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Q-1 (a) $\tilde{\mathbf{K}}$ is hermitian iff:

$$\tilde{\mathbf{K}} = \tilde{\mathbf{K}}^\dagger$$

(b)

$$\tilde{\mathbf{K}} = |\phi\rangle \langle\psi|$$
$$\tilde{\mathbf{K}}^2 = (|\phi\rangle \langle\psi|)(|\phi\rangle \langle\psi|)$$

Because of the associative principle:

$$\tilde{\mathbf{K}}^2 = |\phi\rangle \langle\psi|\phi\rangle \langle\psi|$$

$\tilde{\mathbf{K}}^2$ is a projection operator if $\psi = \phi$

$$\tilde{\mathbf{K}}^2 = |\psi\rangle \langle\psi|\psi\rangle \langle\psi|$$
$$\tilde{\mathbf{K}}^2 = |\psi\rangle\, 1\, \langle\psi|$$
$$\tilde{\mathbf{K}}^2 = |\psi\rangle \langle\psi|$$

(c) Assuming $\tilde{\mathbf{P}}_1 = |\phi\rangle \langle\phi|$ and $\tilde{\mathbf{P}}_2 = |\psi\rangle \langle\psi|$

$$\tilde{\mathbf{K}} = \lambda\, |\phi\rangle \langle\phi|\psi\rangle \langle\psi|$$

From equation 1.25 we know that the product of $\langle\phi|\psi\rangle$ is normally a complex number, therefore, assuming scalar multiplication with vectors is commutative:

$$\langle\phi|\psi\rangle = c$$
$$\tilde{\mathbf{K}} = \lambda\, |\phi\rangle\, c\, \langle\psi|$$
$$\tilde{\mathbf{K}} = \lambda\, |\phi\rangle \langle\psi| \quad \text{We can just combine } c \text{ into } \lambda$$

1.1 Assumptions:

$$v_{th} = \sqrt{\frac{2k_bT}{m}}$$
$$t = \frac{\Delta x}{v_{th}}$$
$$F_z = ma_z = \mu_z \frac{\delta B_z}{\delta z}$$
$$\Delta z = \frac{1}{2} a_z t^2$$

Substituting variables:

$$\Delta z = \frac{1}{2}\mu_z \frac{\delta B_z}{\delta z}\frac{1}{m}\frac{\sqrt{\Delta x}}{2k_bT}m$$
$$\Delta z = \mu_z \frac{\delta B_z}{\delta z}\frac{\sqrt{\Delta x}}{4k_bT}$$
$$\Delta z = 9.27 \cdot 10^{-24} \cdot 10 \cdot \frac{1}{4 \cdot 1.38 \cdot 10^{-23} \cdot 1273.15} \approx 0.0013 \text{ m}$$

1.3

$$\left\langle (\Delta A)^2 \right\rangle \left\langle (\Delta B)^2 \right\rangle$$

1.7

1.8