Maxwell's Equations

$\nabla \cdot E = \frac{1}{\epsilon_0}\rho \mid \oint E \cdot da = \frac{1}{\epsilon_0}Q_{enc}$

$\nabla \cdot B = 0$

$\nabla \times E = -\frac{\delta B}{\delta t} \mid \oint E \cdot dl = -\frac{d\phi}{dt}$

$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\delta E}{\delta t}$

$\oint B \cdot dl = \mu_0 I_{enc} + \mu_0 \epsilon_0 \int (\frac{\delta E}{\delta t}) \cdot da$

Vector Analysis

$A \times B = (A_y B_z - A_z B_y)\hat{x} + (A_z B_x - A_x B_z)\hat{y} + (A_x B_y - A_y B_x)\hat{z}$

$A \times B = (A_\phi B_z - A_z B_\phi)\hat{r} + (A_z B_r - A_r B_z)\hat{\phi} + (A_r B_\phi - A_\phi B_r)\hat{z}$

$A \times B = (A_\theta B_\phi - A_\phi B_\theta)\hat{r} + (A_\phi B_r - A_r B_\phi)\hat{\theta} + (A_r B_\theta - A_\theta B_r)\hat{\phi}$

$\nabla T = (\hat{x}\frac{\delta}{\delta x} + \hat{y}\frac{\delta}{\delta y} + \hat{z}\frac{\delta}{\delta z})$

$r = \sqrt{x^2+y^2+z^2}$

$\hat{r} = \frac{\mathbf{r}}{r} = \frac{x\hat{x}+y\hat{y}+z\hat{z}}{\sqrt{x^2+y^2+z^2}}$

$\mathscr{r} = \mathbf{r} - \mathbf{r'}$ $_{r'=SourcePoint, r=FieldPoint}$

$\hat{\mathscr{r}} = \frac{\mathscr{r}}{\mathscr{r}} = \frac{r-r'}{|r-r'|}$

$f(x)\delta(x) = f(0)\delta(x)$

$\int_{\infty}^{\infty} f(x)\delta(x-a)dx = f(a)$

$\delta(kx) = \frac{1}{|k|}\delta(x)$

Electrostatics

$F = \frac{1}{4\pi\epsilon_0}\frac{qQ}{\mathscr{r}^2}$

$F = QE$

$E(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')}{\mathscr{r}^2}\hat{\mathscr{r}}d\tau'$

$E_{wire} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}$

$E_{point} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

$E = -\nabla V$

$E^{\parallel}_{above} = E^{\parallel}_{below}$

$E^{\perp}_{above} - E^{\perp}_{below} = \frac{1}{\epsilon_0}\sigma$

$E_{dip}(r,\theta) = \frac{\rho}{4\pi\epsilon_0 r^3}(2cos\theta\hat{r} + sin\theta\hat{\theta})$

$E_{dip} = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[3(p\cdot\hat{r})\hat{r} - p]$

$\phi_E = \int_s E \cdot da$ $_{\phi=Flux}$

$\oint E \cdot dl = 0$

$\nabla \times E = 0$

$V(r) \equiv -\int_a^b E \cdot dl$

$V(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')}{\mathscr{r}}d\tau'$

$V_{point} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

$\nabla^2 V = -\frac{\rho}{\epsilon_0}$ $_{Poisson's Equation}$

$V_{above} - V_{below} = -\oint_a^b E \cdot dl$

$W = \int_a^b F \cdot dl = \frac{\epsilon_0}{2}\int E^2 d\tau$

$W = \frac{1}{2}CV^2$

$C \equiv \frac{Q}{V}$

$C = 4\pi\epsilon_0\frac{ab}{(b-a)}$ $_{CocentricShells}$

Potentials

First Uniqueness Theorem: The solution to Laplace equation in some volume V is uniquely determined if V is specified on the boundary surface S

Second Uniqueness Theorem:
In a volume V surrounded by a conductors and containing a specified charge density ρ, the electric field is uniquely determined if the *total charge* on each conductor is given. (The region as a whole can be bounded by another conductor or else unbounded)

