General Formulas

$e^{i\theta} = \cos\theta + i\sin\theta$

$-\frac{\hbar^2}{2m}\frac{\delta\Psi(x,t)}{\delta x^2} + V(x)\Psi(x,t) = i\hbar\frac{\delta\Psi(x,t)}{\delta t}$

$\sin\alpha + \sin\beta = 2\cos\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}$

Light

$\mathcal{E}_0$=*WaveAmplitude*	n=*IndexOfRefraction*
k=*Wavenumber*	a=*SingleSlitWidth*
ω=*Wavelength*	h=*Planck'sConstant*
T=*Period*	K=*KineticEnergy*
ν=*OrdinaryFrequency*	W=*WorkFunction*
ϕ=*PhaseShift*	p=*momentum*
c=*SpeedOfLightVacuum*	

$\mathcal{E} = \mathcal{E}_0\cos(kx - \omega t)$ (1.1)

$k = \frac{2\pi}{\lambda}$ (1.2)

$\omega = \frac{2\pi}{T} = 2\pi\nu$ (1.3)

$\nu = 1/T$

$\mathcal{E} = \mathcal{E}_0\cos(kx - \omega t + \phi)$ (1.6)

$e^{i\theta} = \cos\theta + i\sin\theta$ (1.7)

$\omega = kc$ (1.11)

$\omega\nu = c$ (1.12)

$\frac{\delta^2\mathcal{E}}{\delta x^2} - \frac{n^2}{c}\frac{\delta^2\mathcal{E}}{\delta t^2} = 0$ (1.13)

$\lambda\nu = \frac{c}{n}$ (1.14)

$a\sin\theta = n\lambda$ *(minima)* Single Slit

$E = h\nu$ (1.18)

$K = h\nu - W$ (1.19)

$h\nu_0 = hc/\lambda_0 = W$ (1.20)

$p = \frac{h}{\lambda}$ (1.21)

$\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$ (1.28) **Compton**

The First Principle of Quantum Mechanics

The probability of an event $= z^*z$ (1.32)

The probability of detecting a particle is equal to z^*z, where z is called the probability amplitude and z^* is its conjugate

The Second Principle of Quantum Mechanics

To determine the probability amplitude for a process that can be viewed as taking place in a series of steps we multiply the probability amplitudes for each of these steps. Examples of this are propagation of a photon from a light source or a beam splitter, transmission at the beam splitter, and propagation to a photo detector

$z = z_a z_b \cdots$ (1.38)

The Third Principle of Quantum Mechanics

If there are multiple ways that an event can occur we add the amplitudes for each of these ways.

$z = z_1 + z_2 + \cdots$ (1.47)

$\phi = kx$

$z = x + iy = r\cos\phi + ir\sin\phi = re^{i\phi}$

$z^* = x - iy = r\cos\phi - ir\sin\phi = re^{-i\phi}$

Wave Mechanics

**In this section we assume a free particle, V(x)=0*

j=*ProbabilityCurrent*	Δx=*Uncertainty*
$\langle x\rangle$=*ExpectationValue*	

$\lambda = \frac{h}{p}$ (2.1) **de Broglie wavelength**

$d\sin\theta = n\lambda$ (2.3) *(maxima)* Double Slit

$x_{n+1} - x_n = \frac{L\lambda}{d}$ (2.4)

$2d\sin\theta = n\lambda$ (2.5) **Bragg relation**

$-\frac{\hbar^2}{2m}\frac{\delta\Psi(x,t)}{\delta x^2} + V(x)\Psi(x,t) = i\hbar\frac{\delta\Psi(x,t)}{\delta t}$ (2.6)

$-\frac{\hbar^2}{2m}\frac{\delta\Psi(x,t)}{\delta x^2} = i\hbar\frac{\delta\Psi(x,t)}{\delta t}$ (2.7)

$\frac{\delta^2\mathcal{E}}{\delta x^2} = \frac{n^2}{c}\frac{\delta^2\mathcal{E}}{\delta t^2}$ (2.8)

$E = h\nu - \frac{h}{2\pi}2\pi\nu = \hbar\omega$ (2.9)

