Johannes Byle

10.5(a) By inspection we can know that the center in the xy direction is on the origin. We can find the center in the z direction by integrating from 0 to r in the z direction.

$$\frac{1}{2} = \int \pi(r^2 - z^2)dz$$

$$\frac{1}{2} = \pi r^2 - \pi\frac{1}{3}z^3$$

$$\sqrt[3]{3r^2 - \frac{3}{2\pi}} = z$$

$$CM = (0, 0, \sqrt[3]{3r^2 - \frac{3}{2\pi}})$$

10.14 The moment of inertia of a shell is:

$$I = \frac{2}{5}m\Big(\frac{r_2^5 - r_1^5}{r_2^3 - r_2^3}\Big)$$

Change in Angular momentum due to the flywheel is:

$$L = I\omega = \frac{1}{2}mr^2\omega$$

$$\frac{1}{2}m_1 r^2 \omega_1 = \frac{2}{5}m_2\Big(\frac{r_2^5 - r_1^5}{r_2^3 - r_1^3}\Big)\omega_2$$

$$5\frac{m_1}{m_2}\frac{r^2(r_2^3 - r_1^3)}{r_2^5 - r_1^5}\omega_1 = \omega_2$$

$$5\frac{10}{6000}\frac{0.1^2(6^3 - 5^3)}{6^5 - 5^5}1000 \approx 0.00163$$

$$\frac{10}{360}\frac{1}{0.00163} \approx 17.04\,\mathrm{min}$$

10.5(b)

$$W = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{1}{2}m_1 r^2 \omega_1^2$$

$$W = \frac{10}{4} \times 0.1^2 \times 1000^2 = 2500$$

10.23 Since it lies within the xy plane, $z = 0$ thus:

$$I_{xz} = I_{yz} = I_{zx} = I_{zy} = 0$$

$$I_{xx} = I_{yy} = \sum m_\alpha(y_\alpha^2 + z_\alpha^2) + \sum m_\alpha(x_\alpha^2 + z_\alpha^2)$$

Since z is zero this is equal to I_{zz}