Method of Images

· Replace the conducting plane with a mirror image charge

· Use Gausss law on each charge in isolation

· Sum up the electric field contribution from each charge

$V(x,y) = (Ae^{kx} + B^{-kx})(Csinky + Dcosky)$

$V(r,\theta) = \sum_{l=0}^{\infty}(A_l r^l + \frac{B_l}{r^{l+1}})P_l(cos\theta)$

$V_{mon} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$

$V_{dip} \approx \frac{1}{4\pi\epsilon_0}\frac{qdcos\theta}{r^2}$ $_{d=DipoleDistance}$

$V_{dip} = \frac{1}{4\pi\epsilon_0}\frac{p\cdot\hat{r}}{r^2}$ $_{p=DipoleMoment}$

$p \equiv \int r'\rho(r')d\tau' = \sum_{i=1}^{n} q_i r'_i$

$P_0(x) = 1$

$P_1(x) = x$

$P_2(x) = \frac{1}{2}(3x^2 - 1)$

$P_3(x) = \frac{1}{2}(5x^3 - 3x)$

$E_{dip}(r,\theta) = \frac{\rho}{4\pi\epsilon_0 r^3}(2cos\theta\hat{r} + sin\theta\hat{\theta})$

$E_{dip} = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[3(p\cdot\hat{r})\hat{r} - p]$

Electric Fields In Matter

$p = \alpha E$ $_{p=DipoleMoment, \alpha=Polarizability}$

$N_{dip} = p \times E$ $_{N=Torque}$

$F_{dip} = (p \cdot \nabla)E$

$U_{dip} = -p \cdot E$

$P \equiv_{DipoleMomentPerUnitVolume/Polarization}$

$\sigma_b \equiv P \cdot \hat{n}$ $_{\sigma_b=BoundSurfaceCharge}$

$\rho_b \equiv -\nabla \cdot P$ $_{\rho_b=BoundVolumeCharge}$

$D \equiv \epsilon_0 E + P$ $_{D=ElectricDisplacement}$

$\nabla \cdot D = \rho_f$

$\oint D \cdot da = Q_{f_{enc}}$

$D = \epsilon E$

$\epsilon \equiv \epsilon_0(1+\chi_e)$ $_{\chi_e=ElectricSusceptability}$

$\epsilon_r \equiv 1 + \chi_e = \frac{\epsilon}{\epsilon_0}$ $_{\epsilon_r=RelativePermattivity}$

$C = \epsilon_r C_{vac}$ $_{ForLinearDialectric}$

Magnetostatics

Magnetic forces do no work.

