General Formulas

$e^{i\theta} = \cos\theta + i\sin\theta$

$-\frac{\hbar^2}{2m}\frac{\delta\Psi(x,t)}{\delta x^2} + V(x)\Psi(x,t) = i\hbar\frac{\delta\Psi(x,t)}{\delta t}$

$\sin\alpha + \sin\beta = 2\cos\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}$

Light

$\mathcal{E}_0$=WaveAmplitude	n=IndexOfRefraction
k=Wavenumber	a=SingleSlitWidth
ω=Wavelength	h=Planck'sConstant
T=Period	K=KineticEnergy
ν=OrdinaryFrequency	W=WorkFunction
ϕ=PhaseShift	p=momentum
c=SpeedOfLightVacuum	

$\mathcal{E} = \mathcal{E}_0\cos(kx-\omega t)$ (1.1)

$k = \frac{2\pi}{\lambda}$ (1.2)

$\omega = \frac{2\pi}{T} = 2\pi\nu$ (1.3)

$\nu = 1/T$

$\mathcal{E} = \mathcal{E}_0\cos(kx-\omega t+\phi)$ (1.6)

$e^{i\theta} = \cos\theta + i\sin\theta$ (1.7)

$\omega = kc$ (1.11)

$\omega\nu = c$ (1.12)

$\frac{\delta^2\mathcal{E}}{\delta x^2} - \frac{n^2}{c}\frac{\delta^2\mathcal{E}}{\delta t^2} = 0$ (1.13)

$\lambda\nu = \frac{c}{n}$ (1.14)

$a\sin\theta = n\lambda$ (*minima*)

$E = h\nu$ (1.18)

$K = h\nu - W$ (1.19)

$h\nu_0 = hc/\lambda_0 = W$ (1.20)

$p = \frac{h}{\lambda}$ (1.21)

$\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$ (1.28) **Compton**

The First Principle of Quantum Mechanics

The probability of an event $= z^*z$ (1.32)

The Second Principle of Quantum Mechanics

To determine the probability amplitude for a process that can be viewed as taking place in a series of steps we multiply the probability amplitudes for each of these steps.

$z = z_a z_b \cdots$ (1.38)

The Third Principle of Quantum Mechanics

If there are multiple ways that an event can occur we add the amplitudes for each of these ways.

$z = z_1 + z_2 + \cdots$ (1.47)

$\phi = kx$

$z = x + iy = r\cos\phi + ir\sin\phi = re^{i\phi}$

$z^* = x - iy = r\cos\phi - ir\sin\phi = re^{-i\phi}$

Wave Mechanics

**In this section we assume a free particle, V(x)=0*

j=ProbabilityCurrent	Δx=Uncertainty
$\langle x\rangle$=ExpectationValue	

$\lambda = \frac{h}{p}$ (2.1) **de Broglie wavelength**

$d\sin\theta = n\lambda$ (2.3) (*maxima*)

$x_{n+1} - x_n = \frac{L\lambda}{d}$ (2.4)

$2d\sin\theta = n\lambda$ (2.5) **Bragg relation**

$-\frac{\hbar^2}{2m}\frac{\delta\Psi(x,t)}{\delta x^2} + V(x)\Psi(x,t) = i\hbar\frac{\delta\Psi(x,t)}{\delta t}$ (2.6)

$-\frac{\hbar^2}{2m}\frac{\delta\Psi(x,t)}{\delta x^2} = i\hbar\frac{\delta\Psi(x,t)}{\delta t}$ (2.7)

$\frac{\delta^2\mathcal{E}}{\delta x^2} = \frac{n^2}{c}\frac{\delta^2\mathcal{E}}{\delta t^2}$ (2.8)

$E = h\nu - \frac{h}{2\pi}2\pi\nu = \hbar\omega$ (2.9)

$p = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k$ (2.10)

$\hbar\omega = \hbar kc$ (2.11)

$E = pc$ (2.12)

$\hbar\omega = \frac{\hbar^2k^2}{2m}$ (2.15)

$p = \frac{h}{\lambda} = \hbar k$ (2.16)

$E = h\nu = \hbar\omega$ (2.17)

$E = \frac{p^2}{2m}$ (2.18)

$|\Psi(x,t)|^2dx$ =*the probability of finding the particle between x and x+dx at the time t if a measurement of the particle's position is carried out*

