Lab 5: Cold Pill

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$$\frac{dx}{dt} = -k_1 x$$
$$\frac{dy}{dt} = k_1 x - k_2 y$$

(a) This system of differential equations can be expressed as the matrix:

$$\begin{bmatrix} -k_1 & 0 \\ k_1 & -k_2 \end{bmatrix}$$

Whose eigenvalues and eigenvectors are:

$$\lambda = -k_1, -k_2$$

$$\vec{v_1} = \begin{bmatrix} -\frac{k_1 - k_2}{k_1} \\ 1 \end{bmatrix}$$

$$\vec{v_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Thus the solution to this system of equations can be expressed as:

$$y(t) = C_1 e^{-k_1 t} \vec{v_1} + C_2 e^{-k_2 t} \vec{v_2}$$

(b) In this case our values are:

$$\lambda = -0.9, -0.6$$

$$\vec{v_1} = \begin{bmatrix} -\frac{1}{3} \\ 1 \end{bmatrix}$$

$$\vec{v_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Thus our solutions are:

$$x(t) = -C_1 e^{-0.9t} \frac{1}{3}$$
$$y(t) = C_1 e^{-0.9t} + C_2 e^{-0.6t}$$

Solving for C_1 and C_2 we get:

$$C_1 = -900$$

$$C_2 = 900$$

The plot of both these functions is:

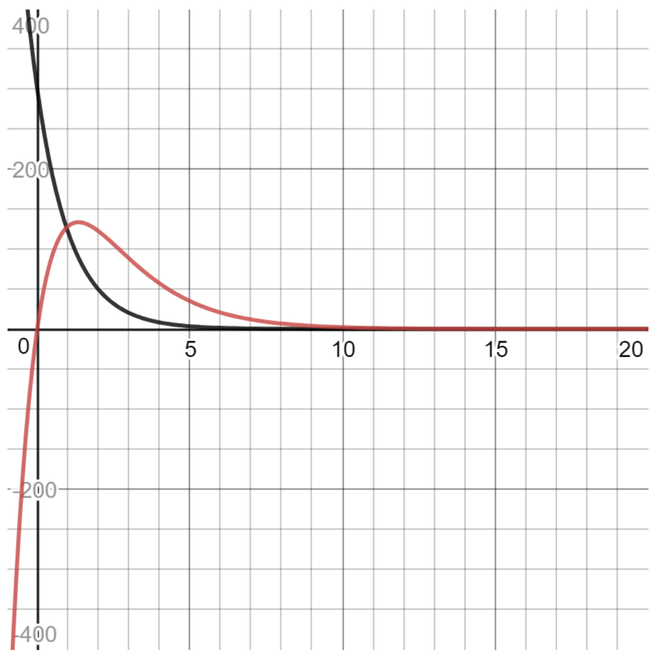

Figure 1: Graph of x(t) (black) and y(t) (red)

(c) The amount of drug in the bloodstream peaks when $\frac{dy}{dt} = 0$ where $y(t) = C_1e^{-0.9t} + C_2e^{-0.6t}$.

$$\frac{dy}{dt} = 810e^{-0.9t} - 540e^{-0.6t} = 0$$

Thus the amount of drug in the bloodstream peaks when $t = \frac{10}{3}ln(\frac{810}{540})$ or about 81 minutes. At this time the amount of drug in the bloodstream is $\frac{400}{3}$ or about 133.3 mg and the amount of blood in the GI-tract is $\frac{80}{9}$ or about 8.9 mg.

(d) To find the sensitivity of the peak times with varying k_1 one needs to define all the variables in terms of k_1.

$$C_1 = -300\frac{k1}{k_1 - 0.6}$$

$$C_2 = 300\frac{k1}{k_1 - 0.6}$$

Thus we can express the time when there it the peak amount of drug in the bloodstream as:

$$t = \frac{ln(\frac{k_1}{k_2})}{k_1 - k_2}$$

The sensitivity can be shown by plotting t vs k_1:

