HW 14

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4.9.1
Proof. Suppose $A \subseteq B$. Further suppose $X \in \mathbb{P}(A)$. Then, by definition of powerset $X \subseteq A$. By definition of subset $X \subseteq B$.
Then by definition of powerset $X \in \mathbb{P}(B)$, therefore by definition of subset $\mathbb{P}(A) \subseteq \mathbb{P}(B)$ $\square$.

4.9.3
Proof. Suppose $X \in \mathbb{P}(A) \cap \mathbb{P}(B)$. Then, by definition of intersect $X \in \mathbb{P}(A)$ and $X \in \mathbb{P}(B)$. By definition of powerset $X \subseteq A$. By definition of powerset $X \subseteq B$. Then, by definition of intersect $X \subseteq A \cap B$.
Therefore by definition of powerset $X \in \mathbb{P}(A \cap B)$ $\square$.

4.9.4
Proof. Suppose $B \in \mathbb{P}(A - C)$. Further suppose $x \in B$. By definition of powerset $B \subseteq A - C$. Then, by definition of subset $x \in A - C$ Then, by definition of difference $x \in A$ and $x \notin C$. Then, by definition of subset $B \subseteq A$. Therefore, by definition of powerset $B \in \mathbb{P}(A)$ $\square$.

4.9.6
Proof. Suppose $a \in A$. Further suppose $X \in \mathbb{P}(A - \{a\}) \cap \{C \cup \{a\} | C \in \mathbb{P}(A - \{a\})\}$. By definition of intersect $X \in \mathbb{P}(A - \{a\})$ and $X \in \{C \cup \{a\} | C \in \mathbb{P}(A - \{a\})\}$ By definition of powerset $X \subseteq A - \{a\}$. Suppose $x \in X$. By definition of subset $x \in A - \{a\}$ Then, by definition of difference, $X \in A$ and $x \notin \{a\}$. By definition of union $a \in X \forall X \in \{C \cup \{a\} | C \in \mathbb{P}(A - \{a\})\}$, therefore $X \in \mathbb{P}(A - \{a\}) \cap \{C \cup \{a\} | C \in \mathbb{P}(A - \{a\})\} = \emptyset$.