General Formulas

$e^{i\theta} = \cos\theta + i\sin\theta$

$-\frac{\hbar^2}{2m}\frac{\delta\Psi(x,t)}{\delta x^2} + V(x)\Psi(x,t) = i\hbar\frac{\delta\Psi(x,t)}{\delta t}$

$\sin\alpha + \sin\beta = 2\cos\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}$

Light

$\mathcal{E}_0$=WaveAmplitude	n=IndexOfRefraction
k=Wavenumber	a=SingleSlitWidth
ω=Wavelength	h=Planck'sConstant
T=Period	K=KineticEnergy
ν=OrdinaryFrequency	W=WorkFunction
ϕ=PhaseShift	p=momentum
c=SpeedOfLightVacuum	

$\mathcal{E} = \mathcal{E}_0\cos(kx - \omega t)$ (1.1)

$k = \frac{2\pi}{\lambda}$ (1.2)

$\omega = \frac{2\pi}{T} = 2\pi\nu$ (1.3)

$\nu = 1/T$

$\mathcal{E} = \mathcal{E}_0\cos(kx - \omega t + \phi)$ (1.6)

$e^{i\theta} = \cos\theta + i\sin\theta$ (1.7)

$\omega = kc$ (1.11)

$\omega\nu = c$ (1.12)

$\frac{\delta^2\mathcal{E}}{\delta x^2} - \frac{n^2}{c}\frac{\delta^2\mathcal{E}}{\delta t^2} = 0$ (1.13)

$\lambda\nu = \frac{c}{n}$ (1.14)

$a\sin\theta = n\lambda$ (minima)

$E = h\nu$ (1.18)

$K = h\nu - W$ (1.19)

$h\nu_0 = hc/\lambda_0 = W$ (1.20)

$p = \frac{h}{\lambda}$ (1.21)

$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$ (1.28) **Compton**

The First Principle of Quantum Mechanics

The probability of an event $= z^*z$ (1.32)

The Second Principle of Quantum Mechanics

To determine the probability amplitude for a process that can be viewed as taking place in a series of steps we multiply the probability amplitudes for each of these steps.

$z = z_a z_b \cdots$ (1.38)

The Third Principle of Quantum Mechanics

If there are multiple ways that an event can occur we add the amplitudes for each of these ways.

$z = z_1 + z_2 + \cdots$ (1.47)

$\phi = kx$

$z = x + iy = r\cos\phi + ir\sin\phi = re^{i\phi}$

$z^* = x - iy = r\cos\phi - ir\sin\phi = re^{-i\phi}$

Wave Mechanics

*In this section we assume a free particle, V(x)=0

j=ProbabilityCurrent	Δx=Uncertainty
$\langle x\rangle$=ExpectationValue	

$\lambda = \frac{h}{p}$ (2.1) **de Broglie wavelength**

$d\sin\theta = n\lambda$ (2.3) (maxima)

$x_{n+1} - x_n = \frac{L\lambda}{d}$ (2.4)

$2d\sin\theta = n\lambda$ (2.5) **Bragg relation**

$-\frac{\hbar^2}{2m}\frac{\delta\Psi(x,t)}{\delta x^2} + V(x)\Psi(x,t) = i\hbar\frac{\delta\Psi(x,t)}{\delta t}$ (2.6)

$-\frac{\hbar^2}{2m}\frac{\delta\Psi(x,t)}{\delta x^2} = i\hbar\frac{\delta\Psi(x,t)}{\delta t}$ (2.7)

$\frac{\delta^2\mathcal{E}}{\delta x^2} = \frac{n^2}{c}\frac{\delta^2\mathcal{E}}{\delta t^2}$ (2.8)

$E = h\nu - \frac{h}{2\pi}2\pi\nu = \hbar\omega$ (2.9)

$p = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k$ (2.10)

$\hbar\omega = \hbar kc$ (2.11)

$E = pc$ (2.12)

$\hbar\omega = \frac{\hbar^2k^2}{2m}$ (2.15)

$p = \frac{h}{\lambda} = \hbar k$ (2.16)