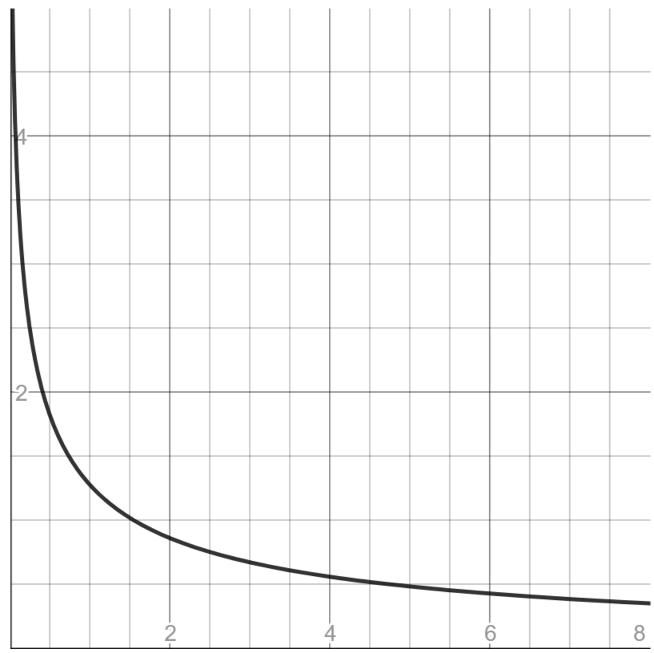

Figure 2: Graph of t vs k_1 where t is the vertical axis and k_1 is the horizontal axis

Plugging this value of t into the general equation for y we get:

$$y(t) = -300\frac{k1}{k_1 - 0.6}e^{-k_1\frac{ln(\frac{k_1}{0.6})}{k_1 - 0.6}}\vec{v_1} + 300\frac{k1}{k_1 - 0.6}e^{-0.6\frac{ln(\frac{k_1}{0.6})}{k_1 - 0.6}}\vec{v_2}$$

Which can be simplified as:

$$y(t) = 300\frac{k1}{k_1 - 0.6}[(\frac{k_1}{0.6})^{-\frac{0.6}{k_1 - 0.6}} - (\frac{k_1}{0.6})^{-\frac{k_1}{k_1 - 0.6}}]$$

Graphing $y(t)$ vs k_1 in this equation we get:

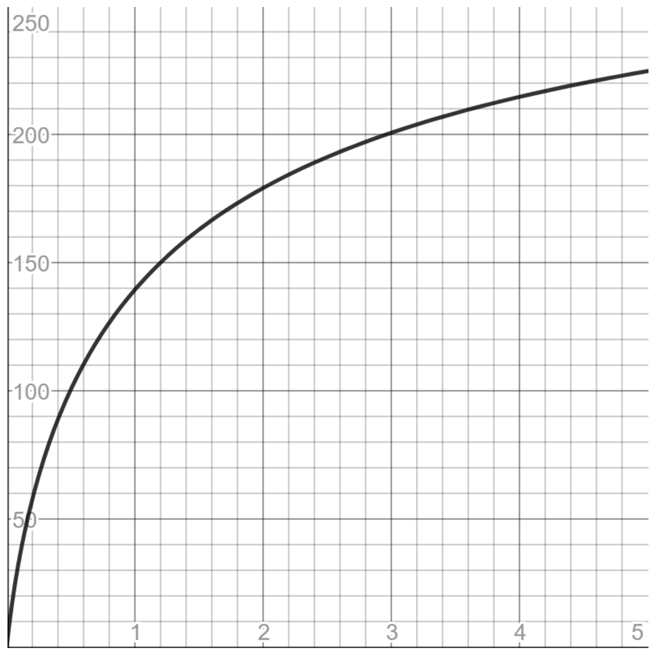

Figure 3: Graph of $y(t)$ vs k_1 where $y(t)$ is the vertical axis and k_1 is the horizontal axis

(e) If we set $t = 1.5$ we get:

$$1.5 = \frac{ln(\frac{k_1}{0.6})}{k_1 - 0.6}$$

Using WolframAlpha we were able to find that in the above equation:

$$k_1 \approx 0.738$$

Plugging this into the equation for y we can find out how many mg of the drug is at the maximum point:

$$y(t) = 300\frac{0.738}{0.738 - 0.6}[(\frac{0.738}{0.6})^{-\frac{0.6}{0.738-0.6}} - (\frac{0.738}{0.6})^{-\frac{0.738}{0.738-0.6}}] \approx 121.97mg$$