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3.1

a)

$$\left[\hat{A},\hat{B}+\hat{C}\right]=\hat{A}\left(\hat{B}+\hat{C}\right)-\left(\hat{B}+\hat{C}\right)\hat{A}$$

$$\left[\hat{A},\hat{B}+\hat{C}\right]=\hat{A}\hat{B}+\hat{A}\hat{C}-\hat{B}\hat{A}-\hat{C}\hat{A}$$

$$\left[\hat{A},\hat{B}\right]+\left[\hat{A},\hat{C}\right]=\hat{A}\hat{B}-\hat{B}\hat{A}+\hat{A}\hat{C}-\hat{C}\hat{A}$$

$$\left[\hat{A},\hat{B}\right]+\left[\hat{A},\hat{C}\right]=\hat{A}\hat{B}+\hat{A}\hat{C}-\hat{B}\hat{A}-\hat{C}\hat{A}$$

b)

$$\left[\hat{A},\hat{B}\hat{C}\right]=\hat{A}\hat{B}\hat{C}-\hat{B}\hat{C}\hat{A}$$

$$\hat{B}\left[\hat{A},\hat{C}\right]+\left[\hat{A},\hat{B}\right]\hat{C}=\hat{B}\left(\hat{A}\hat{C}-\hat{C}\hat{A}\right)+\left(\hat{A}\hat{B}-\hat{B}\hat{A}\right)\hat{C}$$

$$\hat{B}\left[\hat{A},\hat{C}\right]+\left[\hat{A},\hat{B}\right]\hat{C}=\hat{A}\hat{B}\hat{C}-\hat{B}\hat{C}\hat{A}$$

c)

$$\left[\hat{A}\hat{B},\hat{C}\right]=\hat{A}\hat{B}\hat{C}-\hat{C}\hat{A}\hat{B}$$

$$\hat{A}\left[\hat{B},\hat{C}\right]+\left[\hat{A},\hat{C}\right]\hat{B}=\hat{A}\left(\hat{B}\hat{C}-\hat{C}\hat{B}\right)+\left(\hat{A}\hat{C}-\hat{C}\hat{A}\right)\hat{B}$$

$$\hat{A}\left[\hat{B},\hat{C}\right]+\left[\hat{A},\hat{C}\right]\hat{B}=\hat{A}\hat{B}\hat{C}-\hat{C}\hat{A}\hat{B}$$

3.3

$$\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}0&-i\\i&0\end{bmatrix}+\begin{bmatrix}0&-i\\i&0\end{bmatrix}\begin{bmatrix}0&1\\1&0\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$$\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}1&0\\0&-1\end{bmatrix}+\begin{bmatrix}1&0\\0&-1\end{bmatrix}\begin{bmatrix}0&1\\1&0\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$$\begin{bmatrix}0&-i\\i&0\end{bmatrix}\begin{bmatrix}1&0\\0&-1\end{bmatrix}+\begin{bmatrix}1&0\\0&-1\end{bmatrix}\begin{bmatrix}0&-i\\i&0\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$$\begin{bmatrix}0&1\\1&0\end{bmatrix}^2=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$\begin{bmatrix}0&-i\\i&0\end{bmatrix}^2=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$\begin{bmatrix}1&0\\0&-1\end{bmatrix}^2=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

3.4

a)

$$\vec{\sigma}=\sigma_x\hat{i}+\sigma_y\hat{j}+\sigma_z\hat{k}$$

$$\vec{\sigma}\times\vec{\sigma}=\left[\sigma_y,\sigma_z\right]\hat{i}+\left[\sigma_x,\sigma_z\right]\hat{j}+\left[\sigma_x,\sigma_y\right]\hat{k}$$

$$\vec{\sigma}\times\vec{\sigma}=\begin{bmatrix}0&2i\\2i&0\end{bmatrix}\hat{i}+\begin{bmatrix}0&-2\\2&0\end{bmatrix}\hat{j}+\begin{bmatrix}2i&0\\0&-2i\end{bmatrix}\hat{k}$$

b)

$$\left(\sigma\cdot\mathbf{a}\right)\left(\sigma\cdot\mathbf{b}\right)=\left(\sigma_xa_x+\sigma_ya_y+\sigma_za_z\right)\left(\sigma_xb_x+\sigma_yb_y+\sigma_zb_z\right)$$

I am writing $\left(\sigma\cdot\mathbf{a}\right)\left(\sigma\cdot\mathbf{b}\right)$ as a matrix so that the expansion is easier to see.

$$\left(\sigma\cdot\mathbf{a}\right)\left(\sigma\cdot\mathbf{b}\right)=\begin{bmatrix}\sigma_x^2a_xb_x&\sigma_x\sigma_ya_xb_y&\sigma_x\sigma_za_xb_z\\\sigma_y\sigma_xa_yb_x&\sigma_y^2a_yb_y&\sigma_y\sigma_za_yb_z\\\sigma_z\sigma_xa_zb_x&\sigma_z\sigma_ya_zb_y&\sigma_z^2a_zb_z\end{bmatrix}$$

$$\sigma_x^2a_xb_x+\sigma_y^2a_yb_y+\sigma_z^2a_zb_z=\left(\mathbf{a}\cdot\mathbf{b}\right)I$$

$$\sigma_x\sigma_ya_xb_y+\sigma_x\sigma_za_xb_z+\sigma_y\sigma_xa_yb_x+$$

$$\sigma_y\sigma_za_yb_z+\sigma_z\sigma_xa_zb_x+\sigma_z\sigma_ya_zb_y=i\sigma\cdot\left(\mathbf{a}\times\mathbf{b}\right)$$

3.10

$$\left(\frac{\hbar}{2}\right)^2\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}0&-i\\i&0\end{bmatrix}-\left(\frac{\hbar}{2}\right)^2\begin{bmatrix}0&-i\\i&0\end{bmatrix}\begin{bmatrix}0&1\\1&0\end{bmatrix}=\frac{\hbar^2i}{2}\begin{bmatrix}1&0\\0&-1\end{bmatrix}$$

3.15

$$\hat{S}_x\rightarrow\frac{1}{\sqrt{2}}\begin{bmatrix}0&1&0\\1&0&1\\0&1&0\end{bmatrix}$$

$$\left|1,1\right\rangle_x\rightarrow\begin{bmatrix}-1\\0\\1\end{bmatrix}$$

$$\left|1,1\right\rangle_x=-\left|1,1\right\rangle+\left|1,-1\right\rangle$$

Normalized:

$$\left|1,1\right\rangle_x=-\frac{1}{\sqrt{2}}\left|1,1\right\rangle+\frac{1}{\sqrt{2}}\left|1,-1\right\rangle$$

3.23

$$\begin{bmatrix}0&\sqrt{3}&0&0\\\sqrt{3}&0&2&0\\0&2&0&\sqrt{3}\\0&0&\sqrt{3}&0\end{bmatrix}\frac{1}{2\sqrt{2}}\begin{bmatrix}1\\\sqrt{3}\\\sqrt{3}\\1\end{bmatrix}=\frac{3}{2\sqrt{2}}\begin{bmatrix}1\\\sqrt{3}\\\sqrt{3}\\1\end{bmatrix}$$