Stationary charges $\rightarrow$ Electrostatics

Steady currents $\rightarrow$ Magnetostatics

$F_{mag} = Q(v \times B)$

$F_{mag} = \int I(dl \times B)$

$K = \frac{dI}{dl_\perp} = \sigma v$ $_{(K=SurfaceCurrentDensity)}$

$J = \frac{dI}{da_\perp} = \rho v$ $_{(J=VolumeCurrentDensity)}$

$J = \frac{1}{\mu_0}(\nabla \times B)$

$\nabla \cdot J = -\frac{\delta\rho}{\delta t}$

$\nabla \times B = \mu_0 J$

$\oint B \cdot dl = \mu_0 I_{enc}$

$B = \nabla \times A$ $_{(A=VectorPotential)}$

$\nabla \cdot A = 0$

$\nabla^2 A = -\mu_0 J$

$A(r) = \frac{\mu_0}{4\pi}\int \frac{J(r')}{\mathscr{r}}d\tau'$

$A_{above} = A_{below}$

$A_{dip}(r) = \frac{\mu_0}{4\pi}\frac{m\times\hat{r}}{r^2}$

$B^{\perp}_{above} = B^{\perp}_{below}$

$B^{\parallel}_{above} - B^{\parallel}_{below} = \mu_0 K$

$B_{above} - B_{below} = \mu_0(K \times \hat{n})$

$m = I\int da = Ia$ $_{(m=MagneticDipoleMoment)}$

$B_{dip}(r) = \frac{\mu_0}{4\pi}[3(m\cdot\hat{r})\hat{r} - m]$

$B_{line} = \frac{\mu_0 I}{4\pi s}(sin\theta_2 - sin\theta_1)$

$B_{loop} = \frac{\mu_0 I}{2r}$

$B_{wire} = \frac{\mu_0 I}{2\pi r}$

$B_{solenoid} = \mu_0 n I$

$J_b = \nabla \times M$

$K_b = M \times \hat{n}$

Magnetic Fields in Matter

$N = m \times b$ $_{N=Torque}$

$F_{loop} = \nabla(m \cdot B)$

$J_b = \nabla \times M$ $_{J_b=VolumeBoundCurrent}$

$K_b = M \times \hat{n}$ $_{K_b=SurfaceBoundCurrent}$

$H \equiv \frac{1}{\mu_0}B - M$

$\nabla \times H = J_f \mid \oint H \cdot dl = I_{f_{enc}}$

$\nabla \cdot H = -\nabla \cdot M$

$M \equiv_{MagneticDipoleMoment/Magnetization}$

$M = \chi_m H$ $_{\chi_m=MagneticSusceptibility}$

$B = \mu H$

$\mu \equiv \mu_0(1+\chi_m)$

$U_{dip} = -m \cdot B$

Electrodynamics

$F = ma = qE$

$J = \sigma(E + v \times B)$ $_{\sigma=Conductivity}$

$J = \sigma E$ $_{Ohm's Law}$

$J_d = \epsilon_0\frac{\delta E}{\delta t}$

$I = \sigma\int E \cdot da$

$P = VI = I^2 R$

$V = IR$

$\tau = RC$

$\varepsilon \equiv \oint f \cdot dl = \oint f_s \cdot dl$ $_{\varepsilon=ElectromotiveForce}$

$\varepsilon = -\frac{d\Phi}{dt}$

$\varepsilon = IR$

A Changing magnetic field induces an electric field

Nature abhors a change in flux

$F_{mag} = \int I(dl \times B)$

$F_{mag} = Q(v \times B)$

$\Phi = LI$ $_{L=SelfInductance}$

$\varepsilon = -L\frac{dI}{dt}$

$W = \frac{1}{2}LI^2$

$W = \frac{1}{2\mu_0}\int_{allspace} B^2 d\tau$

Conservation Laws

$\frac{\delta\rho}{\delta t} = -\nabla \cdot J$

$u = \frac{1}{2}(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2)$ $_{u=EnergyPerVolume}$

$S \equiv \frac{1}{\mu_0}(E \times B)$ $_{S=PoyntingVector}$

$P = \int S \cdot da$

Electromagnetic Waves

$\omega = 2\pi v = kv$

$\frac{\delta^2 f}{\delta z^2} = \frac{1}{v^2}\frac{\delta^2 f}{\delta t^2}$

$\nabla^2 E = \mu_0\epsilon_0\frac{\delta^2 E}{\delta t^2}$

$\nabla^2 B = \mu_0\epsilon_0\frac{\delta^2 B}{\delta t^2}$

$E(z,t) = E_0 cos(kz - \omega t + \delta)\hat{x}$

$B(z,t) = \frac{1}{c}E_0 cos(kz - \omega t + \delta)\hat{y}$

$\tilde{E}(r,t) = \tilde{E}_0 e^{i(k\cdot r - \omega t)}\hat{n}$

$\tilde{B}(r,t) = \frac{1}{c}\tilde{E}_0 e^{i(k\cdot r - \omega t)}(\hat{k} \times \hat{n}) = \frac{1}{c}\hat{k} \times \tilde{E}$

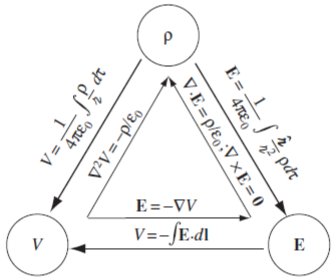


FIGURE 2.35

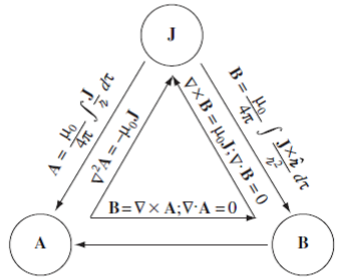
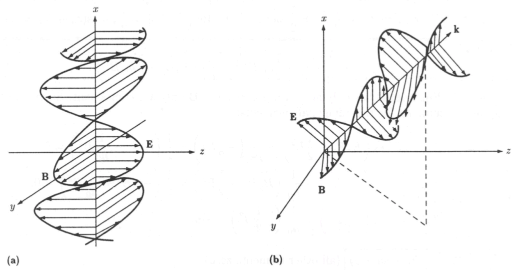