$E = h\nu = \hbar\omega$ (2.17)

$E = \frac{p^2}{2m}$ (2.18)

$|\Psi(x,t)|^2dx$ =*the probability of finding the particle between x and x+dx at the time t if a measurement of the particle's position is carried out*

$|\Psi(x,t)|^2$ **probability density**

$\int_{-\infty}^{\infty}|\Psi(x,t)|^2dx = 1$ (2.19)

$\frac{\delta|\Psi|^2}{\delta t} = \frac{\Psi^*\Psi}{\delta t} = \Psi^*\frac{\delta\Psi}{\delta t} + \Psi\frac{\delta\Psi^*}{\delta t}$ (2.20)

$j_x(x,t) = \frac{\hbar}{2mi}(\Psi^*\frac{\delta\Psi}{\delta t} + \Psi\frac{\delta\Psi^*}{\delta t})$ (2.24)

$\frac{d}{dt}\int_{-\infty}^{\infty}|\Psi(x,t)|^2dx = -j_x(x,t)|_{-\infty}^{\infty} = 0$

$\Psi(x,t) = \int_{-\infty}^{\infty}A(k)e^{i(kx-\omega t)}dk$ (2.29)

$\Delta x\Delta k \geq \frac{1}{2}$ (2.30)

$\Delta x\Delta p_x \geq \frac{\hbar}{2}$ (2.31) **Heisenberg**

$v_{ph} = \frac{\omega}{k} = \frac{2\pi\nu}{(2\pi/\lambda)} = \lambda\nu$ (2.33)

The phase velocity is the speed at which a point on the wave, such as a crest, moves.

$v_{ph} = \frac{\omega}{k} = \frac{\hbar\omega}{\hbar k} = \frac{E}{p} = \frac{mv^2/2}{mv}\frac{v}{2}$ (2.34)

$v_g = \frac{d\omega}{dk}$ (2.36)

The group velocity is the speed of a localized packet of waves that has been generated by superposing many waves together

$\Psi(x,t) = \int_{-\infty}^{\infty}A(k)e^{i(kx-\omega t)}dk$ (2.37)

$\omega \cong \omega_0 + v_g(k - k_0)$ (2.39)

Dispersion relation is the relationship between ω and k $\langle x\rangle = \int_{-\infty}^{\infty}x|\Psi(x,t)|^2dx$ (2.53)

The average values $\langle x\rangle$ are referred to as the expectation values

$\langle x^2\rangle = \int_{-\infty}^{\infty}x^2|\Psi(x,t)|^2dx$ (2.55)

$(\Delta x)^2 = \langle x^2\rangle - \langle x\rangle^2$ (2.56)

Δx, the standard deviation, is also called the uncertainty

$(\Delta p_x)^2 = \langle p_x^2\rangle - \langle p_x\rangle^2$ (2.57)

$\frac{d\langle x\rangle}{dt} = \frac{\langle p_x\rangle}{m}$ (2.58)

$\langle p_x\rangle = \int_{-\infty}^{\infty}\Psi^*\frac{\hbar}{i}\frac{\delta\Psi}{\delta x}dx$ (2.63)

$\frac{d\langle p_x\rangle}{dt} = \langle -\frac{\delta V}{\delta x}\rangle$ (2.64)

The Time-Independent Schrödinger Equation

*In this section we assume V(x) is independent of t

δ_{nm}=KroneckerDelta	ψ_a=Eigenfunction
a=Eigenvalue	T=TransmissionCoef.