$|\Psi(x,t)|^2$ **probability density**

$\int_{-\infty}^{\infty}|\Psi(x,t)|^2dx = 1$ (2.19)

$\frac{\delta|\Psi|^2}{\delta t} = \frac{\Psi^*\Psi}{\delta t} = \Psi^*\frac{\delta\Psi}{\delta t} + \Psi\frac{\delta\Psi^*}{\delta t}$ (2.20)

$j_x(x,t) = \frac{\hbar}{2mi}(\Psi^*\frac{\delta\Psi}{\delta t} + \Psi\frac{\delta\Psi^*}{\delta t})$ (2.24)

$\frac{d}{dt}\int_{-\infty}^{\infty}|\Psi(x,t)|^2dx = -j_x(x,t)|_{-\infty}^{\infty} = 0$

$\Psi(x,t) = \int_{-\infty}^{\infty}A(k)e^{i(kx-\omega t)}dk$ (2.29)

$\Delta x\Delta k \geq \frac{1}{2}$ (2.30)

$\Delta x\Delta p_x \geq \frac{\hbar}{2}$ (2.31) **Heisenberg**

$v_{ph} = \frac{\omega}{k} = \frac{2\pi\nu}{(2\pi/\lambda)} = \lambda\nu$ (2.33)

The phase velocity is the speed at which a point on the wave, such as a crest, moves.

$v_{ph} = \frac{\omega}{k} = \frac{\hbar\omega}{\hbar k} = \frac{E}{p} = \frac{mv^2/2}{mv}\frac{v}{2}$ (2.34)

$v_g = \frac{d\omega}{dk}$ (2.36)

The group velocity is the speed of a localized packet of waves that has been generated by superposing many waves together

$\Psi(x,t) = \int_{-\infty}^{\infty}A(k)e^{i(kx-\omega t)}dk$ (2.37)

$\omega \cong \omega_0 + v_g(k-k_0)$ (2.39)

Dispersion relation is the relationship between ω *and* k $\langle x\rangle = \int_{-\infty}^{\infty}x|\Psi(x,t)|^2dx$ (2.53)

The average values $\langle x\rangle$ *are referred to as the expectation values*

$\langle x^2\rangle = \int_{-\infty}^{\infty}x^2|\Psi(x,t)|^2dx$ (2.55)

$(\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2$ (2.56)

Δx, *the standard deviation, is also called the uncertainty*

$(\Delta p_x)^2 = \langle p_x^2\rangle - \langle p_x\rangle^2$ (2.57)

$\frac{d\langle x\rangle}{dt} = \frac{\langle p_x\rangle}{m}$ (2.58)

$\langle p_x\rangle = \int_{-\infty}^{\infty}\Psi^*\frac{\hbar}{i}\frac{\delta\Psi}{\delta x}dx$ (2.63)

$\frac{d\langle p_x\rangle}{dt} = \langle -\frac{\delta V}{\delta x}\rangle$ (2.64)

The Time-Independent Schrödinger Equation

**In this section we assume V(x) is independent of t*

δ_{nm}=KroneckerDelta	ψ_a=Eigenfunction
a=Eigenvalue	T=TransmissionCoef.

$\Psi(x,t) = \psi(x)f(t)$ (3.2)

$\frac{\delta^2\Psi(x,t)}{\delta x^2} = f(t)\frac{d^2\psi(x)}{dx^2}$ (3.3)

$\frac{\delta\Psi(x,t)}{\delta t} = \psi(x)\frac{df(t)}{dt}$ (3.4)

$\frac{df(t)}{dt} = \frac{-iE}{\hbar}f(t)$ (3.8)

$-\frac{\hbar^2}{2m}\frac{\delta\psi(x)}{\delta x^2} + V(x)\psi(x) = E\psi(x)$ (3.9)

$f(t) = f(0)e^{-iEt/\hbar}$ (3.10)

$f(t) = f(0)e^{-i\omega t}$ (3.11)

$E = \hbar\omega$ (3.12)

$\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$ (3.13)

$|\Psi(x,t)|dd = |\psi(x)|^2$ (3.14)

$$V(x) = \begin{cases} 0, & 0<x<L. \\ \infty, & \text{elsewhere.}\end{cases}$$

$-\frac{\hbar^2}{2m}\frac{\delta\psi}{\delta x^2} = E\psi$ (3.16) $0<x<L$

$k^2 = \frac{2mE}{\hbar^2}$ (3.17)

