Name Solutions

Phys 331 – ST&Q – Exam 1 – Townsend Chapters 1-4
Feb 20, 2019 – 100 Points Total

Instructions:

- You may bring notes on one side of an 8.5"x11" sheet of paper.
- You may use a calculator. No other memory or computational assistance is allowed.
- If you run out of space, you may continue your work on a piece of scratch paper. Please make a note and staple the paper to the back of the test.
- Not all questions are worth the same number of points. Be aware of this as you prioritize your time.
- Read questions fully. For many questions, there are multiple parts.

1.) (20 pts) The wave function for a particle is given by

$$\Psi = \frac{e^{i\pi}}{2}\psi_1 + \frac{\sqrt{3}}{2}\psi_2$$

Where ψ_1 and ψ_2 are energy eigenfunctions with energy eigenvalues E_1 and E_2 respectively. What is the probability that a measurement of the energy yields the value E_1? What are $\langle E \rangle$ and ΔE? For this system, the energy eigenvalues are such that $E_2 = 4E_1$.

$c_1 = \frac{e^{i\pi}}{2}$, $c_2 = \frac{\sqrt{3}}{2}$

(7 pts) Probability of measuring E_1: $P_1 = |c_1|^2 = \boxed{\frac{1}{4}}$

(7 pts) Expectation value $\langle E \rangle = P_1E_1 + P_2E_2 = \frac{E_1 + 3E_2}{4} = \boxed{\frac{13E_1}{4}}$

(3 pts) Expectation value $\langle E^2 \rangle = P_1E_1^2 + P_2E_2^2 = \frac{E_1^2 + 3(16E_1^2)}{4} = \frac{49E_1^2}{4}$

(3 pts) Uncertainty $\Delta E = \sqrt{\langle E^2 \rangle - \langle E \rangle^2} = \sqrt{\frac{49E_1^2}{4} - \frac{169E_1^2}{16}}$

$$\boxed{= \frac{3\sqrt{3}}{4}E_1}$$

2.) (20 pts) In the figure below, each of the beam splitters are half-silvered mirrors. What is the probability amplitude that a photon arrives at PM_2 via path ABD? What is the probability amplitude that a photon arrives at PM_2 via path ACD? What is the probability that a photon arrives at PM_2? Give the condition for 100% probability that a photon arrives at PM_2.

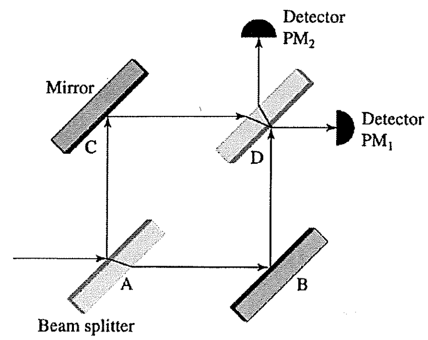

Figure 1.23 copyright 2009 University Science Books

(6 pts) $$Z_{ABD} = \frac{1}{\sqrt{2}} e^{i\pi} \frac{1}{\sqrt{2}} e^{ikl_1} = \boxed{-\frac{1}{2} e^{ikl_1}}$$

Let l_1 be the distance along path ABD, l_2 is distance along ACD.

(6 pts) $$Z_{ACD} = \frac{e^{i\pi}}{\sqrt{2}} e^{i\pi} \frac{1}{\sqrt{2}} e^{ikl_2} = \boxed{\frac{1}{2} e^{ikl_2}}$$

Probability of a photon arriving at PM_2:

$$P_{PM2} = (Z^*_{ABD} + Z^*_{ACD})(Z_{ABD} + Z_{ACD})$$

$$= \frac{1}{4}\left(-e^{-ikl_1} + e^{-ikl_2}\right)\left(-e^{ikl_1} + e^{ikl_2}\right)$$

$$= \frac{1}{4}\left(2 - e^{-ik(l_1-l_2)} - e^{ik(l_1-l_2)}\right)$$

$$= \frac{1}{2}\left[1 - \cos(k(l_1-l_2))\right]$$

(6 pts) $$\boxed{P_{PM2} = \sin^2\left[\frac{k(l_1-l_2)}{2}\right]}$$

(2 pts) For 100% probability $\frac{k(l_1-l_2)}{2} = (n+\frac{1}{2})\pi$ for $n = 0, 1, 2, \ldots$

3.) (20 pts) A free particle from the left is incident ~~from the left~~ on a step potential with energy $E<V_0$. Write the general solution for $\psi(x)$ in both regions. What terms of the general solution can you eliminate immediately? Why? Calculate the transmission and reflection coefficients T and R.

