Math Methods Assignment #6

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1. (a) Starting with Ampere's Law:

$$\begin{aligned}
\nabla \times \mathcal{B} &= \frac{4\pi}{c}\boldsymbol{J} + \frac{1}{c}\frac{\partial \mathcal{E}}{\partial t} \\
\nabla\left(\nabla \times \mathcal{B}\right) &= \frac{4\pi}{c}\nabla \cdot \boldsymbol{J} + \frac{1}{c}\frac{\partial(\nabla \cdot \mathcal{E})}{\partial t} = 0 \quad \text{Using the identity: } \nabla\left(\nabla \cdot \times A\right) = 0 \\
\nabla\left(\nabla \times \mathcal{B}\right) &= \frac{4\pi}{c}\nabla\boldsymbol{J} + \frac{4\pi}{c}\frac{\partial \rho}{\partial t} = 0 \quad \text{Using: } \nabla \cdot \mathcal{E} = 4\pi\rho \\
\nabla\boldsymbol{J} + \frac{\partial \rho}{\partial t} &= 0
\end{aligned}$$

(b) Starting with the curl of the electric field:

$$\begin{aligned}
\nabla \times \mathcal{E} &= -\frac{1}{c}\frac{\partial \mathcal{B}}{\partial t} \\
\nabla \times \mathcal{E} &= -\frac{1}{c}\frac{\partial(\nabla \times \mathcal{A})}{\partial t} \\
\nabla \times \left(\mathcal{E} + \frac{1}{c}\frac{\partial \mathcal{A}}{\partial t}\right) &= 0 \\
\mathcal{E} &= -\nabla\phi - \frac{1}{c}\frac{\partial \mathcal{A}}{\partial t} \quad \text{Rewriting in terms of a scalar potential}
\end{aligned}$$

(c)

$$F = \begin{bmatrix} 0 & -B_3 & B_2 \\ B_3 & 0 & -B_1 \\ -B_2 & B_1 & 0 \end{bmatrix}$$

$$B = \left[\tfrac{1}{2}(B_1 + B_1) \quad \tfrac{1}{2}(B_2 + B_2) \quad \tfrac{1}{2}(B_3 + B_3)\right]$$