FIGURE 5.48

Problem 9.9

(a) $\boxed{\mathbf{k} = -\frac{\omega}{c}\,\hat{\mathbf{x}};\ \hat{\mathbf{n}} = \hat{\mathbf{z}}.}$ $\mathbf{k}\cdot\mathbf{r} = \left(-\frac{\omega}{c}\,\hat{\mathbf{x}}\right)\cdot(x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}) = -\frac{\omega}{c}x;\ \mathbf{k}\times\hat{\mathbf{n}} = -\hat{\mathbf{x}}\times\hat{\mathbf{z}} = \hat{\mathbf{y}}.$

$$\boxed{\mathbf{E}(x,t) = E_0\cos\left(\frac{\omega}{c}x + \omega t\right)\hat{\mathbf{z}};\quad \mathbf{B}(x,t) = \frac{E_0}{c}\cos\left(\frac{\omega}{c}x + \omega t\right)\hat{\mathbf{y}}.}$$



(b) $\boxed{\mathbf{k} = \frac{\omega}{c}\left(\frac{\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}}}{\sqrt{3}}\right);\ \hat{\mathbf{n}} = \frac{\hat{\mathbf{x}}-\hat{\mathbf{z}}}{\sqrt{2}}.}$ (Since $\hat{\mathbf{n}}$ is parallel to the xz plane, it must have the form $\alpha\,\hat{\mathbf{x}} + \beta\,\hat{\mathbf{z}}$; since $\hat{\mathbf{n}}\cdot\mathbf{k} = 0, \beta = -\alpha$; and since it is a unit vector, $\alpha = 1/\sqrt{2}$.)

$$\mathbf{k}\cdot\mathbf{r} = \frac{\omega}{\sqrt{3}c}(\hat{\mathbf{x}}+\hat{\mathbf{y}}+\hat{\mathbf{z}})\cdot(x\,\hat{\mathbf{x}}+y\,\hat{\mathbf{y}}+z\,\hat{\mathbf{z}}) = \frac{\omega}{\sqrt{3}c}(x+y+z);\ \hat{\mathbf{k}}\times\hat{\mathbf{n}} = \frac{1}{\sqrt{6}}\begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ 1 & 1 & 1\\ 1 & 0 & -1\end{vmatrix} = \frac{1}{\sqrt{6}}(-\hat{\mathbf{x}} + 2\,\hat{\mathbf{y}} - \hat{\mathbf{z}}).$$

$$\boxed{\begin{aligned}\mathbf{E}(x,y,z,t) &= E_0\cos\left[\frac{\omega}{\sqrt{3}c}(x+y+z) - \omega t\right]\left(\frac{\hat{\mathbf{x}}-\hat{\mathbf{z}}}{\sqrt{2}}\right);\\ \mathbf{B}(x,y,z,t) &= \frac{E_0}{c}\cos\left[\frac{\omega}{\sqrt{3}c}(x+y+z) - \omega t\right]\left(\frac{-\hat{\mathbf{x}}+2\hat{\mathbf{y}}-\hat{\mathbf{z}}}{\sqrt{6}}\right).\end{aligned}}$$

Problem 8.1

Example 7.13.

$$\left.\begin{aligned}\mathbf{E} &= \frac{\lambda}{2\pi\epsilon_0}\frac{1}{s}\,\hat{\mathbf{s}}\\ \mathbf{B} &= \frac{\mu_0 I}{2\pi}\frac{1}{s}\,\hat{\boldsymbol{\phi}}\end{aligned}\right\}\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}) = \frac{\lambda I}{4\pi^2\epsilon_0}\frac{1}{s^2}\,\hat{\mathbf{z}};$$

$$P = \int \mathbf{S}\cdot d\mathbf{a} = \int_a^b S\,2\pi s\,ds = \frac{\lambda I}{2\pi\epsilon_0}\int_a^b \frac{1}{s}\,ds = \frac{\lambda I}{2\pi\epsilon_0}\ln(b/a).$$

$$\text{But } V = \int_a^b \mathbf{E}\cdot d\mathbf{l} = \frac{\lambda}{2\pi\epsilon_0}\int_a^b \frac{1}{s}\,ds = \frac{\lambda}{2\pi\epsilon_0}\ln(b/a),\ \text{so } \boxed{P = IV.}$$

