Phys 342 – Electromagnetic Theory
Final Review Strategies

Strategies for finding electric fields in electrostatics

1. Is all the charge known everywhere?
   a. Is there symmetry? If so, use Gauss's Law to find E, $\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$
   b. If not enough symmetry, must integrate $\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}\,')(\vec{r}-\vec{r}\,')\,d\tau'}{|\vec{r}-\vec{r}\,'|^3}$ or integrate to find V first: $V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}\,')\,d\tau'}{|\vec{r}-\vec{r}\,'|}$
2. Is the electric potential given on certain boundaries? Use separation of variables in either Cartesian or Spherical Coordinates to get the potential everywhere first, being careful to do appropriate matching of solutions in different regions of space.
   a. Cartesian V=X(x)Y(y)Z(z) with X = $Ae^{kx} + Be^{-kx}$ or X = A sin(kx) + B cos(kx) and similar for Y and Z. Use infinite sums to match boundary conditions.
   b. Spherical $\boldsymbol{V(r,\theta)} = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l\boldsymbol{(\cos\theta)}$
3. Is the field needed only "far away" from the charges? Use the multipole expansion for V. Calculate total charge Q or dipole moment $\vec{p}$ and plug in to the expression for E or V from point charge or dipole.

Strategies for finding electric fields with materials

4. Is the polarization $\vec{P}$ given? If so, calculate the bound charges from P. Now you know all the charge so use step 1 above to get E.
5. Is ε (or ε_r or χ_e) for the material given along with the free charge and you have symmetry? If so, use Gauss's Law to find D first $\oint \vec{D}\cdot d\vec{a} = Q_{f,enc}$. Then use $\vec{D} = \varepsilon\vec{E}$ to find E from D. (You can also get P from E if you need to.)

Strategies for finding magnetic fields with steady currents

6. Are all the currents known everywhere?
   a. Is there symmetry? If so, use Ampere's Law to get B, $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$
   b. If not enough symmetry, must integrate $\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\,'\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$
7. Is the field needed only "far away" from the currents? Use the multipole expansion for A. Calculate dipole moment $\vec{m}$ and plug into the expression for A or B from a dipole.

Strategies for finding magnetic fields with materials

8. Is the magnetization $\vec{M}$ given? If so, calculate the bound currents from M. Now you know all the currents so use step 6.
9. Is μ (or χ_m) for the material given along with the free current and you have symmetry? If so, use Ampere's Law to find H first $\oint \vec{H} \cdot d\vec{l} = I_{f,enc}$. Then use $\vec{B} = \mu \vec{H}$ to get B from H. (You can also get M from H if you need to.)

Strategy for finding E with time changing B

10. Is there symmetry? Use $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_{enc}}{dt}$ to get E from the time changing magnetic flux. Check the sign with Lenz's Law.

Strategy for finding B from time changing E

11. Is there symmetry? Use $\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{a}$ to get B from the displacement current enclosed by a loop.

Strategy for finding Capacitance of a configuration of two conductors

12. Put +Q on one conductor and –Q on the other. Calculate E between the conductors and find the potential difference between them using $\Delta V = -\int \vec{E} \cdot d\vec{l}$. The capacitance is given by $C = \frac{Q}{\Delta V}$.

Strategy for finding Resistance of a configuration of two conductors

13. Set a potential difference ΔV between the conductors and find the total current I that flows. Use Ohm's Law to relate E to the current density $\vec{J} = \sigma \vec{E}$ and integrate to find the total current I flowing from one conductor to the other. The resistance is R = ΔV/I. (Sometimes it's easier to start with the current, find E from Ohm's Law, then find ΔV from E.)

Strategy for finding Self Inductance

14. Run a current I through the wire and calculate the flux through the loop. Use $\Phi = LI$ to find the inductance L. Don't forget to include the number of turns linking the flux.
15. If it's hard to define one single loop, first calculate B everywhere and use magnetic energy to get the inductance from $U_m = \int \frac{B^2 d\tau}{2\mu_0} = \frac{1}{2} LI^2$

Strategy for finding Mutual Inductance

16. Run a current I through one loop and calculate the magnetic flux through the other loop. Use $\Phi = MI$ to find the mutual inductance M.