$\psi(x) = A\sin kx + B\cos kx$ (3.21) $0<x<L$

$k_n = \frac{n\pi}{L}$ (3.26)

$E_n = \frac{\hbar k_n^2}{2m} = \frac{n\hbar^2\pi^2}{2mL^2}$ (3.27)

$\psi(x) = A_n\sin\frac{n\pi x}{L}$ (3.28) $0<x<L$

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}, & 0<x<L. \\ 0, & \text{elsewhere.}\end{cases}$$

$\Psi(x) = c_1\psi_1(x) + c_2\psi_2(x)$ (3.38)

$c_1(t) = \frac{1}{\sqrt{2}}e^{-iE_1t/\hbar}$ (3.39)

$\Psi = \sum_{n=1}^{\infty}c_n\psi_n(x)$ (3.40)

$$\delta_{nm} = \begin{cases} 1, & m=n. \\ 0, & m\neq n.\end{cases}$$

$\int_{-\infty}^{\infty}\psi_x^*(x)\psi_n(x)dx = \delta_{nm}$ (3.49)

$|c_n|^2 = P_n$ (3.59)

The above is the probability of obtaining E_n *if a measurement of the energy of a particle with wave function* Ψ *is carried out*

$\langle E\rangle = \sum_{n=1}^{\infty}|c_n|^2E_n$ (3.61)

$A_{op}\psi_a = a\psi_a$ (3.63)

$x_{op} = x$ (3.64)

$p_{xop} = \frac{\hbar}{i}\frac{\delta}{\delta x}$ (3.65)

$E_{op} = \frac{(p_{xop})^2}{2m} + V(x_{op})$ (3.71)

$H \equiv E_{op} = -\frac{\hbar^2}{2m}\frac{\delta^2}{\delta x^2} + V(x)$ (3.72)

$\langle E\rangle = \int_{-\infty}^{\infty}\Psi^*H\Psi dx$ (3.81)

One-Dimensional Potentials

$$V(x) = \begin{cases} 0, & |x|<a/2. \\ V_0, & |x|>a/2.\end{cases}$$

$k = \frac{\sqrt{2mE}}{\hbar}$

$\psi(x) = Ae^{ikx} + Be^{-ikx}$ $|x|<a/2$

$\kappa = \frac{\sqrt{2m(V_0-E)}}{\hbar} > 0$

$\psi(x) = Ce^{\kappa x} + De^{-\kappa x}$ $|x|>a/2$

$$\psi(x) = \begin{cases} Ce^{\kappa x}, & x\leq -a/2. \\ 2A\cos kx, & -a/2\leq x\leq a/2. \\ Ce^{-\kappa x}, & x\geq a/2.\end{cases}$$

$$V(x) = \begin{cases} 0, & x<0. \\ V_0, & x>0.\end{cases}$$

$k = \frac{\sqrt{2mE}}{\hbar}$

$k_0 = \sqrt{k^2 - \frac{2mV_0}{\hbar^2}}$

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx}, & x<0. \\ Ce^{ik_0x}, & x>0.\end{cases}$$

$$j_x = \begin{cases} \frac{\hbar k}{m}(|A|^2 - |B|^2), & x<0. \\ \frac{\hbar k_0}{m}|C|^2, & x>0.\end{cases}$$

$T \cong (\frac{4\kappa k}{k^2+\kappa^2})^2e^{-2\kappa a}$

Principles of Quantum Mechanics

Constants

$\hbar = 6.582 \times 10^{-16}$

$\epsilon_0 = 8.854 \times 10^{-12}$

$e = 1.602 \times 10^{-19}$

$k_B = 8.617 \times 10^{-5}$

$\Psi^*\Psi dx$ is the probability of finding the particle between x and $x + dx$

$\int_{-\infty}^{\infty} \Psi^*\Psi dx = 1$ (5.97)

A Hermitian operator satisfies:

$\int_{-\infty}^{\infty} \phi^* A_{op}\psi dx = \int_{-\infty}^{\infty} (A_{op}\phi)^*\psi dx$ (5.98)

$A_{op}\psi_a = a\psi_a$ (5.99)

Orthonormal wave functions satisfy:

$\Psi = \sum_a c_a\psi_a$ (5.100)

Probability of obtaining a:

$|c_a|^2 = \left| \int_{-\infty}^{\infty} \psi_a^*\Psi dx \right|^2$ (5.101)

Average or expectation value:

$\langle A \rangle = \sum_a |c_a|^2 a = \int_{-\infty}^{\infty} \Psi^* A_{op}\Psi dx$ (5.102)

Commutator:

$[A_{op}, B_{op}] = A_{op}B_{op} - B_{op}A_{op}$ (5.103)

If: $[A_{op}, B_{op}] = iC_{op}$ (5.104)

Then : $\Delta A \Delta B \geq \frac{|\langle C \rangle|}{2}$ (5.105)

$\Delta x \Delta p_x \geq \frac{\hbar}{2}$ (5.106)

$[x_{op}, p_{x_{op}}] = i\hbar$ (5.107)

$H\Psi(x,t) = i\hbar \frac{\delta\Psi(x,t)}{\delta t}$ (5.108)

Hamiltonian, the energy operator:

$H = -\frac{\hbar^2}{2m}\frac{\delta^2}{\delta x^2} + V(x)$ (5.109)

Expectation values $\frac{d\langle A \rangle}{dt}$ =:

$\frac{i}{\hbar}\int_{-\infty}^{\infty} \Psi^*[H, A_{op}]\Psi dx + \int_{-\infty}^{\infty} \Psi^* \frac{\delta A_{op}}{\delta t}\Psi dx$

If the Hamiltonian commutes withe the operator corresponding to the observable A and $\delta A_{op}/\delta t = 0$, then $\langle A \rangle$ is independent of time.

$\Delta E \frac{\Delta A}{\left|\frac{d<A>}{dt}\right|} \geq \frac{\hbar}{2}$ (5.111)

$\Delta E \Delta t \geq \frac{\hbar}{2}$ (5.112)

Parity operator:

$\Pi\psi(x) = \psi(-x)$ (5.4)

If there exists a single eigenfunction with eigenvalue a then its nondegenerate.

$x_{op} = x$ (3.64)

$p_{x_{op}} = \frac{\hbar}{i}\frac{\delta}{\delta x}$ (3.66)

Quantum Mechanics in Three Dimensions

Cubic Box:

$E_{n_x,n_y,n_z} = \frac{(n_x^2+n_y^2+n_z^2)\hbar^2\pi^2}{2mL^2}$ (6.104)

Hydrogenic Atom:

$E_n = -\frac{mZ^2e^4}{(4\pi\epsilon_0)^2 2\hbar^2 n^2} = -\frac{(1.36eV)Z^2}{n^2}$ (6.105)

Allowed values of l:

$l = 0, 1, 2, ..., n - 1$

Angular momentum operator:

$L_{op}^2 = L_{x_{op}}^2 + L_{y_{op}}^2 + L_{z_{op}}^2$

$L_{op}^2 Y_{l,m_l}(\theta,\phi) = l(l+1)\hbar^2 Y_{l,m_l}(\theta,\phi)$ (6.106)

The eigenfunctions Y_{l,m_l}, the spherical harmonics, also satisfy:

$L_{op}Y_{l,m_l}(\theta,\phi) = m_l\hbar Y_{l,m_l}(\theta,\phi)$ (6.107)

The energies (6.105) are independent of m_l because of the rotational symetry of the potential energy $-Ze^2/4\pi\epsilon_0 r$

The maximum value for L_z is $l\hbar$ which is always less than the total angular momentum $\sqrt{l(l+1)\hbar}$

$[L_{x_{op}}, L_{y_{op}}] = i\hbar L_{z_{op}}$

$[L_{y_{op}}, L_{z_{op}}] = i\hbar L_{x_{op}}$

$[L_{z_{op}}, L_{x_{op}}] = i\hbar L_{y_{op}}$ (6.108)

$\Delta L_x \Delta_y \geq \frac{\hbar}{2}|\langle L_z \rangle|$ (6.109)

Spin angular momentum S:

$\chi\pm$ are two dimensional column vectors:

$S_{op}^2\chi\pm = s(s+1)\hbar^2\chi\pm \ s = 1/2$ (6.110)

$S_{z_{op}}\chi\pm = \pm\frac{\hbar}{2}\chi\pm$ (6.111)

