Using QAOA to solve the Clique-problem

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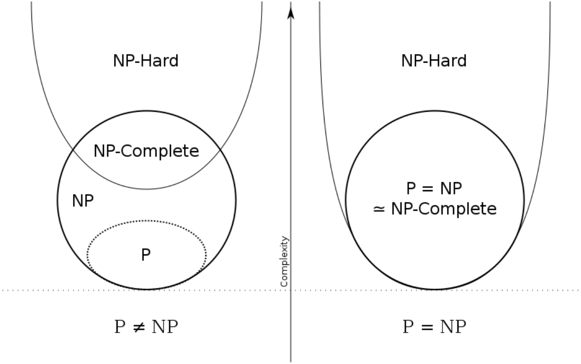
Motivation



NP-Hard
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Complexity
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P = NP
≃ NP-Complete
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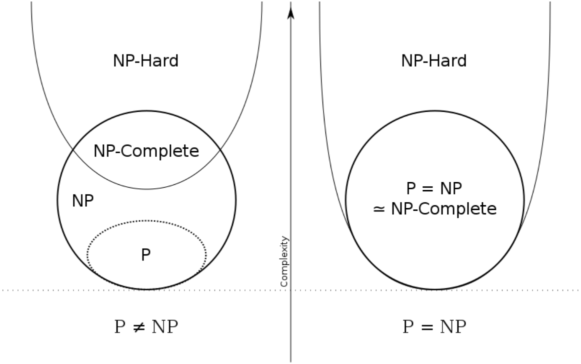

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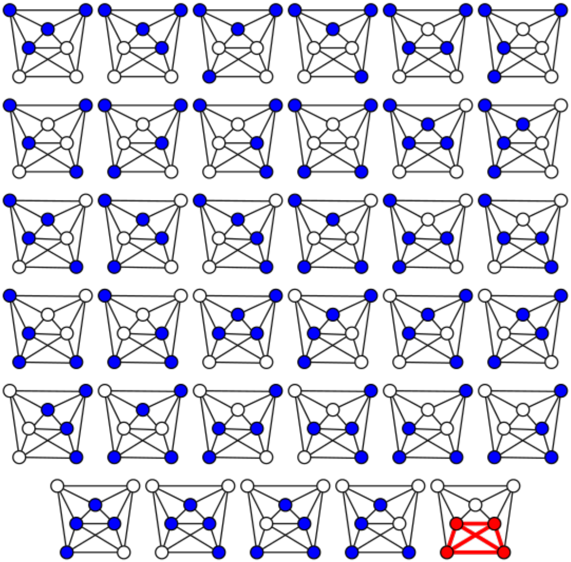

Quantum Approximation Optimization Algorithm

Objective function on bit string z for m clauses:

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Unitary operator $U(B,\beta)$ depending on angle β between 0 and π:

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Initial state uniform superposition:

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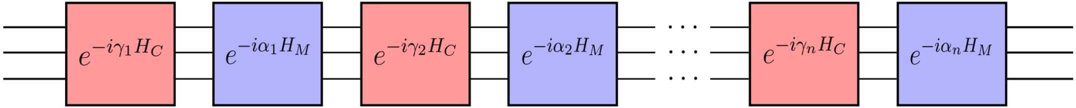

The Optimization part

F_p is the expectation of C in this state:

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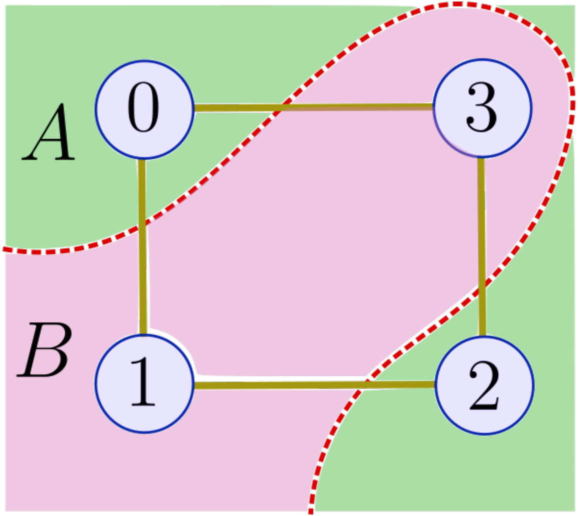
With the number of layer p high enough we can find a good approximation:

$$\lim_{p \to \infty} M_p = \max_z C(z) \tag{10}$$

MaxCut



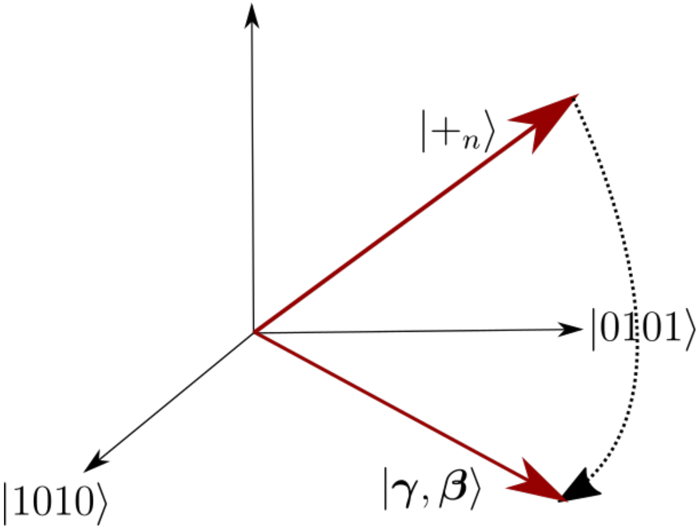
Hamiltonians:

$$H_{cost} = \frac{1}{2} \sum_{(i,j)\in E(G)} (Z_i Z_j - I) \tag{11}$$

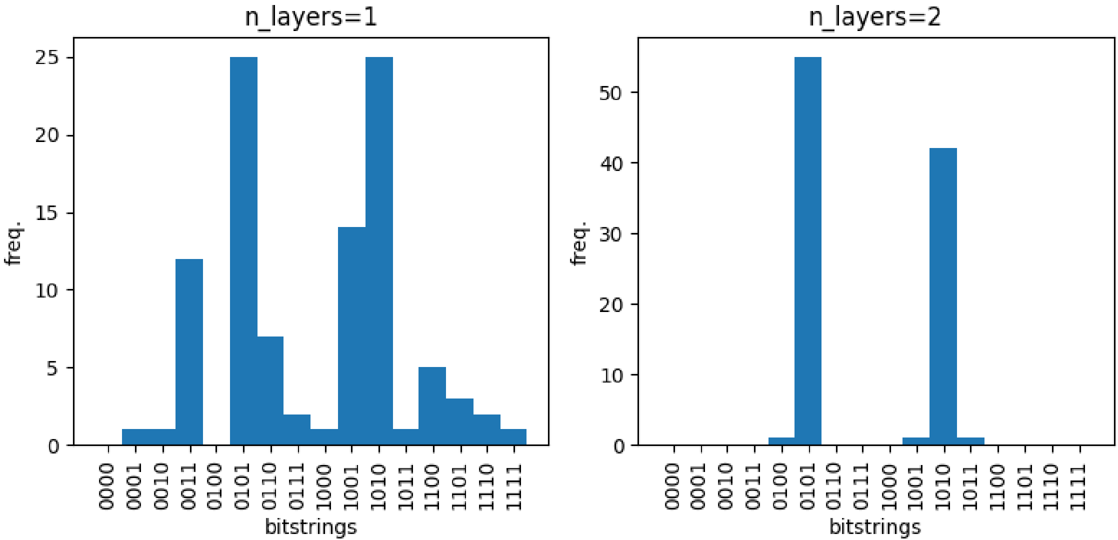
$$H_{mixer} = \sum_{i\in V(G)} X_i \tag{12}$$

Solution for this Graph,
represented by bit string $z = z_0 z_1 z_2 z_3$:
$z = 0101$
and
$z = 1010$