$p = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k$ (2.10)

$\hbar\omega = \hbar kc$ (2.11)

$E = pc$ (2.12)

$\hbar\omega = \frac{\hbar^2k^2}{2m}$ (2.15)

$p = \frac{h}{\lambda} = \hbar k$ (2.16)

$E = h\nu = \hbar\omega$ (2.17)

$E = \frac{p^2}{2m}$ (2.18)

$|\Psi(x,t)|^2dx$ =*the probability of finding the particle between x and x+dx at the time t if a measurement of the particle's position is carried out*

$|\Psi(x,t)|^2$ **probability density**

$\int_{-\infty}^{\infty}|\Psi(x,t)|^2dx = 1$ (2.19)

$\frac{\delta|\Psi|^2}{\delta t} = \frac{\Psi^*\Psi}{\delta t} = \Psi^*\frac{\delta\Psi}{\delta t} + \Psi\frac{\delta\Psi^*}{\delta t}$ (2.20)

$j_x(x,t) = \frac{\hbar}{2mi}(\Psi^*\frac{\delta\Psi}{\delta x} - \Psi\frac{\delta\Psi^*}{\delta x})$ (2.24)

$\frac{d}{dt}\int_{-\infty}^{\infty}|\Psi(x,t)|^2dx = -j_x(x,t)|_{-\infty}^{\infty} = 0$

$\Psi(x,t) = \int_{-\infty}^{\infty}A(k)e^{i(kx-\omega t)}dk$ (2.29)

$\Delta x\Delta k \geq \frac{1}{2}$ (2.30)

$\Delta x\Delta p_x \geq \frac{\hbar}{2}$ (2.31) **Heisenberg**

$v_{ph} = \frac{\omega}{k} = \frac{2\pi\nu}{(2\pi/\lambda)} = \lambda\nu$ (2.33)

The phase velocity is the speed at which a point on the wave, such as a crest, moves.

$v_{ph} = \frac{\omega}{k} = \frac{\hbar\omega}{\hbar k} = \frac{E}{p} = \frac{mv^2/2}{mv}\frac{v}{2}$ (2.34)

$v_g = \frac{d\omega}{dk}$ (2.36)

The group velocity is the speed of a localized packet of waves that has been generated by superposing many waves together

$\Psi(x,t) = \int_{-\infty}^{\infty}A(k)e^{i(kx-\omega t)}dk$ (2.37)

$\omega \cong \omega_0 + v_g(k-k_0)$ (2.39)

Dispersion relation is the relationship between ω and k

$\langle x\rangle = \int_{-\infty}^{\infty}x|\Psi(x,t)|^2dx$ (2.53)

The average values $\langle x\rangle$ are referred to as the expectation values

$\langle x^2\rangle = \int_{-\infty}^{\infty}x^2|\Psi(x,t)|^2dx$ (2.55)

$(\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2$ (2.56)

Δx, the standard deviation, is also called the uncertainty

$(\Delta p_x)^2 = \langle p_x^2\rangle - \langle p_x\rangle^2$ (2.57)

$\frac{d\langle x\rangle}{dt} = \frac{\langle p_x\rangle}{m}$ (2.58)

$\langle p_x\rangle = \int_{-\infty}^{\infty}\Psi^*\frac{\hbar}{i}\frac{\delta\Psi}{\delta x}dx$ (2.63)

$\frac{d\langle p_x\rangle}{dt} = \langle -\frac{\delta V}{\delta x}\rangle$ (2.64)

$\langle p_x\rangle = \int_{-\infty}^{\infty}\Psi\frac{\hbar}{i}\frac{\delta\Psi}{\delta x}dx$

$\Delta p_x = (\langle p_x^2\rangle - \langle p_x\rangle^2)^{1/2}$

Schrodinger equation for a free particle:

$\Psi(x,t) = Ae^{i(kx-\omega t)}$

Where:

$p = \hbar k = h/\lambda$

$E = \hbar\omega - h\nu$

$\Psi(x,t) = \int_{-\infty}^{\infty}A(k)e^{i(kx-\omega t)}dk$

Wave move with group velocity:

