

HW March 1, Johannes Byle

7.8(a)

$$T = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2$$

$$U = \frac{1}{2}k(x_1 - x_2 - l)^2$$

$$\mathcal{L} = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 - \frac{1}{2}k(x_1 - x_2 - l)^2$$

(b)

$$\mathcal{L} = 2m\dot{X}^2 - \frac{1}{2}kx^2$$

(c)

$$\frac{\delta\mathcal{L}}{\delta X} = 0$$

$$\frac{\delta\mathcal{L}}{\delta\dot{X}} = 4m\dot{X} = C$$

$$\frac{\delta\mathcal{L}}{\delta x} = -kx = \ddot{x}$$

$$X(t) = \frac{C}{4m}t$$

$$x(t) = A\sin(kt)$$

7.12

$$\mathcal{L} = T - U$$

In order to add F_{fric} to the above equation we would have to take the integral of the friction force and subtract it from the U term. When we convert the Lagrangian to the derivative form we then convert F_{fric} back to a basic force form, leaving the equation in the book.

7.23

$$\mathcal{L} = T - U$$

$$T = \frac{1}{2}m(\dot{X} + \dot{x})^2 + \frac{1}{2}M\dot{X}^2$$

$$U = \frac{1}{2}kx^2$$

$$\frac{\delta\mathcal{L}}{\delta X} = 0$$

$$\frac{\delta\mathcal{L}}{\delta\dot{X}} = m(\dot{X} + \dot{x}) + M\dot{X} = C$$

$$\frac{\delta\mathcal{L}}{\delta x} = -kx = \frac{d}{dt}m(\dot{X} + \dot{x})$$

$$\ddot{X} = -\omega^2 A \cos(\omega t)$$

$$-kx = m\ddot{X} + m\ddot{x}$$

$$\ddot{x} + \omega_0^2 x = B \cos \omega t$$