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2.2

$$\hat{P}_+ = |+z\rangle\langle+z|$$
$$\hat{P}_+^\dagger = (|+z\rangle)^\dagger(\langle+z|)^\dagger = |+z\rangle\langle+z|$$
$$\hat{P}_+^2\Psi = \lambda^2\Psi = \hat{P}_+\Psi = \lambda\Psi$$
$$\lambda^2 = \lambda$$
$$\lambda = 0, 1$$

2.4

$$\langle+y|+x\rangle = \langle+y|+z\rangle\langle+z|+x\rangle + \langle+y|-z\rangle\langle-z|+x\rangle$$
$$\langle+y|+x\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\frac{1}{\sqrt{2}}$$
$$\langle-y|+x\rangle = \langle-y|+z\rangle\langle+z|+x\rangle + \langle-y|-z\rangle\langle-z|+x\rangle$$
$$\langle-y|+x\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\frac{1}{\sqrt{2}}$$
$$|+x\rangle = \frac{1}{2}\begin{bmatrix}1-i\\1+i\end{bmatrix}$$
$$\langle+y|-x\rangle = \langle+y|+z\rangle\langle+z|-x\rangle + \langle+y|-z\rangle\langle-z|-x\rangle$$
$$\langle+y|-x\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\frac{1}{\sqrt{2}}$$
$$\langle-y|-x\rangle = \langle-y|+z\rangle\langle+z|-x\rangle + \langle-y|-z\rangle\langle-z|-x\rangle$$
$$\langle-y|-x\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\frac{1}{\sqrt{2}}$$
$$|-x\rangle = \frac{1}{2}\begin{bmatrix}1-i\\1+i\end{bmatrix}$$

2.9

$$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}^\dagger\begin{bmatrix}5 & 7\\6 & 8\end{bmatrix}^\dagger = \begin{bmatrix}5 & 21\\12 & 31\end{bmatrix}$$
$$\left(\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}5 & 7\\6 & 8\end{bmatrix}\right)^\dagger = \begin{bmatrix}5 & 21\\12 & 31\end{bmatrix}$$

2.11

$$|\psi\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}\begin{bmatrix}1\\\sqrt{2}\end{bmatrix}$$
$$\langle S_x\rangle = \left(\frac{1}{\sqrt{2}\sqrt{3}}\right)^2\left(1+\sqrt{2}\right) \approx 0.40$$

2.12

a)

$$\left(\frac{i}{\sqrt{3}}\right)^2 = \frac{1}{3}$$

b)

$$-\frac{2}{3}\sin\theta\,|x\rangle + \frac{i}{\sqrt{3}}\cos\theta\,|y\rangle$$
$$|\langle y'|\psi\rangle|^2 = \frac{2}{3}\sin\theta\,|x\rangle + \frac{1}{3}\cos\theta\,|y\rangle$$

c)

$$|R\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle + i\,|y\rangle\right) = \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{2}\sqrt{3}}$$
$$|L\rangle = 1 - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{2}\sqrt{3}}$$

Since $|L\rangle$ is larger it will spin in that direction

d)

$$\left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$$
$$-\frac{2}{3}\sin\theta\,|x\rangle + \frac{1}{\sqrt{3}}\cos\theta\,|y\rangle$$
$$|\langle y'|\psi\rangle|^2 = \frac{2}{3}\sin\theta\,|x\rangle + \frac{1}{3}\cos\theta\,|y\rangle$$
$$|R\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle + i\,|y\rangle\right) = \frac{1}{\sqrt{3}} + \frac{i}{\sqrt{2}\sqrt{3}}$$
$$|L\rangle = 1 - \frac{1}{\sqrt{3}} - \frac{i}{\sqrt{2}\sqrt{3}}$$