$v_g = d\omega/dk = \hbar k/m = p/m$

$\omega = \hbar k^2/2m$

The Time-Independent Schrödinger Equation

**In this section we assume V(x) is independent of t*

δ_{nm}=*KroneckerDelta*	ψ_a=*Eigenfunction*
a=*Eigenvalue*	T=*TransmissionCoef.*

$\Psi(x,t) = \psi(x)f(t)$ (3.2)

$\frac{\delta^2\Psi(x,t)}{\delta x^2} = f(t)\frac{d^2\psi(x)}{dx^2}$ (3.3)

$\frac{\delta\Psi(x,t)}{\delta t} = \psi(x)\frac{df(t)}{dt}$ (3.4)

$\frac{df(t)}{dt} = \frac{-iE}{\hbar}f(t)$ (3.8)

$-\frac{\hbar^2}{2m}\frac{\delta\psi(x)}{\delta x^2} + V(x)\psi(x) = E\psi(x)$ (3.9)

$f(t) = f(0)e^{-iEt/\hbar}$ (3.10)

$f(t) = f(0)e^{-i\omega t}$ (3.11)

$E = \hbar\omega$ (3.12)

$\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$ (3.13)

$|\Psi(x,t)|dd = |\psi(x)|^2$ (3.14)

$$V(x) = \begin{cases} 0, & 0 < x < L. \\ \infty, & \text{elsewhere.} \end{cases}$$

$-\frac{\hbar^2}{2m}\frac{\delta\psi}{\delta x^2} = E\psi$ (3.16) $0 < x < L$

$k^2 = \frac{2mE}{\hbar^2}$ (3.17)

$\psi(x) = A\sin kx + B\cos kx$ (3.21) $0 < x < L$

$k_n = \frac{n\pi}{L}$ (3.26)

$E_n = \frac{\hbar k_n^2}{2m} = \frac{n^2\hbar^2\pi^2}{2mL^2}$ (3.27)

$\psi(x) = A_n\sin\frac{n\pi x}{L}$ (3.28) $0 < x < L$

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}, & 0 < x < L. \\ 0, & \text{elsewhere.} \end{cases}$$

$\Psi(x) = c_1\psi_1(x) + c_2\psi_2(x)$ (3.38)

$c_1(t) = \frac{1}{\sqrt{2}}e^{-iE_1t/\hbar}$ (3.39)

$\Psi = \sum_{n=1}^{\infty}c_n\psi_n(x)$ (3.40)

$$\delta_{nm} = \begin{cases} 1, & m = n. \\ 0, & m \neq n. \end{cases}$$

ψ_n's are orthanormal if they satisfy:

$\int_{-\infty}^{\infty}\psi_x^*(x)\psi_n(x)dx = \delta_{nm}$ (3.49)

$|c_n|^2 = P_n$ (3.59)

The above is the probability of obtaining E_n if a measurement of the energy of a particle with wave function Ψ is carried out

$\langle E\rangle = \sum_{n=1}^{\infty}|c_n|^2E_n$ (3.61)

$c_n = \int_{-\infty}^{\infty}\psi^*(x)\psi(x)dx$

$H\psi = E\psi$

Where H is the energy operator:

$H = \frac{(p_{x_{op}})^2}{2m} + V(x)$

$A_{op}\psi_a = a\psi_a$ (3.63)

$x_{op} = x$ (3.64)

$p_{xop} = \frac{\hbar}{i}\frac{\delta}{\delta x}$ (3.65)

$E_{op} = \frac{(p_{xop})^2}{2m} + V(x_{op})$ (3.71)

$H \equiv E_{op} = -\frac{\hbar^2}{2m}\frac{\delta^2}{\delta x^2} + V(x)$ (3.72)

$\langle E\rangle = \int_{-\infty}^{\infty}\Psi^*H\Psi dx$ (3.81)

One-Dimensional Potentials

Energy eigenfunctions are oscillatory in regions where E>V and exponential in V>E.

$$V(x) = \begin{cases} 0, & |x| < a/2. \\ V_0, & |x| > a/2. \end{cases}$$

$k = \frac{\sqrt{2mE}}{\hbar}$
$\psi(x) = Ae^{ikx} + Be^{-ikx}$ $|x| < a/2$
$\kappa = \frac{\sqrt{2m(V_0-E)}}{\hbar} > 0$

