

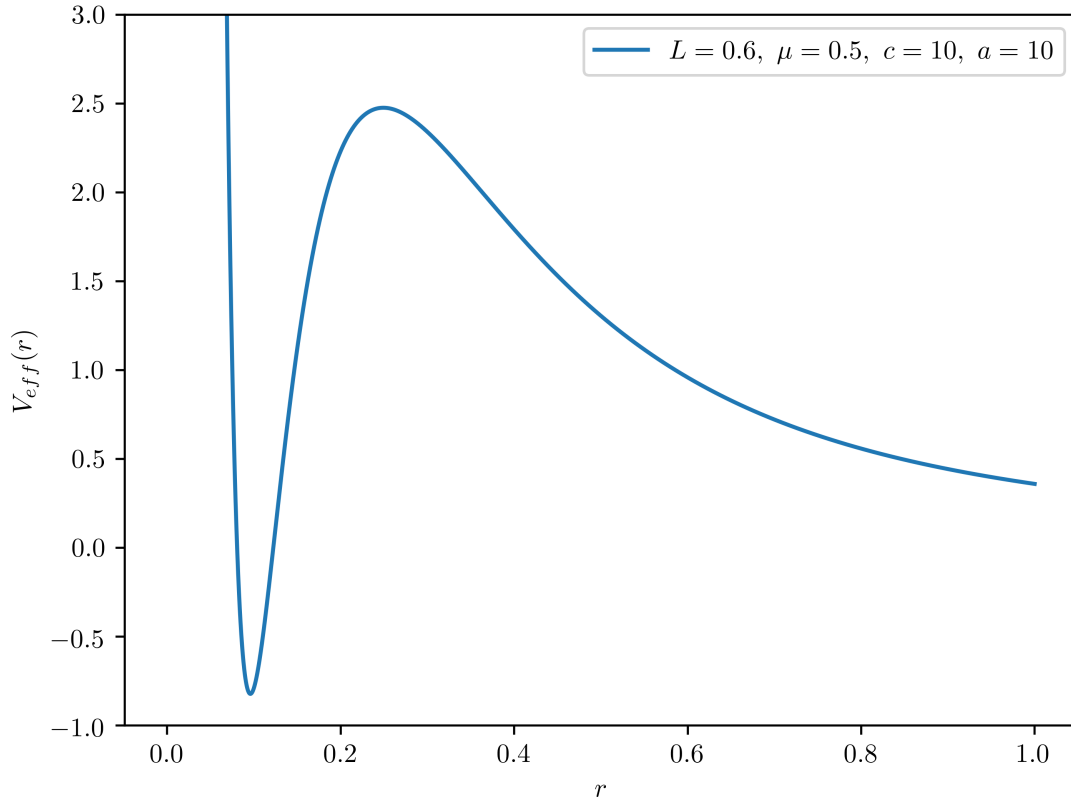
Classical Assignment #6

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1. (a) The effective potential is simply:

$$V_{\text{eff}}(r) = \frac{L^2}{2\mu r^2} - C \frac{e^{-\alpha r}}{r}$$



- (b) There are 3 distinct ranges of energies: The first range of energies are those above the “hump” where the particle will simply be scattered. Below this the particle will either, depending on the initial value of r , scatter off the hump, or “orbit” around the central force while oscillating between two different values of r . The third range is simply the bottom of the well, the lowest allowed energy, where the particle will cleanly orbit the central force.
- (a) Since the particle is scattering off of a sphere, $r_{\min} = a_0$, and since the origin is at the center of the sphere $\Psi = \arcsin\left(\frac{s}{a_0}\right)$. Using the relation $\Theta = \pi - 2\Psi$ we get

$\Theta = \pi - 2 \arcsin \left(\frac{s}{a_0} \right)$. Rearranging this equation in terms of s :

$$s = \sin \left(\frac{\pi - \Theta}{2} \right) = \cos \left(\frac{\Theta}{2} \right)$$
$$\frac{ds}{d\Theta} = -\frac{1}{2} \sin \left(\frac{\Theta}{2} \right)$$