Quiz 5

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1

$$\delta_l = -\frac{2mk}{\hbar^2}\int_0^\infty V(r) j_l(kr)^2 r^2 dr$$
$$V = V_0\theta(R-r)$$
$$\delta_0 = -\frac{2mkV_0}{\hbar^2}\int_0^R j_0(kr)^2 r^2 dr$$
$$\delta_0 = -\frac{2mkV_0}{\hbar^2}\int_0^R \frac{\sin^2(kr)}{k^2r^2} r^2 dr$$
$$\delta_0 = \frac{2mkV_0}{\hbar^2}\left(\frac{\sin(2kR)-2kR}{4k^3}\right) = \frac{2mkV_0}{\hbar^2}\left(\frac{2kRj_0(2\chi)-2kR}{4k^3}\right)$$
$$\delta_0 = \frac{mV_0R^2}{\hbar^2}\left(\frac{j_0(2\chi)-1}{\chi}\right) = -\frac{mV_0R^2}{\hbar^2}\left(\frac{1-j_0(2\chi)}{\chi}\right)$$

2

$$\delta_l = -\frac{2mk}{\hbar^2}\int_0^\infty V(r)\left(\frac{1}{n!}\left(\frac{kr}{2}\right)^n\right)^2 r^2 dr$$
$$V = V_0\theta(R-r)$$
$$\delta_l = -\frac{2mkV_0}{\hbar^2}\int_0^R \left(\frac{1}{l!}\left(\frac{kr}{2}\right)^l\right)^2 r^2 dr$$
$$\delta_l = -\frac{2mkV_0}{\hbar^2}\frac{4^{-l}R^3\left(kR\right)^{2l}}{(2l+3)(l!)^2}$$