Johannes Byle

10.6

$$u = \rho e^{-\rho/2} F(\rho)$$

$$F(\rho) = \sum_{k=0}^{\infty} = c_k \rho^k$$

$$c_{k+1} = c_k \frac{k + l + 1 - \lambda}{(k+1)(k+2l+2)}$$

$$c_0 = 1$$

$$c_1 = \frac{1-3}{(1)(2)} = -1$$

$$c_2 = -\frac{1+1-3}{(1+1)(1+2)} = \frac{1}{6}$$

$$c_3 = \frac{1}{6}\frac{2+1-3}{(2+1)(2+2)} = 0$$

$$u = \rho e^{-\rho/2}\left(1 - \rho + \frac{1}{6}\rho^2\right)$$

10.9

$$u = A \sin k_0 x + B \cos k_0 x \quad |x| < a$$

$$u = C e^{-qx} \quad x > a$$

$$u = D e^{qx} \quad x < -a$$

Assuming the wavefunction is symmetric (A=0):

$$B \cos k_0 a = C e^{-qa}$$

$$B \cos -k_0 a = D e^{-qa}$$

Thus $C = D$

$$-k_0 B \sin k_0 a = -qDe^{-qa}$$

$$-k_0 B \sin -k_0 a = qDe^{-qa}$$

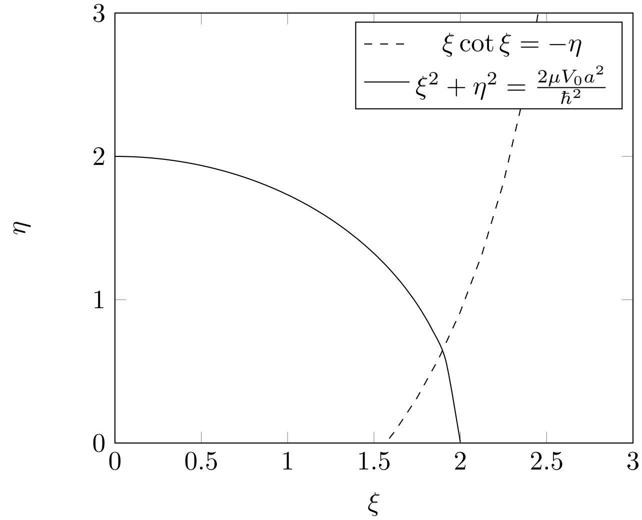
$$\frac{-k_0 B \sin -k_0 a}{B \cos -k_0 a} = \frac{qDe^{-qa}}{De^{-qa}}$$

$$-k_0 \tan -k_0 a = q$$

$$q = \sqrt{n^2}$$

$$\xi = k_0 a$$

$$\eta = qa$$



10.15
$n = 1$
$n_r = 0, l = 1$
Magic number= 6
$n = 2$
$n_r = 1, l = 0$
$n_r = 0, l = 2$
Magic number= 12
$n = 3$
$n_r = 1, l = 1$
$n_r = 0, l = 3$
Magic number= 22
$n = 4$
$n_r = 2, l = 0$
$n_r = 1, l = 2$
$n_r = 0, l = 4$
Magic number= 30

10.19

a)

$$\hat{H} = \frac{\hat{\mathbf{p}}_1^2}{2m_1} + \frac{\hat{\mathbf{p}}_2^2}{2m_2} + V_a(|\hat{\mathbf{r}}|) + \left(\frac{1}{4} - \frac{\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2}{\hbar^2}\right) V_b(|\hat{\mathbf{r}}|)$$

In the triplet state:

$$\hat{H} = \frac{\hat{\mathbf{p}}_1^2}{2m_1} + \frac{\hat{\mathbf{p}}_2^2}{2m_2} + V_a(|\hat{\mathbf{r}}|)$$

In the singlet state:

$$\hat{H} = \frac{\hat{\mathbf{p}}_1^2}{2m_1} + \frac{\hat{\mathbf{p}}_2^2}{2m_2} + V_a(|\hat{\mathbf{r}}|) + \frac{1}{4} V_b(|\hat{\mathbf{r}}|)$$

