# Quantum I Assignment #4

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1.13 Assuming that the observable is $|E\rangle$ with eigenkets $a|1\rangle + b|2\rangle$:

$$H|E\rangle = (H_{11}|1\rangle\langle 1| + H_{22}|2\rangle\langle 2| + H_{12}[|1\rangle\langle 2| + |2\rangle\langle 1|])(a|1\rangle + b|2\rangle) \tag{1}$$

$$H|E\rangle = H_{11}a|1\rangle + H_{22}b|2\rangle + H_{12}b|1\rangle + H_{12}a|2\rangle \tag{2}$$

Since $H|E\rangle = \lambda|E\rangle = \lambda a|1\rangle + \lambda b|2\rangle$:

$$\lambda a = H_{11}a + H_{12}b \tag{3}$$

$$\lambda b = H_{22}b + H_{12}a \tag{4}$$

Using substition to solve for $\lambda$:

$$\frac{\lambda(a - H_{11})}{H_{12}} = b \tag{5}$$

$$\frac{\lambda(a - H_{11})}{H_{12}}(\lambda - H_{22}) = H_{12}a \tag{6}$$

1.18 If $|\psi\rangle$ is a eigenket of both $A$ and $B$ then:

$$A|\psi\rangle = a|\psi\rangle$$
$$B|\psi\rangle = b|\psi\rangle$$
$$(AB + BA)|\psi\rangle = 2ab|\psi\rangle$$

The only way this is true is if either $a$ or $b$ is 0.[1]

1.25 (a) No:

$$\begin{bmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = b \tag{7}$$

$$\begin{bmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = -ib \tag{8}$$

$$\begin{bmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = ib \tag{9}$$

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[1]http://peeterjoot.com/2015/09/28/can-anticommuting-operators-have-a-simulaneous-eigenket/

(b)

$$\begin{bmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{bmatrix} \begin{bmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{bmatrix} = \begin{bmatrix} ab & 0 & 0 \\ 0 & 0 & iab \\ 0 & -iab & 0 \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{bmatrix} = \begin{bmatrix} ab & 0 & 0 \\ 0 & 0 & iab \\ 0 & -iab & 0 \end{bmatrix} \tag{11}$$