Classical Assignment #7

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October 25, 2021

1. From Goldstein page 153 we know that we can go from body coordinates to space axes using the following relation $\mathbf{x} = \mathbf{A}^{-1}\mathbf{x'}$. Since $\mathbf{A} = \mathbf{BCD}$:

$$\mathbf{BCD} = \begin{pmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} \cos(\psi)\cos(\phi) - \cos(\theta)\sin(\psi)\sin(\phi) & \cos(\theta)\sin(\psi)\cos(\phi) + \cos(\psi)\sin(\phi) & \sin(\theta)\sin(\psi) \\ -\cos(\theta)\cos(\psi)\sin(\phi) - \sin(\psi)\cos(\phi) & \cos(\theta)\cos(\psi)\cos(\phi) - \sin(\psi)\sin(\phi) & \sin(\theta)\cos(\psi) \\ \sin(\theta)\sin(\phi) & \sin(\theta)(-\cos(\phi)) & \cos(\theta) \end{pmatrix}$$

$$\mathbf{A}^{-1} = \begin{pmatrix} \cos(\psi)\cos(\phi) - \cos(\theta)\sin(\psi)\sin(\phi) & -\cos(\theta)\cos(\psi)\sin(\phi) - \sin(\psi)\cos(\phi) & \sin(\theta)\sin(\phi) \\ \cos(\theta)\sin(\psi)\cos(\phi) + \cos(\psi)\sin(\phi) & \cos(\theta)\cos(\psi)\cos(\phi) - \sin(\psi)\sin(\phi) & \sin(\theta)(-\cos(\phi)) \\ \sin(\theta)\sin(\psi) & \sin(\theta)\cos(\psi) & \cos(\theta) \end{pmatrix}$$

$$\omega_{bf} = \begin{pmatrix} \theta'\cos(\psi) + \sin(\theta)\sin(\psi)\phi' \\ \sin(\theta)\cos(\psi)\phi' - \theta'\sin(\psi) \\ \cos(\theta)\phi' + \psi' \end{pmatrix}$$

$$\mathbf{A}^{-1}\omega_{bf} = \begin{pmatrix} \theta'\cos(\phi) + \sin(\theta)\psi'\sin(\phi) \\ \theta'\sin(\phi) - \sin(\theta)\psi'\cos(\phi) \\ \cos(\theta)\psi' + \phi' \end{pmatrix}$$