HW 16

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5.3.8
$\mathcal{I}_{R^{-1}}(\mathcal{I}_R(A)) \subseteq A$ is false
Proof. Counter example $X = \{x, a\}$, $Y = \{y\}$, $R = \{(x, y), (a, y)\}$ and $A = \{a\}$.
$\mathcal{I}_R(A) = \{y\}$
$\mathcal{I}_{R^{-1}}(\{y\}) = \{x, a\}$
$\{x, a\} \nsubseteq \{a\}$

Proof. Suppose $a \in A$. By definition of subset $a \in X$, then by definition of relation and by definition of image $Y \subseteq \mathcal{I}_R(A)$. Then by definition of image and by definition of relation $X \subseteq \mathcal{I}_{R^{-1}}(\mathcal{I}_R(A))$. By definition of subset $a \in \mathcal{I}_{R^{-1}}(\mathcal{I}_R(A))$, therefore $A \subseteq \mathcal{I}_{R^{-1}}(\mathcal{I}_R(A))$.

5.3.10
Proof. Suppose $(a, b) \in R$. By definition of inverse $(b, a) \in R^{-1}$, by definition again $(a, b) \in (R^{-1})^{-1}$. Therefore $R \subseteq (R^{-1})^{-1}$. Further suppose $(a, b) \in (R^{-1})^{-1}$. By definition of inverse $(b, a) \in R^{-1}$, by definition again $(a, b) \in R$. Therefore $(R^{-1})^{-1} \subseteq R$. Therefore, by definition of subset $(R^{-1})^{-1} = R$