(10 pts) $\psi = \begin{cases} Ae^{ikx} + Be^{-ikx} & \text{for } x<0 \\ Ce^{\kappa x} + De^{-\kappa x} & \text{for } x>0 \end{cases}$

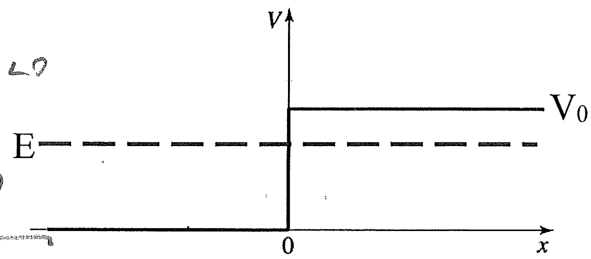

Figure 4.21 copyright 2009 University Science Books

$k = \sqrt{2mE/\hbar^2}$ $\kappa = \sqrt{2m(V_0-E)/\hbar^2}$

(2 pts) Can immediately say $C=0$ b/c otherwise $\psi = \infty$ at $x=\infty$.

Calculate T and R

For $x>0$,

$$j_x = \frac{\hbar}{2mi}\left[D^* e^{-\kappa x}\left(-\kappa D e^{-\kappa x}\right) - De^{-\kappa x}\left(-\kappa D^* e^{-\kappa x}\right)\right]$$

$$= 0$$

(4 pts) Because $j_x = 0$ for $x>0$, $\boxed{T=0.}$

(4 pts) Because $T+R=1$, $\boxed{R=1}$

Also, see example 4.3 p139

4.) (20 pts) At t=0, a particle is in an infinite square well

$V = 0$ for 0<x<L

$V = \infty$ elsewhere

with an unnormalized wavefunction $\Psi(x) = \delta(x - \frac{L}{2})$. Compared to the probability of measuring E_1, find the relative probability that a measurement of the particle's energy will be E_2.

To calculate P_2/P_1 we will need $|c_2|^2/|c_1|^2$

(7 pts)

$$c_1 = \int_{-\infty}^{\infty} \psi_1^* \, \delta(x - \tfrac{L}{2})\,dx = \int_0^L \sin\left(\frac{\pi x}{L}\right) \delta(x - \tfrac{L}{2})\,dx$$

$$= \sin\left(\frac{\pi L}{2L}\right) = 1$$

(7 pts)

$$c_2 = \int_{-\infty}^{\infty} \psi_2^* \, \delta(x - \tfrac{L}{2})\,dx = \int_0^L \sin\left(\frac{2\pi x}{L}\right) \delta(x - \tfrac{L}{2})\,dx$$

$$= \sin\left(\frac{2\pi L}{2L}\right) = 0$$

(6 pts)

$$\boxed{\frac{P_2}{P_1} = \frac{|c_2|^2}{|c_1|^2} = \frac{0}{1} = 0}$$

5.) (10 pts) What experimental evidence is there that light is a wave? What experimental evidence is there that light is a particle?

(5 pts) Light is a wave: interference seen with double slit, single slit, interferometers.

(5 pts) Light is a particle: scattering like a particle (Compton scattering), arriving at a detector like a particle i.e. being at a single location and not spread out like a wave (photon anticoincidence experiments).

6.) (10 pts) Why is the energy of a bound particle quantized while the energy of a free particle is not? Feel free to use a specific example (e.g. particle in an infinite square well) to support your argument.

(7 pts) To satisfy boundary conditions on ψ and $d\psi/dx$ only certain values of k and κ are allowed. Because k and κ both contain E, only certain values of E are allowed.

For example, in an infinite sq well, $\psi = 0$ at the boundaries. To make this happen, only an integer number of half wavelengths is allowed. This means there are only certain allowed values of λ, and therefore k, and therefore E. E is quantized.

(3 pts) A free particle is not subject to these restrictions.