Problem 7.62.

$$\left.\begin{aligned}\mathbf{E} &= \frac{\sigma}{\epsilon_0}\,\hat{\mathbf{z}}\\ \mathbf{B} &= \mu_0 K\,\hat{\mathbf{x}} = \frac{\mu_0 I}{w}\,\hat{\mathbf{x}}\end{aligned}\right\}\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}) = \frac{\sigma I}{\epsilon_0 w}\,\hat{\mathbf{y}};$$

$$P = \int \mathbf{S}\cdot d\mathbf{a} = Swh = \frac{\sigma I h}{\epsilon_0},\ \text{but } V = \int \mathbf{E}\cdot d\mathbf{l} = \frac{\sigma}{\epsilon_0}h,\ \text{so } \boxed{P = IV.}$$

Problem 8.2

(a) $\mathbf{E} = \frac{\sigma}{\epsilon_0}\,\hat{\mathbf{z}};\ \sigma = \frac{Q}{\pi a^2};\ Q(t) = It\ \Rightarrow\ \mathbf{E}(t) = \boxed{\frac{It}{\pi\epsilon_0 a^2}\,\hat{\mathbf{z}}.}$

$$B\,2\pi s = \mu_0\epsilon_0\frac{\partial E}{\partial t}\pi s^2 = \mu_0\epsilon_0\frac{I\pi s^2}{\pi\epsilon_0 a^2}\ \Rightarrow\ \mathbf{B}(s,t) = \boxed{\frac{\mu_0 I s}{2\pi a^2}\,\hat{\boldsymbol{\phi}}.}$$

(b) $u_{\text{em}} = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) = \frac{1}{2}\left[\epsilon_0\left(\frac{It}{\pi\epsilon_0 a^2}\right)^2 + \frac{1}{\mu_0}\left(\frac{\mu_0 I s}{2\pi a^2}\right)^2\right] = \boxed{\frac{\mu_0 I^2}{2\pi^2 a^4}\left[(ct)^2 + (s/2)^2\right].}$

$$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}) = \frac{1}{\mu_0}\left(\frac{It}{\pi\epsilon_0 a^2}\right)\left(\frac{\mu_0 I s}{2\pi a^2}\right)(-\hat{\mathbf{s}}) = \boxed{-\frac{I^2 t}{2\pi^2\epsilon_0 a^4}s\,\hat{\mathbf{s}}.}$$

$$\frac{\partial u_{\text{em}}}{\partial t} = \frac{\mu_0 I^2}{2\pi^2 a^4}2c^2 t = \frac{I^2 t}{\pi^2\epsilon_0 a^4};\quad -\boldsymbol{\nabla}\cdot\mathbf{S} = \frac{I^2 t}{2\pi^2\epsilon_0 a^4}\boldsymbol{\nabla}\cdot(s\,\hat{\mathbf{s}}) = \frac{I^2 t}{\pi^2\epsilon_0 a^4} = \frac{\partial u_{\text{em}}}{\partial t}.\ \checkmark$$

(c) $U_{\text{em}} = \int u_{\text{em}} w 2\pi s\,ds = 2\pi w\frac{\mu_0 I^2}{2\pi^2 a^4}\int_0^b[(ct)^2 + (s/2)^2]s\,ds = \frac{\mu_0 w I^2}{\pi a^4}\left[(ct)^2\frac{s^2}{2} + \frac{1}{4}\frac{s^4}{4}\right]\Big|_0^b$

$= \boxed{\frac{\mu_0 w I^2 b^2}{2\pi a^4}\left[(ct)^2 + \frac{b^2}{8}\right].}$ Over a surface at radius b: $P_{\text{in}} = -\int \mathbf{S}\cdot d\mathbf{a} = \frac{I^2 t}{2\pi^2\epsilon_0 a^4}\left[b\,\hat{\mathbf{s}}\cdot(2\pi b w\,\hat{\mathbf{s}})\right] = \boxed{\frac{I^2 w t b^2}{\pi\epsilon_0 a^4}.}$

$$\frac{dU_{\text{em}}}{dt} = \frac{\mu_0 w I^2 b^2}{2\pi a^4}2c^2 t = \frac{I^2 w t b^2}{\pi\epsilon_0 a^4} = P_{\text{in}}.\ \checkmark\ \text{(Set } b = a \text{ for } \textit{total.}\text{)}$$

