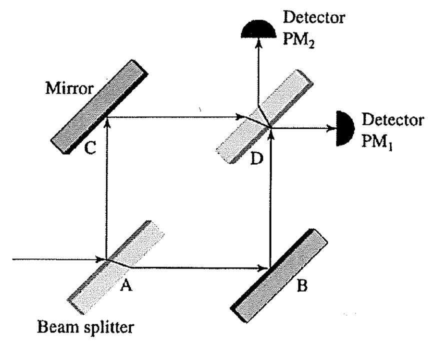
2.) (20 pts) In the figure below, each of the beam splitters are half-silvered mirrors. What is the probability amplitude that a photon arrives at PM_2 via path ABD? What is the probability amplitude that a photon arrives at PM_2 via path ACD? What is the probability that a photon arrives at PM_2? Give the condition for 100% probability that a photon arrives at PM_2.



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(6 pts) $Z_{ABD} = \frac{1}{\sqrt{2}} e^{i\pi} \frac{1}{\sqrt{2}} e^{ikl_1} = \boxed{-\frac{1}{2} e^{ikl_1}}$

Let l_1 be the distance along path ABD, l_2 is distance along ACD.

(6 pts) $Z_{ACD} = \frac{e^{i\pi}}{\sqrt{2}} e^{i\pi} \frac{1}{\sqrt{2}} e^{ikl_2} = \boxed{\frac{1}{2} e^{ikl_2}}$

Probability of a photon arriving at PM_2:

$$P_{PM2} = (Z^*_{ABD} + Z^*_{ACD})(Z_{ABD} + Z_{ACD})$$

$$= \frac{1}{4}\left(-e^{-ikl_1} + e^{-ikl_2}\right)\left(-e^{ikl_1} + e^{ikl_2}\right)$$

$$= \frac{1}{4}\left(2 - e^{-ik(l_1-l_2)} - e^{ik(l_1-l_2)}\right)$$

$$= \frac{1}{2}\left[1 - \cos(k(l_1 - l_2))\right]$$

(6 pts) $\boxed{P_{PM2} = \sin^2\left[\frac{k(l_1-l_2)}{2}\right]}$

(2 pts) For 100% probability $\frac{k(l_1-l_2)}{2} = (n+\frac{1}{2})\pi$ for $n = 0, 1, 2, \ldots$