The magnetic moment:

$\mu = 2.00232\left(\frac{-e}{2m}\right)S$ (6.112)

The spherical harmonics with $l = 0, l = 1, andl = 2$

$Y_{0,0}(\theta,\phi) = \sqrt{\frac{1}{4\pi}}$

$Y_{1,\pm1}(\theta,\phi) = \sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\theta}$

$Y_{1,0}(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\cos\theta$

$Y_{2,\pm2}(\theta,\phi) = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{\pm 2i\theta}$

$Y_{2,\pm1}(\theta,\phi) = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{\pm i\theta}$

$Y_{2,0}(\theta,\phi) = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$

These spherical harmonics are the eigenfunctions.

For a three dimensional box from 0 to L the energy eigenfunctions (ψ_{n_x,n_y,n_z}) are:

$\left(\frac{2}{L}\right)^{3/2} \sin\frac{n_x\pi x}{L}\sin\frac{n_y\pi y}{L}\sin\frac{n_z\pi z}{L}$

The Energy Eigenvalues are:

$E_{n_x,n_y,n_z} = \frac{(n_x^2+n_y^2+n_z^2)\hbar^2\pi^2}{2mL^2}$

Identical Particles

Bosons are symmetric under particle exchange, Fermions are anti-symmetric. Bosons have integral intrinsic spin, Fermions have half integral intrinsic spin.

Valid Boson states:

$\Psi_S(1,2) = \psi_\alpha(1)\psi_\alpha(2)$ (7.104)

$\Psi_S(1,2) = \frac{1}{\sqrt{2}}[\psi_\alpha(1)\psi_\beta(2) + \psi_\beta(1)\psi_\alpha(2)]$

Bosons are more likely to be in the same state than are distinguishable particles.

The average number of identical Bosons in a state with energy E in thermal equilibrium at temperature T is given by:

$n(E) = \frac{1}{e^{(E-\mu)/k_bT}-1}$ (7.106)

Where μ is the chemical potential.

Valid Fermions:

$\Psi_A(1,2) = \frac{1}{\sqrt{2}}[\psi_\alpha(1)\psi_\beta(2) - \psi_\beta(1)\psi_\alpha(2)]$

The average number of identical Fermions in a state with energy E in thermal equilibrium at temperature T is given by:

$n(E) = \frac{1}{e^{(E-E_F)/k_bT}+1}$ (7.108)

Where E_F is the Fermi energy. Photons are Bosons, thus the distribution of electromagnetic energy in a cavity:

$\rho(\nu)d\nu = \frac{8\pi h\nu^3}{c^3(e^{h\nu/k_bT}-1)}$ (7.109)

Total energy per unit area per unit time:

σT^4 (7.67)

$\sigma = 5.67 \times 10^{-8}$ (7.68)

$\lambda_{max}T = 2.9 \times 10^{-3}$ (7.69)

$E_F = \frac{1}{2}mV_F^2$

$E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$

$E_{total} = \frac{3}{5}NE_F$

Solid State Physics

Bloch ansatz:

$\psi(x+a) = e^{i\theta}\psi(x)$ (8.4)

$\frac{\sqrt{2mE}}{\hbar} = k$ (8.10)

$\cos\theta = \cos ka + \frac{\alpha\sin ka}{2ka}$ (8.26)

Contact Potential: $(W_b - W_a)/e$

For semiconductors: $E_F = \frac{E_g}{2}$ (8.33)

$n(E) = \frac{1}{e^{E_g/2k_bT}+1}$ (8.34)

Nuclear Physics

$m_{nucleus} = Zm_p + (A-Z)m_n - B.E./c^2$

Where $B.E. \times (1, 0, -1)$ is:

$a_1A - a_2A^{2/3} - a_3\frac{Z^2}{A^{1/3}} - a_4\frac{(z-\frac{A}{2})^2}{A} + \frac{a_5}{\sqrt{A}}$

$N(t) = N(0)e^{-Rt} = N(0)e^{-t/\tau}$ (9.79)

The lifetime of an unstable nucleus:

$\tau = 1/R$

$t_{1/2} = \tau\ln 2$

$a_1 = 15.75MeV$

$a_2 = 17.8MeV$

$a_3 = 0.711MeV$

$a_4 = 94.8MeV$

$a_5 = 11.2MeV$