R8B.9 **(a)** According to equation R8.33, we have

$$\vec{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \frac{1}{E}\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \frac{1}{18\text{ kg}}\begin{bmatrix} 9\text{ kg} \\ 15\text{ kg} \\ 1\text{ kg} \end{bmatrix} = \begin{bmatrix} 1/2 \\ 5/6 \\ 1/18 \end{bmatrix} = \begin{bmatrix} 0.500 \\ 0.833 \\ 0.055 \end{bmatrix} \tag{1}$$

(b) The speed of this object is thus

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{(\tfrac{1}{2})^2 + (\tfrac{5}{6})^2 + (\tfrac{1}{18})^2} = \sqrt{(\tfrac{9}{18})^2 + (\tfrac{15}{18})^2 + (\tfrac{1}{18})^2} = \tfrac{1}{18}\sqrt{81 + 225 + 1} = 0.973 \tag{2}$$

(c) According to equation R8.31, we have

$$m = \sqrt{E^2 - p_x^2 - p_y^2 - p_z^2} = \sqrt{(18\text{ kg})^2 - (9\text{ kg})^2 - (15\text{ kg})^2 - (1\text{ kg})^2} = \sqrt{324 - 81 - 225 - 1} = 4.12\text{ kg} \tag{3}$$

(d) According to equation R8.29, we have

$$|\vec{p}| = \sqrt{p_x^2 + p_y^2 + p_z^2} = \sqrt{(9\text{ kg})^2 + (15\text{ kg})^2 + (1\text{ kg})^2} = \sqrt{81 + 225 + 1}\text{ kg} = \sqrt{307}\text{ kg} = 17.5\text{ kg} \tag{4}$$

(e) According to equation R8.26, $K = E - m = 18\text{ kg} - 4.12\text{ kg} = 13.88\text{ kg}$.

R8M.1 The three things wrong with this four-momentum vector are the following. (1) All four components of a four-momentum vector have units of kilograms, not just the first. (2) The time component of a four-momentum vector, which is equal to $m[1 - |\vec{v}|^2]^{-1/2}$, can never be negative. (3) Since $[p_t^2 - p_x^2 - p_y^2 - p_z^2]^{1/2} = m$ must be a positive real number, p_t^2 must be larger in magnitude than each of the other squared components. Even if the other components are given units of kilograms, this requirement will not be satisfied.

R8M.5 My rest mass is about 70 kg. Converting this to energy units in the SI system, I find that if the price of electricity is \$0.06 MJ, my total energy is worth

$$70\,\cancel{\text{kg}}\left(\frac{3.0\times10^8\,\cancel{\text{m}}}{1\,\cancel{\text{s}}}\right)^2\left(\frac{1\,\cancel{\text{J}}}{1\,\cancel{\text{kg}}\cdot\cancel{\text{m}}^2/\cancel{\text{s}}^2}\right)\left(\frac{\$0.06}{10^6\,\cancel{\text{J}}}\right) = \$3.8\times10^{11} = \$380\text{ billion.}$$

R8M.7 Note that if the object's speed is $|\vec{v}| = \frac{3}{5}$, then $\sqrt{1 - |\vec{v}|^2} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$. The kinetic energy that a 100,000-kg train moving at such a speed would cost

$$K = \frac{m}{\sqrt{1 - |\vec{v}|^2}} - m = \frac{5}{4}m - m = \frac{m}{4} = \frac{100{,}000\,\cancel{\text{kg}}}{4}\left(\frac{3.0\times10^8\,\cancel{\text{m}}}{1\,\cancel{\text{s}}}\right)^2\left(\frac{1\,\cancel{\text{J}}}{1\,\cancel{\text{kg}}\cdot\cancel{\text{m}}^2/\cancel{\text{s}}^2}\right)\left(\frac{\$0.06}{10^6\,\cancel{\text{J}}}\right) = \$1.35\times10^{14}$$

This is \$135 trillion, which is in the ballpark of 8 times the United States' 2014 gross domestic product. I don't think that Congress will spring for this experiment.