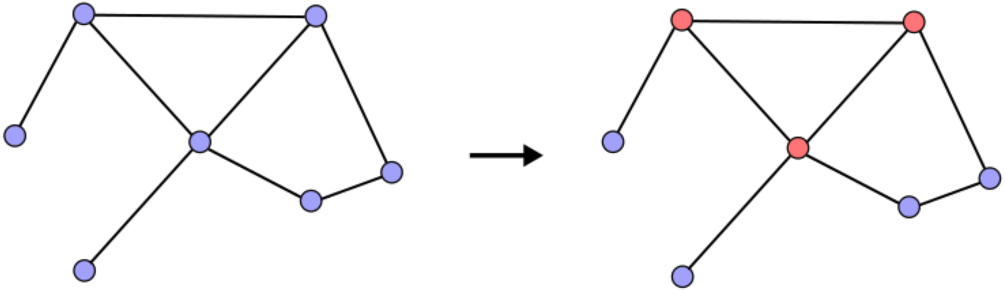
Optimal State on Bloch Sphere



MaxCut result for 1 and 2 layers



MaxClique

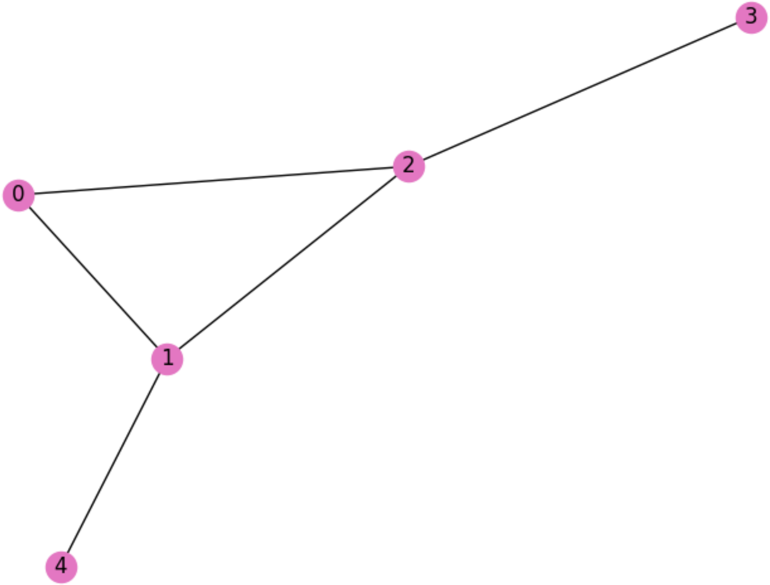


Hamiltionians

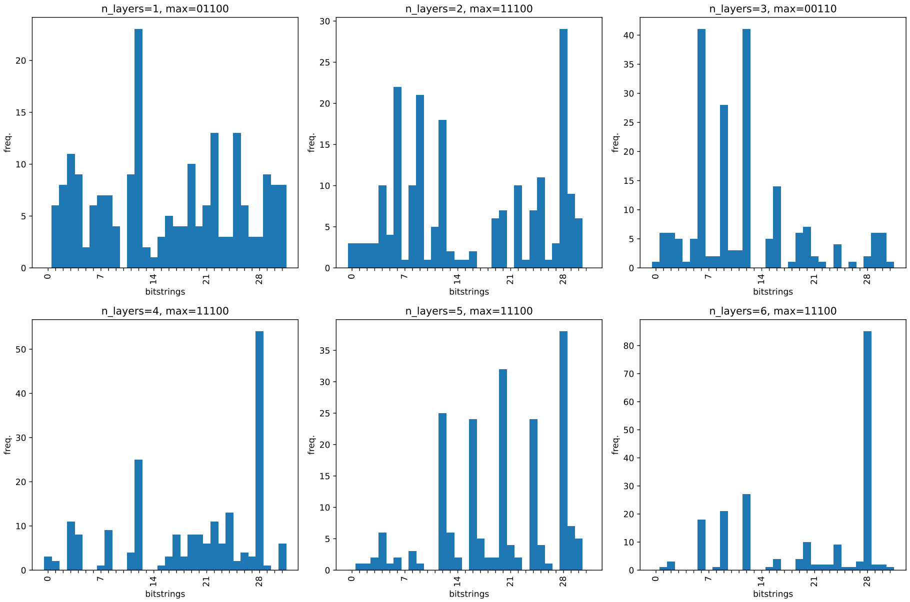
$$H_{\text{cost}} = 3 \sum_{(i,j)\in E(\overline{G})} (Z_i Z_j - Z_i - Z_j) + \sum_{i\in V(G)} Z_i \tag{13}$$

