Quiz 3

Johannes Byle

November 2, 2019

a)

Normalization

$$\psi_0 = Ae^{-ar^2/2}$$

$$a = \frac{m\omega}{\hbar} = \frac{\sqrt{mk}}{\hbar}$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} A^2 e^{-ar^2} r^2 \sin\phi dr d\phi d\theta = 1$$

$$\frac{\sqrt{\pi}A^2}{4a^{3/2}}\int_0^{2\pi}\int_0^{\pi} \sin\phi d\phi d\theta = 1$$

$$\frac{\sqrt{\pi}A^2}{2a^{3/2}}\int_0^{2\pi} d\theta = 1$$

$$\frac{\pi^{3/2}A^2}{a^{3/2}} = 1$$

$$A = \left(\frac{a}{\pi}\right)^{3/4}$$

Units

Variable	Units
m	kg
ω	rad/s
r	m
$\hbar$	$kg \cdot m^2 \cdot s^{-1}$

$$\psi_0 = \left(\frac{m\omega}{\hbar}\right)^{3/4} e^{-\frac{m\omega}{\hbar}r^2/2}$$

$$\psi_0 = \left(\frac{kg \cdot rad/s}{kg \cdot m^2 \cdot s^{-1}}\right)^{3/4} e^{-\frac{kg \cdot rad/s}{kg \cdot m^2 \cdot s^{-1}}m^2/2}$$

$$\psi_0 = \left(\frac{rad}{m^2}\right)^{3/4}$$

b)

$$\langle\psi_0|\, z^2\, |\psi_0\rangle = \langle\psi_0|\, r^2\cos^2\theta\, |\psi_0\rangle$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} A^2 e^{-ar^2} r^4\cos^2\theta\sin\phi dr d\phi d\theta$$

$$\frac{3\sqrt{\pi}A^2}{8a^{5/2}}\int_0^{2\pi}\int_0^{\pi}\cos^2\theta\sin\phi d\phi d\theta$$

$$\frac{3\sqrt{\pi}A^2}{4a^{5/2}}\int_0^{2\pi}\cos^2\theta d\theta$$

$$\frac{3\pi^{3/2}A^2}{4a^{5/2}} = \frac{3\pi^{3/2}}{4a^{5/2}}\left(\frac{a}{\pi}\right)^{3/2} = \frac{3}{4a}$$

c)

$$\langle\psi_0|\, x^2\, |\psi_0\rangle = \langle\psi_0|\, r^2\sin^2\theta\cos^2\phi\, |\psi_0\rangle$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} A^2 e^{-ar^2} r^4\sin^2\theta\cos^2\phi\sin\phi dr d\phi d\theta$$

$$\frac{3\sqrt{\pi}A^2}{8a^{5/2}}\int_0^{2\pi}\int_0^{\pi}\sin^2\theta\cos^2\phi\sin\phi d\phi d\theta$$

$$\frac{\sqrt{\pi}A^2}{4a^{5/2}}\int_0^{2\pi}\sin^2\theta d\theta$$

$$\frac{\pi^{3/2}A^2}{4a^{5/2}} = \frac{\pi^{3/2}}{4a^{5/2}}\left(\frac{a}{\pi}\right)^{3/2} = \frac{1}{4a}$$

d)

$$\langle\psi_0|\, \frac{1}{2}kr^2\, |\psi_0\rangle$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} A^2 e^{-ar^2}\frac{1}{2}kr^4\sin\phi dr d\phi d\theta$$

$$\frac{3\sqrt{\pi}kA^2}{16a^{5/2}}\int_0^{2\pi}\int_0^{\pi}\sin\phi d\phi d\theta$$

$$\frac{3\sqrt{\pi}kA^2}{8a^{5/2}}\int_0^{2\pi} d\theta$$

$$\frac{3\pi^{3/2}kA^2}{4a^{5/2}} = \frac{3\pi^{3/2}k}{4a^{5/2}}\left(\frac{a}{\pi}\right)^{3/2} = \frac{3k}{4a}$$

$$V = kg\cdot m^2\cdot s^{-3}\cdot A^{-1}$$

$$\frac{k\hbar}{m\omega} = \frac{k\cdot kg\cdot m^2\cdot s^{-1}}{kg\cdot rad/s} = k\cdot m^2$$

$$k = kg\cdot s^{-3}\cdot A^{-1}$$