Phys 342 – Electromagnetic Theory
Final Review Strategies

Strategies for finding electric fields in electrostatics

1. Is all the charge known everywhere?
   a. Is there symmetry? If so, use Gauss's Law to find E, $\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$
   b. If not enough symmetry, must integrate $\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}')(\vec{r}-\vec{r}')d\tau'}{|\vec{r}-\vec{r}'|^3}$ or integrate to find V first: $V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}')d\tau'}{|\vec{r}-\vec{r}'|}$
2. Is the electric potential given on certain boundaries? Use separation of variables in either Cartesian or Spherical Coordinates to get the potential everywhere first, being careful to do appropriate matching of solutions in different regions of space.
   a. Cartesian V=X(x)Y(y)Z(z) with X = Ae^{kx} + Be^{-kx} or X = A sin(kx) + B cos(kx) and similar for Y and Z. Use infinite sums to match boundary conditions.
   b. Spherical $V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$
3. Is the field needed only "far away" from the charges? Use the multipole expansion for V. Calculate total charge Q or dipole moment $\vec{p}$ and plug in to the expression for E or V from point charge or dipole.

Strategies for finding electric fields with materials

4. Is the polarization $\vec{P}$ given? If so, calculate the bound charges from P. Now you know all the charge so use step 1 above to get E.
5. Is ε (or ε_r or χ_e) for the material given along with the free charge and you have symmetry? If so, use Gauss's Law to find D first $\oint \vec{D}\cdot d\vec{a} = Q_{f,enc}$. Then use $\vec{D} = \varepsilon\vec{E}$ to find E from D. (You can also get P from E if you need to.)

Strategies for finding magnetic fields with steady currents

6. Are all the currents known everywhere?
   a. Is there symmetry? If so, use Ampere's Law to get B, $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$
   b. If not enough symmetry, must integrate $\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}'\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$
7. Is the field needed only "far away" from the currents? Use the multipole expansion for A. Calculate dipole moment $\vec{m}$ and plug into the expression for A or B from a dipole.

Strategies for finding magnetic fields with materials

8. Is the magnetization $\vec{M}$ given? If so, calculate the bound currents from M. Now you know all the currents so use step 6.
9. Is μ (or χ_m) for the material given along with the free current and you have symmetry? If so, use Ampere's Law to find H first $\oint \vec{H}\cdot d\vec{l} = I_{f,enc}$. Then use $\vec{B} = \mu\vec{H}$ to get B from H. (You can also get M from H if you need to.)

Strategy for finding E with time changing B

10. Is there symmetry? Use $\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_{enc}}{dt}$ to get E from the time changing magnetic flux. Check the sign with Lenz's Law.

Strategy for finding B from time changing E

11. Is there symmetry? Use $\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0\frac{\partial}{\partial t}\int \vec{E}\cdot d\vec{a}$ to get B from the displacement current enclosed by a loop.

Strategy for finding Capacitance of a configuration of two conductors

12. Put +Q on one conductor and –Q on the other. Calculate E between the conductors and find the potential difference between them using $\Delta V = -\int \vec{E}\cdot d\vec{l}$. The capacitance is given by $C = \frac{Q}{\Delta V}$.

Strategy for finding Resistance of a configuration of two conductors

13. Set a potential difference ΔV between the conductors and find the total current I that flows. Use Ohm's Law to relate E to the current density $\vec{J} = \sigma\vec{E}$ and integrate to find the total current I flowing from one conductor to the other. The resistance is R = ΔV/I. (Sometimes it's easier to start with the current, find E from Ohm's Law, then find ΔV from E.)

Strategy for finding Self Inductance

14. Run a current I through the wire and calculate the flux through the loop. Use $\Phi = LI$ to find the inductance L. Don't forget to include the number of turns linking the flux.
15. If it's hard to define one single loop, first calculate B everywhere and use magnetic energy to get the inductance from $U_m = \int \frac{B^2 d\tau}{2\mu_0} = \frac{1}{2}LI^2$

Strategy for finding Mutual Inductance

16. Run a current I through one loop and calculate the magnetic flux through the other loop. Use $\Phi = MI$ to find the mutual inductance M.