$$H_{\text{mixer}} = \sum_{i\in V(G)} X_i \tag{14}$$

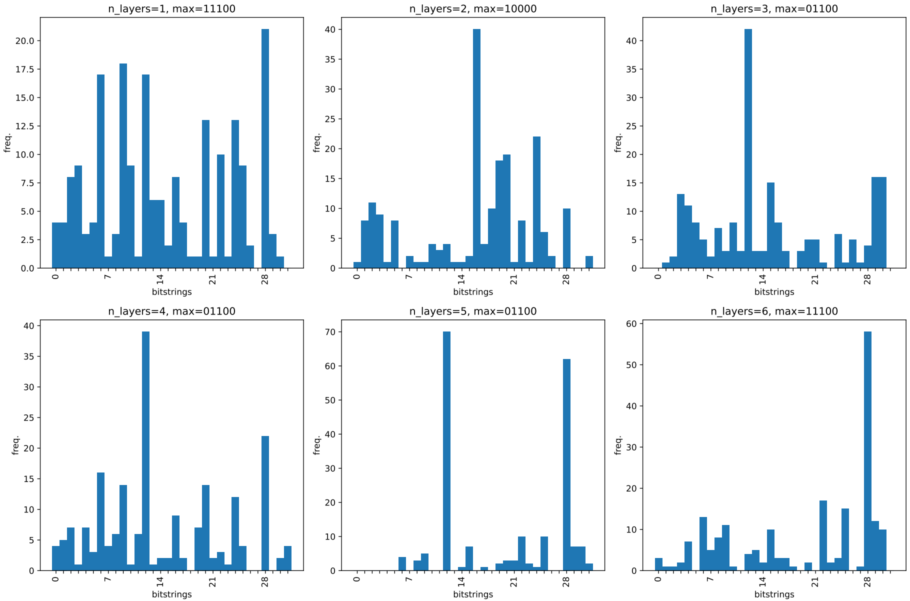
MaxClique example



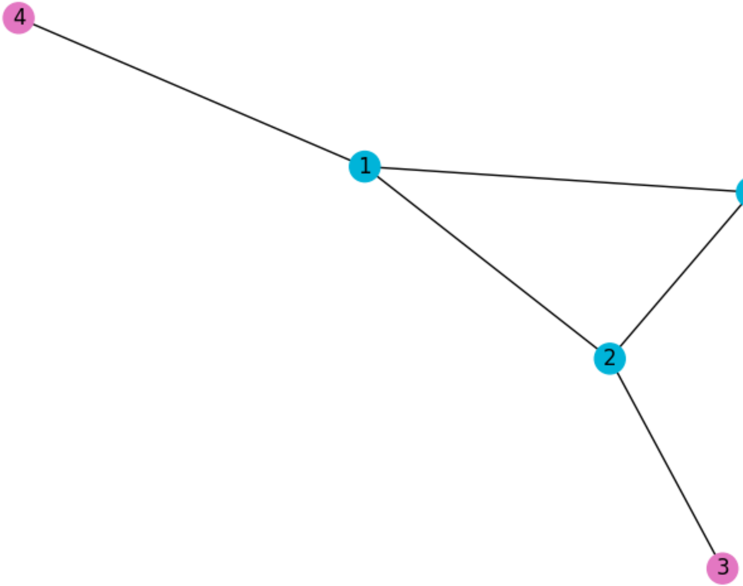
MaxClique result for example with different layers



