Quiz 2

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a)

The time-independent Schrödinger equation in one dimension:

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right]\psi(x) = E\psi(x)$$

Converting the above to two-dimensions:

$$\left[-\frac{\hbar^2}{2m}\left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2}\right) + V(x,y)\right]\psi(x,y) = E\psi(x,y)$$

Using separation of variables we can write $\psi(x)$ as $X(x)Y(y)$:

$$X(x) = A\sin(k_x x) + B\cos(k_x x)$$

$$Y(y) = C\sin(k_y y) + D\cos(k_y y)$$

Applying boundary conditions:

$$\begin{bmatrix} \sin(ka/2) & \cos(ka/2) \\ \sin(-ka/2) & \cos(-ka/2) \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = 0$$

For odd values of n:

$$\psi_n(x) = B_n \cos\left(\frac{n\pi x}{a}\right)$$

For even values of n:

$$\psi_n(x) = A_n \sin\left(\frac{n\pi x}{a}\right)$$

Applying the normalization condition:

$$1 = \int_{-a/2}^{a/2}\int_{-a/2}^{a/2} A^2C^2\sin^2\left(\frac{n\pi x}{a}\right)\sin^2\left(\frac{n\pi y}{a}\right)dxdy = \frac{1}{4}a^2A^2C^2$$

$$1 = \int_{-a/2}^{a/2}\int_{-a/2}^{a/2} B^2D^2\cos^2\left(\frac{n\pi x}{a}\right)\cos^2\left(\frac{n\pi y}{a}\right)dxdy = \frac{1}{4}a^2B^2D^2$$

For even n:

$$\psi(x,y) = \frac{2}{a} \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right)$$

For odd n:

$$\psi(x,y) = \frac{2}{a} \cos\left(\frac{n\pi x}{a}\right) \cos\left(\frac{n\pi y}{a}\right)$$

The energy eigenvalue for the one dimensional system:

$$E_1 = \frac{\hbar^2\pi^2}{2m}\left(\frac{1}{a^2} + \frac{1}{a^2}\right) = \frac{\hbar^2\pi^2}{ma}$$

b)

From the book we know that for a 1D well:

$$E_n = \frac{\hbar^2\pi^2 n^2}{2ma^2}$$

From section a we know that for a 2D well:

$$E_n = \frac{\hbar^2\pi^2}{2m}\left(\frac{n^2}{a^2} + \frac{n^2}{a^2}\right)$$

We can infer that for a 3D well:

$$E_n = \frac{\hbar^2\pi^2}{2m}\left(\frac{n^2}{a^2} + \frac{n^2}{a^2} + \frac{n^2}{a^2}\right)$$

c)

$$E_1 = \frac{\hbar^2\pi^2}{2m}\left(\frac{1^2}{a^2} + \frac{1^2}{a^2} + \frac{1^2}{a^2}\right) = \frac{3\hbar^2\pi^2}{2ma^2}$$

$$E_1 = \frac{3(1.1 \cdot 10^{-34})^2\pi^2}{2(9.1 \cdot 10^{-31})(10^{-9})^2} \approx 2.0 \cdot 10^{-19} J$$