end{bmatrix}, \quad \vec{p}_3 = 2m\begin{bmatrix} v'_{3x} \\ v'_{3y} \\ v'_{3z} \end{bmatrix} = 2m\begin{bmatrix} -0.60 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1.20m \\ 0 \\ 0 \end{bmatrix} \qquad (3)$$

We see here that $\vec{p}'_1 + \vec{p}'_2 \neq \vec{p}'_3$. so momentum is *not* conserved in this frame. The law of conservation of momentum is thus incompatible with the Einstein velocity transformation, a problem that we will address in the next chapter.