$\psi(x) = Ce^{\kappa x} + De^{-\kappa x}$ $|x| > a/2$

$$\psi(x) = \begin{cases} Ce^{\kappa x}, & x \leq -a/2. \\ 2A\cos kx, & -a/2 \leq x \leq a/2. \\ Ce^{-\kappa x}, & x \geq a/2. \end{cases}$$

$$V(x) = \begin{cases} 0, & x < 0. \\ V_0, & x > 0. \end{cases}$$

$k = \frac{\sqrt{2mE}}{\hbar}$
$k_0 = \sqrt{k^2 - \frac{2mV_0}{\hbar^2}}$

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx}, & x < 0. \\ Ce^{ik_0 x}, & x > 0. \end{cases}$$

$$j_x = \begin{cases} \frac{\hbar k}{m}(|A|^2 - |B|^2), & x < 0. \\ \frac{\hbar k_0}{m}|C|^2, & x > 0. \end{cases}$$

$T \cong (\frac{4\kappa k}{k^2+\kappa^2})^2 e^{-2\kappa a}$

Principles of Quantum Mechanics

Constants
$\hbar = 6.582 \times 10^{-16}$
$\epsilon_0 = 8.854 \times 10^{-12}$
$e = 1.602 \times 10^{-19}$
$k_B = 8.617 \times 10^{-5}$
$\Psi^*\Psi dx$ is the probability of finding the particle between x and $x + dx$
$\int_{-\infty}^{\infty} \Psi^*\Psi dx = 1$ (5.97)
A Hermitian operator satisfies:
$\int_{-\infty}^{\infty} \phi^* A_{op}\psi dx = \int_{-\infty}^{\infty} (A_{op}\phi)^*\psi dx$ (5.98)
$A_{op}\psi_a = a\psi_a$ (5.99)
Orthonormal wave functions satisfy:
$\Psi = \sum_a c_a\psi_a$ (5.100)
Probability of obtaining a:
$|c_a|^2 = \left| \int_{-\infty}^{\infty} \psi_a^* \Psi dx \right|^2$ (5.101)
Average or expectation value:
$\langle A \rangle = \sum_a |c_a|^2 a = \int_{-\infty}^{\infty} \Psi^* A_{op}\Psi dx$ (5.102)
Commutator:
$[A_{op}, B_{op}] = A_{op}B_{op} - B_{op}A_{op}$ (5.103)
If: $[A_{op}, B_{op}] = iC_{op}$ (5.104)
Then : $\Delta A \Delta B \geq \frac{|\langle C \rangle|}{2}$ (5.105)
$\Delta x \Delta p_x \geq \frac{\hbar}{2}$ (5.106)
$[x_{op}, p_{x_{op}}] = i\hbar$ (5.107)
$H\Psi(x,t) = i\hbar\frac{\delta\Psi(x,t)}{\delta t}$ (5.108)
Hamiltonian, the energy operator:
$H = -\frac{\hbar^2}{2m}\frac{\delta^2}{\delta x^2} + V(x)$ (5.109)
Expectation values $\frac{d\langle A \rangle}{dt}$ =:
$\frac{i}{\hbar}\int_{-\infty}^{\infty} \Psi^*[H, A_{op}]\Psi dx + \int_{-\infty}^{\infty} \Psi^* \frac{\delta A_{op}}{\delta t}\Psi dx$
If the Hamiltonian commutes withe the operator corresponding to the observable A and $\delta A_{op}/\delta t = 0$, then $\langle A \rangle$ is independent of time.
$\Delta E \frac{\Delta A}{\left|\frac{d<A>}{dt}\right|} \geq \frac{\hbar}{2}$ (5.111)
$\Delta E \Delta t \geq \frac{\hbar}{2}$ (5.112)
Parity operator:
$\Pi\psi(x) = \psi(-x)$ (5.4)
If there exists a single eigenfunction with eigenvalue a then its nondegenerate.
$x_{op} = x$ (3.64)
$p_{x_{op}} = \frac{\hbar}{i}\frac{\delta}{\delta x}$ (3.66)

