MATHEMATICS IN QUANT FINANCE

A Comprehensive Math Map

Stochastic Processes, Topology, Game Theory, and Beyond

Core Mathematical Pillars

Fundamental Disciplines

The four major mathematical areas that form the quantitative foundation

1. Analysis & Dynamics

- Calculus, PDEs
- Stochastic Processes

2. Algebra & Optimization

- Linear Algebra
- Convex Optimization

3. Discrete & Advanced

- Game Theory
- Topology & Measure

4. Statistics & Computation

- Probability & Inference
- Numerical Methods

Topic Focus 1: Stochastic Calculus

Modeling Randomness

Math for asset movement and continuous time modeling

Key Concepts:

- **Stochastic Processes** — Wiener processes (Brownian motion), Lévy processes
- **Ito's Calculus** — Specific differentiation rule for random functions
- **Martingales** — Definition of a fair game, used for risk-neutral pricing
- **Fokker-Planck Eq.** — Forward equation for probability density

Use in Quant Finance:

- Derivatives Pricing, Volatility Modeling

Stochastic Calculus: The Core SDE

Geometric Brownian Motion (GBM) - SDE

Defines how a non-dividend paying stock price S_t moves

Formula:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

Term Breakdown:

- dS_t: Small price change
- μ: Drift (expected return)
- σ: Volatility (standard deviation)
- dW_t: Wiener process increment (random noise)

Topic Focus 2: Linear Algebra

The Math of Data Structure

Foundation for multidimensional data, risk, and portfolio structure

Key Concepts:

- **Matrix Theory** — Covariance matrices, matrix operations
- **Eigen-Decomposition** — Principal Component Analysis (PCA) for factor extraction
- **Matrix Decompositions** — Cholesky decomposition for correlated random number generation
- **Solving Linear Systems** — Numerical methods for hedging parameter calculation

Use in Quant Finance:

- Risk Modeling, Factor Investing, Portfolio Construction

Topic Focus 3: Optimization

The Search for the Optimal Solution

Finding the best portfolio, trading speed, or parameter set

Key Concepts:

- **Convex Programming** — Essential for finding unique, reliable optimal portfolios
- **Quadratic Programming (QP)** — Used for Mean-Variance optimization (risk minimization)
- **Lagrangian Multipliers** — Solving constrained optimization (return targets, limits)
- **Linear Programming (LP)** — Used for budget and cost minimization problems

Use in Quant Finance:

- Portfolio Construction, Algorithmic Execution

Markowitz Portfolio Variance

The core formula minimized in classical portfolio optimization

Formula:

$$\sigma_P^2 = w^T \Sigma w$$

Term Breakdown:

- σ_P^2: Portfolio Variance (Risk)
- w: Vector of Portfolio Weights
- Σ: Covariance Matrix of asset returns

Minimizing this quadratic function is the essence of QP in finance

Topic Focus 4: Probability & Measure Theory

The Foundation of Risk

Rigorous framework for statistics, expectation, and pricing

Key Concepts:

- **Probability Space** — Defining the set of outcomes, events, and probability measure $(\Omega, \mathcal{F}, P)$
- **Conditional Expectation** — Central to pricing and hedging
- **Girsanov Theorem** — Links real-world and risk-neutral measures
- **Central Limit Theorem** — Used for statistical approximations and inference

Use in Quant Finance:

- Statistical Inference, Risk-Neutral Pricing Theory

Topic Focus 5: Time Series Analysis

The Math of Prediction

Modeling data points indexed by time for forecasting and strategy

Key Concepts:

- **Stationarity** — Identifying mean-reversion, reliable statistical properties
- **ARIMA/GARCH** — Models for conditional mean and conditional volatility forecasting
- **Cointegration** — Modeling relationships between non-stationary series (pair trading)
- **Kalman Filtering** — State-space models for parameter estimation in noisy data

Use in Quant Finance:

- Volatility Forecasting, Statistical Arbitrage Strategy Development

Topic Focus 6: Discrete & Numerical Methods

Computation and Strategy

Tools for solving problems lacking closed-form solutions or involving strategy

Key Concepts:

- **Monte Carlo Methods** — Simulation for valuation and complex risk metrics (VaR/CVaR)
- **Finite Difference Methods** — Grid-based solutions for PDEs (e.g., American options)
- **Game Theory** — Nash Equilibrium, repeated games for execution strategies
- **Graph Theory** — Modeling interconnectedness, network risk

Use in Quant Finance:

- Exotic Option Pricing, High-Frequency Trading (HFT) Microstructure