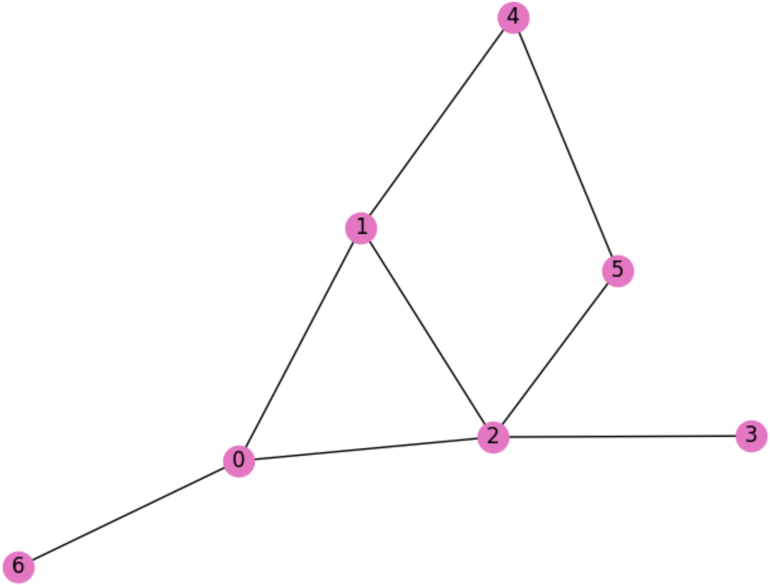
MaxClique result with different initial values