Quantum Mechanics in Three Dimensions

Cubic Box:
$E_{n_x,n_y,n_z} = \frac{(n_x^2+n_y^2+n_z^2)\hbar^2\pi^2}{2mL^2}$ (6.104)
Hydrogenic Atom:
$E_n = -\frac{mZ^2e^4}{(4\pi\epsilon_0)^2 2\hbar^2 n^2} = -\frac{(1.36eV)Z^2}{n^2}$ (6.105)
Allowed values of l:
$l = 0, 1, 2, ..., n-1$
$\nabla^2 = \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} + \frac{\delta^2}{\delta z^2}$
$\nabla^2 =$
$\frac{1}{r^2}\frac{\delta}{\delta r}(r^2\frac{\delta}{\delta r}) + \frac{1}{r^2\sin\theta}\frac{\delta}{\delta\theta}(\sin\theta\frac{\delta}{\delta\theta}) + \frac{1}{r^2\sin^2\theta}\frac{\delta^2}{\delta\phi^2}$
Angular momentum operator:
$L_{op}^2 = L_{x_{op}}^2 + L_{y_{op}}^2 + L_{z_{op}}^2$
$L_{op}^2 Y_{l,m_l}(\theta,\phi) = l(l+1)\hbar^2 Y_{l,m_l}(\theta,\phi)$ (6.106)
$L_{op}^2 = -\hbar^2[\frac{1}{\sin\theta}\frac{\delta}{\delta\theta}(\sin\theta\frac{\delta}{\delta\theta}) + \frac{1}{\sin^2\theta}\frac{\delta^2}{\delta\phi^2}]$
$L = r \times p$
$L_x = yp_z - zp_y$
The eigenfunctions Y_{l,m_l}, the spherical harmonics, also satisfy:
$L_{op}Y_{l,m_l}(\theta,\phi) = m_l\hbar Y_{l,m_l}(\theta,\phi)$ (6.107)
The energies (6.105) are independent of m_l because of the rotational symetry of the potential energy $-Ze^2/4\pi\epsilon_0 r$
The maximum value for L_z is $l\hbar$ which is always less than the total angular momentum $\sqrt{l(l+1)}\hbar$
$[L_{x_{op}}, L_{y_{op}}] = i\hbar L_{z_{op}}$
$[L_{y_{op}}, L_{z_{op}}] = i\hbar L_{x_{op}}$
$[L_{z_{op}}, L_{x_{op}}] = i\hbar L_{y_{op}}$ (6.108)
$\Delta L_x \Delta_y \geq \frac{\hbar}{2}|\langle L_z \rangle|$ (6.109)
$L_{z_{op}} = \frac{\hbar}{i}\frac{\delta}{\delta\phi}$
Spin angular momentum S:
$\chi\pm$ are two dimensional column vectors:
$S_{op}^2\chi\pm = s(s+1)\hbar^2\chi\pm$ $s = 1/2$ (6.110)
$S_{z_{op}}\chi\pm = \pm\frac{\hbar}{2}\chi\pm$ (6.111)
The magnetic moment:
$\mu = 2.00232\left(\frac{-e}{2m}\right)S$ (6.112)
The spherical harmonics with $l = 0, l = 1, andl = 2$
$Y_{0,0}(\theta,\phi) = \sqrt{\frac{1}{4\pi}}$
$Y_{1,\pm1}(\theta,\phi) = \sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\theta}$
$Y_{1,0}(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\cos\theta$
$Y_{2,\pm2}(\theta,\phi) = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{\pm 2i\theta}$
$Y_{2,\pm1}(\theta,\phi) = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{\pm i\theta}$
$Y_{2,0}(\theta,\phi) = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$
These spherical harmonics are the eigenfunctions.
For a three dimensional box from 0 to L the energy eigenfunctions (ψ_{n_x,n_y,n_z}) are:
$\left(\frac{2}{L}\right)^{3/2}\sin\frac{n_x\pi x}{L}\sin\frac{n_y\pi y}{L}\sin\frac{n_z\pi z}{L}$
The Energy Eigenvalues are:
$E_{n_x,n_y,n_z} = \frac{(n_x^2+n_y^2+n_z^2)\hbar^2\pi^2}{2mL^2}$

