Johannes Byle

7.2

$$\hat{a}^\dagger \to \begin{bmatrix} 0 & 0 & 0 & \cdots \\ \sqrt{1} & 0 & 0 & \\ 0 & \sqrt{2} & 0 & \\ 0 & 0 & \sqrt{3} & \\ \vdots & & & \end{bmatrix},\ \hat{a} \to \begin{bmatrix} 0 & \sqrt{1} & 0 & 0 & \cdots \\ 0 & 0 & \sqrt{2} & 0 & \\ 0 & 0 & 0 & \sqrt{3} & \\ \vdots & & & & \end{bmatrix}$$

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a} + \hat{a}^\dagger\right) \to \sqrt{\frac{\hbar}{2m\omega}} \begin{bmatrix} 0 & \sqrt{1} & 0 & 0 & \cdots \\ \sqrt{1} & 0 & \sqrt{2} & 0 & \\ 0 & \sqrt{2} & 0 & \sqrt{3} & \\ 0 & 0 & \sqrt{3} & 0 & \\ \vdots & & & & \end{bmatrix}$$

$$\hat{p}_x = -i\sqrt{\frac{m\omega\hbar}{2}}\left(\hat{a} - \hat{a}^\dagger\right) \to -i\sqrt{\frac{m\omega\hbar}{2}} \begin{bmatrix} 0 & \sqrt{1} & 0 & 0 & \cdots \\ -\sqrt{1} & 0 & \sqrt{2} & 0 & \\ 0 & -\sqrt{2} & 0 & \sqrt{3} & \\ 0 & 0 & -\sqrt{3} & 0 & \\ \vdots & & & & \end{bmatrix}$$

$$[\hat{x}, \hat{p}_x] = \hat{x}\hat{p}_x - \hat{p}_x\hat{x} = i\hbar$$

$$\sqrt{\frac{\hbar}{2m\omega}}\left(-i\sqrt{\frac{m\omega\hbar}{2}}\right) = -i\frac{\hbar}{2}$$

$$\left(\hat{a} + \hat{a}^\dagger\right)\left(\hat{a} - \hat{a}^\dagger\right) - \left(\hat{a} + \hat{a}^\dagger\right)\left(\hat{a} - \hat{a}^\dagger\right) = \begin{bmatrix} 2 & 0 & 0 & 0 & \cdots \\ 0 & 2 & 0 & 0 & \\ 0 & 0 & 2 & 0 & \\ 0 & 0 & 0 & 2 & \\ \vdots & & & & \end{bmatrix}$$

$$-i\frac{\hbar}{2} \begin{bmatrix} 2 & 0 & 0 & 0 & \cdots \\ 0 & 2 & 0 & 0 & \\ 0 & 0 & 2 & 0 & \\ 0 & 0 & 0 & 2 & \\ \vdots & & & & \end{bmatrix} = i\hbar$$

7.4

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}_x\right)$$

$$\langle p| \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}_x\right)|0\rangle = \sqrt{\frac{m\omega}{2\hbar}}\left(\langle p|\,\hat{x}\,|0\rangle + \langle p|\,\frac{i}{m\omega}\hat{p}_x\,|0\rangle\right)$$

$$\sqrt{\frac{m\omega}{2\hbar}}\left(i\hbar\frac{\delta}{\delta p}\langle p|0\rangle + \frac{i\hat{p}_x}{m\omega}\langle p|0\rangle\right) = 0$$

$$\frac{\delta}{\delta p}\langle p|0\rangle = -\frac{\hat{p}_x}{m\omega\hbar}\langle p|0\rangle$$

$$\int \frac{1}{\psi}d\psi = -\frac{1}{m\omega\hbar}\int p dp$$

$$C\ln\psi = -\frac{p^2}{2m\omega\hbar}$$

$$\psi = Ce^{-p^2/2m\omega\hbar}$$

$$\int_{-\infty}^{\infty} Ce^{-p^2/2m\omega\hbar}dp = C\sqrt{4\pi m\omega\hbar} = 1$$

$$\psi = \frac{1}{\sqrt{4\pi m\omega\hbar}}e^{-p^2/2m\omega\hbar}$$

7.12

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\langle x|E\rangle + \frac{1}{2}m\omega^2x^2\langle x|E\rangle = E\langle x|E\rangle$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}Ne^{-ax^2} + \frac{1}{2}m\omega^2x^2Ne^{-ax^2} = ENe^{-ax^2}$$

$$-\frac{\hbar^2}{2m}2aNe^{-ax^2}\left(2ax^2 - 1\right) + \frac{1}{2}m\omega^2x^2Ne^{-ax^2} = ENe^{-ax^2}$$

$$-\frac{\hbar^2}{2m}2a\left(2ax^2 - 1\right) + \frac{1}{2}m\omega^2x^2 = E$$

$$-\frac{\hbar^2}{2m}4a^2x^2 - \frac{\hbar^2}{2m}2a + \frac{1}{2}m\omega^2x^2 - E = 0$$

$$\left(\frac{1}{2}m\omega^2 - \frac{\hbar^2}{m}2a^2\right)x^2 + \frac{\hbar^2}{m}a - E = 0$$

$$a = \frac{m\omega}{2\hbar}$$

$$E = \frac{\hbar^2}{m}a = \frac{\hbar\omega}{2}$$

7.15

$$\frac{a_{k+2}}{a_k} = \frac{2k+1-\varepsilon}{(k+2)(k+1)}$$

$$\varepsilon = 2n+1$$

$$\frac{a_{k+2}}{a_k} = \frac{2(k-n)}{(k+2)(k+1)}$$

$$n=0,\ \frac{a_2}{a_0} = 0,\ 1$$

$$n=1,\ \frac{a_2}{a_0} = -1,\ y$$

$$n=2,\ \frac{a_2}{a_0} = -2,\ 1-2y^2$$

$$n=3,\ \frac{a_3}{a_1} = -\frac{2}{3},\ 6-4y^3$$

$$n=4,\ \frac{a_3}{a_1} = -1,\ \frac{a_2}{a_0} = -4,\ y^3+4y^2+y$$

7.17

$$|\langle x|0\rangle|^2 = \frac{m\omega}{\pi\hbar}e^{-m\omega x^2/\hbar}$$

Equation C.18 in appendix C:

$$\delta(x) = \lim_{\alpha\to\infty}\sqrt{\frac{\alpha}{\pi}}e^{-\alpha x^2}$$

We know that:

$$\lim_{\hbar\to 0}\frac{m\omega}{\hbar} = \lim_{\alpha\to\infty}\alpha$$

Thus:

$$\lim_{\hbar\to 0}\frac{m\omega}{\pi\hbar}e^{-m\omega x^2/\hbar} = \lim_{\alpha\to\infty}\sqrt{\frac{\alpha}{\pi}}e^{-\alpha x^2}$$