$V_a(|\hat{\mathbf{r}}|)$ can be ignored because $b < a$ Because this is a particle in a box:

$$\psi_n = A \sin\left(\frac{n\pi r}{a}\right)$$

Normalized:

$$\int_0^a A^2 \sin^2\left(\frac{\pi r}{a}\right) dr = 1$$

$$A=\sqrt{\frac{2}{a}}$$

Which is a singlet state.
b) The energies are:

$$E_n=\frac{\hbar^2\pi^2n^2}{2ma^2}$$

The results don't depend on how much larger a is than b, as long as a is greater we can ignore b.

11.3

$$\langle n|\hat{H}_1|n\rangle=\int_{-\infty}^{\infty}\left(\frac{1}{2}m\omega_1^2\frac{\hbar}{2m\omega}\left(\hat{a}+\hat{a}^\dagger\right)^2\right)^2dx=\frac{\hbar}{2m\omega}$$

$$E_n^{(2)}=\sum_{k\neq n}\frac{\left|\langle k|\hat{H}_1|n\rangle\right|^2}{\left(n+\frac{1}{2}\right)\hbar\omega-\left(k+\frac{1}{2}\right)\hbar\omega}$$

$$\langle k|\hat{H}_1|n\rangle=\frac{1}{2}m\omega_1^2\frac{\hbar}{2m\omega}\langle k|\hat{a}^2+2\hat{N}+\hat{a}^{\dagger 2}|n\rangle$$

$$\langle k|\hat{H}_1|n\rangle=\xi\sqrt{n+1}\sqrt{n+2}\langle k|n+2\rangle+2n\langle k|n\rangle+\sqrt{n}\sqrt{n-1}\langle k|n-2\rangle$$

$k\neq n$, thus:

$$\langle k|\hat{H}_1|n\rangle=\xi\sqrt{n+1}\sqrt{n+2}\langle k|n+2\rangle+\sqrt{n}\sqrt{n-1}\langle k|n-2\rangle$$

This compares to the real eigenvalues:

$$\hat{H}|n\rangle=\left(n+\frac{1}{2}\right)\hbar\omega|n\rangle$$

11.8

Equation 9.148 is a long equation, but the only ϕ dependence is $e^{im\phi}$

$$\langle n,l',m'|\hat{z}|n,l,m\rangle=C\int_0^\pi e^{im\phi}e^{-im'\phi}d\phi$$

$$C\int_0^\pi e^{im\phi}e^{im'\phi}d\phi=\frac{i\left(-1+e^{i\pi(m-m')}\right)}{m-m'}$$

Because $m-m'$ always is a integer:

$$\frac{i\left(-1+e^{i\pi(m-m')}\right)}{m-m'}=0$$

11.19

$$\frac{d}{dl'}\left(\frac{l'(l'+1)\hbar^2}{2\mu r}\right)=\frac{(2l'+1)\hbar^2}{2\mu r}$$

$$E=-\frac{\mu c^2Z^2\alpha^2}{2\left(n_r+l'+1\right)^2}$$

$$\frac{dE}{dl'}=-\frac{\mu c^2Z^2\alpha^2}{\left(n_r+l'+1\right)^3}$$

$$\frac{dl'}{d\gamma}=\frac{Z\left(n_r+l'+1\right)^3}{\mu c^2\alpha^2n^3a_0^2(l+\frac{1}{2})}$$

$$\frac{dE}{dl'\frac{dl'}{d\gamma}}=-\frac{\mu c^2Z^2\alpha^2}{\left(n_r+l'+1\right)^3}\frac{Z\left(n_r+l'+1\right)^3}{\mu c^2\alpha^2n^3a_0^2(l+\frac{1}{2})}=\frac{\gamma Z^3}{n^3a_0^2(l+\frac{1}{2})}$$