Identical Particles

Bosons are symmetric under particle exchange, Fermions are anti-symmetric. Bosons have integral intrinsic spin, Fermions have half integral intrinsic spin.
Valid Boson states:
$\Psi_S(1,2) = \psi_\alpha(1)\psi_\alpha(2)$ (7.104)
$\Psi_S(1,2) = \frac{1}{\sqrt{2}}[\psi_\alpha(1)\psi_\beta(2) + \psi_\beta(1)\psi_\alpha(2)]$
Bosons are more likely to be in the same state than are distinguishable particles.
The average number of identical Bosons in a state with energy E in thermal equilibrium at temperature T is given by:
$n(E) = \frac{1}{e^{(E-\mu)/k_bT}-1}$ (7.106)
Where μ is the chemical potential.
Valid Fermions:
$\Psi_A(1,2) = \frac{1}{\sqrt{2}}[\psi_\alpha(1)\psi_\beta(2) - \psi_\beta(1)\psi_\alpha(2)]$
The average number of identical Fermions in a state with energy E in thermal equilibrium at temperature T is given by:
$n(E) = \frac{1}{e^{(E-E_F)/k_bT}+1}$ (7.108)
Where E_F is the Fermi energy. Photons are Bosons, thus the distribution of electromagnetic energy in a cavity:
$\rho(\nu)d\nu = \frac{8\pi h\nu^3}{c^3(e^{h\nu/k_bT}-1)}$ (7.109)
Total energy per unit area per unit time:
σT^4 (7.67)
$\sigma = 5.67 \times 10^{-8}$ (7.68)
$\lambda_{max}T = 2.9 \times 10^{-3}$ (7.69)
$E_F = \frac{1}{2}mV_F^2$
$E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$
$E_{total} = \frac{3}{5}NE_F$
Degeneracy Pressure:
$P = -\frac{dE_{total}}{dV} = \frac{2}{5}(3\pi^2)^{2/3}\frac{\hbar^2}{2m}(\frac{N}{V})^{5/3}$
Compressibility:
$K = (-V\frac{dP}{dV})^{-1}$
Ways of choosing N_1 from group of N:
$\frac{N!}{N_1!(N-N_1)!}$
Fermi temperature: $E_F = k_B T_F$
$I = I_0(e^{e\varphi/k_bT-1})$
e in exponent is charge on electron.

Solid State Physics

Bloch ansatz:
$\psi(x+a) = e^{i\theta}\psi(x)$ (8.4)
$\frac{\sqrt{2mE}}{\hbar} = k$ (8.10)
$\cos\theta = \cos ka + \frac{\alpha\sin ka}{2ka}$ (8.26)
Contact Potential: $(W_b - W_a)/e$
For semiconductors: $E_F = \frac{E_g}{2}$ (8.33)

$n(E) = \frac{1}{e^{E_g/2k_bT}+1}$ (8.34)

Nuclear Physics

$m_{nucleus} = Zm_p + (A-Z)m_n - B.E./c^2$

Where $B.E. \times (1, 0, -1)$ is:

$a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(z-\frac{A}{2})^2}{A} + \frac{a_5}{\sqrt{A}}$

$N(t) = N(0)e^{-Rt} = N(0)e^{-t/\tau}$ (9.79)

Radius of a nucleus composed of A nucleons:

$R = r_0 A^{1/3}$

$r_0 = 1.2fm = 12 \times 10^{-15}$

The lifetime of an unstable nucleus:

$\tau = 1/R$

$t_{1/2} = \tau \ln 2$

$a_1 = 15.75MeV$

$a_2 = 17.8MeV$

$a_3 = 0.711MeV$

$a_4 = 94.8MeV$

$a_5 = 11.2MeV$

Relativity

$c = 3 \times 10^8 m/s$

Conservation of Kinetic Energy is not a thing.

Galilean relativity:

$t' = t$

$x' = x - \beta t$

Coordinate time is Δt between two events recorded by a pair of clocks synchronized at rest in a given inertial reference frame (one clock present at each event). This is basically the difference on the time axis.