$\Psi(x,t) = \psi(x)f(t)$ (3.2)

$\frac{\delta^2\Psi(x,t)}{\delta x^2} = f(t)\frac{d^2\psi(x)}{dx^2}$ (3.3)

$\frac{\delta\Psi(x,t)}{\delta t} = \psi(x)\frac{df(t)}{dt}$ (3.4)

$\frac{df(t)}{dt} = \frac{-iE}{\hbar}f(t)$ (3.8)

$-\frac{\hbar^2}{2m}\frac{\delta\psi(x)}{\delta x^2} + V(x)\psi(x) = E\psi(x)$ (3.9)

$f(t) = f(0)e^{-iEt/\hbar}$ (3.10)

$f(t) = f(0)e^{-i\omega t}$ (3.11)

$E = \hbar\omega$ (3.12)

$\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$ (3.13)

$|\Psi(x,t)|dd = |\psi(x)|^2$ (3.14)

$$V(x) = \begin{cases} 0, & 0 < x < L. \\ \infty, & \text{elsewhere.} \end{cases}$$

$-\frac{\hbar^2}{2m}\frac{\delta\psi}{\delta x^2} = E\psi$ (3.16) $0 < x < L$

$k^2 = \frac{2mE}{\hbar^2}$ (3.17)

$\psi(x) = A\sin kx + B\cos kx$ (3.21) $0 < x < L$

$k_n = \frac{n\pi}{L}$ (3.26)

$E_n = \frac{\hbar k_n^2}{2m} = \frac{n\hbar^2\pi^2}{2mL^2}$ (3.27)

$\psi(x) = A_n\sin\frac{n\pi x}{L}$ (3.28) $0 < x < L$

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}, & 0 < x < L. \\ 0, & \text{elsewhere.} \end{cases}$$

$\Psi(x) = c_1\psi_1(x) + c_2\psi_2(x)$ (3.38)

$c_1(t) = \frac{1}{\sqrt{2}}e^{-iE_1t/\hbar}$ (3.39)

$\Psi = \sum_{n=1}^{\infty}c_n\psi_n(x)$ (3.40)

$$\delta_{nm} = \begin{cases} 1, & m = n. \\ 0, & m \neq n. \end{cases}$$

$\int_{-\infty}^{\infty}\psi_x^*(x)\psi_n(x)dx = \delta_{nm}$ (3.49)

$|c_n|^2 = P_n$ (3.59)

The above is the probability of obtaining E_n if a measurement of the energy of a particle with wave function Ψ is carried out

$\langle E\rangle = \sum_{n=1}^{\infty}|c_n|^2E_n$ (3.61)

$A_{op}\psi_a = a\psi_a$ (3.63)

$x_{op} = x$ (3.64)

$p_{xop} = \frac{\hbar}{i}\frac{\delta}{\delta x}$ (3.65)

$E_{op} = \frac{(p_{xop})^2}{2m} + V(x_{op})$ (3.71)

$H \equiv E_{op} = -\frac{\hbar^2}{2m}\frac{\delta^2}{\delta x^2} + V(x)$ (3.72)

$\langle E\rangle = \int_{-\infty}^{\infty}\Psi^*H\Psi dx$ (3.81)

One-Dimensional Potentials

$$V(x) = \begin{cases} 0, & |x| < a/2. \\ V_0, & |x| > a/2. \end{cases}$$

$k = \frac{\sqrt{2mE}}{\hbar}$

$\psi(x) = Ae^{ikx} + Be^{-ikx}$ $|x| < a/2$

$\kappa = \frac{\sqrt{2m(V_0-E)}}{\hbar} > 0$

$\psi(x) = Ce^{\kappa x} + De^{-\kappa x}$ $|x| > a/2$

$$\psi(x) = \begin{cases} Ce^{\kappa x}, & x \leq -a/2. \\ 2A\cos kx, & -a/2 \leq x \leq a/2. \\ Ce^{-\kappa x}, & x \geq a/2. \end{cases}$$

$$V(x) = \begin{cases} 0, & x < 0. \\ V_0, & x > 0. \end{cases}$$

$k = \frac{\sqrt{2mE}}{\hbar}$

$k_0 = \sqrt{k^2 - \frac{2mV_0}{\hbar^2}}$

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx}, & x < 0. \\ Ce^{ik_0x}, & x > 0. \end{cases}$$

$$j_x = \begin{cases} \frac{\hbar k}{m}(|A|^2 - |B|^2), & x < 0. \\ \frac{\hbar k_0}{m}|C|^2, & x > 0. \end{cases}$$

$T \cong (\frac{4\kappa k}{k^2+\kappa^2})^2e^{-2\kappa a}$