

Problems from 8.1

Problem 8.1

Example 7.13.

$$\left. \begin{aligned} \mathbf{E} &= \frac{\lambda}{2\pi\epsilon_0} \frac{1}{s} \hat{\mathbf{s}} \\ \mathbf{B} &= \frac{\mu_0 I}{2\pi} \frac{1}{s} \hat{\phi} \end{aligned} \right\} \mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B}) = \frac{\lambda I}{4\pi^2 \epsilon_0} \frac{1}{s^2} \hat{\mathbf{z}};$$

$$P = \int \mathbf{S} \cdot d\mathbf{a} = \int_a^b S 2\pi s ds = \frac{\lambda I}{2\pi\epsilon_0} \int_a^b \frac{1}{s} ds = \frac{\lambda I}{2\pi\epsilon_0} \ln(b/a).$$

$$\text{But } V = \int_a^b \mathbf{E} \cdot d\mathbf{l} = \frac{\lambda}{2\pi\epsilon_0} \int_a^b \frac{1}{s} ds = \frac{\lambda}{2\pi\epsilon_0} \ln(b/a), \text{ so } \boxed{P = IV.}$$

Problem 7.62.

$$\left. \begin{aligned} \mathbf{E} &= \frac{\sigma}{\epsilon_0} \hat{\mathbf{z}} \\ \mathbf{B} &= \mu_0 K \hat{\mathbf{x}} = \frac{\mu_0 I}{w} \hat{\mathbf{x}} \end{aligned} \right\} \mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B}) = \frac{\sigma I}{\epsilon_0 w} \hat{\mathbf{y}};$$

$$P = \int \mathbf{S} \cdot d\mathbf{a} = Swh = \frac{\sigma I h}{\epsilon_0}, \text{ but } V = \int \mathbf{E} \cdot d\mathbf{l} = \frac{\sigma}{\epsilon_0} h, \text{ so } \boxed{P = IV.}$$

Problem 8.2

$$(a) \mathbf{E} = \frac{\sigma}{\epsilon_0} \hat{\mathbf{z}}; \sigma = \frac{Q}{\pi a^2}; Q(t) = It \Rightarrow \mathbf{E}(t) = \boxed{\frac{It}{\pi\epsilon_0 a^2} \hat{\mathbf{z}}}.$$

$$B 2\pi s = \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \pi s^2 = \mu_0 \epsilon_0 \frac{I \pi s^2}{\pi \epsilon_0 a^2} \Rightarrow \mathbf{B}(s, t) = \boxed{\frac{\mu_0 I s}{2\pi a^2} \hat{\phi}}.$$

$$(b) u_{\text{em}} = \frac{1}{2} \left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) = \frac{1}{2} \left[\epsilon_0 \left(\frac{It}{\pi\epsilon_0 a^2} \right)^2 + \frac{1}{\mu_0} \left(\frac{\mu_0 I s}{2\pi a^2} \right)^2 \right] = \boxed{\frac{\mu_0 I^2}{2\pi^2 a^4} [(ct)^2 + (s/2)^2]}.$$

$$\mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B}) = \frac{1}{\mu_0} \left(\frac{It}{\pi\epsilon_0 a^2} \right) \left(\frac{\mu_0 I s}{2\pi a^2} \right) (-\hat{\mathbf{s}}) = \boxed{-\frac{I^2 t}{2\pi^2 \epsilon_0 a^4} s \hat{\mathbf{s}}}.$$

$$\frac{\partial u_{\text{em}}}{\partial t} = \frac{\mu_0 I^2}{2\pi^2 a^4} 2ct = \frac{I^2 t}{\pi^2 \epsilon_0 a^4}; \quad -\nabla \cdot \mathbf{S} = \frac{I^2 t}{2\pi^2 \epsilon_0 a^4} \nabla \cdot (s \hat{\mathbf{s}}) = \frac{I^2 t}{\pi^2 \epsilon_0 a^4} = \frac{\partial u_{\text{em}}}{\partial t}. \checkmark$$

$$(c) U_{\text{em}} = \int u_{\text{em}} w 2\pi s ds = 2\pi w \frac{\mu_0 I^2}{2\pi^2 a^4} \int_0^b [(ct)^2 + (s/2)^2] s ds = \frac{\mu_0 w I^2}{\pi a^4} \left[(ct)^2 \frac{s^2}{2} + \frac{1}{4} \frac{s^4}{4} \right]_0^b$$

$$= \boxed{\frac{\mu_0 w I^2 b^2}{2\pi a^4} \left[(ct)^2 + \frac{b^2}{8} \right]}. \text{ Over a surface at radius } b: P_{\text{in}} = - \int \mathbf{S} \cdot d\mathbf{a} = \frac{I^2 t}{2\pi^2 \epsilon_0 a^4} [b \hat{\mathbf{s}} \cdot (2\pi b w \hat{\mathbf{s}})] = \boxed{\frac{I^2 w t b^2}{\pi \epsilon_0 a^4}}.$$

$$\frac{dU_{\text{em}}}{dt} = \frac{\mu_0 w I^2 b^2}{2\pi a^4} 2ct = \frac{I^2 w t b^2}{\pi \epsilon_0 a^4} = P_{\text{in}}. \checkmark \text{ (Set } b = a \text{ for total.)}$$