$\Delta t = t_A - t_B$

The spacetime interval Δs can never be larger than the coordinate time Δt.

Coordinate time Δt

$t_1 - t_2$ as measured by clocks in any inertial frame.

The clocks for the two events could be different.

Many possible values for this (many inertial frames).

Proper time $\Delta\tau$

Measured by a single clock present at both events.

This clock could be in a non-inertial frame.

Many possible different values for this (many possible clock paths).

Spacetime Interval Δs

$t_1 - t_2$ as measured by one clock present at both events and attached to an inertial frame.

Unique for any pair of events.

$\Delta t \geq \Delta s \geq \Delta\tau$

$\Delta\tau = \int \sqrt{1-v^2}dt$

$\Delta s^2 = \Delta t^2 - \Delta d^2$

Spacetime interval is the same in any inertial frame, the others are different.

If speed is constant: $\Delta\tau = \sqrt{1-v^2}\Delta t$

Lorentz Transformation Equations

$t' = \gamma(t - \beta x)$

$x' = \gamma(x - \beta t)$

Lorentz Contraction

$L = \sqrt{1-\beta^2}L_R$

$\Delta s^2 \geq 0$ Time-like.

$\Delta s^2 = 0$ Light-like.

$\Delta s^2 \leq 0$ Space-like.

If casually connected the order matters, otherwise the order can be switched in different reference frames.

Einstein Velocity Transformation

$v'_x = \frac{v_x - \beta}{1-\beta v_x}$

$v'_y = \frac{v_y\sqrt{1-\beta^2}}{1-\beta v_x}$

$v'_z = \frac{v_z\sqrt{1-\beta^2}}{1-\beta v_x}$

For inverse transformation swap primes with unprime and change β to $-\beta$.

$$\vec{p}_{tot} = \sum m_i \begin{bmatrix} v_{ix} \\ v_{iy} \\ v_{iz} \end{bmatrix}$$

Four-momentum:

$$P_{tot} = \sum \frac{m_i}{\sqrt{1-v_i^2}} \begin{bmatrix} 1 \\ v_{ix} \\ v_{iy} \\ v_{iz} \end{bmatrix}$$

At low speeds:

$$P = \sum \frac{m}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ v_x \\ v_y \\ v_z \end{bmatrix} \approx \begin{bmatrix} m + \frac{1}{2}mv^2 \\ mv_x \\ mv_y \\ mv_z \end{bmatrix}$$

Relativistic K: $k = \frac{m}{\sqrt{1-v^2}} - m$

Binomial approximation:

$(1+x)^\alpha \approx 1 + \alpha x$

$$\begin{bmatrix} f & 10^{-15} \\ p & 10^{-12} \\ n & 10^{-9} \\ \mu & 10^{-6} \\ m & 10^{-3} \\ M & 10^{6} \end{bmatrix}$$

Name Solutions

Phys 331 – ST&Q – Exam 1 – Townsend Chapters 1-4
Feb 20, 2019 – 100 Points Total

Instructions:
- You may bring notes on one side of an 8.5"x11" sheet of paper.
- You may use a calculator. No other memory or computational assistance is allowed.
- If you run out of space, you may continue your work on a piece of scratch paper. Please make a note and staple the paper to the back of the test.
- Not all questions are worth the same number of points. Be aware of this as you prioritize your time.
- Read questions fully. For many questions, there are multiple parts.

1.) (20 pts) The wave function for a particle is given by

$$\Psi = \frac{e^{i\pi}}{2}\psi_1 + \frac{\sqrt{3}}{2}\psi_2$$

Where ψ_1 and ψ_2 are energy eigenfunctions with energy eigenvalues E_1 and E_2 respectively. What is the probability that a measurement of the energy yields the value E_1? What are $\langle E\rangle$ and ΔE? For this system, the energy eigenvalues are such that $E_2 = 4E_1$.

