HW Feb 27, Johannes Byle

6.11

$$\frac{\delta f}{\delta y} - \frac{d}{dx}\frac{\delta f}{\delta y'} = 0$$

Since $\frac{\delta f}{\delta y}$ is 0, $\frac{\delta f}{\delta y'}$ must be constant:

$$f = \sqrt{x}\sqrt{1+y'^2}$$

$$\frac{\delta f}{\delta y'} = \frac{\sqrt{x}y'}{\sqrt{1+y'^2}} = C$$

$$\frac{xy'^2}{1+y'^2} = C^2$$

$$\frac{1}{y'^2} = \frac{x}{C^2} - 1$$

$$y' = \sqrt{\frac{C^2}{x-C^2}}$$

$$\int dy = \int \sqrt{\frac{C^2}{x-C^2}}dx$$

$$y = 2C\sqrt{x-C^2} + C_0$$

Which is a parabola.

6.18

$$\frac{\delta f}{\delta \theta} - \frac{d}{dr}\frac{\delta f}{\delta \theta'} = 0$$

$$dL = \sqrt{dr^2 + r^2 d\theta^2}$$

$$f = \sqrt{1 + r^2\theta'^2}$$

$$\frac{\delta f}{\delta \theta} = 0$$

$$\frac{\delta f}{\delta \theta'} = \frac{r^2\theta'}{\sqrt{1+r^2\theta'^2}} = C$$

7.1

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

$$u = mgy$$

$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - mgz$$

$$\frac{\delta \mathcal{L}}{\delta q_i} = \frac{d}{dt}\frac{\delta \mathcal{L}}{\delta \dot{q}_i}$$

$$\frac{\delta \mathcal{L}}{\delta \dot{x}} = m\dot{x} = constant$$

$$\frac{\delta \mathcal{L}}{\delta \dot{y}} = m\dot{y} = constant$$

$$\frac{\delta \mathcal{L}}{\delta \dot{z}} = m\dot{z}$$

$$\frac{\delta \mathcal{L}}{\delta z} = -mg = \frac{d}{dt}m\dot{z}$$

$$z = -\frac{1}{2}gt^2$$

7.1

$$\mathcal{L} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$$

$$kx = \frac{d}{dt}m\dot{x}$$

$$\frac{k}{m}x = \ddot{x}$$

$$x = A\sin\sqrt{\frac{k}{m}}t$$