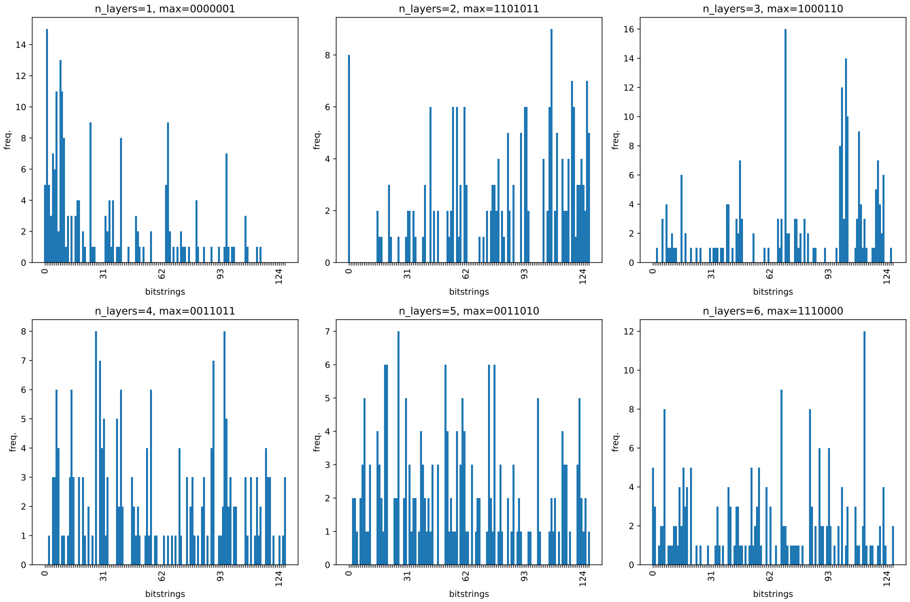
MaxClique example solution



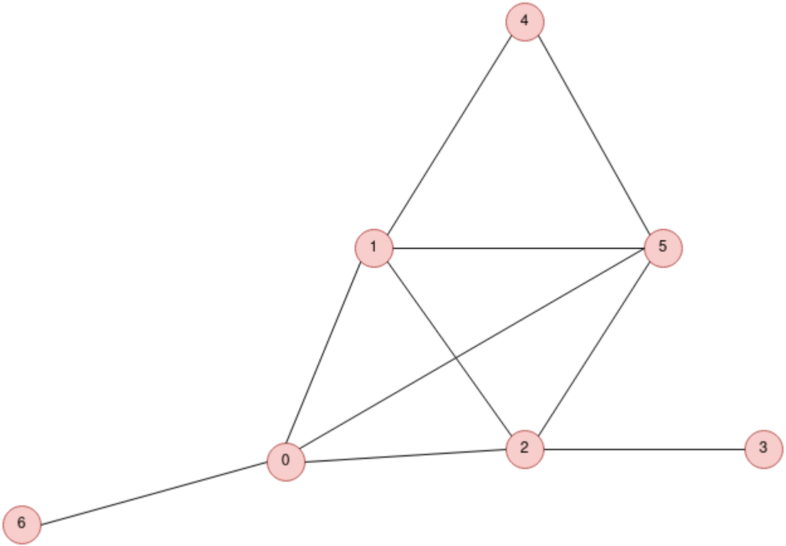
MaxClique large example



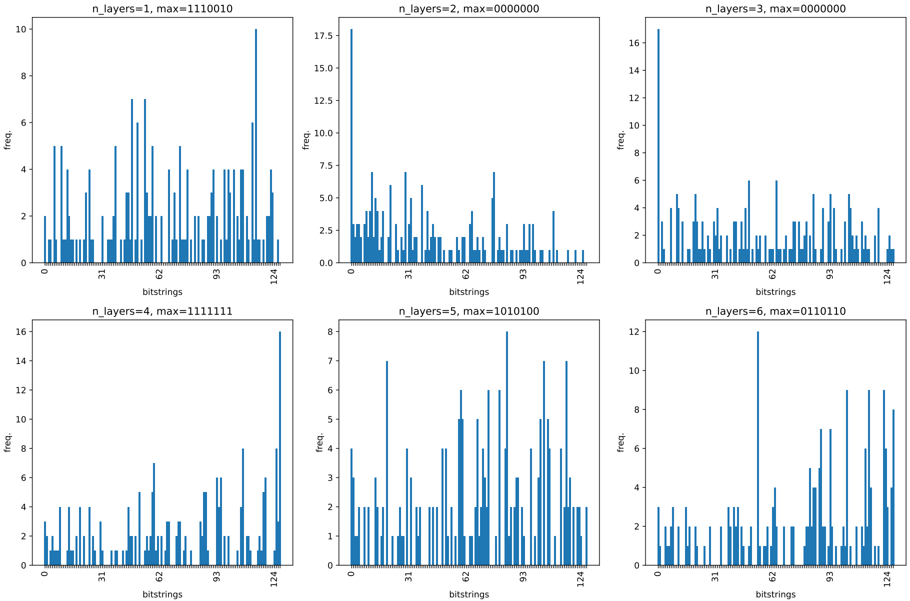
MaxClique result for large example with different layers