$c_1 = \frac{e^{i\pi}}{2}$, $c_2 = \frac{\sqrt{3}}{2}$

(7 pts) Probability of measuring E_1: $P_1 = |c_1|^2 \boxed{= \frac{1}{4}}$

(7 pts) Expectation value $\langle E\rangle = P_1E_1 + P_2E_2 = \frac{E_1 + 3E_2}{4} \boxed{= \frac{13E_1}{4}}$

(3 pts) Expectation value $\langle E^2\rangle = P_1E_1^2 + P_2E_2^2 = \frac{E_1^2 + 3(16E_1^2)}{4} = \frac{49E_1^2}{4}$

(3 pts) Uncertainty $\Delta E = \sqrt{\langle E^2\rangle - \langle E\rangle^2} = \sqrt{\frac{49E_1^2}{4} - \frac{169E_1^2}{16}}$

$\boxed{= \frac{3\sqrt{3}}{4}E_1}$

2.) (20 pts) In the figure below, each of the beam splitters are half-silvered mirrors. What is the probability amplitude that a photon arrives at PM_2 via path ABD? What is the probability amplitude that a photon arrives at PM_2 via path ACD? What is the probability that a photon arrives at PM_2? Give the condition for 100% probability that a photon arrives at PM_2.

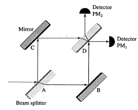

Figure 1.23 copyright 2009 University Science Books

(6 pts) $Z_{ABD} = \frac{1}{\sqrt{2}}e^{i\pi}\frac{1}{\sqrt{2}}e^{ikl_1} \boxed{= -\frac{1}{2}e^{ikl_1}}$

Let l_1 be the distance along path ABD, l_2 is distance along ACD.

(6 pts) $Z_{ACD} = \frac{e^{i\pi}}{\sqrt{2}}e^{i\pi}\frac{1}{\sqrt{2}}e^{ikl_2} \boxed{= \frac{1}{2}e^{ikl_2}}$

Probability of a photon arriving at PM_2:

$P_{pm2} = (Z^*_{ABD} + Z^*_{ACD})(Z_{ABD} + Z_{ACD})$

$= \frac{1}{4}(-e^{-ikl_1} + e^{-ikl_2})(-e^{ikl_1} + e^{ikl_2})$

$= \frac{1}{4}(2 - e^{-ik(l_1-l_2)} - e^{ik(l_1-l_2)})$

$= \frac{1}{2}[1 - \cos(k(l_1-l_2))]$

(6 pts) $\boxed{P_{pm2} = \sin^2\left[\frac{k(l_1-l_2)}{2}\right]}$

(2 pts) $\boxed{\text{For 100\% probability } \frac{k(l_1-l_2)}{2} = (n+\tfrac{1}{2})\pi \text{ for } n = 0, 1, 2, \ldots}$

4.)

$E_F = \frac{1}{2}m_eV_F^2$, $E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2N}{V}\right)^{2/3}$

Once E_F is known, can calculate V_F

$\frac{N}{V} = \frac{0.97\,g}{1\,cm^3} \times \frac{1\,mole}{23\,g} \times \left(\frac{100\,cm}{1m}\right)^3 \times \frac{6.023\times10^{23}\,particles}{1\,mole}$

$= 2.54\times10^{28}\,\frac{electrons}{m^3}$

$E_F = \frac{(1.055\times10^{-34})^2}{2\times9.1\times10^{-31}}\left(3\pi^2 \times 2.54\times10^{28}\right)^{2/3}$

$= 5.058\times10^{-19}\,J$

$V_F = \sqrt{2E_F/m_e} = 1.05\times10^6\,m/s$

$\boxed{V_F = 1.1\times10^6\,m/s}$

P.5

5.) N-type semiconductors are doped to have more electrons in the conduction band. This means E_F is higher than in pure Silicon

P-type semiconductors are doped to have more holes in the valence band. This means E_F is lower than in pure silicon.

When a piece of P-type is joined to a piece of N-type, a contact potential forms. The P-type ends up at higher potential energy as a result.

Forward biasing pushes the plentiful N-type conduction electrons over the contact potential. Current flows freely.

Reverse biasing pushes whatever P-type conduction electrons there are toward the N-type side. Aren't many P-type conduction electrons. Small amount of current (effectively none) flows.

P.6