Johannes Byle

11.15

$U = (m_1+m_2)gL_1(1-\cos\phi_1)+m_2gL_2(1-\cos\phi_2)$

$T = \frac{1}{2}m_1L_1^2\dot{\phi_1}^2+\frac{1}{2}m_2[L_1^2\dot{\phi_1}^2+L_2^2\dot{\phi_2}^2+2L_1L_2\dot{\phi_1}\dot{\phi_2}\cos(\phi_1-\phi_2)]$

Assuming the small angle approximation:

$U = (m_1+m_2)gL_1\frac{\phi_1^2}{2}+m_2gL_2\frac{phi_2^2}{2}$

$T = \frac{1}{2}L_1^2\phi_1^2(m_1+m_2)+\frac{1}{2}m_2L_2^2\dot{\phi_2}^2+m_2L_1L_2\dot{\phi_1}\dot{\phi_2}$

Which give us the equations of motion which match (11.41) and (11.42):

$$\begin{bmatrix}(m_1+m_2)L_1^2 & m_2L_1L_2\\ m_2L_1L_2 & m_2L_2^2\end{bmatrix}\ddot{\phi} = -\begin{bmatrix}(m_1+m_2)gL_1 & 0\\ 0 & m_2gL_2\end{bmatrix}\phi$$

11.16(a)

$$K-\omega^2M = \begin{bmatrix}(m_1+m_2)(gL_1-\omega^2L_1^2) & -\omega^2m_2L_1L_2\\ -\omega^2m_2L_1L_2 & m_2(gL_2-\omega^2L_2^2)\end{bmatrix}$$

$\det(K-\omega^2M) = (m_1+m_2)(gL_1-\omega^2L_1^2)m_2(gL_2-\omega^2L_2^2)-(-\omega^2m_2L_1L_2)^2$