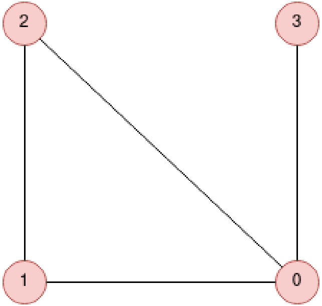
MaxClique large and complex example



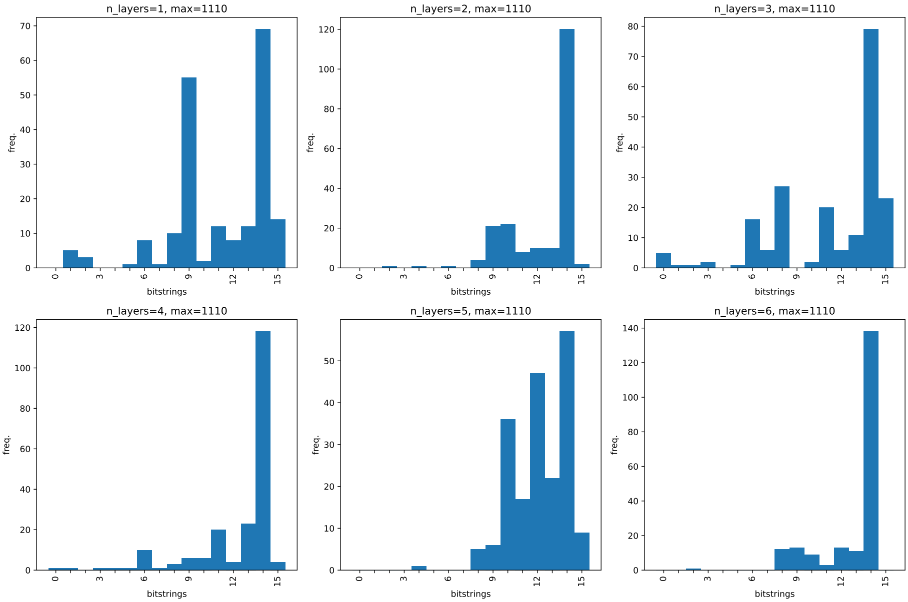
MaxClique result for large and complex example with different layers



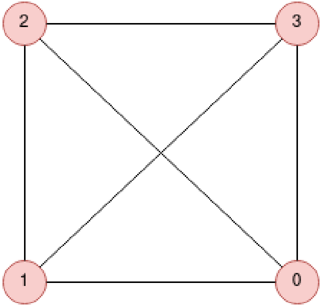
MaxClique small example



MaxClique result for small example with different layers



MaxClique simple example



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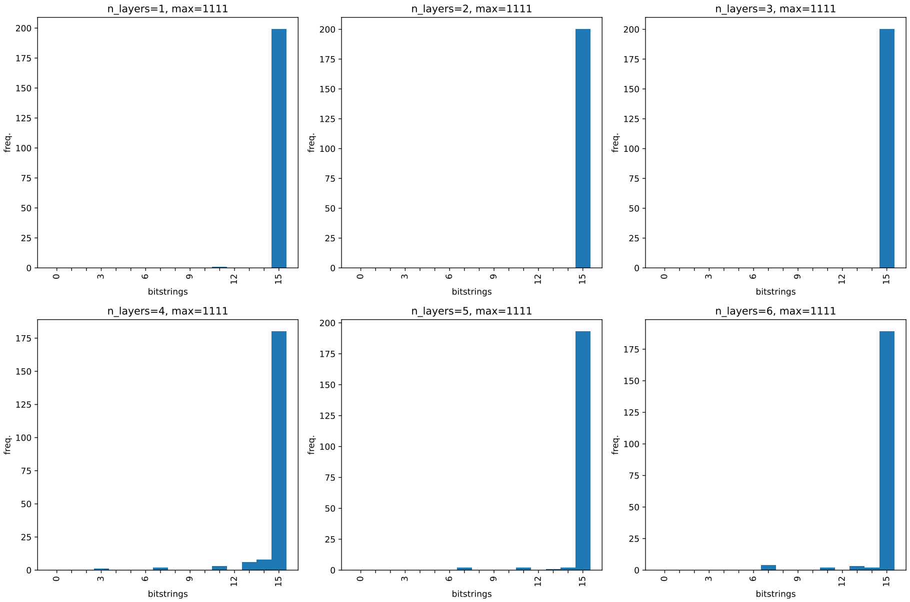

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- Then Optimize the angles of each layer
- Experiment with different optimizers, layers and optimization steps
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- Further work: Find optimal parameters for MaxClique

Thank you for your attention!

Any questions?

References

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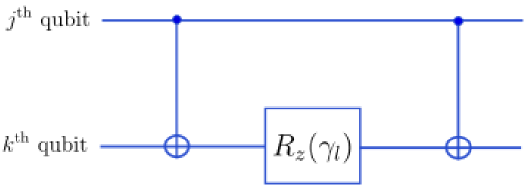
Gate representation MaxCut

$$e^{-i\gamma_l(1-\sigma_z^j\sigma_z^k)/2}$$



$$e^{-i\beta_l\sigma_x^j}$$