Quiz 4

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1 a)

$$E_n^{(2)} = \sum_{k \neq n} \frac{\left| \langle k | \hat{H}_1 | n \rangle \right|^2}{\left(n + \frac{1}{2}\right) \hbar\omega - \left(k + \frac{1}{2}\right) \hbar\omega}$$

$$\langle k | \hat{H}_1 | n \rangle = -q\mathcal{E} \frac{\hbar}{2m\omega} \left(\sqrt{n+1} \langle k | n+1 \rangle + \sqrt{n} \langle k | n-1 \rangle \right)$$

$$E_n^{(2)} = \frac{q^2 \mathcal{E}^2 \hbar}{2m\omega} \left(\frac{n+1}{-\hbar\omega} + \frac{n}{\hbar\omega} \right) = \frac{q^2 \mathcal{E}^2}{2m\omega^2}$$

b)

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2 - q\mathcal{E}\hat{x}$$

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{1}{2} m\omega^2 \left(\hat{x} - \frac{q\mathcal{E}}{m\omega^2} \right)^2 - \frac{q^2 \mathcal{E}^2}{2m\omega^2}$$

$$\hat{x}_s = \hat{x} - \frac{q\mathcal{E}}{m\omega^2}$$

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}_s^2 - \frac{q^2 \mathcal{E}^2}{2m\omega^2} = \hat{H}_0 + \hat{H}_2$$

$$E_n = \left(n + \frac{1}{2} \right) \hbar\omega - \frac{q^2 \mathcal{E}^2}{2m\omega^2}$$

2 a)

$$\psi = Are^{-r/a}$$

$$\int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} \left(Are^{-r/a} \right)^2 r^2 \sin\phi \, dr d\phi d\theta = 1$$

$$\frac{3a^5 A^2}{4} \int_0^{2\pi} \int_0^{\pi} \sin\phi \, d\phi d\theta = 1$$

$$3\pi a^5 A^2 = 1$$

$$A = \sqrt{\frac{1}{3\pi a^5}}$$

b)

$$\langle E \rangle = \langle \psi | \hat{H} | \psi \rangle$$

$$\langle E \rangle = \int d^3 r \psi^* \left[-\frac{\hbar^2}{2\mu} \left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} \right) - \frac{e^2}{r} \right] \psi$$

From example 12.2:

$$\langle E \rangle = \frac{\hbar^2}{2\mu a^2} - \frac{e^2}{a}$$

$$\frac{d \langle E \rangle}{da} = -\frac{\hbar^2}{\mu a^3} + \frac{e^2}{a^2} = 0$$

$$a = \frac{\hbar^2}{e^2 \mu}$$

$$E_0 \leq \langle E \rangle = -\frac{e^4 \mu}{2\hbar^2}$$

c)

$$\mathbf{L}^2 |\psi\rangle = l \left(l + \frac{1}{2} \right) \hbar^2 |\psi\rangle$$

$$\langle E \rangle = l \left(l + \frac{1}{2} \right) \hbar^2 \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} \left(A r e^{-r/a} \right)^2 r^2 \sin \phi \, dr d\phi d\theta$$

$$E_2 = 2 \left(2 + \frac{1}{2} \right) \hbar^2 = 5\hbar^2$$