

2.2

$$\begin{aligned}\hat{P}_+ &= | +z \rangle \langle +z | \\ \hat{P}_+^\dagger &= (| +z \rangle)^\dagger (\langle +z |)^\dagger = | +z \rangle \langle +z | \\ \hat{P}_+^2 \Psi &= \lambda^2 \Psi = \hat{P}_+ \Psi = \lambda \Psi \\ \lambda^2 &= \lambda \\ \lambda &= 0, 1\end{aligned}$$

2.4

$$\begin{aligned}\langle +y | +x \rangle &= \langle +y | +z \rangle \langle +z | +x \rangle + \\ &\quad \langle +y | -z \rangle \langle -z | +x \rangle \\ \langle +y | +x \rangle &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \frac{1}{\sqrt{2}} \\ \langle -y | +x \rangle &= \langle -y | +z \rangle \langle +z | +x \rangle + \\ &\quad \langle -y | -z \rangle \langle -z | +x \rangle \\ \langle -y | +x \rangle &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \frac{1}{\sqrt{2}} \\ | +x \rangle &= \frac{1}{2} \begin{bmatrix} 1 - i \\ 1 + i \end{bmatrix} \\ \langle +y | -x \rangle &= \langle +y | +z \rangle \langle +z | -x \rangle + \\ &\quad \langle +y | -z \rangle \langle -z | -x \rangle \\ \langle +y | -x \rangle &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \frac{1}{\sqrt{2}} \\ \langle -y | -x \rangle &= \langle -y | +z \rangle \langle +z | -x \rangle + \\ &\quad \langle -y | -z \rangle \langle -z | -x \rangle \\ \langle -y | -x \rangle &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \frac{1}{\sqrt{2}} \\ | -x \rangle &= \frac{1}{2} \begin{bmatrix} 1 - i \\ 1 + i \end{bmatrix}\end{aligned}$$

2.9

$$\begin{aligned}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^\dagger \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}^\dagger &= \begin{bmatrix} 5 & 21 \\ 12 & 31 \end{bmatrix} \\ \left( \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix} \right)^\dagger &= \begin{bmatrix} 5 & 21 \\ 12 & 31 \end{bmatrix}\end{aligned}$$

2.11

$$\begin{aligned}|\psi\rangle &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ \sqrt{2} \end{bmatrix} \\ \langle S_x \rangle &= \left( \frac{1}{\sqrt{2}\sqrt{3}} \right)^2 (1 + \sqrt{2}) \approx 0.40\end{aligned}$$

2.12

a)

$$\left( \frac{i}{\sqrt{3}} \right)^2 = \frac{1}{3}$$

b)

$$\begin{aligned}-\frac{2}{3} \sin \theta |x\rangle + \frac{i}{\sqrt{3}} \cos \theta |y\rangle \\ |\langle y' | \psi \rangle|^2 = \frac{2}{3} \sin^2 \theta + \frac{1}{3} \cos^2 \theta\end{aligned}$$

c)

$$\begin{aligned}|R\rangle &= \frac{1}{\sqrt{2}} (|x\rangle + i |y\rangle) = \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{2}\sqrt{3}} \\ |L\rangle &= 1 - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{2}\sqrt{3}}\end{aligned}$$

Since  $|L\rangle$  is larger it will spin in that direction

d)

$$\begin{aligned}\left( \frac{1}{\sqrt{3}} \right)^2 &= \frac{1}{3} \\ -\frac{2}{3} \sin \theta |x\rangle + \frac{1}{\sqrt{3}} \cos \theta |y\rangle \\ |\langle y' | \psi \rangle|^2 &= \frac{2}{3} \sin^2 \theta + \frac{1}{3} \cos^2 \theta \\ |R\rangle &= \frac{1}{\sqrt{2}} (|x\rangle + i |y\rangle) = \frac{1}{\sqrt{3}} + \frac{i}{\sqrt{2}\sqrt{3}} \\ |L\rangle &= 1 - \frac{1}{\sqrt{3}} - \frac{i}{\sqrt{2}\sqrt{3}}\end{aligned}$$