Quantum I Assignment #4

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1.13 Assuming that the observable is $|E\rangle$ with eigenkets $a|1\rangle + b|2\rangle$:

$$H|E\rangle = (H_{11}|1\rangle\langle 1| + H_{22}|2\rangle\langle 2| + H_{12}[|1\rangle\langle 2| + |2\rangle\langle 1|])(a|1\rangle + b|2\rangle) \tag{1}$$
$$H|E\rangle = H_{11}a|1\rangle + H_{22}b|2\rangle + H_{12}b|1\rangle + H_{12}a|2\rangle \tag{2}$$

Since $H|E\rangle = \lambda|E\rangle = \lambda a|1\rangle + \lambda b|2\rangle$:

$$\lambda a = H_{11}a + H_{12}b \tag{3}$$
$$\lambda b = H_{22}b + H_{12}a \tag{4}$$

Using substition to solve for λ:

$$\frac{\lambda(a - H_{11})}{H_{12}} = b \tag{5}$$
$$\frac{\lambda(a - H_{11})}{H_{12}}(\lambda - H_{22}) = H_{12}a \tag{6}$$
$$\lambda \to \frac{aH_{11} - aH_{12} + bH_{12} - bH_{22}}{a - b} \tag{7}$$

1.18 If $|\psi\rangle$ is a eigenket of both A and B then:

$$A|\psi\rangle = a|\psi\rangle$$
$$B|\psi\rangle = b|\psi\rangle$$
$$(AB + BA)|\psi\rangle = 2ab|\psi\rangle$$

The only way this is true is if either a or b is 0.[1]

1.25 (a) No:

$$\begin{bmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = b \tag{8}$$
$$\begin{bmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = -ib \tag{9}$$
$$\begin{bmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = ib \tag{10}$$

[1] http://peeterjoot.com/2015/09/28/can-anticommuting-operators-have-a-simulaneous-eigenket/

(b)

$$\begin{bmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{bmatrix} \begin{bmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{bmatrix} = \begin{bmatrix} ab & 0 & 0 \\ 0 & 0 & iab \\ 0 & -iab & 0 \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{bmatrix} = \begin{bmatrix} ab & 0 & 0 \\ 0 & 0 & iab \\ 0 & -iab & 0 \end{bmatrix} \tag{12}$$

(c) Choosing the eigenkets: $|1\rangle$, $|2\rangle$, $|3\rangle$ we get the following eigenvalues:

$$ab\,|1\rangle - iab\,|2\rangle + iab\,|3\rangle \tag{13}$$

1.28

$$S_z = \frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad S_x = \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{14}$$

$$U = S_x \cdot S_z^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{15}$$

This is consistent with:

$$U = +\frac{1}{\sqrt{2}}\,|+\rangle - \frac{1}{2}\,|-\rangle \tag{16}$$