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9.15 From equation 9.53 we know that the radial acceleration is:

$$\ddot{z} = -g + 2\Omega\dot{x}\sin\theta$$

At the north pole θ is 0

$$\ddot{z} = -g = g_0$$

At the equator θ is π

$$\ddot{z} = -g + 2\Omega\dot{x} = g_0\left(1 + \frac{2\Omega\dot{x}}{g_0}\right) = \lambda g_0$$

9.20 (a)

$$m\ddot{r} = F + 2m\dot{r} \times \Omega + m(\Omega \times r) \times \Omega$$

Since $r = x\hat{x} + y\hat{y}$

$$m\ddot{x} = F_x + 2m\dot{y}\Omega + m\Omega^2x$$

$$m\ddot{y} = F_y + 2m\dot{x}\Omega + m\Omega^2y$$

(b)

$$\eta = x + iy$$

$$\ddot{\eta} = 2i\Omega\dot{\eta} + \Omega^2\eta$$

$$\eta = e^{-i\alpha t}$$

$$\dot{\eta} = -i\alpha e^{-i\alpha t}$$

$$\ddot{\eta} = -\alpha^2 e^{-i\alpha t}$$

$$-\alpha^2 = 2\Omega\alpha + \Omega^2$$

$$\alpha = \Omega$$

Since this is only one solution we need to guess $te^{-i\alpha t}$

$$\eta = e^{-i\Omega t}(A + Bt)$$

(c)

$$\eta(0) = e^{-i\Omega(0)}(A + B(0)) = A = x_0$$

$$\dot{\eta}(0) = -i\Omega e^{-i\Omega(0)}(x_0 + B(0)) + Be^{-i\Omega(0)}$$

$$\dot{\eta}(0) = B - \Omega x_0 i = v_{x0} + v_{y0}i$$

$$B = v_{x0} + (v_{y0} + \Omega x_0)i$$

$$e^{it} = \cos t + i \sin t$$

$$x(t) = (x_0 + v_{x0}t) \cos \Omega t + (v_{y0} + \Omega x_0)t \sin \Omega t$$

$$y(t) = -(x_0 + v_{x0}t) \sin \Omega t + (v_{y0} + \Omega x_0)t \cos \Omega t$$

(d) It moves in a spiral

9.22

$$m\ddot{r} = q(Q + \dot{r} \times B)$$

$$m\ddot{r} = q(Q + \dot{r} \times B) + 2m\dot{r} \times \Omega + m(\Omega \times r) \times \Omega$$

Which is an ellipse